Extraction of information on transersity GPDs from π^0 and η production on EIC of China

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Based on arXiv: 2206.06547 and 2209.14493

Nov 11, 2023

Outline

This slide focus on the extraction of transversity GPDs from pseudoscalar meson production. It contains follow sections:

- Theoretical frame
- Compare with experimental data
- Meson production at EicC
- Transversity GPDs extracted from EicC
- Summary

Introduction to GPDs

Generalized Parton Distributions (GPDs) can be extracted from deep virtual Compton Scattering (DVCS), Time-like Compton Scattering (TCS) and Hard Exclusive Meson Production (HEMP) processes.

GPDs can be employed to study

- Spin puzzle
- Energy Momentum tensor
- Mass radius, distributions and pressure

Quark helicity conservation distributions

The quark helicity conservation distributions go with the Dirac matrix γ^+ and $\gamma^+\gamma_5$, where $i = 1, 2$ is a transverse index, it is defined as[EPJC-19-485]

$$\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\langle p', \lambda'|\bar{\psi}(-\frac{1}{2}z)\gamma^+\psi(\frac{1}{2}z)|P, \lambda\rangle|_{z^+=0, z_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p', \lambda')\left[H^q\gamma^+ + E^q\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}\right]u(p, \lambda). \quad (1)$$

$$\frac{1}{2}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\langle p', \lambda'|\bar{\psi}(-\frac{1}{2}z)\gamma^+\gamma_5\psi(\frac{1}{2}z)|P, \lambda\rangle|_{z^+=0, z_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p', \lambda')\left[\widetilde{H}^q\gamma^+\gamma_5 + \widetilde{E}^q\frac{\gamma_5\Delta^+}{2m}\right]u(p, \lambda). \quad (2)$$

H^q, E^q, $\widetilde{H}^q$ and $\widetilde{E}^q$ are quark helicity conservation distributions.

Quark helicity flip distributions

The quark helicity flip distributions go with the Dirac matrix σ^{+i}, where $i = 1, 2$ is a transverse index, it is defined as[EPJC-19-485]

$$\frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) i\sigma^{+i} \psi(\frac{1}{2}z) | P, \lambda \rangle |_{z^+=0, z_T=0}$$
$$= \frac{1}{2P^+} \bar{u}(p', \lambda') \Big[H_T^q i\sigma^{+i} + \widetilde{H}_T^q \frac{P^+\Delta^i - \Delta^+ P^i}{m^2}$$
$$+ E_T^q \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \widetilde{E}_T^q \frac{\gamma^+ P^i - P^+\gamma^i}{m} \Big] u(p, \lambda). \quad (3)$$

H_T^q, $\widetilde{H}_T^q$, E_T^q and $\widetilde{E}_T^q$ are quark helicity flip distributions.

Sum rules of GPDs

GPD connects parton distribution via $H(x,0,0) = xf(x)$. Hadron Form factor can be obtain from GPDs

$$\int dx H^q(x,\xi,t) = F_1^q(t), \qquad \int dx E_q(x,\xi,t) = F_2^q(t); \tag{4}$$

$$\int dx \tilde{H}^q(x,\xi,t) = G_A^q(t), \qquad \int dx \tilde{E}^q(x,\xi,t) = G_p^q(t). \tag{5}$$

Ji sum rules for the proton angular memonta

$$\int x dx (H^q(x,\xi,0) + E^q(x,\xi,0) = 2J^q. \tag{6}$$

$J_q = \frac{1}{2}\Delta q + L_q$. L_q is key quantity to solve the spin puzzle.

Pseudoscalar meson production diagram

HEMP can be adopted to study the GPDs via handbag approach. Employing the handbag approach, we can calculate the meson production in $\gamma^* + p$ scattering

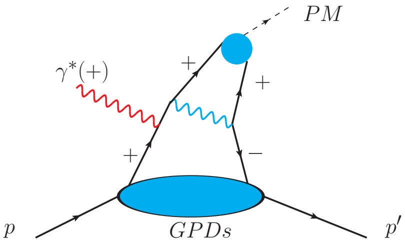

Figure 1: Diagram of PM production in handbag approach.

