Institute of High Energy Physics Chinese Academy of Sciences

B工厂中$e^+e^- \to J/\psi + J/\psi$两圈QCD修正

黄旭东

中国科学院高能物理研究所(IHEP)

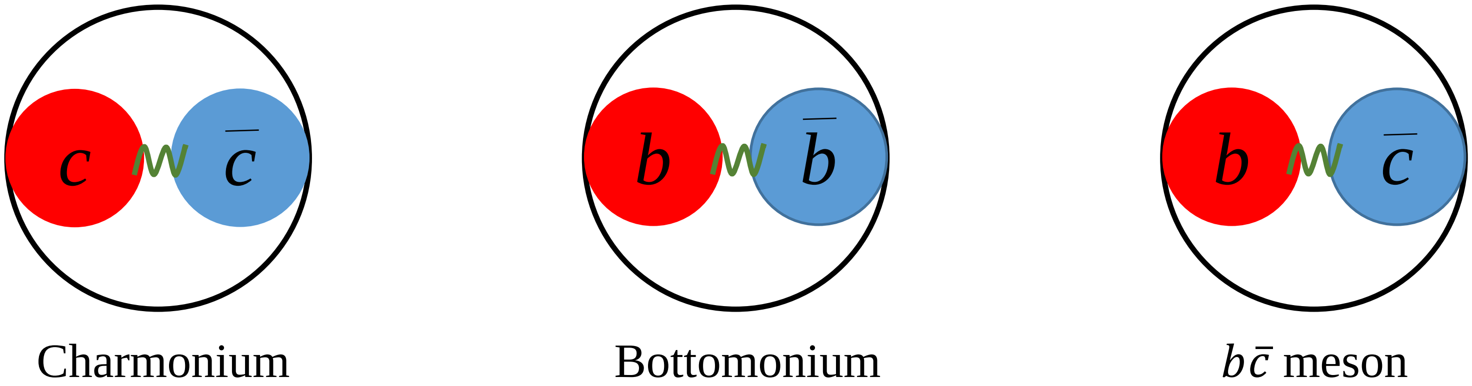
In collaboration with Bin Gong, Rui-Chang Niu, Huai-Min Yu, and Jian-Xiong Wang (arXiv:2311.04751)

第五届粒子物理天问论坛

12th Nov., 2023, 湖南. 长沙

Contents

Background



- Since the discovery of J/ψ in 1974, the heavy quarkonium production has been a focus of theoretical and experimental researches.
- Heavy quarkonium is the simplest hadron in QCD, similar to the hydrogen atom in QED.
- Heavy quarkonium present an ideal laboratory for the studying QCD, which can help to understand the interplay between the perturbative effects and nonperturbative effects.

Background

Experiment

- $\sigma(e^+e^- \to J/\psi + J/\psi) \times \mathcal{B}_{>2} < 9.1\text{fb}$ (Belle, PRD 2004)
- $\sigma(e^+e^- \to J/\psi + \eta_c) \times B_{\geq 2}$=25.6 $\pm$ 2.8 $\pm$ 3.4fb (Belle, PRD 2004)
- $\sigma(e^+e^- \to J/\psi + \chi_{c0}) \times B_{\geq 2}$=6.4 $\pm$ 1.7 $\pm$ 1.0fb (Belle, PRD 2004)

- $\sigma(e^+e^- \to J/\psi + \eta_c) \times B_{\geq 2}$=17.6 $\pm 2.8^{+1.5}_{-2.1}$fb (BaBar, PRD 2005)
- $\sigma(e^+e^- \to \rho^0 + \rho^0) = 20.7 \pm 0.7 \pm 2.7\text{fb}$ (BaBar, PRL 2006)
- $\sigma(e^+e^- \to \rho^0 + \phi) = 5.7 \pm 0.5 \pm 0.8\text{fb}$ (BaBar, PRL 2006)

Theoretical Calculation

- The LO NRQCD predictions is even greater than the LO NRQCD prediction for $e^+e^- \to J/\psi + \eta_c$
  G. T. Bodwin, J. Lee and E. Braaten, PRL 2003 8.7fb
  G. T. Bodwin, J. Lee and E. Braaten, PRD 2003 6.65fb
- Two-photon exchange model, considered the photon fragmentation contribution only
  M. Davier, M. E. Peskin and A. Snyder, arXiv:hep-ph/0606155 2006 2.38fb
- Further took into account the non-fragmentation contribution within the NRQCD factorization framework,
  G. T. Bodwin, E. Braaten, J. Lee and C. Yu, PRD 2006 1.69±0.35fb

Motivation

Theoretical Calculation

- The NLO NRQCD predictions, the combined NLO perturbative and relativistic corrections
  B. Gong and J. X. Wang, PRL 2008 -3.4~2.3fb
  Y. Fan, J. Lee and C. Yu, PRD 2013 -12~-0.43fb
- Following the recipe practised in PRD 74, 074014 (2006), splitting the amplitude into the photon-fragmentation and non-fragmentation parts
  Y. Fan, J. Lee and C. Yu, PRD 2013 1~1.5fb
- Following PRD 74, 074014 (2006), the interference and the non-fragmentation parts are then computed through NNLO within NRQCD
  W. L.Sang, F. Feng, Y. Jia, Z. Mo, J. Pan and J. Y. Zhang, PRL 2023 $2.13^{+0.30}_{-0.06}$fb

Motivation

1、The NLO perturbative correction turns out to be negative and significant, the NNLO correction in the standard NRQCD?

