第五届粒子物理天问论坛

Light scalar meson LCDAs and semileptonic decays within QCD sum rule approach

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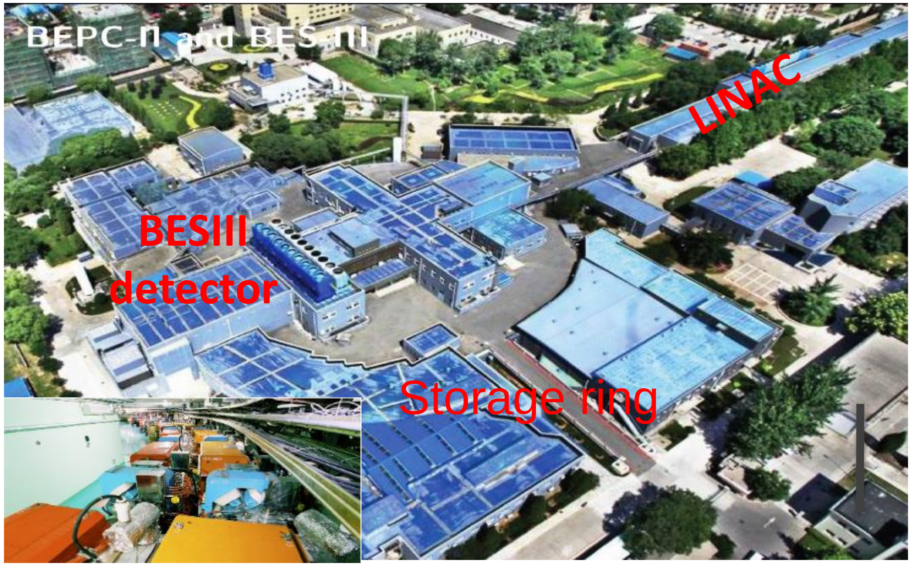
湖南师范大学 2023.11.11

Outline

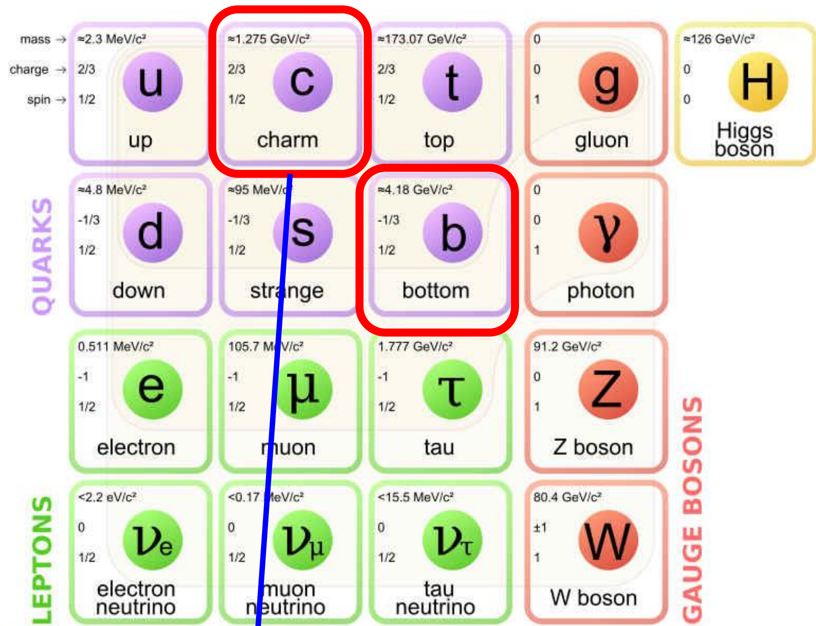
- Motivations
- a0(980) - meson twist-2 DA within QCDSR
- K0*(1430) twist-2 DA and semileptonic decays
- Summary

Run 3 (2022-)



another
7 years

I. Motivation



Standard_Model_of_Elementary_Particles.svg (SVG file, nominally 774 × 581 pixels, file size: 419 KB)

B/D factory
LHCb
Belle-II
BESIII

Pure leptonic and semileptonic D/B decay

- **Decay constant**
- **Transition form factors**
- **CKM matrix element**
- **Decay width and anomalous**
- **Other Observables**

$$U = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{tb} & V_{ts} & V_{tb} \end{pmatrix}$$

1. Scalar meson a0(980)

PRL 121 (2018), 081802

Observation of the Semileptonic Decay $D^0 \to a_0(980)^- e^+ \nu_e$ and Evidence for $D^+ \to a_0(980)^0 e^+ \nu_e$

Using an e^+e^- collision data sample of 2.93 fb^{-1} collected at a center-of-mass energy of 3.773 GeV by the BESIII detector at BEPCII, we report the observation of $D^0 \to a_0(980)^- e^+ \nu_e$ and evidence for $D^+ \to a_0(980)^0 e^+ \nu_e$ with significances of 6.4σ and 2.9σ, respectively. The absolute branching fractions are determined to be $\mathcal{B}(D^0 \to a_0(980)^- e^+ \nu_e) \times \mathcal{B}(a_0(980)^- \to \eta\pi^-) = (1.33^{+0.33}_{-0.29}(\text{stat}) \pm 0.09(\text{syst})) \times 10^{-4}$ and $\mathcal{B}(D^+ \to a_0(980)^0 e^+ \nu_e) \times \mathcal{B}(a_0(980)^0 \to \eta\pi^0) = (1.66^{+0.81}_{-0.66}(\text{stat}) \pm 0.11(\text{syst})) \times 10^{-4}$. An upper limit of $\mathcal{B}(D^+ \to a_0(980)^0 e^+ \nu_e) \times \mathcal{B}(a_0(980)^0 \to \eta\pi^0) < 3.0 \times 10^{-4}$ is also determined at the 90% confidence level.

TABLE II. The relative systematic uncertainties (in %) on the branching fraction measurements. Items marked with $*$ are derived from the fit procedure and are not used when evaluating the upper limit of the branching fraction.

Source	$D^0 \to a_0(980)^- e^+ \nu_e$	$D^+ \to a_0(980)^0 e^+ \nu_e$
Tracking	2.0	1.0
π PID	0.5	-
π^0 reconstruction	-	1.0
η reconstruction	1.0	1.0
Positron PID	0.6	0.6
The best $\eta\pi^0$ combination	-	0.3
Lateral moment requirement	1.6	1.6
Form factor model	5.3	5.6
η and π^0 branching fraction	0.5	0.5
MC statistics	0.6	0.9
*U resolution	2.7	1.1
*$a_0(980)$ line shape	0.2	0.3
*Background modeling	0.3	2.0
Total	6.7	6.6

$$\mathcal{B}(D^+ \to a_0(980)^0 e^+ \nu_e) \times \mathcal{B}(a_0(980)^0 \to \eta\pi^0) = (1.66^{+0.81}_{-0.66} \pm 0.11) \times 10^{-4},$$

$$\frac{\Gamma(D^0 \to a_0(980)^- e^+ \nu_e)}{\Gamma(D^+ \to a_0(980)^0 e^+ \nu_e)} = 2.03 \pm 0.95 \pm 0.06,$$

consistent with the prediction of isospin symmetry, where the shared systematic uncertainties have been canceled. The two branching fractions provide information about the $d\bar{u}$ and $(u\bar{u} - d\bar{d})/\sqrt{2}$ components in the $a_0(980)^-$ and $a_0(980)^0$ wave functions, respectively [4]. Along with the result of the branching fraction of $D^+ \to f_0 e^+ \nu_e$, a result in preparation at BESIII, we will have a valuable input for understanding the nature of the light scalar mesons.

2. f0(980)

Study of the $f_0(980)$ through the decay $D_s^+ \to \pi^+\pi^- e^+\nu_e$

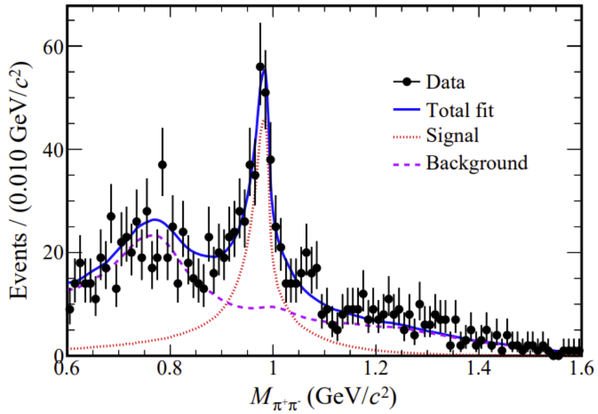


FIG. 1. Fit to the $M_{\pi^+\pi^-}$ distribution of the accepted candidates for the decay $D_s^+ \to f_0(980)e^+\nu_e$. The points with error bars are data, and the blue line is the total fit. The red dotted and violet dashed lines are the signal and background shapes, respectively.

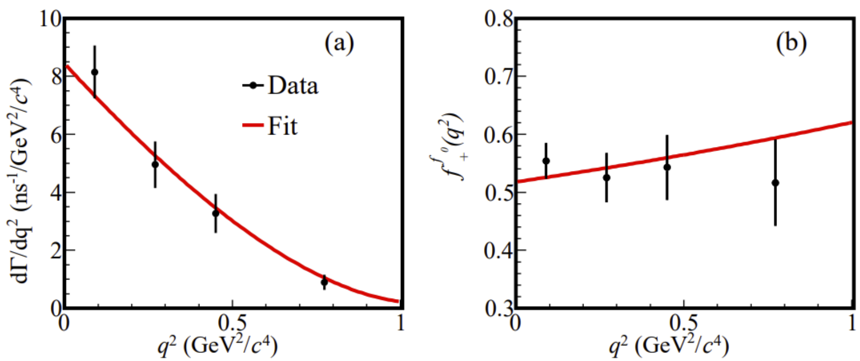


FIG. 2. Fit to the differential decay rate as a function of q^2 (a) and projection to the FF $f_+^{f_0}(q^2)$ (b). The points with error bars are data, and the red line is the fit.

In summary, using e^+e^- collision data corresponding to an integrated luminosity of 7.33 fb^{-1} collected at $E_{\rm CM} = 4.128 - 4.226$ GeV by the BESIII detector, we measure the BF of $D_s^+ \to f_0(980)e^+\nu_e$, $f_0(980) \to \pi^+\pi^-$ decay to be $(1.72 \pm 0.13_{\rm stat} \pm 0.10_{\rm syst}) \times 10^{-3}$, which is 2.6 times more accurate than the previous measurement [23]. Using the relation between the BF and the mixing angle ϕ involved in the $q\bar{q}$ mixture picture for $f_0(980)$ as $\sin\phi \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) + \cos\phi s\bar{s}$ [8, 11], we find that the $s\bar{s}$ component is dominant. This conclusion disagrees with that in Ref. [11] where their calculation is based on the CLEO measurement [21].

arXiv: 2303.12927

3. a0(980) structure

Scalar meson under 1GeV

quark-antiquark
tetraquark states
two-meson molecule bound states
hybrid states

64. Scalar Mesons below 1 GeV

Revised August 2021 by S. Eidelman (Budker Inst., Novosibirsk; Novosibirsk U.), T. Gutsche (Tübingen U.), C. Hanhart (Jülich), R.E. Mitchell (Indiana U.) and S. Spanier (Tennessee U.).

PDG

64.1 Introduction

In contrast to the vector and tensor mesons, the identification of the scalar mesons is a long-standing puzzle. Scalar resonances are difficult to resolve because some of them have large decay widths, which cause a strong overlap between resonances and background. In addition, in some cases, several decay channels open up within a short mass interval (*e.g.* at the $K\bar{K}$ and $\eta\eta$ thresholds), producing cusps in the line shapes of the nearby resonances. Furthermore, one expects non-$q\bar{q}$ scalar objects, such as hadronic molecules and multiquark states, in the mass range of interest (for reviews see, *e.g.*, Refs. [1–5]).

3. a0(980) structure

(1) four-quark scenario

$$a_0^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})s\bar{s}, \qquad a_0^+ = u\bar{d}s\bar{s}, \qquad a_0^- = d\bar{u}s\bar{s},$$

PRD 15 (1977) 267
PRD 15 (1977) 281.
NPB 578 (2000) 367 – *Lattice*
PRD 97 (2018) 034506 -- Lattice
PLB 608 (2005) 69 – QCD sum rule
…

Long-Standing Puzzle

(2) two-quark states

$$a_0^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \qquad a_0^+ = u\bar{d}, \qquad a_0^- = d\bar{u},$$

PRD 73 (2006)014017 -- QCD sum rule
PRD 102 (2020) 016013 -- CCQM
EPJC 80 (2020) 554
PRL 121 (2018) 081802 – BES III
EPJC 82 (2022) 473 – AdS/QCD
PRD 99 (2019) 093007 – pQCD
...

