A Lattice QCD study of the low-energy interactions of doubly charmed baryons

Jing-Yu Yi（义景宇）
湖南大学

Collaborators: Ze-Rui Liang（梁泽锐）, Qu-Zhi Li（李衢智）, Liuming Liu（刘柳明）,
Xiaonu Xiong（熊小努）, Yi-Bo Yang（杨一玻）, Peng Sun（孙鹏）, De-Liang Yao（姚德良）

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第五届粒子物理天问论坛

Outline

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- Doubly charmed baryons
- Simulation in Lattice QCD

Lattice setup

- Ensembles
- Distillation quark smearing

Results

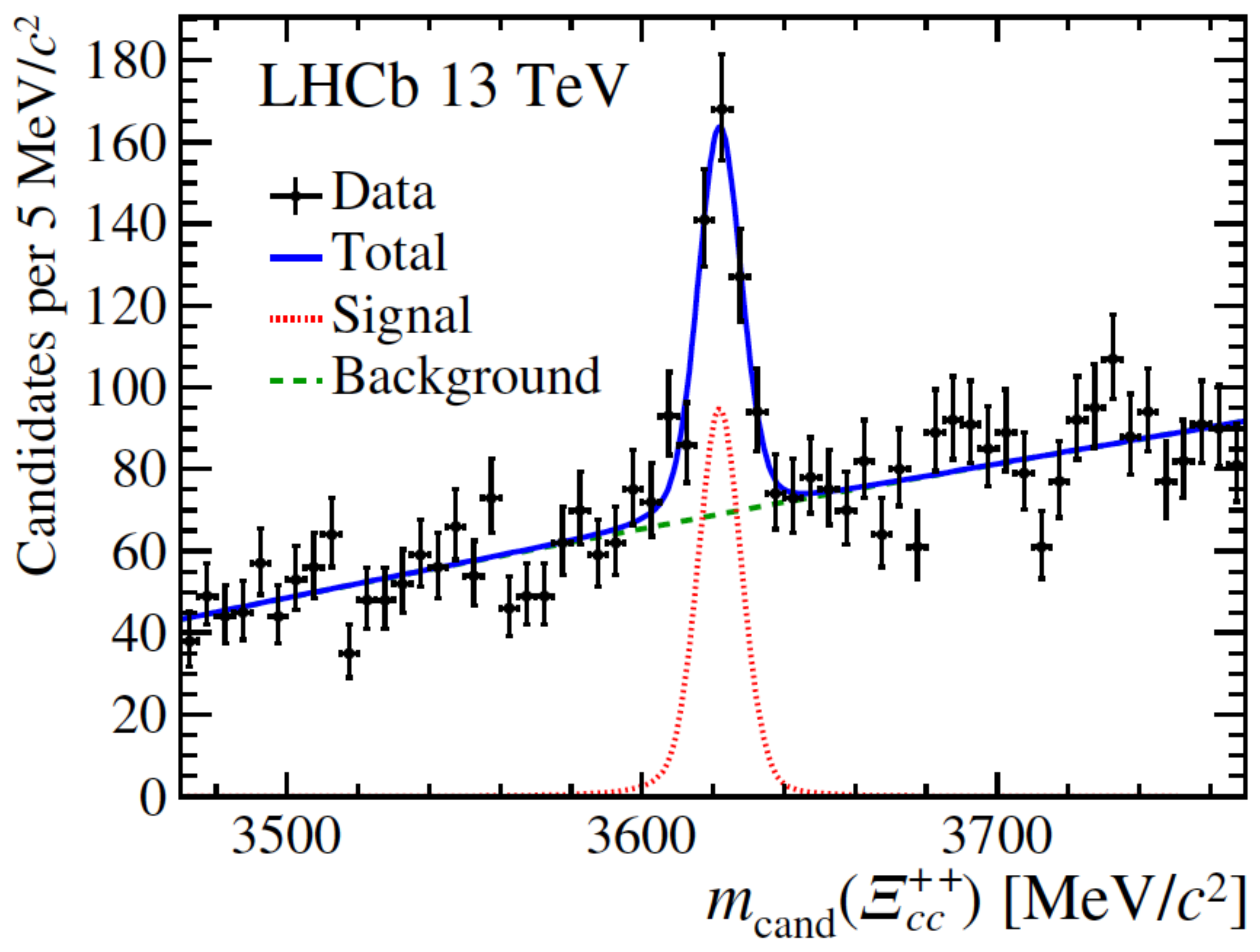
- Energy levels
- Scattering lengths

Summary and Outlook

Doubly charmed baryons

Observation of doubly charmed baryons

- SELEX M. Mattson and others, Phys. Rev. Lett. 89, 112001 (2002).
- FOCUS S. P. Ratti, Nucl. Phys. B Proc. Suppl. 115, 33 (2003).
- Belle R. Chistov and others, Phys. Rev. Lett. 97, 162001 (2006).
- BABAR B. Aubert and others, Phys. Rev. D 74, 011103 (2006).
- LHCb R. Aaij and others, Phys. Rev. Lett. 119, 112001 (2017).
- Searching for Ξ_{cc}^{+}, Ω_{cc}^{+}, …



Theoretical studies

- Quark model
  - Q.-F. Lü, K.-L. Wang, L.-Y. Xiao, and X.-H. Zhong, Phys. Rev. D 96, 114006 (2017).
  - W. Wang, F.-S. Yu, and Z.-X. Zhao, Eur. Phys. J. C 77, 781 (2017).
  - H.-W. Ke, F. Lu, X.-H. Liu, and X.-Q. Li, Eur. Phys. J. C 80, 140 (2020).
  - H.-W. Ke and X.-Q. Li, Phys. Rev. D 105, 096011 (2022).