2、How to obtain an positive, physical cross section in the standard NRQCD?

Cross Sections and NRQCD Factorization

G.T. Bodwin, E. Braaten and G.P. Lepage, PRD 1995

- Under the NRQCD factorization, the cross section for $e^+e^- \to J/\psi + J/\psi$ can be written as

$$\sigma(e^+e^- \to J/\psi + J/\psi)$$
$$= \hat{\sigma}(e^+e^- \to (c\bar{c})[{}^3S_1^{[1]}] + (c\bar{c})[{}^1S_0^{[1]}]) \times \langle \mathcal{O}^{J/\psi} \rangle^2$$

Short-distance coefficients (SDCs)
perturbative calculable

Long-distance matrix elements (LDMEs)
nonperturbative, universal

Separate the short-distance effect and long-distance dynamics

- At B factories, the process and expressed as

$$\sigma(e^+e^- \to J/\psi + J/\psi) = \frac{1}{8s} \sum |A|^2 \, d\Phi_2$$

Calculation of the NNLO SDCs

- The SDCs can be derived by the perturbative matching procedure.
- In the lowest-order nonrelativistic approximation, only the color-singlet contribution need to be considered.
- Nearly 600 two-loop diagrams for the processes $e^+e^- \to (c\bar{c})[\,^3S_1^{[1]}] + (c\bar{c})[\,^3S_1^{[1]}]$(FeynArts) T. Hahn, CPC 2001

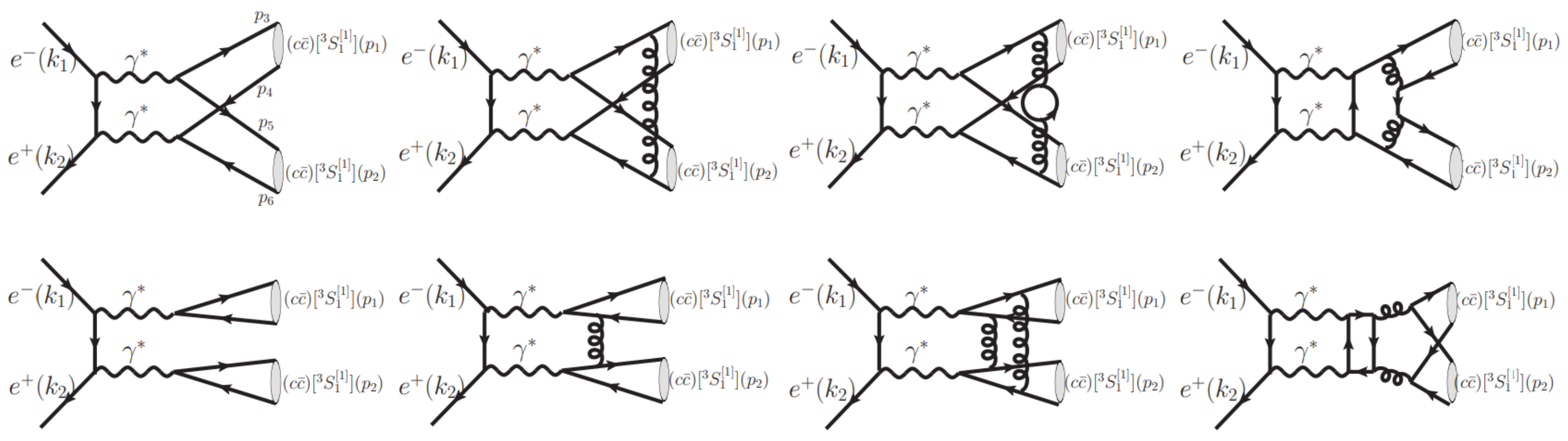
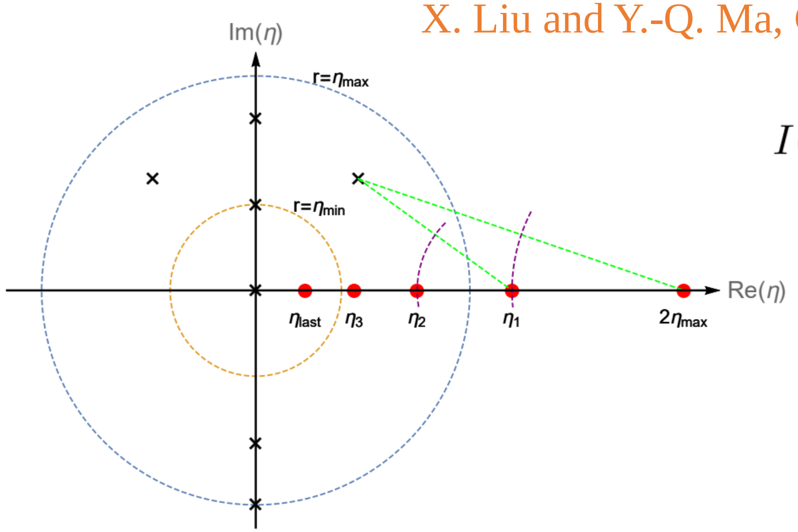


Figure 1. Several representative Feynman diagrams for $e^+e^- \to (c\bar{c})[^3S_1^{[1]}] + (c\bar{c})[^3S_1^{[1]}]$.

