CP-VIOLATING OBSERVABLES IN FOUR-BODY B DECAYS

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Based on 2103.00642, 2107.10684, 2112.10939, and 2204.01092.

第5届粒子物理天问论坛，2023/11/11@湖南师范大学

OUTLINE

- MOTIVATION
- FRAMEWORK FOR THE FOUR-BODY DECAYS
- RESULTS AND DISCUAAIONS
- SUMMARY AND OUTLOOK

MOTIVATION

- Charge conjugation (C), parity (P), and time reversal (T) are three fundamental discrete transformations in nature.

- CP violation (CPV) is a key requirement for generating the matter-antimatter asymmetry in the universe.
  [REV. MOD. PHYS.76 (2003) 1]

- In SM, CPV is attributed to an irreducible phase in the CKM quark-mixing matrix.

- CPVs in K-, B-, D-meson have been confirmed by experiments.
  Phys. Rev. Lett. 13(1964) ; Belle, Phys. Rev. Lett.(2001) LHCb; Phys. Rev. Lett.(2019)

- First evidence (3.3σ) for CPV in $\Lambda_b \to p\pi^+\pi^-\pi^-$ decays.
  [LHCb, Nature Physics 13 (2017) 391]

- T-odd triple productstake the form $(\vec{p}_a \times \vec{p}_b)\cdot\vec{p}_c$ which arises from $\epsilon_{\mu\nu\rho\sigma} p_a^\mu p_b^\nu p_c^\rho p_d^\sigma$ contraction. It is also CP-odd.

- Triple-product asymmetries (TPAs) may reveal potential signals of CPV. *[Valencia, 1989; Datta, London, 2003]*

$$A_T \equiv \frac{\Gamma(TP>0)-\Gamma(TP<0)}{\Gamma(TP>0)+\Gamma(TP<0)}.$$

- Compare this asymmetry with a corresponding quantity in the CP conjugate process to obtain the "true" CPV signal.

$$A_T(true) \propto \sin(\phi)\cos(\delta)$$

- The construction of a scalar triple product requires a final state with at least four particles, e.g. four-body decays.

- D, B mesons and b-baryons.

Four-body B meson decays are rich in CPV phenomena in the quark sector.

- Rich resonance structures.
- Interference between different resonances might induce lager CPV.
- Angular analysis could provide a lot of meaningful CP asymmetries.

Several Collaborations have observed B meson decays into various four-body charmless hadronic final states in certain two body invariant mass regions.

- ✓ $B_s \to (K\pi)(K\pi)$, $JHEP03(2018)140$
- ✓ $B_{(s)} \to (K\pi)(K\pi)$, $JHEP07(2019)032$
- ✓ $B^0 \to (\pi\pi)(K\pi)$, $JHEP05(2019)026$
- ✓ $B_s \to (KK)(K\pi)$, $JHEP11(2013)092$
- ✓ $B_s \to (KK)(KK)$, $JHEP12(2019)155$

Opportunities for CPV searches but also modelling challenges

- Receive both resonant and nonresonant contributions.
- Suffer substantial final-state interactions.
- More complicated strong dynamics than two-body ones.
- A factorization formalism in full phase space is not yet available.

PQCD successfully predicted CPV in the two-body B meson decays

$B \to \pi\pi, K\pi$ [Keum, H.n.Li, Sanda, 2000; C.D.Lu, Ukai, M.Z.Yang, 2000]

In the multi-body sector

- The leading-power regions of a Dalitz plot.

  [C.-H. Chen and H.-N. Li, PLB 561 (2003) 258]
- Quasi-two-body mechanism.

  [NPB 899 (2015) 247,NPB 555 (1999) 231]
- Two-meson distribution amplitudes (TMDAs).

  [Sov. J. Nucl. Phys. 38 (1983) 289, PRD 91 (2015) 094024,PLB 763 (2016) 29…]
- The scenario has been successful applied to three-body decay, encourages us to extended it to the four-body ones.

FRAMEWORK

Based on the quasi-two-body mechanism, four-body processes are assumed to proceed dominantly with two intermediate resonances.

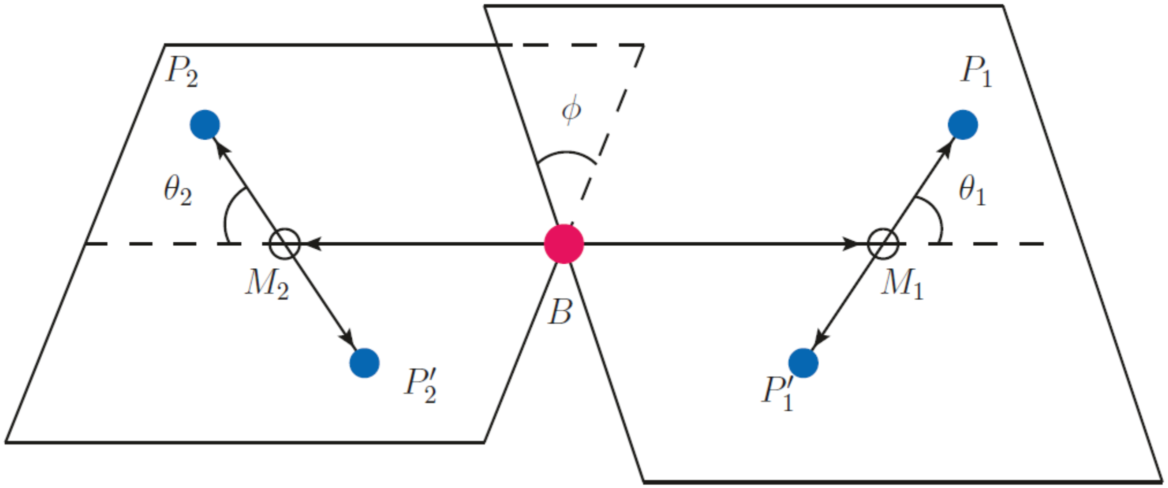


Figure 1. Helicity angles θ_1, θ_2 and ϕ for the $B \to M_1 M_2$ decay, with each intermediate resonance decaying into two pseudoscalars, $M_1 \to P_1 P_1'$ and $M_2 \to P_2 P_2'$.

A decay amplitude is written as

TMDAs

$$A \propto \Phi_B \otimes H \otimes \Phi_{P_1 P_1'} \otimes \Phi_{P_2 P_2'},$$

$\Phi_{PP'}$ absorbs the nonperturbative dynamics involved in the meson pairs.

- **S-wave:** $$\Phi_S(x,\omega) = \frac{1}{\sqrt{2N_c}}[\not p \phi_S(x,\omega) + \omega\phi_S^s(x,\omega) + \omega(\not n \not v - 1)\phi_S^t(x,\omega)],$$

$$\phi^0_{(PP')_S}(x,\omega) = \begin{cases} \frac{9F_{(PP')_S}(\omega)}{\sqrt{2N_c}} a_{PP'} x(1-x)(1-2x), & PP' = K\bar{K}, \\ \frac{3F_{(PP')_S}(\omega)}{\sqrt{2N_c}} x(1-x)\left[\frac{1}{\mu_S} + B_1 3(1-2x) + B_3 \frac{5}{2}(1-2x)(7(1-2x)^2-3)\right], & PP' = K\pi, \end{cases}$$

$$\phi^s_{(PP')_S}(x,\omega) = \frac{F_{(PP')_S}(\omega)}{2\sqrt{2N_c}},$$

$$\mu_S = \omega/(m_s - m_d)$$

$$\phi^t_{(PP')_S}(x,\omega) = \frac{F_{(PP')_S}(\omega)}{2\sqrt{2N_c}}(1-2x),$$

The parametrization of S-wave time-like form factors $F_{PP'}$:

$f_0(980)$: Flatté parametrization [Phys. Lett. B63, 228 (1976)]

$K_0^*(1430)$: LASS line shape [Nucl. Phys. B296, 493 (1988)]

The Gegenbauer moments are chosen to be the same as those of the corresponding scaler mesons.

- **P-wave:**

$$\Phi_P^L(x,\zeta,\omega) = \frac{1}{\sqrt{2N_c}}\left[\omega \not\epsilon_p \phi_P(x,\omega) + \omega\phi_P^s(x,\omega) + \frac{\not p_1 \not p_1' - \not p_1' \not p_1}{\omega(2\zeta-1)}\phi_P^t(x,\omega)\right](2\zeta-1),$$
$$\Phi_P^T(x,\zeta,\omega) = \frac{1}{\sqrt{2N_c}}\left[\gamma_5 \not\epsilon_T \not p \phi_P^T(x,\omega) + \omega\gamma_5\not\epsilon_T \phi_P^a(x,\omega) + i\omega\frac{\epsilon^{\mu\nu\rho\sigma}\gamma_\mu \epsilon_{T\nu} p_\rho n_{-\sigma}}{p\cdot n_-}\phi_P^v(x,\omega)\right]$$
$$\times\sqrt{\zeta(1-\zeta)+\alpha}, \qquad (2.20)$$

$$\phi_P(x,\omega) = \frac{3F_{K\pi}^{\parallel}(\omega^2)}{\sqrt{2N_c}}x(1-x)\left\{1 + a_{1K^*}^{\parallel}3(1-2x) + a_{2K^*}^{\parallel}\frac{3}{2}(5(1-2x)^2-1)\right\},$$
$$\phi_P^s(x,\omega) = \frac{3F_{K\pi}^{\perp}(\omega^2)}{2\sqrt{2N_c}}(1-2x),$$
$$\phi_P^t(x,\omega) = \frac{3F_{K\pi}^{\perp}(\omega^2)}{2\sqrt{2N_c}}(1-2x)^2,$$

$$F_P^{\perp}/F_P^{\parallel} \sim f_V^T/f_V$$

The parametrization of P-wave time-like form factors $F_{PP'}^{\parallel}$:

$K^*(892), \varphi(1020)$: Relativistic Breit-Wigner model.

The Gegenbauer moments have been fitted from a global analysis of three-body charmless hadronic decays in PQCD.

[Ya Li et al, PHYSICAL REVIEW D 104, 096014 (2021)]

FEYNMAN DIAGRAMS

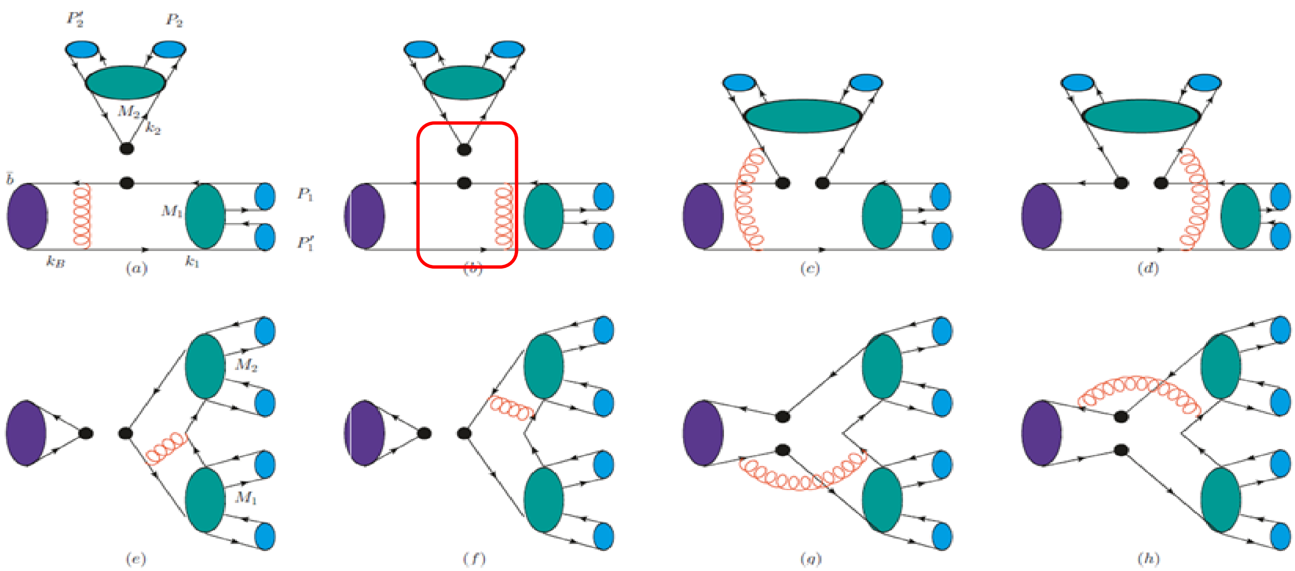


Figure 2. Leading-order diagrams for the $B \to M_1 M_2 \to P_1 P_1' P_2 P_2'$ decays, where the symbol • denotes a weak vertex, and the cyan oval represents either a scalar or vector resonance.

- The hard amplitude involves six external on shell quarks, four of which correspond to the four-fermion operators and two of which are the spectator quarks in the initial and final states.
- The hard kernels start at α_s in the PQCD approach.