  - ……

Doubly charmed baryons

Theoretical studies

- Lattice QCD
  - L. Liu, H.-W. Lin, K. Orginos, and A. Walker-Loud, Phys. Rev. D 81, 094505 (2010).
  - Z. S. Brown, W. Detmold, S. Meinel, and K. Orginos, Phys. Rev. D 90, 094507 (2014).
  - N. Mathur and M. Padmanath, Phys. Rev. D 99, 031501 (2019).
  - H. Bahtiyar, K. U. Can, G. Erkol, P. Gubler, M. Oka, and T. T. Takahashi, Phys. Rev. D 102, 054513 (2020).
- QCD sum rules
  - H.-X. Chen, Q. Mao, W. Chen, X. Liu, and S.-L. Zhu, Phys. Rev. D 96, 031501 (2017).
  - Z.-G. Wang, Eur. Phys. J. C 78, 826 (2018).
  - C.-Y. Wang, C. Meng, Y.-Q. Ma, and K.-T. Chao, Phys. Rev. D 99, 014018 (2019).
  - Y.-J. Shi, W. Wang, and Z.-X. Zhao, Eur. Phys. J. C 80, 568 (2020).
- Chiral perturbation theory
  - Z.-H. Guo, Phys. Rev. D 96, 074004 (2017).
  - Z.-F. Sun and M. J. Vicente Vacas, Phys. Rev. D 93, 094002 (2016).
  - A. N. Hiller Blin, Z.-F. Sun, and M. J. Vicente Vacas, Phys. Rev. D 98, 054025 (2018).
  - M.-J. Yan, X.-H. Liu, S. Gonzàlez-Solís, F.-K. Guo, C. Hanhart, U.-G. Meißner, and B.-S. Zou, Phys. Rev. D 98, 091502 (2018).
  - M.-Z. Liu, Y. Xiao, and L.-S. Geng, Phys. Rev. D 98, 014040 (2018).
  - D.-L. Yao, Phys. Rev. D 97, 034012 (2018).
  - L. Meng and S.-L. Zhu, Phys. Rev. D 100, 014006 (2019).
- ……

Doubly charmed baryons

Baryon chiral perturbation theory (BChPT)