Calculation of the NNLO SDCs

- Handle the Lorentz index contraction and Dirac/SU(Nc) traces (FeynCalc) V. Shtabovenko, R. Mertig and F. Orellana, CPC 2016
- Decompose the Feynman amplitudes into 234 Feynman integral families (CalcLoop) Yan-Qing Ma, https://gitlab.com/multiloop-pku/calcloop
- Calculate the Feynman integrals (87287 FIs) (Kira, AMFlow) J. Klappert, F. Lange, P. Maierhöfer and J. Usovitsch, CPC 2021; X. Liu and Y.-Q. Ma, CPC 2023



$$I(D;\{\nu_\alpha\};\eta) \equiv \int \prod_{i=1}^{L} \frac{\mathrm{d}^D \ell_i}{\mathrm{i}\pi^{D/2}} \prod_{\alpha=1}^{N} \frac{1}{(\mathcal{D}_\alpha + \mathrm{i}\eta)^{\nu_\alpha}} ,$$

$$\frac{\partial}{\partial \eta} \vec{I}(\eta) = A(\eta) \vec{I}(\eta) ,$$

X. Liu, Y.-Q. Ma and C.-Y. Wang, PLB 2018

- We demand 10-digit precision for each Feynman integral family.

Renormalization

- The amplitudes are renormalized according to

$$\mathcal{A}(\alpha_s, m_Q) = Z_{2,c}^2 \left[\mathcal{A}_{bare}^{0l} + \mathcal{A}_{bare}^{1l}(\alpha_{s,bare}, m_{Q,bare}) + \mathcal{A}_{bare}^{2l}(\alpha_{s,bare}, m_{Q,bare}) \right]$$

where $m_{Q,\mathrm{bare}} = Z_{m,Q} m_Q \quad \alpha_{s,\mathrm{bare}} = \left(\frac{e^{\gamma_E}}{4\pi}\right)^{\epsilon} \mu_R^{2\epsilon} Z_{\alpha_s}^{\overline{\mathrm{MS}}} \alpha_s(\mu_R),$

P. Bärnreuther, M. Czakon and P. Fiedler, JHEP 2014
W. Tao, R. Zhu and Z.-J. Xiao, PRD 2022

$$Z_{\alpha_s} = 1 - \left(\frac{\alpha_s^{(n_f)}}{2\pi}\right) \frac{b_0}{2\epsilon} + \left(\frac{\alpha_s^{(n_f)}}{2\pi}\right)^2 \left(\frac{b_0^2}{4\epsilon^2} - \frac{b_1}{8\epsilon}\right)$$

- The renormalized $\mathcal{A}(\alpha_s, m_Q)$ can be obtained by expanding the r.h.s. of such equation over renormalized quantities to $\mathcal{O}(\alpha_s^3)$,

$$\mathcal{A}(\alpha_s, m_Q) = \mathcal{A}^{0l}(m_Q) + \mathcal{A}^{1l}(\alpha_s, m_Q) + \mathcal{A}^{2l}(\alpha_s, m_Q) + \mathcal{O}(\alpha_s^3)$$

Calculating amplitudes

- Complete-basis space

$$\begin{pmatrix} |e_1\rangle \\ |e_2\rangle \\ |e_3\rangle \\ |e_4\rangle \\ |e_5\rangle \\ |e_6\rangle \\ |e_7\rangle \\ |e_8\rangle \\ |e_9\rangle \\ |e_{10}\rangle \end{pmatrix} = \begin{pmatrix} g^{\rho_1\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) \\ k_1^{\rho_1}k_1^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) + k_2^{\rho_1}k_2^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) \\ k_1^{\rho_1}k_1^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) - k_2^{\rho_1}k_2^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) \\ k_1^{\rho_1}k_2^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) \\ k_2^{\rho_1}k_1^{\rho_2}\bar{v}_{m_e}(k_2).\not{p}_2.u_{m_e}(k_1) \\ k_1^{\rho_2}\bar{v}_{m_e}(k_2).\gamma^{\rho_1}.u_{m_e}(k_1) + k_2^{\rho_1}\bar{v}_{m_e}(k_2).\gamma^{\rho_2}.u_{m_e}(k_1) \\ k_1^{\rho_2}\bar{v}_{m_e}(k_2).\gamma^{\rho_1}.u_{m_e}(k_1) - k_2^{\rho_1}\bar{v}_{m_e}(k_2).\gamma^{\rho_2}.u_{m_e}(k_1) \\ k_1^{\rho_1}\bar{v}_{m_e}(k_2).\gamma^{\rho_2}.u_{m_e}(k_1) + k_2^{\rho_2}\bar{v}_{m_e}(k_2).\gamma^{\rho_1}.u_{m_e}(k_1) \\ k_1^{\rho_1}\bar{v}_{m_e}(k_2).\gamma^{\rho_2}.u_{m_e}(k_1) - k_2^{\rho_2}\bar{v}_{m_e}(k_2).\gamma^{\rho_1}.u_{m_e}(k_1) \\ \bar{v}_{m_e}(k_2).\not{p}_2.\gamma^{\rho_1}.\gamma^{\rho_2}.u_{m_e}(k_1) \end{pmatrix}$$