Kinematics including the final state masses

$$B(p_B) \to M_1(p)M_2(q) \to P_1(p_1)P_1'(p_1')P_2(p_2)P_2'(p_2'),$$

In the rest frame of the B meson in the light-cone coordinates $p_B = (M/\sqrt{2})(1, 1, \mathbf{0}_T)$

$$p = \frac{M}{\sqrt{2}}(g^+, g^-, \mathbf{0}_T), \quad q = \frac{M}{\sqrt{2}}(f^-, f^+, \mathbf{0}_T), \quad p^2 = \omega_1^2,\, q^2 = \omega_2^2$$

$$\epsilon_p = \frac{1}{\sqrt{2\eta_1}}(g^+, -g^-, \mathbf{0}_T), \quad \epsilon_q = \frac{1}{\sqrt{2\eta_2}}(-f^-, f^+, \mathbf{0}_T), \quad \eta_{1,2} = \omega_{1,2}^2/M^2$$

The meson momentum fractions up to corrections from the meson masses

$$\frac{p_1^+}{p^+} = \zeta_1 + \frac{r_1 - r_1'}{2\eta_1}, \quad \frac{p_2^-}{q^-} = \zeta_2 + \frac{r_2 - r_2'}{2\eta_2}, \qquad r_i^{(\prime)} = m_i^{(\prime)2}/M^2$$

The momenta of four final-state mesons:

$$p_1 = \left(\frac{M}{\sqrt{2}}\left(\zeta_1 + \frac{r_1 - r_1'}{2\eta_1}\right) g^+, \frac{M}{\sqrt{2}}\left(1-\zeta_1 + \frac{r_1 - r_1'}{2\eta_1}\right) g^-, \mathbf{p}_T\right),$$

$$p_1' = \left(\frac{M}{\sqrt{2}}\left(1-\zeta_1 - \frac{r_1 - r_1'}{2\eta_1}\right) g^+, \frac{M}{\sqrt{2}}\left(\zeta_1 - \frac{r_1 - r_1'}{2\eta_1}\right) g^-, -\mathbf{p}_T\right),$$

$$p_2 = \left(\frac{M}{\sqrt{2}}\left(1-\zeta_2 + \frac{r_2 - r_2'}{2\eta_2}\right) f^-, \frac{M}{\sqrt{2}}\left(\zeta_2 + \frac{r_2 - r_2'}{2\eta_2}\right) f^+, \mathbf{q}_T\right),$$

$$p_2' = \left(\frac{M}{\sqrt{2}}\left(\zeta_2 - \frac{r_2 - r_2'}{2\eta_2}\right) f^-, \frac{M}{\sqrt{2}}\left(1-\zeta_2 - \frac{r_2 - r_2'}{2\eta_2}\right) f^+, -\mathbf{q}_T\right),$$

$$|\mathbf{p}_T|^2 = \omega_1^2[\zeta_1(1-\zeta_1)+\alpha_1],$$

$$|\mathbf{q}_T|^2 = \omega_2^2[\zeta_2(1-\zeta_2)+\alpha_2]$$

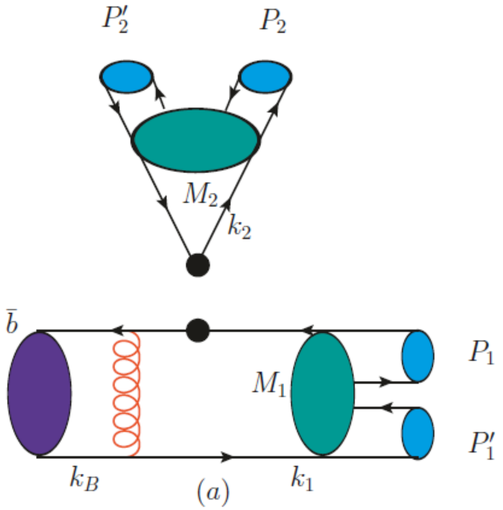
$$\alpha_i = \frac{(r_i - r_i')^2}{4\eta_i^2} - \frac{r_i + r_i'}{2\eta_i}.$$

Three valence quarks momenta are parametrized as

$$k_B = (0, x_B p_B^-, \mathbf{k}_{BT})$$

$$k_1 = (x_1 p^+, 0, \mathbf{k}_{1T})$$

$$k_2 = (0, x_2 q^-, \mathbf{k}_{2T})$$



The differential rate for the decay

$$\frac{d^5\Gamma}{d\Omega} = \frac{k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)}{16(2\pi)^6 M^2}|A|^2, \qquad \text{with } \Omega \equiv \{\theta_1, \theta_2, \phi, \omega_1, \omega_2\}$$

$$k(\omega_1,\omega_2) = \frac{\sqrt{[M^2-(\omega_1+\omega_2)^2][M^2-(\omega_1-\omega_2)^2]}}{2M}, \qquad k(\omega) = \frac{\sqrt{[\omega^2-(m_K+m_\pi)^2][\omega^2-(m_K-m_\pi)^2]}}{2\omega};$$

The relations between zeta and theta:

$$2\zeta_i - 1 = \sqrt{1+4\alpha_i}\cos\theta_i, \qquad \zeta_i \in \left[\frac{1-\sqrt{1+4\alpha_i}}{2}, \frac{1+\sqrt{1+4\alpha_i}}{2}\right]$$

The differential rate read as

$$\frac{d^5\Gamma}{d\zeta_1 d\zeta_2 d\omega_1 d\omega_2 d\phi} = \frac{k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)}{4(2\pi)^6 M^2\sqrt{1+4\alpha_1}\sqrt{1+4\alpha_2}}|A|^2.$$

Six helicity amplitudes:

$$A_{VV}: B_s^0 \to \bar{K}^{*0}(\to K^-\pi^+)K^{*0}(\to K^+\pi^-),$$
$$A_{VS}: B_s^0 \to \bar{K}^{*0}(\to K^-\pi^+)(K^+\pi^-)_0,$$
$$A_{SV}: B_s^0 \to \overline{(K^-\pi^+)}_0 K^{*0}(\to K^+\pi^-),$$
$$A_{SS}: B_s^0 \to \overline{(K^-\pi^+)}_0 (K^+\pi^-)_0,$$

$$A_0, A_\parallel, \boxed{A_\perp} \to \text{CP-odd}$$

$$A_{S-} = \frac{A_{SV} - A_{VS}}{\sqrt{2}}, \quad \boxed{A_{S+} = \frac{A_{VS} + A_{SV}}{\sqrt{2}},}$$

The full decay amplitude:

$$\begin{aligned} A = & \frac{2\zeta_1 - 1}{\sqrt{1+4\alpha_1}} \frac{2\zeta_2 - 1}{\sqrt{1+4\alpha_2}} A_0 + 2\sqrt{2}\sqrt{\frac{\zeta_1(1-\zeta_1)+\alpha_1}{1+4\alpha_1}}\sqrt{\frac{\zeta_2(1-\zeta_2)+\alpha_2}{1+4\alpha_2}} \cos(\phi) A_\parallel \\ & + i2\sqrt{2}\sqrt{\frac{\zeta_1(1-\zeta_1)+\alpha_1}{1+4\alpha_1}}\sqrt{\frac{\zeta_2(1-\zeta_2)+\alpha_2}{1+4\alpha_2}} \sin(\phi) A_\perp + A_{SS} \\ & + \frac{1}{\sqrt{2}}\left(\frac{2\zeta_2 - 1}{\sqrt{1+4\alpha_2}} - \frac{2\zeta_1 - 1}{\sqrt{1+4\alpha_1}}\right) A_{S-} + \frac{1}{\sqrt{2}}\left(\frac{2\zeta_2 - 1}{\sqrt{1+4\alpha_2}} + \frac{2\zeta_1 - 1}{\sqrt{1+4\alpha_1}}\right) A_{S+}. \end{aligned}$$

$$A_h = X_h \int dx_B dx_1 dx_2 b_B db_B b_1 db_1 b_2 db_2 \mathrm{Tr}[C(t)\Phi_B(x_B,b_B)\Phi_{K\pi}(x_1)\Phi_{K\pi}(x_2)H(x_i,b_i,t)S_t(x_i)e^{-S(t)}]$$

$$X_h = \begin{cases} \sqrt{1+4\alpha_1}\sqrt{1+4\alpha_2} & h = 0, \parallel, \perp \\ \sqrt{1+4\alpha_{1,2}} & h = VS, SV \\ 1 & h = SS, \end{cases}$$

Triple product asymmetries

$$
\begin{aligned}
A_T^1 &= \frac{\Gamma((2\zeta_1-1)(2\zeta_2-1)\sin\phi>0)-\Gamma((2\zeta_1-1)(2\zeta_2-1)\sin\phi<0)}{\Gamma((2\zeta_1-1)(2\zeta_2-1)\sin\phi>0)+\Gamma((2\zeta_1-1)(2\zeta_2-1)\sin\phi<0)} \\
&= -\frac{2\sqrt{2}}{\pi\mathcal{D}}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)Im[A_\perp A_0^*], \\
A_T^2 &= \frac{\Gamma(\sin(2\phi)>0)-\Gamma(\sin(2\phi)<0)}{\Gamma(\sin(2\phi)>0)+\Gamma(\sin(2\phi)<0)} \\
&= -\frac{4}{\pi\mathcal{D}}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)Im[A_\perp A_\parallel^*], \\
A_T^3 &= \frac{\Gamma\left(\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}-\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)\sin\phi>0\right)-\Gamma\left(\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}-\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)\sin\phi<0\right)}{\Gamma\left(\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}-\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)\sin\phi>0\right)+\Gamma\left(\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}-\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)\sin\phi<0\right)} \\
&= \frac{32}{5\pi\mathcal{D}}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)Im[A_\perp A_{S-}^*], \\
A_T^4 &= \frac{\Gamma(\sin\phi>0)-\Gamma(\sin\phi<0)}{\Gamma(\sin\phi>0)+\Gamma(\sin\phi<0)} \\
&= -\frac{9\pi}{4\sqrt{2}\mathcal{D}}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)Im[A_\perp A_{SS}^*],
\end{aligned}
$$

$$Im(A_\perp A_h^*) = |A_\perp||A_h^*|\sin(\Delta\phi+\Delta\delta).$$

Strong phases can produce a nonzero value

To identify a true CP violation signal, one has to compare the TPAs in B and Bbar meson decays.

$$A_T^i(true) \equiv \frac{\Gamma(TP>0)-\Gamma(TP<0)+\bar{\Gamma}(TP>0)-\bar{\Gamma}(TP<0)}{\Gamma(TP>0)+\Gamma(TP<0)+\bar{\Gamma}(TP>0)+\bar{\Gamma}(TP<0)} \propto \sin(\Delta\phi)\cos(\Delta\delta),$$

$$A_T^i(fake) \equiv \frac{\Gamma(TP>0)-\Gamma(TP<0)-\bar{\Gamma}(TP>0)+\bar{\Gamma}(TP<0)}{\Gamma(TP>0)+\Gamma(TP<0)+\bar{\Gamma}(TP>0)+\bar{\Gamma}(TP<0)} \propto \cos(\Delta\phi)\sin(\Delta\delta),$$

S-wave-induced direct CP asymmetries

$$
\begin{aligned}
A_D^1 &= \frac{\Gamma\left((2\zeta_1-1)(2\zeta_2-1)\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)>0\right)-\Gamma\left((2\zeta_1-1)(2\zeta_2-1)\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)<0\right)}{\Gamma\left((2\zeta_1-1)(2\zeta_2-1)\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)>0\right)+\Gamma\left((2\zeta_1-1)(2\zeta_2-1)\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)<0\right)} \\
&= \frac{3\sqrt{2}}{5\mathcal{D}}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)[3Re(A_{S^+}A_0^*)+5Re(A_{S^+}A_{SS}^*)], \\
A_D^2 &= \frac{\Gamma\left(\cos(\phi)\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)>0\right)-\Gamma\left(\cos(\phi)\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)<0\right)}{\Gamma\left(\cos(\phi)\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)>0\right)+\Gamma\left(\cos(\phi)\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)<0\right)} \\
&= \frac{32}{5\pi\mathcal{D}}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)Re[A_{S^+}A_{\parallel}^*], \\
A_D^3 &= \frac{\Gamma\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}>0\right)-\Gamma\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}<0\right)}{\Gamma\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}>0\right)+\Gamma\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}+\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}<0\right)} \\
&= \frac{6\sqrt{2}}{5\mathcal{D}}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)[Re(A_{S^+}A_0^*)+5Re(A_{S^+}A_{SS}^*)], \\
A_D^4 &= \frac{\Gamma\left(\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}\right)^2-\left(\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)^2>0\right)-\Gamma\left(\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}\right)^2-\left(\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)^2<0\right)}{\Gamma\left(\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}\right)^2-\left(\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)^2>0\right)+\Gamma\left(\left(\frac{2\zeta_1-1}{\sqrt{1+4\alpha_1}}\right)^2-\left(\frac{2\zeta_2-1}{\sqrt{1+4\alpha_2}}\right)^2<0\right)} \\
&= -\frac{3}{\mathcal{D}}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)Re[A_{S^+}A_{S^-}^*]. \qquad (4.4)
\end{aligned}
$$

Combining the quantities for the corresponding CP-conjugate process, we define the S-wave-induced direct CP asymmetries

$$A_S^i = A_D^i + \overline{A_D^i} \propto Sin(\Delta\phi)Sin(\Delta\delta)$$

It can be measured using untagged samples, in which CP-conjugate processes need not be distinguished.

NUMERICAL RESULTS AND DISCUSSIONS

1. CP averaged four-body branching ratios

$\omega \in [m_{K^*} - 0.15, m_{K^*} + 0.15]$GeV   JHEP05(2021)082

Theoretical uncertainties:

- B meson LCDAs
- TMDAs
- Hard scale

$b \to s$ transitions $\sim 10^{-6}$

$b \to d$ transitions $\sim 10^{-7}$

Components	$B_s^0 \to (K^+\pi^-)(K^-\pi^+)$	$B_s^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	
$\mathcal{B}_0$	$(2.7^{+0.4+0.3+1.3}_{-0.5-0.4-0.8})\times 10^{-6}$	$(2.1^{+0.4+0.3+1.0}_{-0.3-0.2-0.4})\times 10^{-6}$	
$\mathcal{B}_\parallel$	$(7.7^{+0.1+1.2+3.4}_{-0.2-1.2-2.1})\times 10^{-7}$	$(7.4^{+0.1+1.3+3.3}_{-0.1-1.2-1.9})\times 10^{-7}$	
$\mathcal{B}_\perp$	$(7.7^{+0.3+1.3+3.5}_{-0.2-1.1-2.0})\times 10^{-7}$	$(7.3^{+0.1+1.2+3.2}_{-0.2-1.2-1.9})\times 10^{-7}$	
$\mathcal{B}_{SS}$	$(5.1^{+0.6+3.5+1.6}_{-0.6-3.0-1.1})\times 10^{-7}$	$(4.7^{+0.5+3.3+1.2}_{-0.4-2.5-0.8})\times 10^{-7}$	
$\mathcal{B}_{S+}$	$(6.8^{+1.3+1.0+3.2}_{-1.1-1.1-1.7})\times 10^{-7}$	$(7.5^{+1.4+1.0+2.7}_{-1.5-1.1-1.7})\times 10^{-7}$	
$\mathcal{B}_{S-}$	$(2.7^{+0.6+0.5+1.1}_{-0.4-0.3-0.6})\times 10^{-6}$	$(2.7^{+0.5+0.4+0.9}_{-0.2-0.3-0.5})\times 10^{-6}$	
$\mathcal{B}_{\rm total}$	$(8.1^{+1.3+1.1+3.6}_{-1.1-1.2-2.2})\times 10^{-6}$	$(7.5^{+1.2+1.0+2.7}_{-0.9-0.6-1.5})\times 10^{-6}$	
Components	$B^0 \to (K^-\pi^+)(K^+\pi^-)$	$B^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$B^+ \to (K^0\pi^+)(K^-\pi^+)$
$\mathcal{B}_0$	$(1.0^{+0.4+0.3+0.5}_{-0.2-0.1-0.2})\times 10^{-7}$	$(9.3^{+3.2+3.0+0.4}_{-2.4-0.7-0.6})\times 10^{-8}$	$(2.9^{+1.2+0.3+1.0}_{-0.6-0.2-0.6})\times 10^{-7}$
$\mathcal{B}_\parallel$	$(2.1^{+0.2+0.4+0.8}_{-0.1-0.3-0.5})\times 10^{-8}$	$(1.4^{+0.2+1.5+0.1}_{-0.4-0.9-0.2})\times 10^{-10}$	$(2.0^{+0.3+0.4+0.9}_{-0.1-0.3-0.5})\times 10^{-8}$
$\mathcal{B}_\perp$	$(2.0^{+0.2+0.4+0.9}_{-0.1-0.3-0.5})\times 10^{-8}$	$(2.1^{+0.0+0.4+0.1}_{-0.2-0.8-0.0})\times 10^{-11}$	$(2.1^{+0.2+0.3+0.9}_{-0.2-0.4-0.5})\times 10^{-8}$
$\mathcal{B}_{SS}$	$(2.2^{+0.3+1.6+0.8}_{-0.1-1.2-0.4})\times 10^{-8}$	$(1.2^{+0.3+1.1+0.0}_{-0.3-0.3-0.0})\times 10^{-8}$	$(2.4^{+0.5+1.6+0.7}_{-0.3-1.1-0.5})\times 10^{-8}$
$\mathcal{B}_{S+}$	$(2.3^{+0.8+0.4+0.8}_{-0.6-0.2-0.4})\times 10^{-8}$	$(1.5^{+0.4+1.1+0.2}_{-0.3-0.0-0.1})\times 10^{-8}$	$(2.5^{+0.9+1.1+1.0}_{-0.5-0.2-0.5})\times 10^{-8}$
$\mathcal{B}_{S-}$	$(6.1^{+2.5+1.2+2.9}_{-1.5-0.9-1.6})\times 10^{-8}$	$(4.5^{+1.3+2.2+0.1}_{-0.9-1.0-0.0})\times 10^{-8}$	$(9.0^{+3.3+1.4+3.8}_{-2.0-1.7-2.0})\times 10^{-8}$
$\mathcal{B}_{\rm total}$	$(2.5^{+0.8+0.3+1.1}_{-0.6-0.4-0.7})\times 10^{-7}$	$(1.7^{+0.5+0.4+0.0}_{-0.3-0.3-0.0})\times 10^{-7}$	$(4.7^{+1.6+0.7+1.5}_{-1.0-0.5-1.0})\times 10^{-7}$