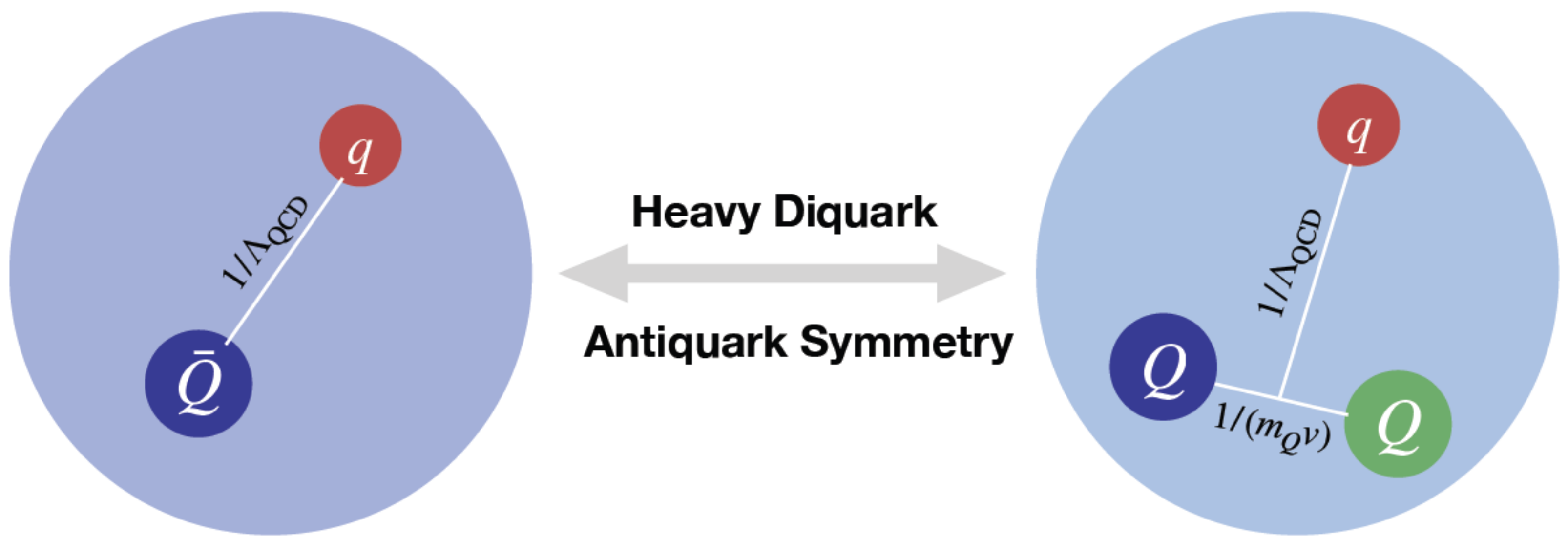
$$
\begin{aligned}
\mathcal{L}^{(1)}_{\psi\phi} &= \bar{\psi}\left(i\not{D} - m\right)\psi + \frac{g}{2}\bar{\psi}\not{u}\gamma_5\psi \ , \\
\mathcal{L}^{(2)}_{\psi\phi} &= b_1\bar{\psi}\left\langle\chi_+\right\rangle\psi + b_2\bar{\psi}\widetilde{\chi}_+\psi + b_3\bar{\psi}u^2\psi + b_4\bar{\psi}\left\langle u^2\right\rangle\psi + \frac{b_5}{m^2}\bar{\psi}\left(\{u^\mu, u^\nu\}D_{\mu\nu} + H.c.\right)\psi \\
&\quad + \frac{b_6}{m^2}\bar{\psi}\left(\left\langle u^\mu u^\nu\right\rangle D_{\mu\nu} + H.c.\right)\psi + ib_7\bar{\psi}\left[u^\mu, u^\nu\right]\sigma_{\mu\nu}\psi \ , \\
\mathcal{L}^{(3)}_{\psi\phi} &= ic_{11}\bar{\psi}\left[u_\mu, h^{\mu\nu}\right]\gamma_\nu\psi + \frac{c_{12}}{m^2}\bar{\psi}\left(i\left[u^\mu, h^{\nu\rho}\right]\gamma_\mu D_{\nu\rho} + H.c.\right)\psi + \frac{c_{13}}{m}\bar{\psi}\left(i\left\{u^\mu, h^{\nu\rho}\right\}\right. \\
&\quad \left.\times\sigma_{\mu\nu}D_\rho + H.c.\right)\psi + \frac{c_{14}}{m}\bar{\psi}\left(i\sigma_{\mu\nu}\left\langle u^\mu h^{\nu\rho}\right\rangle D_\rho + H.c.\right)\psi + c_{15}\bar{\psi}\left\{u^\mu, \widetilde{\chi}_+\right\}\gamma_5\gamma_\mu\psi \\
&\quad + c_{16}\bar{\psi}u^\mu\gamma_5\gamma_\mu\left\langle\chi_+\right\rangle\psi + c_{17}\bar{\psi}\gamma_5\gamma_\mu\left\langle u^\mu\widetilde{\chi}_+\right\rangle\psi + ic_{18}\bar{\psi}\gamma_5\gamma_\mu\left[D^\mu, \widetilde{\chi}_-\right]\psi \\
&\quad + ic_{19}\bar{\psi}\gamma_5\gamma_\mu\left\langle\left[D^\mu, \chi_-\right]\right\rangle\psi + c_{20}\bar{\psi}\left[\widetilde{\chi}_-, u^\mu\right]\gamma_\mu\psi \ .
\end{aligned}
$$