- Amplitudes

$$\mathcal{A}^{nl}|_{n=0,1,2} = \sum_{i=1}^{10} c_i^{nl}|e_i\rangle$$

$$\mathcal{A}^{ml}\mathcal{A}^{nl,*} = \sum_{i=1}^{10}\sum_{j=1}^{10} c_i^{ml}G_{i,j}c_j^{nl,*}$$

$$c_i^{nl}|_{n=0,1,2} = \sum_{i=1}^{10} G_{i,j}^{-1}d_j^{nl}$$

$$d_i^{nl}|_{n=0,1,2} = \langle\mathcal{A}^{nl}|e_i\rangle$$

$$G_{i,j} = \langle e_i|e_j\rangle$$

Differential cross section

- Then, the differential cross section can be written as

$$\frac{d\sigma_{e^+e^-\to(c\bar{c})[^3S_1^{[1]}]+(c\bar{c})[^3S_1^{[1]}]}}{d|\cos\theta|} = \frac{1}{8s}\frac{\kappa}{16\pi}\left|\mathcal{A}^{0l}+\mathcal{A}^{1l}+\mathcal{A}^{2l}+\mathcal{O}(\alpha_s^3)\right|^2$$
$$= \frac{1}{8s}\frac{\kappa}{16\pi}\left(|\mathcal{A}^{0l}|^2+2\mathrm{Re}(\mathcal{A}^{1l}\mathcal{A}^{0l,*})+2\mathrm{Re}(\mathcal{A}^{2l}\mathcal{A}^{0l,*})+|\mathcal{A}^{1l}|^2\right.$$
$$\left.+2\mathrm{Re}(\mathcal{A}^{2l}\mathcal{A}^{1l,*})+|\mathcal{A}^{2l}|^2+\cdots\right),$$

where $\kappa = \sqrt{1-(16m_c^2)/s}$ and θ is the angle between the J/ψ and the beam.

- The square of NNLO amplitude (S-NNLO)

$$\boxed{|\mathcal{A}^{0l}|^2}+\boxed{2\mathrm{Re}(\mathcal{A}^{1l}\mathcal{A}^{0l,*})}+\boxed{2\mathrm{Re}(\mathcal{A}^{2l}\mathcal{A}^{0l,*})+|\mathcal{A}^{1l}|^2}+2\mathrm{Re}(\mathcal{A}^{2l}\mathcal{A}^{1l,*})+|\mathcal{A}^{2l}|^2+\cdots$$

LO NLO NNLO

- There still remains IR divergence in $\mathcal{A}^{2l}\mathcal{A}^{0l,*}$, $\mathcal{A}^{2l}\mathcal{A}^{1l,*}$ $|\mathcal{A}^{2l}|^2$.

Differential cross section

- The anomalous dimension for the NRQCD current J

$$\gamma_J = \frac{\mathrm{d}\ln Z_J}{\mathrm{d}\ln\mu} = -C_F\left(2C_F + 3C_A\right)\frac{\pi^2}{6}\left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3).$$

A. Czarnecki and K. Melnikov, PRL 1998、M. Beneke, A. Signer and V.A. Smirnov, PRL 1998
A. Czarnecki and K. Melnikov, PLB 2001

- By including the two-loop corrections to the NRQCD bilinear operators carrying the quantum number of J/ψ in $\overline{\mathrm{MS}}$ scheme

$$\langle \mathcal{O}^{(c\bar{c})[^3S_1^{[1]}]}(^3S_1^{[1]})\rangle|_{\overline{\mathrm{MS}}} = 2N_c\left[1 - \alpha_s^2(\mu_R)\left(\frac{\mu_\Lambda^2 e^{\gamma_E}}{\mu_R^2 4\pi}\right)^{-2\epsilon}\left(\frac{C_F^2}{3} + \frac{C_F C_A}{2}\right)\frac{1}{2\epsilon}\right]$$

H.S. Chung, JHEP 2020

- The differential cross section for $e^+e^- \to J/\psi + J/\psi$ can be written as

$$\frac{d\sigma_{e^+e^-\to J/\psi+J/\psi}}{d|\cos\theta|} = \frac{d\sigma_{e^+e^-\to(c\bar{c})[^3S_1^{[1]}]+(c\bar{c})[^3S_1^{[1]}]}}{d|\cos\theta|}\frac{\langle\mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle^2}{\langle\mathcal{O}^{(c\bar{c})[^3S_1^{[1]}]}(^3S_1^{[1]})\rangle^2|_{\overline{\mathrm{MS}}}}$$
$$= (f_0 + f_1\alpha_s + f_2\alpha_s^2 + \boxed{f_3\alpha_s^3 + f_4\alpha_s^4} + \cdots)|R_s^{J/\psi}(0)|^4$$

where $\langle\mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle \approx N_c|R_s^{J/\psi}(0)|^2/(2\pi)$ incomplete