$\omega_1 \in [m_{K^*} - 0.15, m_{K^*} + 0.15]$ GeV
$\omega_2 \in [m_\phi - 0.015, m_\phi + 0.015]$ GeV

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Components	$B^0 \to (K^+K^-)(K^+\pi^-)$	$B_s^0 \to (K^+K^-)(K^-\pi^+)$	$B^+ \to (K^+K^-)(K^0\pi^+)$
$\mathcal{B}_0$	$1.8^{+0.7+0.3+0.6}_{-0.6-0.4-0.4} \times 10^{-6}$	$3.1^{+1.2+1.0+1.6}_{-0.8-0.9-1.2} \times 10^{-8}$	$1.8^{+0.9+0.4+0.7}_{-0.5-0.3-0.4} \times 10^{-6}$
$\mathcal{B}_\parallel$	$3.1^{+0.5+0.4+1.3}_{-0.4-0.4-0.8} \times 10^{-7}$	$5.3^{+0.6+2.0+2.5}_{-0.3-1.4-1.3} \times 10^{-9}$	$3.4^{+0.6+0.5+1.3}_{-0.4-0.4-0.8} \times 10^{-7}$
$\mathcal{B}_\perp$	$3.3^{+0.6+0.5+1.3}_{-0.4-0.4-0.8} \times 10^{-7}$	$5.2^{+0.3+1.9+2.6}_{-0.3-1.6-1.5} \times 10^{-9}$	$3.6^{+0.6+0.4+1.3}_{-0.5-0.5-0.9} \times 10^{-7}$
$\mathcal{B}_{SS}$	$4.7^{+2.0+2.4+1.8}_{-1.4-1.9-1.4} \times 10^{-8}$	$1.3^{+0.7+0.6+0.5}_{-0.5-0.5-0.4} \times 10^{-9}$	$5.2^{+2.1+2.6+2.0}_{-1.5-2.1-1.6} \times 10^{-8}$
$\mathcal{B}_{VS}$	$4.6^{+1.2+1.0+1.7}_{-0.9-1.0-1.4} \times 10^{-7}$	$5.0^{+2.3+1.2+2.7}_{-1.8-1.1-2.1} \times 10^{-9}$	$4.8^{+1.5+1.0+1.8}_{-1.1-1.0-1.3} \times 10^{-7}$
$\mathcal{B}_{SV}$	$2.2^{+0.6+0.4+0.8}_{-0.5-0.4-0.6} \times 10^{-7}$	$1.6^{+0.5+0.8+0.5}_{-0.4-0.6-0.4} \times 10^{-9}$	$2.5^{+0.7+0.5+0.9}_{-0.5-0.4-0.7} \times 10^{-7}$
$\mathcal{B}_{\text{total}}$	$3.2^{+1.0+0.4+1.1}_{-0.9-0.5-0.8} \times 10^{-6}$	$4.9^{+1.6+1.4+2.4}_{-1.1-1.2-1.6} \times 10^{-8}$	$3.3^{+1.2+0.6+1.2}_{-0.8-0.5-0.8} \times 10^{-6}$

$\omega \in [m_\phi - 0.03, m_\phi + 0.03]$GeV

Phys.Rev.D 105 (2022) 093001

Components	$B_s^0 \to (K^+K^-)(K^+K^-)$	$B^0 \to (K^+K^-)(K^+K^-)$
$\mathcal{B}_0$	$(1.73^{+0.62+0.13+0.77}_{-0.43-0.13-0.65}) \times 10^{-6}$	$(3.98^{+0.06+0.07+0.06}_{-0.05-0.06-0.07}) \times 10^{-9}$
$\mathcal{B}_\parallel$	$(1.40^{+0.10+0.11+0.58}_{-0.09-0.10-0.59}) \times 10^{-6}$	$(4.48^{+0.08+0.05+0.08}_{-0.12-0.04-0.05}) \times 10^{-11}$
$\mathcal{B}_\perp$	$(1.40^{+0.13+0.10+0.58}_{-0.11-0.10-0.62}) \times 10^{-6}$	$(1.01^{+0.08+0.13+0.37}_{-0.02-0.24-0.47}) \times 10^{-12}$
$\mathcal{B}_{VS}$	$(3.66^{+1.80+0.56+1.36}_{-1.02-0.52-0.98}) \times 10^{-8}$	$(5.50^{+0.60+1.70+1.30}_{-0.20-1.46-1.16}) \times 10^{-11}$
$\mathcal{B}_{SS}$	$(4.38^{+2.18+2.05+2.60}_{-1.35-1.40-2.00}) \times 10^{-9}$	$(1.17^{+0.22+0.96+0.14}_{-0.20-0.59-0.20}) \times 10^{-11}$
$\mathcal{B}_{total}$	$(4.57^{+0.86+0.35+1.95}_{-0.64-0.34-1.87}) \times 10^{-6}$	$(4.09^{+0.07+0.10+0.08}_{-0.06-0.08-0.08}) \times 10^{-9}$

2. S-wave fractions

$$f_{SS} = \frac{\mathcal{B}_{SS}}{\mathcal{B}_{\text{total}}}, \quad f_{S\pm} = \frac{\mathcal{B}_{S\pm}}{\mathcal{B}_{\text{total}}},$$

Modes	$f_{SS}(\%)$	$f_{S^+}(\%)$	$f_{S^-}(\%)$	$f_{S\text{-wave}}(\%)$
$B_s^0 \to (K^+\pi^-)(K^-\pi^+)$	$6.3^{+0.1+3.5+0.2}_{-0.2-3.4-0.4}$	$8.4^{+0.2+1.2+0.4}_{-0.1-0.9-0.0}$	$33.3^{+1.6+2.6+0.1}_{-1.2-2.4-0.2}$	$48.0^{+1.9+7.3+0.7}_{-1.5-6.7-0.6}$
LHCb [27]	$6.6\pm2.2\pm0.7$	$11.4\pm3.7\pm2.3$	$48.5\pm5.1\pm1.9$	$\cdots$
LHCb [31]	$8.7\pm1.1\pm1.1$	$4.8\pm1.4\pm1.1$	$55.8\pm2.1\pm1.4$	$69.4\pm1.6\pm1.0$
$B^0 \to (K^-\pi^+)(K^+\pi^-)$	$8.8^{+1.5+5.5+0.2}_{-1.1-4.9-0.4}$	$9.2^{+0.4+1.0+0.2}_{-0.9-0.7-0.2}$	$24.4^{+1.5+1.7+0.5}_{-1.1-2.0-0.5}$	$42.4^{+3.4+8.2+0.9}_{-3.1-7.6-1.1}$
LHCb [31]	$2.3\pm1.4\pm0.4$	$0.8\pm1.3\pm0.7$	$37.7\pm5.2\pm2.4$	$40.8\pm5.0\pm1.7$
$B_s^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$6.4^{+0.3+4.1+0.1}_{-0.2-3.4-0.3}$	$9.9^{+0.1+0.3+0.0}_{-0.8-0.9-0.5}$	$35.1^{+0.7+1.7+0.0}_{-0.7-3.0-0.6}$	$51.4^{+1.1+6.1+0.1}_{-1.7-7.3-1.4}$
$B^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$7.1^{+0.0+5.1+0.0}_{-0.4-1.9-0.1}$	$8.8^{+0.2+5.3+0.4}_{-0.1-0.0-0.4}$	$26.5^{+0.0+6.8+0.0}_{-1.0-3.3-0.4}$	$42.4^{+0.2+17.2+0.4}_{-1.5-5.2-0.9}$
$B^+ \to (K^0\pi^+)(K^-\pi^+)$	$5.1^{+0.3+3.0+0.0}_{-0.5-2.6-0.2}$	$5.3^{+0.2+1.9+0.5}_{-0.3-0.4-0.3}$	$19.1^{+0.0+1.3+0.9}_{-0.2-3.4-0.0}$	$29.5^{+0.5+6.2+1.4}_{-1.0-6.4-0.5}$

Modes	f_{SS}	f_{VS}	f_{SV}	$f_{S\text{-wave}}$
$B^0 \to (K^+K^-)(K^+\pi^-)$	$0.015^{+0.001+0.007+0.002}_{-0.000-0.006-0.002}$	$0.144^{+0.013+0.022+0.016}_{-0.007-0.020-0.015}$	$0.069^{+0.006+0.014+0.002}_{-0.002-0.011-0.005}$	$0.228^{+0.020+0.037+0.018}_{-0.009-0.032-0.022}$
LHCb [58]	$\cdots$	$0.143\pm0.013\pm0.012$	$0.122\pm0.013\pm0.008$	$\cdots$
$B_s^0 \to (K^+K^-)(K^-\pi^+)$	$0.027^{+0.004+0.013+0.002}_{-0.004-0.011-0.002}$	$0.102^{+0.010+0.032+0.022}_{-0.018-0.030-0.029}$	$0.033^{+0.000+0.016+0.004}_{-0.001-0.014-0.007}$	$0.162^{+0.014+0.057+0.027}_{-0.023-0.050-0.037}$
$B^+ \to (K^+K^-)(K^0\pi^+)$	$0.016^{+0.000+0.008+0.001}_{-0.001-0.006-0.002}$	$0.146^{+0.003+0.021+0.014}_{-0.006-0.021-0.009}$	$0.076^{+0.003+0.014+0.001}_{-0.005-0.013-0.007}$	$0.238^{+0.006+0.035+0.015}_{-0.012-0.035-0.018}$

Modes	$f_{VS}(\%)$	$f_{SS}(\%)$	$f_{S-wave}(\%)$
$B_s^0 \to (K^+K^-)(K^+K^-)$	$0.801^{+0.205+0.057+0.192}_{-0.129-0.059-0.031}$	$0.096^{+0.025+0.035+0.011}_{-0.019-0.026-0.008}$	$0.897^{+0.230+0.092+0.203}_{-0.148-0.085-0.039}$
$B^0 \to (K^+K^-)(K^+K^-)$	$1.345^{+0.121+0.373+0.286}_{-0.030-0.338-0.263}$	$0.286^{+0.048+0.222+0.028}_{-0.045-0.141-0.044}$	$1.631^{+0.169+0.595+0.314}_{-0.075-0.479-0.307}$

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- A global fitting to the Gegenbauer moments of the S-wave DAs in PQCD has not been carried out yet.
- The S-wave contribution in the $K\pi$ pair is greater than that of KK one.
- The double S-wave contributions are smaller than the single S-wave ones.

3. Triple product asymmetries (10^{-2})

Asymmetries	$B_s^0 \to (K^+\pi^-)(K^-\pi^+)$	$B_s^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$B^0 \to (K^-\pi^+)(K^+\pi^-)$	$B^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$B^+ \to (K^0\pi^+)(K^+\pi^-)$
A_T^1	$11.8^{+0.8}_{-1.1}$	$9.7^{+0.5}_{-0.6}$	$10.6^{+1.3}_{-1.7}$	~ 0	$8.5^{+0.9}_{-0.3}$
$\bar{A}_T^1$	$-11.8^{+0.8}_{-1.1}$	$-9.9^{+0.5}_{-0.6}$	$-10.6^{+1.3}_{-1.7}$	~ 0	$-9.2^{+3.5}_{-0.2}$
$A_T^1(\text{true})$	0	$-0.1^{+0.0}_{-0.1}$	0	~ 0	$-0.2^{+0.2}_{-0.1}$
$A_T^1(\text{fake})$	$11.8^{+0.8}_{-1.1}$	$9.8^{+0.3}_{-0.5}$	$10.6^{+1.3}_{-1.7}$	~ 0	$8.7^{+1.5}_{-0.1}$
A_T^2	$0.2^{+0.1}_{-0.1}$	$0.2^{+0.1}_{-0.1}$	$0.3^{+0.1}_{-0.1}$	~ 0	$0.2^{+0.0}_{-0.1}$
$\bar{A}_T^2$	$-0.2^{+0.1}_{-0.1}$	$-0.2^{+0.0}_{-0.1}$	$-0.3^{+0.1}_{-0.1}$	~ 0	$0.3^{+0.2}_{-0.2}$
$A_T^2(\text{true})$	0	0	0	~ 0	$0.25^{+0.10}_{-0.10}$
$A_T^2(\text{fake})$	$0.2^{+0.1}_{-0.1}$	$0.2^{+0.1}_{-0.1}$	$0.3^{+0.1}_{-0.1}$	~ 0	$-0.05^{+0.00}_{-0.00}$
A_T^3	$2.1^{+1.0}_{-0.6}$	$3.3^{+0.4}_{-0.7}$	$2.0^{+0.9}_{-1.0}$	~ 0	$3.6^{+0.3}_{-0.7}$
$\bar{A}_T^3$	$-2.1^{+1.0}_{-0.6}$	$-1.0^{+0.0}_{-0.1}$	$-2.0^{+0.9}_{-1.0}$	~ 0	$-0.8^{+0.1}_{-0.2}$
$A_T^3(\text{true})$	0	$1.2^{+0.5}_{-0.2}$	0	~ 0	$1.6^{+0.6}_{-0.1}$
$A_T^3(\text{fake})$	$2.1^{+1.0}_{-0.6}$	$2.2^{+0.7}_{-1.0}$	$2.0^{+0.9}_{-1.0}$	~ 0	$2.3^{+0.1}_{-0.5}$
A_T^4	$-4.6^{+2.0}_{-1.5}$	$-5.1^{+2.0}_{-1.4}$	$-5.1^{+2.3}_{-1.9}$	~ 0	$-2.7^{+0.5}_{-0.5}$
$\bar{A}_T^4$	$4.6^{+2.0}_{-1.5}$	$4.1^{+1.6}_{-1.4}$	$5.1^{+2.3}_{-1.9}$	~ 0	$1.0^{+0.5}_{-0.5}$
$A_T^4(\text{true})$	0	$-0.5^{+0.2}_{-0.2}$	0	~ 0	$-0.9^{+0.5}_{-0.5}$
$A_T^4(\text{fake})$	$-4.6^{+2.0}_{-1.5}$	$-4.6^{+1.8}_{-1.5}$	$-5.1^{+2.3}_{-1.9}$	~ 0	$-1.9^{+0.3}_{-0.9}$

Asymmetry	Data
A_T^1(true)	$0.003 \pm 0.041 \pm 0.009$
A_T^2(true)	$0.009 \pm 0.041 \pm 0.009$
A_T^3(true)	$0.019 \pm 0.041 \pm 0.008$
A_T^4(true)	$-0.040 \pm 0.041 \pm 0.008$

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The LHCb measurements show no manifest deviation from zero.