Unknown Low Energy Constants

- Experimental data
- Lattice QCD data
- Symmetries
- ……

Heavy diquark-antiquark symmetry

- M. J. Savage and M. B. Wise, Phys. Lett. B 248, 177 (1990).
- M.-J. Yan, X.-H. Liu, S. Gonzàlez-Solís, F.-K. Guo, C. Hanhart, U.-G. Meißner, and B.-S. Zou, Phys. Rev. D 98, 091502 (2018).
- L. Meng and S.-L. Zhu, Phys. Rev. D 100, 014006 (2019).
- Z.-R. Liang, P.-C. Qiu, and D.-L. Yao, JHEP 07, 124 (2023).
- ……



Doubly charmed baryons

Scattering lengths in BChPT

Z.-R. Liang, P.-C. Qiu, and D.-L. Yao, JHEP 07, 124 (2023).

$$a_{0+}^{(S,I)} = \frac{m_\psi}{4\pi\left(m_\psi + m_\phi\right)} \left\{ \left[A^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0} + m_\phi \left[B^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0} \right\}$$

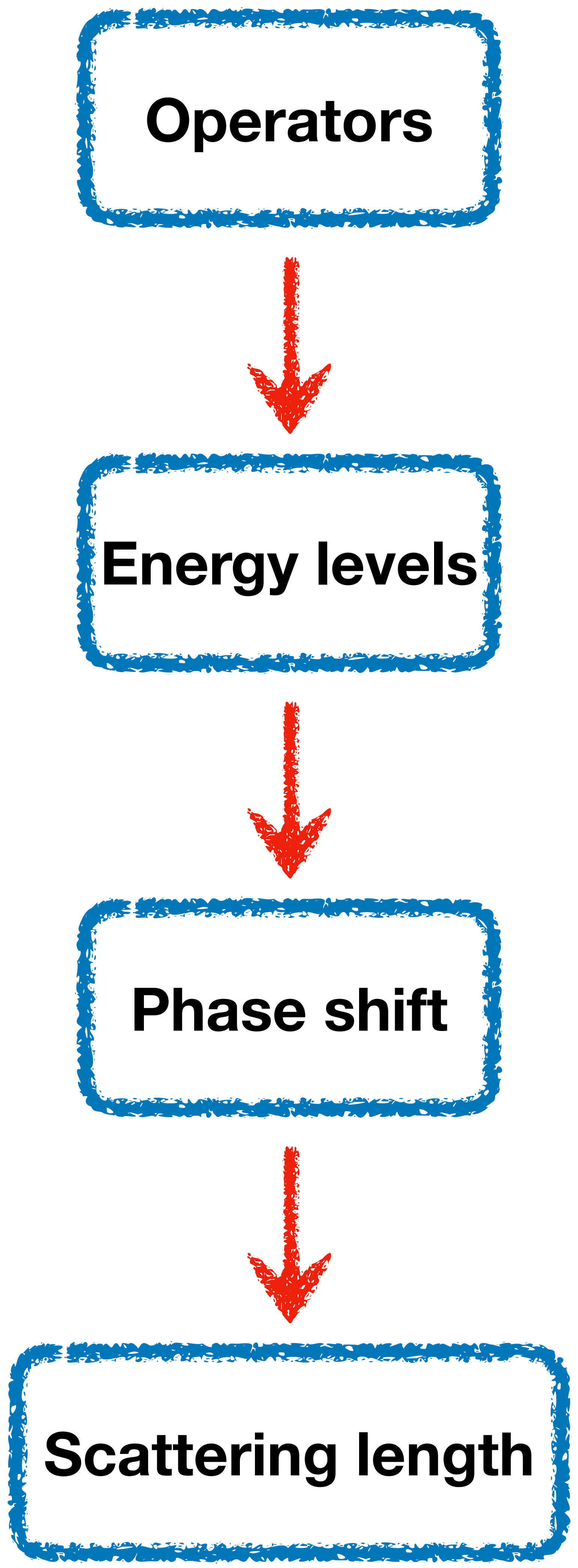
$$a_{1+}^{(S,I)} = \frac{m_\psi}{6\pi\left(m_\psi + m_\phi\right)} \left\{ \left[\partial_t A^{(S,I)}(s,t)\right]_{t=0,\mathbf{q}^2=0} + m_\phi \left[\partial_t B^{(S,I)}(s,t)\right]_{t=0,\mathbf{q}^2=0} \right\}$$

$$a_{1-}^{(S,I)} = a_{1+}^{(S,I)} - \frac{1}{16\pi m_\psi\left(m_\psi + m_\phi\right)} \left\{ \left[A^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0} - \left(2m_\psi + m_\phi\right) \left[B^{(S,I)}(s,0)\right]_{\mathbf{q}^2=0} \right\}$$