Phenomenological results

- Input parameters:

PDG, PTEP 2022
G.T. Bodwin, J. Lee and C. Yu, PRD 2008

$\sqrt{s} = 10.58\text{GeV}$， $m_b = 4.8\text{GeV}$， $m_c = 1.5\text{GeV}$， $\alpha(2m_c) = 1/132.6$，
$\alpha_s(M_z) = 0.1179$, $\left|R_s^{J/\psi}(0)\right|^2_{LO} = 0.492\text{GeV}^3$， $\left|R_s^{J/\psi}(0)\right|^2_{NLO} = 0.796\text{GeV}^3$，
$\left|R_s^{J/\psi}(0)\right|^2_{NNLO,\mu_\Lambda=1GeV} = 1.810\text{GeV}^3$，

- The leptonic decay widths : $\Gamma_{J/\psi\to e^+e^-} = 5.53\text{keV}$

$$\Gamma_{J/\psi\to e^+e^-} = \frac{4\alpha^2 e_c^2}{m_{J/\psi}^2}|R_s^{J/\psi}(0)|^2\Big\{1 - 2C_F\frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2\Big[-2C_F\beta_0\ln\frac{\mu_R^2}{m_c^2} - 3\pi^2 C_F\left(\frac{1}{18}C_F + \frac{1}{12}C_A\right)\ln\frac{\mu_\Lambda^2}{m_c^2} + C_A C_F\left(\frac{89\pi^2}{144} - \frac{151}{72} - \frac{5\pi^2}{6}\ln 2 - \frac{13}{4}\zeta_3\right) + C_F^2\left(\frac{23}{8} - \frac{79\pi^2}{36} + \pi^2\ln 2 - \frac{1}{2}\zeta_3\right) + C_F T_F n_H\left(\frac{22}{9} - \frac{2\pi^2}{9}\right) + \frac{11}{18}C_F T_F n_L\Big]\Big\}^2,$$

F. Feng, Y. Jia, Z. Mo, J. Pan, W.-L. Sang, and J.-Y. Zhang, arXiv:2207.14259

Phenomenological results

- The differential cross section for $e^+e^- \to J/\psi + J/\psi$ can be written as

$$\frac{d\sigma_{e^+e^-\to J/\psi+J/\psi}}{d|\cos\theta|} = \frac{d\sigma_{e^+e^-\to(c\bar{c})[^3S_1^{[1]}]+(c\bar{c})[^3S_1^{[1]}]}}{d|\cos\theta|}\frac{\langle\mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle^2}{\langle\mathcal{O}^{(c\bar{c})[^3S_1^{[1]}]}(^3S_1^{[1]})\rangle^2|_{\overline{\mathrm{MS}}}}$$
$$= (f_0 + f_1\alpha_s + f_2\alpha_s^2 + f_3\alpha_s^3 + f_4\alpha_s^4 + \cdots)|R_s^{J/\psi}(0)|^4$$

$\|\cos\theta\|$	f_0	f_1	f_2
0.193	3.0687	−11.1472	$-43.3988 + 0.5647 n_f - 11.1472\beta_0 L_\mu - 15.9116 L_{\mu_\Lambda}$
0.402	3.8973	−14.2469	$-54.8858 + 0.7247 n_f - 14.2469\beta_0 L_\mu - 20.2080 L_{\mu_\Lambda}$
0.601	5.9069	−21.6244	$-83.0903 + 1.1036 n_f - 21.6244\beta_0 L_\mu - 30.6282 L_{\mu_\Lambda}$
0.698	7.9392	−28.9429	$-111.9326 + 1.4775 n_f - 28.9429\beta_0 L_\mu - 41.1664 L_{\mu_\Lambda}$
0.800	12.0746	−43.5649	$-171.1529 + 2.2221 n_f - 43.5649\beta_0 L_\mu - 62.6088 L_{\mu_\Lambda}$
0.849	15.7238	−56.2870	$-223.7382 + 2.8694 n_f - 56.2870\beta_0 L_\mu - 81.5310 L_{\mu_\Lambda}$
0.902	22.8893	−80.9980	$-327.4304 + 4.1287 n_f - 80.9980\beta_0 L_\mu - 118.6851 L_{\mu_\Lambda}$
0.922	27.1569	−95.6123	$-389.3525 + 4.8758 n_f - 95.6123\beta_0 L_\mu - 140.8136 L_{\mu_\Lambda}$
0.951	37.0190	−129.2083	$-532.7535 + 6.6029 n_f - 129.2083\beta_0 L_\mu - 191.9502 L_{\mu_\Lambda}$
0.975	50.9428	−176.3416	$-735.9322 + 9.0683 n_f - 176.3416\beta_0 L_\mu - 264.1479 L_{\mu_\Lambda}$
0.999	54.7376	−187.8744	$-797.7502 + 10.2369 n_f - 187.8744\beta_0 L_\mu - 283.8247 L_{\mu_\Lambda}$

where $\beta_0 = \frac{1}{4\pi}\left(11 - \frac{2}{3}n_f\right)$ $L_\mu = \ln\frac{\mu_R^2}{m_c^2}$ $L_{\mu_\Lambda} = \ln\frac{\mu_\Lambda^2}{m_c^2}$

