HEAVY QUARK PHYSICS (ch. 2)

- 第一章 回顾
- 重味强子简介
- 重夸克对称性 < 味道 / 自旋
- 有效拉氏密度
- 对称性的应用
  - 衰变常数
  - 形状因子

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第一章回顾

△ QCD

- 拉氏密度: $\mathcal{L}_{QCD} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} + \bar{q}(i\not{D} - m_q) q$ + Counter terms 协变导数 $D_\mu = \partial_\mu + i g_s T^A A^A_\mu$

  $G^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu - g f^{ABC} A^B_\mu A^C_\nu$, $[T^A, T^B] = i f^{ABC} T^C$

- β函数: $\beta(g) = -\frac{g^3}{16\pi^2}\left(11 - \frac{2}{3} N_q\right) + O(g^5)$. $N_q < 16$, 负数, 渐近自由. (Nobel prize '04)

  $$\alpha_s(\mu_2) = \frac{1}{\left[1/\alpha_s(\mu_1) + \beta_0 \ln(\mu_2^2/\mu_1^2)\right]}, \quad \beta_0 = \frac{33 - 2N_q}{12\pi}.$$

  $$\Lambda_{QCD} = \mu_1 e^{-1/[2\beta_0 \alpha_s(\mu_1)]}$$

  $$\alpha_s(\mu) = \frac{12\pi}{(33 - 2N_q)\ln(\mu^2/\Lambda_{QCD}^2)}$$

  $\mu \to \Lambda_{QCD}$, $\alpha_s(\mu) \to \infty$ (微扰论失效)

  $\mu \sim \Lambda_{QCD}$, 非微扰变得重要.

  dimensional transmutation ($m_q = 0$. g_s is dimensionless. but QCD is characterized by a dimensional parameter)

  Even no quarks!

- 禁闭: 没有自由的夸克胶子, 没有带颜色的强子

  介子: 2 夸克 $\bar{q}^\alpha q_\alpha$

  重子 3 夸克 $\epsilon^{\alpha\beta\gamma} q_\alpha q_\beta q_\gamma$

△ 弱衰变有效哈密顿量

Ref: A.J. Buras, 9806471

△ QCD的两个极限

— m_u m_d m_s $\lesssim \Lambda_{QCD}$, 轻夸克, $m_q \to 0$, $SU(3)_L \otimes SU(3)_R$ 手征对称性

— m_c, m_b, $m_t \gtrsim \Lambda_{QCD}$ 重夸克, $m_Q \to \infty$, 重夸克对称性

重夸克

$m_c \sim 1.4$ GeV, $m_b \sim 4.8$ GeV, $m_t \sim 175$ GeV

第二章　重夸克，对称性与有效理论

1、引论

- 自由度、标度：重夸克 m_Q，轻夸克 m_q，Λ_{QCD}

  $\Lambda_{QCD} \sim 200\,\mathrm{MeV}$

  $m_t \sim 170\,\mathrm{GeV}$

  $m_q \ll \Lambda_{QCD} \ll m_Q$（重夸克：c, b, t，t 不能形成强子：$t \to bW$，$\Gamma_t \sim 1.5\,\mathrm{GeV}$）

  $\sim 10^{-25}\,s$　　强子化 $\sim 10^{-24}\,s$

  特征尺度：$1/\Lambda_{QCD}$

  重夸克速度 v 在 QCD 动力学下不变：$\Delta v = \frac{\Delta p}{m_Q} \sim \frac{\Lambda_{QCD}}{m_Q}$.

  当 $m_Q \to \infty$，v 可用来标记重夸克场，v 不随时间改变.（速度和位置可同时测准：

  $[x, v] = \frac{1}{m_Q}[x, p] = \frac{i\hbar}{m_Q} \to 0$,

  存在坐标空间传播子可以再次看到）

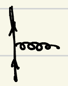
  $\langle Q_v(x)\,\bar{Q}_v(y)\rangle \to \theta(t_1 - t_2)\,\delta^3(\vec{x}-\vec{y})$

  重夸克场："静"态外源，轻夸克在此场中运动（QCD 原子）

- 对称性：
  - 味对称（heavy quark flavor symmetry）：动力学与夸克质量无关.

    破坏：考虑质量修正 $1/m_Q$.　破坏程度 $\sim 1/m_{Q_i} - \frac{1}{m_{Q_j}}$

  - 自旋对称：（heavy quark spin-symmetry）：相互作用不依赖质量，对重夸克自旋作任意变换动力学不变.



$\bar{u}(p)\gamma^\mu u(p')A_\mu(k)$　in Dirac representation:

"静电学"

$$u(p,s) = \sqrt{\frac{p^0+m}{2m}}\begin{pmatrix} \xi_s \\ \frac{\vec{p}\cdot\vec{\sigma}}{p^0+m}\xi_s \end{pmatrix},\quad \gamma^0 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix},\quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}.$$

$$\bar{u}(p,s)\gamma^0 u(p',s') \sim \sqrt{\frac{p^0+m}{2m}}\sqrt{\frac{p'^0+m}{2m}}\left(\xi_s^+,\ \xi_s^+\frac{\vec{p}\cdot\vec{\sigma}}{p^0+m}\right)\cdot\begin{pmatrix} \xi_{s'} \\ \frac{\vec{p}'\cdot\vec{\sigma}}{p'^0+m}\xi_{s'} \end{pmatrix}$$

spin-symmetry 破坏：

不正比于 $1/m_{Q_i} - 1/m_{Q_j}$

$$\sim \quad \xi_s^+\ \xi_{s'} \sim \delta_{ss'}.$$

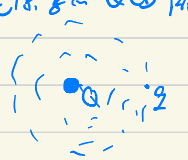
works when $|p'| \sim |p| \ll m$

For a general v: boost.

2. 重味强子的量子数（含有一个重夸克的强子）

- 重味强子的结构：（以介子为例）

  氢原子和 QCD 版本：



  重夸克：Q

  轻自由度：轻夸克、胶子 g、夸克对 $q\bar{q}$ } 具有轻夸克相同的量子数

- 角动量
  - 总角动量守恒 $\hat{J}$
  - 重夸克自旋 $\hat{S}_Q$，$m_Q \to \infty$ 时守恒（重夸克自旋对称性）
  - 轻自由度的总自旋 $\hat{S}_\ell$　$\hat{S}_\ell = \hat{J} - \hat{S}_Q$，$m_Q \to \infty$ 极限下守恒.

- 好量子数：j，s_Q，s_ℓ

$$\hat{J}^2\,|H^{(Q)}\rangle = j(j+1)\,|H^{(Q)}\rangle$$

$$\hat{S}_Q^2\,|H^{(Q)}\rangle = s_Q(s_Q+1)\,|H^{(Q)}\rangle$$

$$\hat{S}_\ell^2\,|H^{(Q)}\rangle = s_\ell(s_\ell+1)\,|H^{(Q)}\rangle$$

- 重味介子

  —— 重夸克 + 轻夸克（胶子、夸克对）

  基态 $s_Q = 1/2$，$s_\ell = 1/2$，$j = 1/2 \otimes 1/2 = 0 \oplus 1$.　$J^P = 0^-, 1^-$

— 宇称：—. 夸克与反夸克的内禀宇称相反

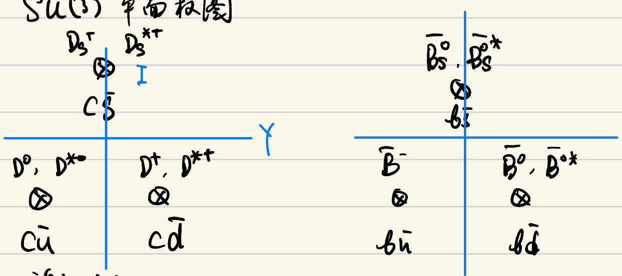
— D, D^* : $Q=c$ 场: $P_v^{(Q)}$ $P_{v\mu}^{*(Q)}$

$\bar{B}, \bar{B}^*$: $Q=b$ 赝标 矢量

轻夸克: $\bar{u}, \bar{d}, \bar{s}$.