- In the modes with the neutral intermediate states, the true TPAs vanish without the weak phase difference.
- The tiny transverse polarization component in the pure annihilation mode leads to the vanishing TPAs.
- The smallness of A_T^2 is attributed to the suppression from the strong phase difference between the perpendicular and parallel polarization amplitudes.
- The predicted nonvanishing fake TPAs can be tested if flavor tagged measurements are available in the future.

- The true TPAs of the B^0mode is measured to be consistent with zero, showing no evidence for CPV.
- A significant fake asymmetry $A_T^1(fake)$ is observed in all three measurements.
- The only available measurements for the S-wave induced TPAs are from LHCb.

Asymmetries	$B^0 \to (K^+K^-)(K^+\pi^-)$	$B_s^0 \to (K^+K^-)(K^-\pi^+)$	$B^+ \to (K^+K^-)(K^0\pi^+)$
A_T^1	$-13.8^{+4.8}_{-4.3}$	$-24.0^{+6.6}_{-3.8}$	$-14.1^{+5.0}_{-3.8}$
$\bar{A}_T^1$	$13.8^{+4.8}_{-4.3}$	$24.0^{+6.6}_{-3.8}$	$+13.8^{+5.6}_{-3.9}$
A_T^1(true)	0.0	0.0	$-0.15^{+0.05}_{-0.30}$
A_T^1(fake)	$\boxed{-13.8^{+4.8}_{-4.3}}$	$-24.0^{+6.6}_{-3.8}$	$-14.0^{+5.3}_{-3.9}$
A_T^2	$-0.3^{+0.1}_{-0.1}$	$-0.1^{+0.0}_{-0.0}$	$-0.3^{+0.1}_{-0.1}$
$\bar{A}_T^2$	$0.3^{+0.1}_{-0.1}$	$0.1^{+0.0}_{-0.0}$	$0.2^{+0.1}_{-0.0}$
A_T^2(true)	0.0	0.0	$-0.05^{+0.00}_{-0.05}$
A_T^2(fake)	$-0.3^{+0.1}_{-0.1}$	$-0.1^{+0.0}_{-0.0}$	$-0.3^{+0.1}_{-0.1}$
A_T^3	$-5.4^{+1.0}_{-0.6}$	$-6.4^{+2.1}_{-2.2}$	$-5.6^{+1.0}_{-0.6}$
$\bar{A}_T^3$	$5.4^{+1.0}_{-0.6}$	$6.4^{+2.1}_{-2.2}$	$5.5^{+1.0}_{-0.6}$
A_T^3(true)	0.0	0.0	$-0.05^{+0.00}_{-0.00}$
A_T^3(fake)	$-5.4^{+1.0}_{-0.6}$	$-6.4^{+2.1}_{-2.2}$	$-5.6^{+1.0}_{-0.6}$
A_T^4	$1.6^{+3.0}_{-3.0}$	$-8.1^{+3.2}_{-3.3}$	$1.6^{+3.2}_{-2.7}$
$\bar{A}_T^4$	$-1.6^{+3.0}_{-3.0}$	$8.1^{+3.2}_{-3.3}$	$-1.8^{+3.1}_{-2.8}$
A_T^4(true)	0.0	0.0	$-0.10^{+0.05}_{-0.00}$
A_T^4(fake)	$1.6^{+3.0}_{-3.0}$	$-8.1^{+3.2}_{-3.3}$	$1.7^{+3.2}_{-2.8}$
A_T^5	$-4.3^{+1.1}_{-0.8}$	$-6.7^{+1.8}_{-1.9}$	$-4.2^{+1.1}_{-0.8}$
$\bar{A}_T^5$	$4.3^{+1.1}_{-0.8}$	$6.7^{+1.8}_{-1.9}$	$4.3^{+1.2}_{-0.8}$
A_T^5(true)	0.0	0.0	$0.05^{+0.00}_{-0.05}$
A_T^5(fake)	$-4.3^{+1.1}_{-0.8}$	$-6.7^{+1.8}_{-1.9}$	$-4.3^{+1.2}_{-0.8}$

Asymmetries	*BABAR*	Belle	LHCb
A_T^1(true)	$-0.046 \pm 0.031 \pm 0.017$	$-0.029 \pm 0.025 \pm 0.005$	$-0.007 \pm 0.012 \pm 0.002$
A_T^2(true)	$-0.003 \pm 0.056 \pm 0.036$	$0.021 \pm 0.040 \pm 0.006$	$+0.004 \pm 0.014 \pm 0.002$
A_T^3(true)	...	...	$+0.004 \pm 0.006 \pm 0.001$
A_T^4(true)	...	...	$+0.002 \pm 0.006 \pm 0.001$
A_T^1(fake)	$-0.203 \pm 0.031 \pm 0.019$	$-0.211 \pm 0.025 \pm 0.010$	$-0.105 \pm 0.012 \pm 0.006$
A_T^2(fake)	$0.016 \pm 0.058 \pm 0.038$	$-0.041 \pm 0.040 \pm 0.013$	$-0.017 \pm 0.014 \pm 0.003$
A_T^3(fake)	...	...	$-0.063 \pm 0.006 \pm 0.005$
A_T^4(fake)	...	...	$-0.019 \pm 0.006 \pm 0.007$

Asymmetries	*BABAR*
A_T^1(true)	$-0.025 \pm 0.056 \pm 0.019$
A_T^2(true)	$0.028 \pm 0.084 \pm 0.026$
A_T^1(fake)	$-0.114 \pm 0.056 \pm 0.011$
A_T^2(fake)	$-0.061 \pm 0.084 \pm 0.023$

- The only available amplitude analysis of B^+ was performed by the BABAR, but the S-wave component have not been determined.
- The full angular analysis of the B_s mode has not been done because of the limited signal events.

4. S-wave-induced direct CP asymmetries (10^{-2})

Asymmetries	$B_s^0 \to (K^+\pi^-)(K^-\pi^+)$	$B_s^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$B^0 \to (K^-\pi^+)(K^+\pi^-)$	$B^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$B^+ \to (K^0\pi^+)(K^+\pi^-)$
A_D^1	$6.1^{+0.6}_{-1.5}$	$-1.0^{+1.0}_{-1.6}$	$9.6^{+1.6}_{-3.3}$	$5.4^{+3.0}_{-5.4}$	$6.5^{+2.6}_{-3.3}$
$\bar{A}_D^1$	$-6.1^{+0.6}_{-1.5}$	$-15.1^{+1.5}_{-1.3}$	$-9.6^{+1.6}_{-3.3}$	$-7.0^{+10.3}_{-6.7}$	$-4.2^{+2.7}_{-5.1}$
A_S^1	0	$\boxed{-16.1^{+3.0}_{-2.6}}$	0	$-1.6^{+12.7}_{-12.9}$	$2.3^{+4.7}_{-6.6}$
A_D^2	$-1.9^{+1.2}_{-1.5}$	$-2.5^{+1.0}_{-1.1}$	$-0.7^{+1.3}_{-1.0}$	~ 0	$-1.3^{+0.3}_{-0.5}$
$\bar{A}_D^2$	$1.9^{+1.2}_{-1.5}$	$3.8^{+0.9}_{-0.8}$	$0.7^{+1.3}_{-1.0}$	~ 0	$-0.8^{+1.2}_{-1.0}$
A_S^2	0	$1.3^{+0.5}_{-0.3}$	0	~ 0	$-2.1^{+0.9}_{-0.8}$
A_D^3	$3.4^{+0.9}_{-2.0}$	$0.8^{+0.2}_{-0.2}$	$1.6^{+1.7}_{-1.3}$	$6.6^{+4.9}_{-3.8}$	$4.8^{+2.1}_{-1.9}$
$\bar{A}_D^3$	$-3.4^{+0.9}_{-2.0}$	$-8.8^{+2.4}_{-2.3}$	$-1.6^{+1.7}_{-1.3}$	$-4.6^{+4.1}_{-9.3}$	$-6.5^{+4.0}_{-2.5}$
A_S^3	0	$-8.0^{+2.3}_{-2.6}$	0	$2.0^{+7.2}_{-11.9}$	$-1.7^{+5.9}_{-4.8}$
A_D^4	$5.9^{+1.6}_{-1.0}$	$5.8^{+1.4}_{-1.3}$	$4.3^{+2.6}_{-2.2}$	$-3.1^{+2.7}_{-1.7}$	$3.2^{+2.2}_{-0.6}$
$\bar{A}_D^4$	$-5.9^{+1.6}_{-1.0}$	$-9.4^{+1.7}_{-1.5}$	$-4.3^{+2.6}_{-2.2}$	$-12.0^{+6.5}_{-7.2}$	$1.5^{+2.0}_{-6.6}$
A_S^4	0	$-3.6^{+1.0}_{-0.8}$	0	$\boxed{-15.1^{+7.1}_{-3.9}}$	$4.7^{+2.5}_{-1.2}$

Asymmetry	Data
A_S^1	$-0.061 \pm 0.041 \pm 0.012$
A_S^2	$0.081 \pm 0.041 \pm 0.008$
A_S^3	$-0.079 \pm 0.041 \pm 0.023$
A_S^4	$-0.081 \pm 0.041 \pm 0.010$

LHCb,JHEP07(2015)166

The LHCb measurements show no manifest deviation from zero.

- The fractions f_{S^+} are less than 10%, most values of A_D are only a few percent.
- The observed two largest S-wave-induced direct CP asymmetries are over 15% in magnitude, which could be searched in the future.
- The S-wave-induced direct CPVs in the considered decays still acquire less theoretical and experimental attention, we will wait for the confrontation with future data.

5. Direct CP asymmetries (10^{-2})

$$\mathcal{A}_h^{\mathrm{dir}} = \frac{\bar{\mathcal{B}}_h - \mathcal{B}_h}{\bar{\mathcal{B}}_h + \mathcal{B}_h},$$

- There are no direct CPVs in neutral resonance modes.
- However, the charged modes receive an additional tree contribution and the direct CPVs arises from the interference between the tree and penguin amplitudes.

$\mathcal{A}_h^{\mathrm{dir}}$	$B_s^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$B^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$B^+ \to (K^0\pi^+)(K^+\pi^-)$
$h = 0$	$29.9^{+4.3}_{-2.8}$	$-29.4^{+41.8}_{-12.0}$	$-10.7^{+11.3}_{-10.1}$
$h = \parallel$	$-18.1^{+4.6}_{-4.4}$	$-3.3^{+2.3}_{-3.0}$	$-10.0^{+1.7}_{-1.4}$
$h = \perp$	$-18.1^{+4.5}_{-4.0}$	$6.5^{+8.3}_{-9.2}$	$-2.2^{+1.9}_{-1.4}$
$h = SS$	$1.6^{+12.6}_{-2.4}$	$24.4^{+13.4}_{-8.1}$	$-61.8^{+9.2}_{-15.4}$
$h = SV$	$-30.1^{+2.8}_{-8.4}$	$-55.4^{+21.1}_{-29.6}$	$-4.6^{+15.9}_{-5.4}$
$h = VS$	$0.3^{+0.1}_{-0.6}$	$11.0^{+2.2}_{-1.4}$	$-44.3^{+7.8}_{-13.1}$

$B^+ \to (K^+K^-)(K^0\pi^+)$

A_0^{CP}	$A_\parallel^{CP}$	$A_\perp^{CP}$	A_{SS}^{CP}	A_{VS}^{CP}	A_{SV}^{CP}	A_{total}^{CP}
$-4.1^{+6.1}_{-4.6}$	$5.8^{+2.1}_{-3.8}$	$4.9^{+3.8}_{-4.4}$	$4.5^{+2.0}_{-2.5}$	$3.8^{+2.2}_{-4.2}$	$3.2^{+0.1}_{-4.4}$	$-0.3^{+3.2}_{-2.5}$

The direct CP asymmetries for various helicity states turn out to be small.

- The tree contribution only appears in the annihilation diagrams, which are power suppressed with respect to the emission ones.
- The CKM element of tree diagram is smaller than that of penguin.

SUMMARY AND OUTLOOK

- The PQCD factorization formalism based on the quasi-two-body decay mechanism for four-body B meson decays has been well established and have many potential applications.

- The S-wave contributions were found to be substantial especially in the $K\pi$ modes, which should be considered in the angular analysis.

- Various CP asymmetries are presented. In particular, by including the S-wave components, we have estimated the S-wave induced TPAs and direct CPV for the first time.

- The true TPAs in most of the considered channels are tiny, of order 10^{-2} or even lower. The fake ones could be larger, but they do not reflect CPV.

- The angular analysis may be improved by expanding the mass region, which provide more meaningful asymmetries in the future.

Thanks for your attention!

