The nuts and bolts of muon precession frequency measurements in the Fermilab Muon g-2 experiment

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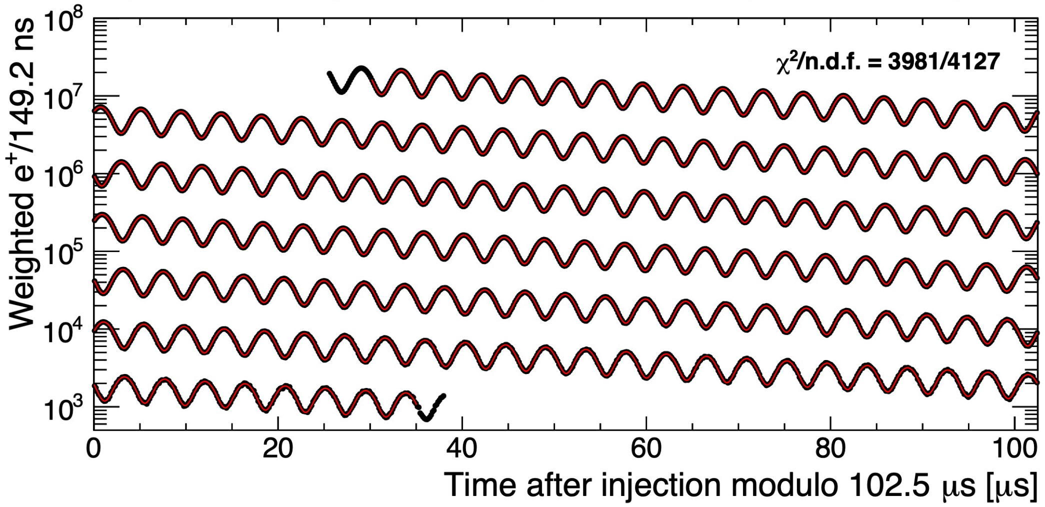
Muon g-2 collaboration @ Liverpool

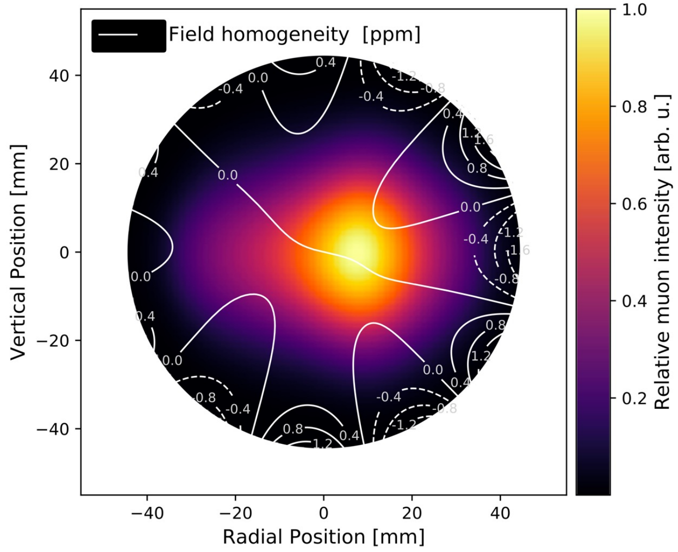
李政道研究所
TSUNG-DAO LEE INSTITUTE

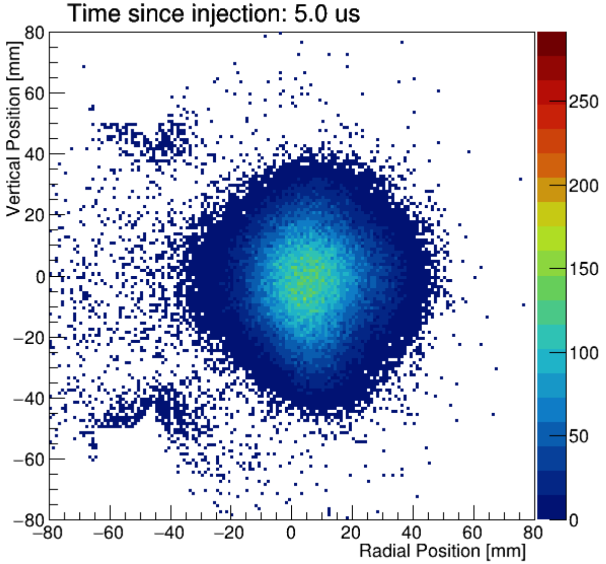
Muon g-2 Experiment in a Nutshell

$$\frac{\omega_a}{\omega_p} = \frac{\omega_a^m}{\omega_p^m}$$

Measured Values

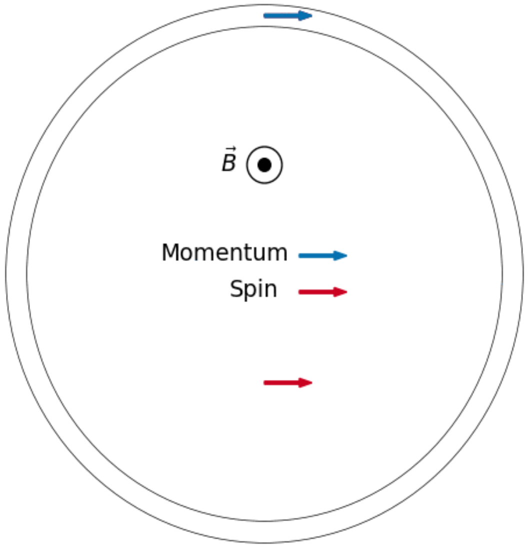


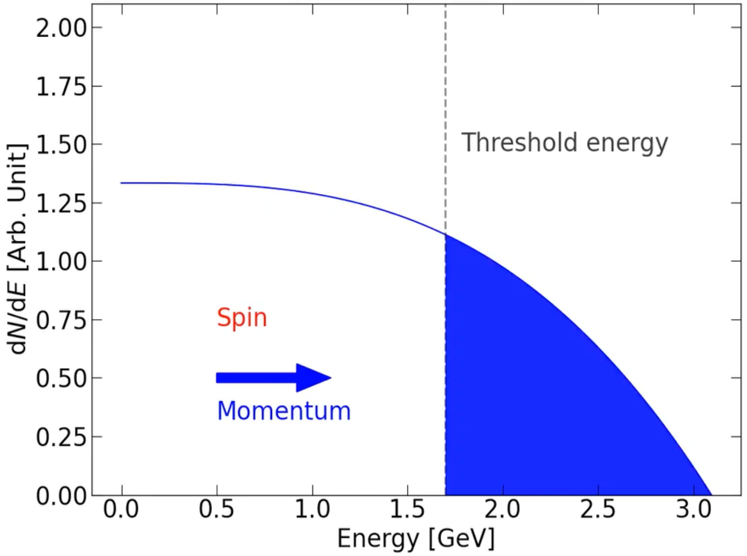


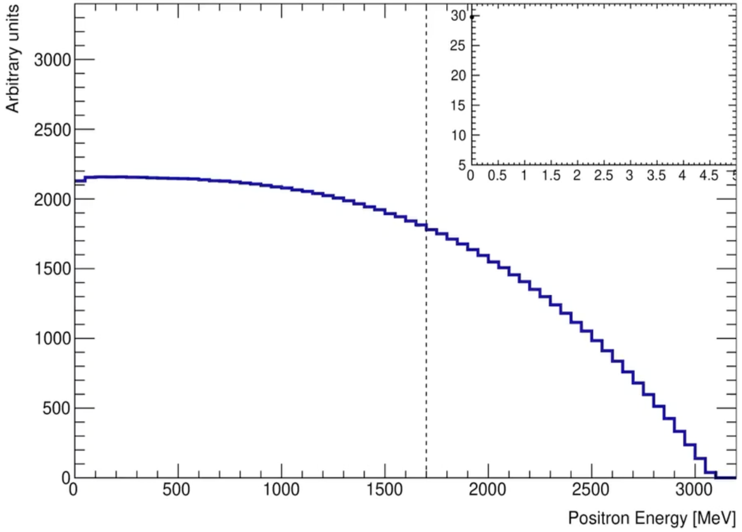


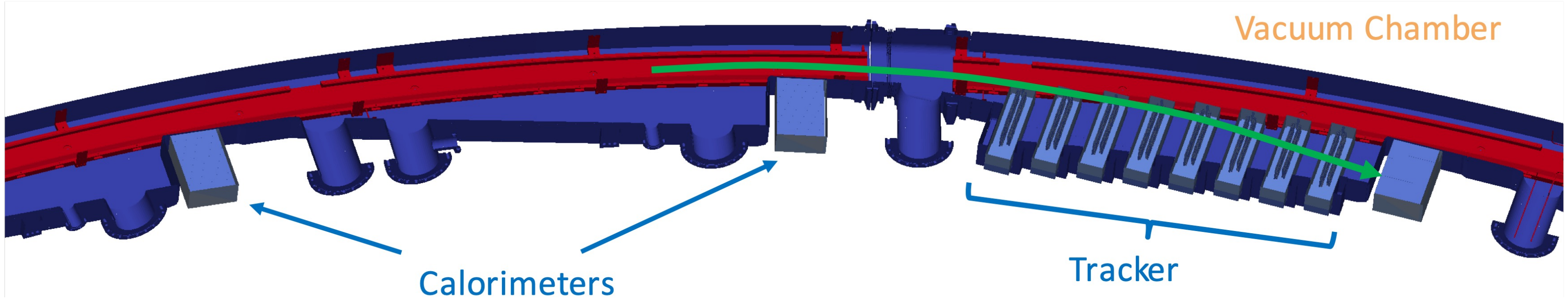
Muon precession frequency

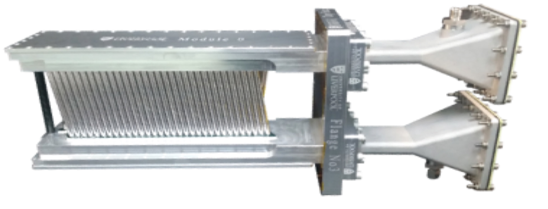
$$N(t) = N_0 e^{-t/\tau}\left[1 + A_\mu \cos(\omega_a t + \phi)\right]$$