4. Scalar meson twist-2 DA

$$\langle 0|\bar{q}_1(z)\gamma_\mu q_2(-z)|S(p)\rangle = p_\mu \bar{f}_S \int_0^1 dx e^{i(2u-1)(p\cdot z)} \phi_{2;S}(x,\mu)$$

$$\phi_{2;S}(x,\mu) = 6x\bar{x}\left[a^0_{2;S}(\mu) + \sum_{n=1}^{\infty} a^n_{2;S}(\mu) C^{3/2}_n(\xi)\right]$$

$$\langle \xi^n_{2;S}\rangle|_\mu \longleftrightarrow a^n_{2;S}(\mu)$$

State	$\langle\xi\rangle$	$\langle\xi^3\rangle$
$a_0(980)$	-0.56 ± 0.05	-0.21 ± 0.03
$a_0(1450)$	0.53 ± 0.20	0.00 ± 0.04
$f_0(980)$	-0.47 ± 0.05	-0.20 ± 0.03
$f_0(1500)$	0.48 ± 0.24	-0.05 ± 0.04
$\kappa(800)$	-0.55 ± 0.07	-0.21 ± 0.05
$K_0^*(1430)$	0.35 ± 0.07	-0.08 ± 0.06

$$\begin{aligned}
&\langle \xi^l_S\rangle m_S \bar{f}_S^2 e^{-m_S^2/M^2} + \langle \xi^l_{S'}\rangle m_{S'} \bar{f}_{S'}^2 e^{-m_{S'}^2/M^2} \\
&= \Bigg\{ -\frac{3}{16\pi^2} M^2 \left(\frac{m_{q_2}+m_{q_1}}{l+2} + \frac{m_{q_2}-m_{q_1}}{l+1}\right) f(0) + \langle \bar{q}_2 q_2\rangle + \frac{10l-3}{24}\frac{\langle \bar{q}_2 g_s\sigma\cdot G q_2\rangle}{M^2} \\
&\quad + \frac{l(4l-5)}{36}\frac{\langle g_s^2 G^2\rangle\langle \bar{q}_2 q_2\rangle}{M^4} + (-1)^{l+1}\Bigg[-\frac{3}{16\pi^2} M^2 \left(\frac{m_{q_2}+m_{q_1}}{l+2} - \frac{m_{q_2}-m_{q_1}}{l+1}\right) f(0) \\
&\quad + \langle \bar{q}_1 q_1\rangle + \frac{10l-3}{24}\frac{\langle \bar{q}_1 g_s\sigma\cdot G q_1\rangle}{M^2} + \frac{l(4l-5)}{36}\frac{\langle g_s^2 G^2\rangle\langle \bar{q}_1 q_1\rangle}{M^4}\Bigg]\Bigg\},
\end{aligned}$$

Hai-Yang Cheng, PRD73 (2006) 014017

I. Motivation

$$\phi_{2;a_0}(x,\mu) = 6x\bar{x}\left[a^0_{2;a_0}(\mu) + \sum_{n=1}^{\infty} a^n_{2;a_0}(\mu) C^{3/2}_n(\xi)\right]$$

⇓

$$a^0_{2;a_0}, a^1_{2;a_0}, a^2_{2;a_0}, a^3_{2;a_0}, \ldots$$

⇓

$$\xi^0_{2;a_0}, \xi^1_{2;a_0}, \xi^2_{2;a_0}, \xi^3_{2;a_0}, \ldots$$

Anti-symmetry

$$\xi^1_{2;a_0}, \xi^3_{2;a_0}, \ldots$$

⇓

QCD sum rule approach

⇓

Background field theory

1. Making the calculated physical quantity independent of the gauge
2. Theory be invariant under the background field gauge
3. Propagators in momentum space can be treated directly

1. Correlation function

$$\langle 0|\bar{q}_1(z)\gamma_\mu q_2(-z)|a_0(p)\rangle = p_\mu \bar{f}_{a_0}\int_0^1 dx e^{i(2u-1)(p\cdot z)}\phi_{2;a_0}(x,\mu),$$

$$\langle 0|\bar{q}_1(z)q_2(-z)|a_0(p)\rangle = m_{a_0}\bar{f}_{a_0}\int_0^1 dx e^{i(2u-1)(p\cdot z)}\phi^p_{3,a_0}(x,\mu),$$

Charge conjugation invariance

$$\phi_{2;a_0}(x,\mu) = -\phi_{2;a_0}(1-x,\mu) \text{ and } \int_0^1 dx\phi_{2;a_0}(x,\mu) = 0$$

$$\int_0^1 dx\phi^p_{3;a_0}(x,\mu) = 1$$

$$\Pi^{(n,0)}_{2;a_0}(z,q) = i\int d^4x e^{iq\cdot x}\langle 0|T\{J^V_n(x), J^{S,\dagger}_0(0)\}|0\rangle = (z\cdot q)^{n+1}I_{2;a_0}(q^2),$$

$$J^V_n(x) = \bar{q}_1(x)\not{z}(iz\cdot\overleftrightarrow{D})^n q_2(x)$$

$$J^S_0(0) = \bar{q}_1(0)q_2(0),$$

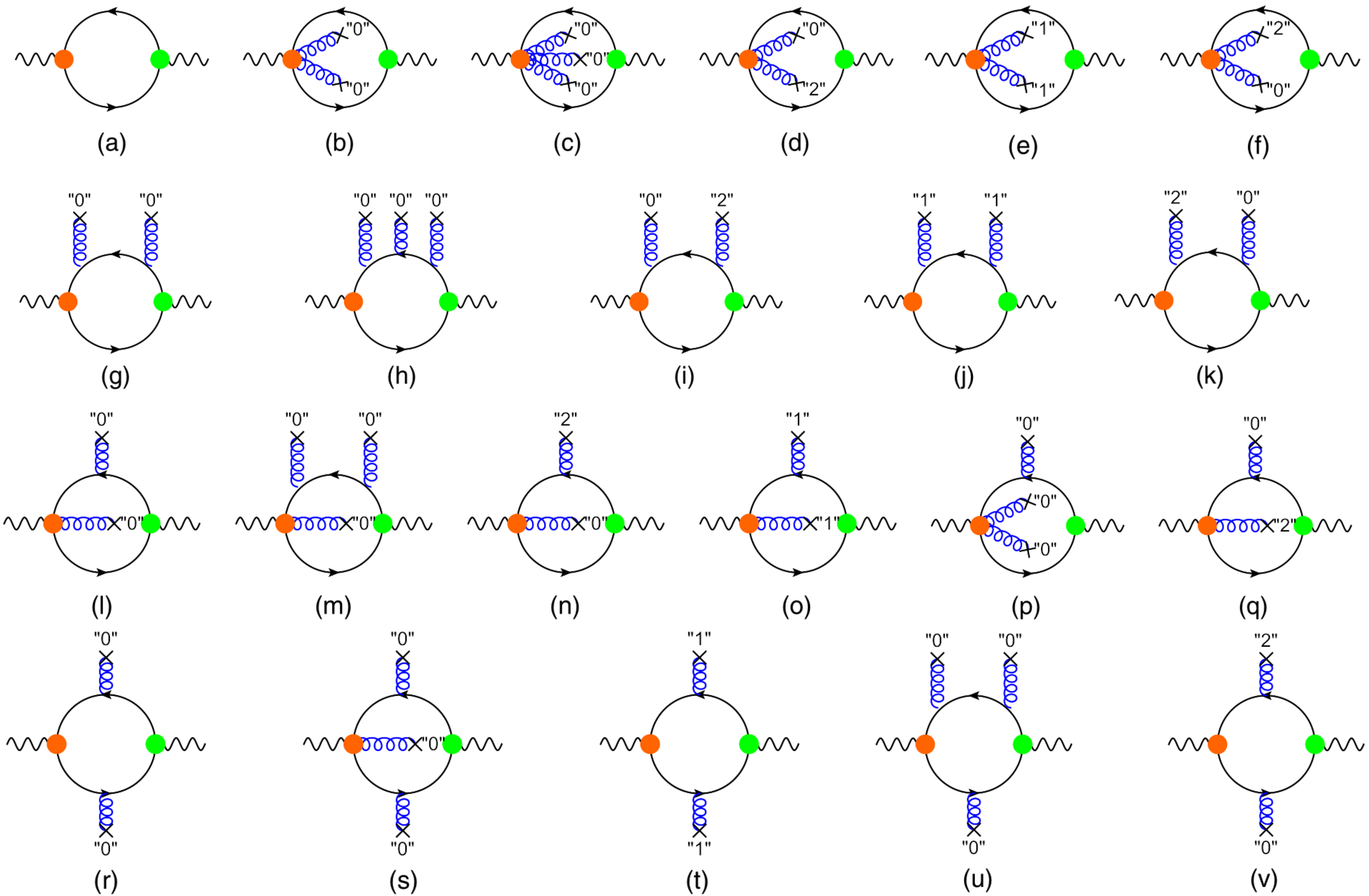
2. Operator product expansion(OPE)

$$
\begin{aligned}
\Pi^{(n,0)}_{2;a_0}(z,q) &= i\int d^4x e^{iq\cdot x}\langle 0|T\{\bar q_1(x)\not z(iz\cdot\overleftrightarrow{D})^n q_2(x), \bar q_2(0)q_1(0)\}|0\rangle \\
&\Downarrow q_{1,2}\to q_{1,2}+\eta_{1,2} \\
&= i\int d^4x e^{iq\cdot x}\langle 0|T\{[\bar q_1(x)+\bar\eta_1(x)]\not z(iz\cdot\overleftrightarrow{D})^n[q_2(x)+\eta_2(x)], \\
&\qquad [\bar q_2(0)+\bar\eta_2(0)][q_1(0)+\eta_1(0)]\}|0\rangle
\end{aligned}
$$

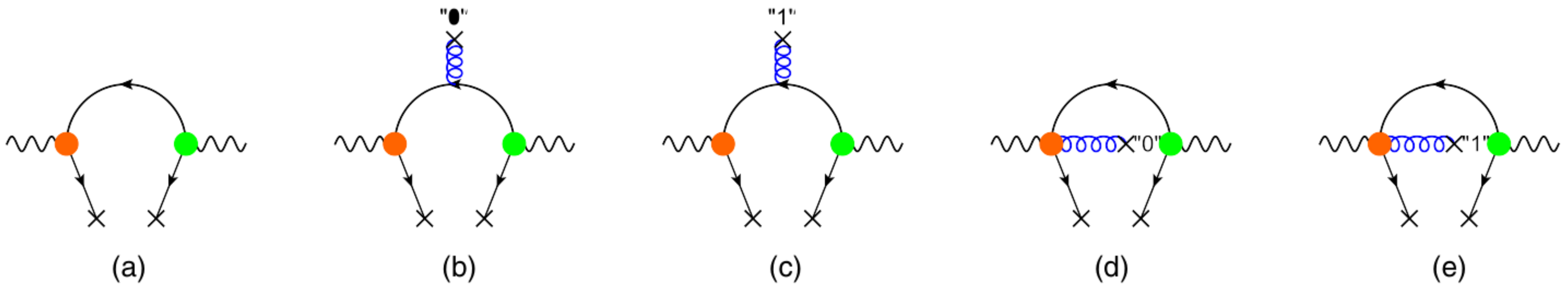
$$
\begin{aligned}
\Pi^{(n,0)}_{2;a_0}(z,q) = i\int d^4x e^{iq\cdot x}\Big\{ &-\mathrm{Tr}\langle 0|S_F^{q_1}(0,x)\not z(iz\cdot\overleftrightarrow{D})^n S_F^{q_2}(x,0)|0\rangle \\
&+\mathrm{Tr}\langle 0|\bar q_1(x)q_1(0)\not z(iz\cdot\overleftrightarrow{D})^n S_F^{q_2}(x,0)|0\rangle \\
&+\mathrm{Tr}\langle 0|S_F^{q_1}(0,x)\not z(iz\cdot\overleftrightarrow{D})^n \bar q_2(x)q_2(0)|0\rangle+\cdots\Big\}
\end{aligned}
$$

(1) Feynman diagrams for the first term

Phys. Rev. D 90, (2014) 016004



(2) Feynman diagrams for the second and third terms



Basic calculation procedure:

- Substitution and trace of vertex operator and propagator
- Momentum integrals in D-dimensional space
- Renormalization and $\overline{MS}$-schemes
- Borel transform.