Spare slides

$$F_s(\omega) = \frac{\omega}{k(\omega)} \cdot \frac{1}{\cot\delta_B - i} + e^{2i\delta_B} \frac{m_0^2\Gamma_0/k(m_0)}{m_0^2 - \omega^2 - i m_0^2 \frac{\Gamma_0}{\omega} \frac{k(\omega)}{k(m_0)}},$$

$$\cot\delta_B = \frac{1}{a k(\omega)} + \frac{1}{2} b k(\omega),$$

$$F^{\parallel}_{K\pi}(\omega) = \frac{m_{K^*}^2}{m_{K^*}^2 - \omega^2 - i m_{K^*}\Gamma(\omega)},$$

$$F_{(K\bar{K})_S}(\omega) = \frac{m_{f_0(980)}^2}{m_{f_0(980)}^2 - \omega^2 - i m_{f_0(980)}(g_{\pi\pi}\rho_{\pi\pi} + g_{KK}\rho_{KK}F_{KK}^2)}$$

P-wave $K\pi$ DAs

$$\Phi_P^L(x,\zeta,\omega) = \frac{1}{\sqrt{2N_c}}\left[\omega\not\epsilon_p\phi_P(x,\omega) + \omega\phi_P^s(x,\omega) + \frac{\not p_1\not p_1' - \not p_1'\not p_1}{\omega(2\zeta-1)}\phi_P^t(x,\omega)\right](2\zeta-1),$$

$$\Phi_P^T(x,\zeta,\omega) = \frac{1}{\sqrt{2N_c}}\left[\gamma_5\not\epsilon_T\not p\phi_P^T(x,\omega) + \omega\gamma_5\not\epsilon_T\phi_P^a(x,\omega) + i\omega\frac{\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\epsilon_{T\nu}p_\rho n_{-\sigma}}{p\cdot n_-}\phi_P^v(x,\omega)\right]$$
$$\times\sqrt{\zeta(1-\zeta)+\alpha}, \quad (2.20)$$

The factor 2zeta − 1corresponds exactly to the structure for a time-like vector form factor

$$(p_1 - p_1')_\mu - \frac{m_1^2 - m_1'^2}{p^2}p_\mu = (2\zeta_1 - 1)\omega_1\epsilon_{p\mu},$$

The relativistic Breit-Wigner (RBW) function

$$F_{K\pi}^{\parallel}(\omega^2) = \frac{m_{K^*}^2}{m_{K^*}^2 - \omega^2 - im_{K^*}\Gamma(\omega^2)}, \quad \Rightarrow \quad F_{K\pi}^{\parallel}(\omega^2) \to e^{i\beta(m_{K^*}^2-\omega^2)}F_{K\pi}^{\parallel}(\omega^2)$$

$B_{(s)} \to (\pi\pi)(K\pi)$

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Modes	PQCD	Experiment[a]
$B^+ \to (f_0(500) \to)\pi^+\pi^-(K_0^*(1430)^+ \to)K^0\pi^+$ BW	$17.5^{+8.1+12.0+6.3}_{-5.3-8.1-5.5}$	
$B^+ \to (f_0(500) \to)\pi^+\pi^-(K_0^*(1430)^+ \to)K^0\pi^+$ Bugg	$19.2^{+8.9+12.1+7.0}_{-5.9-8.2-6.2}$	
$B^0 \to (f_0(500) \to)\pi^+\pi^-(K_0^*(1430)^0 \to)K^+\pi^-$ BW	$18.5^{+8.5+12.2+6.6}_{-5.6-8.3-5.9}$	
$B^0 \to (f_0(500) \to)\pi^+\pi^-(K_0^*(1430)^0 \to)K^+\pi^-$ Bugg	$20.3^{+9.4+12.3+7.4}_{-6.2-6.5-9.8}$	
$B_s^0 \to (f_0(500) \to)\pi^+\pi^-(\bar{K}_0^*(1430)^0 \to)K^-\pi^+$ BW	$0.4^{+0.1+0.6+0.1}_{-0.1-0.3-0.1}$	
$B_s^0 \to (f_0(500) \to)\pi^+\pi^-(\bar{K}_0^*(1430)^0 \to)K^-\pi^+$ Bugg	$0.4^{+0.1+0.6+0.1}_{-0.1-0.3-0.1}$	
$B^+ \to (f_0(980) \to)\pi^+\pi^-(K_0^*(1430)^+ \to)K^0\pi^+$	$1.6^{+0.4+0.6+0.8}_{-0.3-0.5-0.5}$	
$B^0 \to (f_0(980) \to)\pi^+\pi^-(K_0^*(1430)^0 \to)K^+\pi^-$	$1.5^{+0.4+0.6+0.8}_{-0.3-0.5-0.5}$	1.2 ± 0.4
$B_s^0 \to (f_0(980) \to)\pi^+\pi^-(\bar{K}_0^*(1430)^0 \to)K^-\pi^+$	$0.07^{+0.03+0.03+0.05}_{-0.03-0.03-0.03}$	
$B^+ \to (f_0(500) \to)\pi^+\pi^-(K^{*+} \to)K^0\pi^+$ BW	$1.1^{+0.2+1.5+0.4}_{-0.2-0.2-0.2}$	
$B^+ \to (f_0(500) \to)\pi^+\pi^-(K^{*+} \to)K^0\pi^+$ Bugg	$1.1^{+0.2+1.6+0.4}_{-0.2-0.2-0.2}$	
$B^0 \to (f_0(500) \to)\pi^+\pi^-(K^{*0} \to)K^+\pi^-$ BW	$1.0^{+0.2+1.4+0.4}_{-0.2-0.2-0.2}$	
$B^0 \to (f_0(500) \to)\pi^+\pi^-(K^{*0} \to)K^+\pi^-$ Bugg	$1.0^{+0.2+1.2+0.4}_{-0.2-0.2-0.2}$	
$B_s^0 \to (f_0(500) \to)\pi^+\pi^-(\bar{K}^{*0} \to)K^-\pi^+$ BW	$0.17^{+0.04+0.22+0.04}_{-0.04-0.10-0.06}$	
$B_s^0 \to (f_0(500) \to)\pi^+\pi^-(\bar{K}^{*0} \to)K^-\pi^+$ Bugg	$0.17^{+0.04+0.22+0.04}_{-0.04-0.08-0.04}$	
$B^+ \to (f_0(980) \to)\pi^+\pi^-(K^{*+} \to)K^0\pi^+$	$3.1^{+0.9+0.7+0.7}_{-0.7-0.7-0.9}$	2.8 ± 0.5
$B^0 \to (f_0(980) \to)\pi^+\pi^-(K^{*0} \to)K^+\pi^-$	$2.9^{+0.9+0.5+1.2}_{-0.7-0.6-0.8}$	$2.6^{+1.4}_{-1.2}$
$B_s^0 \to (f_0(980) \to)\pi^+\pi^-(\bar{K}^{*0} \to)K^-\pi^+$	$0.02^{+0.01+0.01+0.01}_{-0.01-0.01-0.01}$	
$B^+ \to (\rho^+ \to)\pi^+\pi^0(K_0^*(1430)^0 \to)K^+\pi^-$	$14.1^{+6.1+4.9+5.5}_{-3.9-4.3-3.4}$	
$B^+ \to (\rho^0 \to)\pi^+\pi^-(K_0^*(1430)^+ \to)K^0\pi^+$	$5.1^{+2.2+2.1+2.3}_{-1.5-1.8-1.3}$	
$B^0 \to (\rho^0 \to)\pi^+\pi^-(K_0^*(1430)^0 \to)K^+\pi^-$	$7.9^{+3.3+2.3+2.9}_{-2.3-2.2-1.7}$	18 ± 4
$B^0 \to (\rho^- \to)\pi^-\pi^0(K_0^*(1430)^+ \to)K^0\pi^+$	$11.8^{+5.1+4.2+4.7}_{-3.4-3.7-2.9}$	19 ± 8
$B_s^0 \to (\rho^+ \to)\pi^+\pi^0(K_0^*(1430)^- \to)\bar{K}^0\pi^-$	$14.1^{+4.3+4.4+1.0}_{-3.2-3.9-1.0}$	
$B_s^0 \to (\rho^0 \to)\pi^+\pi^-(\bar{K}_0^*(1430)^0 \to)K^-\pi^+$	$0.4^{+0.1+0.2+0.1}_{-0.1-0.2-0.1}$	

$B_{(s)} \to (K\bar{K})(K\pi)$ Phys.Rev.D 105 (2022) 053002

Components	$B^0 \to (K^+K^-)(K^+\pi^-)$	$B_s^0 \to (K^+K^-)(K^-\pi^+)$	$B^+ \to (K^+K^-)(K^0\pi^+)$
$\mathcal{B}_0$	$1.8^{+0.7+0.3+0.6}_{-0.6-0.4-0.4} \times 10^{-6}$	$3.1^{+1.2+1.0+1.6}_{-0.8-0.9-1.2} \times 10^{-8}$	$1.8^{+0.9+0.4+0.7}_{-0.5-0.3-0.4} \times 10^{-6}$
$\mathcal{B}_\parallel$	$3.1^{+0.5+0.4+1.3}_{-0.4-0.4-0.8} \times 10^{-7}$	$5.3^{+0.6+2.0+2.5}_{-0.3-1.4-1.3} \times 10^{-9}$	$3.4^{+0.6+0.5+1.3}_{-0.4-0.4-0.8} \times 10^{-7}$
$\mathcal{B}_\perp$	$3.3^{+0.6+0.5+1.3}_{-0.4-0.4-0.8} \times 10^{-7}$	$5.2^{+0.3+1.9+2.6}_{-0.3-1.6-1.5} \times 10^{-9}$	$3.6^{+0.6+0.4+1.3}_{-0.5-0.5-0.9} \times 10^{-7}$
$\mathcal{B}_{SS}$	$4.7^{+2.0+2.4+1.8}_{-1.4-1.9-1.4} \times 10^{-8}$	$1.3^{+0.7+0.6+0.5}_{-0.5-0.5-0.4} \times 10^{-9}$	$5.2^{+2.1+2.6+2.0}_{-1.5-2.1-1.6} \times 10^{-8}$
$\mathcal{B}_{VS}$	$4.6^{+1.2+1.0+1.7}_{-0.9-1.0-1.4} \times 10^{-7}$	$5.0^{+2.3+1.2+2.7}_{-1.8-1.1-2.1} \times 10^{-9}$	$4.8^{+1.5+1.0+1.8}_{-1.1-1.0-1.3} \times 10^{-7}$
$\mathcal{B}_{SV}$	$2.2^{+0.6+0.4+0.8}_{-0.5-0.4-0.6} \times 10^{-7}$	$1.6^{+0.5+0.8+0.5}_{-0.4-0.6-0.4} \times 10^{-9}$	$2.5^{+0.7+0.5+0.9}_{-0.5-0.4-0.7} \times 10^{-7}$
$\mathcal{B}_{\rm total}$	$3.2^{+1.0+0.4+1.1}_{-0.9-0.5-0.8} \times 10^{-6}$	$4.9^{+1.6+1.4+2.4}_{-1.1-1.2-1.6} \times 10^{-8}$	$3.3^{+1.2+0.6+1.2}_{-0.8-0.5-0.8} \times 10^{-6}$

$B^0_{(s)} \to \phi\phi \to (K^+K^-)(K^+K^-)$ arXiv:2204.01092

TABLE III: PQCD predictions for the branching ratios of various components and their sum in the $B^0_{(s)} \to (K^+K^-)(K^+K^-)$ decays. The theoretical uncertainties are attributed to the variations of the shape parameter $\omega_{B_{(s)}}$ in the $B_{(s)}$ meson DA, of the Gegenbauer moments in various twist DAs of KK pair, and of the hard scale t and the QCD scale $\Lambda_{\rm QCD}$.

Components	$B_s^0 \to (K^+K^-)(K^+K^-)$	$B^0 \to (K^+K^-)(K^+K^-)$
$\mathcal{B}_0$	$(1.73^{+0.62+0.13+0.77}_{-0.43-0.13-0.65}) \times 10^{-6}$	$(3.98^{+0.06+0.07+0.06}_{-0.05-0.06-0.07}) \times 10^{-9}$
$\mathcal{B}_\parallel$	$(1.40^{+0.10+0.11+0.58}_{-0.09-0.10-0.59}) \times 10^{-6}$	$(4.48^{+0.08+0.05+0.08}_{-0.12-0.04-0.05}) \times 10^{-11}$
$\mathcal{B}_\perp$	$(1.40^{+0.13+0.10+0.58}_{-0.11-0.10-0.62}) \times 10^{-6}$	$(1.01^{+0.08+0.13+0.37}_{-0.02-0.24-0.47}) \times 10^{-12}$
$\mathcal{B}_{VS}$	$(3.66^{+1.80+0.56+1.36}_{-1.02-0.52-0.98}) \times 10^{-8}$	$(5.50^{+0.60+1.70+1.30}_{-0.20-1.46-1.16}) \times 10^{-11}$
$\mathcal{B}_{SS}$	$(4.38^{+2.18+2.05+2.60}_{-1.35-1.40-2.00}) \times 10^{-9}$	$(1.17^{+0.22+0.96+0.14}_{-0.20-0.59-0.20}) \times 10^{-11}$
$\mathcal{B}_{total}$	$(4.57^{+0.86+0.35+1.95}_{-0.64-0.34-1.87}) \times 10^{-6}$	$(4.09^{+0.07+0.10+0.08}_{-0.06-0.08-0.08}) \times 10^{-9}$