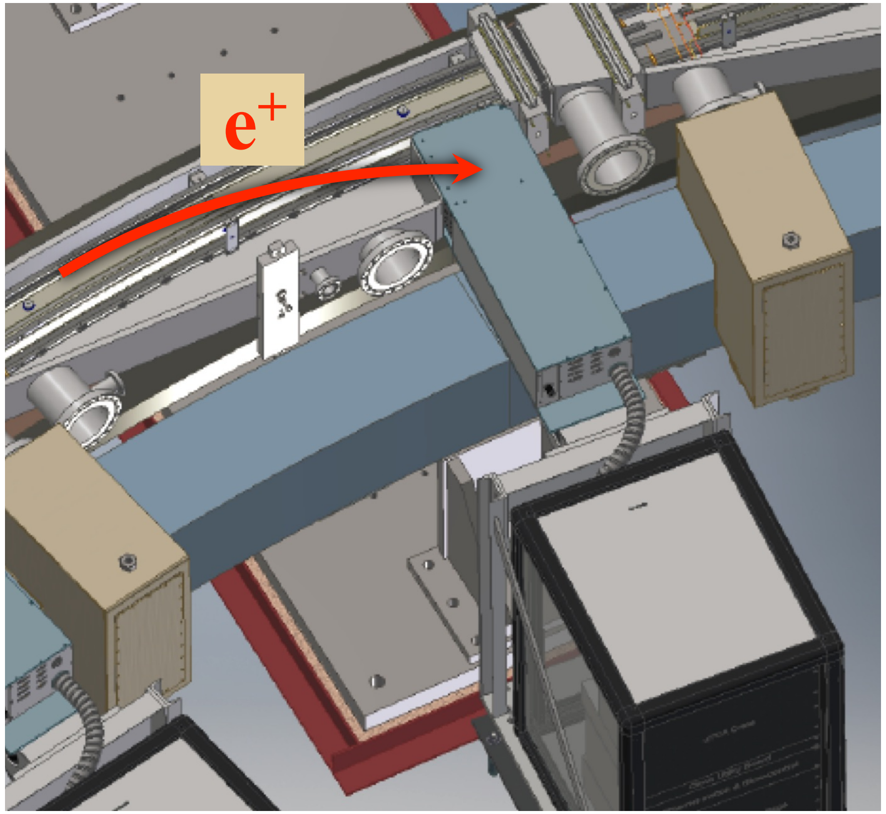






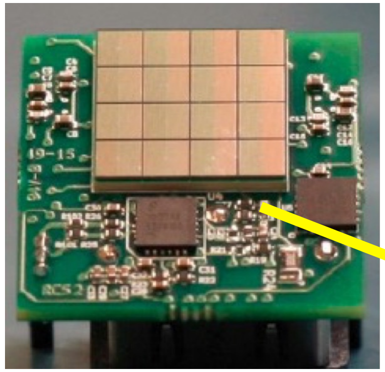
The SWISS KNIFE for g-2 experiment

Calo measure positron time and energy

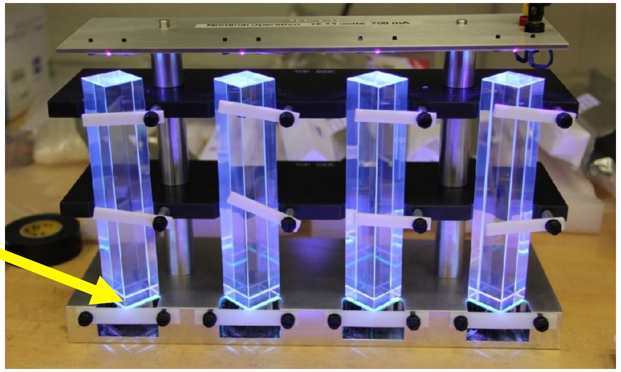


Decay positron curving in and striking a calorimeter

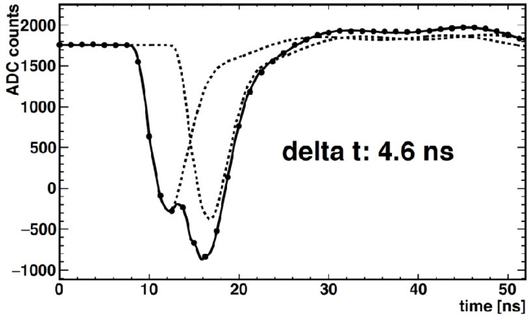
SiPM



PbF₂

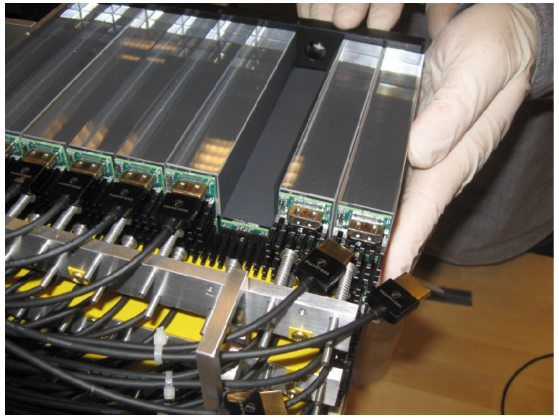


pileup separation



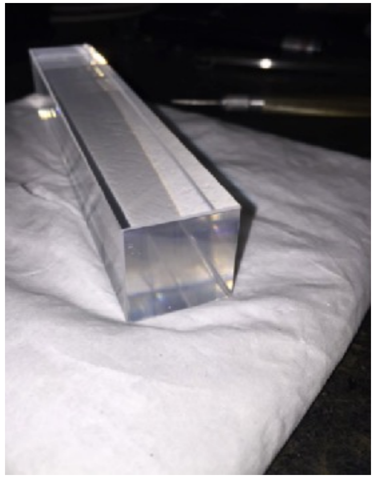
PMT-like signal, B-field operation, 100% separation > 2.5 ns

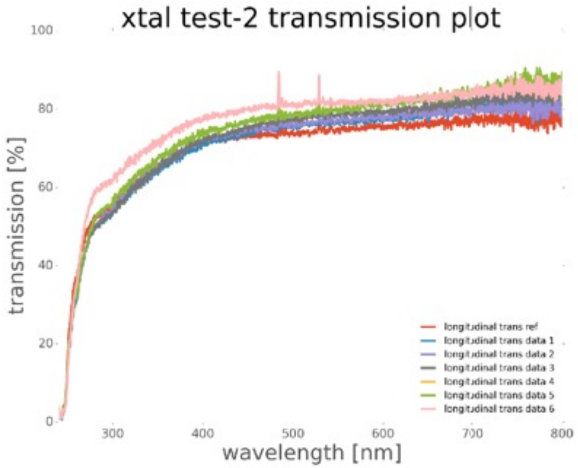
Opened up calorimeter



Stacking crystals

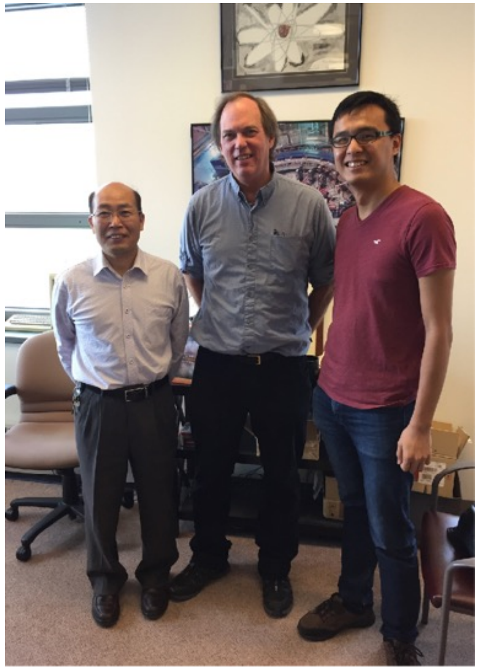
Calorimeter: Quality control systems

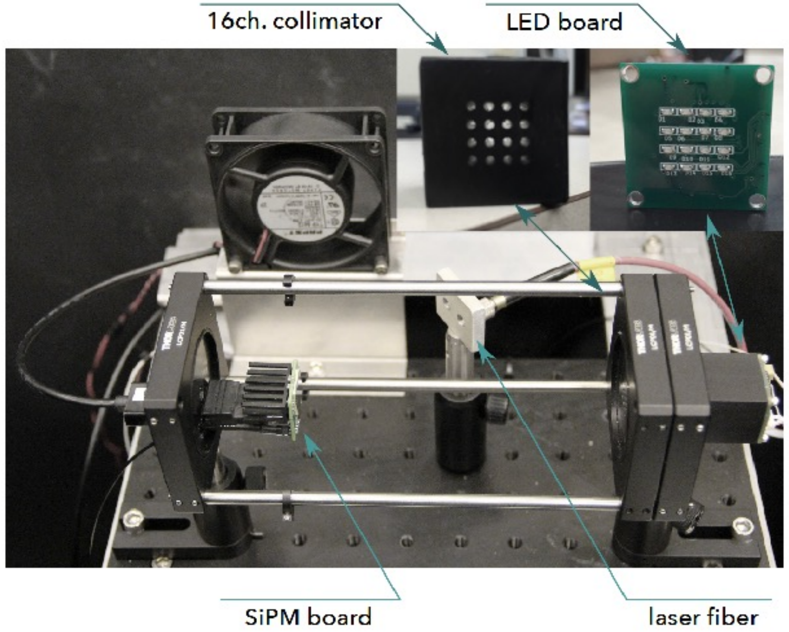


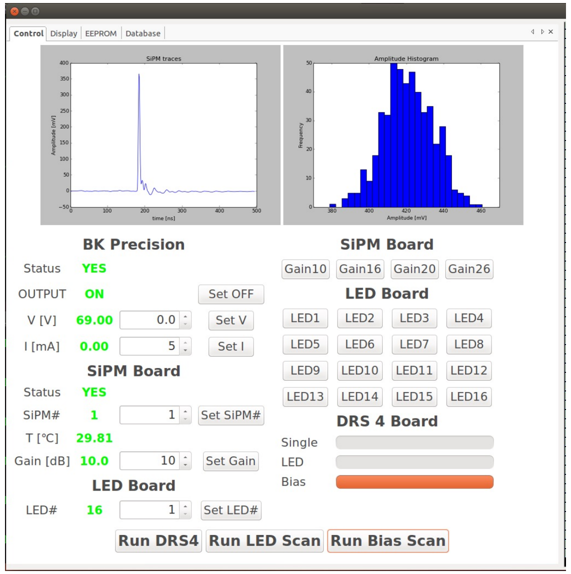


Inspection procedure and transmission measurement

Guohao Ren
Head of Inorganic Scintillator
(SICCAS,上海硅酸盐研究所)







BK Precision

Status YES
OUTPUT ON
V [V] 69.00 0.0
I [mA] 0.00 5

SiPM Board

Status YES
SiPM# 1 1
T [°C] 29.81
Gain [dB] 10.0 10

LED Board

LED# 16 1

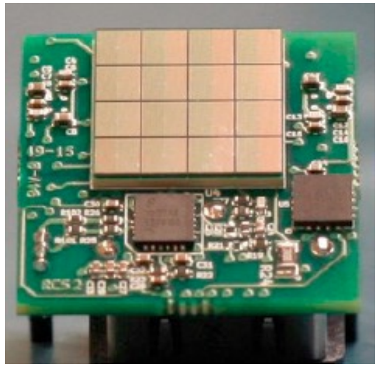
SiPM Board

LED Board

DRS 4 Board

Single
LED
Bias

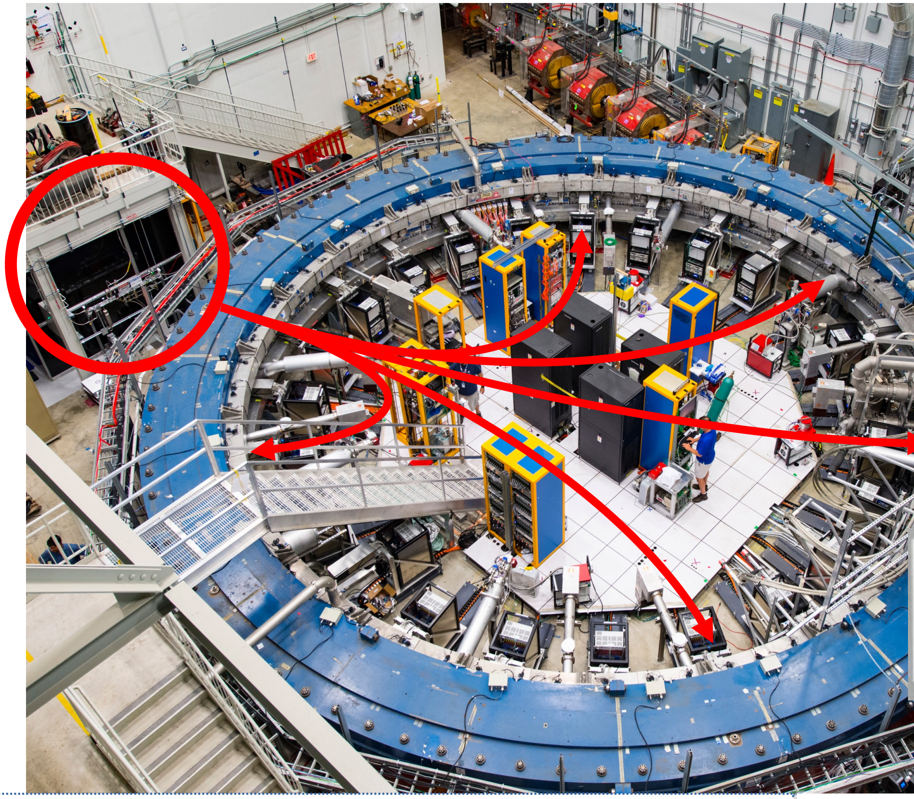
Good transmission and in-range dimension needed!
1350 of them