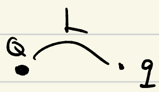
— 介子构成SU(3)味对称的 $\bar{3}$ 表示。 (3 的复共轭表示)

SU(3) 平面权图



— 激发态：

在非相对论夸克模型下：轨道角动量



$P=(-1)^{L+1}$

如果 $L=1$, $\begin{cases} S_l = 1/2 : & 0^+ \ (D_0^*), \quad 1^+ \ (D_1^*) \\ S_l = 3/2 : & 1^+ \ (D_1), \quad 2^+ \ (D_2^*) \end{cases}$

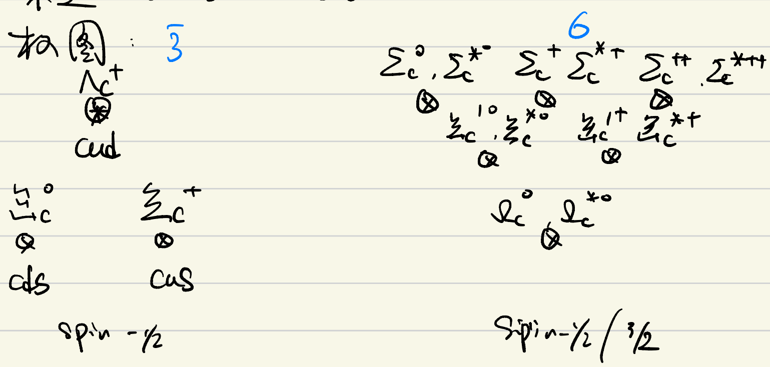
• 重味重子

— 1 重夸克 + 2 轻夸克 + (胶子、夸克对)

基态: $S_l=0$ $(\Lambda^{(Q)})$, or $S_l=1$ $(\Sigma_v^{(Q)}, \Sigma_{v\mu}^{*(Q)})$

味道: $3 \otimes 3 = \bar{3} \oplus 6$

权图: $\bar{3}$ 6



— 从非相对论夸克模型理解：

基态: $L=0$. 空间波函数对称

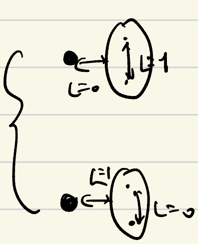
颜色空间全反称.

色单态

$\begin{cases} S_l=0, \text{自旋空间反称} \Rightarrow \text{味道波函数反称} \rightarrow \bar{3} & J^P = \frac{1}{2}^+ \quad \Lambda_v^{(Q)} \\ S_l=1, \text{自旋: 对称} \Rightarrow \text{味道: 对称} \rightarrow 6 & J^P = \frac{3}{2}^+, \frac{1}{2}^+ \quad \Sigma_v^{(Q)}, \Sigma_{v\mu}^{*(Q)} \end{cases}$

费米统计

$L=1$ 的激发态：



更加复杂

3、激发态重味强子的强衰变 （选讲）

4、重余总碎裂到重强子（选讲）

5. 场的协变表示

- 重夸克对称性：B 和 B^* 形成简并的多重态.

  动机：将 B 和 B^* 用同一量表示，在重夸克对称性下线性变换.

- 重介子情形：

  B 场（赝标量场）：$P_v^{(Q)}$，B^* 场（矢量场）：$P_{v,\mu}^{*(Q)}$ $(\epsilon\cdot\epsilon^*=-1,\ v\cdot\epsilon=0)$

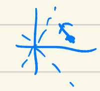
- 场量 $H_v^{(Q)}$：湮灭 $Q\bar{q}$ 介子.

  - 用夸克场表示 $Q\bar{q}$，$H_v^{(Q)}$ 的变换模式应与 $\bar{Q}Q$ 旋量 $Q\bar{q}$ 相同.
  - 洛伦兹变换下：$H_v^{(Q)'}(x') = D(\Lambda)H_v^{(Q)}(x)D(\Lambda)^{-1}$ ~~双旋量变换规则~~

    $v' = \Lambda v$，$x' = \Lambda x$，$D(\Lambda)$ 为旋量在洛伦兹变换下的变换矩阵

    $$H_v^{(Q)}(x) \rightarrow H_v^{(Q)'}(x) = D(\Lambda)\, H^{(Q)}_{\Lambda^{-1}v}(\Lambda^{-1}x)\, D(\Lambda)^{-1}$$

$\phi'(x') = \phi(x)$



$(A^\mu)'(x) = \Lambda^\mu{}_\nu A^\nu(\Lambda^{-1}x)$

  - 将 $H_v^{(Q)}(x)$ 表示成 $P_v^{(Q)}(x)$ 和 $P_{v,\mu}^{*(Q)}(x)$ 的线性组合.

    $$H_v^{(Q)} = \frac{1+\slashed{v}}{2}\left[\slashed{P}_v^{*(Q)} + i P_v^{(Q)}\gamma_5\right]$$

    投影出 "good component"　arbitrary. 取决于赝标和矢量相对相位的选取

  - 宇称变换：$H_v^{(Q)}(x) \rightarrow \gamma_0 H_{v_R}^{(Q)}(x_R)\gamma_0$

    $$= \gamma_0 \frac{1+\slashed{v}_R}{2}\left[\slashed{P}_{v_R}^{*(Q)}(x_R) + iP_{v_R}^{(Q)}\gamma_5\right]\gamma_0$$

    $$= \gamma_0\gamma_0\frac{1+\slashed{v}}{2}\gamma_0\gamma_0\left[\bar{\slashed{P}}_{v_R}^{*(Q)}(x_R) - iP_{v_R}^{(Q)}\gamma_5\right]\gamma_0\gamma_0$$

    $$= \frac{1+\slashed{v}}{2}\left[\bar{\slashed{P}}_{v_R}^{*(Q)}(x_R) - iP_{v_R}^{(Q)}\gamma_5\right]$$

    why bispinor?

    $\bar{Q}[\cdots]q$ is scalar under Lorentz transform

    $\bar{P}_{v_R} \equiv (P^0_{v_R}, -\vec{P}_{v_R})$

  - H_v 满足约束方程 $\slashed{v}H_v^{(Q)} = H_v^{(Q)}$，$H_v^{(Q)}\slashed{v} = -H_v^{(Q)}$
  - 共轭场：$\bar{H}_v^{(Q)} = \gamma^0 H_v^{(Q)\dagger}\gamma^0 = \left[P_{v\mu}^{*(Q)\dagger}\gamma^\mu + iP_v^{\dagger}\gamma_5\right]\frac{1+\slashed{v}}{2}$，同样按照双旋量变换
  - 静止系：$v = v_r = (1,\vec{0})$，此时 $H_{v_r}^{(Q)} = \begin{pmatrix} 0 & iP_{v_r}^{(Q)} - \vec{\sigma}\cdot\vec{P}_{v_R}^{*(Q)} \\ 0 & 0 \end{pmatrix}$

- 重夸克自旋变换规则：

  - $H_{v_r}^{(Q)}$ 作为 $S_Q\otimes S_\ell$ 的 $(1/2, 1/2)$ 表示

    S_Q：重夸克自旋算符，S_ℓ：轻自由度自旋算符

    $[\hat{S}_Q, H_{v_r}^{(Q)}] = \frac{1}{2}\sigma_{4\times4}H_{v_r}^{(Q)}$，$[\hat{S}_\ell, H_{v_r}^{(Q)}] = -\frac{1}{2}H_{v_r}^{(Q)}\sigma_{4\times4}$　$\sigma_{4\times4} = \begin{pmatrix}\vec{\sigma} & \\ & \vec{\sigma}\end{pmatrix}$