Scattering lengths in Lattice QCD

- Liu et al., “Interactions of Charmed Mesons with Light Pseudoscalar Mesons from Lattice QCD and Implications on the Nature of the $D_{s0}^*(2317)$.”
- Lyu et al., “Doubly Charmed Tetraquark T_{cc}^+ from Lattice QCD near Physical Point.”
- Xing et al., “First Observation of the Hidden-Charm Pentaquarks on Lattice.”
- Padmanath and Prelovsek, “Signature of a Doubly Charm Tetraquark Pole in DD* Scattering on the Lattice.”
- Bulava et al., “The Two-Pole Nature of the $\Lambda(1405)$ from Lattice QCD.”
- ……

Determination of LECs on Lattice



Simulation in Lattice QCD

Determination of LECs on Lattice

Operators → Energy levels → Phase shift → Scattering length

Goldstone boson operators

$$\mathcal{O}_M(x) \equiv \bar{\psi}^{f_1}(x)\Gamma\psi^{f_2}(x)$$

$$\mathcal{O}_{\pi^+}(x) = \bar{d}(x)^a_\alpha\left(\gamma_5\right)_{\alpha\beta}u(x)^a_\beta$$

$$\mathcal{O}_{K^+}(x) = \bar{s}(x)^a_\alpha\left(\gamma_5\right)_{\alpha\beta}u(x)^a_\beta$$

$$\mathcal{O}_{\pi^0}(x) = \frac{1}{\sqrt{2}}\left(\bar{u}(x)^a_\alpha\left(\gamma_5\right)_{\alpha\beta}u(x)^a_\beta - \bar{d}(x)^a_\alpha\left(\gamma_5\right)_{\alpha\beta}d(x)^a_\beta\right)$$

$$\mathcal{O}_{K^0}(x) = \bar{s}(x)^a_\alpha\left(\gamma_5\right)_{\alpha\beta}d(x)^a_\beta$$

$$\mathcal{O}_{\eta}(x) = \frac{1}{\sqrt{2}}\left(\bar{u}(x)^a_\alpha\left(\gamma_5\right)_{\alpha\beta}u(x)^a_\beta + \bar{d}(x)^a_\alpha\left(\gamma_5\right)_{\alpha\beta}d(x)^a_\beta\right)$$

Doubly Charmed Baryon operators

$$\mathcal{O}^B_\gamma(x) = \epsilon^{ijk}P_\pm\left[\psi^{f_1,i}_\alpha(x)\left(C\gamma_5\right)_{\alpha\beta}\psi^{f_2,j}_\beta(x)\right]\psi^{f_3,k}_\gamma(x)$$

$$\mathcal{O}_{\Xi^{++}_{cc}(ccu)}(x) = \epsilon^{ijk}P_+\left[Q^{iT}_c(x)C\gamma_5 q^j_u(x)\right]Q^k_c(x)$$

$$\mathcal{O}_{\Xi^{+}_{cc}(ccd)}(x) = \epsilon^{ijk}P_+\left[Q^{iT}_c(x)C\gamma_5 q^j_d(x)\right]Q^k_c(x)$$

$$\mathcal{O}_{\Omega^{+}_{cc}(ccs)}(x) = \epsilon^{ijk}P_+\left[Q^{iT}_c(x)C\gamma_5 q^j_s(x)\right]Q^k_c(x)$$

Simulation in Lattice QCD

Determination of LECs on Lattice

- **Operators**
- Energy levels
- Phase shift
- Scattering length

Two-particle operators

$$\mathcal{O}^{I=0}_{\Xi_{cc}K}(t) = \frac{1}{\sqrt{2}}\mathcal{O}_{\Xi_{cc}^{++}}(t)\mathcal{O}_{K^0}(t) - \frac{1}{\sqrt{2}}\mathcal{O}_{\Xi_{cc}^{+}}(t)\mathcal{O}_{K^+}(t)$$

$$\mathcal{O}^{I=1}_{\Xi_{cc}K}(t) = \mathcal{O}_{\Xi_{cc}^{++}}(t)\mathcal{O}_{K^+}(t) \qquad \mathcal{O}^{I=3/2}_{\Xi_{cc}\pi}(t) = \mathcal{O}_{\Xi_{cc}^{++}}(t)\mathcal{O}_{\pi^+}(t)$$

$$\mathcal{O}^{I=1/2}_{\Omega_{cc}\bar{K}}(t) = \mathcal{O}_{\Omega_{cc}^{+}}(t)\mathcal{O}_{\bar{K}^0}(t)$$