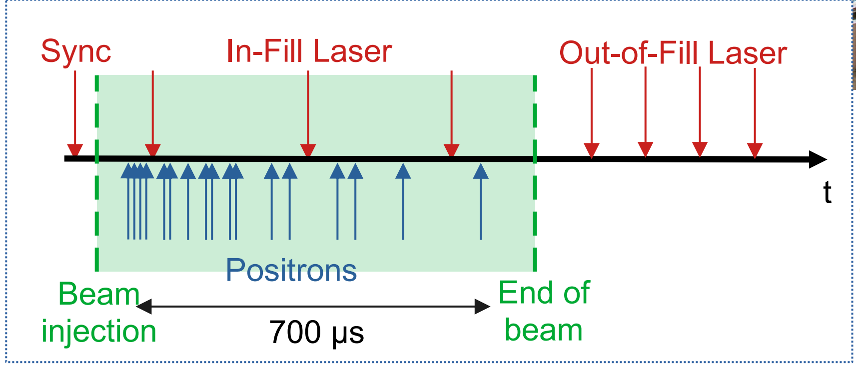


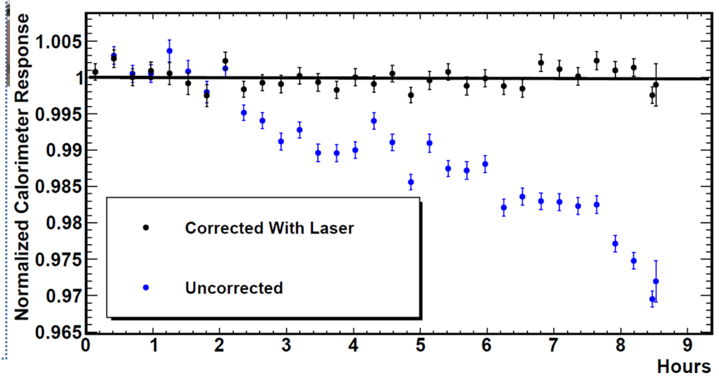
All 16 CHs,
on-chip T-sensor,
memory have to work!
1400 of them

Gain monitoring and calibration system

- Laser light @405 nm sent to all 1296 crystals with distribution panels
- Time synchronization at the ~50 ps
- Gain calibration of the SiPMs at the 10^{-4} level (< 20 ppb systematics in gain)







Real World Complications: Corrections

- We need to make corrections for seven small effects:

E-field & vertical motion:
Spin precesses slower
than in basic equation

Phase changes over each fill:
Phase-Acceptance, Differential
Decay, Muon Losses

$$\frac{\omega_a}{\omega_p} = \frac{\omega_a^m}{\omega_p^m}\,\frac{1+\overbrace{C_e+C_p}+\overbrace{C_{pa}+C_{dd}+C_{ml}}}{1+\underbrace{B_k+B_q}}$$

Measured Values

Transient Magnetic Fields:
Quad Vibrations,
Kicker Eddy Current,

- ω_a corrections are small, but dominated Run-1 systematics: **500(93) ppb**
- For Run-2/3 they are **580(40) ppb**, dominated by C_e and C_p

Spin precession in B and E-field

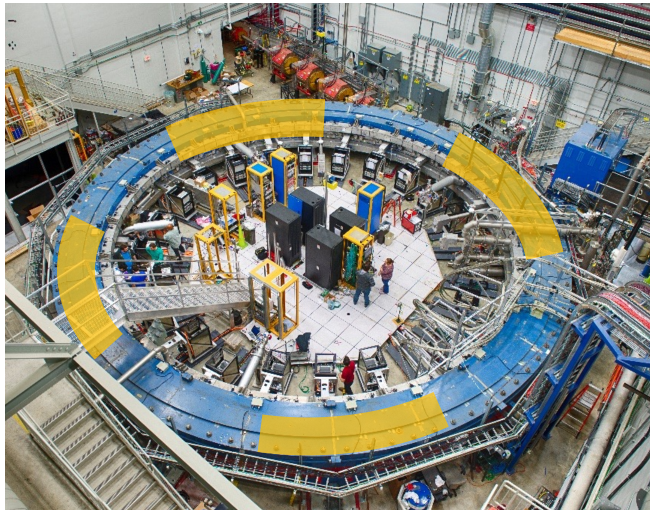
- Generic spin-motion is described by BMT equation:

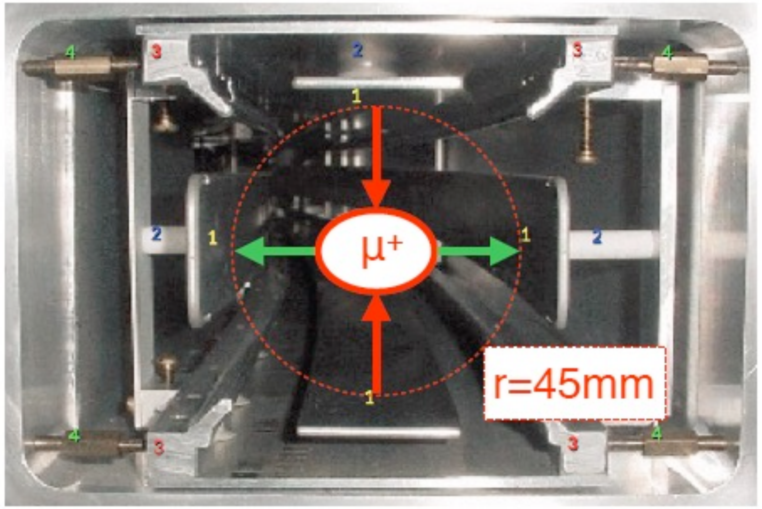
$$\frac{d(\hat{\beta}\cdot\vec{S})}{dt} = -\frac{q}{m}\vec{S}_T \cdot \left[a_\mu \hat{\beta}\times\vec{B} + \beta \cancelto{p_\mu = 3.094\text{ GeV}}{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)} \frac{\vec{E}}{c}\right]$$

simplified equation

$$\omega_a = a_\mu \frac{eB}{m}$$

- Muons travel in **E-field** from focusing quadrupoles: experience a **motional magnetic field** in their rest frame





- The electrostatic quadrupoles (ESQ) are used to focus the beam vertically.
- Four sections cover 43% of the circumference.
- Each plate is charged to ±18 kV.

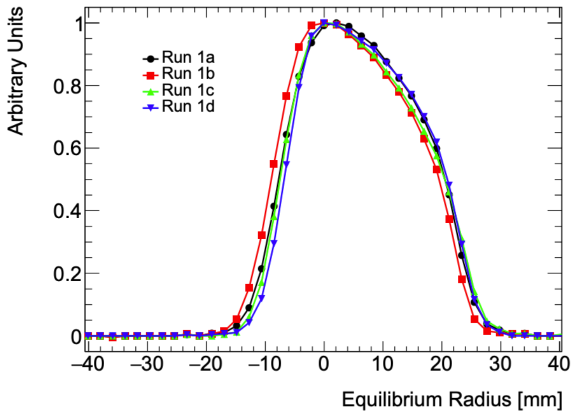
Finite width of the p-distribution

- Generic spin-motion is described by BMT equation:

$$\frac{d(\hat{\beta}\cdot\vec{S})}{dt} = -\frac{q}{m}\vec{S}_T\cdot\left[a_\mu\hat{\beta}\times\vec{B} + \underline{\beta\cancelto{p_\mu = 3.094\text{ GeV}}{\left(a_\mu - \frac{1}{\gamma^2-1}\right)}\frac{\vec{E}}{c}}\right]$$

- Muons travel in **E-field** from focusing quadrupoles: experience a **motional magnetic field** in their rest frame
- Term vanishes at "magic" momentum (p_μ = 3.094 GeV)
- But not all muons are at p_{magic}, hence corrections are needed
- C_E comes from p_μ distribution measured using timing data from calorimeters

p_μ distribution

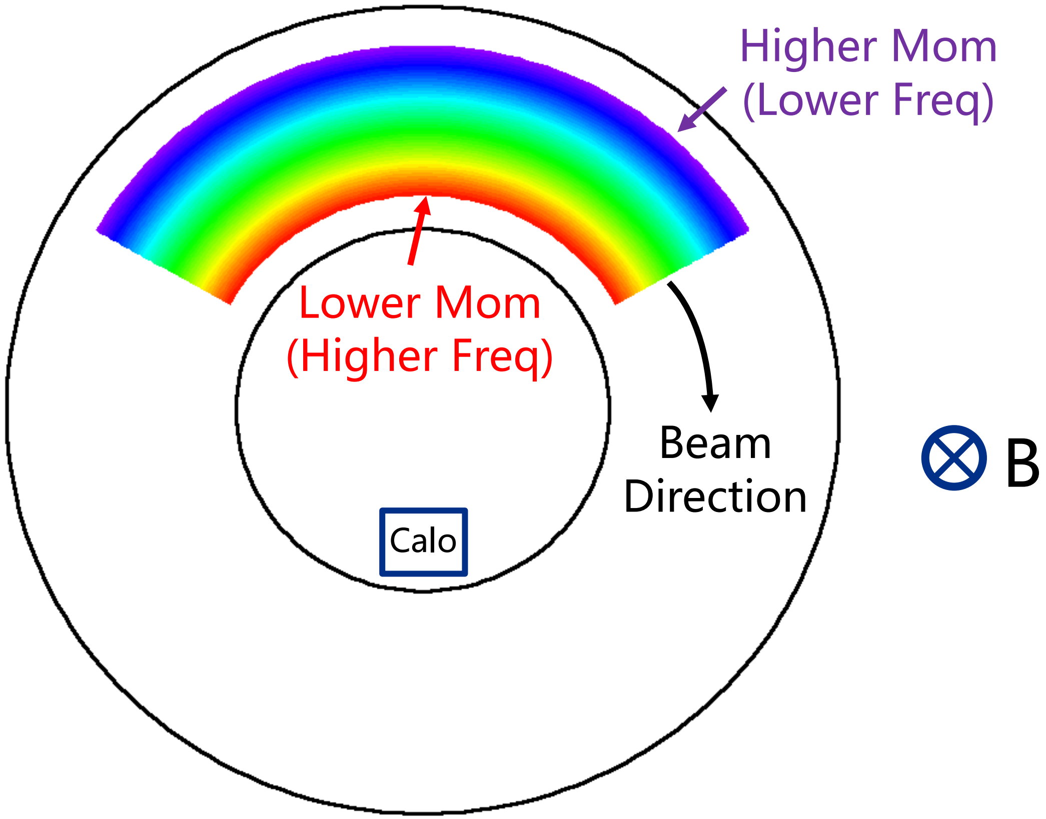


$$\boxed{C_E = 451 \pm 32\text{ ppb}}$$

Fast rotation signal

- Imagine injecting uniform momentum & time distributions:

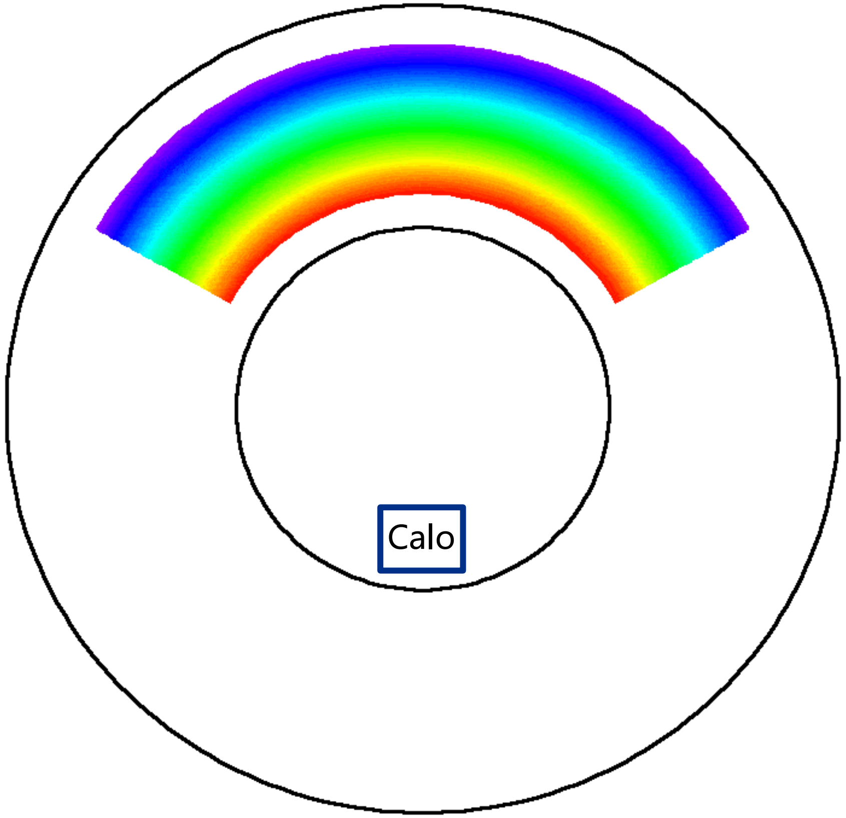
v ~ c for
all muons



- Higher momentum muons have further to travel, so have lower cyclotron frequencies

Fast rotation signal

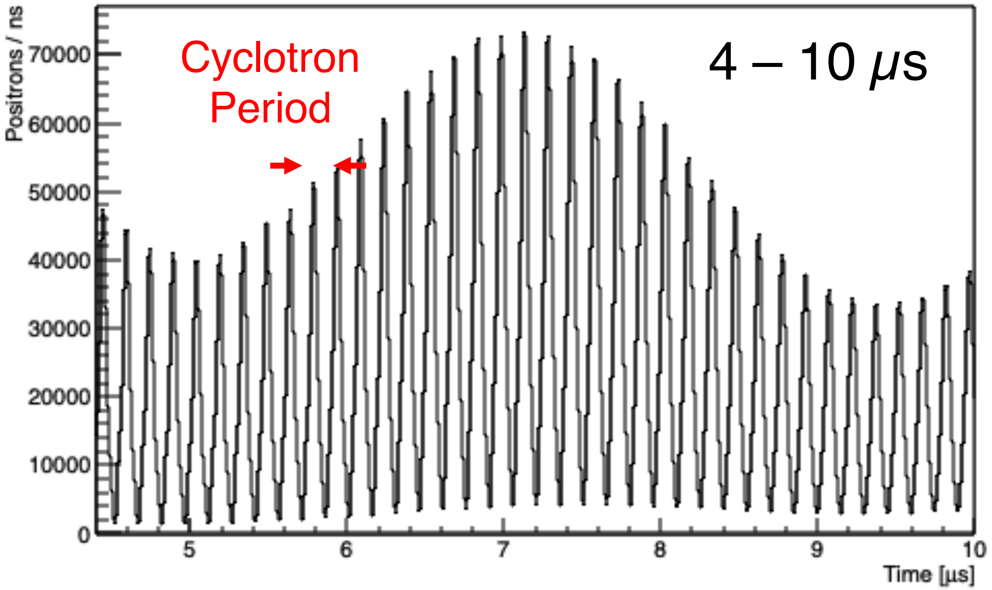
- Over time, lower momentum muons will catch up with higher momentum muons:

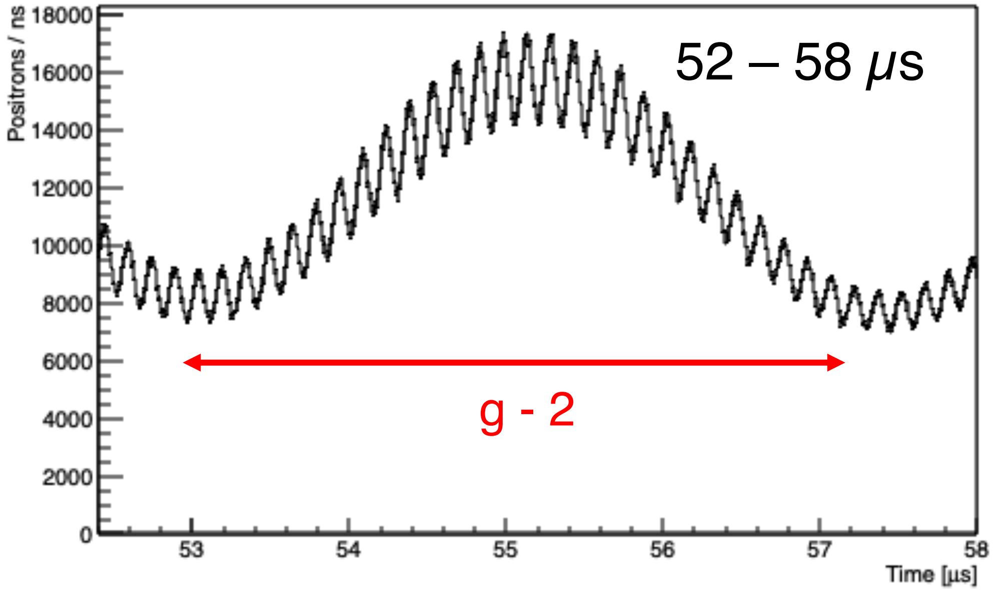


- The way that the gaps are filled in is related to the momentum distribution of the stored beam

Fast rotation signal in the calorimeter

- Effect is a strong feature of the calorimeter timing data at early times:

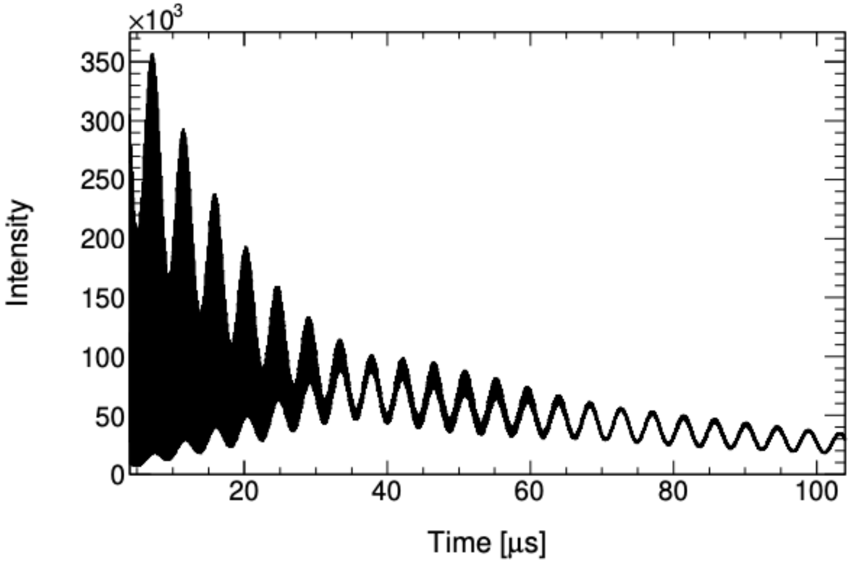




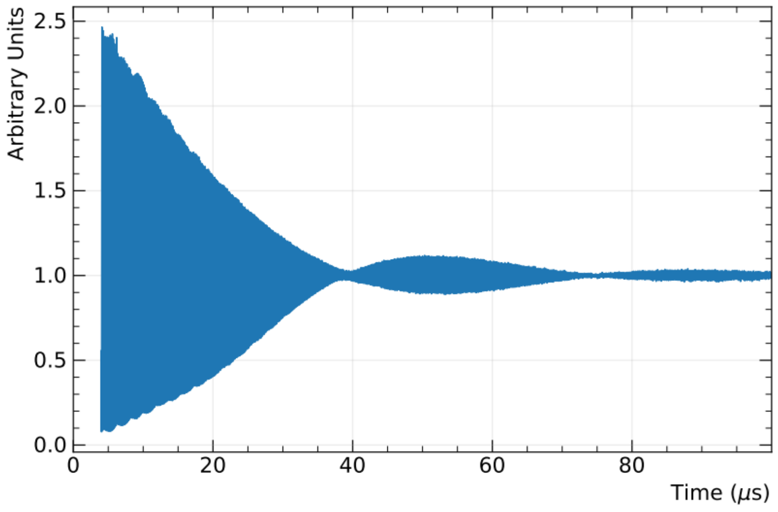
- Less pronounced when all calos are added together
- We utilized this feature to measure the muon momentum distribution or equilibrium position

E-field Correction: C_e

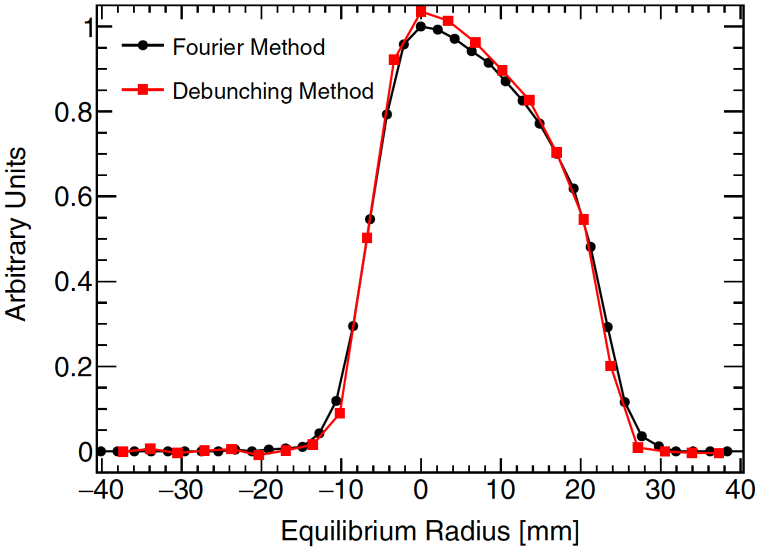
Finely-binned decay e^+ spectrum



Isolated FR signal



Reconstructed x_e distribution



Equilibrium position reconstruction approaches:

1. Fourier method (frequency domain)
2. Debunching method (time domain)

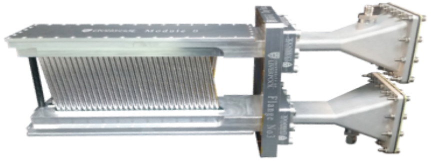
$$C_e \approx 2n(1-n)\beta_0^2 \frac{\langle x_e^2 \rangle}{R_0^2}$$

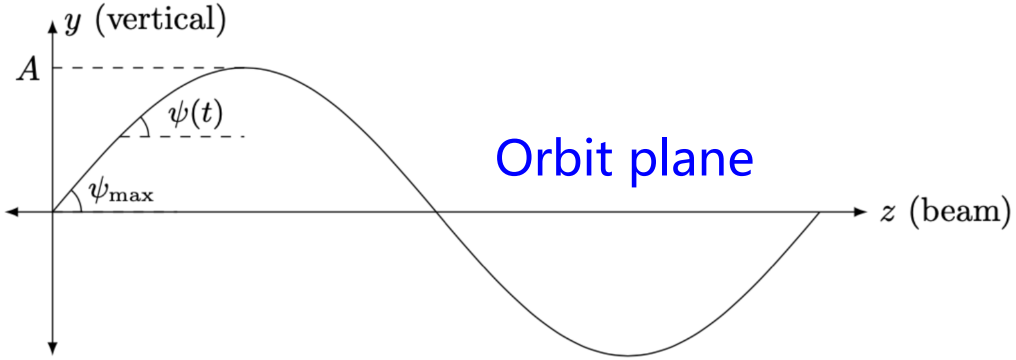
Beam vertical oscillation

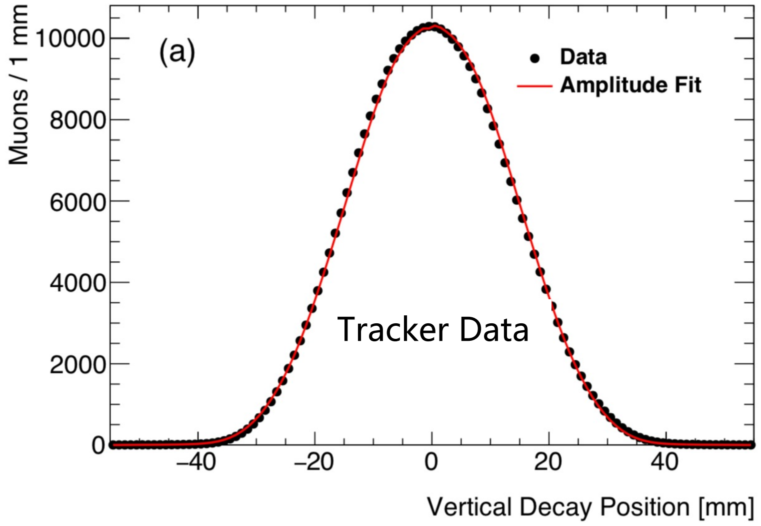
- Generic spin-motion is described by BMT equation:

$$\frac{d(\hat{\beta}\cdot\vec{S})}{dt} = -\frac{q}{m}\vec{S}_T\cdot\left[a_\mu\hat{\beta}\times\vec{B} + \beta\left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{E}}{c}\right]$$