(3). Propagator up to six-dimension

$$S_F^0(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\Big[-\frac{m+\not p}{m^2-p^2}\Big],$$

$$S_F^2(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\Big[-\frac{i}{2}\frac{\gamma^\mu(m-\not p)\gamma^\nu}{(m^2-p^2)^2}G_{\mu\nu}\Big],$$

$$S_F^3(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\Big\{-\frac{2}{3}\Big[\frac{(\gamma^\mu p^\rho+\gamma^\rho p^\mu)(m-\not p)}{(m^2-p^2)^3} - \frac{g^{\mu\rho}}{(m^2-p^2)^2}\Big]\gamma^\nu G_{\mu\nu;\rho}\Big\},$$

$$S_F^{4(1)}(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\Big\{\frac{1}{4}\Big[\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^3} - \frac{2p^\mu}{(m^2-p^2)^3}\Big]\gamma^\nu\gamma^\rho\gamma^\sigma + \frac{1}{2}\Big[\frac{(m+\not p)\gamma^\mu}{(m^2-p^2)^3}\, g^{\nu\sigma} + 4\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^4}p^\nu p^\sigma\Big]\gamma^\rho\Big\}G_{\mu\nu}G_{\rho\sigma},$$

$$S_F^{4(2)}(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\Big\{\frac{i}{4}\Big[\frac{g^{\{\mu\rho}\gamma^{\sigma\}}(m-\not p)}{(m^2-p^2)^3} - \frac{2g^{\{\mu\rho}p^{\sigma\}}}{(m^2-p^2)^3} + 4\frac{\gamma^{\{\mu}p^\rho p^{\sigma\}}(m-\not p)}{(m^2-p^2)^4}\Big]\gamma^\nu \times G_{\mu\nu;\rho\sigma}\Big\},$$

$$S_F^{5(1)}(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\Big\{-\frac{i}{3}\Big[\Big(3\frac{\gamma^\mu(m-\not p)\gamma^\nu}{(m^2-p^2)^4}(p^\lambda\gamma^\rho+p^\rho\gamma^\lambda) - \frac{\gamma^\nu(g^{\mu\lambda}\gamma^\rho+g^{\mu\rho}\gamma^\lambda)}{(m^2-p^2)^3}\Big) \cdot\gamma^\sigma + 4\Big(\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^4}g^{\{\nu\sigma}p^{\lambda\}} + 2\frac{p^\mu g^{\{\nu\sigma}p^{\lambda\}}}{(m^2-p^2)^4} + 6\,\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^5}p^\nu p^\sigma p^\lambda\Big)\gamma^\rho\Big]\,G_{\mu\nu} \times G_{\rho\sigma;\lambda}\Big\},$$

(3). Propagator up to six-dimension

$$S_F^{5(2)}(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\bigg\{\frac{2i}{3}\bigg[\bigg(\frac{g^{\mu\lambda}}{(m^2-p^2)^3} + 6\frac{p^\mu p^\lambda}{(m^2-p^2)^4}\bigg)\gamma^\nu\gamma^\rho\gamma^\sigma - 2\,\bigg(\frac{\gamma^\mu(m-\not{p})}{(m^2-p^2)^4} \times g^{\{\nu\sigma}p^{\lambda\}} + 2\frac{p^\mu g^{\{\nu\sigma}p^{\lambda\}}}{(m^2-p^2)^4} + 6\frac{\gamma^\mu(m-\not{p})}{(m^2-p^2)^5}p^\nu p^\sigma p^\lambda\bigg)\gamma^\rho\bigg]G_{\mu\nu;\lambda}G_{\rho\sigma}\bigg\},$$

$$S_F^{5(3)}(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\ \frac{4}{15}\bigg[\frac{g^{\{\rho\sigma}p^\lambda\gamma^{\mu\}}(m-\not{p})}{(m^2-p^2)^4} - \frac{2g^{\{\rho\sigma}p^\lambda p^{\mu\}}}{(m^2-p^2)^4} + 6\frac{\gamma^{\{\mu}p^\rho p^\sigma p^{\lambda\}}(m-\not{p})}{(m^2-p^2)^5} - \frac{g^{(\mu\nu\sigma\lambda)}}{(m^2-p^2)^3}\bigg]\gamma^\nu G_{\mu\nu;\rho\sigma\lambda},$$

$$S_F^{6(1)}(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\frac{i}{8}\bigg\{\bigg[\frac{\gamma^\mu(m-\not{p})}{(m^2-p^2)^4} - 4\frac{p^\mu}{(m^2-p^2)^4}\bigg]\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\lambda\gamma^\tau + 2\bigg[\frac{3\gamma^\mu(m-\not{p})}{(m^2-p^2)^4} \times g^{\sigma\tau} + 16\frac{\gamma^\mu(m-\not{p})}{(m^2-p^2)^5}p^\sigma p^\tau - 4\frac{g^{\mu\sigma}p^\tau + g^{\mu\tau}p^\sigma}{(m^2-p^2)^4}\bigg]\gamma^\nu\gamma^\rho\gamma^\lambda\bigg\}G_{\mu\nu}G_{\rho\sigma}G_{\lambda\tau},$$

$$S_F^{6(2)}(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\bigg(-\frac{1}{8}\bigg)\bigg\{\bigg[3\frac{\gamma^\mu(m-\not{p})\gamma^\nu}{(m^2-p^2)^4}g^{\{\lambda\tau}\gamma^{\rho\}} + 16\frac{\gamma^\mu(m-\not{p})\gamma^\nu}{(m^2-p^2)^5}\ \gamma^{\{\rho}p^\lambda p^{\tau\}} - 4\frac{\gamma^\nu}{(m^2-p^2)^4}g^{\mu\{\lambda}p^\tau\gamma^{\rho\}}\bigg]\gamma^\sigma + 4\bigg[\frac{m+\not{p}}{(m^2-p^2)^4}\ g^{(\nu\sigma\tau\lambda)} + 6\frac{m+\not{p}}{(m^2-p^2)^5}\ g^{\{\nu\sigma}p^\tau p^{\lambda\}} + 48\frac{m+\not{p}}{(m^2-p^2)^6}p^\nu p^\sigma p^\tau p^\lambda\bigg]\gamma^\mu\gamma^\rho\bigg\}G_{\mu\nu}G_{\rho\sigma;\lambda\tau},$$

(3). Propagator up to six-dimension

$$S_F^{6(3)}(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\left(-\frac{2}{9}\right)\Bigg\{3\Bigg[\left(2\frac{\gamma^\mu(m-\not p)}{(m^2-p^2)^5}p^\lambda p^\tau - \frac{g^{\{\mu\lambda}p^{\tau\}}}{(m^2-p^2)^4} - \frac{4p^\mu p^\lambda p^\tau}{(m^2-p^2)^5}\right)$$

$$\cdot\ \gamma^\nu\gamma^\rho + (\mu\leftrightarrow\lambda) + (\rho\leftrightarrow\tau) + (\mu\leftrightarrow\lambda, \rho\leftrightarrow\tau)\Bigg]\gamma^\sigma + 4\Bigg[\frac{m+\not p}{(m^2-p^2)^4}g^{(\nu\lambda\sigma\tau)} + 6$$

$$\times\frac{m+\not p}{(m^2-p^2)^5}g^{\{\nu\lambda}p^\sigma p^{\tau\}} + 48\frac{m+\not p}{(m^2-p^2)^6}p^\nu p^\lambda p^\sigma p^\tau\Bigg]\gamma^\mu\gamma^\rho\Bigg\}G_{\mu\nu;\lambda}G_{\rho\sigma;\tau},$$

$$S_F^{6(4)}(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\left(-\frac{1}{2}\right)\Bigg\{\Bigg[\frac{m+\not p}{(m^2-p^2)^4}g^{\{\nu\tau\lambda\sigma\}} + 6\frac{m+\not p}{(m^2-p^2)^5}g^{\{\nu\tau}p^\lambda p^{\sigma\}} + 48$$

$$\times\frac{(m+\not p)p^\nu p^\tau p^\lambda p^\sigma}{(m^2-p^2)^6}\Bigg]\gamma^\mu\gamma^\rho - 3\Bigg[\frac{g^{\{\mu\lambda}p^{\tau\}}}{(m^2-p^2)^4} + \frac{8p^\mu p^\lambda p^\tau}{(m^2-p^2)^5}\Bigg]\gamma^\nu\gamma^\rho\gamma^\sigma\Bigg\}G_{\mu\nu;\lambda\tau}G_{\rho\sigma},$$

$$S_F^{6(5)}(x,0) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x}\Bigg\{-\frac{i}{18}\Bigg[\frac{g^{[\rho\sigma\lambda\tau}\gamma^{\mu]}(m-\not p)}{(m^2-p^2)^4} - 4\frac{g^{\{\rho\sigma\lambda\tau}p^{\mu\}}}{(m^2-p^2)^4} + 6\ g^{\{\rho\sigma}p^\lambda p^\tau\gamma^{\mu\}}$$

$$\times\frac{m-\not p}{(m^2-p^2)^5} - 12\frac{g^{\{\rho\sigma}p^\lambda p^\tau p^{\mu\}}}{(m^2-p^2)^5} + 48\frac{p^{\{\rho}p^\sigma p^\lambda p^\tau p^{\mu\}}(m-\not p)}{(m^2-p^2)^6}\Bigg]\gamma^\nu G_{\mu\nu;\rho\sigma\lambda\tau}\Bigg\}.$$

(4) Vertex up to six-dimension

$$(z\cdot\overleftrightarrow{D})^n_0 = (z\cdot\overleftrightarrow{\partial})^n,$$

$$(z\cdot\overleftrightarrow{D})^n_2 = -i(z\cdot\overleftrightarrow{\partial})^{n-1}\underline{x}^\mu z^\nu G_{\mu\nu},$$

$$(z\cdot\overleftrightarrow{D})^n_3 = -\frac{2i}{3}(z\cdot\overleftrightarrow{\partial})^{n-1}\underline{x}^\mu \underline{x}^\rho z^\nu G_{\mu\nu;\rho},$$

$$(z\cdot\overleftrightarrow{D})^n_{4(1)} = -\frac{n(n-1)}{2}(z\cdot\overleftrightarrow{\partial})^{n-2}\underline{x}^\mu \underline{x}^\rho z^\nu z^\sigma G_{\mu\nu}G_{\rho\sigma},$$

$$(z\cdot\overleftrightarrow{D})^n_{4(2)} = \left[-\frac{i}{4}n(z\cdot\overleftrightarrow{\partial})^{n-2}\underline{x}^\mu \underline{x}^\rho \underline{x}^\sigma - \frac{i}{12}(n-1)(n-2)(z\cdot\overleftrightarrow{\partial})^{n-3}\underline{x}^\mu z^\rho z^\sigma z^\nu\right]G_{\mu\nu;\rho\sigma},$$

$$(z\cdot\overleftrightarrow{D})^n_{5(1)} = -\frac{n(n-1)}{3}(z\cdot\overleftrightarrow{\partial})^{n-2}\underline{x}^\mu \underline{x}^\rho \underline{x}^\sigma \underline{x}^\lambda z^\nu z^\sigma G_{\mu\nu}G_{\rho\sigma;\lambda},$$

$$(z\cdot\overleftrightarrow{D})^n_{5(2)} = -\frac{n(n-1)}{3}(z\cdot\overleftrightarrow{\partial})^{n-2}\underline{x}^\mu \underline{x}^\rho \underline{x}^\sigma \underline{x}^\lambda z^\nu z^\sigma G_{\mu\nu;\lambda}G_{\rho\sigma},$$

$$(z\cdot\overleftrightarrow{D})^n_{5(3)} = \left[-\frac{i}{15}n(z\cdot\overleftrightarrow{\partial})^{n-1}\underline{x}^\mu \underline{x}^\rho \underline{x}^\sigma \underline{x}^\lambda z^\nu - \frac{i}{45}n(n-1)(n-2)(z\cdot\overleftrightarrow{\partial})^{n-2}\underline{x}^\mu(\underline{x}^\rho z^\sigma z^\lambda + \underline{x}^\sigma z^\rho z^\lambda + \underline{x}^\lambda z^\sigma z^\rho)z^\nu\right]G_{\mu\nu;\rho\sigma\lambda},$$

$$(z\cdot\overleftrightarrow{D})^n_{6(1)} = \frac{i}{6}n(n-1)(n-2)(z\cdot\overleftrightarrow{\partial})^{n-3}\underline{x}^\mu \underline{x}^\rho \underline{x}^\lambda z^\nu z^\sigma z^\tau G_{\mu\nu}G_{\rho\sigma}G_{\lambda\tau},$$

$$(z\cdot\overleftrightarrow{D})^n_{6(2)} = \left[-\frac{n(n-1)}{8}(z\cdot\overleftrightarrow{\partial})^{n-2}\underline{x}^\mu \underline{x}^\rho \underline{x}^\lambda \underline{x}^\tau z^\nu z^\sigma - \frac{1}{12}n(n-1)(n-2)(n-3)(z\cdot\overleftrightarrow{\partial})^{n-4} \times \underline{x}^\mu \underline{x}^\rho z^\lambda z^\tau z^\nu z^\sigma\right]G_{\mu\nu}G_{\rho\sigma;\lambda\tau},$$

$$(z\cdot\overleftrightarrow{D})^n_{6(3)} = -\frac{2}{9}n(n-1)(z\cdot\overleftrightarrow{\partial})^{n-2}\underline{x}^\mu \underline{x}^\rho \underline{x}^\lambda \underline{x}^\tau z^\nu z^\sigma G_{\mu\nu;\lambda}G_{\rho\sigma;\tau},$$

$$(z\cdot\overleftrightarrow{D})^n_{6(4)} = -\frac{n(n-1)}{8}(z\cdot\overleftrightarrow{\partial})^{n-2}\underline{x}^\mu \underline{x}^\rho \underline{x}^\lambda \underline{x}^\tau z^\nu z^\sigma G_{\mu\nu;\lambda\tau}G_{\rho\sigma}, \quad \ldots\ldots$$

3. Hadronic Expression

$$\langle 0|\bar{q}_1(0)\not{z}(iz\cdot\overleftrightarrow{D})^n q_2(0)|a_0(p)\rangle = (z\cdot p)^{n+1}\bar{f}_{a_0}\langle\xi^n_{2;a_0}\rangle|_\mu,$$
$$\langle 0|\bar{q}_1(0)q_2(0)|a_0(p)\rangle = m_{a_0}\bar{f}_{a_0}\langle\xi^{p,0}_{3;a_0}\rangle|_\mu.$$

$$\mathrm{Im}I^{(n,0)}_{2;a_0,\mathrm{had}}(q^2) = \pi\delta(q^2-m^2_{a_0})m_{a_0}\bar{f}^2_{a_0}\langle\xi^n_{2;a_0}\rangle|_\mu\langle\xi^{p;0}_{3;a_0}\rangle|_\mu - \frac{3m_q}{4\pi^2(n+2)}\theta(q^2-s_{a_0}).$$

