Light meson light cone distribution on Lattice QCD

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Out Line

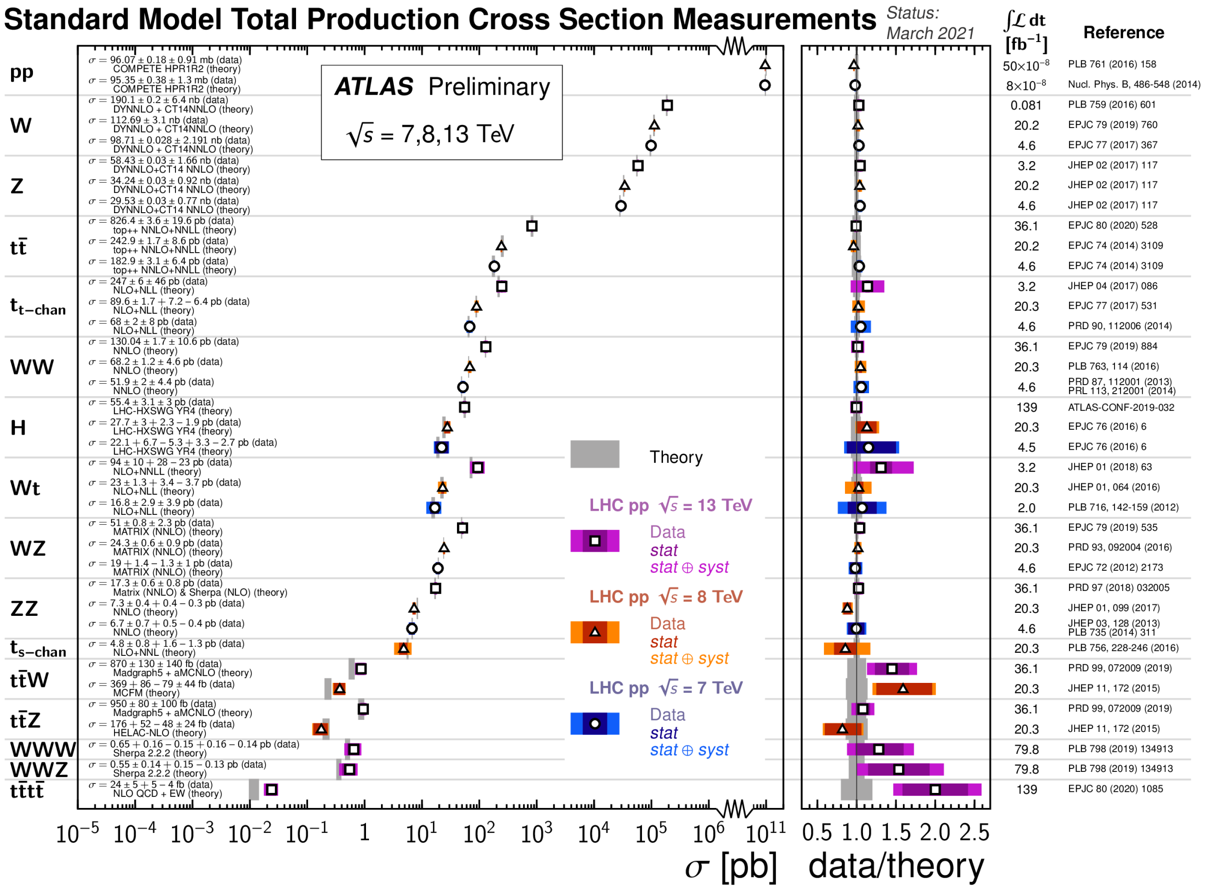
01 Motivation of LCDA

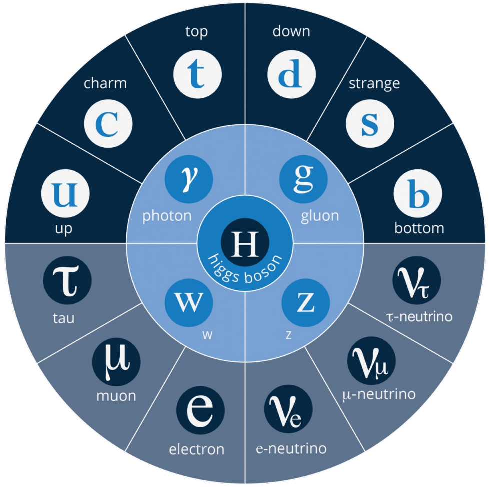
02 LCDA on Lattice

03 Research by LaMET

04 Outlook and Summary

The Standard Model: describes physics across 13 orders





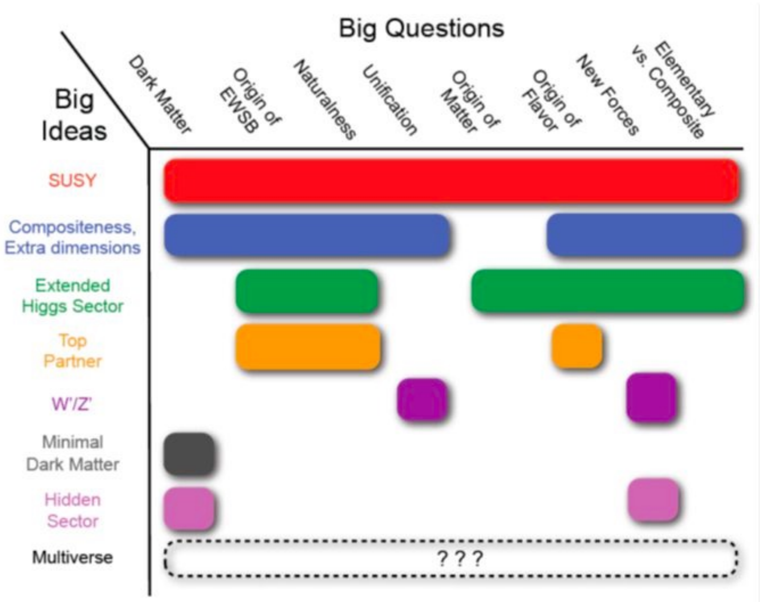
$SU(3) \otimes SU(2) \otimes U(1)$

A unified theory describing strong, weak and electromagnetic interactions

Current Difficulties：high energy – search for problem；

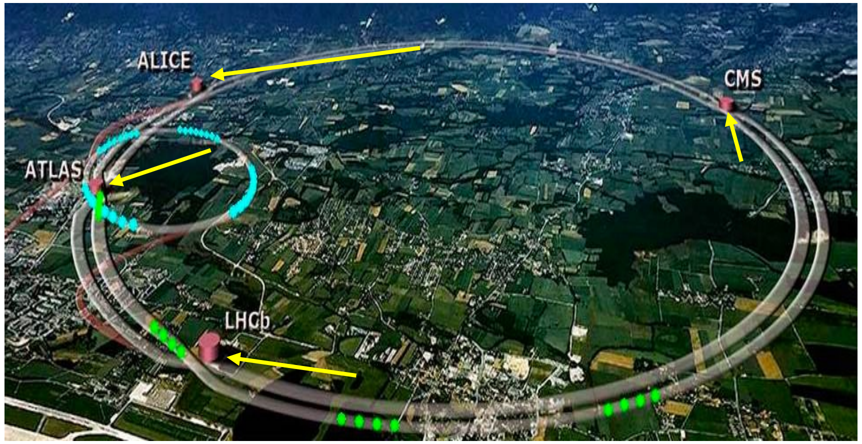
low energy – search for answers

- **New physics beyond the standard model(BSM)?**
- **Final theory (Theory of everything)?**



Snowmass '13 Energy Frontier Report arXiv:1401.6081v1 [hep-ex]

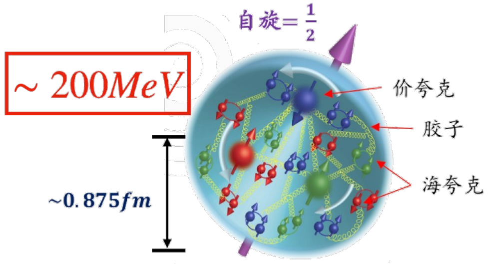
High Energy Collider



Current Difficulties： high energy – search for problem；

low energy – search for answers

- (~**1GeV**) QCD cannot be computed analytically at the proton radius scale.