- Muons oscillate vertically **(pitch)** so the $\hat{\beta}\times\vec{B}$ term is reduced



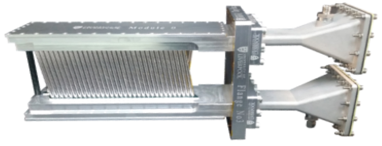




- C_P is extracted from vertical width measured by the trackers

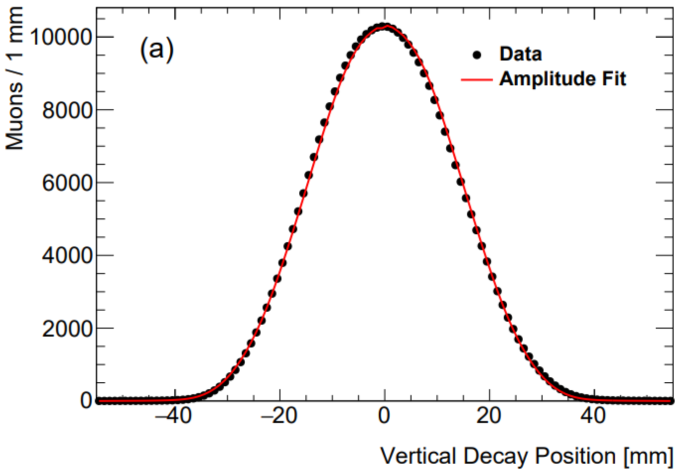
$$\boxed{C_P = 170 \pm 10 \text{ ppb}}$$

Pitch correction: C_p

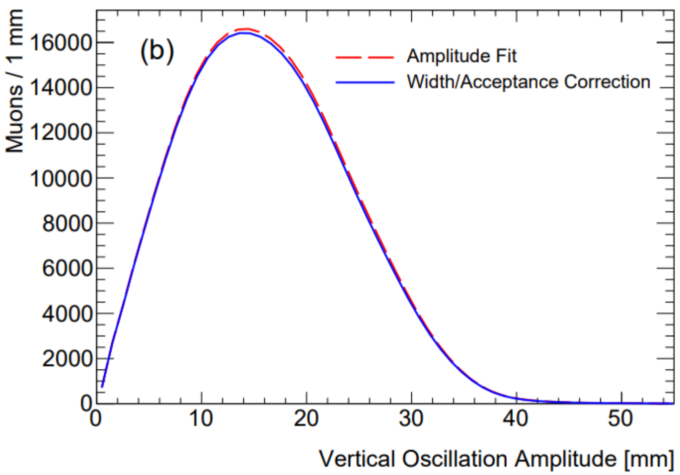


- C_p is calculated from the amplitude distribution of the vertical motion
- The amplitude is reconstructed from the position distribution, including the acceptance correction.
- Dominant systematic error source: tracker alignment and reconstruction.
- Improvement after Run-1: Independent & different method analysis cross-check.

Vertical position



Vertical oscillation amplitude



$$C_p = \frac{n}{2}\frac{\langle y^2\rangle}{R_0^2} = \frac{n}{4}\frac{\langle A^2\rangle}{R_0^2}$$

C_p	Correction [ppb]	Uncertainty [ppb]
Run-1	180	13
Run-2/3	170	10

Real World Complications: Corrections

- We need to make corrections for seven small effects:

E-field & vertical motion:
Spin precesses slower
than in basic equation

Phase changes over each fill:
Phase-Acceptance, Differential
Decay, Muon Losses

$$\frac{\omega_a}{\omega_p} = \frac{\omega_a^m}{\omega_p^m}\,\frac{1+\overbrace{C_e+C_p}+\overbrace{C_{pa}+C_{dd}+C_{ml}}}{1+\underbrace{B_k+B_q}}$$

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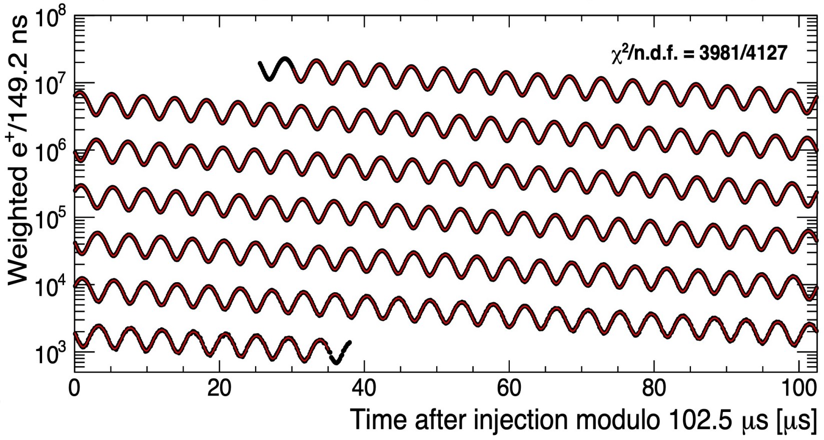
- ω_a corrections are small, but dominated Run-1 systematics: **500(93) ppb**
- For Run-2/3 they are **580(40) ppb**, dominated by C_e and C_p

ω_a systematics due to phase shifts

$$N(t) = N_0 e^{-t/\tau}\left[1 + A_\mu \cos(\omega_a t + \phi)\right]$$



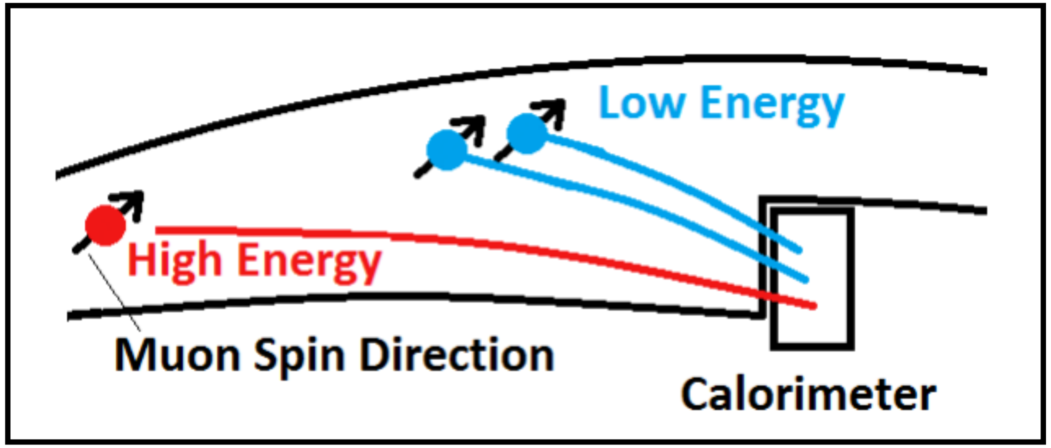
- If there are effects that **change the g-2 phase** of the detected e^+ over time
- These make us mis-measure ω_a with no indications that we're getting it wrong

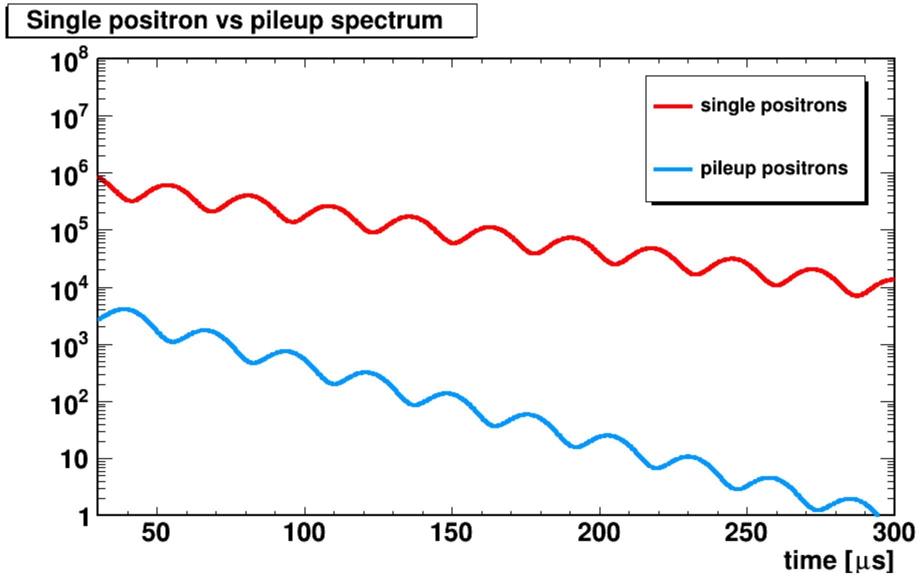
$$\begin{aligned} cos(\omega_a t + \phi(t)) &= cos(\omega_a t + \phi_0 + \phi' t + \ldots) \\ &= cos((\omega_a + \phi')t + \phi_0 + \ldots) \end{aligned}$$

- In general, anything that changes from **early-to-late** within each muon fill can be a cause of systematic error.
- Most phase shifts are eliminated by design or before fitting the data

ω_a systematics

- **Detector Effects**
  - Positron event pileups
  - Gain instability
- **Beam Dynamics**
  - Horizontal betatron motion
  - Vertical betatron motion
  - Beam de-bunching