$\langle\xi^{p,0}_{3;a_0}\rangle|_\mu \neq 1$

- calculate to infinity order = 1
- calculate to finite order ≠ 1

4. Dispersion Relation

$$\frac{1}{\pi}\int_{4m_q^2}^{\infty} ds\frac{\mathrm{Im}I^{(n,0)}_{2;a_0,\mathrm{had}}(s)}{s-q^2} = I^{(n,0)}_{2;a_0,\mathrm{QCD}}(q^2)$$

5. QCD sum rule for $\langle\xi^n_{2;a_0}\rangle|_\mu \langle\xi^{p;0}_{3;a_0}\rangle|_\mu$

$$
\begin{aligned}
\frac{\langle\xi^n_{2;a_0}\rangle|_\mu\langle\xi^{p;0}_{3;a_0}\rangle|_\mu m_{a_0}\bar f^2_{a_0}}{e^{m^2_{a_0}/M^2}} &= -\frac{3m_q}{4\pi^2(n+2)}\left(1-e^{-s_0/M^2}\right)+2\langle\bar qq\rangle+\frac{\langle g_s\bar qq\rangle^2}{81M^4}4m_q(n+3)-\frac{\langle g_s^2\bar qq\rangle^2}{1944\pi^2M^4}m_q(2+\kappa^2)\\
&\times\left\{\delta^{0n}\left[-24\left(-\ln\frac{M^2}{\mu^2}\right)-148\right]+\delta^{1n}\left[128\left(-\ln\frac{M^2}{\mu^2}\right)-692\right]+\theta(n-1)\left[8\left(6n^2+34n\right)\left(-\ln\frac{M^2}{\mu^2}\right)\right.\right.\\
&\left.+4n\tilde\psi(n)-2(6n^2+96n+212)\right]+\theta(n-2)\left[8(33n^2-17n)\left(-\ln\frac{M^2}{\mu^2}\right)-2(6n^2+71n)\tilde\psi(n)-\frac{1}{n(n-1)}\right.\\
&\left.\times(231n^4+520n^3-1101n^2+230n)\right]+\theta(n-3)\left[(74n-144n^2)\tilde\psi(n)-\frac{1}{n-1}(169n^3-348n^2+245n\right.\\
&\left.\left.+60)\right]+4(n+5)\right\}-\frac{\langle\alpha_sG^2\rangle}{24\pi M^2}m_q\left\{12n\left(-\ln\frac{M^2}{\mu^2}\right)-6(n+2)+\theta(n-1)\left[4n\left(-\ln\frac{M^2}{\mu^2}\right)+3\tilde\psi(n)-\frac{6}{n}\right]\right.\\
&\left.+\theta(n-2)\left[-(8n+3)\tilde\psi(n)-2(2n+1)+\frac{6}{n}\right]\right\}-\frac{\langle g_s^3fG^3\rangle}{192\pi^2M^4}m_q\left\{\delta^{1n}\left[-24\left(-\ln\frac{M^2}{\mu^2}\right)+84\right]+\theta(n-1)\right.\\
&\times\left[-4n(3n-5)\left(-\ln\frac{M^2}{\mu^2}\right)+2(2n^2+5n-13)\right]+\theta(n-2)\left[-24n^2\left(-\ln\frac{M^2}{\mu^2}\right)+2n\,(n-4)\,\tilde\psi(n)\right.\\
&\left.\left.+17n^2+55n+12\right]+\theta(n-3)\left[2n(n-4)\tilde\psi(n)+\frac{1}{n-1}(19n^3-32n^2+7n+6)\right]\right\}-\frac{\langle g_s\bar q\sigma TGq\rangle}{3M^2}4n, \qquad (9)
\end{aligned}
$$

6. QCD sum rule for $\langle\xi_{3;a_0}^{p;0}\rangle|_\mu$

$$\frac{(\langle\xi_{3;a_0}^{p;0}\rangle|_\mu)^2 m_{a_0}^2 \bar{f}_{a_0}^2}{M^2 e^{m_{a_0}^2/M^2}} = \frac{3}{8\pi^2(n+1)}\left[M^2-(M^2+s_0)e^{-s_0/M^2}\right] + \frac{\langle\alpha_s G^2\rangle}{8\pi M^2} + \frac{\langle g_s^2\bar{q}q\rangle^2(2+\kappa^2)}{486\pi^2 M^4}\left[35-16\left(-\ln\frac{M^2}{\mu^2}\right)\right] - \frac{8\langle g_s\bar{q}q\rangle^2}{27M^4} + \frac{\langle g_s\bar{q}\sigma TGq\rangle}{M^4} + 3m_q\frac{\langle\bar{q}q\rangle}{M^2}$$

⇩

Changed with M2 and s0

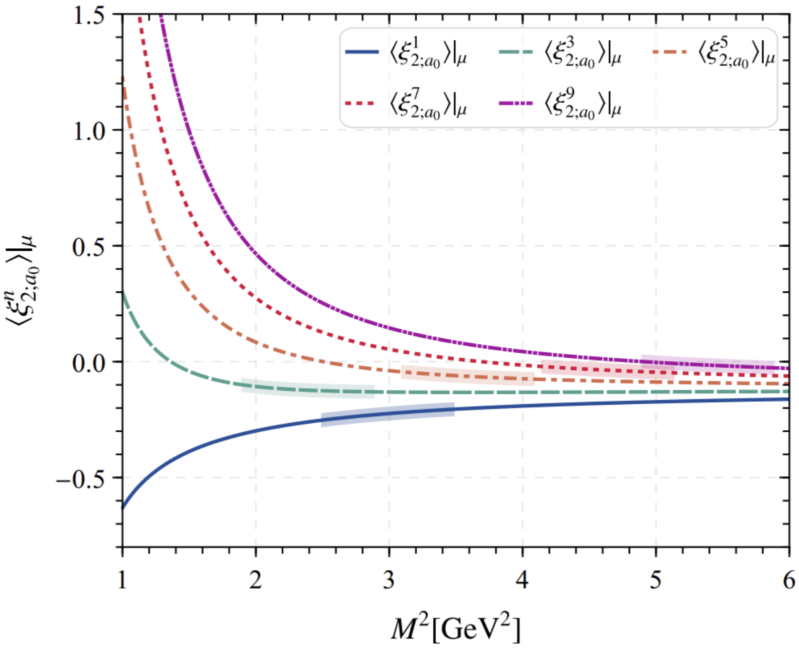
⇩

$$\langle\xi_{2;a_0}^{n}\rangle|_\mu = \frac{\langle\xi_{2;a_0}^{n}\rangle|_\mu\langle\xi_{3;a_0}^{p;0}\rangle|_\mu}{\sqrt{\langle\xi_{3;a_0}^{p;0}\rangle^2|_\mu}}$$

II. Scalar meson a0(980) twist-2 DA and semileptonic decays

n	M^2	$\langle\xi^n_{2;a_0}\rangle\vert_\mu$	Con.
1	[2.483, 3.483]	[−0.250, −0.203]	< 30%
3	[1.869, 2.869]	[−0.095, −0.129]	< 20%
5	[3.057, 4.057]	[−0.041, −0.074]	< 40%
7	[4.143, 5.143]	[−0.020, −0.048]	< 60%
9	[4.916, 5.916]	[−0.000, −0.027]	< 75%

$$\langle\xi^1_{2;a_0}\rangle|_{\mu_0} = -0.310(43), \quad \langle\xi^1_{2;a_0}\rangle|_{\mu_k} = -0.250(34),$$
$$\langle\xi^3_{2;a_0}\rangle|_{\mu_0} = -0.184(32), \quad \langle\xi^3_{2;a_0}\rangle|_{\mu_k} = -0.126(22),$$
$$\langle\xi^5_{2;a_0}\rangle|_{\mu_0} = -0.082(27), \quad \langle\xi^5_{2;a_0}\rangle|_{\mu_k} = -0.067(21),$$
$$\langle\xi^7_{2;a_0}\rangle|_{\mu_0} = -0.053(25), \quad \langle\xi^7_{2;a_0}\rangle|_{\mu_k} = -0.044(18),$$
$$\langle\xi^9_{2;a_0}\rangle|_{\mu_0} = -0.043(23), \quad \langle\xi^9_{2;a_0}\rangle|_{\mu_k} = -0.024(15),$$



Our results for the first two order are slightly smaller than the Cheng's predictions, e.g. $\langle\xi^1_{2;a_0}\rangle|_{\mu_0} = -0.56(5)$ and $\langle\xi^3_{2;a_0}\rangle|_{\mu_0} = -0.21(3)$ by using the QCDSR approach from Ref. [3], which is more likely to be antisymmetric

Higher-order moments will bring false oscillation

7. LCHO model for $\phi_{2;a_0}(x,\mu)$

$$\Psi_{2;a_0}(x,\mathbf{k}_\perp)=\sum_{\lambda_1\lambda_2}\chi_{2;a_0}^{\lambda_1\lambda_2}(x,\mathbf{k}_\perp)\Psi_{2;a_0}^R(x,\mathbf{k}_\perp),$$

$$\sum_{\lambda_1\lambda_2}\chi_{2;a_0}^{\lambda_1\lambda_2}(x,\mathbf{k}_\perp)=\frac{\hat{m}_q^2}{(\mathbf{k}_\perp^2+\hat{m}_q^2)^{1/2}}$$

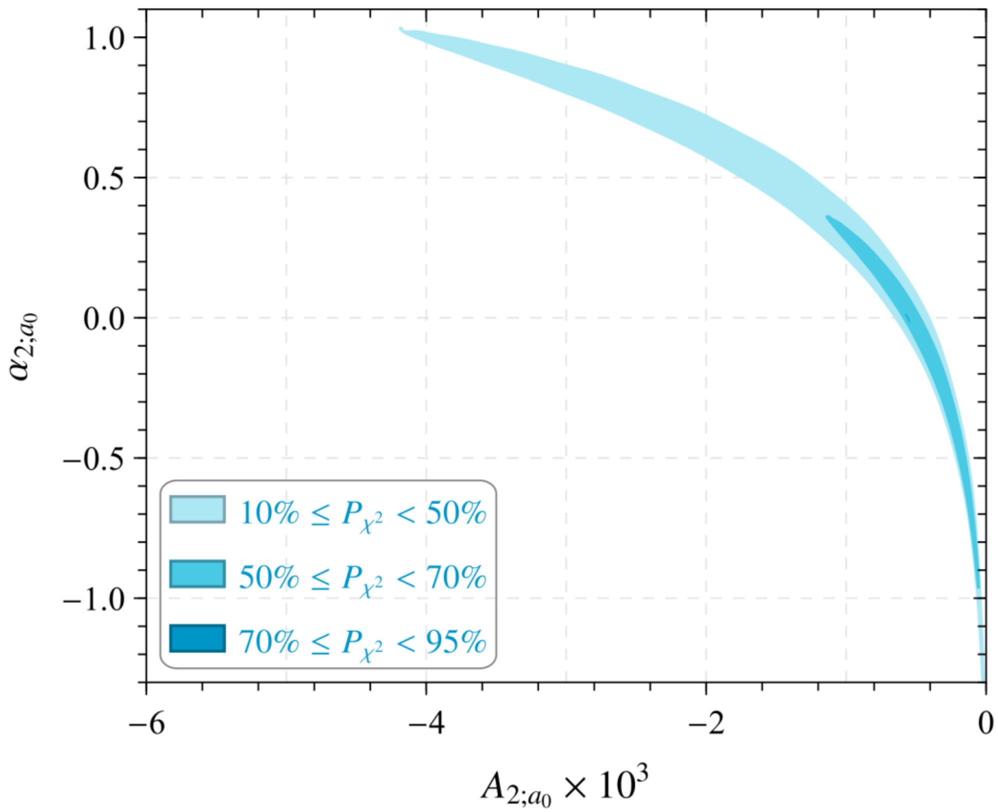
$\lambda_1\lambda_2$	$\chi_{2;a_0}^{\lambda_1\lambda_2}(x,\mathbf{k}_\perp)$	$\lambda_1\lambda_2$	$\chi_{2;a_0}^{\lambda_1\lambda_2}(x,\mathbf{k}_\perp)$
$\downarrow\downarrow$	$-\frac{k_x+ik_y}{\sqrt{2(\hat{m}_q^2+\mathbf{k}_\perp^2)}}$	$\uparrow\uparrow$	$-\frac{k_x-ik_y}{\sqrt{2(\hat{m}_q^2+\mathbf{k}_\perp^2)}}$
$\uparrow\downarrow$	$+\frac{\hat{m}_q}{\sqrt{2(\hat{m}_q^2+\mathbf{k}_\perp^2)}}$	$\downarrow\uparrow$	$-\frac{\hat{m}_q}{\sqrt{2(\hat{m}_q^2+\mathbf{k}_\perp^2)}}$

$$\Psi_{2;a_0}^R(x,\mathbf{k}_\perp)=A_{2;a_0}\varphi_{2;a_0}(x)\exp[-(\mathbf{k}_\perp^2+\hat{m}_q^2)/(8\beta_{2;a_0}^2 x\bar{x})]$$