    其中 $\sigma_{4\times4} = i\epsilon^{ijk}[\gamma^j,\gamma^k]/4$，是通常的旋量表示下的转动矩阵，$\gamma^0 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ $\gamma^i = \begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix}$ $\gamma_5 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

    自旋转动 $\hat{R}$ 作用
    在 Hilbert 空间：$\hat{R}|H\rangle$
    作用在 Hilbert 空间算符上：$\hat{R}^{-1}H_{v_r}\hat{R}$
    对应的无穷小转动则为
    $[\hat{S}, H_{v_r}^{(Q)}]$
    其中 $\hat{S}_Q$ 为 $\hat{R}$ 的无穷小算符
    另一方面：$\hat{R}^{-1}H_{v_r}^{(Q)}\hat{R} = D(R)H_{v_r}^{(Q)}$
    对应的无穷小算符关系为
    $[\hat{S}_Q, H_{v_r}^{(Q)}] = \frac{1}{2}\sigma_{4\times4}H_{v_r}$
    See peskin p.59 for details

    无穷小转动下，$\delta H_{v_r}^{(Q)} = i\left[\vec{\theta}\cdot(\hat{\vec{S}}_Q + \hat{\vec{S}}_\ell), H_{v_r}^{(Q)}\right] = \frac{i}{2}\left[\vec{\theta}\cdot\vec{\sigma}_{4\times4}, H_{v_r}^{(Q)}\right]$

    可以推导出：$\delta P_{v_r}^{(Q)} = 0$　$\delta\vec{P}_{v_r}^{*(Q)} = \vec{\theta}\times\vec{P}_{v_R}^{*(Q)}$　$\vec{v} = \vec{r}\times\vec{\omega}$

  - 重夸克自旋转动：

    $$\delta H_{v_r}^{(Q)} = i\left[\vec{\theta}\cdot\vec{S}_Q, H_{v_r}^{(Q)}\right] = \frac{i}{2}\,\vec{\theta}\cdot\vec{\sigma}_{4\times4}H_{v_r}^{(Q)}$$

    故有 $\delta P_{v_r}^{(Q)} = -\frac{1}{2}\vec{\theta}\cdot\vec{P}_{v_r}^{*(Q)}$，$\delta\vec{P}_{v_r}^{*(Q)} = \frac{1}{2}\vec{\theta}\times\vec{P}_{v_r}^{*(Q)} - \frac{1}{2}\vec{\theta}P_{v_r}^{(Q)}$

    有限大转动矩阵：$H_v^{(Q)} \rightarrow D(R)_Q H_v^{(Q)}$

  - 重夸克味道转动：$H_v^{(Q_i)} \rightarrow U_{ij}H_v^{(Q_j)}$　U_{ij} 为任意幺正矩阵
  - 重夸克对称性：自旋和味道变换后不变.

- 重味重子情形：以 Λ_Q 为例：

  场：$\Lambda_v^{(Q)}(x)$，满足 $\slashed{v}\Lambda_v^{(Q)}(x) = \Lambda_v^{(Q)}(x)$

  Lorentz 群作用下：$\Lambda_v^{(Q)}(x) \rightarrow D(\Lambda)\Lambda_{\Lambda^{-1}v}^{(Q)}(\Lambda^{-1}x)$

  重夸克自旋转动下：$\Lambda_v^{(Q)} \rightarrow D(R)_Q\Lambda_v^{(Q)}$

  极化：用旋量 $u(v,s)$ 表示，v：速度，s：自旋

  归一化：$\bar{u}(v,s)\gamma^\mu u(v,s) = 2v^\mu$.　$\bar{u}(v,s)\gamma^\mu\gamma_5 u(v,s) = s^\mu$

  自旋矢量，$v\cdot s = 0$，$s^2 = -1$

6. 有效拉氏量

- 动机：推导 HQET 有效拉氏量。
  描述 m_Q 以下动力学。
  具有重夸克对称性。

- 历史：Bloch – Nordsieck EFT for soft photon. (heavy electron effective theory)

- 考虑重夸克 Q，速度 v，$p = m_Q v$.
  离壳重夸克，$p = m_Q v + k$. $k \sim \Lambda_{QCD}$

- 传播子：$\dfrac{i(\not{p}+m_Q)}{p^2-m_Q^2+i\varepsilon} = \dfrac{i(m_Q\not{v}+m_Q+\not{k})}{2m_Q v\cdot k + k^2 + i\varepsilon} = i\dfrac{1+\not{v}}{2v\cdot k+i\varepsilon}$

  剩余动量 (residue momentum) 反映离壳度

  Question：反重夸克传播子？

  坐标空间传播子 ($S_Q(x) \propto \theta(t)\,\delta^3(\vec{x})$) 仅沿时间正方向传播！

  传播子含有投影算符 $\dfrac{1+\not{v}}{2}$，如果 $v=(1,\vec{0})$，$\dfrac{1+\not{v}}{2} = \dfrac{1+\gamma^0}{2}$，投影出 Dirac 旋量的粒子部分

- 重夸克场 $Q(x) = e^{-im_Q v\cdot x}[Q_v(x) + \tilde{Q}_v(x)]$

  $Q_v(x) = e^{im_Q v\cdot x}\dfrac{1+\not{v}}{2}Q(x)$

  $\tilde{Q}_v(x) = e^{im_Q v\cdot x}\dfrac{1-\not{v}}{2}Q(x)$

  $e^{im_Q v\cdot x}$ 减除重夸克动量。

  Q_v: leading. $\tilde{Q}_v(x)$: $1/m_Q$ suppressed. $1/m_Q$ 修正在第四章中讨论。

- QCD 拉氏密度：$\mathcal{L}_{QCD} \supset \bar{Q}(x)(i\not{D}-m_Q)Q(x)$

  $\bar{Q}(x) \propto e^{-im_Q v\cdot x}Q_v(x)$

  $\bar{Q}(x)(i\not{D}-m_Q)Q(x)$

  $= e^{im_Q v\cdot x}\bar{Q}_v(x)(i\not{D}-m_Q)e^{-im_Q v\cdot x}Q_v(x) + O(1/m_Q)$

  $= e^{im_Q v\cdot x}\bar{Q}_v(x)e^{-im_Q v\cdot x}(m_Q\not{v}+i\not{D}-m_Q)Q_v(x) + O(1/m_Q)$

  $= \bar{Q}_v(x)\, i\not{D}\, Q_v(x) + O(1/m_Q)$

  另一方面：$\dfrac{1+\not{v}}{2}Q_v(x) = Q_v(x)$，

  于是：$\bar{Q}_v(x)\dfrac{1+\not{v}}{2}i\not{D}\dfrac{1+\not{v}}{2}Q_v(x)$

  $= \bar{Q}_v(x)\, i v\cdot D\, Q_v(x)$

  $\mathcal{L}_{HQET} \supset \bar{Q}_v(x)\, iv\cdot D\, Q_v(x) + \cdots + O(1/m_Q)$

- 重夸克传播子：$\dfrac{i}{v\cdot k+i\varepsilon}\dfrac{1+\not{v}}{2} \longrightarrow Q_v(x) = \dfrac{1+\not{v}}{2}Q_v(x)$

  重夸克—胶子相互作用：$(-ig\,T^A v^\mu)$

  $i\,v\cdot(\partial_\mu + igT^a A^\mu)$

  From full theory: $\dfrac{1+\not{v}}{2}\gamma^\mu\dfrac{1+\not{v}}{2}$ $\dfrac{1+\not{v}}{2}\gamma^\mu\dfrac{1+\not{v}}{2} = v^\mu\dfrac{1+\not{v}}{2} \to v^\mu$.