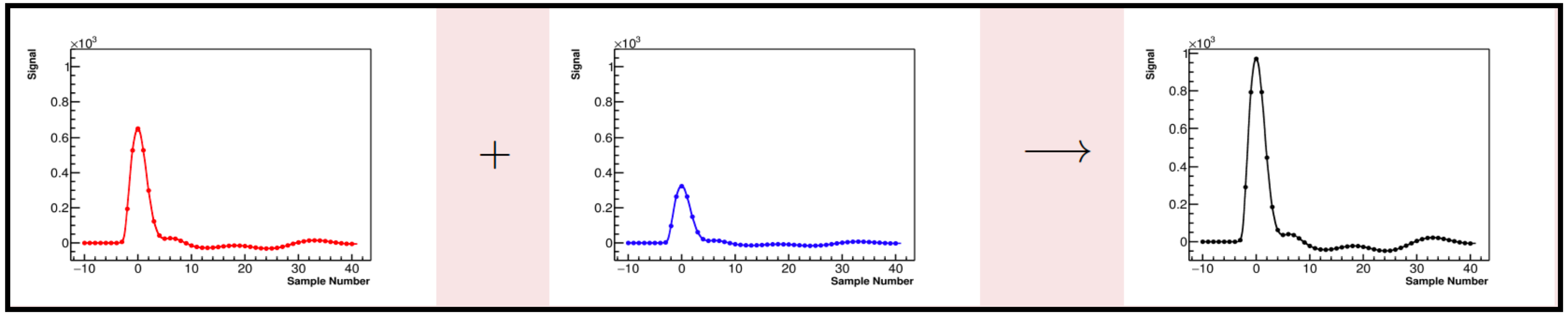


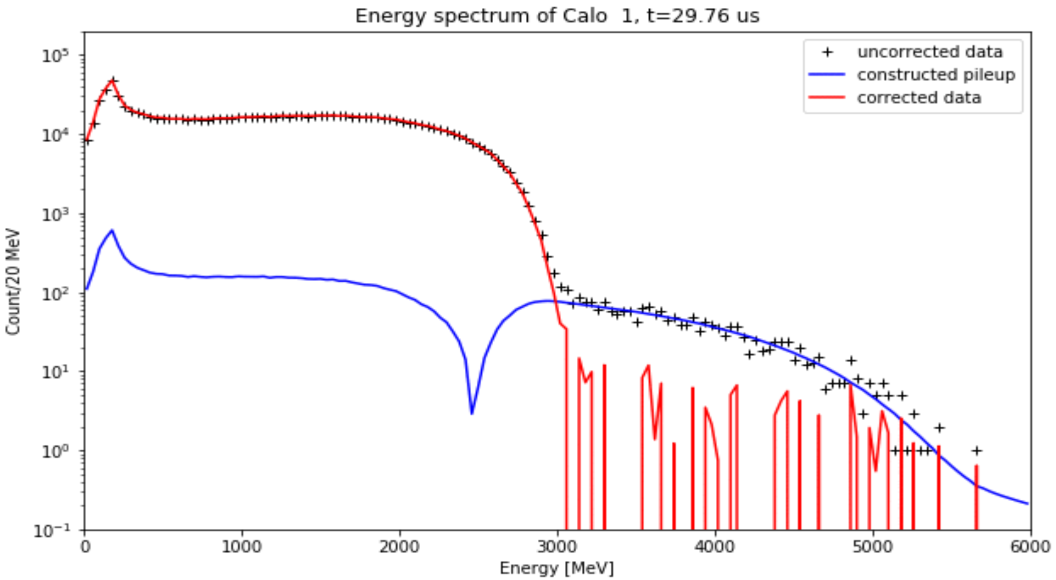
$$A_1 \cos(\omega t + \phi_1) + A_2 \cos(\omega t + \phi_2) = A_3 \cos(\omega t + \phi_3)$$

ω_a systematics

- **Detector Effects**
  - Positron event pileups
  - Gain instability
- **Beam Dynamics**
  - Horizontal betatron motion
  - Vertical betatron motion
  - Beam de-bunching

Artificially constructing pileup spectrum and removing it from the raw data





ω_a systematics

$$N(t) = N_0 e^{-t/\tau}\left[1 + A_\mu \cos(\omega_a t + \phi)\right]$$

- **Detector Effects**
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  - Gain instability
- **Beam Dynamics**
  - Horizontal betatron motion
  - Vertical betatron motion
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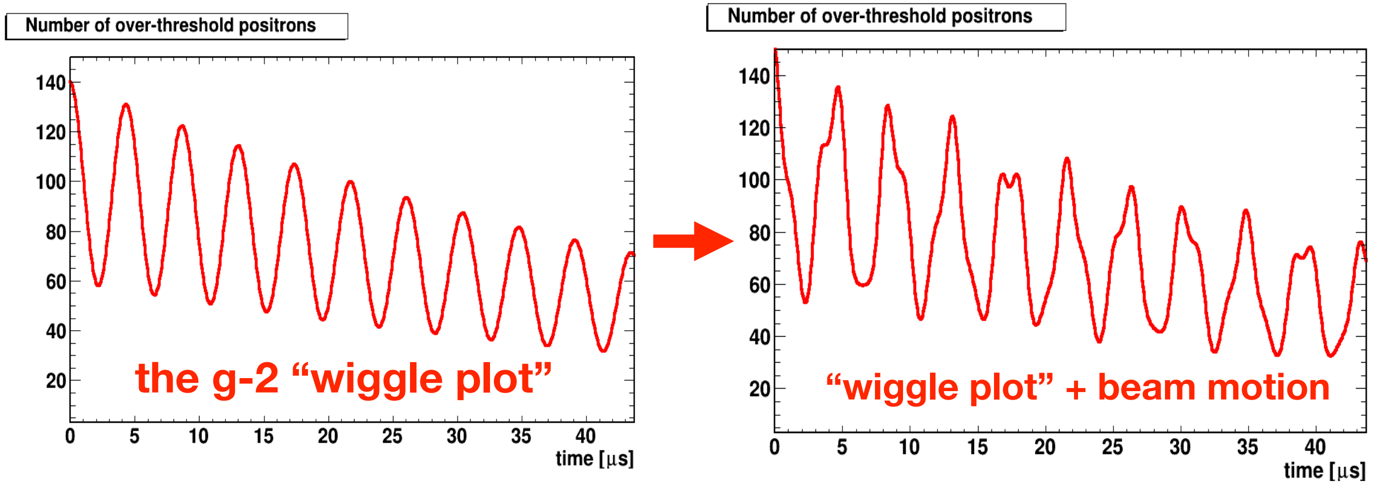
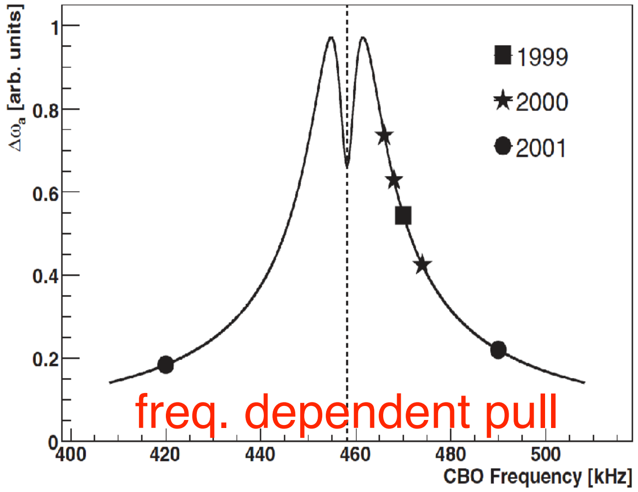
Detector acceptance effect



$$\boxed{C(t) = 1 - e^{-t/\tau_{\text{cbo}}} A_1 \cos(\omega_{\text{cbo}} t + \phi_1)}$$



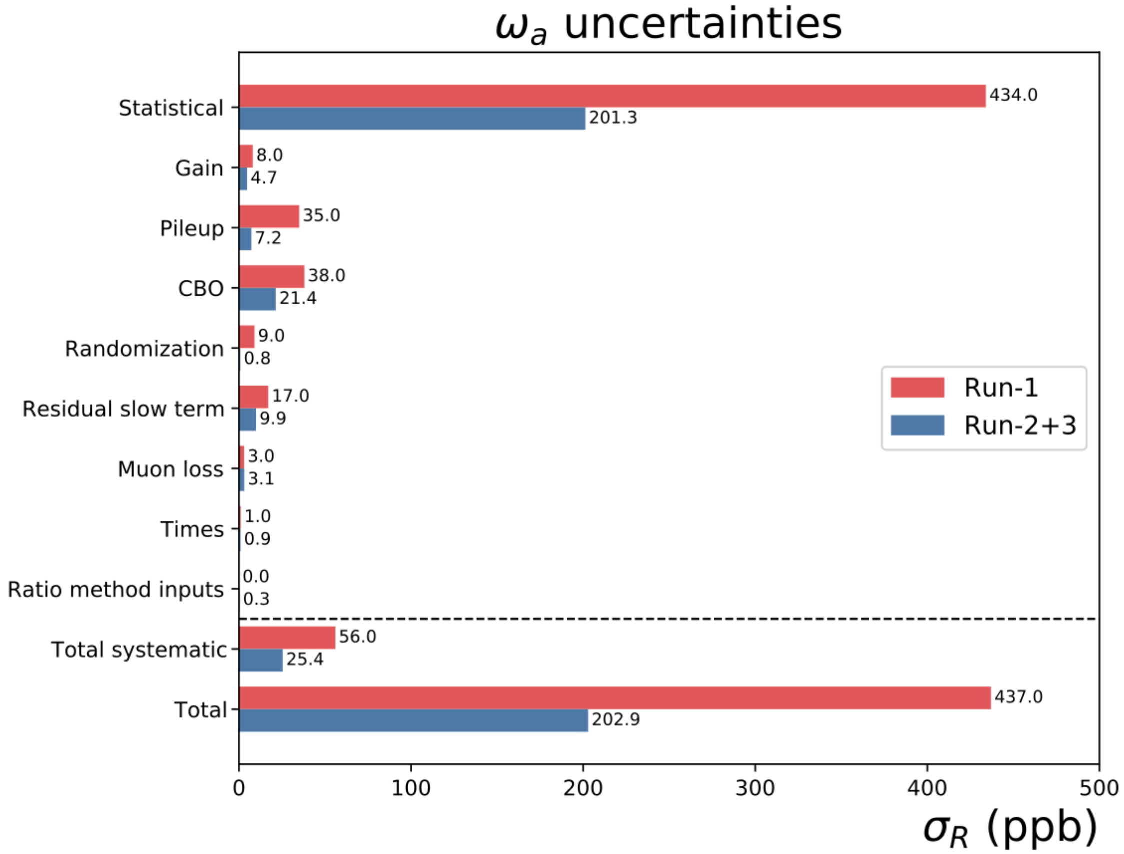
Summary of ω_a systematics for Run 2/3

- With respect to Run-1:
  - Statistical uncertainty reduced by **2.2x**
  - Systematic uncertainty reduced by **2.2x**
- Statistical uncertainty still completely dominates the final Run-2/3 error

TDR (2015) Run-2/3

Category	Syst goal [ppb]	Actual [ppb]	
Gain	20	5	✓
Pileup	40	7	✓
CBO	<30	21	
Lost muons	20	3	✓
Other	-	10	
Total	57	25	✓

KSK – Task force leader for reducing CBO systematics in Run 4/5/6



ω_a systematics due to phase shifts

$$\frac{\omega_a}{\omega_p} = \underbrace{\frac{\omega_a^m}{\omega_p^m}}_{\text{Measured Values}} \frac{1+\overbrace{C_e+C_p}^{\substack{\text{E-field \& vertical motion:}\\ \text{Spin precesses slower}\\ \text{than in basic equation}}}+\overbrace{C_{pa}+C_{dd}+C_{ml}}^{\substack{\text{Phase changes over each fill:}\\ \text{Phase-Acceptance, Differential}\\ \text{Decay, Muon Losses}}}}{1+\underbrace{B_k+B_q}_{\substack{\text{Transient Magnetic Fields:}\\ \text{Quad Vibrations,}\\ \text{Kicker Eddy Current,}}}}$$

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- These make us mis-measure ω_a with no indications that we're getting it wrong

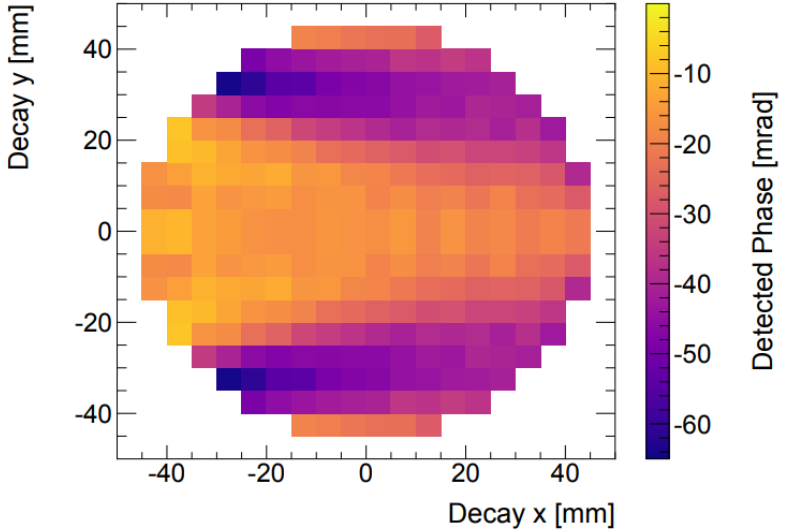
$$\begin{aligned} cos(\omega_a t + \phi(t)) &= cos(\omega_a t + \phi_0 + \phi' t + \ldots) \\ &= cos((\omega_a + \phi')t + \phi_0 + \ldots) \end{aligned}$$