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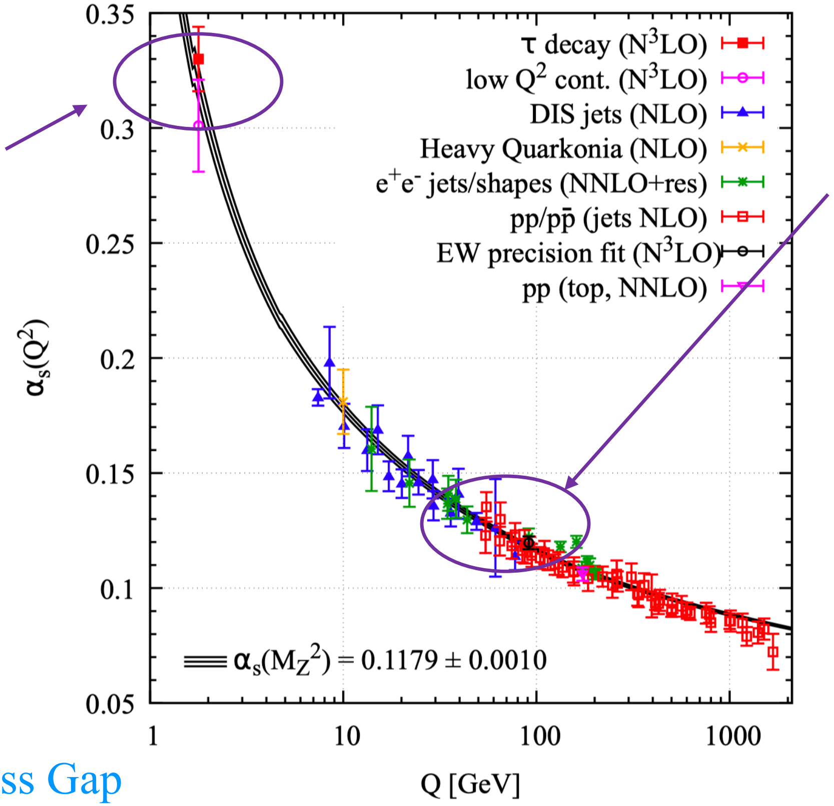
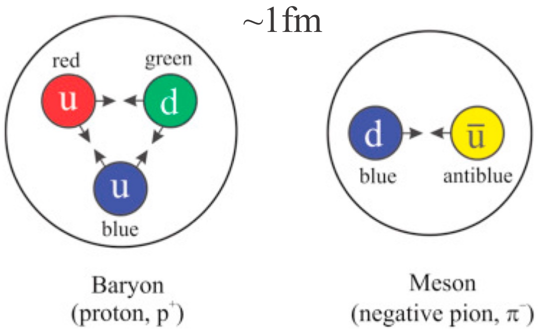
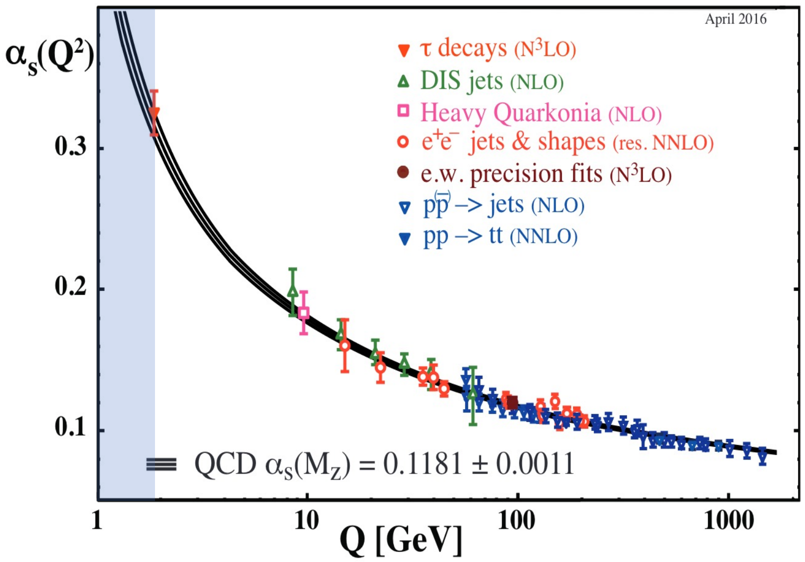
Yang-Mills Existence and Mass Gap



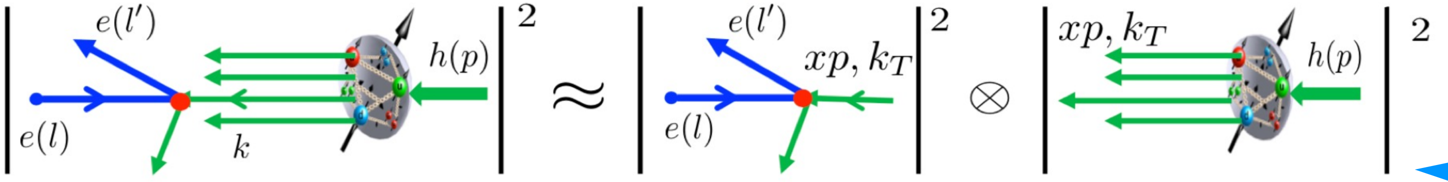
- Experiments and analytical calculations in Asymptotic freedom region.
- Nobel prize 2004

Limited by QCD confinement





- ➢ **QCD factorization** **(1982)**



Cross section

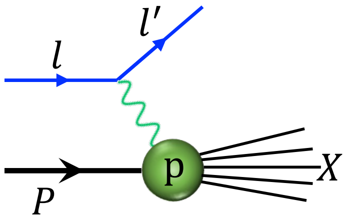
Asymptotic freedom
(hard kernel)

Color confinement
(Parton in hadron)

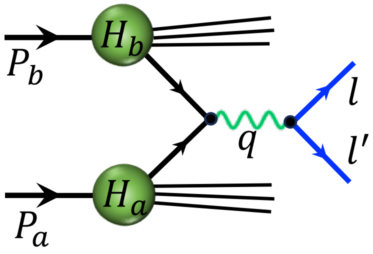
parton density amplitude
(PDF, LCDA)

◆ **PDFs: the probability distribution of partons (quarks and gluons) within a hadron —— Inclusive process**

DIS

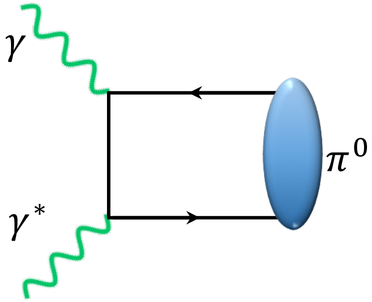


Drell-Yan

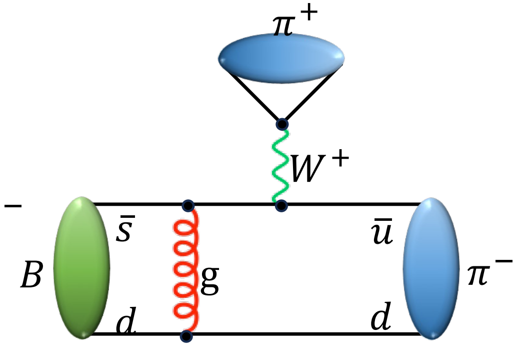


◆ **LCDAs: the probability amplitude for partons within a hadron ——Exclusive process**