- 多个重夸克：$\mathcal{L}_{eff} = \sum\limits_{i=1}^{N_h}\bar{Q}_v^{(i)}(iv\cdot D)Q_v^{(i)}$ v should be the same one!

- 从完整理论到有效理论

两种途径：1. 积分掉重场（使用运动方程）

2. 相对论理论的非相对论约化（FWT变换）

- 两种途径得到Lagrangian略有不同
- 可用场重定义联系，物理结果相同

(Thomas Mannel: Effective Field Theories for Heavy Quarks, Lectures at Les Houches. 2017)

① • 路径积分：

$$Q(x) = e^{-im_Q v\cdot x}[Q_v(x) + \tilde{Q}_v(x)]$$

$$\bar Q(x)(i\not D - m_Q)Q(x) = [\bar Q_v(x) + \bar{\tilde Q}_v(x)]e^{im_Q v\cdot x}[i\not D - m_Q]e^{-im_Q v\cdot x}[Q_v(x)+\tilde Q_v(x)]$$

$$= [\bar Q_v(x) + \bar{\tilde Q}_v(x)]e^{im_Q v\cdot x}[m_Q\not v e^{-im_Q v\cdot x} - m_Q e^{-im_Q v\cdot x} + e^{-im_Q v\cdot x}i\not D][Q_v(x)+\tilde Q_v(x)]$$

$$= [\bar Q_v(x) + \bar{\tilde Q}_v(x)][m_Q(\not v - 1) + i\not D][Q_v(x) + \tilde Q_v(x)]$$

$$\begin{cases} \bar Q_v(x)\, m_Q(\not v - 1)\, Q_v(x) = 0 \\ \bar Q_v(x)\, m_Q(\not v-1)\, \tilde Q_v(x) = 0 \\ \bar{\tilde Q}_v(x)\, m_Q(\not v - 1)\, Q_v(x) = 0 \\ \bar{\tilde Q}_v(x)\, m_Q(\not v-1)\, \tilde Q_v(x) = \bar{\tilde Q}_v(-2m_Q)\tilde Q_v(x) \end{cases}$$

$$\bar Q_v(x)\, i\not D\, Q_v(x) = \bar Q_v(x)\frac{1+\not v}{2}i\not D\frac{1+\not v}{2}Q_v(x) = \bar Q_v(x)\, iv\cdot D\, Q_v(x)$$

$$\bar{\tilde Q}_v(x)\, i\not D\, \tilde Q_v(x) = \bar{\tilde Q}_v(x)\frac{1-\not v}{2}i\not D\frac{1-\not v}{2}\tilde Q_v(x) = -\bar{\tilde Q}_v(x)\, iv\cdot D\, \tilde Q_v(x)$$

$$\bar Q_v(x)\, i\not D\, \tilde Q_v(x) = \bar Q_v(x)\frac{1+\not v}{2}i\not D\frac{1-\not v}{2}\tilde Q_v(x) = \bar Q_v(x)\left(i\not D\frac{1-\not v}{2} + iv\cdot D\right)\frac{1-\not v}{2}\tilde Q_v(x) = \bar Q_v(x)(i\not D + iv\cdot D)\frac{1-\not v}{2}\tilde Q_v(x) = \bar Q_v(x)\, i\not D_\perp\, \tilde Q_v(x)$$

$$\bar{\tilde Q}_v(x)\, i\not D\, Q_v(x) = \bar Q_v(x)\, i\not D_\perp\, \tilde Q_v(x)$$

其中 $D_\perp^\mu = D^\mu - v\cdot D\, v^\mu$

$$\mathcal L \to \bar Q_v(x)\, iv\cdot D\, Q_v(x) - \bar{\tilde Q}_v(x)(iv\cdot D + 2m_Q)\tilde Q_v(x) + \bar Q_v(x)\, i\not D_\perp\, \tilde Q_v(x) + \bar{\tilde Q}_v(x)\, i\not D_\perp\, Q_v(x)$$

$Q_v(x)$：无质量项，轻自由度。$\tilde Q_v(x)$，有质量项，重自由度，被积分掉

- 生成泛函：$Z[\eta,\bar\eta,\lambda] = \int[DQ][D\bar Q][Dq]\exp\left\{i\int d^4x\left[\bar Q(x)(i\not D - m_Q)Q(x) + \mathcal L_{light} + \bar Q(x)\eta(x) + Q(x)\bar\eta(x) + q(x)\lambda(x)\right]\right\}$

$$\bar Q(x)\eta(x) \to \bar Q_v(x)\eta(x)$$

$$Q(x)\bar\eta(x) \to Q_v(x)\bar\eta(x)$$

(当 scale $\ll 2m_Q$，Green function 中 $\tilde Q_v(x)$ is irrelevant. 去掉相应 in source.)

$$Z[\rho,\bar\rho,\lambda] = \int[DQ_v(x)][D\bar Q_v(x)][D\tilde Q_v(x)][D\bar{\tilde Q}_v(x)]\exp\{\cdots\}$$

积分掉：$\tilde Q_v(x)$，$\bar{\tilde Q}_v(x)$

$$\int[D\tilde Q_v(x)][D\bar{\tilde Q}_v(x)]\exp\left\{i\int d^4x\left(-\bar{\tilde Q}_v(x)(iv\cdot D + 2m_Q)\tilde Q_v(x)\right) + \bar Q_v(x)\, i\not D_\perp\, \tilde Q_v(x)\right.$$

$$= \Delta\exp\left\{\int d^4x\, \bar Q_v(x)\, i\not D_\perp\frac{1}{iv\cdot D + 2m_Q + i\epsilon}i\not D_\perp\, Q_v(x)\right\} + \bar{\tilde Q}_v(x)(i\not D_\perp)Q_v(x)\Big\}$$

$$\Delta = \exp\left[\frac12\mathrm{Tr}\ln[iv\cdot D + 2m_Q + i\epsilon]\right]$$

$v\cdot A = 0$ gauge, $\ln\Delta$ is a constant

$$D\to k, \quad \mathcal L_{eff} \sim \bar Q_v(x)(iv\cdot D)Q_v(x) + \frac{1}{2m_Q}\sum_{n=0}^{\infty}\bar Q_v\, i\not D_\perp\left(-\frac{iv\cdot D}{2m_Q}\right)^n i\not D_\perp Q_v$$

② • Foldy-Wouthuysen-Tani 变换 (Vout - high - sen) (Dirac eq. → Schrödinger eq.)

$L_{QCD} \supset \bar\psi(i\not D - m_Q)\psi$. $\psi = \frac{1+\not v}{2}\psi + \frac{1-\not v}{2}\psi$

take $v = (1,0,0,0)$

多用于NRQCD的推导.