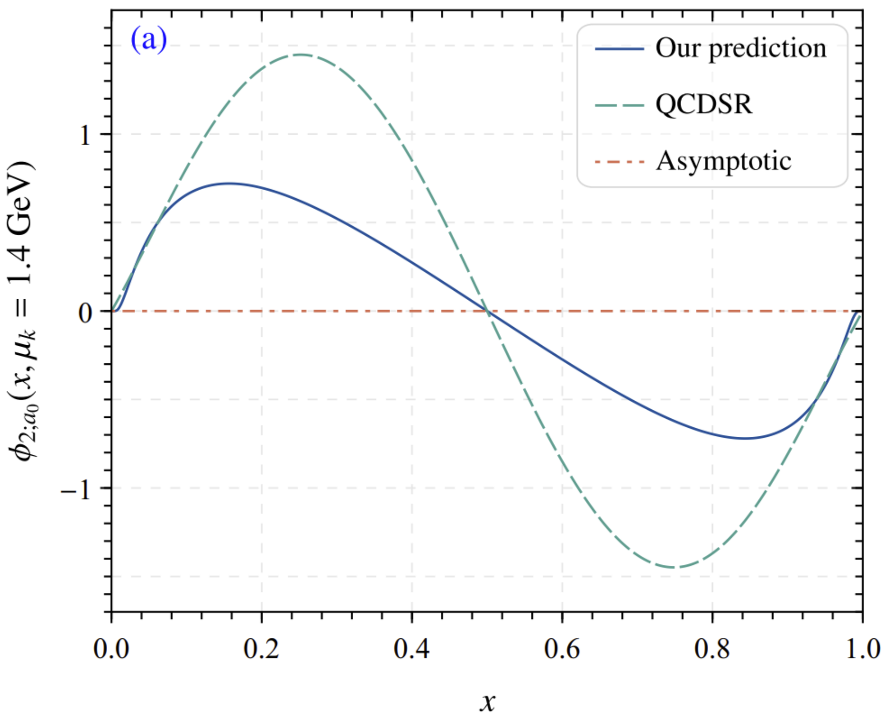
$$\varphi_{2;a_0}(x)=(x\bar{x})^{\alpha_{2;a_0}}C_1^{3/2}(2x-1)$$

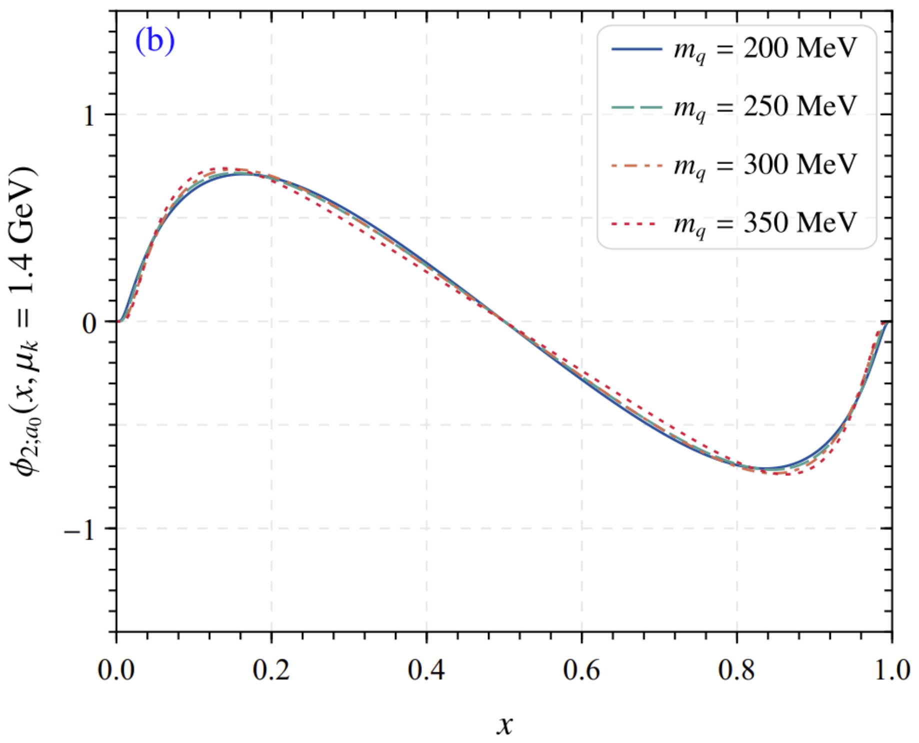
$$\phi_{2;a_0}(x,\mu)=\frac{A_{2;a_0}\hat{m}_q\beta_{2;a_0}}{4\sqrt{2}\pi^{3/2}}\sqrt{x\bar{x}}\varphi_{2;a_0}(x)\left\{\mathrm{Erf}\left[\sqrt{\frac{\hat{m}_q^2+\mu^2}{8\beta_{2;a_0}^2 x\bar{x}}}\right]-\mathrm{Erf}\left[\sqrt{\frac{\hat{m}_q^2}{8\beta_{2;a_0}^2 x\bar{x}}}\right]\right\}$$

7. LCHO model for $\phi_{2;a_0}(x,\mu)$



7. LCHO model for $\phi_{2;a_0}(x,\mu)$





8. D-a0(980) transition form factors

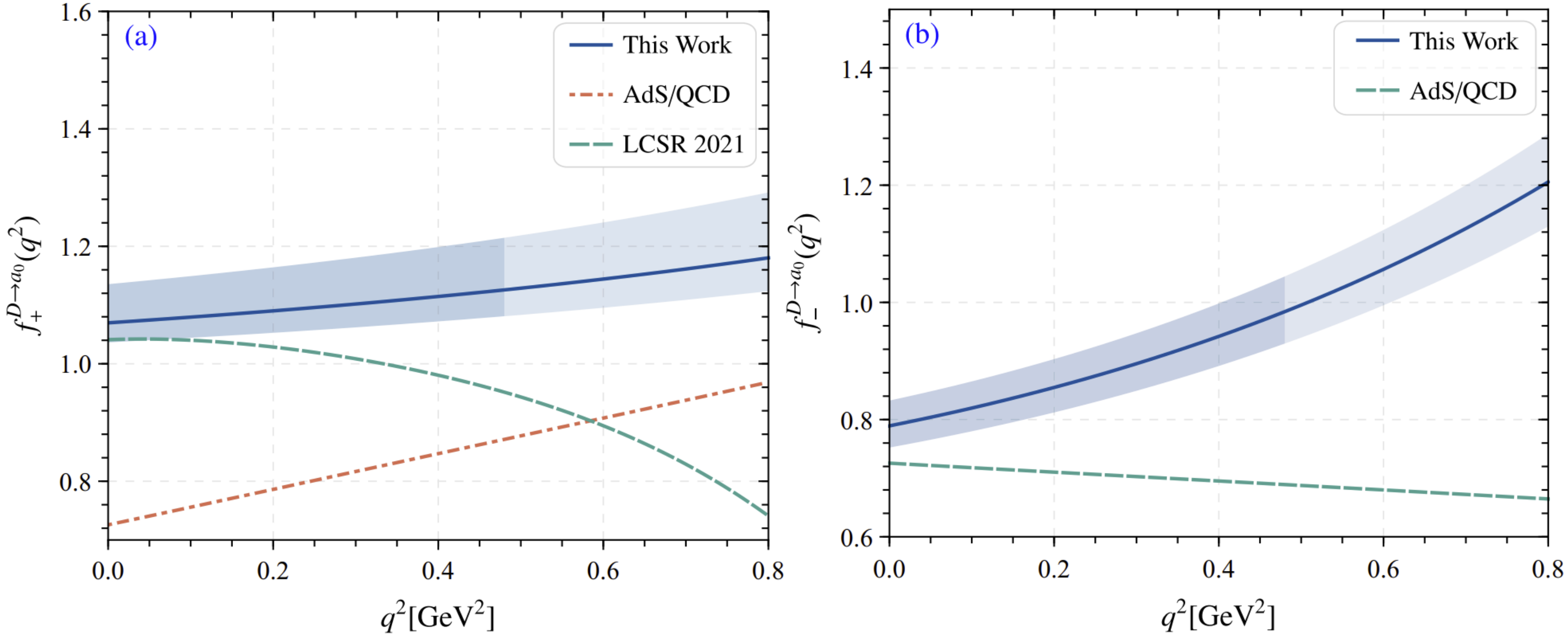
$$\Pi_\mu(p,q) = i\int d^4x e^{iq\cdot x}\langle a_0|T\{\bar{q}_1(x)\gamma_\mu\gamma_5 c(x), \bar{c}i\gamma_5 q_2(0)\}|0\rangle,$$

$$f_+^{D\to a_0}(q^2) = \frac{m_c \bar{f}_{a_0}}{m_D^2 f_D}\int_{u_0}^1 du e^{(m_{a_0}^2 - s(u))/M^2}\bigg\{-\frac{m_c}{u}\phi_{2;a_0}(u) + m_{a_0}\phi_{3,a_0}^p(u) + \frac{m_{a_0}}{6}\bigg[\frac{2}{u}\phi_{3,a_0}^\sigma(u) - \frac{1}{m_c^2 + u^2 m_{a_0}^2 - q^2} \times\bigg((m_c^2 - u^2 m_{a_0}^2 + q^2)\frac{d\phi_{3,a_0}^\sigma(u)}{du} - \frac{4um_c^2 m_{a_0}^2}{m_c^2 + u^2 m_{a_0}^2 - q^2}\phi_{3,a_0}^\sigma(u)\bigg)\bigg]\bigg\},$$

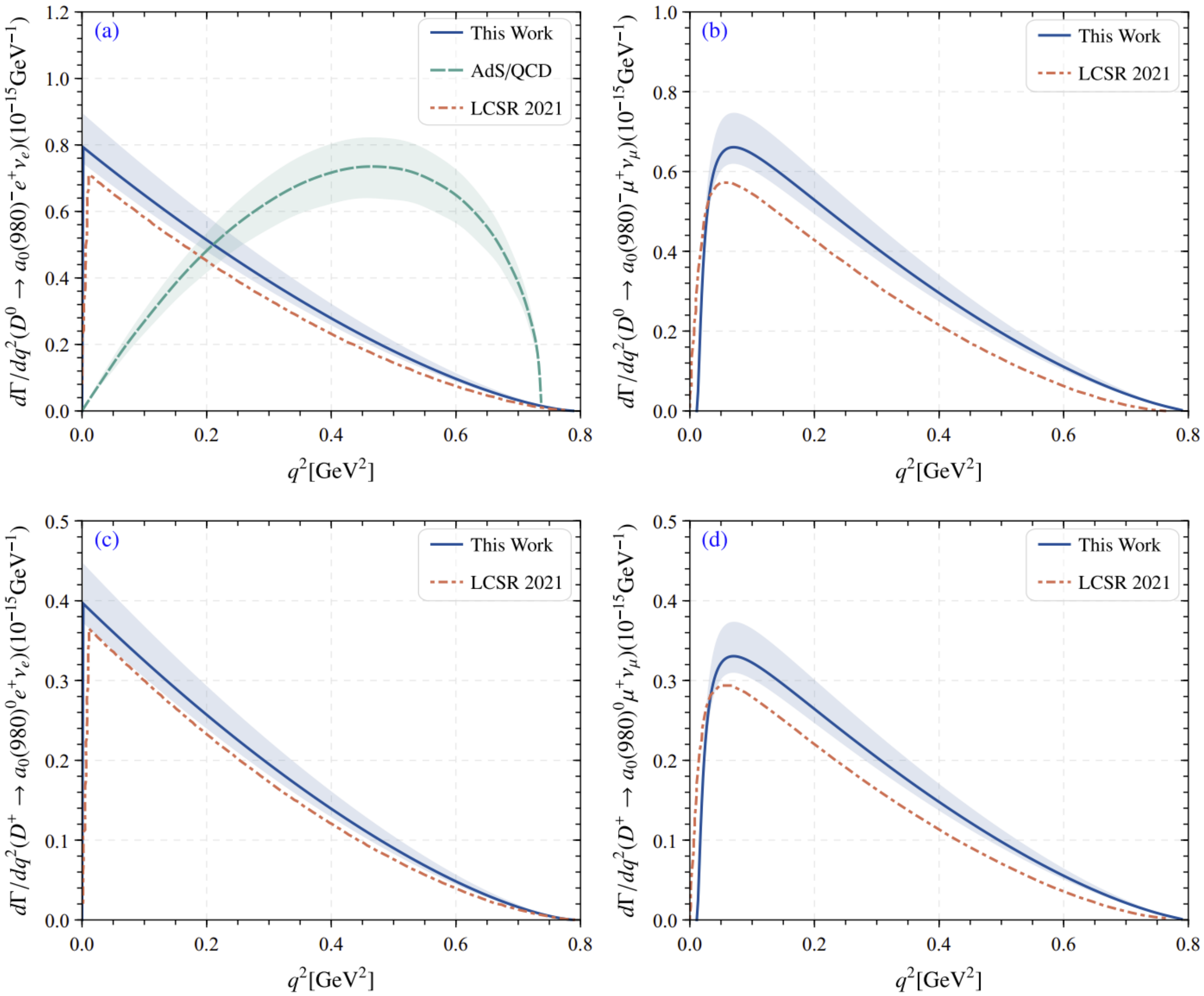
$$f_-^{D\to a_0}(q^2) = \frac{m_c\ \bar{f}_{a_0}}{m_D^2 f_D}\int_{u_0}^1 du\ e^{(m_{a_0}^2 - s(u))/M^2}\bigg[\frac{\phi_{3;a_0}^p(u)}{u} + \frac{1}{6u}\frac{d\phi_{3,a_0}^\sigma(u)}{du}\bigg].$$