Differential cross section of meson

The unpolarized PM cross section can be decomposed into a number of partial cross sections which are observables of the process $\gamma^* p \to \pi^0 p$ [EPJC-65-137]. Two vector meson production can refer to [PLB-550-65].

$$\frac{d^2\sigma}{dt d\phi} = \frac{1}{2\pi}\left(\frac{d\sigma_T}{dt} + \varepsilon \frac{d\sigma_L}{dt} + \varepsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi \frac{d\sigma_{LT}}{dt}\right) \quad (7)$$

Here ε represents the ratio of fluxes of longitudinally and transversely polarized virtual photons $\varepsilon \approx \frac{1-y}{1-y+y^2/2}$. $y = Q^2/(x_B s)$ with $x_B = Q^2/(2pq)$.

Differential cross section of meson

The partial cross sections are expressed in terms of the $\gamma^* p \to \pi^0 p$ helicity amplitudes. When we omit small $M_{0-,-+}$ amplitude, they can be written as follows[EPJC-65-137]

$$
\begin{aligned}
\frac{d\sigma_L}{dt} &= \frac{1}{\kappa}[\mid M_{0+,0+} \mid^2 + \mid M_{0-,0+} \mid^2], \\
\frac{d\sigma_T}{dt} &= \frac{1}{2\kappa}(\mid M_{0-,++} \mid^2 + 2 \mid M_{0+,++} \mid^2), \\
\frac{d\sigma_{LT}}{dt} &= -\frac{1}{\sqrt{2}\kappa}\mathrm{Re}\left[M^*{}_{0-,++}M_{0-,0+}\right], \\
\frac{d\sigma_{TT}}{dt} &= -\frac{1}{\kappa} \mid M_{0+,++} \mid^2 .
\end{aligned}
\tag{8}
$$

where $\kappa = 16\pi(W^2 - m^2)\sqrt{\Lambda(W^2, -Q^2, m^2)}$. $\Lambda(x,y,z)$ is defined as $\Lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz$.

Scattering amplitudes

The amplitudes can be written as

$$
\begin{aligned}
M_{0-,0+} &= \frac{e_0}{Q}\frac{\sqrt{-t'}}{2m}\langle\tilde{E}\rangle,\\
M_{0+,0+} &= \sqrt{1-\xi^2}\frac{e_0}{Q}[\langle\tilde{H}\rangle - \frac{\xi^2}{1-\xi^2}\langle\tilde{E}\rangle],\\
M_{0-,++} &= \frac{e_0}{Q}\sqrt{1-\xi^2}\langle H_T\rangle,\\
M_{0+,++} &= -\frac{e_0}{Q}\frac{\sqrt{-t'}}{4m}\langle\tilde{E}_T\rangle,
\end{aligned}
\tag{9}
$$

where $e_0 = \sqrt{4\pi\alpha}$ with $\alpha = \frac{1}{137}$ is the fine structure constant. And ξ is defined as

$$
\xi = \frac{x_B}{2-x_B}(1+\frac{m_P^2}{Q^2}),\ \ t' = t - t_0,\ \ t_0 = -\frac{4m^2\xi^2}{1-\xi^2}. \tag{10}
$$

pseudoscalar meson production in electron-ion collider

The convolution function is calculated as

$$\langle H(\xi,t,Q^2)\rangle = \int dx H(x,\xi,t,Q^2)\mathscr{H}(x,\xi,t,Q^2). \tag{11}$$

$H(x,\xi,t,Q^2)$ is the GPD functions which is dependent on models and $\mathscr{H}(x,\xi,t,Q^2)$ is the hard part of the amplitude which can be calculated perturbatively. However, the factorization is not proven now.
$\langle\tilde{H}\rangle$ and $\langle\tilde{E}\rangle$ are the convolutions of twist-2 while $\langle H_T\rangle$ and $\langle\tilde{E}_T\rangle$ are the convolutions of twist-3.

Hard part of scattering amplitude in twist-2

The hard part is calculated employing the k-dependent wave function, describing the longitudinally polarized mesons. The twist-2 hard part is given as

$$\mathscr{H}^{\pi^0}_{0\lambda,0\lambda} = C_F\sqrt{\frac{2Q^2}{N_c\xi}}\int d\tau d^2b\phi_{\pi^0}(\tau,b)\alpha_s(\mu_R)e^{-S}[T_s - T_u]. \quad (12)$$

$\phi_M(\tau,b)$ is the wave function of the PM. S is the Sudakov factor and T_s is the propagator.