Phenomenological results

$\lvert\cos\theta\rvert$	f_3
0.193	$92.0656 + 12.3660 L_\mu + 28.9002 L_{\mu_\Lambda}$
0.402	$117.4510 + 15.9175 L_\mu + 36.9363 L_{\mu_\Lambda}$
0.601	$178.1131 + 24.2128 L_\mu + 56.0634 L_{\mu_\Lambda}$
0.698	$238.4933 + 32.2766 L_\mu + 75.0372 L_{\mu_\Lambda}$
0.800	$359.3808 + 48.0764 L_\mu + 112.9460 L_{\mu_\Lambda}$
0.849	$464.6776 + 61.6160 L_\mu + 145.9294 L_{\mu_\Lambda}$
0.902	$669.2768 + 87.6152 L_\mu + 209.9949 L_{\mu_\Lambda}$
0.922	$790.2901 + 102.8788 L_\mu + 247.8839 L_{\mu_\Lambda}$
0.951	$1068.4986 + 137.7803 L_\mu + 334.9846 L_{\mu_\Lambda}$
0.975	$1458.9501 + 186.4323 L_\mu + 457.1819 L_{\mu_\Lambda}$
0.999	$1557.3600 + 196.9479 L_\mu + 487.0818 L_{\mu_\Lambda}$