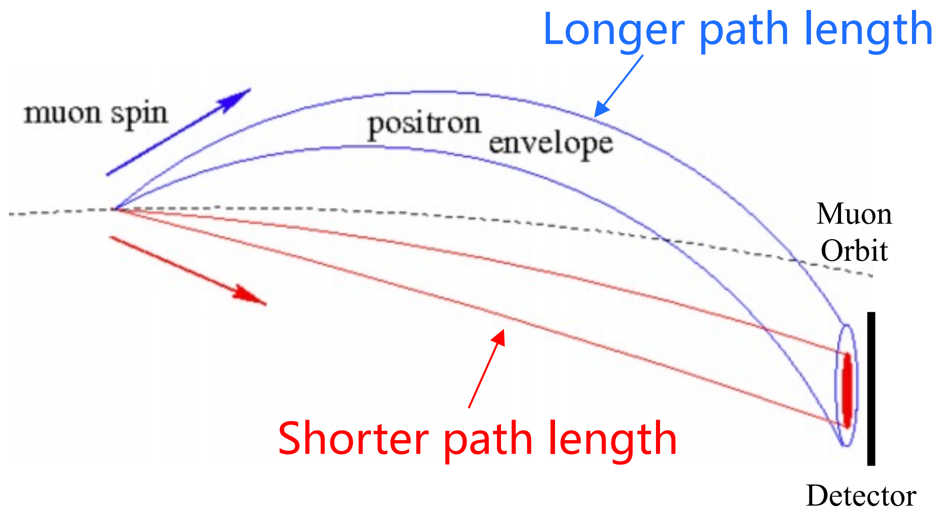
- In general, anything that changes from **early-to-late** within each muon fill can be a cause of systematic error.
- Most phase shifts are eliminated by design or before fitting the data, but we must correct for three effects **(C_{pa} , C_{ml} , C_{dd})**

g-2 phase and detector acceptance

- Average detected g-2 phase changes with decay position:

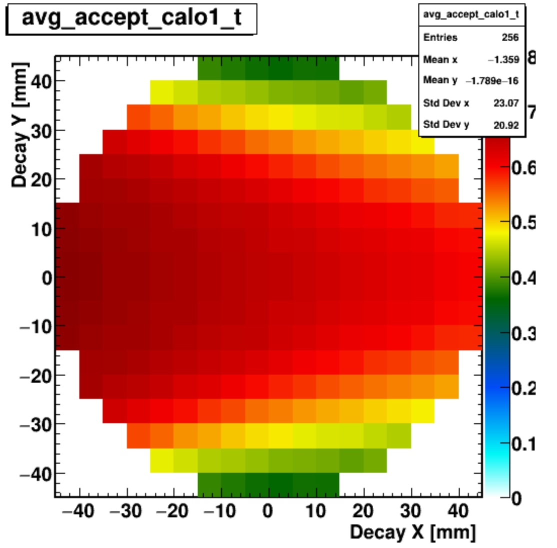
Phase map (Geant4)




Longer path length

Shorter path length

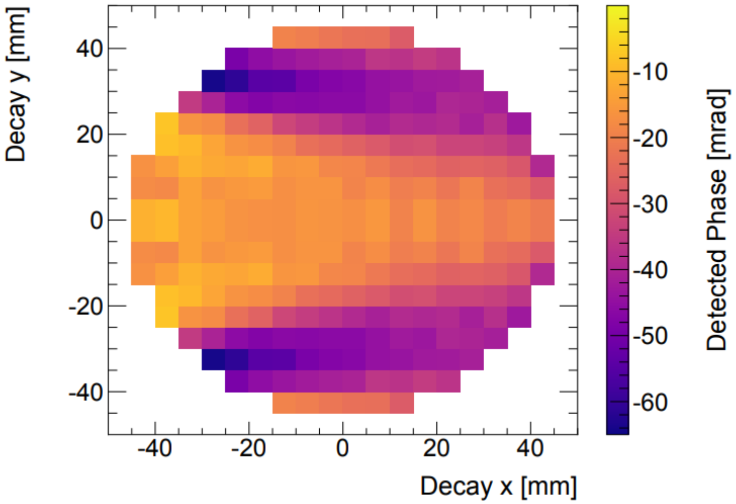
Acceptance map (Geant4)



- Origin is acceptance: if e^+ decays outwards then it will have a longer path length to a detector
- We see fewer events from top/bottom of storage region as they miss the detectors vertically

Beam instability and phase-acceptance

Phase map (Geant4)

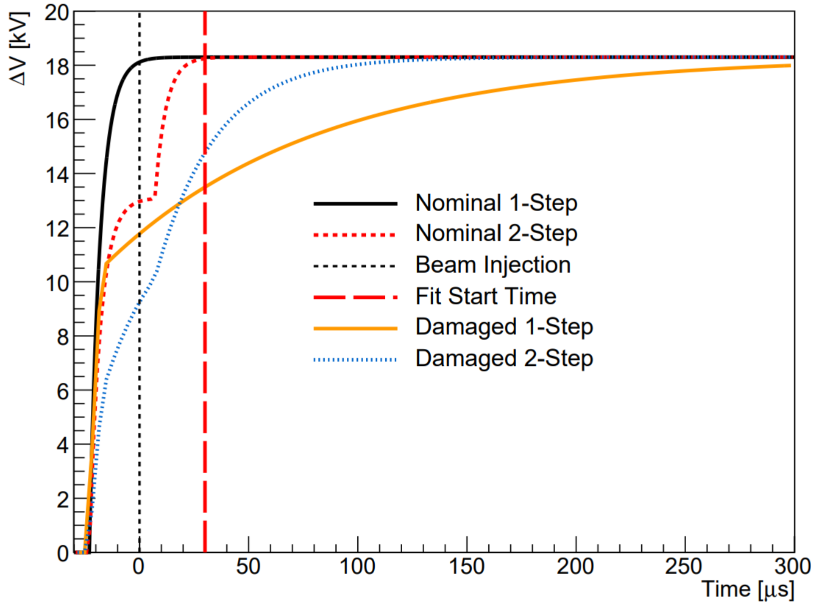


- Due to acceptance, ϕ depends on muon decay position (x,y)
- Not an issue if the muon distribution doesn't change shape over a fill

- But in Run 1, equipment failure led to beam instability
  - 2/32 quad HV resistors died

→ Focusing E-field changed

→ Beam width changed

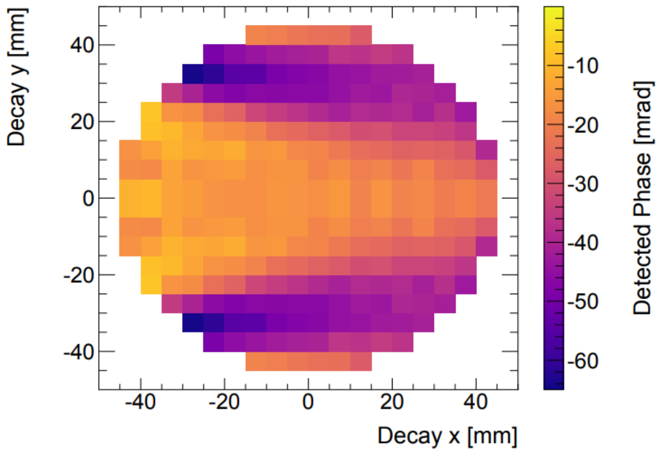


Phase versus time

$$\varphi^c_{pa}(t) = \arctan \frac{\sum_{ij} \overbrace{M^c(x_i,y_j,t)}^{\text{Decay distribution }(t)} \cdot \varepsilon^c(x_i,y_j)\cdot \overbrace{A^c(x_i,y_j)}^{\text{Asymmetry map}} \cdot \sin(\varphi^c_{pa}(x_i,y_j))}{\sum_{ij} M^c(x_i,y_j,t)\cdot \underbrace{\varepsilon^c(x_i,y_j)}_{\text{Acceptance map}}\cdot A^c(x_i,y_j)\cdot\cos(\underbrace{\varphi^c_{pa}(x_i,y_j)}_{\text{Phase map}})}$$

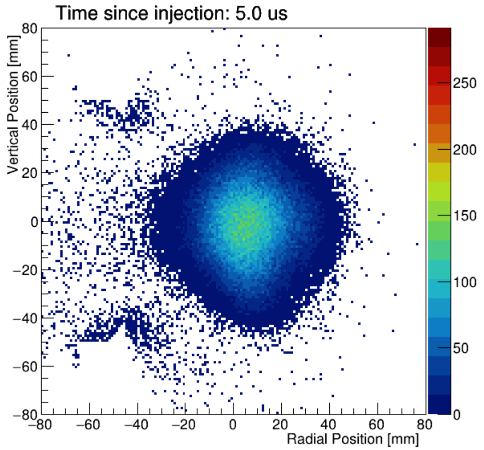
- The (azimuthally-averaged) muon position distribution does change over time.

Phase (x, y)



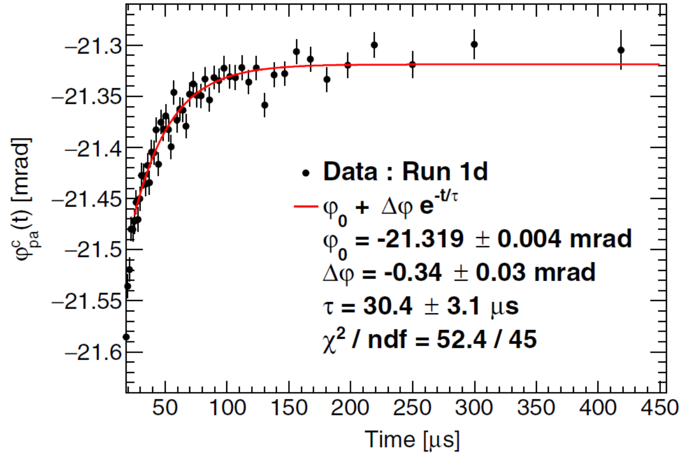
$\otimes$

Muon distribution



$=$

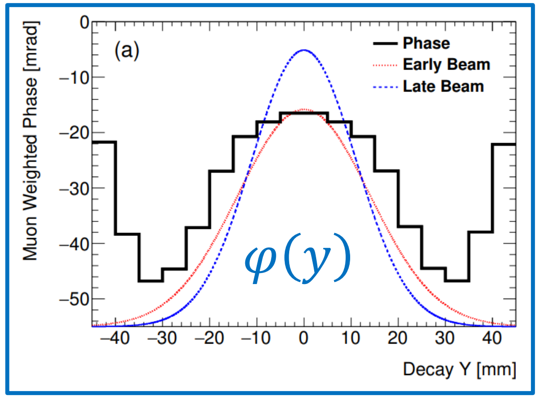
Phase (t)

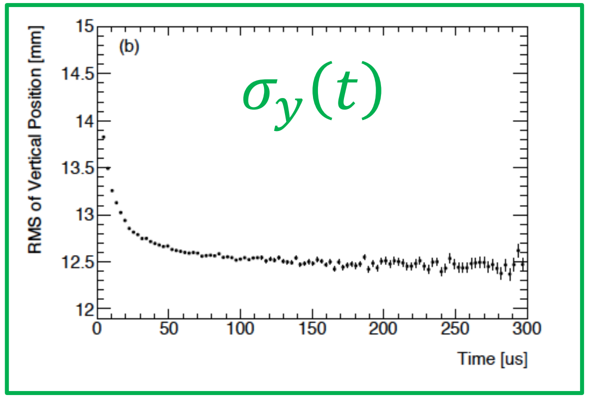


Phase-acceptance correction

- The dominant effect from Run-1 came from the early-to-late vertical distribution change.