Propagator in twist-2

T_s and T_u are the propagators which includes Bessel functions. They are written as

$$T_s = -\frac{i}{4}H_0^{(1)}(\sqrt{(1-\tau)(x-\xi)/(2\xi)}bQ)\Theta(x-\xi) - \frac{1}{2\pi}K_0(\sqrt{(1-\tau)(\xi-x)/(2\xi)}bQ)\Theta(\xi-x). \quad (13)$$

and

$$T_u = -\frac{1}{2\pi}K_0(\sqrt{\tau(x+\xi)/(2\xi)}bQ). \quad (14)$$

Hard part of scattering amplitude in twist-3

The hard part is calculated employing the k-dependent wave function, describing the longitudinally polarized mesons. The twist-3 hard part is given as

$$\mathscr{H}^{\pi^0}_{0-,++} = \frac{8C_F}{\sqrt{2N_c}} \int d\tau d^2 b \phi_{\pi^0}(\tau, b) \alpha_s(\mu_R) e^{-S} \Big[\frac{-e_u}{x - \xi + i\varepsilon} \delta^2(b) + \frac{e_d}{x + \xi - i\varepsilon} \delta^2(b) + \frac{(1-\tau)Q^2}{2\xi} e_u T_s - \frac{\tau Q^2}{2\xi} e_d T_u \Big]. \quad (15)$$

$\phi_M(\tau, b)$ is the wave function of the PM in twist-3. S is the Sudakov factor and T_s is the propagator.

GPDs function definitions

The GPDs are constructed adopting the double distribution representation

$$F(x,\xi,t)=\int_{-1}^{1}d\rho\int_{-1+|\rho|}^{1-|\rho|}d\gamma\delta(\rho+\xi\gamma-x)\,\omega(\rho,\gamma,t), \tag{16}$$

F with PDFs h via the double distribution functions ω. For the valence quark double distribution functions, it is

$$\omega(\rho,\gamma,t)=h(\rho,t)\,\frac{3}{4}\,\frac{[(1-|\rho|)^2-\gamma^2]}{(1-|\rho|)^3}. \tag{17}$$

The t- dependence in PDFs $h(\rho,t)$ is expressed as the Regge form

$$h(\rho,t)=N\,e^{(b-\alpha'\ln\rho)t}\rho^{-\alpha(0)}\,(1-\rho)^{\beta}, \tag{18}$$

GPD functions

The H_T GPDs are connected with transversity PDFs as following

$$h_T(\rho,0)=\delta(\rho); \quad \text{and} \quad \delta(\rho)=N_T\,\rho^{1/2}\,(1-\rho)\,[q(\rho)+\Delta q(\rho)], \qquad (19)$$

The detail information of the transversity GPDs can be referred to [EPJC-73-2278].

π^0 and η amplitude

The flavor factors for π^0 production appear in combinations

$$F^{\pi^0} = \frac{1}{\sqrt{2}}(e_u F^u - e_d F^d). \quad (20)$$

Considering η, there are two states of η[EPJA-47-112]

$$F^{\eta 8} = \frac{1}{\sqrt{6}}(e_u F^u + e_d F_d) \quad (21)$$

$$F^{\eta 1} = \frac{1}{\sqrt{3}}(e_u F^u + e_d F_d). \quad (22)$$

Here the explicit values $e_u = 2/3$, $e_d = -1/3$ of quark charges will be adopted.

η amplitude

Calculation of the amplitudes of η production is based on the singlet-octet decomposition of η-state where the amplitude is presented in the form

$$M_\eta = \cos\theta_8 M^{(8)} - \sin\theta_1 M^{(1)}. \quad (23)$$

n the case if we omit the strange sea contribution which is small and can be neglected, the GPDs contribution to these amplitudes has a form

$$F^{(\eta 8)} = \frac{1}{3\sqrt{6}}(2F^u - F^d);\;\; F^{(\eta 1)} = \sqrt{2}F^{(\eta 8)} \quad (24)$$

We use the values of mixing angles and decay coupling constant from [PRD-58-114006].

$$\theta_8 = -21.2^\circ,\;\; \theta_1 = -9.2^\circ; f_8 = 1.26 f_\pi,\;\; f_1 = 1.17 f_\pi. \quad (25)$$

Parameters of the GK model

GPD	$\alpha(0)$	β^u	β^d	$\alpha'[\mathrm{GeV}^{-2}]$	$b[\mathrm{GeV}^{-2}]$	N^u	N^d
$\widetilde{E}$	0.48	5	5	0.45	0.9	14.0	4.0
$\bar{E}_T$	0.3	4	5	0.45	0.5	6.83	5.05
H_T	-	-	-	0.45	0.3	1.1	-0.3

Table 1: Regge parameters and normalizations of the GPDs, at a scale of $2\,\mathrm{GeV}$. Model I.