- FWT变换：作一幺正变换，使"好"和"坏"分量 decouple.
- "Good" "bad" coupling:

$$\bar\psi\frac{1+\gamma_0}{2}(i\not D - m_Q)\frac{1-\gamma_0}{2}\psi$$

$$= \bar\psi\frac{1+\gamma_0}{2}(iD^0\gamma^0 - iD^i\gamma^i - m_Q)\frac{1-\gamma_0}{2}\psi$$

$$= \bar\psi\frac{1+\gamma_0}{2}(-iD^i\gamma^i)\frac{1-\gamma_0}{2}\psi$$

$$\psi\to\psi = e^{-\frac{i}{2m}D^i\gamma^i}\psi_{(1)} \qquad \mathcal L\sim\bar\psi_{(1)}(iD^0\gamma_0 - m)\psi_{(1)} + \frac1m\left(-\frac12 D_jD^j - \frac{ie}{8}[\gamma^j,\gamma^k]F_{jk} - \frac{ie}{2}\gamma^j\gamma^0F_{j0}\right) + O\left(\frac{1}{m^2}\right)$$

$$\psi_{(1)}\to\psi_{(1)} = e^{\frac{i}{2m^2}\left(-\frac{ie}{2}\gamma^j\gamma^0F_{j0}\right)}\psi_{(2)},\ \dots \qquad \psi = e^{-imt}\psi'$$

Ref: A. Das. 9310372

S. Balk, J.G. Körner, D. Pirjol, 9307230.

7. 态归一化

- 相对论性归一：$\langle H(p')|H(p)\rangle = 2E_p(2\pi)^3\delta^3(\vec{p}-\vec{p}')$，$E_p=\sqrt{|\vec{p}|^2+m_H^2}$
  态带有质量量纲 -1.
- HQET 下，态与完整QCD差 $1/m_Q$ 修正和归一化因子.
  $\langle H(v',k')|H(v,k)\rangle = 2v^0\delta_{vv'}(2\pi)^3\delta^3(\vec{k}-\vec{k}')$ 与 m_Q 无关. 态带有量纲 $-3/2$.
- 重参数化不变性：v 和 k 的分解有自由度：$v+O(\Lambda_{QCD}/m_Q)$，$k-O(\Lambda_{QCD}/m_Q)$
  在矩阵元中经常将初态末态剩余动量取成0，$|H(v)\rangle=|H(v,k=0)\rangle$
- 相对论和 HQET 归一：相对论性的（完整QCD）的态用动量 p 标记
  HQET 的态用速度 v 标记
  $$|H(p)\rangle = \sqrt{m_H}\,[\,|H(v)\rangle + O(1/m_Q)]$$
- 旋量归一：$\bar{u}(p,s)\gamma^\mu u(p,s) = 2p^\mu$
  $\bar{u}(v,s)\gamma^\mu u(v,s) = 2v^\mu$
  $u(p,s)=\sqrt{m_H}\,u(v,s)$

8. 重味介子衰变常数

- 最简单的应用 HQET 的例子：介子衰变常数.
- 赝标介子 $(\bar{B}, D)$ 衰变常数：
  $$\underbrace{\langle 0|\bar{q}\gamma^\mu\gamma_5 Q(0)}_{3}|\underbrace{P(p)\rangle}_{-} = -i\underbrace{f_P}_{\text{量纲 }1}p^\mu$$
  矢量介子衰变常数：
  $$\langle 0|\bar{q}\gamma^\mu Q(0)|P^*(p,\epsilon)\rangle = \underbrace{f_{P^*}}_{\text{量纲 }2}\epsilon^\mu$$
  (or $f_{P^*}m_V\epsilon^\mu$, where m_V is the mass of P^*. In this notation f_{P^*} shares the same dimension with f_P)
- HQET 下，算符关系：
  $$\bar{q}\Gamma^\mu Q(0) = \bar{q}\Gamma^\mu Q_v(0) + \underbrace{\alpha_s \text{ corrections}}_{\text{perturbative matching}} + \underbrace{1/m_Q \text{ corrections}}_{\text{power corrections}}$$
- 用协变張子场表示：
  - $\bar{q}\Gamma^\mu Q_v$ ：Lorentz 4 矢量. 重夸克自旋变换下：$\bar{q}\Gamma^\mu Q_v \to \bar{q}\Gamma^\mu D(R)_Q Q_v$
  - 用 $H_v^{(Q)}$ 表示的流应有同样变换规律！
  - 构造：1个 $H_v^{(Q)}$ （初态只有一个介子）
    $\Gamma^\mu H_v^{(Q)}$ ：假定 $\Gamma^\mu \to \Gamma^\mu D(R)_Q^{-1}$，则 $\Gamma^\mu H_v^{(Q)}$ 不变；（$\bar{q}\Gamma^\mu Q_v$ 不变）
    固定 $\Gamma^\mu=\gamma^\mu$ or $\gamma^\mu\gamma_5$ 不变，则 $\Gamma^\mu H_v^{(Q)}$ 和 $\bar{q}\Gamma^\mu Q_v$ 一样变.
    $\mathrm{Tr}[X\Gamma^\mu H_v]$ ：X 是 Lorentz aa 旋量，H_v 也是
    X 只能依赖 v，所以 $X=a_0(v^2)+a_1(v^2)\cancel{v}$.
    $$\bar{q}\Gamma^\mu Q_v = \frac{a}{2}\mathrm{Tr}\,\Gamma^\mu H_v^{(Q)},\quad a=[a_0(v)-a_1(v)]$$
    Why $a/2$?
    $H_v^{(Q)}=H_v^{(Q)}\frac{1-\cancel{v}}{2}$, so
    $\mathrm{Tr}[X\Gamma^\mu H_v]$
    $=\mathrm{Tr}[(a_0+a_1\cancel{v})\Gamma^\mu H_v\frac{1-\cancel{v}}{2}]$
    $=\mathrm{Tr}[[\frac{1}{2}(a_0+a_1\cancel{v})-\frac{1}{2}(a_0\cancel{v}+a_1)]\Gamma^\mu H_v]$
    $=\frac{1}{2}(a_0-a_1)\mathrm{Tr}[\Gamma^\mu H_v]-\frac{1}{2}(a_0-a_1)\mathrm{Tr}[\Gamma^\mu H_v\cancel{v}]$
    $=(a_0-a_1)\mathrm{Tr}[\Gamma^\mu H_v]$. A factor 1/2 is introduced so that $\frac{1}{2}\mathrm{Tr}[\Gamma^\mu H_v] \to \begin{cases} -iv^\mu P_v^{(Q)}, & \Gamma^\mu=\gamma^\mu\gamma_5 \\ P_v^{*(Q)\mu}, & \Gamma^\mu=\gamma^\mu \end{cases}$
  - 求迹：$a\begin{cases} -iv^\mu P_v^{(Q)}, & \Gamma^\mu=\gamma^\mu\gamma_5 \\ P_v^{*(Q)\mu}, & \Gamma^\mu=\gamma^\mu \end{cases}$
  - 矩阵元：$\langle 0|\bar{q}\gamma^\mu\gamma_5 Q_v|P(v)\rangle = -iav^\mu$
    $\langle 0|\bar{q}\gamma^\mu Q_v|P^*(v)\rangle = a\epsilon^\mu$
- 与前面定义对比：
  $$f_P=\frac{a}{\sqrt{m_P}},\qquad f_{P^*}=a\sqrt{m_{P^*}}$$
  重夸克极限下：$m_P=m_{P^*}$，所以 $f_P=\frac{a}{\sqrt{m_P}}$，$f_{P^*}=m_P f_P$

这意味着 $f_P \propto m_P^{-1/2}$，$f_{P^*} \propto m_P^{1/2}$

对于 B和D，$\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}}$，$f_{D^*} = m_D f_D$，$f_{B^*} = m_B f_B$

- 实验测量：

$D \to \bar{\ell} \nu_\ell$，$\bar{B} \to \ell \bar{\nu}_\ell$

$$\Gamma = \frac{G_F^2 |V_{Q\bar{q}}|^2}{8\pi} f_P^2 m_\ell^2 m_P \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2$$

9. $\bar{B} \to D^{(*)}$ 形状因子

- $\bar{B} \to D / D^*$ 半轻衰变：抽取 V_{cb}.
- $\bar{B}$ 半轻衰变振幅：由弱有效 Hamiltonian 的矩阵元决定：

  $$H_W = \frac{4G_F}{\sqrt{2}} V_{cb} [\bar{c}\gamma_\mu P_L b][\bar{e}\gamma^\mu P_L \nu_e]$$

  不考虑辐射修正. 矩阵元可分成强子部分 · 轻子部分.