	$f_+^{D\to a_0}(0)$	$f_-^{D\to a_0}(0)$
This work	$1.070^{+0.066}_{-0.033}$	$0.789^{+0.043}_{-0.037}$
CCQM [19]	$0.55^{+0.02}_{-0.02}$	$0.03^{+0.01}_{-0.01}$
LCSR 2021 [21]	$0.85^{+0.10}_{-0.11}$	$-0.85^{+0.10}_{-0.11}$
LCSR 2017 [20]	1.76(26)	0.31(13)
AdS/QCD [22]	0.72(9)	-

8. D-a0(980) transition form factors



9. Semileptonic decay widths



10. Branching fraction

	$D^0 \to a_0(980)^- e^+ \nu_e$	$D^0 \to a_0(980)^- \mu^+ \nu_\mu$	$D^+ \to a_0(980)^0 e^+ \nu_e$	$D^+ \to a_0(980)^0 \mu^+ \nu_\mu$
This work	$1.574^{+0.254}_{-0.156}$	$1.496^{+0.240}_{-0.147}$	$1.982^{+0.320}_{-0.196}$	$1.885^{+0.302}_{-0.186}$
CCQM [19]	1.68 ± 0.15	1.63 ± 0.14	2.18 ± 0.38	2.12 ± 0.37
LCSR 2017 [20]	$4.08^{+1.37}_{-1.22}$	-	$5.40^{+1.78}_{-1.59}$	-
LCSR 2021 [21]	1.36	1.21	1.79	1.59
AdS/QCD [22]	2.44 ± 0.30	-	-	-

$$\mathcal{B}(D \to a_0(980)(\to \eta\pi)e^+\nu_e) = \mathcal{B}(D \to a_0(980)e^+\nu_e) \times \mathcal{B}(a_0(980) \to \eta\pi).$$

	$\mathcal{B}(D^0 \to a_0(980)^-(\to \eta\pi^-)e^+\nu_e)$	$\mathcal{B}(D^+ \to a_0(980)^0(\to \eta\pi^0)e^+\nu_e)$
This work	$1.330^{+0.216}_{-0.134}$	$1.675^{+0.272}_{-0.169}$
BESIII [16]	$1.33^{+0.33}_{-0.29}$	$1.66^{+0.81}_{-0.66}$
LCSR 2021 [21]	1.15	1.51
PDG [17]	$1.33^{+0.30}_{-0.29}$	$1.7^{+0.8}_{-0.7}$

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1. Correlation function

$$\langle 0\,|\bar{s}(z)\gamma_\mu u(-z)|\,K_0^{*+}\rangle = \bar{f}_{K_0^*} p_\mu \int_0^1 du e^{i(2u-1)p\cdot z}\phi_{2;K_0^*}(u,\mu),$$

$$\langle 0\,|\bar{s}(z)u(-z)|\,K_0^{*+}\rangle = m_{K_0^*}\bar{f}_{K_0^*} \int_0^1 du e^{i(2u-1)p\cdot z}\phi^p_{3;K_0^*}(u,\mu),$$

$$\int_0^1 dx\phi_{2;K_0^*}(x,\mu) = 0$$

$$\Pi_{2;K_0^*}(z,q) = i\int d^4x e^{iq\cdot x}\langle 0|T\{J_n(x),\hat{J}_0^\dagger(0)\}|0\rangle$$
$$= (z\cdot q)^{n+1} I_{2;K_0^*}(q^2)$$

$$J_n(x) = \bar{s}(x)\not{z}(iz\cdot \overleftrightarrow{D})^n u(x),$$
$$\hat{J}_0^\dagger(0) = \bar{u}(0)s(0).$$

$$\Pi_{2;K_0^*}(z,q) = i\int d^4x e^{iq\cdot x}\langle 0|T\{\bar{s}(x)\not{z}(iz\cdot \overleftrightarrow{D})^n u(x),\bar{u}(0)s(0)\}|0\rangle$$

2. QCD sum rule for $\langle\xi^{n}_{2;\,K_0^*}\rangle|_\mu\ \langle\xi^{p;0}_{3;K_0^*}\rangle|_\mu$

$$\frac{\langle\xi^n\rangle_{2;K_0^*}\langle\xi_p^0\rangle_{3;K_0^*}m_{K_0^*}\bar f_{K_0^*}^2}{M^2e^{m_{K_0^*}^2/M^2}}=\frac{1}{\pi}\frac{1}{M^2}\int_{m_s^2}^{s_{K_0^*}}dse^{-s/M^2}\mathrm{Im}I_{2;K_0^*}^{\rm pert}(s)+\Big(1+\frac{m_sm_u}{2M^2}+\frac{2n+1}{2}\frac{m_s^2}{M^2}\Big)\frac{\langle\bar ss\rangle}{M^2}+\Big(-1-\frac{m_sm_u}{2M^2}+\frac{m_s^2}{M^2}\Big)\frac{(-1)^n\langle\bar uu\rangle}{M^2}+\hat I_{\langle G^2\rangle}(M^2)+\hat I_{\langle G^2\rangle}^{m_s^3}(M^2)+\Big(-\frac{2n}{3}-\frac{8n-9}{36}\frac{m_sm_u}{M^2}\Big)\frac{\langle g_s\bar s\sigma TGs\rangle}{(M^2)^2}+\Big[\frac{2n}{3}\Big(1-\frac{m_s^2}{M^2}\Big)+\frac{8n-9}{36}\frac{m_sm_u}{M^2}+\frac{m_s^2}{4M^2}\Big]\frac{(-1)^n\langle g_s\bar u\sigma TGu\rangle}{(M^2)^2}+\frac{2(n+3)}{81}m_u\frac{\langle g_s\bar ss\rangle^2}{(M^2)^3}+\frac{-2(n+3)}{81}m_s\Big(1-\frac{m_s^2}{M^2}\Big)\frac{(-1)^n\langle g_s\bar uu\rangle^2}{(M^2)^3}+\hat I_{\langle G^3\rangle}(M^2)+\hat I_{\langle G^3\rangle}^{m_s^3}(M^2)+\hat I_{\langle q^4\rangle}(M^2)+\hat I_{\langle q^4\rangle}^{m_s^3}(M^2), \tag{9}$$

$$\mathrm{Im}I_{2;K_0^*}^{\rm pert}(s)=-\frac{3}{16\pi(n+1)(n+2)}\Big\{m_s\Big[\Big(1-\frac{2m_s^2}{s}\Big)^{n+1}\Big(2(n+1)\Big(1-\frac{m_s^2}{s}\Big)+1\Big)+(-1)^n\Big]-m_u\Big[\Big(1-\frac{2m_s^2}{s}\Big)^{n+1}\Big(-2(n+1)\Big(1-\frac{m_s^2}{s}\Big)+2n+3\Big)+(-1)^n(2n+3)\Big]\Big\},$$

$$\hat{I}_{\langle G^2\rangle}(M^2) = \frac{\langle\alpha_s G^2\rangle}{(M^2)^2}((-1)^n m_s - m_u)\frac{1}{48\pi}\Big\{-12(-1)^n n\Big(-\ln\frac{M^2}{\mu^2}\Big) + 6(-1)^n(n+2) + \theta(n-1)\Big[-4(-1)^n n\Big(-\ln\frac{M^2}{\mu^2}\Big) - 3\widetilde{\psi}_3(n)\Big] + \theta(n-2)\Big[-(8n+3)\widetilde{\psi}_2(n) + (-1)^n(4n+9) + 7 + \frac{6}{n}\Big]\Big\}, \tag{A1}$$

$$\hat{I}_{\langle G^3\rangle}(M^2) = \frac{\langle g_s^3 f G^3\rangle}{(M^2)^3}((-1)^n m_s - m_u)\frac{1}{384\pi^2}\Big\{\delta^{n1}\Big[-24\Big(-\ln\frac{M^2}{\mu^2}\Big) + 84\Big] + \theta(n-1)\Big[4(-1)^n n(3n-5)\Big(-\ln\frac{M^2}{\mu^2}\Big) - 2(-1)^n(2n^2+5n-13)\Big] + \theta(n-2)\Big[24(-1)^n n^2\Big(-\ln\frac{M^2}{\mu^2}\Big) + 2n(n-4)\widetilde{\psi}_2(n) - 17(-1)^n n^2 - 55(-1)^n n - 6(-1)^n + 6\Big] + \theta(n-3)\Big[-2n(8n-1)\widetilde{\psi}_1(n) + \frac{1}{n-1}\Big(-19(-1)^n n^3 + 16(1+(-1)^n)n^2 + 3(2+(-1)^n)n + 6\Big)\Big]\Big\}, \tag{A2}$$

$$\hat{I}_{\langle q^4\rangle}(M^2) = \frac{(2+\kappa^2)\langle g_s^2\bar{u}u\rangle^2}{(M^2)^3}((-1)^n m_s - m_u)\frac{1}{3888\pi^2}\Big\{8(-1)^n n\Big(-\ln\frac{M^2}{\mu^2}\Big) - 4(-1)^n(n+5) + \delta^{n0}\Big[-24\Big(-\ln\frac{M^2}{\mu^2}\Big) - 148\Big] + \delta^{n1}\Big[128\Big(-\ln\frac{M^2}{\mu^2}\Big) - 692\Big] + \theta(n-1)\Big[-8\Big(6(-1)^n n^2 + (-25+9(-1)^n)n - 6(1+(-1)^n)\Big)\Big(-\ln\frac{M^2}{\mu^2}\Big) - 4n\widetilde{\psi}_3(n) + 2\Big(6(-1)^n n^2 + (-47+49(-1)^n)n + 57(-1)^n - 151 - \frac{24}{n}(1+(-1)^n)\Big)\Big] + \theta(n-2)\Big[-4\Big(66(-1)^n n^2 - 34(-1)^n n + 15(1+(-1)^n)\Big)\Big(-\ln\frac{M^2}{\mu^2}\Big) + 2\Big(-6n^2 + (-21+50(-1)^n)n + 12(1+(-1)^n)\Big)\widetilde{\psi}_2(n) + \frac{1}{n(n-1)}\Big(231(-1)^n n^4 + (-94+426(-1)^n)n^3 + (116-985(-1)^n)n^2 + (98+328(-1)^n)n - 60(1+(-1)^n)\Big)\Big] + \theta(n-3)\Big[\Big(144n^2 - 74n + 30(1+(-1)^n)\Big)\widetilde{\psi}_1(n) + \frac{1}{n-1}\Big(169(-1)^n n^3 - 12(12+17(-1)^n)n^2 + (10+111(-1)^n)n - 2(63+38(-1)^n)\Big)\Big]\Big\}, \tag{A3}$$

$$\hat{I}^{m_s^3}_{\langle G^2\rangle}(M^2) = \frac{\langle\alpha_s G^2\rangle}{(M^2)^3}m_s^3\frac{1}{24\pi}\Big\{-2\delta^{n0} - 10\delta^{n1}\Big[\Big(-\ln\frac{M^2}{\mu^2}\Big) - \frac{17}{5}\Big] + \theta(n-1)\Big[6n\Big(-\ln\frac{M^2}{\mu^2}\Big) - 3(n+3)\Big] + \theta(n-2)\times\Big[-2n(7n-2)\Big(-\ln\frac{M^2}{\mu^2}\Big) + 3(-1)^n n\widetilde{\psi}_2(n) + 9n^2 + 24n - 3(-1)^n - 2\Big] + \theta(n-3)\Big[(-1)^n n(7n-2)\widetilde{\psi}_1(n) - \frac{1}{n-1}\Big(-7n^3 + (5+7(-1)^n)n^2 + (2+(-1)^n)n + 2(-1)^n\Big)\Big]\Big\}, \tag{A4}$$