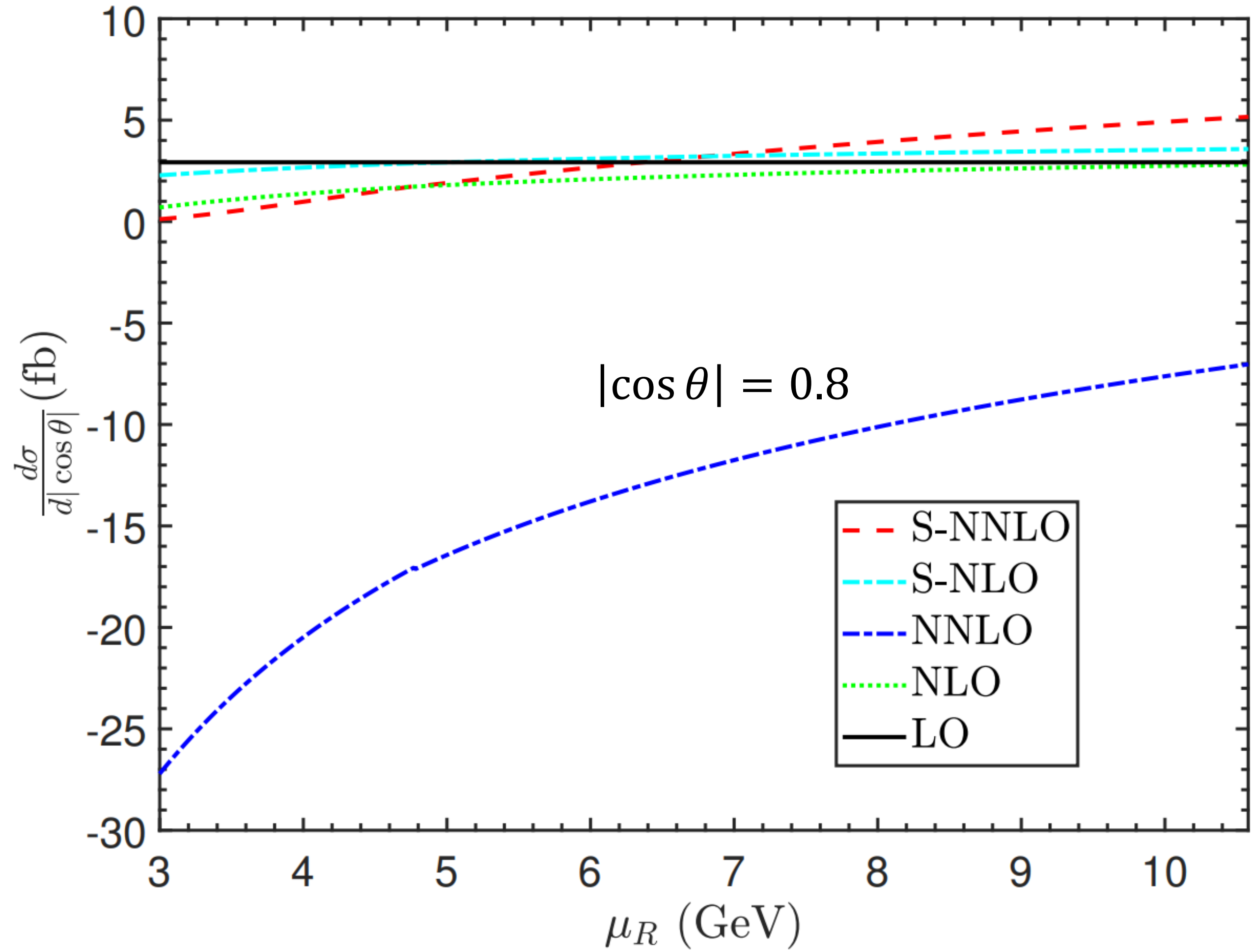
Phenomenological results

$\lvert\cos\theta\rvert$	f_4
0.193	$209.5396 + 56.1687L_\mu + 131.4701L_{\mu_\Lambda} + 17.6318L_\mu L_{\mu_\Lambda} + 3.7722L_\mu^2 + 20.6262L_{\mu_\Lambda}^2$
0.402	$265.3937 + 71.6562L_\mu + 166.7230L_{\mu_\Lambda} + 22.5346L_\mu L_{\mu_\Lambda} + 4.8556L_\mu^2 + 26.1955L_{\mu_\Lambda}^2$
0.601	$401.8986 + 108.6657L_\mu + 252.5597L_{\mu_\Lambda} + 34.2039L_\mu L_{\mu_\Lambda} + 7.3860L_\mu^2 + 39.7032L_{\mu_\Lambda}^2$
0.698	$540.8027 + 145.5020L_\mu + 339.6221L_{\mu_\Lambda} + 45.7797L_\mu L_{\mu_\Lambda} + 9.8459L_\mu^2 + 53.3639L_{\mu_\Lambda}^2$
0.800	$824.0584 + 219.2561L_\mu + 517.0743L_{\mu_\Lambda} + 68.9077L_\mu L_{\mu_\Lambda} + 14.6655L_\mu^2 + 81.1595L_{\mu_\Lambda}^2$
0.849	$1074.4249 + 283.4970L_\mu + 673.7844L_{\mu_\Lambda} + 89.0306L_\mu L_{\mu_\Lambda} + 18.7958L_\mu^2 + 105.6884L_{\mu_\Lambda}^2$
0.902	$1566.1342 + 408.3217L_\mu + 981.5335L_{\mu_\Lambda} + 128.1166L_\mu L_{\mu_\Lambda} + 26.7267L_\mu^2 + 153.8510L_{\mu_\Lambda}^2$
0.922	$1858.9494 + 482.1512L_\mu + 1164.8197L_{\mu_\Lambda} + 151.2324L_\mu L_{\mu_\Lambda} + 31.3829L_\mu^2 + 182.5361L_{\mu_\Lambda}^2$
0.951	$2536.2604 + 651.8733L_\mu + 1588.3677L_{\mu_\Lambda} + 204.3721L_\mu L_{\mu_\Lambda} + 42.0295L_\mu^2 + 248.8244L_{\mu_\Lambda}^2$
0.975	$3491.4667 + 890.0966L_\mu + 2186.5424L_{\mu_\Lambda} + 278.9239L_\mu L_{\mu_\Lambda} + 56.8706L_\mu^2 + 342.4140L_{\mu_\Lambda}^2$
0.999	$3766.2171 + 950.1359L_\mu + 2354.0053L_{\mu_\Lambda} + 297.1657L_\mu L_{\mu_\Lambda} + 60.0783L_\mu^2 + 367.9209L_{\mu_\Lambda}^2$

- They are not changed when we demand 10-digit or 20-digit precision for each Feynman integral family