  强子部分由 $V^\mu = \bar{c}\gamma_\mu b$ 和 $A^\mu = \bar{c}\gamma_\mu\gamma_5 b$ 的矩阵元构成
- 矩阵元 $\longrightarrow$ 几个 Lorentz 不变振幅：形状因子.
- $\langle D(p') | V^\mu | \bar{B}(p) \rangle = a p^\mu + b p'^\mu$

  a, b：Lorentz 标量的函数：如 p^2, p'^2, $p \cdot p'$. $p^2 = m_B^2$, $p'^2 = m_D^2$. $p \cdot p' \to (p-p')^2 = q^2$

  $\langle D(p') | V^\mu | \bar{B}(p) \rangle = f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu$

  $\langle D^*(p', \epsilon) | V^\mu | \bar{B}(p) \rangle = g(q^2) \epsilon^{\mu\nu\alpha\tau} \epsilon_\nu^* (p+p')_\alpha (p-p')_\tau$

  $\langle D^*(p', \epsilon) | A^\mu | \bar{B}(p) \rangle = -if(q^2)\epsilon^{*\mu} - i\epsilon^* \cdot p [a_+(q^2)(p+p')^\mu + a_-(q^2)(p-p')^\mu]$
- 离散变换：　P：宇称　T：时间反演

  $P | D(p) \rangle = - | D(p_P) \rangle$, $T | D(p) \rangle = - | D(p_T) \rangle$

  $P | D^*(p, \epsilon) \rangle = | D^*(p_P, \epsilon_P) \rangle$, $T | D^*(p, \epsilon) \rangle = | D^*(p_T, \epsilon_T) \rangle$

  $p = (p^0, \vec{p})$, $\epsilon(\epsilon^0, \vec{\epsilon})$, $p_P = p_T = (p^0, -\vec{p})$, $\epsilon_P = \epsilon_T = (\epsilon^0, -\vec{\epsilon})$

  强相互作用的宇称和时间反演不变性要求：

  $\langle \psi | J^0 | \chi \rangle = \eta_P \langle \psi_P | J^0 | \chi_P \rangle$, $\langle \psi | J^0 | \chi \rangle^* = \eta_T \langle \psi_T | J^0 | \chi_T \rangle$

  $\langle \psi | J^i | \chi \rangle = -\eta_P \langle \psi_P | J^i | \chi_P \rangle$, $\langle \psi | J^i | \chi \rangle^* = -\eta_T \langle \psi_T | J^i | \chi_T \rangle$

  $J = V$: $\eta_P = 1$, $\eta_T = 1$.

  $J = A$: $\eta_P = -1$, $\eta_T = 1$

  $|\chi_P\rangle = P|\chi\rangle$, $|\chi_T\rangle = T|\chi\rangle$

  例子：P：$\langle D^*(p', \epsilon) | V^0 | \bar{B}(p) \rangle = - \langle D^*(p'_P, \epsilon_P) | V^0 | \bar{B}(p_P) \rangle$

  $$\langle D^*(p', \epsilon) | V^0 | \bar{B}(p) \rangle \sim \epsilon^{0\nu\alpha\beta} \underset{-1}{\epsilon_\nu^*} \underset{-1}{p_\alpha} \underset{-1}{p'_\beta}$$

  T：$\langle D^*(p', \epsilon) | V^0 | \bar{B}(p) \rangle^* = - \langle D^*(p'_T, \epsilon_T) | V^0 | \bar{B}(p_T) \rangle$

  要求系数 $g(q^2)$ 为实数

  $\langle D^*(p', \epsilon) | A^0 | \bar{B}(p) \rangle^* = -\langle D^*(p'_T, \epsilon_T) | A^0 | \bar{B}(p_T) \rangle$
- 振幅：

  $$M(\bar{B} \to D e \bar{\nu}_e) = \langle \underset{e\bar{\nu}_e}{D(p')} | \frac{4G_F}{\sqrt{2}} V_{cb} [\bar{c}\gamma_\mu P_L b][\bar{e}\gamma^\mu P_L \nu_e] | \bar{B}(p) \rangle$$

  $p - p' = q = p_e + p_{\nu_e}$

  $$= \sqrt{2} G_F V_{cb} [f_+(q^2)(p+p')^\mu + \underbrace{f_-(q^2)(p-p')^\mu}_{0}] \bar{u}(p_e)\gamma_\mu P_L v(p_{\nu_e})$$

  振幅模平方：$\sum_{\text{spins}} |M(\bar{B} \to D e \bar{\nu}_e)|^2$

  $$= 2G_F^2 |V_{cb}|^2 |f_+(q^2)|^2 (p+p')^{\mu_1} (p+p')^{\mu_2} \mathrm{Tr}[\not{p}_e \gamma_{\mu_1} P_L \not{p}_{\nu_e} P_R \gamma_{\mu_2}]$$

  $$= 2G_F^2 |V_{cb}|^2 |f_+(q^2)|^2 (p+p')^{\mu_1} (p+p')^{\mu_2} \mathrm{Tr}[\not{p}_e \gamma_{\mu_1} \not{p}_{\nu_e} \gamma_{\mu_2} P_L]$$
- 衰变宽度：

  $$\frac{d\Gamma}{dq^2} = \frac{1}{2m_B} \int \frac{d^3p'}{(2\pi)^3 2p'^0} \int \frac{d^3p_e}{(2\pi)^3 2p_e^0} \int \frac{d^3p_{\nu_e}}{(2\pi)^3 2p_{\nu_e}^0} |M|^2 (2\pi)^4 \delta^{(4)}(q - p_e - p_{\nu_e}) \delta(q^2 - (p-p')^2)$$

以下推导衰变宽度表达式.

$$\Gamma \quad \int \frac{d^3 p_e}{(2\pi)^3 2p_e^0} \int \frac{d^3 p_{\nu_e}}{(2\pi)^3 2p_{\nu_e}^0} \mathrm{Tr}[\not p_e \gamma_{\mu_1} \not p_{\nu_e} \gamma_{\mu_2}] (2\pi)^4 \delta^4[q-(p_e+p_{\nu_e})]$$

$$= 4\int \frac{d^3 p_e}{(2\pi)^3 2p_e^0} \int \frac{d^3 p_{\nu_e}}{(2\pi)^3 2p_{\nu_e}^0} [p_e^{\mu_1} p_{\nu_e}^{\mu_2} - p_e\cdot p_{\nu_e} g^{\mu_1\mu_2} + p_e^{\mu_2} p_{\nu_e}^{\mu_1}] (2\pi)^4 \delta^4(q-(p_e+p_{\nu_e}))$$

$$= a\, q^{\mu_1} q^{\mu_2} + b q^2 g^{\mu_1\mu_2}$$

$$p_e\cdot p_{\nu_e} = \frac{1}{2} q^2$$

两边同时点乘 $q_{\mu_1} q_{\mu_2}$： $a\, q^4 + b q^4 = 4\int [dp_e][dp_{\nu_e}] [2(p_e\cdot q)(p_{\nu_e}\cdot q) - p_e\cdot p_{\nu_e} q^2](2\pi)^4\delta^4(q-(p_e+p_{\nu_e}))$

$$= 4\int [dp_e][dp_{\nu_e}] [2 (p_e\cdot p_{\nu_e})^2 - p_e\cdot p_{\nu_e} q^2](2\pi)^4\delta^4(q-(p_e+p_{\nu_e}))$$

$$= 4\int [dp_e][dp_{\nu_e}] [\tfrac{1}{2} q^4 - \tfrac{1}{2} q^4](2\pi)^4\delta(\ ) = 0$$

So. $a = -b$.