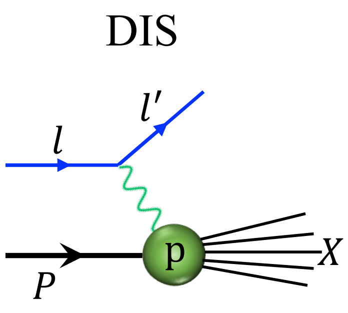
$\gamma\gamma^* \to \pi$

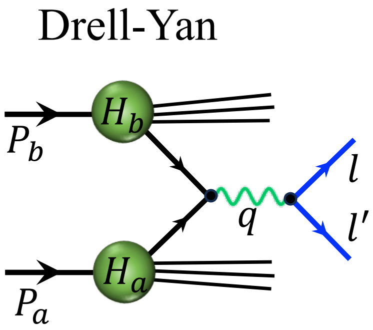


$B \to \pi^+\pi^-$

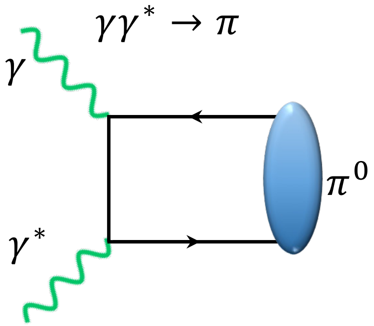


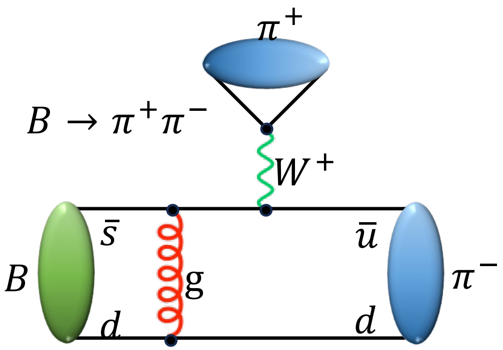
➢ PDF



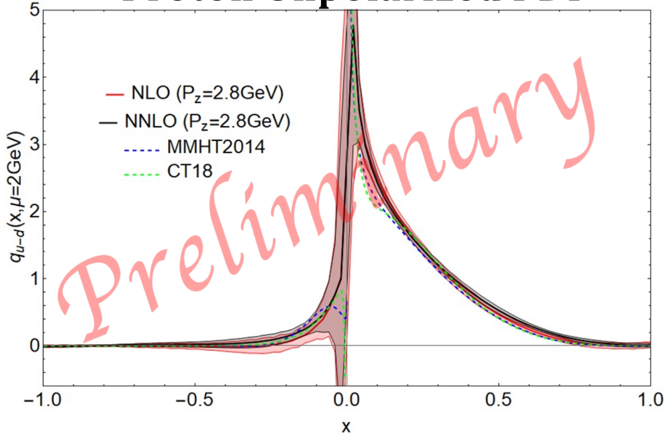


➢ LCDA



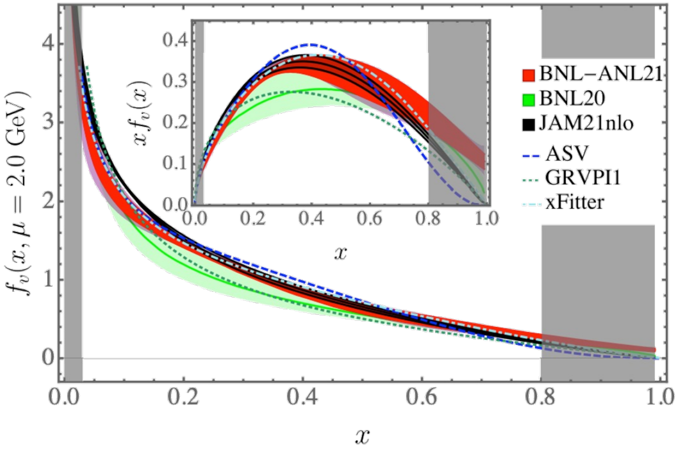


Proton Unpolarized PDF



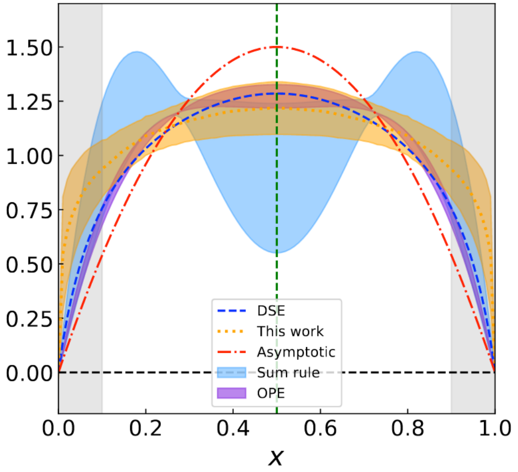
Preliminary from LPC

Pion valancePDF



X.Gao, et.al. PRL.128.142003 (2022)

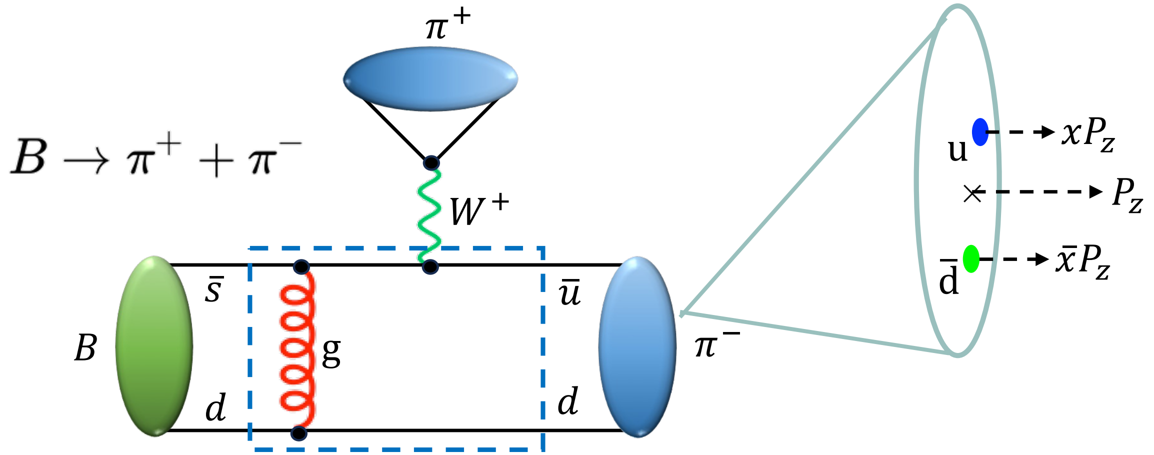
Pion LCDA



(LPC) J.Hua et.al. PRL.129,132001 (2022)

LCDA as most important input in flavor physics：

- $B \to \pi l \nu_l, B \to \pi\pi, \ldots$
- $\gamma^* \to \gamma\pi, \gamma\gamma \to \pi\pi$
- $eN \to eN\pi$
- $B \to K^* l^+ l^-$
- $B \to \phi l^+ l^-$
- $\ldots$



CKM matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Anomalous ⋯

Test of lepton universality with $B^0 \to K^{*0}\ell^+\ell^-$ decays #2

LHCb Collaboration • R. Aaij (CERN) et al. (May 16, 2017)

Published in: *JHEP* 08 (2017) 055 • e-Print: 1705.05802 [hep-ex]

1979 → **2023**

- **Asymptotic LCDAs**
  G. P. Lepage et.al., Phys. Rev. Lett. 43 (1979)
  G. P. Lepage et.al., Phys.Lett.B 87B(1979)
- **Sum rules**
  V.L. Chernyak et. al., Nucl.Phys.B 201 (1982)
  Vladimir M. Braun et. al., Z.Phys.C 44 (1989)
  Patricia Ball et. al., JHEP 08 (2007)
- **Lattice calculation by OPE**
  G. Martinelli et. al., Phys.Lett.B 190 (1987)
  RQCD Collaboration, JHEP 11 (2020)
- **Quantum Computing**
  QuNu Collaboration arXiv:2207.13258(2022)
- **Quark model**
  Choi, Phys.Rev.D 75 (2007)
- **Dyson-Schwinger Equation**
  F. Gao, L. Chang et.al.Phys.Rev.D 90 (2014)
  Craig D.et.al., Prog.Part.Nucl.Phys. (2021)
- **Light-cone sum rule**
  S. Cheng et.al. Phys.Rev.D 102 (2020)
- **Lattice calculation by LaMET**
  Zhang, et. al., Phys.Rev.D 95 (2017)
  R. Zhang et.al., Phys.Rev.D 102 (2020)
  J.Hua et.al(LPC)., Pev.Lett.127 (2021)

➢ Pion DA:

$$\int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle 0 | \bar{\psi}_1(0) n \cdot \gamma\gamma_5 U(0,\xi^-) \psi_2(\xi^-) | \pi(p) \rangle = i f_\pi \Phi_\pi(x)$$

$$\phi_\pi(x) = 6x(1-x) \sum_{n=1,2,\cdots} a^{\pi}_{2n-2} C^{(3/2)}_{2n-2}(2x-1)$$

Gegenbauer expansion

- **Sum rules limited to first few moments and large uncertainties**

  P. Ball et.al. PRD71,014015 (2005), P. Ball et.al. JHEP. 03069 (2007)

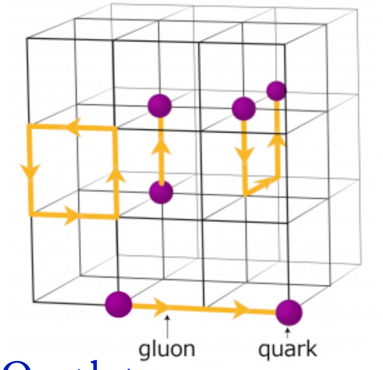
- **Solutions for DAs from Dyson-Schwinger equations depend on kernels**

  F. Gao, L. Chang et.al. PRD 90,014001 (2014), Craig D.et.al., PPNP.120, 103883 (2021)

- **Global fits rely on theoretical and experimental precisions**

  N. G. Stefanis PRD102.034022(2020), C.Jian, S.Chen and J.Hua EPJC83.7556(2022)

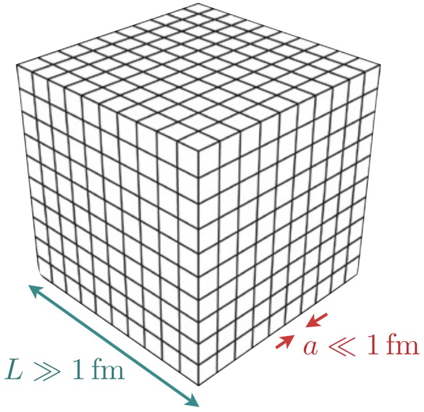
LQCD **is formulated as a Feynman path integral on a 4D Euclidean grid.**



Simulations provide a stochastic computation follows QCD Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{4}G^{a}_{\mu\nu}G^{a,\mu\nu}$$

- Gluon fields on links of a hypercube;
- Quark fields on sites: approaches to fermion discretization – Wilson, Staggered, Overlap;



☞ **Discrete: lattice spacing a → UV regulator; box length L → IR regulator;**

☞ **Derivatives: discretization errors ($a \to 0$); $\mathcal{O}(a)$ improved actions; ……**

☞ **Finite volume ($M_{\pi}L \to \infty$): FV errors exponentially small for $M_{\pi}L > 4$;**

☞ **Chiral extrapolation ($M_{\pi} \to 135\mathrm{MeV}$);**

☞ **Numerical importance sampling of path integral: statistical errors.**

LQCD Observables:

- Building blocks: ensembles of gauge configurations; quark propagators
- Hadron & interactions put in as external probes: N-point correlation function

LQCD Methodology:

☞ Generate gauge configurations;

☞ Calculate quark propagators on the gauge configurations;