Modes	$\mathcal{B}(10^{-6})$	$f_0(\%)$	$f_{\parallel}(\%)$	$f_{\perp}(\%)$
$B^+ \to \rho^+ K^{*0}$	$11.6^{+1.5+2.2+4.8}_{-1.2-2.3-3.5}$	$73.5^{+2.5+8.9+2.4}_{-2.3-9.4-3.3}$	$13.4^{+1.2+4.7+1.1}_{-1.3-4.6-1.2}$	$13.1^{+1.1+4.7+2.2}_{-1.2-4.3-1.2}$
PQCD (former)	$9.9^{+4.7}_{-4.1}$	70 ± 5		$13.7^{+2.1}_{-1.9}$
QCDF	$9.2^{+3.8}_{-5.5}$	48^{+52}_{-40}		
SCET	8.93 ± 3.18	45 ± 18		24.9 ± 11.1
FAT	10.4 ± 2.6	46.0 ± 12.9		27.2 ± 7.0
Expt.[a]	9.2 ± 1.5	48 ± 8		
$B^+ \to \rho^0 K^{*+}$	$7.5^{+1.3+1.3+2.7}_{-0.9-1.2-2.2}$	$78.4^{+2.3+6.6+2.5}_{-2.1-7.1-3.4}$	$13.3^{+1.0+3.9+2.3}_{-1.2-3.9-1.9}$	$8.3^{+1.1+3.3+1.2}_{-1.1-2.9-0.8}$
PQCD (former)	$6.1^{+2.8}_{-2.4}$	75^{+4}_{-5}		$11.9^{+2.3}_{-2.0}$
QCDF	$5.5^{+1.4}_{-2.5}$	67^{+31}_{-48}		
SCET	4.64 ± 1.37	42 ± 14		26.6 ± 9.9
FAT	5.83 ± 1.20	40.7 ± 10.6		29.8 ± 5.9
Expt. [a]	4.6 ± 1.1	78 ± 12		
$B^0 \to \rho^0 K^{*0}$	$4.4^{+0.4+0.9+1.9}_{-0.3-0.8-1.3}$	$63.3^{+1.3+10.3+1.3}_{-1.2-9.9-1.5}$	$13.9^{+1.1+4.5+0.7}_{-1.1-4.4-1.0}$	$22.8^{+0.1+5.5+2.4}_{-0.2-5.9-2.0}$
PQCD (former)	$3.3^{+1.7}_{-1.4}$	65^{+4}_{-5}		$16.9^{+2.7}_{-1.8}$
QCDF	$4.6^{+3.6}_{-3.5}$	39^{+60}_{-31}		
SCET	5.87 ± 1.87	61 ± 13		17.6 ± 7.9
FAT	5.09 ± 1.23	48.7 ± 12.3		25.8 ± 6.7
Expt.[a]	3.9 ± 1.3	40 ± 14		
$B^0 \to \rho^- K^{*+}$	$10.5^{+1.2+2.2+4.3}_{-0.9-1.6-3.1}$	$72.9^{+2.2+8.7+2.1}_{-2.1-8.8-3.0}$	$13.6^{+1.1+4.4+1.1}_{-1.2-4.5-1.0}$	$13.5^{+1.0+4.3+1.9}_{-1.1-4.3-1.1}$
PQCD (former)	$8.4^{+3.8}_{-3.5}$	68 ± 5		15.6 ± 2.5
QCDF	$8.9^{+4.9}_{-5.6}$	53^{+45}_{-32}		
SCET	10.6 ± 3.2	55 ± 14		20.3 ± 8.6
FAT	10.5 ± 2.3	38.9 ± 11.3		30.8 ± 6.3
Expt.[a]	10.3 ± 2.6	38 ± 13		
$B_s^0 \to \rho^+ K^{*-}$	$34.2^{+12.2+3.4+2.4}_{-8.5-3.3-2.2}$	$91.2^{+0.1+1.0+0.3}_{-0.2-1.3-0.4}$	$6.8^{+0.1+1.0+0.3}_{-0.0-0.7-0.1}$	$2.0^{+0.1+0.4+0.1}_{-0.1-0.4-0.1}$
PQCD (former)	$24.0^{+11.0}_{-9.1}$	95 ± 1		$2.31^{+0.22}_{-0.21}$
QCDF	$21.6^{+1.6}_{-3.2}$	92^{+1}_{-4}		
SCET	28.1 ± 4.2	99.1 ± 0.3		0.4 ± 0.18
FAT	38.6 ± 8.3	94.4 ± 1.2		2.74 ± 0.64
$B_s^0 \to \rho^0 \bar{K}^{*0}$	$1.3^{+0.4+0.1+0.3}_{-0.4-0.3-0.4}$	$53.4^{+0.7+6.9+5.4}_{-0.4-6.5-5.2}$	$25.2^{+0.0+3.3+2.3}_{-0.2-3.4-2.8}$	$21.4^{+0.4+3.3+2.9}_{-0.5-3.5-2.8}$
PQCD (former)	$0.40^{+0.22}_{-0.17}$	57^{+9}_{-13}		$22.5^{+7.3}_{-4.7}$
QCDF	$1.3^{+2.6}_{-0.7}$	90^{+5}_{-24}		
SCET	1.04 ± 0.30	87 ± 5		5.81 ± 2.84
FAT	1.18 ± 0.46	79.8 ± 8.0		10.2 ± 4.1
Expt.[a]	< 767			

Modes	TPAs-1					
	$\mathcal{A}_T^1$	$\bar{\mathcal{A}}_T^1$	$\mathcal{A}_{\text{T-true}}^1$	$\mathcal{A}_{\text{T-fake}}^1$	$\mathcal{A}_{\text{T-True}}^{(1)\text{ave}}$	$\mathcal{A}_{\text{T-fake}}^{(1)\text{ave}}$
$B^+ \to (\rho^+ \to)\pi^+\pi^0(K^{*0} \to)K^+\pi^-$	$24.94^{+2.05}_{-3.27}$	$-25.65^{+3.81}_{-2.72}$	$-0.36^{+0.28}_{-0.54}$	$25.29^{+2.37}_{-3.33}$	$-0.07^{+0.29}_{-0.56}$	$25.29^{+2.36}_{-3.34}$
$B^+ \to (\rho^0 \to)\pi^+\pi^-(K^{*+} \to)K^0\pi^+$	$14.51^{+3.55}_{-4.06}$	$-24.52^{+3.37}_{-2.35}$	$-5.00^{+1.44}_{-1.33}$	$19.52^{+2.64}_{-3.47}$	$-2.79^{+1.20}_{-1.22}$	$18.95^{+2.87}_{-3.67}$
$B^0 \to (\rho^0 \to)\pi^+\pi^-(K^{*0} \to)K^+\pi^-$	$23.55^{+3.59}_{-4.74}$	$-29.96^{+2.49}_{-2.44}$	$-3.20^{+1.55}_{-2.19}$	$26.76^{+2.62}_{-3.12}$	$-3.14^{+1.58}_{-2.04}$	$26.75^{+2.66}_{-3.20}$
$B^0 \to (\rho^- \to)\pi^-\pi^0(K^{*+} \to)K^0\pi^+$	$19.50^{+2.41}_{-3.09}$	$-28.19^{+2.35}_{-2.02}$	$-4.34^{+1.02}_{-1.20}$	$23.84^{+1.94}_{-2.51}$	$-1.56^{+1.33}_{-1.70}$	$23.34^{+2.00}_{-2.73}$
$B_s^0 \to (\rho^+ \to)\pi^+\pi^0(K^{*-} \to)\bar{K}^0\pi^-$	$-3.79^{+1.61}_{-1.53}$	$3.72^{+0.76}_{-0.73}$	$-0.04^{+0.86}_{-0.88}$	$-3.75^{+0.91}_{-0.85}$	$0.30^{+0.85}_{-0.93}$	$-3.75^{+0.87}_{-0.81}$
$B_s^0 \to (\rho^0 \to)\pi^+\pi^-(\bar{K}^{*0} \to)K^-\pi^+$	$-30.96^{+1.31}_{-0.42}$	$29.56^{+2.57}_{-4.69}$	$-0.70^{+1.68}_{-2.10}$	$-30.26^{+2.67}_{-0.94}$	$-8.97^{+3.06}_{-1.89}$	$-30.45^{+2.31}_{-0.51}$
Modes	TPAs-2					
	$\mathcal{A}_T^2$	$\bar{\mathcal{A}}_T^2$	$\mathcal{A}_{\text{T-true}}^2$	$\mathcal{A}_{\text{T-fake}}^2$	$\mathcal{A}_{\text{T-True}}^{(2)\text{ave}}$	$\mathcal{A}_{\text{T-fake}}^{(2)\text{ave}}$
$B^+ \to (\rho^+ \to)\pi^+\pi^0(K^{*0} \to)K^+\pi^-$	$-1.44^{+1.08}_{-1.07}$	$1.54^{+1.10}_{-1.04}$	$0.05^{+0.12}_{-0.04}$	$-1.49^{+1.06}_{-1.08}$	$0.04^{+0.11}_{-0.07}$	$-1.49^{+1.06}_{-1.08}$
$B^+ \to (\rho^0 \to)\pi^+\pi^-(K^{*+} \to)K^0\pi^+$	$-1.18^{+1.32}_{-1.39}$	$-9.81^{+2.51}_{-2.56}$	$-5.49^{+1.63}_{-1.72}$	$4.31^{+1.14}_{-1.17}$	$-5.00^{+1.53}_{-1.61}$	$3.69^{+1.08}_{-1.16}$
$B^0 \to (\rho^0 \to)\pi^+\pi^-(K^{*0} \to)K^+\pi^-$	$-1.73^{+1.51}_{-1.27}$	$16.27^{+3.73}_{-3.97}$	$7.27^{+2.07}_{-2.06}$	$-9.00^{+2.12}_{-1.93}$	$7.25^{+2.07}_{-2.18}$	$-8.98^{+2.21}_{-1.98}$
$B^0 \to (\rho^- \to)\pi^-\pi^0(K^{*+} \to)K^0\pi^+$	$-0.93^{+0.65}_{-1.03}$	$1.55^{+1.05}_{-1.02}$	$0.31^{+0.25}_{-0.21}$	$-1.24^{+0.83}_{-0.83}$	$0.16^{+0.21}_{-0.11}$	$-1.20^{+0.80}_{-0.82}$
$B_s^0 \to (\rho^+ \to)\pi^+\pi^0(K^{*-} \to)\bar{K}^0\pi^-$	$-0.58^{+0.28}_{-0.28}$	$0.64^{+0.13}_{-0.16}$	$0.04^{+0.10}_{-0.16}$	$-0.61^{+0.18}_{-0.16}$	$0.08^{+0.12}_{-0.13}$	$-0.61^{+0.17}_{-0.16}$
$B_s^0 \to (\rho^0 \to)\pi^+\pi^-(\bar{K}^{*0} \to)K^-\pi^+$	$-3.43^{+0.49}_{-0.39}$	$1.80^{+0.45}_{-0.72}$	$-0.81^{+0.25}_{-0.37}$	$-2.61^{+0.50}_{-0.35}$	$-1.53^{+0.39}_{-0.28}$	$-2.84^{+0.46}_{-0.29}$

channel	TPAs-1			
	$\mathcal{A}_{\text{T}}^1$	$\bar{\mathcal{A}}_{\text{T}}^1$	$\mathcal{A}_{\text{T-true}}^1$	$\mathcal{A}_{\text{T-fake}}^1$
$B_s^0 \to \phi\phi \to (K^+K^-)(K^+K^-)$	$30.38^{+1.16}_{-2.39}$	$-30.38^{+2.39}_{-1.16}$	0	$30.38^{+1.16}_{-2.39}$
$B^0 \to \phi\phi \to (K^+K^-)(K^+K^-)$	$0.67^{+0.21}_{-0.14}$	$-0.67^{+0.14}_{-0.21}$	0	$0.67^{+0.21}_{-0.14}$
channel	TPAs-2			
	$\mathcal{A}_{\text{T}}^2$	$\bar{\mathcal{A}}_{\text{T}}^2$	$\mathcal{A}_{\text{T-true}}^2$	$\mathcal{A}_{\text{T-fake}}^2$
$B_s^0 \to \phi\phi \to (K^+K^-)(K^+K^-)$	$0.15^{+0.03}_{-0.10}$	$-0.15^{+0.10}_{-0.03}$	0	$0.15^{+0.03}_{-0.10}$
$B^0 \to \phi\phi \to (K^+K^-)(K^+K^-)$	$-0.11^{+0.02}_{-0.04}$	$0.11^{+0.04}_{-0.02}$	0	$-0.11^{+0.02}_{-0.04}$

Modes	PQCD
$B^+ \to (f_0(500) \to)\pi^+\pi^-(K_0^*(1430)^+ \to)K^0\pi^+$ BW	$4.1^{+0.8+1.9+0.6}_{-0.7-2.6-0.4}$
$B^+ \to (f_0(500) \to)\pi^+\pi^-(K_0^*(1430)^+ \to)K^0\pi^+$ Bugg	$4.1^{+0.9+1.7+0.5}_{-0.7-2.9-0.4}$
$B^0 \to (f_0(500) \to)\pi^+\pi^-(K_0^*(1430)^0 \to)K^+\pi^-$ BW	$3.4^{+0.6+2.1+0.9}_{-0.5-3.2-0.2}$
$B^0 \to (f_0(500) \to)\pi^+\pi^-(K_0^*(1430)^0 \to)K^+\pi^-$ Bugg	$3.4^{+0.6+2.0+0.6}_{-0.5-3.2-0.2}$
$B_s^0 \to (f_0(500) \to)\pi^+\pi^-(\bar{K}_0^*(1430)^0 \to)K^-\pi^+$ BW	$-77.0^{+7.9+22.1+13.9}_{-6.8-4.1-5.3}$
$B_s^0 \to (f_0(500) \to)\pi^+\pi^-(\bar{K}_0^*(1430)^0 \to)K^-\pi^+$ Bugg	$-76.9^{+8.5+20.8+13.7}_{-6.8-4.3-5.3}$
$B^+ \to (f_0(980) \to)\pi^+\pi^-(K_0^*(1430)^+ \to)K^0\pi^+$	$-0.3^{+0.8+1.2+1.1}_{-0.0-1.2-2.7}$
$B^0 \to (f_0(980) \to)\pi^+\pi^-(K_0^*(1430)^0 \to)K^+\pi^-$	$-0.3^{+0.3+1.1+3.3}_{-0.0-0.9-2.3}$
$B_s^0 \to (f_0(980) \to)\pi^+\pi^-(\bar{K}_0^*(1430)^0 \to)K^-\pi^+$	$2.8^{+0.8+0.7+1.9}_{-0.6-1.5-1.6}$
$B^+ \to (f_0(500) \to)\pi^+\pi^-(K^{*+} \to)K^0\pi^+$ BW	$-31.4^{+3.5+19.3+4.7}_{-4.5-4.7-7.6}$
$B^+ \to (f_0(500) \to)\pi^+\pi^-(K^{*+} \to)K^0\pi^+$ Bugg	$-34.4^{+3.5+19.6+3.1}_{-4.5-5.4-7.2}$
$B^0 \to (f_0(500) \to)\pi^+\pi^-(K^{*0} \to)K^+\pi^-$ BW	$17.9^{+0.5+0.6+2.4}_{-1.4-17.3-6.4}$
$B^0 \to (f_0(500) \to)\pi^+\pi^-(K^{*0} \to)K^+\pi^-$ Bugg	$18.3^{+0.1+0.6+1.7}_{-0.3-18.0-6.3}$
$B_s^0 \to (f_0(500) \to)\pi^+\pi^-(\bar{K}^{*0} \to)K^-\pi^+$ BW	$41.3^{+1.0+2.9+6.3}_{-1.2-41.4-7.5}$
$B_s^0 \to (f_0(500) \to)\pi^+\pi^-(\bar{K}^{*0} \to)K^-\pi^+$ Bugg	$39.1^{+1.1+4.2+7.5}_{-0.5-39.6-7.2}$
$B^+ \to (f_0(980) \to)\pi^+\pi^-(K^{*+} \to)K^0\pi^+$	$0.2^{+0.3+0.7+0.5}_{-0.4-0.9-0.4}$
$B^0 \to (f_0(980) \to)\pi^+\pi^-(K^{*0} \to)K^+\pi^-$	$-0.2^{+0.0+0.9+1.4}_{-0.0-0.6-0.4}$
$B_s^0 \to (f_0(980) \to)\pi^+\pi^-(\bar{K}^{*0} \to)K^-\pi^+$	$1.3^{+0.2+0.8+0.1}_{-0.4-0.1-1.8}$
$B^+ \to (\rho^+ \to)\pi^+\pi^0(K_0^*(1430)^0 \to)K^+\pi^-$	$2.5^{+1.0+1.2+1.2}_{-0.4-1.2-1.2}$
$B^+ \to (\rho^0 \to)\pi^+\pi^-(K_0^*(1430)^+ \to)K^0\pi^+$	$-3.6^{+0.0+1.6+0.0}_{-1.4-7.3-4.8}$
$B^0 \to (\rho^0 \to)\pi^+\pi^-(K_0^*(1430)^0 \to)K^+\pi^-$	$6.8^{+1.5+4.7+0.0}_{-1.3-3.9-1.6}$
$B^0 \to (\rho^- \to)\pi^-\pi^0(K_0^*(1430)^+ \to)K^0\pi^+$	$2.4^{+0.1+1.0+0.0}_{-1.4-3.8-3.4}$
$B_s^0 \to (\rho^+ \to)\pi^+\pi^0(K_0^*(1430)^- \to)\bar{K}^0\pi^-$	$7.8^{+1.1+0.7+1.6}_{-1.0-1.0-1.3}$
$B_s^0 \to (\rho^0 \to)\pi^+\pi^-(\bar{K}_0^*(1430)^0 \to)K^-\pi^+$	$56.3^{+4.0+10.4+12.1}_{-3.1-9.7-5.6}$

The QCDF evaluation gives a low longitudinal polarization fraction by including weak annihilation corrections with the best-fit endpoint parameters. But an obvious tension between the data and the prediction for the branching ratio B(Bs ! K_x0003_) is invoked accordingly.