点乘 $g_{\mu_1\mu_2}$： $a q^2 + 4b q^2 = 4\int [dp_e][dp_{\nu_e}] [2p_e\cdot p_{\nu_e} - 4 p_e\cdot p_{\nu_e}](2\pi)^4\delta^4(\ )$

$$(a+4b) = -\cancel{4}\int [dp_e][dp_{\nu_e}] (2\pi)^4 \delta^4(q-(p_e+p_{\nu_e}))$$

$$b = -\frac{4}{3}\int [dp_e][dp_{\nu_e}] (2\pi)^4 \delta^4(q-(p_e+p_{\nu_e}))$$

$$= -\frac{4}{3}\int \frac{d^3 p_e}{(2\pi)^3 2p_e^0} \frac{1}{2p_{\nu_e}^0} 2\pi\, \delta(q^0 - (p_e^0 + p_{\nu_e}^0))$$

$$= -\frac{4}{3}\int \frac{d^3 p_e}{(2\pi)^3 2p_e^0} \frac{1}{2p_{\nu_e}^0} 2\pi\, \delta(q^0 - 2|\vec p_e|)$$

因为积分是 q^2 的函数，可选择 $\vec q = 0$ 而保持 q^2 不变

$$= -\frac{4}{3}\int \frac{d|\vec p|\, |\vec p_e|^2\, d\Omega^{(3)}}{(2\pi)^2\, 4|\vec p_e|^2} \delta(q^0 - 2|\vec p_e|)$$

$\int d\Omega^{(3)} = 4\pi$

$$= -\frac{4}{3}\cdot\frac{1}{4\pi^2}\cdot \pi \cdot \frac{1}{2}$$

$$= -\frac{1}{6\pi}, \quad a = \frac{1}{6\pi}$$

所以积分结果为： $\frac{1}{6\pi}(q^{\mu_1} q^{\mu_2} - q^2 g^{\mu_1\mu_2})$ $\lrcorner$

然后：由于 $(p+p')^{\mu_1}(p+p')^{\mu_2}(q_{\mu_1}q_{\mu_2} - q^2 g_{\mu_1\mu_2})$

$$= (p+p')\cdot(p-p')\,(p+p')\cdot(p-p') - (p-p')^2(p+p')^2 \qquad q^2 = (p-p')^2 = m_B^2 + m_D^2 - 2p\cdot p'$$

$$= (m_B^2 - m_D^2)^2 - q^2(2m_B^2 + 2m_D^2 - q^2)$$

$$= (m_B^2 + m_D^2 - q^2)^2 - 4m_B^2 m_D^2$$

以及积分：

$$\int \frac{d^3 p'}{(2\pi)^3 2p'^0} \delta(q^2 - (p-p')^2) = \frac{1}{16\pi^2 m_B^2}\sqrt{(q^2 - m_B^2 - m_D^2)^2 - 4m_B^2 m_D^2}$$

$$\Gamma \quad \int \frac{d^3 \vec p'}{(2\pi)^3 2p'^0} \delta(q^2 - (p-p')^2)$$

$$= \int \frac{d^3 \vec p'}{(2\pi)^3 2p'^0} \delta(q^2 - m_B^2 - m_D^2 + 2p\cdot p')$$

$$= \int \frac{d^3 \vec p'}{(2\pi)^3 2p'^0} \delta(q^2 - m_B^2 - m_D^2 + 2p^0 p'^0 - 2|\vec p||\vec p'|\cos\theta)$$

$$= \int \frac{d^3 \vec p'}{(2\pi)^3 2p'^0} \delta(q^2 - m_B^2 - m_D^2 + 2m_B\sqrt{|\vec p'|^2 + m_D^2})$$

$$= \int \frac{d|\vec p'|\, |\vec p'|^2\, d\Omega^{(3)}}{(2\pi)^3\, 2\sqrt{|\vec p'|^2 + m_D^2}} \delta(q^2 - m_B^2 - m_D^2 + 2m_B\sqrt{|\vec p'|^2 + m_D^2})$$

令 $\sqrt{|\vec p'|^2 + m_D^2} = x$ $\quad d|\vec p'| = dx\cdot\frac{d|\vec p'|}{dx} = dx\cdot\frac{1}{\frac{dx}{d|\vec p'|}} = dx\cdot\frac{1}{\frac{|\vec p'|}{\sqrt{|\vec p'|^2 + m_D^2}}} = dx\cdot\frac{x}{|\vec p'|}$

$$\int \frac{dx\cdot\frac{x}{|\vec p'|}\, |\vec p'|^2\, 4\pi}{2(2\pi)^3\, x} \delta(q^2 - m_B^2 - m_D^2 + 2m_B x)$$

$$= \int \frac{dx}{4\pi^2} |\vec p'|\, \delta(q^2 - m_B^2 - m_D^2 + 2m_B x)$$

$$= \frac{1}{2m_B}\frac{1}{4\pi^2}\sqrt{\frac{m_B^4 + (m_D^2 - q^2)^2 - 2m_B^2(m_D^2 + q^2)}{4m_B^2}} = \frac{1}{4m_B^2}\frac{1}{4\pi^2}\sqrt{(m_B^2 - m_D^2 + q^2)^2 - 4m_B^2 m_D^2}$$ $\lrcorner$

最后，微分衰变宽度为：

$$\frac{d\Gamma}{dq^2}(\bar{B}\to De\bar{\nu}_e) = \frac{G_F^2|V_{cb}|^2|f_+|^2}{192\pi^3 m_B^3}\left[(q^2-m_B^2-m_D^2)^2-4m_B^2m_D^2\right]^{3/2}$$

$q^2=(p-p')^2=(m_B-p'^0)^2-\vec{p}'^2=m_B^2+m_D^2-m_Bp'^0$

- HQET 下的形状因子

  - B介子、D介子速度： $v=p/m_B$， $v'=\frac{p'}{m_D}$． $w=v\cdot v'=\frac{p\cdot p'}{m_B\cdot m_D}=\frac{1}{2}\frac{m_B^2+m_D^2-q^2}{m_Bm_D}$

    $w-1=\frac{m_B^2+m_D^2-2m_Bm_D-q^2}{2m_Bm_D}=\frac{(m_B-m_D)^2-q^2}{2m_Bm_D}$， $0<w-1<\frac{(m_B-m_D)^2}{2m_Bm_D}$

    零反冲(zero recoil)： D 在 B 的静止系下静止． $w=1$

  - 形状因子：

$$\frac{\langle D(p')|V^\mu|\bar{B}(p)\rangle}{\sqrt{m_Bm_D}} = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu$$

$$\frac{\langle D^*(p',\epsilon)|V^\mu|\bar{B}(p)\rangle}{\sqrt{m_Bm_{D^*}}} = h_V(w)\epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta$$

$$\frac{\langle D^*(p',\epsilon)|A^\mu|\bar{B}(p)\rangle}{\sqrt{m_Bm_{D^*}}} = -ih_{A_1}(w)(w+1)\epsilon^{*\mu}+ih_{A_2}(w)(\epsilon^*\cdot v)v^\mu+ih_{A}(w)(\epsilon^*\cdot v)v'^\mu$$

Must be linear in ϵ^*, thus there are no more $\epsilon^*\cdot v$ dependences in form factors.