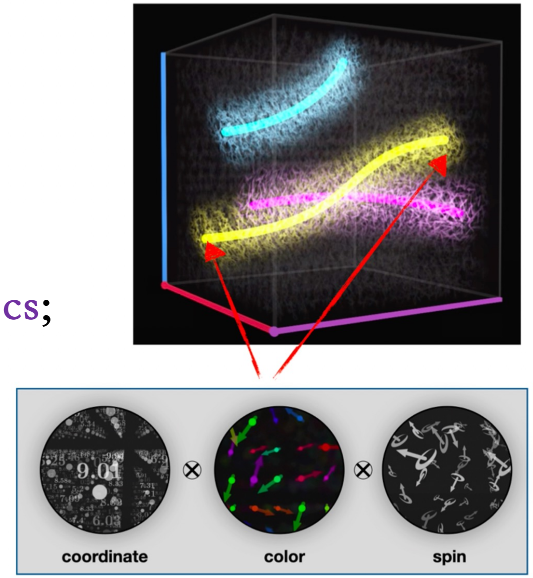
☞ Formulate operators that best probe the physics:

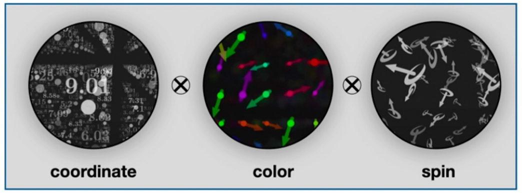
  - Low energy effective operators encapsulating SM/BSM physics;

☞ Construct hadronic correlation functions by the building blocks;

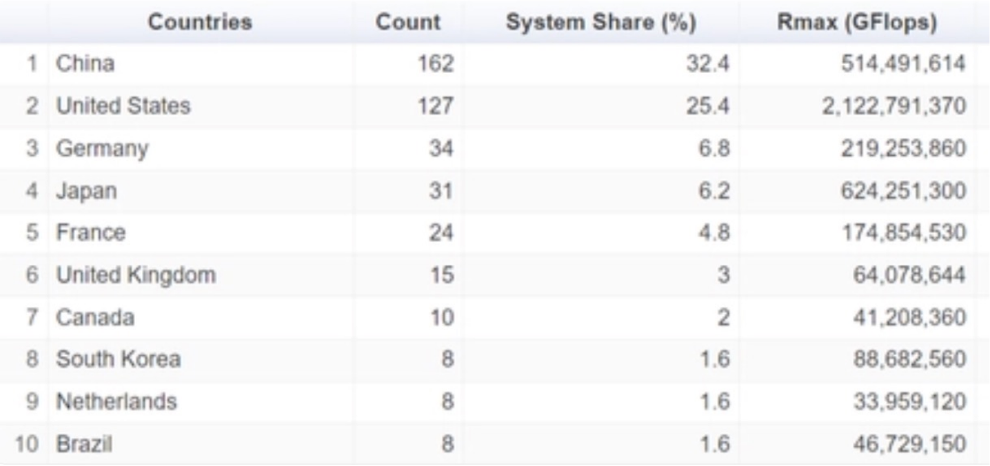
☞ Extract hadron ground states by reduction formula;

☞ Evaluate the hadronic matric elements.





Top 500



	Countries	Count	System Share (%)	Rmax (GFlops)
1	China	162	32.4	514,491,614
2	United States	127	25.4	2,122,791,370
3	Germany	34	6.8	219,253,860
4	Japan	31	6.2	624,251,300
5	France	24	4.8	174,854,530
6	United Kingdom	15	3	64,078,644
7	Canada	10	2	41,208,360
8	South Korea	8	1.6	88,682,560
9	Netherlands	8	1.6	33,959,120
10	Brazil	8	1.6	46,729,150

Millions of dimensions or even more !

Super computers

Lattice v.s. Continuum

We simulate:	We want:
😀 At finite lattice spacing a	🤔 $a \to 0$
😀 In finite volume L^3	🤔 $L \to \infty$
😀 Euclidean space	🤔 Minkowski space
😀 Lattice regularization	🤔 Some continuum scheme
😀 Some bare input quark masses: $am_l, \quad am_s, \quad am_c, \quad am_b$ In general, $m_\pi^{\text{lat}} \neq m_\pi^{\text{phy}}$	🤔 $m_q^{\text{lat}} = m_q^{\text{phy}}$

⇨ **Lost the real time information!**

- ➩ **Need to <u>control all limits</u>:** particularly simultaneously control FV and discretization
- ➩ **<u>Universality</u>:** different input parameters **must** give converge results.

- **Pion DA:**

$$\int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \left\langle 0 \middle| \bar{\psi}_1(0) n \cdot \gamma\gamma_5 U\left(0, \xi^-\right) \psi_2\left(\xi^-\right) \middle| \pi(p) \right\rangle = i f_\pi \Phi_\pi(x)$$

$$\phi_\pi(x) = 6x(1-x) \sum_{n=1,2,\cdots} a^\pi_{2n-2} C^{(3/2)}_{2n-2}(2x-1)$$

Gegenbauer expansion

- **Lattice by OPE limited to first few moments**

V.M.Braun et.al. PRD 92.014504 (2015) , V.M.Braun et.al. JHEP 04082 (2017),
(RQCD) G.S.Bali et.al. JHEP 08065 (2019)

- **Quasi-correlation(LaMET) allows access to entire x range, but not reliable near endpoints of x**

J.H.Zhang PRD95. 094514(2017), R.Zhang H.W.Lin et.al. PRD102. 094519(2020),
(LPC)J.Hua et.al. PRL127. 062002(2021), (LPC)J.Hua et.al. PRL129. 132001(2022)

Lattice with OPE

$$\langle \xi^n \rangle \equiv \int_0^1 dx(2x-1)^n \phi_\pi(x)$$

- The nonlocal operator can be defined as a generating function for renormalized local operators:

$$\bar{d}(z_2 n)\not{n}\gamma_5[z_2 n, z_1 n]u(z_1 n) = \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k!l!} n^\rho n^{\mu_1} \ldots n^{\mu_{k+l}} \mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}},$$

$$\mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} = \bar{d}(0)\boxed{\overleftarrow{D}_{(\mu_1}\ldots\overleftarrow{D}_{\mu_k}\overrightarrow{D}_{\mu_{k+1}}\ldots\overrightarrow{D}_{\mu_{k+l}}}\gamma_{\rho)}\gamma_5 u(0)\,.$$

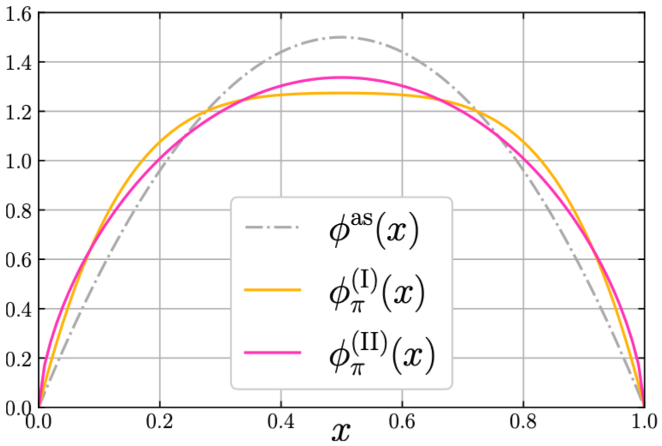
All enclosed Lorentz indices and the subtraction of traces

- Moments of the pion DA are given by matrix elements of local operators:

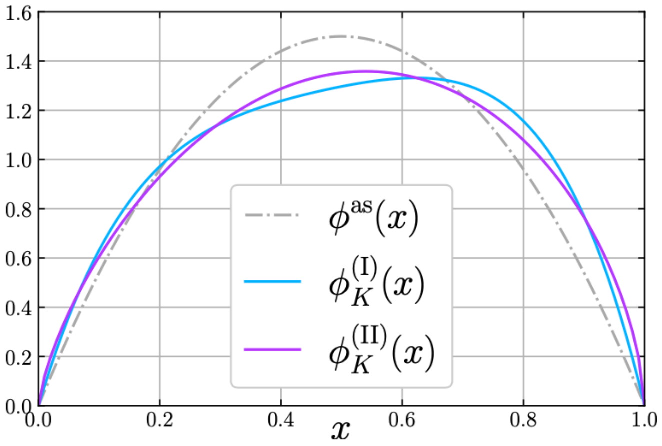
$$i^{k+l}\langle 0|\mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}}|\pi(p)\rangle = if_\pi p_{(\rho}p_{\mu_1}\ldots p_{\mu_{k+l})}\langle x^l(1-x)^k\rangle.$$

$l = k$ for pion

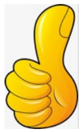
$a_2^\pi = 0.101^{+24}_{-24}$



$a_1^K = 0.0533^{+34}_{-35}$ $a_2^K = 0.090^{+19}_{-20}$



(RQCD) G.S.Bali et.al. JHEP 08065 (2019)



Very precise low order moment!

How to push high order moment ?