III. K0*(1430) twist-2 DA and semileptonic decays

$$\hat{I}^{m_s^3}_{\langle G^3\rangle}(M^2) = \frac{\langle g_s^3 fG^3\rangle}{(M^2)^4} m_s^3 \frac{1}{6912\pi^2}\Big\{ -24\delta^{n0} - 288\delta^{n1} + \delta^{n2}\Big[2352\Big(-\ln\frac{M^2}{\mu^2}\Big) - 10188\Big] + \theta(n-2)\Big[144n(n-1) \times (2n-5)\Big(-\ln\frac{M^2}{\mu^2}\Big) - 9(8n^3+21n^2-205n+90)\Big] + \theta(n-3)\Big[24n(n-1)(36n-23)\Big(-\ln\frac{M^2}{\mu^2}\Big) - 144(-1)^n n(n-1)(n-2)\widetilde{\psi}_1(n) - 36(-1)^n n(n-1)\widetilde{\psi}_5(n) - 72(-1)^n n(n-1)\widetilde{\psi}_6(n) + \frac{2}{n-2}\Big(-505n^4 + (281+72(-1)^n)n^3 - 2(-1274+99(-1)^n)n^2 + (-2417+135(-1)^n)n + 90(-1)^n + 474\Big)\Big] + \theta(n-4) \times \Big[12(-1)^n n(n-1)(36n-23)\widetilde{\psi}_4(n) + \frac{1}{n-2}\Big(-566n^4 + (1987-432(-1)^n)n^3 + (-1999+492(-1)^n)n^2 + (578-354(-1)^n)n - 156(-1)^n\Big)\Big]\Big\}, \tag{A5}$$

$$\hat{I}^{m_s^3}_{\langle q^4\rangle}(M^2) = \frac{(2+\kappa^2)\langle g_s^2\bar{u}u\rangle^2}{(M^2)^4} m_s^3 \frac{1}{23328\pi^2}\Big\{\delta^{n0}\Big[1368\Big(-\ln\frac{M^2}{\mu^2}\Big) + 1512\Big] + \delta^{n1}\Big[1032\Big(-\ln\frac{M^2}{\mu^2}\Big) - 1272\Big] + \delta^{n2} \times \Big[-3504\Big(-\ln\frac{M^2}{\mu^2}\Big) + 13980\Big] - 48(-1)^n n\theta(n-1) + \theta(n-2)\Big[-12\Big(-8n^3 + 86n^2 + 4(-39+25(-1)^n)n - 9(1+(-1)^n)\Big)\Big(-\ln\frac{M^2}{\mu^2}\Big) + \frac{3}{n(n-1)}\Big(-48n^5 + 429n^4 + (70+204(-1)^n)n^3 + (-1473+596(-1)^n)n^2 + (622-472(-1)^n)n + 36(1+(-1)^n)\Big)\Big] + \theta(n-3)\Big[24\Big(-76n^3 + 177n^2 - 147n + 24(1+(-1)^n)\Big) \times \Big(-\ln\frac{M^2}{\mu^2}\Big) - 6\Big(8(-1)^n n^3 - 86(-1)^n n^2 + 4(-25+39(-1)^n)n + 9(1+(-1)^n)\Big)\widetilde{\psi}_1(n) - \frac{2}{n(n-1)(n-2)} \times \Big(-943n^6 - 3(-519+8(-1)^n)n^5 + (6041-24(-1)^n)n^4 - 3(5720-117(-1)^n)n^3 + (15359+363(-1)^n)n^2 - 6(857+259(-1)^n)n + 576(1+(-1)^n)\Big)\Big] + \theta(n-4)\Big[-12\Big(76(-1)^n n^3 - 177(-1)^n n^2 + 147(-1)^n n - 24(1+(-1)^n)\Big)\widetilde{\psi}_4(n) + \frac{1}{(n-1)(n-2)}\Big(1106n^5 + (-5469+912(-1)^n)n^4 + (10358-2580(-1)^n)n^3 + (-9669+3294(-1)^n)n^2 + (4250-798(-1)^n)n - 12(48+73(-1)^n)\Big)\Big]\Big\}, \tag{A6}$$

3. QCD sum rule for $\langle\xi^{p;0}_{3;K_0^*}\rangle|_\mu$

$\langle\xi^{p,0}_{3;K_0^*}\rangle|_\mu \neq 1$
- calculate to infinite order = 1
- calculate to finite order ≠ 1

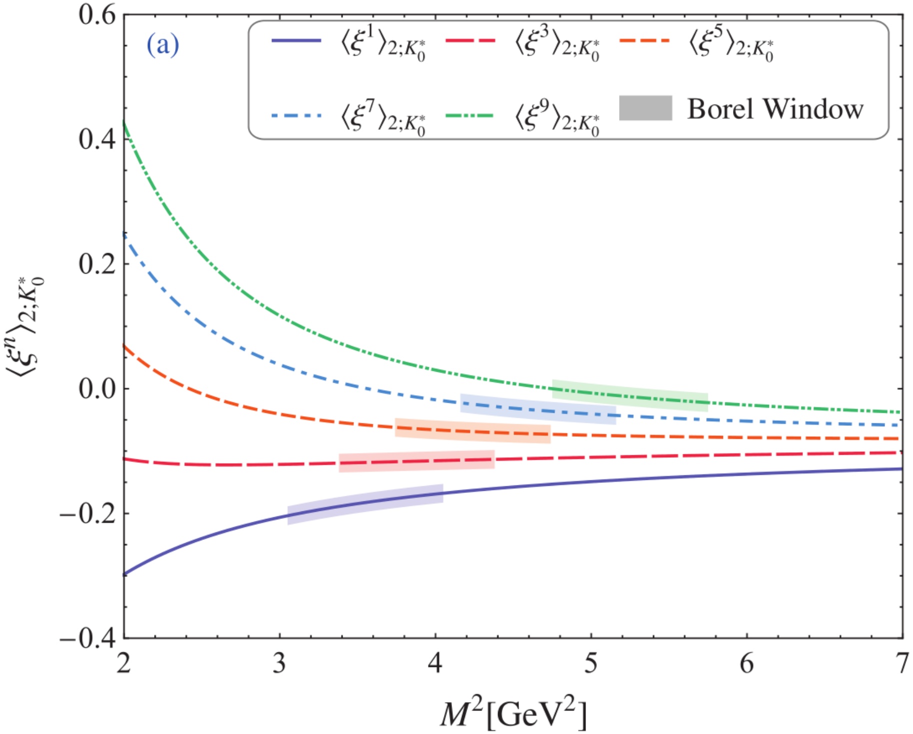
$$\frac{m^2_{K_0^*}\bar f^2_{K_0^*}\langle\xi_p^0\rangle^2_{3;K_0^*}}{M^2 e^{m^2_{K_0^*}/M^2}} = \frac{1}{\pi}\frac{1}{M^2}\int_{m_s^2}^{s_{K_0^*}} ds e^{-s/M^2}\mathrm{Im}I^{\rm pert}_{3;K_0^*}(s) + \Big(\frac{m_s}{2}+m_u\Big)\frac{\langle\bar ss\rangle}{M^2} + \Big(\frac{m_u}{2}+m_s-\frac{m_s^3}{M^2}\Big)\frac{\langle\bar uu\rangle}{M^2} + \frac{1}{24\pi}\Big(3-\frac{4m_s^2}{M^2}\Big)$$
$$\times\frac{\langle\alpha_s G^2\rangle}{M^2} + \frac{m_u}{2}\frac{\langle g_s\bar s\sigma TGs\rangle}{(M^2)^2} + \frac{m_s}{2}\Big(1-\frac{3m_s^2}{2M^2}\Big)\frac{\langle g_s\bar u\sigma TGu\rangle}{(M^2)^2} - \frac{4}{27}\frac{\langle g_s\bar ss\rangle^2}{(M^2)^2} - \frac{4}{27}\Big(1-\frac{5m_s^2}{4M^2}\Big)\frac{\langle g_s\bar uu\rangle^2}{(M^2)^2} - \frac{1}{96\pi^2}\frac{m_s^2}{M^2}\frac{\langle g_s^3 fG^3\rangle}{(M^2)^2}$$
$$+\frac{1}{972\pi^2}\Big\{-12\Big(-\ln\frac{M^2}{\mu^2}\Big)+70+\frac{m_s^2}{M^2}\Big[15\Big(-\ln\frac{M^2}{\mu^2}\Big)+93\Big]\Big\}\frac{(2+\kappa^2)\langle g_s^2\bar uu\rangle^2}{(M^2)^2}, \tag{11}$$

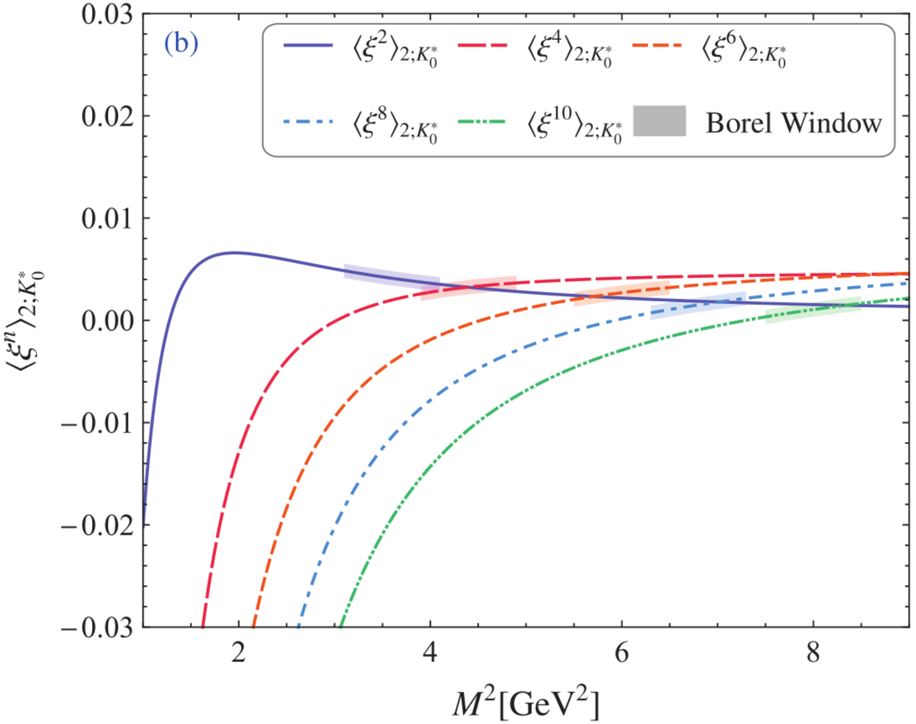
with

$$\mathrm{Im}I^{\rm pert}_{3;K_0^*}(s) = \frac{3s}{8\pi}\Big(1-\frac{m_s^2}{s}\Big)^2\Big[3-2\Big(1-\frac{m_s^2}{s}\Big)\Big] - \frac{3}{4\pi}m_s^2\Big(1-\frac{m_s^2}{s}\Big)^2. \tag{12}$$

$$\langle\xi^n\rangle_{2;K_0^*} = \frac{\langle\xi^n\rangle_{2;K_0^*}\times\langle\xi_p^0\rangle_{3;K_0^*}}{\sqrt{\langle\xi_p^0\rangle^2_{3;K_0^*}}}$$

4. Borel Windows





5. Result for first 10th order ξ-moments

$\langle\xi^n\rangle_{2;K_0^*}$	1 GeV	1.4 GeV	3 GeV
$\langle\xi^1\rangle_{2;K_0^*}$	$-0.261^{+0.056}_{-0.071}$	$-0.211^{+0.045}_{-0.057}$	$-0.156^{+0.033}_{-0.043}$
$\langle\xi^2\rangle_{2;K_0^*}$	$0.0065^{+0.0046}_{-0.0057}$	$0.0050^{+0.0035}_{-0.0043}$	$0.0034^{+0.0024}_{-0.0030}$
$\langle\xi^3\rangle_{2;K_0^*}$	$-0.177^{+0.034}_{-0.045}$	$-0.138^{+0.026}_{-0.035}$	$-0.098^{+0.019}_{-0.025}$
$\langle\xi^4\rangle_{2;K_0^*}$	$0.0052^{+0.0031}_{-0.0037}$	$0.0039^{+0.0024}_{-0.0028}$	$0.0026^{+0.0016}_{-0.0019}$
$\langle\xi^5\rangle_{2;K_0^*}$	$-0.103^{+0.022}_{-0.028}$	$-0.081^{+0.017}_{-0.022}$	$-0.058^{+0.012}_{-0.016}$
$\langle\xi^6\rangle_{2;K_0^*}$	$0.0044^{+0.0024}_{-0.0027}$	$0.0033^{+0.0018}_{-0.0020}$	$0.0022^{+0.0012}_{-0.0014}$
$\langle\xi^7\rangle_{2;K_0^*}$	$-0.045^{+0.021}_{-0.022}$	$-0.038^{+0.016}_{-0.017}$	$-0.030^{+0.011}_{-0.012}$
$\langle\xi^8\rangle_{2;K_0^*}$	$0.0025^{+0.0024}_{-0.0026}$	$0.0020^{+0.0017}_{-0.0019}$	$0.0014^{+0.0011}_{-0.0013}$
$\langle\xi^9\rangle_{2;K_0^*}$	$-0.018^{+0.023}_{-0.023}$	$-0.018^{+0.016}_{-0.017}$	$-0.016^{+0.010}_{-0.011}$
$\langle\xi^{10}\rangle_{2;K_0^*}$	$0.0018^{+0.0021}_{-0.0024}$	$0.0014^{+0.0015}_{-0.0017}$	$0.0010^{+0.0010}_{-0.0011}$