Parameters of the GK model

GPD	$\alpha(0)$	$\alpha'[\mathrm{GeV}^{-2}]$	$b[\mathrm{GeV}^{-2}]$	N^u	N^d
$\widetilde{E}_{\mathrm{n.p.}}$	0.32	0.45	0.6	18.2	5.2
$\bar{E}_T$	-0.1	0.45	0.67	29.23	21.61
H_T	-	0.45	0.04	0.68	-0.186

Table 2: Regge parameters and normalizations of the GPDs at a scale of $2\,\mathrm{GeV}$. Model II.

Parameters of the GK model

GPD	$\alpha(0)$	β^u	β^d	$\alpha'[\mathrm{GeV}^{-2}]$	$b[\mathrm{GeV}^{-2}]$	N^u	N^d
$\widetilde{E}$	0.48	5	5	0.45	0.9	14.0	4.0
$\bar{E}_T$	-0.1	4	5	0.45	0.77	20.91	15.46
H_T	-	-	-	0.45	0.3	1.1	-0.3

Table 3: Regge parameters and normalization of the GPDs, at a scale of $2\,\mathrm{GeV}$. Model III.

π^0 production at CLAS

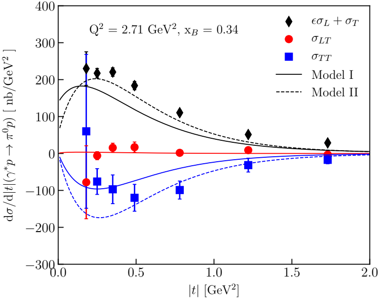

Figure 2: Cross section of π^0 production in the CLAS energy range together with the data [PRC-90-025205]. Black lines describe $\sigma = \sigma_T + \varepsilon\sigma_L$, red lines represent σ_{LT}, blue lines depict σ_{TT}.

π^0 production at COMPASS

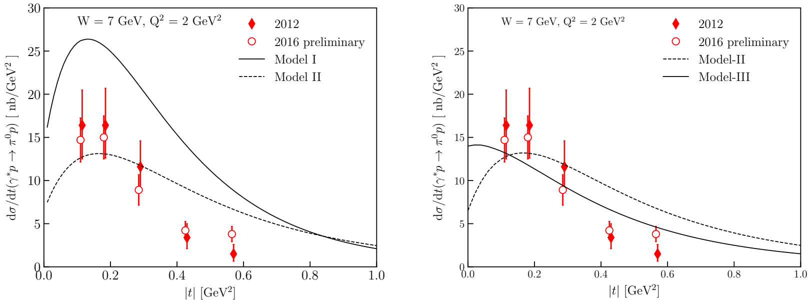

Figure 3: Models results at COMPASS kinematics. Experimental data are from COMPASS[PLB-805-135454].

η production at CLAS

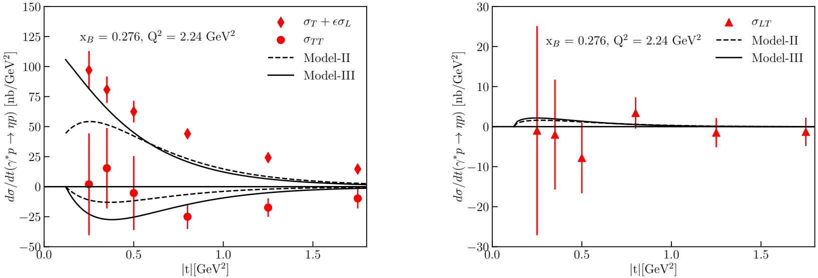

Figure 4: Cross section of η production in the CLAS energy range together with the data[PRC-95-035202]. Left graph is for σ and σ_{TT} while right graph is for σ_{LT}.