1. Operator mixing
2. Computing power

$\langle \xi^2 \rangle_\pi = 0.235^{+8}_{-8}$

$\langle \xi^4 \rangle_\pi = 0.109^{+5}_{-5} \Rightarrow a_4^\pi = 0.002^{+71}_{-71}$

Bad convergence problem

- **Gegenbauer moments:**

$$\phi_\pi(x) = 6x(1-x)\sum_{n=1,2,\cdots} a^\pi_{2n-2}C^{(3/2)}_{2n-2}(2x-1)$$

- **OPE moments:**

$$\langle \xi^n \rangle \equiv \int_0^1 dx(2x-1)^n \phi_\pi(x)$$

Limitations of OPE !

$$\begin{aligned}
a_0^\pi &= \langle\xi^0\rangle, \\
a_2^\pi &= \frac{7}{12}\left(5\langle\xi^2\rangle - \langle\xi^0\rangle\right), \\
a_4^\pi &= \frac{11}{24}\left(21\langle\xi^4\rangle - 14\langle\xi^2\rangle + \langle\xi^0\rangle\right), \\
a_6^\pi &= \frac{5}{64}\left(429\langle\xi^6\rangle - 495\langle\xi^4\rangle + 135\langle\xi^2\rangle - 5\langle\xi^0\rangle\right), \\
a_8^\pi &= \frac{19}{384}\left(2431\langle\xi^8\rangle - 4004\langle\xi^6\rangle + 2002\langle\xi^4\rangle - 308\langle\xi^2\rangle + 7\langle\xi^0\rangle\right), \\
a_{10}^\pi &= \frac{23}{1536}\left(29393\langle\xi^{10}\rangle - 62985\langle\xi^8\rangle + 46410\langle\xi^6\rangle - 13650\langle\xi^4\rangle + 1365\langle\xi^2\rangle - 21\langle\xi^0\rangle\right)
\end{aligned}$$

Huge coefficients !

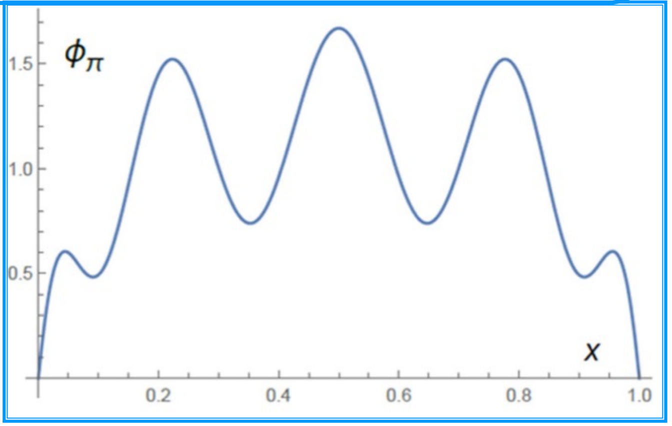
Good convergence

$$(\langle\xi^0\rangle, \langle\xi^2\rangle, \langle\xi^4\rangle, \langle\xi^6\rangle, \langle\xi^8\rangle, \langle\xi^{10}\rangle)|_{\mu=2\,\mathrm{GeV}} = (1, 0.254, 0.125, 0.077, 0.054, 0.041)$$

T.Zhong PRD104. 016021(2021)

Bad convergence

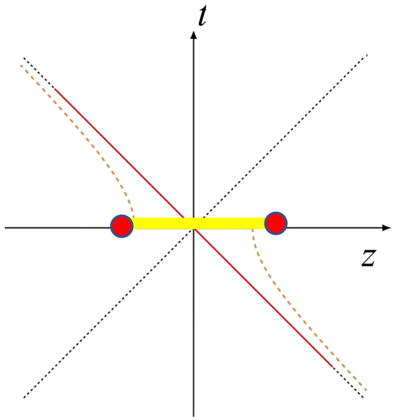
$$(a_0^\pi, a_2^\pi, a_4^\pi, a_6^\pi, a_8^\pi, a_{10}^\pi)|_{\mu=2\,\mathrm{GeV}} = (1, 0.157, 0.032, 0.035, 0.098, -0.046)$$



Hsiang-nan Li PRD106. 034015(2022)

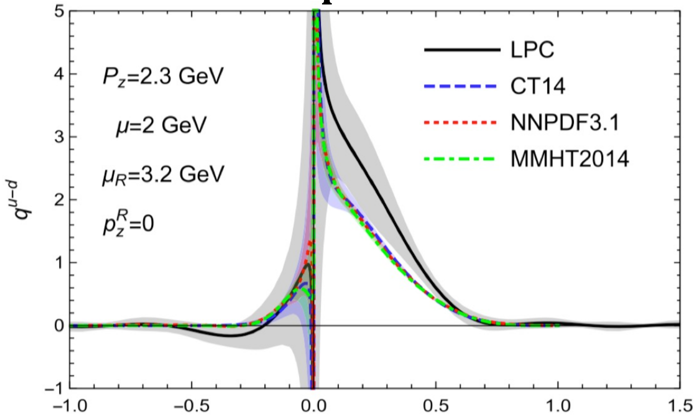
Large Momentum Effective Theory

- X. Ji, PRL 110 (2013);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and Y. Zhao, RMP 93 (2021).



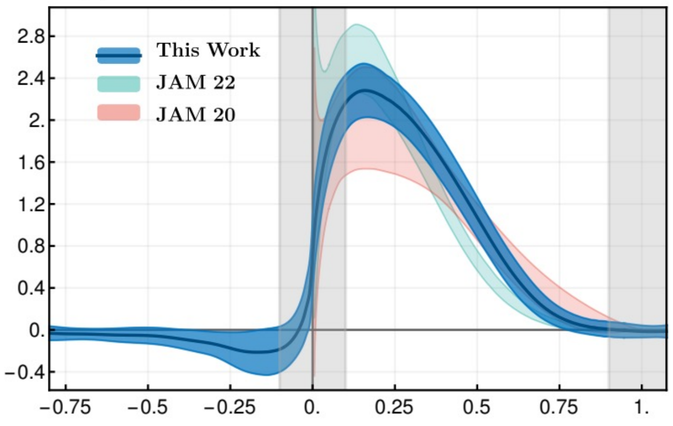
- Entire x dependence distributions

Proton Unpolarized PDF



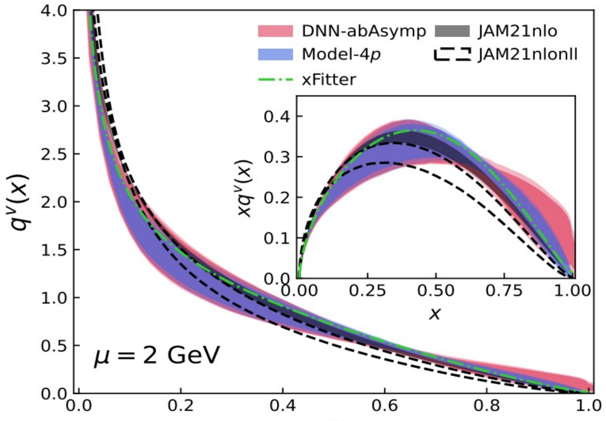
LPC, PRD101,034020 (2020)

Proton transversity PDF



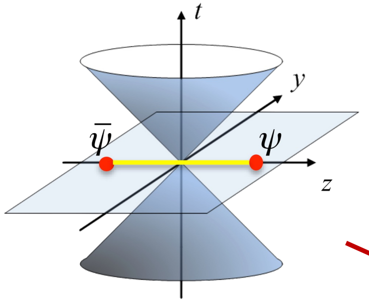
LPC, arXiv 2208.08008((2022)

Pion valancePDF

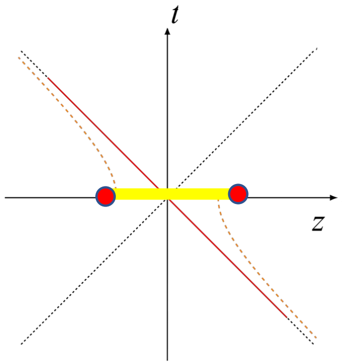


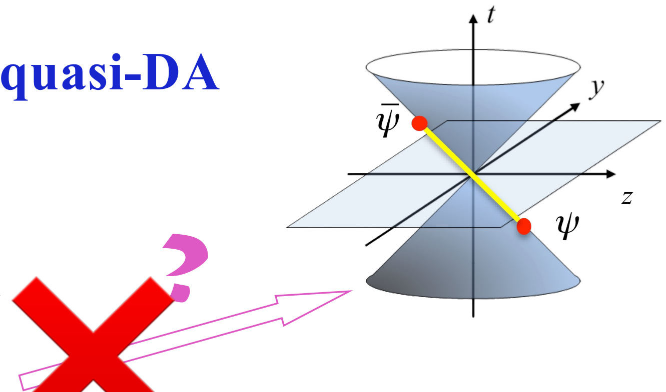
X.Gao, et.al. PRD106,114510 (2022)