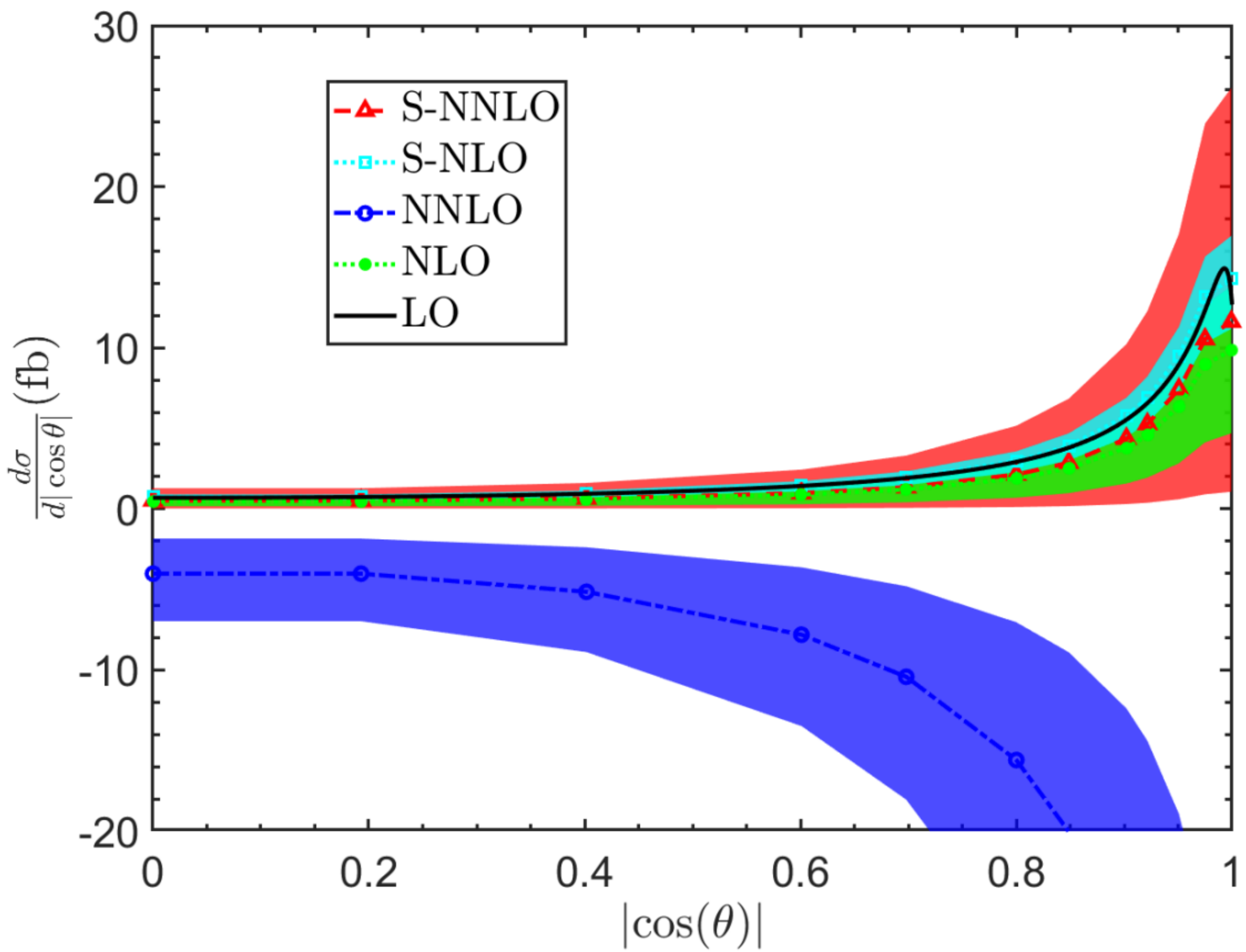
renormalization scales μ_R dependence

- The μ_R dependence of the differential cross section at LO, NLO, NNLO, the square of NLO amplitude (S-NLO), and the square of NNLO amplitude (S-NNLO).



Differential cross section

- The differential cross section for $e^+e^- \to J/\psi + J/\psi$. (central value for $\mu_R = \sqrt{s}/2$, bound for $\mu_R \in [2m_c, \sqrt{s}]$)



Integrated cross section

- The integrated cross section can be obtained by using the trapezoidal rule approximately

Ueberhuber, C. W. Berlin: Springer-Verlag, 1997.

$$\int_{x_1}^{x_2} f(x)dx = \frac{f(x_2)+f(x_1)}{2}(x_2-x_1)$$

- The integrated cross section (in fb) of $e^+e^- \to J/\psi + J/\psi$ with three typical renormalization scales μ_R.

$\sigma(fb)$	LO	NLO	NNLO	S-NLO	S-NNLO
$\mu_R = 2m_c$	2.29	0.61	−21.10	1.83	0.12
$\mu_R = \sqrt{s}/2$	2.29	1.54	−11.97	2.37	1.76
$\mu_R = \sqrt{s}$	2.29	2.25	−5.27	2.84	4.17

Scale uncertainties are ~ 106%, 132%, 43%, 230%

Integrated cross section

- The integrated cross section (in fb) of $e^+e^- \to J/\psi + J/\psi$ at the B factories :

$$\sigma_{\mathrm{S-NNLO}} = 1.76^{+2.41+0.25}_{-1.64-0.25} = 1.76^{+2.42}_{-1.66}\ (\mathrm{fb}),$$

Uncertainties caused by: $\mu_R \in [2m_c, \sqrt{s}]$ and the method for estimating the integrated cross section from the differential cross section.

$$\frac{|f(x_2)-f(x_1)|}{2}(x_2 - x_1)$$

- Results of PRL 131 (2023) 161904

σ (fb)	Fragmentation	LO	NLO	NNLO
Optimized NRQCD	2.52	1.85	$1.93^{+0.05}_{-0.01}$	$2.13^{+0.30}_{-0.06}$
Traditional NRQCD		6.12	$1.56^{+0.73}_{-2.95}$	$-2.38^{+1.27}_{-5.35}$

- Exp: an upper limit is placed, $\sigma(e^+e^- \to J/\psi + J/\psi)\, \mathcal{B}_{>2} < 9.1$ fb at the 90% confidence level,

Summary

- The μ_R dependence of the cross section for $e^+e^- \to J/\psi + J/\psi$ becomes larger at the NNLO level.
- The NNLO prediction for $e^+e^- \to J/\psi + J/\psi$ suffers from an unphysical, negative cross section.
- We obtain a physical prediction of the cross section for $e^+e^- \to J/\psi + J/\psi$ in the standard NRQCD method.

Thanks!