π^0 production at EicC

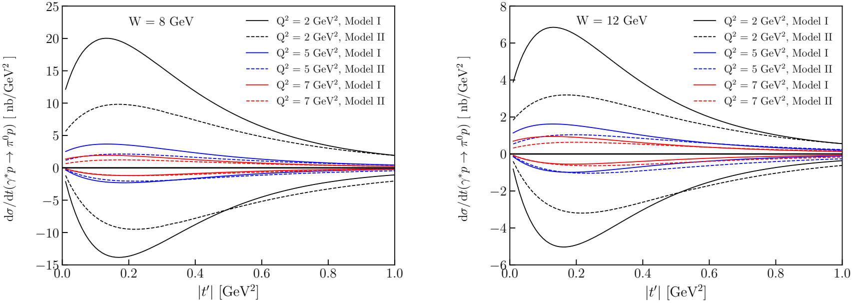

Figure 5: Cross section of π^0 production in EicC energy range, Lines in upper part describe $\sigma = \sigma_T + \varepsilon\sigma_L$ while lines in down part depict σ_{TT}.

η production at EicC

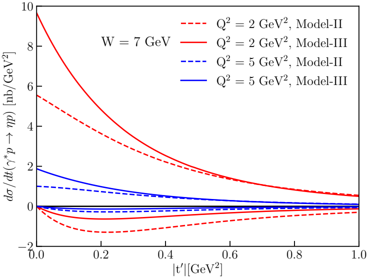

Figure 6: Cross section of η production in the EicC energy ranges. Lines in upper part describe $\sigma = \sigma_T + \varepsilon\sigma_L$ while lines in down part depict σ_{TT}.

Extraction of GPD from meson cross section

On the other hand, we can extract convolution function from PM cross sections.

$$
\begin{aligned}
|M_{0+++}| &= \sqrt{-\kappa\frac{\mathrm{d}\sigma_{TT}}{\mathrm{d}t}}, \\
|M_{0-++}| &= \sqrt{2\kappa(\frac{\mathrm{d}\sigma_{T}}{\mathrm{d}t}+\frac{\mathrm{d}\sigma_{TT}}{\mathrm{d}t})},
\end{aligned} \tag{26}
$$

Using the relationship, we can obtain the convolution functions.

$$
\begin{aligned}
M_{0-,++} &= \frac{e_0}{Q}\sqrt{1-\xi^2}\langle H_T\rangle, \\
M_{0+,++} &= -\frac{e_0}{Q}\frac{\sqrt{-t'}}{4m}\langle \tilde{E}_T\rangle,
\end{aligned} \tag{27}
$$

Transversity GPDs from π^0 cross section

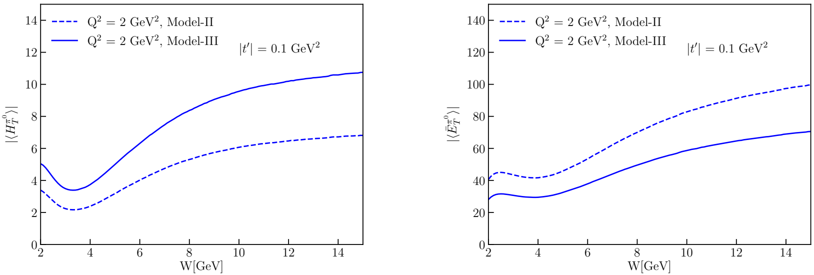

Figure 7: GPDs extracted from .

Transversity GPDs from η cross section

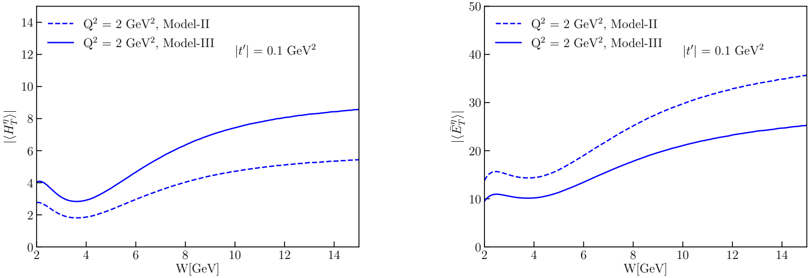

Figure 8: GPDs extracted from .

Summary

We can conclude following conclusions:

- π^0 and η production can be employed to study transversity GPDs.
- We confirm that π^0 and η transversity dominance $\sigma_T \gg \sigma_L$, observed at low CLAS energies is valid up to EicC energies range.
- Results of this work can be applied in future EicC experiments to give additional essential constraints on transversity GPDs at EicC energies range.
- EicC will be good perform to study transversity GPDs adopting HEMP at the future.

Thanks for your attentions !