➢ Define a lattice calculable, equal-time correlation: **quasi-DA**



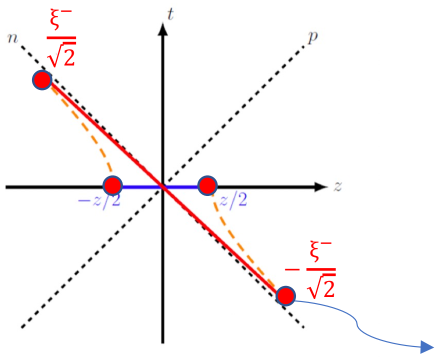
Boosting P^z





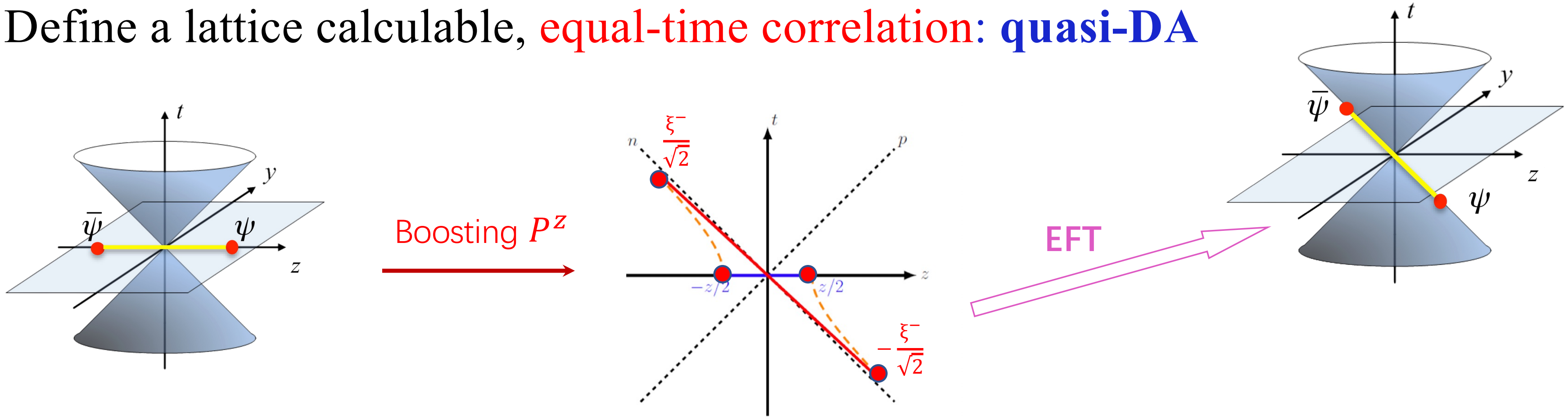
$\lim_{P^z} \tilde{q}(x, P^z) = q(x)$

- z^2 is Lorentz invariant, time-like correlator cannot become to light-like;
- In the large P^z limit, the deviation from light-cone is power suppressed by $m^2/(P^z)^2$ and $\Lambda^2/(P^z)^2$.



Power suppressed by $m^2/(P^z)$^2, $\Lambda^2/(P^z)$^2

- Define a lattice calculable, equal-time correlation: **quasi-DA**



- Effective field theory:
  - Instead of taking $P^z \to \infty$ calcuation, one can perform an expansion for large but finite P^z:

$$\tilde{q}(x, P^z, \mu) = \int \frac{dy}{|y|} \boxed{C(x, y, P^z, \mu)}\, q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2, M^2}{(P^z)^2}\right)$$

Quasi-DA LCDA

Matching kernel

- **Pion LCDA:**

I. Calculate the bare quasi-DA correlation

$$\tilde{h}(z,a,P_z) = \langle 0|\bar{\psi}_1(0)n_z\cdot\gamma\gamma_5 U(0,z)\psi_2(z)|\pi(P)\rangle$$

II. Non-perturbative renormalization

$$\tilde{h}(z,a,P_z) = Z(z,a)\tilde{h}_R(z,a,P_z)$$

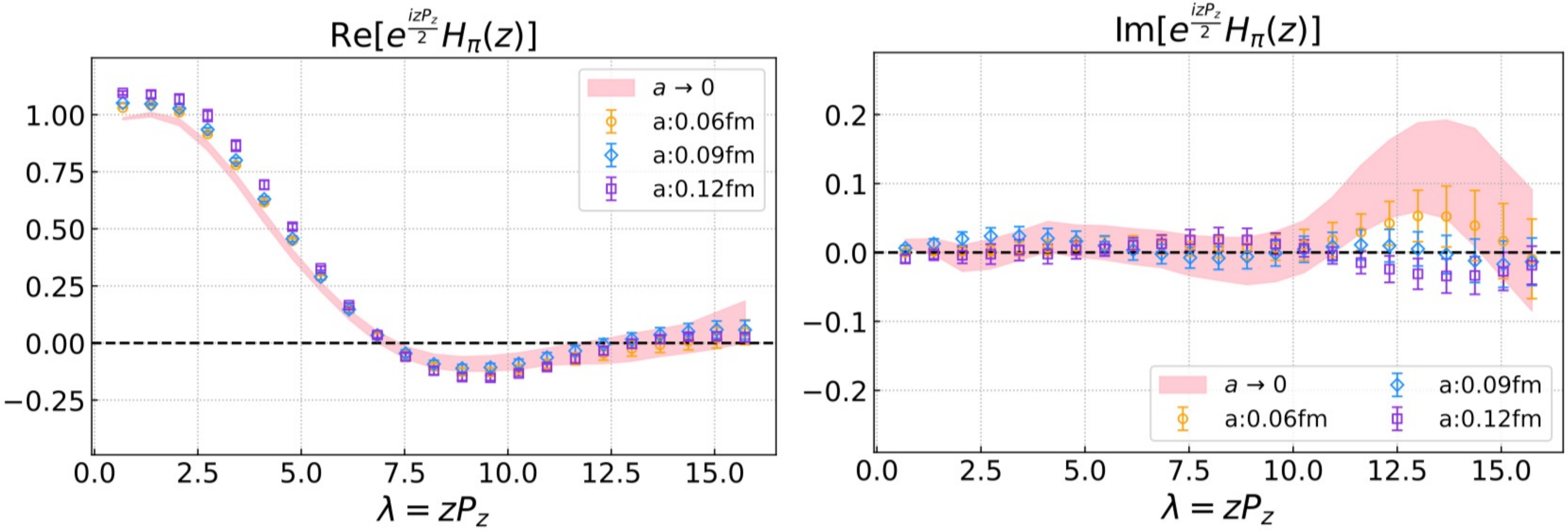
III. Fourier transform (Extrapolation)

$$if_\pi\tilde{\phi}_\pi(x,P_z) = \int \frac{d_z}{2\pi} e^{-ixzP_z}\tilde{h}_R(z,a\to 0,P_z)$$

IV. Matching to light clone

$$\tilde{\phi}_\pi(x,P_z) = \int dy\, Z(x,y,Pz,\mu)\phi(x,\mu) + p.c.$$

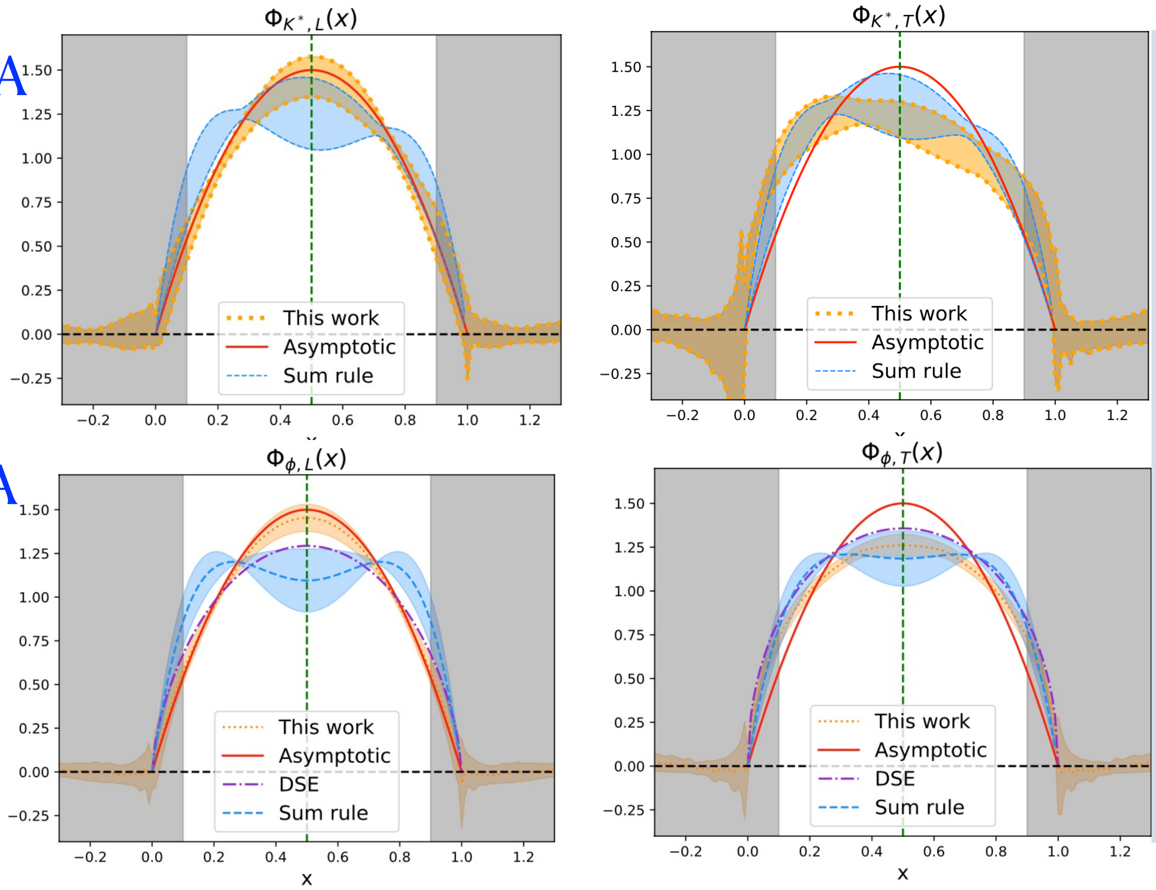
➢ Renormalized quasi-DA in coordinate space:



$P_z = 2.15GeV$ (LPC)J.Hua et.al. PRL129. 132001(2022)

K^* LCDA

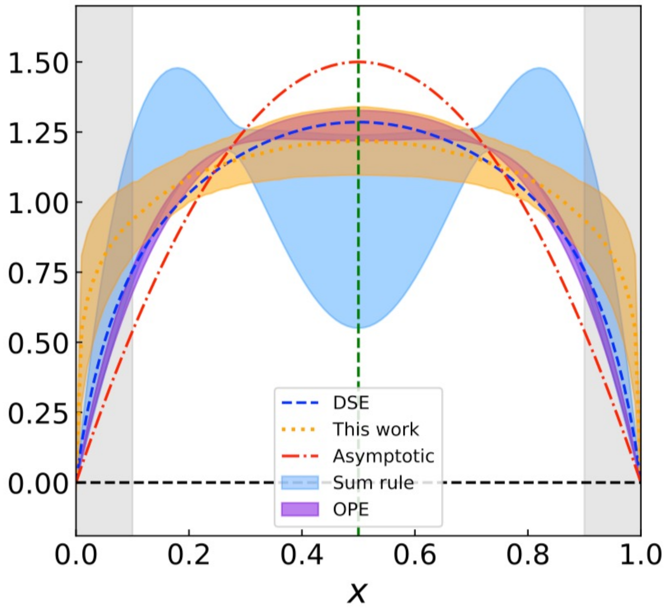
ϕ LCDA



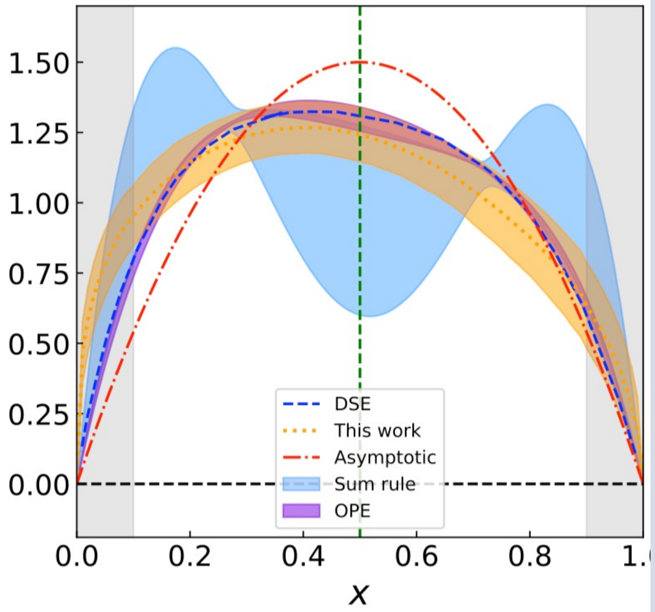
- 3 lattice spacings: (0.12, 0.09, 0.06) fm, largest volume ($96^3 \times 192$)
- 3 momentum: (1.29, 1.72, 2.15) GeV
- mass: K^*: 0.89GeV, ϕ: 1.02GeV
- Hybrid scheme (based on RI/MOM)

(LPC)J.Hua et.al. PRL127. 062002(2021)

π LCDA:



K LCDA:



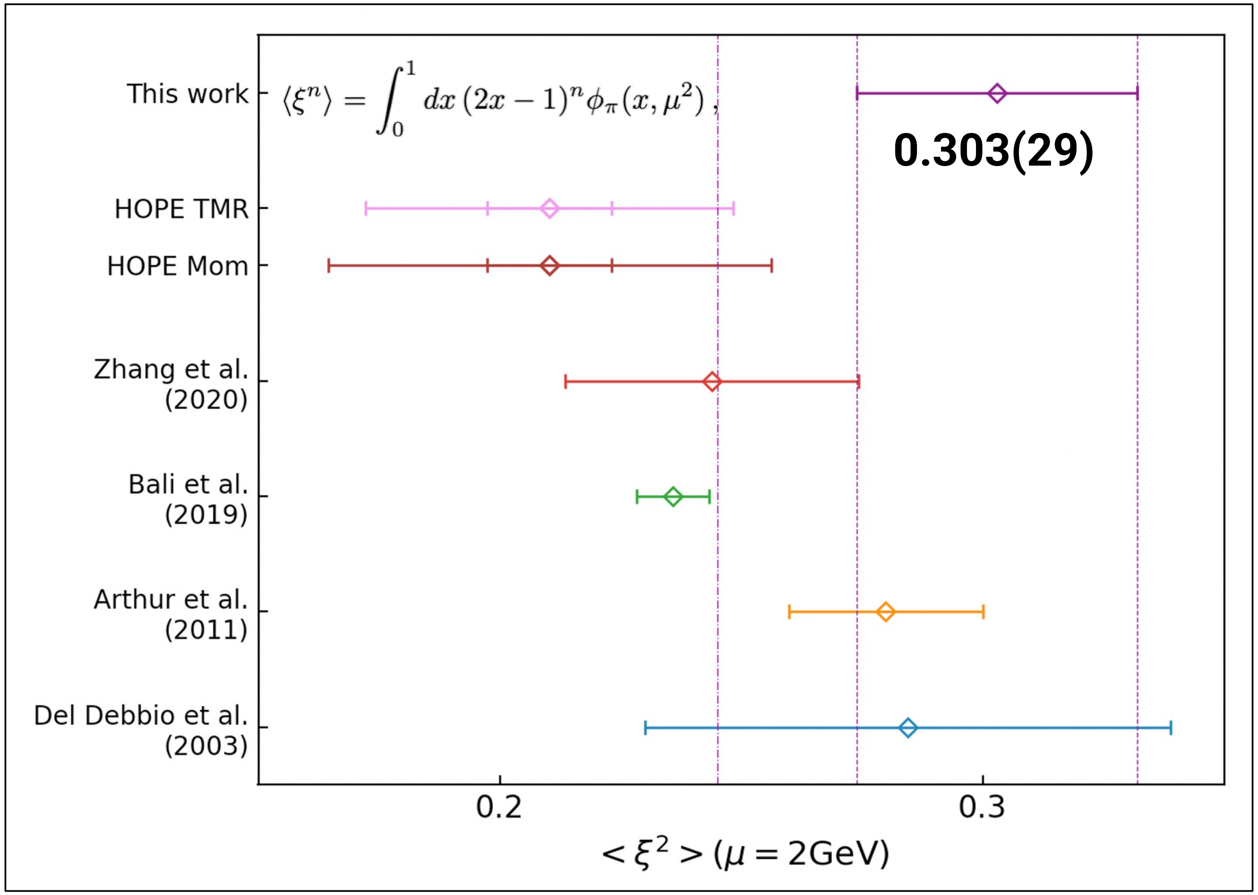
(LPC)J.Hua et.al. PRL129. 132001(2022)

- **3 lattice spacings: (0.12, 0.09, 0.06) fm, largest volume ($96^3\times192$)**
- **3 momentum: (1.29, 1.72, 2.15) GeV**
- **mass:** π: 0.13GeV, K: 0.49GeV
- **Hybrid scheme (Self renormalization)**

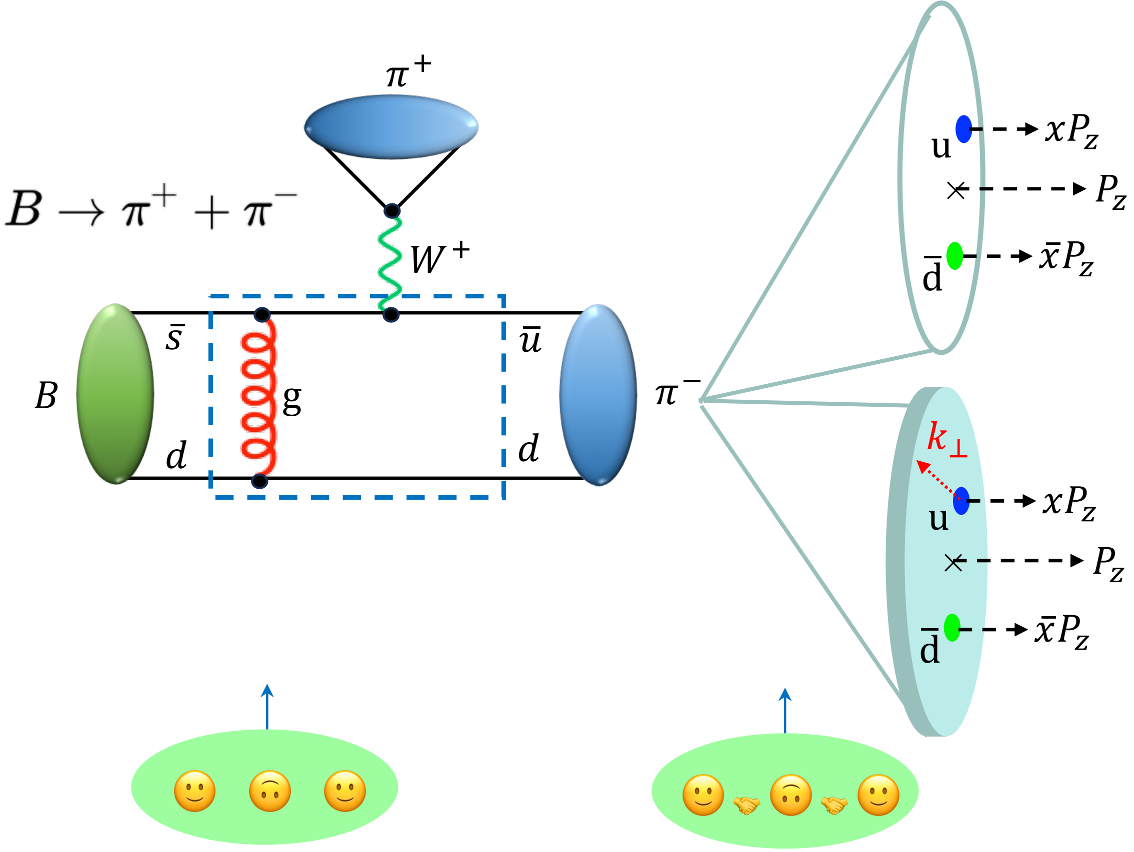
Some outlook

- ➢ **Moments tension between OPE and LaMET**
  - **High moments in OPE**
  - **Extend endpoint region in LaMET**
  - **Combine analysis**