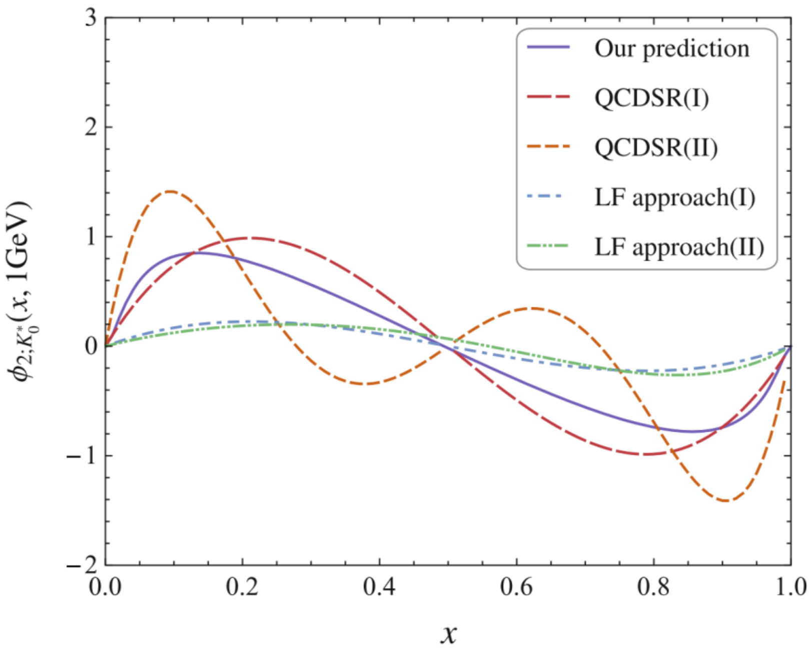
	$\langle\xi^1\rangle_{2;K_0^*}$	$\langle\xi^2\rangle_{2;K_0^*}$	$\langle\xi^3\rangle_{2;K_0^*}$	$a_1^{2;K_0^*}$	$a_2^{2;K_0^*}$	$a_3^{2;K_0^*}$
This Work	$-0.261^{+0.056}_{-0.071}$	$0.0065^{+0.0046}_{-0.0057}$	$-0.177^{+0.034}_{-0.045}$	$-0.435^{+0.093}_{-0.118}$	$0.019^{+0.014}_{-0.017}$	$-0.342^{+0.051}_{-0.076}$
QCD SR [17]	$-0.35^{+0.08}_{-0.08}$	–	$-0.23^{+0.06}_{-0.06}$	$-0.57^{+0.13}_{-0.13}$	–	$-0.42^{+0.22}_{-0.22}$
LF Holographic [5]	$-0.078^{+0.018}_{-0.018}$	$-0.010^{+0.001}_{-0.001}$	~ -0.034	$-0.130^{+0.030}_{-0.030}$	$-0.030^{+0.002}_{-0.002}$	$-0.005^{+0.001}_{-0.001}$

6. LCHO model

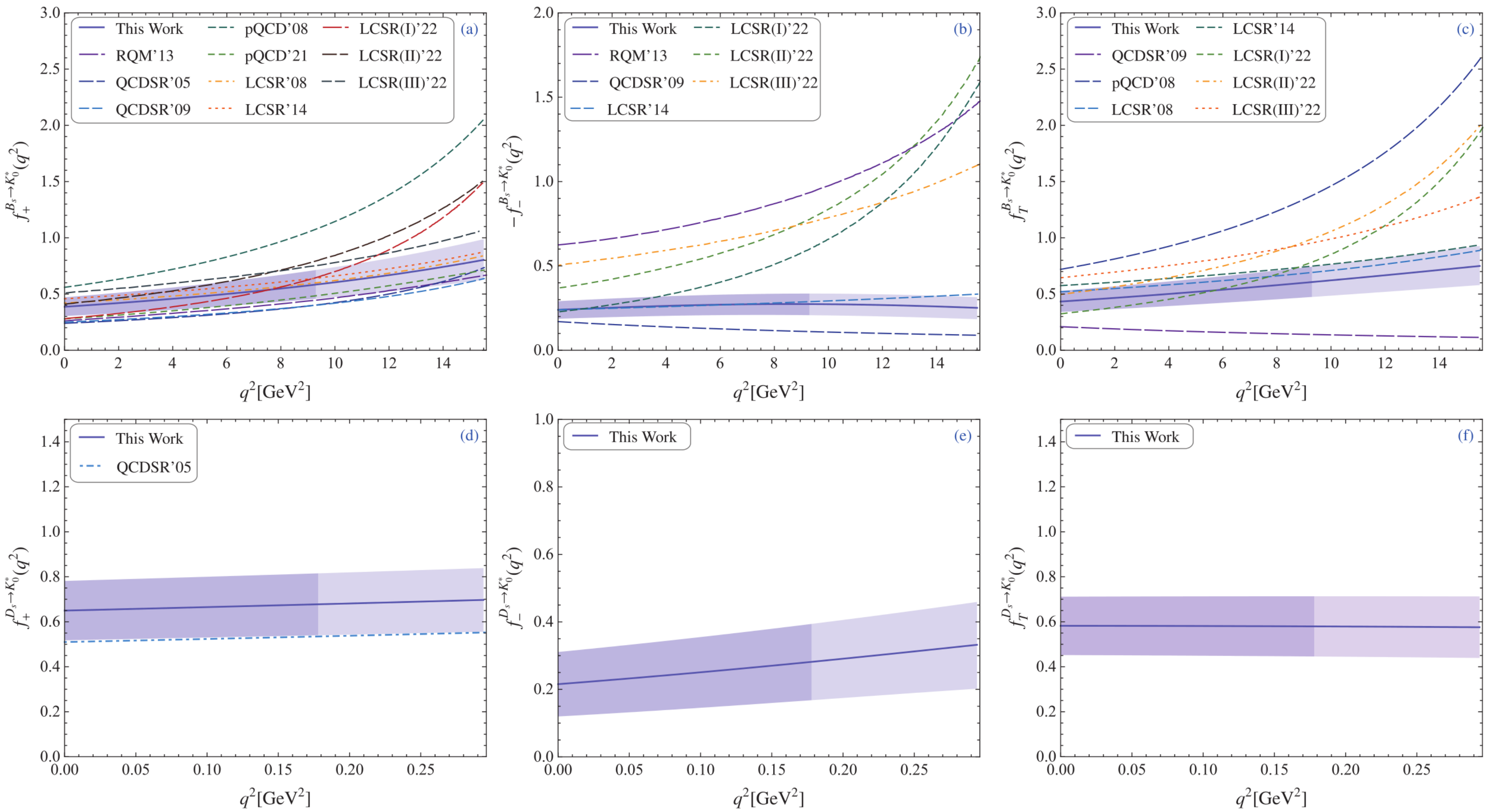
$$\Psi^R_{2;K_0^*}(x,\mathbf{k}_\perp) = A_{2;K_0^*}\varphi_{2;K_0^*}(x)\exp\Big[-\frac{1}{8\beta^2_{2;K_0^*}}\Big(\frac{\mathbf{k}_\perp^2+\hat{m}_s^2}{x}+\frac{\mathbf{k}_\perp^2+\hat{m}_q^2}{\bar{x}}\Big)\Big]$$

$$\varphi_{2;K_0^*}(x) = (x\bar{x})^{\alpha_{2;K_0^*}}\Big[C_1^{3/2}(2x-1)+\hat{B}_{2;K_0^*}C_2^{3/2}(2x-1)\Big],$$

$$\phi_{2;K_0^*}(x,\mu) = \frac{A_{2;K_0^*}\beta_{2;K_0^*}\widetilde{m}}{4\sqrt{2}\pi^{3/2}}\sqrt{x\bar{x}}\varphi_{2;K_0^*}(x)\exp\Big[-\frac{\hat{m}_q^2x+\hat{m}_s^2\bar{x}-\widetilde{m}^2}{8\beta^2_{2;K_0^*}x\bar{x}}\Big]$$
$$\times\Big\{\mathrm{Erf}\Big(\sqrt{\frac{\widetilde{m}^2+\mu^2}{8\beta^2_{2;K_0^*}x\bar{x}}}\Big)-\mathrm{Erf}\Big(\sqrt{\frac{\widetilde{m}^2}{8\beta^2_{2;K_0^*}x\bar{x}}}\Big)\Big\}$$



7. Transition Form Factors



8. Decay Widths

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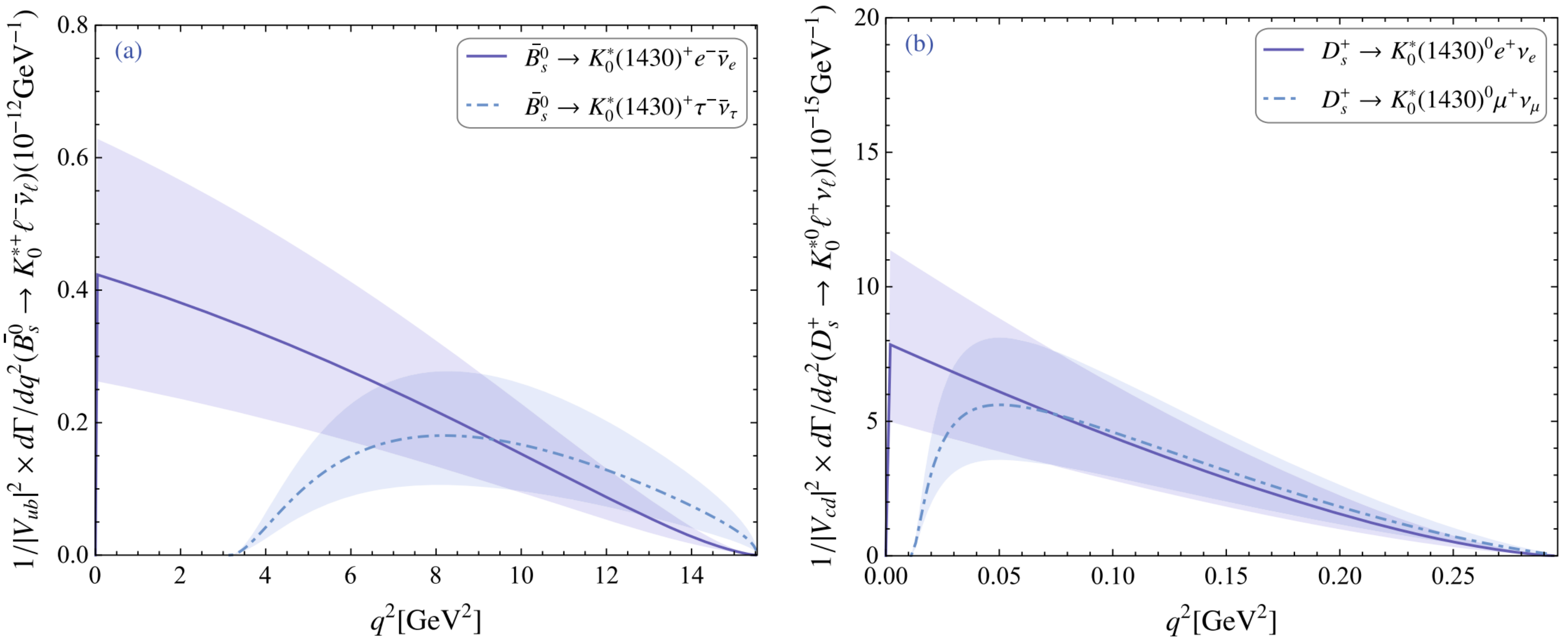

TABLE VIII: Branching fractions ($\times 10^4$) of the semileptonic decays $\overline{B}_s^0 \to K_0^*(1430)^+ \ell^- \bar{\nu}_\ell$ with $\ell = e, \mu$ and τ, respectvely.

	$\mathcal{B}(\overline{B}_s^0 \to K_0^*(1430)^+ e^- \bar{\nu}_e)$	$\mathcal{B}(\overline{B}_s^0 \to K_0^*(1430)^+ \mu^- \bar{\nu}_\mu)$	$\mathcal{B}(\overline{B}_s^0 \to K_0^*(1430)^+ \tau^- \bar{\nu}_\tau)$
This work	$1.13^{+0.74}_{-0.51}$	$1.13^{+0.74}_{-0.51}$	$0.50^{+0.40}_{-0.25}$
Ref. [2]	$0.71^{+0.14}_{-0.14}$	$0.71^{+0.14}_{-0.14}$	$0.21^{+0.04}_{-0.04}$
Ref. [3]	$2.45^{+1.77}_{-1.05}$	$2.45^{+1.77}_{-1.05}$	$1.09^{+0.82}_{-0.47}$
Ref. [8](I)	–	$0.99^{+0.89}_{-0.37}$	$0.49^{+0.33}_{-0.17}$
Ref. [8](II)	–	$1.67^{+1.32}_{-0.53}$	$0.71^{+0.57}_{-0.26}$
Ref. [8](III)	–	$1.90^{+1.48}_{-0.63}$	$0.65^{+0.55}_{-0.24}$
Ref. [9]	$1.3^{+1.3}_{-0.4}$	$1.3^{+1.2}_{-0.4}$	$0.52^{+0.57}_{-0.18}$
Ref. [11]	$1.27^{+0.36}_{-0.36}$	$1.27^{+0.36}_{-0.36}$	$0.54^{+0.16}_{-0.16}$

Summary

- a0(980) meson twist-2 DAs within BFTSR and D->a0(980)lv
- K0*(1430) twist-2 LCDA

Prospect

- a0(980) and K0*(1430) twist-3 DAs
- f0(980) meson DAs and mixture with a0(980)
- B/D→Sl+l-, B/D→S+gamma observables

Thanks for your attention !