  - 微分宽度： $\frac{d\Gamma}{dw}(\bar{B}\to De\bar{\nu}_e)=\frac{G_F^2|V_{cb}|^2m_B^5}{48\pi^3}(w^2-1)^{3/2}r^3(1+r)^2\mathcal{F}_D(w)^2$

$$\frac{d\Gamma}{dw}(\bar{B}\to D^*e\bar{\nu}_e)=\frac{G_F^2|V_{cb}|^2m_B^5}{48\pi^3}(w^2-1)^{1/2}(w+1)^2r^{*3}(1-r^*)^2\left[1+\frac{4w}{w+1}\frac{1-2wr^*+r^{*2}}{(1-r^*)^2}\right]\mathcal{F}_{D^*}(w)^2$$

其中 $r=\frac{m_D}{m_B}$， $r^*=\frac{m_{D^*}}{m_B}$

$$\mathcal{F}_D(w)^2=\left[h_++\left(\frac{r-r}{1+r}\right)h_-\right]^2$$

$$\mathcal{F}_{D^*}(w)^2=\left\{2(1-2wr^*+r^{*2})\left[h_{A_1}^2+\left(\frac{w-1}{w+1}\right)h_V^2\right]+\left[(1-r^*)h_{A_1}+(w-1)(h_{A_1}-h_{A_3}-r^*h_{A_2})\right]^2\right\}\cdot\left\{(1-r^*)^2+\frac{4w}{w+1}(1-2wr^*+r^{*2})\right\}^{-1}$$

- 重夸克对称性对形状因子的限制与 Isgur-Wise 函数

  - 重夸克对称性的适用性： 对于轻自由度的动量传递 << 重夸克质量，但 q^2 并不小！

    传递给轻自由度的动量应使其随 D 反冲．

    B 中轻自由度： $\Lambda_{QCD}v$ ． D 中： $\Lambda_{QCD}v'$ ，

    动量转移： $q^2_{light}\sim\Lambda_{QCD}^2(v-v')^2=2\Lambda_{QCD}^2(1-w)$ if $w\sim O(1)$

  - 诱导场表示：

    QCD矩阵元 ： $\langle H^{(c)}(p')|\bar{c}\Gamma b|H^{(b)}(p)\rangle$

    HQET ： $\langle H^{(c)}(v')|\bar{c}_{v'}\Gamma b_v|H^{(b)}(v)\rangle$

    与重→轻同样技巧： ① 有 $H_{v'}^{(c)}$， $H_v^{(b)}$　　用来建更初末态介子

    ② 重夸克变换 → $\bar{H}_{v'}^{(c)}\Gamma H_v^{(b)}$

    ③ Lorentz 协变 : → $\mathrm{Tr}[X\bar{H}_{v'}^{(c)}\Gamma H_v^{(b)}]$，X只依赖 v,v' → $X=X_0+X_1\not{v}+X_2\not{v}'+X_3\not{v}\not{v}'$

    X_i 是 $v\cdot v'=w$ 的函数．

    ④ $\not{v}H_v^{(b)}=H_v^{(b)}$　$\not{v}'\bar{H}_{v'}^{(c)}=-\bar{H}_{v'}^{(c)}$ ⇒ $\bar{c}_{v'}\Gamma b_v=-\xi(w)\mathrm{Tr}[\bar{H}_{v'}^{(c)}\Gamma H_v^{(b)}]$

  求迹得到：

$$\langle D(v')|\bar{c}_{v'}\gamma_\mu b_v|\bar{B}(v)\rangle=\xi(w)[v_\mu+v'_\mu]$$

$$\langle D^*(v',\epsilon)|\bar{c}_{v'}\gamma_\mu\gamma_5b_v|\bar{B}(v)\rangle=-i\xi(w)[(1+w)\epsilon^*_\mu-(\epsilon^*\cdot v)v'_\mu]$$

$$\langle D^*(v',\epsilon)|\bar{c}_{v'}\gamma_\mu b_v|\bar{B}(v)\rangle=\xi(w)\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v'^\alpha v^\beta$$

  6个形状因子可用1个函数表示！

  - 边界条件： $\xi(1)=1$

    考虑 $\langle\bar{B}(v)|\bar{b}_v\gamma_\mu b_v|\bar{B}(v)\rangle=\frac{\langle\bar{B}(p)|\bar{b}\gamma_\mu b|\bar{B}(p)\rangle}{m_B}=2v_\mu\xi(1)$

    $\mu=0$: $\frac{1}{m_B}\langle\bar{B}(p)|\bar{b}\gamma_0 b|\bar{B}(p)\rangle$ = b quark number

    $=2v^0$.　　thus, $\xi(1)=1$．

  - 形状因子间的关系：

$$h_+(w)=h_V(w)=h_{A_1}(w)=h_{A_3}(w)=\xi(w)\text{，}\quad h_-(w)=h_{A_2}(w)=0$$

并且有: $\mathcal{F}_D(w) = \mathcal{F}_{D^*}(w) = \zeta(w)$

ALEPH 实验验证

- 形状因子可以用一个 Isgur-Wise 函数表示的物理原因:

10. $\Lambda_c \to \Lambda$ 形状因子

- $\Lambda_c \to \Lambda e \bar{\nu}_e$ 重到轻衰变

  $\langle \Lambda(p',s') | \bar{s}\gamma^\mu c | \Lambda_c(p,s)\rangle = \bar{u}(p',s')[f_1\gamma^\mu + i f_2 \sigma^{\mu\nu} q_\nu + f_3 q^\mu] u(p,s)$

  $\langle \Lambda(p',s') | \bar{s}\gamma^\mu\gamma_5 c | \Lambda_c(p,s)\rangle = \bar{u}(p',s')[g_1\gamma^\mu + i g_2 \sigma^{\mu\nu} q_\nu + g_3 q^\mu]\gamma_5 u(p,s)$

  q: 动量转移. $q = p' - p$. $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$. f_i, g_i 都是 q^2 的函数.

- HQET 下的形状因子,

  $\langle \Lambda(p',s') | \bar{s}\Gamma c_v | \Lambda_c(v,s)\rangle = \bar{u}(p',s') X \Gamma u(v,s)$, $X = F_1 + F_2 \slashed{v}$

  $\slashed{v} u(v,s) = u(v,s)$, $\slashed{p}' u(p',s') = m_\Lambda u(p',s')$

  $f_1 = g_1 = F_1 + \frac{m_\Lambda}{m_{\Lambda_c}} F_2$

  $f_2 = f_3 = g_2 = g_3 = \frac{1}{m_{\Lambda_c}} F_2$

  6个形状因子可以用两个函数 $F_{1,2}$ 表示.

- 对 Λ_c 衰变中 Λ 产生的作用.

11. $\Lambda_b \to \Lambda_c$ 形状因子:

- $\langle \Lambda_c(p',s') | \bar{c}\gamma^\mu b | \Lambda_b(p,s)\rangle = \bar{u}(p',s')[f_1\gamma^\mu + f_2 v^\mu + f_3 v'^\mu] u(p,s)$

  $\langle \Lambda_c(p',s') | \bar{c}\gamma^\mu\gamma_5 b | \Lambda_b(p,s)\rangle = \bar{u}(p',s')[g_1\gamma^\mu + g_2 v^\mu + g_3 v'^\mu]\gamma_5 u(p,s)$

- HQET 下:

  $\langle \Lambda_c(v',s') | \bar{c}_{v'}\Gamma b_v | \Lambda_b(v,s) = \zeta(\omega)\bar{u}(v',s')\Gamma u(v,s)$

  $f_1(\omega) = g_1(\omega) = \zeta(\omega)$, $f_2 = f_3 = g_2 = g_3 = 0$

  6个形状因子可以用1个 Isgur-Wise 函数表示.

  $\zeta(1) = 1$. ($\omega = 1$ 处表示 b 夸克的)