Final target:
Precise and accurate LCDA



Three dimensional / TMD Structures



Collinear Factorization

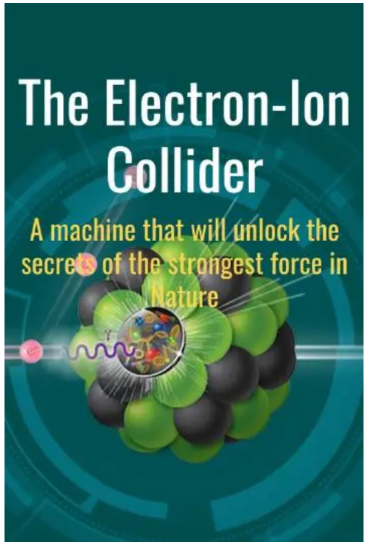
TMD Factorization

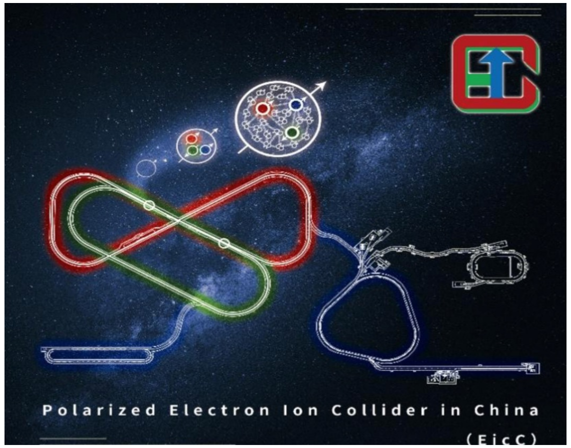
One dimensional LCDA

Three dimensional TMDWF

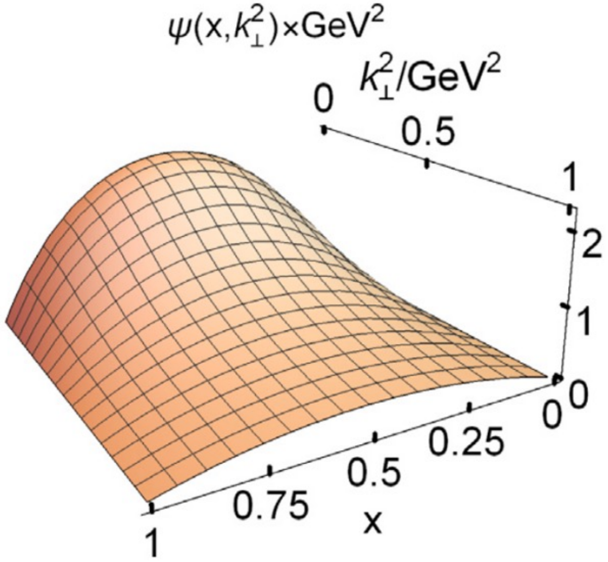
EIC &

EICC



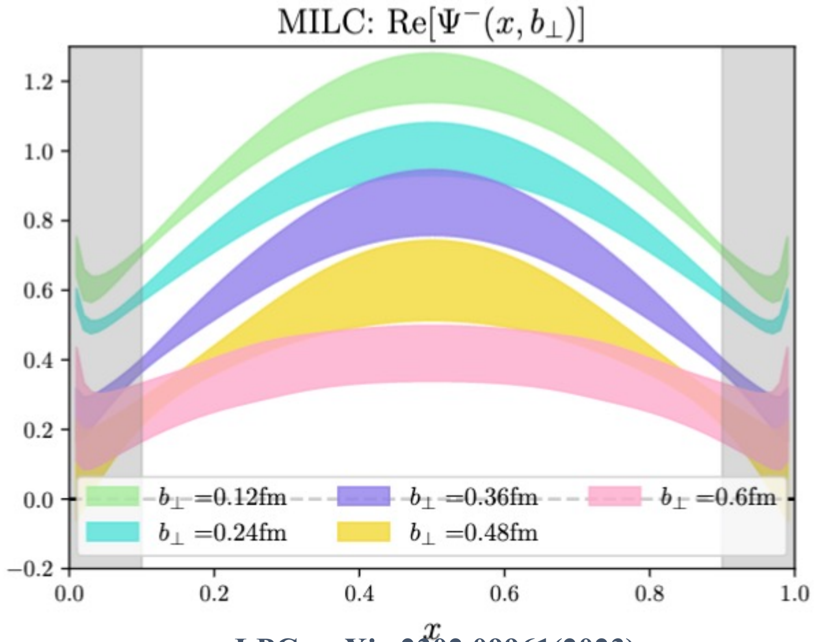


- **3-D curved surface（expected）with $x \to P_z$ and $k_\perp$**



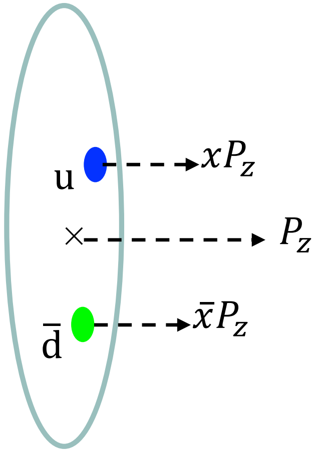
C.D.Roberts et.al. PPNP.120, 138883 (2021)

- **$b_\perp$ (FT → $k_\perp$)dependent TMDWF (currently available)**

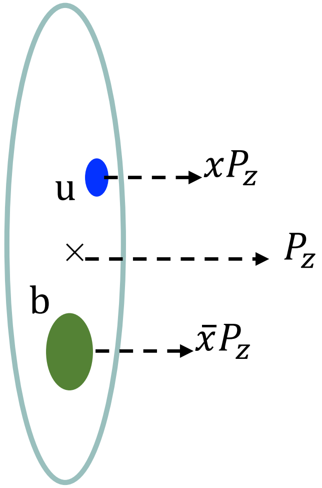


LPC, arXiv 2302.09961(2023)

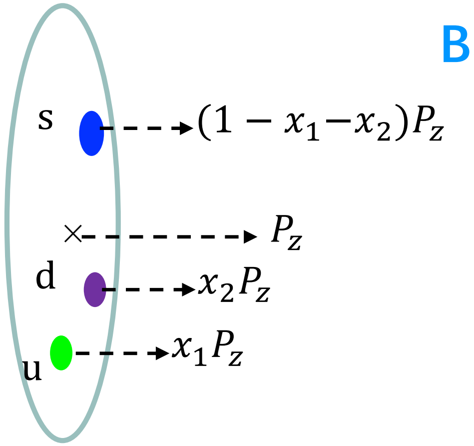
Light meson π/K

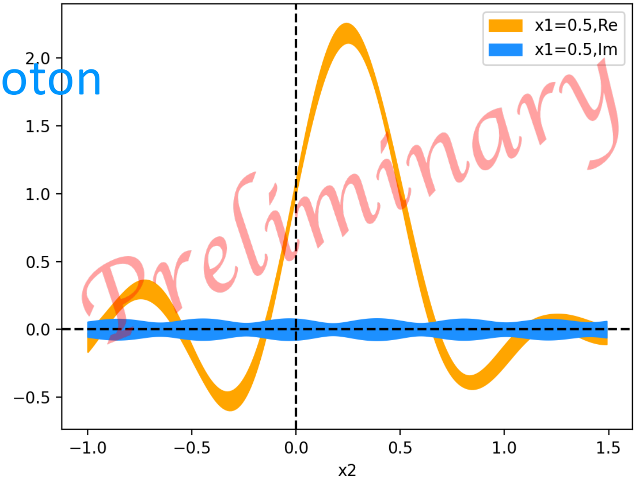


Heavy meson D/B



Baryon Λ, proton





Summary

＊ LCDA describes both the internal structure of the meson and is an important input to the exclusive process.

＊ Lattice QCD provides methods for numerically calculating the internal structure of hadrons from the first principle.

＊ We have multiple methods for calculating LCDA on lattice(OPE, LaMET). The next target is to get an precisely calculated LCDA.

＊ From the LCDA, there are many topics need to move towards, TMDWF, baryon LCDA, heavy meson LCDA …

Thanks for your attentions!