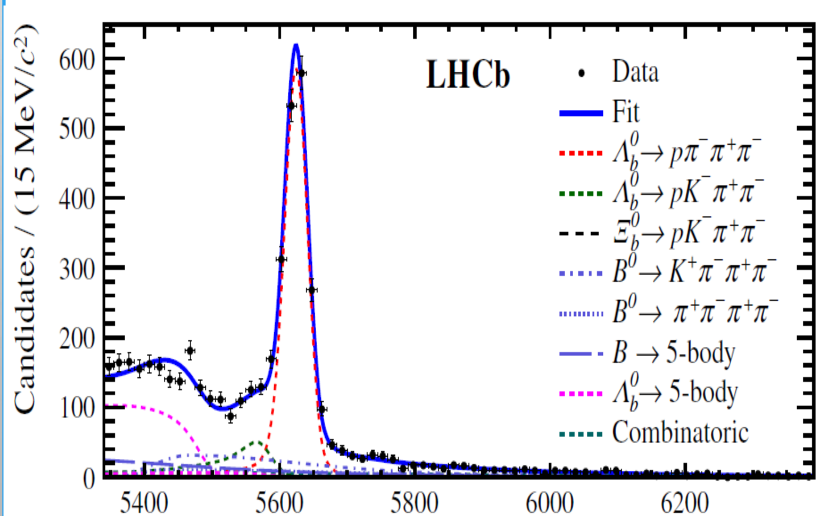
TABLE IV. Same as Table II but for the $B_{(s)} \to P(\phi \to)KK$ decays with $P = \pi, K$.

Modes		Results	Data
$B^+ \to K^+(\phi\to)KK$	$\mathcal{B}(10^{-6})$	$8.46^{+3.57+0.41+2.65}_{-2.70-0.45-1.95}$	$8.8^{+0.7\dagger}_{-0.6}$
	$\mathcal{A}_{CP}(\%)$	$1.4^{+0.8+0.1+0.0}_{-0.3-1.7-0.8}$	2.4 ± 2.8
$B^0 \to K^0(\phi\to)KK$	$\mathcal{B}(10^{-6})$	$7.82^{+3.18+0.40+2.40}_{-2.50-0.19-1.71}$	$7.3 \pm 0.7^{\dagger}$
	$\mathcal{A}_{CP}(\%)$	0	1 ± 14
$B_s^0 \to \bar{K}^0(\phi\to)KK$	$\mathcal{B}(10^{-8})$	$3.52^{+1.30+1.50+2.30}_{-0.64-0.02-1.27}$	...
	$\mathcal{A}_{CP}(\%)$	0	...
$B^+ \to \pi^+(\phi\to)KK$	$\mathcal{B}(10^{-8})$	$1.15^{+0.46+0.02+0.34}_{-0.33-0.20-0.28}$	3.2 ± 1.5
	$\mathcal{A}_{CP}(\%)$	0	10 ± 50
$B^0 \to \pi^0(\phi\to)KK$	$\mathcal{B}(10^{-9})$	$5.32^{+2.21+0.14+1.61}_{-1.53-0.91-1.27}$	< 15
	$\mathcal{A}_{CP}(\%)$	0	...
$B_s^0 \to \pi^0(\phi\to)KK$	$\mathcal{B}(10^{-7})$	$1.06^{+0.41+0.15+0.07}_{-0.34-0.20-0.14}$	...
	$\mathcal{A}_{CP}(\%)$	$27.3^{+1.1+3.2+3.5}_{-1.0-1.4-5.8}$	...

MOTIVATION

- **And also in bayonic sector** [JHEP02(2018)098]

Decay mode	Signal yield	S/B	$\pm 3\sigma$ range (MeV/c^2)
$\Lambda_b^0 \to p\pi^-\pi^+\pi^-$	1809 ± 48	4.9 ± 0.3	[5573.9, 5674.6]
$\Lambda_b^0 \to pK^-\pi^+\pi^-$	5193 ± 76	7.7 ± 0.4	[5574.4, 5674.2]
$\Lambda_b^0 \to pK^-K^+\pi^-$	444 ± 30	0.71 ± 0.06	[5577.4, 5671.1]
$\Lambda_b^0 \to pK^-K^+K^-$	1706 ± 46	8.1 ± 0.7	[5579.0, 5674.6]
$\Xi_b^0 \to pK^-\pi^+\pi^-$	183 ± 22	0.59 ± 0.09	[5747.9, 5846.2]
$\Xi_b^0 \to pK^-\pi^+K^-$	199 ± 21	0.81 ± 0.10	[5747.4, 5846.2]
$\Xi_b^0 \to pK^-K^+K^-$	27 ± 14	0.14 ± 0.08	[5752.7, 5840.8]
$\Lambda_b^0 \to (\Lambda_c^+ \to pK^-\pi^+)\pi^-$	16518 ± 133	—	[5573.7, 5674.8]



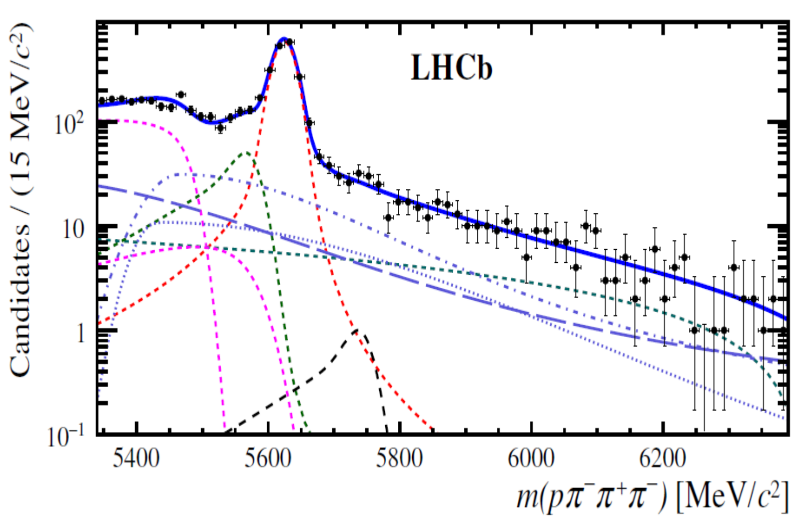


TABLE VI. Updated percentage of the transverse polarizations $f_\perp(\%)$, relative phases $\phi_\parallel(\text{rad})$, $\phi_\perp(\text{rad})$, $\Delta\phi_\parallel(10^{-2}\ \text{rad})$, $\Delta\phi_\perp(10^{-2}\ \text{rad})$, and the *CP* asymmetry parameters $A^0_{CP}(\%)$ and $A^\perp_{CP}(\%)$ in $B \to VV$ decays calculated in the PQCD approach.

Decay modes	$f_\perp$	$\phi_\parallel$	$\phi_\perp$	A^0_{CP}	$A^\perp_{CP}$	$\Delta\phi_\parallel$	$\Delta\phi_\perp$
$B^0 \to \rho^0\rho^0$	$45.9^{+1.1}_{-8.2}$	$2.68^{+1.90}_{-1.09}$	$2.81^{+0.95}_{-1.95}$	$88.9^{+9.0}_{-120.7}$	$-11.6^{+16.2}_{-2.9}$	$-98.9^{+251.9}_{-69.6}$	-105^{+266}_{-41}
$B^0 \to \rho^+\rho^-$	$2.42^{+0.21}_{-0.19}$	$3.12^{+0.06}_{-0.06}$	$3.16^{+0.06}_{-0.05}$	$-2.05^{+0.53}_{-0.55}$	$39.0^{+7.6}_{-8.4}$	$10.2^{+3.0}_{-3.1}$	$9.58^{+2.93}_{-3.19}$
$B^0 \to \rho^0\omega$	$16.7^{+5.0}_{-3.6}$	$3.13^{+0.17}_{-0.19}$	$3.13^{+0.17}_{-0.19}$	$26.6^{+19.8}_{-12.2}$	$-60.0^{+11.8}_{-12.1}$	$-87.8^{+13.7}_{-15.3}$	$-98.4^{+12.9}_{-15.1}$
$B^0 \to \omega\omega$	$18.2^{+6.1}_{-5.3}$	$3.20^{+0.25}_{-0.20}$	$3.21^{+0.24}_{-0.22}$	$-5.70^{+11.8}_{-16.2}$	$17.0^{+19.1}_{-22.1}$	$105^{+13.2}_{-10.4}$	$108^{+13.8}_{-11.1}$
$B^0 \to K^{*0}\rho^0$	$16.9^{+2.7}_{-1.8}$	$4.67^{+0.02}_{-3.06}$	$4.66^{+0.01}_{-3.06}$	$3.64^{+1.20}_{-1.07}$	$-7.71^{+1.97}_{-1.86}$	$-0.12^{+1.72}_{-1.79}$	$0.22^{+1.85}_{-1.65}$
$B^0 \to K^{*+}\rho^-$	$15.6^{+2.5}_{-2.5}$	$3.31^{+0.23}_{-0.21}$	$3.30^{+0.22}_{-0.21}$	$23.8^{+4.7}_{-5.1}$	$-50.9^{+4.9}_{-3.9}$	$128^{+4.1}_{-4.4}$	$127^{+43}_{-4.3}$
$B^0 \to K^{*0}\omega$	$18.3^{+2.6}_{-2.3}$	$2.18^{+0.21}_{-0.20}$	$2.14^{+0.21}_{-0.19}$	$1.46^{+1.44}_{-1.62}$	$-8.92^{+5.01}_{-4.01}$	$-2.28^{+1.79}_{-1.89}$	$-12.0^{+3.5}_{-4.9}$
$B^0 \to K^{*0}\bar{K}^{*0}$	$19.7^{+4.0}_{-3.6}$	$2.26^{+0.20}_{-0.16}$	$2.31^{+0.19}_{-0.15}$	~0.0	~0.0	~0.0	~0.0
$B^0 \to K^{*+}K^{*-}$	~0.0	$3.34^{+0.08}_{-0.06}$	$3.37^{+2.60}_{-0.09}$	$0.02^{+0.02}_{-0.01}$	$-75.3^{+21.1}_{-10.5}$	$56.4^{+10.9}_{-9.7}$	$-129^{+258}_{-2.0}$

TABLE V. Direct *CP* asymmetries (%) in the $B \to VV$ decays and comparison with the predictions from QCDF [4]. Experimental data are from the Particle Data Group [40]. For $B^0 \to K^{*0(+)}\rho^{0(-)}$, the data is from Ref. [49].

Decay modes	This work	QCDF	Expt.
$B^0 \to \rho^0\rho^0$	$70.7^{+2.9+0.8+3.8}_{-5.2-5.4-6.0}$	30^{+17+14}_{-16-26}	
$B^0 \to \rho^+\rho^-$	$0.83^{+0.50+0.66+0.00}_{-0.59-0.31-0.00}$	-4^{+0+3}_{-0-3}	
$B^0 \to \rho^0\omega$	$59.4^{+11.9+6.3+5.0}_{-8.3-5.5-6.3}$	3^{+2+51}_{-6-76}	
$B^0 \to \omega\omega$	$-73.7^{+6.7+2.6+3.3}_{-6.2-6.0-0.9}$	-30^{+15+16}_{-14-18}	
$B^0 \to K^{*0}\rho^0$	$-8.9^{+0.6+2.8+1.1}_{-0.6-2.8-1.0}$	-15^{+4+16}_{-8-14}	$-6 \pm 9 \pm 2$
$B^0 \to K^{*+}\rho^-$	$24.5^{+1.2+2.9+0.0}_{-1.5-3.4-0.6}$	32^{+1+2}_{-3-14}	$21 \pm 15 \pm 2$
$B^0 \to K^{*0}\omega$	$5.6^{+0.3+1.2+0.8}_{-0.3-1.3-0.9}$	23^{+9+5}_{-5-18}	45 ± 25
$B^0 \to K^{*0}\bar{K}^{*0}$	0.0	-14^{+1+6}_{-1-2}	
$B^0 \to K^{*+}K^{*-}$	$29.8^{+2.0+6.4+4.6}_{-5.7-9.5-4.7}$	0	

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TABLE IV. Updated percentage of the longitudinal polarizations f_L of $B \to VV$ decays calculated in the PQCD approach compared with the updated theoretical predictions in the QCD factorization (QCDF) approach [4] and the previous predictions in the PQCD approach [7]. Experimental data are from the Particle Data Group [40].