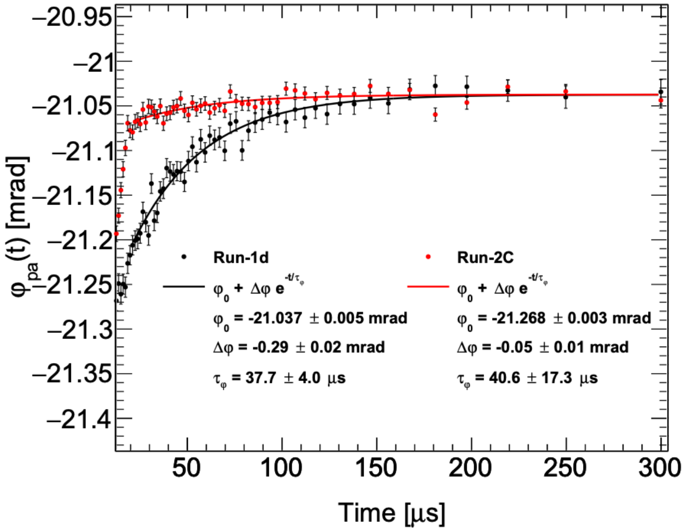
$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \boxed{\frac{\mathrm{d}\varphi}{\mathrm{d}\langle\sigma_y^2\rangle}}\boxed{\frac{\mathrm{d}\langle\sigma_y^2\rangle}{\mathrm{d}t}}$$





- The damaged resistors were replaced before Run-2.
- It significantly improved the beam early-to-late stability, and hence reduced the correction and its uncertainty

C_{pa}	**Correction [ppb]**	**Uncertainty [ppb]**
Run-1	-158	75
Run-2/3	-27	13



Spin/Phase-Momentum Correlations

- Phase changes due to the coupled effects from ϕ-$\langle p\rangle$ correlation & $\langle p\rangle$-t correlation.

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \boxed{\frac{\mathrm{d}\phi}{\mathrm{d}\langle p\rangle}}\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}$$

- Each correlation can be decomposed as follows:

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}\langle p\rangle}\right)_{bml}$$

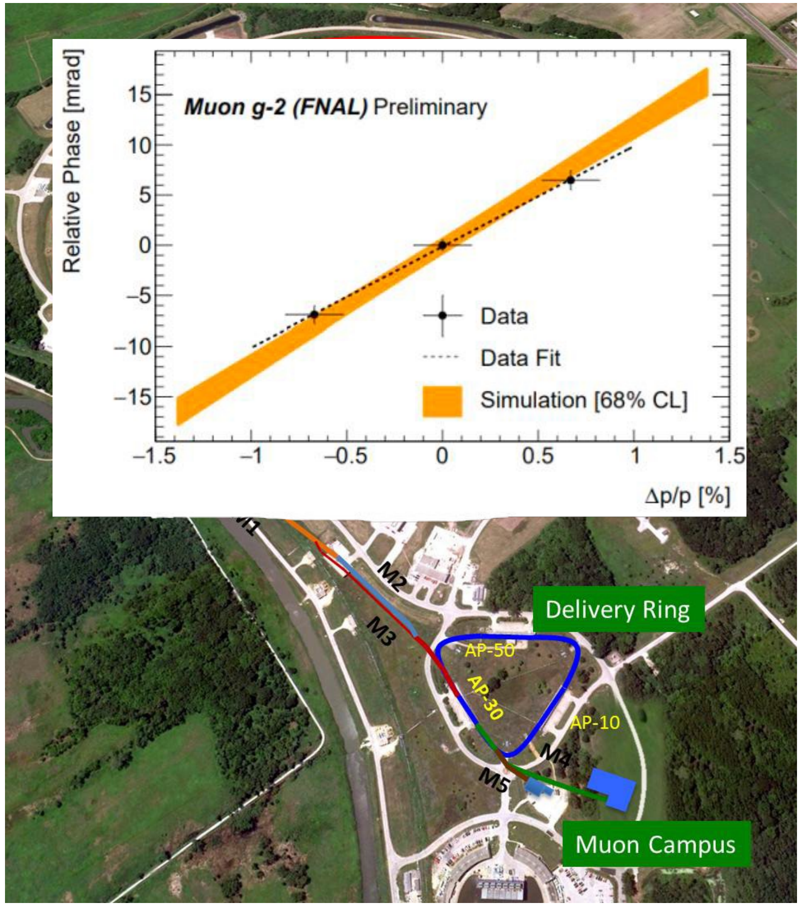
Beamline
Upstream delivery ring dipole bending magnet

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}\langle p\rangle}\right)_{p\text{-}x}$$

p - x/x'
Spin-Transverse coordinate correlation

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}\langle p\rangle}\right)_{p\text{-}t_0}$$

p - t_0
Head-to-tail phase difference &
Head-to-tail stored momentum distribution.



Momentum-Time Correlations

- Phase changes due to the coupled effects from ϕ-$\langle p\rangle$ correlation & $\langle p\rangle$-t correlation.

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\mathrm{d}\phi}{\mathrm{d}\langle p\rangle}\boxed{\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}}$$

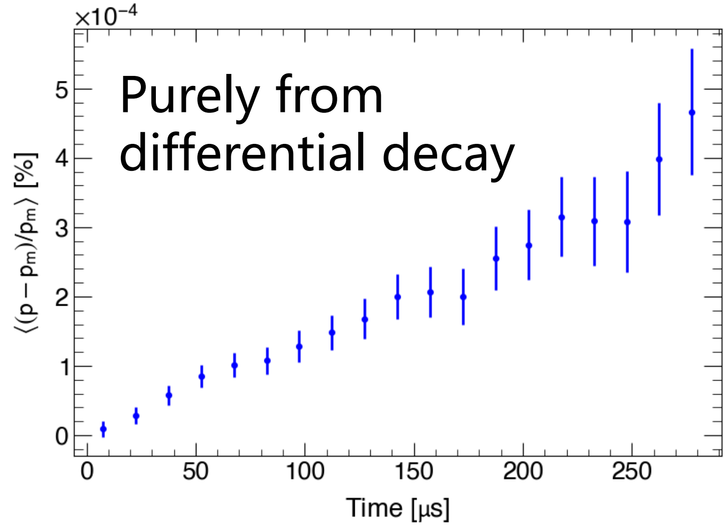
- Each correlation can be decomposed as follows:

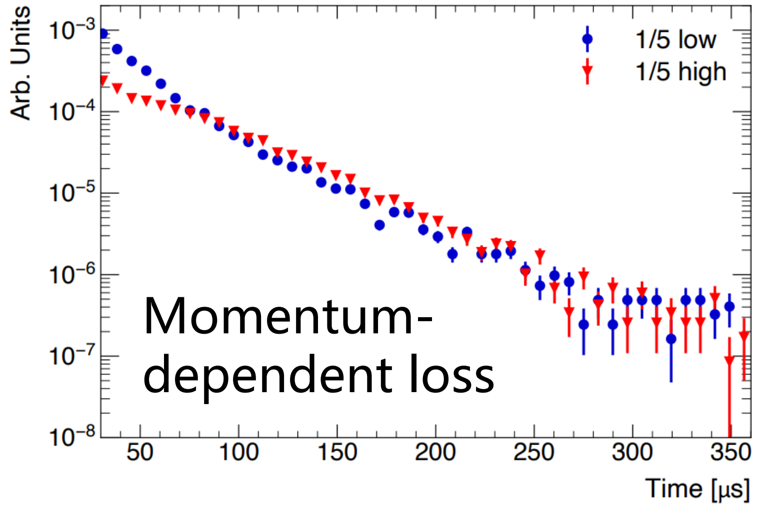
$$\left(\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}\right)_{dd}$$

C_{dd}: Differential decay
Muons have different lifetimes depending on their energies.

$$\left(\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}\right)_{ml}$$

C_{ml}: Momentum-dependent loss
Muon loss spectrum depends on the momentum.





C_{ml} and C_{dd} – Muon Losses Correction

- In Run-1, we neglected C_{dd}.
- We were at the early stage of understanding the p-x/x' and p-t_0 effects
- The beamline C_{dd} was negligible compared to C_{ml} which was enhanced due to the damaged resistors
- Dominant systematics comes from the bunch-by-bunch deviations in $\left(\frac{\mathrm{d}\phi}{\mathrm{d}\langle p\rangle}\right)_{p\text{-}t_0}$

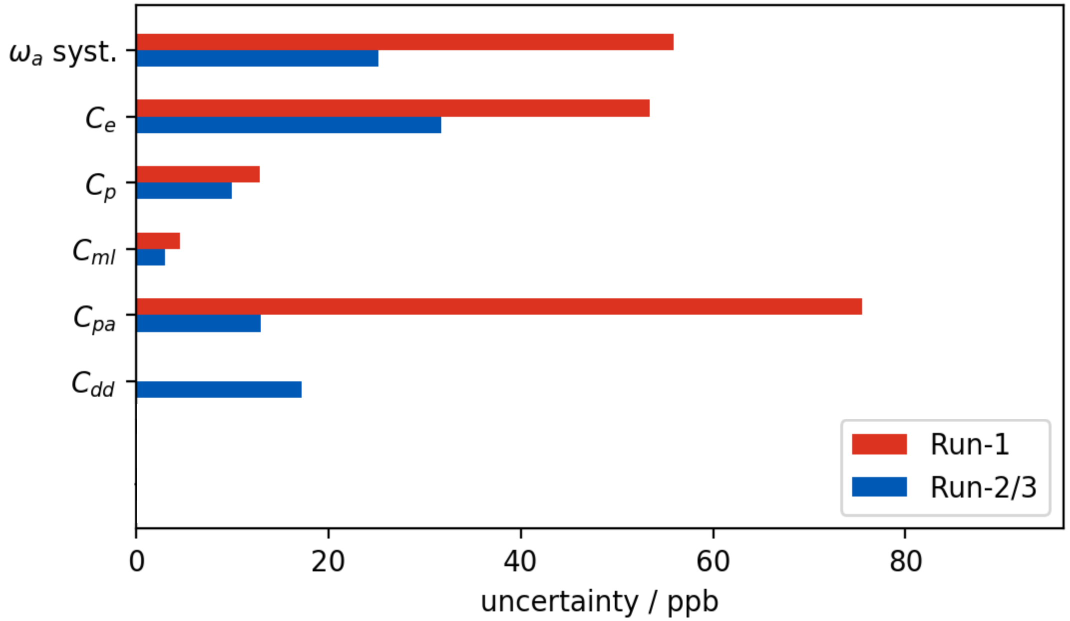
C_{dd}	**Correction [ppb]**	**Uncertainty [ppb]**
Run-1	0	0
Run-2/3	-15	17

C_{ml}	**Correction [ppb]**	**Uncertainty [ppb]**
Run-1	-11	5
Run-2/3	0	3

Summary of BD corrections

- Beam dynamics corrections to the anomalous spin precession frequency ω_a^m were reviewed
- Improvement to experimental conditions have significantly reduced the major systematics in these corrections
- The net uncertainty of the BD corrections was reduced by more than a factor of 2 in Run-2/3.

Corrections [ppb]	**Run-1**	**Run-2/3**
C_e	489 ± 53	451 ± 32
C_p	180 ± 13	170 ± 10
C_{pa}	-158 ± 75	-27 ± 13
C_{dd}	-	-15 ± 17
C_{ml}	-11 ± 5	0 ± 3
Sum	$\mathbf{500 \pm 93}$	$\mathbf{580 \pm 40}$



Summary and outlook

- Muon g-2 is now measured to 200 ppb using Run 1 to Run 3 of our collected data
  - Just published recently in PRL 131 (2023) 16, 161802
- Current systematics in precession frequency measurement have exceeded our expected goal of 70 ppb
  - 25 ppb vs 57 ppb in TDR
- There are still rooms for improvement for the beam dynamics corrections
  - 40 ppb vs 30 ppb in TDR
- Beam dynamics systematics are the limiting factor down the road
  - A very important aspect to be considered and studied in details for future g-2 experiments using magic momentum approach (HIAF?)

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