Decay modes	This work	QCDF	PQCD(former)	Expt.
$B^0 \to \rho^0\rho^0$	$0.12^{+0.04+0.15+0.00}_{-0.02-0.01-0.00}$	$0.92^{+0.03+0.06}_{-0.04-0.37}$	0.60	0.75 ± 0.14[a]
$B^0 \to \rho^+\rho^-$	$0.95^{+0.01+0.01+0.00}_{-0.01-0.01-0.00}$	$0.92^{+0.01+0.01}_{-0.02-0.02}$	0.94	0.977 ± 0.026
$B^0 \to \rho^0\omega$	$0.67^{+0.04+0.03+0.06}_{-0.06-0.04-0.06}$	$0.52^{-0.11+0.50}_{-0.25-0.36}$	0.87	
$B^0 \to \omega\omega$	$0.66^{+0.07+0.04+0.06}_{-0.10-0.02-0.04}$	$0.94^{+0.01+0.04}_{-0.01-0.20}$	0.82	
$B^0 \to K^{*0}\rho^0$	$0.65^{+0.03+0.03+0.00}_{-0.03-0.04-0.00}$	$0.39^{+0.00+0.60}_{-0.00-0.31}$	0.74	0.57 ± 0.10
$B^0 \to K^{*+}\rho^-$	$0.68^{+0.04+0.03+0.02}_{-0.03-0.03-0.02}$	$0.53^{+0.02+0.45}_{-0.03-0.32}$	0.78	
$B^0 \to K^{*0}\omega$	$0.65^{+0.05+0.02+0.00}_{-0.05-0.02-0.00}$	$0.58^{+0.07+0.43}_{-0.10-0.14}$	0.82	0.69 ± 0.13
$B^0 \to K^{*0}\bar{K}^{*0}$	$0.58^{+0.07+0.02+0.02}_{-0.08-0.02-0.01}$	$0.52^{+0.04+0.48}_{-0.07-0.48}$	0.78	0.80 ± 0.13
$B^0 \to K^{*+}K^{*-}$	~1.0	~1.0	0.99	

MOTIVATION

- **And also in bayonic sector** [JHEP02(2018)098,LHCb]

Some charmless four body decays are observed

Decay mode	Signal yield	S/B	$\pm3\sigma$ range (MeV/c^2)
$\Lambda_b^0 \to p\pi^-\pi^+\pi^-$	1809 ± 48	4.9 ± 0.3	[5573.9, 5674.6]
$\Lambda_b^0 \to pK^-\pi^+\pi^-$	5193 ± 76	7.7 ± 0.4	[5574.4, 5674.2]
$\Lambda_b^0 \to pK^-K^+\pi^-$	444 ± 30	0.71 ± 0.06	[5577.4, 5671.1]
$\Lambda_b^0 \to pK^-K^+K^-$	1706 ± 46	8.1 ± 0.7	[5579.0, 5674.6]
$\Xi_b^0 \to pK^-\pi^+\pi^-$	183 ± 22	0.59 ± 0.09	[5747.9, 5846.2]
$\Xi_b^0 \to pK^-\pi^+K^-$	199 ± 21	0.81 ± 0.10	[5747.4, 5846.2]
$\Xi_b^0 \to pK^-K^+K^-$	27 ± 14	0.14 ± 0.08	[5752.7, 5840.8]
$\Lambda_b^0 \to (\Lambda_c^+ \to pK^-\pi^+)\pi^-$	16518 ± 133	—	[5573.7, 5674.8]

No significant CPV effect is observed

Eur. Phys. J. C (2019) 79:745
https://doi.org/10.1140/epjc/s10052-019-7218-1

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Experimental Physics

Measurements of *CP* asymmetries in charmless four-body Λ_b^0 and Ξ_b^0 decays

LHCb Collaboration*

Search for CP violation using triple product asymmetries in $\Lambda_b^0 \to pK^-\pi^+\pi^-$, $\Lambda_b^0 \to pK^-K^+K^-$ and $\Xi_b^0 \to pK^-K^-\pi^+$ decays

JHEP08(2018)039

The LHCb collaboration

TABLE VII. PQCD predictions for the TPAs (%) of the four-body $B^0_{(s)} \to (K^+K^-)(K^+K^-)$ decays. The sources of theoretical errors are the same as in Table III but added in quadrature.

Channel	TPAs − 1			
	$\mathcal{A}^1_{\rm T}$	$\bar{\mathcal{A}}^1_{\rm T}$	$\mathcal{A}^1_{\rm T\text{-true}}$	$\mathcal{A}^1_{\rm T\text{-fake}}$
$B^0_s \to \phi\phi \to (K^+K^-)(K^+K^-)$	$30.38^{+1.16}_{-2.39}$	$-30.38^{+2.39}_{-1.16}$	0	$30.38^{+1.16}_{-2.39}$
$B^0 \to \phi\phi \to (K^+K^-)(K^+K^-)$	$0.67^{+0.21}_{-0.14}$	$-0.67^{+0.14}_{-0.21}$	0	$0.67^{+0.21}_{-0.14}$
	TPAs − 2			
Channel	$\mathcal{A}^2_{\rm T}$	$\bar{\mathcal{A}}^2_{\rm T}$	$\mathcal{A}^2_{\rm T\text{-true}}$	$\mathcal{A}^2_{\rm T\text{-fake}}$
$B^0_s \to \phi\phi \to (K^+K^-)(K^+K^-)$	$0.15^{+0.03}_{-0.10}$	$-0.15^{+0.10}_{-0.03}$	0	$0.15^{+0.03}_{-0.10}$
$B^0 \to \phi\phi \to (K^+K^-)(K^+K^-)$	$-0.11^{+0.02}_{-0.04}$	$0.11^{+0.04}_{-0.02}$	0	$-0.11^{+0.02}_{-0.04}$

Comparing with the BR of the scalar-scalar and scalar-vector modes derived in the two-body PQCD framework, our numbers are generally smaller.

- A global fitting to the Gegenbauer moments of the S-wave DAs in PQCD has not been carried out yet.
- The narrow-width approximation should be corrected by including finite-width effects for the broad scalar intermediate states. [H. Y. Cheng et al. Phys. Rev. D 103, 036017 (2021), Phys. Lett. B 813, 136058 (2021).]

3. Two-body branching ratios and polarization fractions

In the narrow-width limit:

$$\mathcal{B}(B \to K^*(\to K\pi)\bar{K}^*(\to K\pi)) \approx \mathcal{B}(B \to K^*\bar{K}^*) \times \mathcal{B}(K^* \to K\pi) \times \mathcal{B}(\bar{K}^* \to K\pi)$$

Naive SM (V-A) helicity counting rules: $f_L \sim 1 - \mathcal{O}(m_V^2/m_B^2)$, $f_\parallel \sim f_\perp \sim \mathcal{O}(m_V^2/m_B^2)$

Modes	$\mathcal{B}(10^{-6})$	$f_0(\%)$	$f_\parallel(\%)$	$f_\perp(\%)$
$B_s^0 \to K^{*+}K^{*-}$	$8.0^{+0.8+1.2+3.2}_{-0.6-0.8-1.6}$	$58.8^{+2.9+3.0+1.0}_{-3.4-1.3-0.0}$	$20.7^{+1.8+0.7+0.0}_{-1.6-0.7-0.5}$	$20.5^{+1.6+0.5+0.0}_{-1.3-1.4-0.5}$
PQCD-I [32]	$5.4^{+3.3}_{-2.3}$	$42.0^{+14.2}_{-11.2}$	$\cdots$	$27.7^{+5.2}_{-7.0}$
PQCD-II [33]	$6.7^{+3.7}_{-1.9}$	$43.8^{+6.6}_{-4.9}$	$30.1^{+2.4}_{-3.4}$	$26.1^{+2.4}_{-3.2}$
PQCD-NLO [34]	$6.5^{+2.8}_{-2.1}$	$48.1^{+9.7}_{-8.9}$	$\cdots$	$23.9^{+4.4}_{-5.2}$
QCDF-I [35]	$7.6^{+2.5}_{-2.1}$	52^{+20}_{-21}	$\cdots$	$\cdots$
QCDF-II [36]	$9.1^{+10.5}_{-6.3}$	67^{+31}_{-26}	$\cdots$	$\cdots$
SCET [37]	11.0 ± 3.3	55 ± 14	$\cdots$	20.3 ± 8.6
FAT [38]	15.9 ± 3.5	30.9 ± 10.4	$\cdots$	34.9 ± 5.8
$B^0 \to K^{*+}K^{*-}$	$0.21^{+0.06+0.06+0.01}_{-0.05-0.03-0.01}$	~ 100	~ 0.0	~ 0.0
PQCD-I [32]	0.21 ± 0.10	~ 100	~ 0.0	~ 0.0
PQCD-II [119]	$0.064^{+0.005}_{-0.010}$	99	0.5	0.4
QCDF-I [120, 121]	0.1 ± 0.1	~ 100	~ 0.0	~ 0.0
FAT [38]	1.43 ± 0.96	$\cdots$	$\cdots$	$\cdots$
Data [5, 6]	< 2.0	$\cdots$	$\cdots$	$\cdots$
$B^+ \to K^{*+}\bar{K}^{*0}$	$0.74^{+0.24+0.06+0.22}_{-0.14-0.08-0.16}$	$87.6^{+2.3+0.8+0.1}_{-2.3-1.3-0.9}$	$6.0^{+1.2+0.7+0.5}_{-1.0-0.3-0.0}$	$6.4^{+1.0+0.4+0.3}_{-1.4-0.6-0.2}$
PQCD-I [32]	$0.56^{+0.26}_{-0.22}$	74^{+4}_{-5}	$\cdots$	$12.9^{+1.7}_{-2.4}$
PQCD-II [119]	$0.48^{+0.12}_{-0.08}$	81.5	9.0	9.5
QCDF-I [120, 121]	0.6 ± 0.3	45^{+55}_{-38}	$\cdots$	27^{+19}_{-27}
QCDF-II [36]	$0.5^{+0.4}_{-0.3}$	62^{+42}_{-33}	$\cdots$	$\cdots$
SCET [37]	0.52 ± 0.18	50 ± 16	$\cdots$	22.9 ± 10.0
FAT [38]	0.66 ± 0.18	58.3 ± 11.1	$\cdots$	20.8 ± 6.0
Data [5, 6]	0.91 ± 0.29	82^{+15}_{-21}	$\cdots$	$\cdots$

Modes	$\mathcal{B}(10^{-6})$	$f_0(\%)$	$f_{\parallel}(\%)$	$f_{\perp}(\%)$
$B_s^0 \to K^{*0}\bar{K}^{*0}$	$9.5^{+1.1+1.1+4.6}_{-1.1-1.1-2.6}$	$63.6^{+2.7+3.3+1.0}_{-4.2-3.9-1.0}$	$18.2^{+2.1+1.9+0.4}_{-1.5-1.7-0.5}$	$18.2^{+2.0+1.8+0.5}_{-1.3-1.6-0.6}$
PQCD-I [32]	$5.4^{+3.0}_{-2.4}$	$38.3^{+12.1}_{-10.5}$	$\cdots$	$30.0^{+5.3}_{-6.1}$
PQCD-II [33]	$7.8^{+4.2}_{-2.7}$	$49.7^{+5.7}_{-6.1}$	$26.8^{+3.3}_{-3.0}$	$23.5^{+2.8}_{-2.7}$
PQCD-NLO [34]	$6.7^{+2.9}_{-2.2}$	$43.4^{+12.7}_{-12.9}$	$\cdots$	$23.5^{+5.8}_{-5.9}$
QCDF-I [35]	6.6 ± 2.2	56^{+22}_{-27}	$\cdots$	$\cdots$
QCDF-II [36]	$9.1^{+11.3}_{-6.8}$	63^{+42}_{-29}	$\cdots$	$\cdots$
SCET [37]	8.6 ± 3.1	44.9 ± 18.3	$\cdots$	24.9 ± 11.1
FAT [38]	14.9 ± 3.6	34.3 ± 12.6	$\cdots$	33.2 ± 6.9
Data [5, 6]	11.1 ± 2.7	24 ± 4	30 ± 5	38 ± 12
$B^0 \to K^{*0}\bar{K}^{*0}$	$0.32^{+0.09+0.09+0.16}_{-0.04-0.04-0.08}$	$71.5^{+3.8+2.7+1.5}_{-2.6-1.6-0.8}$	$14.6^{+1.3+0.8+0.4}_{-1.9-1.4-0.7}$	$13.9^{+1.3+0.8+0.4}_{-1.9-1.4-0.8}$
PQCD-I [32]	$0.34^{+0.16}_{-0.15}$	58 ± 8	$\cdots$	$19.7^{+4.0}_{-3.6}$
PQCD-II [119]	0.35	78	12	10
QCDF-I [120, 121]	$0.6^{+0.2}_{-0.3}$	52 ± 48	$\cdots$	24 ± 24
QCDF-II [36]	$0.6^{+0.5}_{-0.3}$	69^{+34}_{-27}	$\cdots$	$\cdots$
SCET [37]	0.48 ± 0.16	50 ± 16	$\cdots$	22.9 ± 10.0
FAT [38]	0.61 ± 0.17	58.3 ± 11.1	$\cdots$	20.8 ± 6.0
Data [5, 6]	0.83 ± 0.24	74 ± 5	$\cdots$	$\cdots$

The polarization puzzle associated with the two U-spin related channels $B_s^0 \to \overline{K^*}K^*$ and $B^0 \to \overline{K^*}K^*$ still exists.

$$a^0_{2\phi} = -0.31 \pm 0.19$$

Modes	$\mathcal{B}\,(10^{-6})$	f_0 (%)	$f_\perp$ (%)
$B^0 \to \phi K^{*0}$	$7.4^{+2.5+1.1+2.6}_{-2.1-1.2-1.8}$	$74.1^{+3.1+1.5+1.1}_{-5.8-3.0-1.2}$	$13.3^{+3.0+1.6+0.6}_{-1.6-0.8-0.5}$
PQCD-I [52]	14.86	75.0	11.5
PQCD-II [62]	$9.8^{+4.9}_{-3.8}$	$56.5^{+5.8}_{-5.9}$	$21.3^{+2.8}_{-2.9}$
QCDF-I [38]	$9.3^{+0.5+11.4}_{-0.5-6.5}$	44^{+0+59}_{-0-36}	...
QCDF-II [39]	$9.5^{+1.3+11.9}_{-1.2-5.9}$	50^{+50}_{-42}	25^{+21}_{-25}
SCET [63]	9.14 ± 3.14	51.0 ± 16.4	22.2 ± 9.9
FAT [64]	$8.64 \pm 1.76 \pm 1.70 \pm 0.90$	48.0 ± 16.0	26.0 ± 8.6

Asymmetry	Value
A_T^1	$0.003 \pm 0.041 \pm 0.009$
A_T^2	$0.009 \pm 0.041 \pm 0.009$
A_T^3	$0.019 \pm 0.041 \pm 0.008$
A_T^4	$-0.040 \pm 0.041 \pm 0.008$
A_D^1	$-0.061 \pm 0.041 \pm 0.012$
A_D^2	$0.081 \pm 0.041 \pm 0.008$
A_D^3	$-0.079 \pm 0.041 \pm 0.023$
A_D^4	$-0.081 \pm 0.041 \pm 0.010$

Table 3. Triple product and direct *CP* asymmetries measured in this analysis. The first uncertainties are statistical and the second systematic.

SCET [30]	19.0 ± 6.5	51.0 ± 16.4	22.2 ± 9.9	2.41 ± 0.62	2.54 ± 0.62
FAT [31]	26.4 ± 7.6	39.7 ± 16.0	31.2 ± 8.9	2.53 ± 0.28	2.56 ± 0.27
Data	18.7 ± 1.5	37.8 ± 1.3	29.2 ± 0.9	2.56 ± 0.06	2.82 ± 0.19
$B^0 \to \phi\phi$	$0.016^{+0.005}_{-0.004}$	$98.9^{+0.1}_{-0.7}$	$0.02^{+0.01}_{-0.00}$	$2.38^{+0.21}_{-0.10}$	$4.39^{+0.21}_{-0.27}$
PQCD [28]	$0.012^{+0.006}_{-0.005}$	97 ± 1	0.05 ± 0.02	$3.26^{+0.20}_{-0.14}$	3.50 ± 0.17
Data	< 0.027	...	...	...	...