For s-wave scattering, $J^P = \frac{1}{2}^- \longrightarrow G_1^-$

$$\mathcal{O}^{\Xi_{cc}K,\ I=0}_{\mathbf{p_1},\mathbf{p_2}} = \sum_{\alpha,\mathbf{p_1},\mathbf{p_2}} C_{\alpha,\mathbf{p_1},\mathbf{p_2}}\left(\frac{1}{\sqrt{2}}\mathcal{O}_{\Xi_{cc}^{++},\alpha}(\mathbf{p_1})\mathcal{O}_{K^0}(\mathbf{p_2}) - \frac{1}{\sqrt{2}}\mathcal{O}_{\Xi_{cc}^{+},\alpha}(\mathbf{p_1})\mathcal{O}_{K^+}(\mathbf{p_2})\right)$$

$$\mathcal{O}^{\Xi_{cc}K,\ I=1}_{\mathbf{p_1},\mathbf{p_2}} = \sum_{\alpha,\mathbf{p_1},\mathbf{p_2}} C_{\alpha,\mathbf{p_1},\mathbf{p_2}}\left(\mathcal{O}_{\Xi_{cc}^{++},\alpha}(\mathbf{p_1})\mathcal{O}_{K^+}(\mathbf{p_2})\right)$$

$$\mathcal{O}^{\Omega_{cc}\bar{K},\ I=1/2}_{\mathbf{p_1},\mathbf{p_2}} = \sum_{\alpha,\mathbf{p_1},\mathbf{p_2}} C_{\alpha,\mathbf{p_1},\mathbf{p_2}}\left(\mathcal{O}_{\Omega_{cc}^{+},\alpha}(\mathbf{p_1})\mathcal{O}_{\bar{K}^0}(\mathbf{p_2})\right) \qquad \mathcal{O}^{\Xi_{cc}\pi,\ I=3/2}_{\mathbf{p_1},\mathbf{p_2}} = \sum_{\alpha,\mathbf{p_1},\mathbf{p_2}} C_{\alpha,\mathbf{p_1},\mathbf{p_2}}\left(\mathcal{O}_{\Xi_{cc}^{++},\alpha}(\mathbf{p_1})\mathcal{O}_{\pi^+}(\mathbf{p_2})\right)$$

	α	$\mathbf{p_1}$	$\mathbf{p_2}$	$C_{\alpha,\mathbf{p_1},\mathbf{p_2}}$
$\lvert\mathbf{p_{1,2}}\rvert = 0$	1	(0,0,0)	(0,0,0)	1
$\lvert\mathbf{p_{1,2}}\rvert = 1$	1	(-1,0,0)	(1,0,0)	1
	1	(1,0,0)	(-1,0,0)	1
	1	(0,-1,0)	(0,1,0)	1
	1	(0,1,0)	(0,-1,0)	1
	1	(0,0,-1)	(0,0,1)	1
	1	(0,0,1)	(0,0,-1)	1
$\lvert\mathbf{p_{1,2}}\rvert = \sqrt{2}$	1	(-1,-1,0)	(1,1,0)	1
	1	(1,1,0)	(-1,-1,0)	1
	1	(-1,0,-1)	(1,0,1)	1
	1	(1,0,1)	(-1,0,-1)	1
	1	(0,-1,-1)	(0,1,1)	1
	1	(0,1,1)	(0,-1,-1)	1
	1	(-1,1,0)	(1,-1,0)	1
	1	(1,-1,0)	(-1,1,0)	1
	1	(-1,0,1)	(1,0,-1)	1
	1	(1,0,-1)	(-1,0,1)	1
	1	(0,1,-1)	(0,-1,1)	1
	1	(0,-1,1)	(0,1,-1)	1

H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).

Simulation in Lattice QCD

Determination of LECs on Lattice

Operators

↓

Energy levels

↓

Phase shift

↓

Scattering length

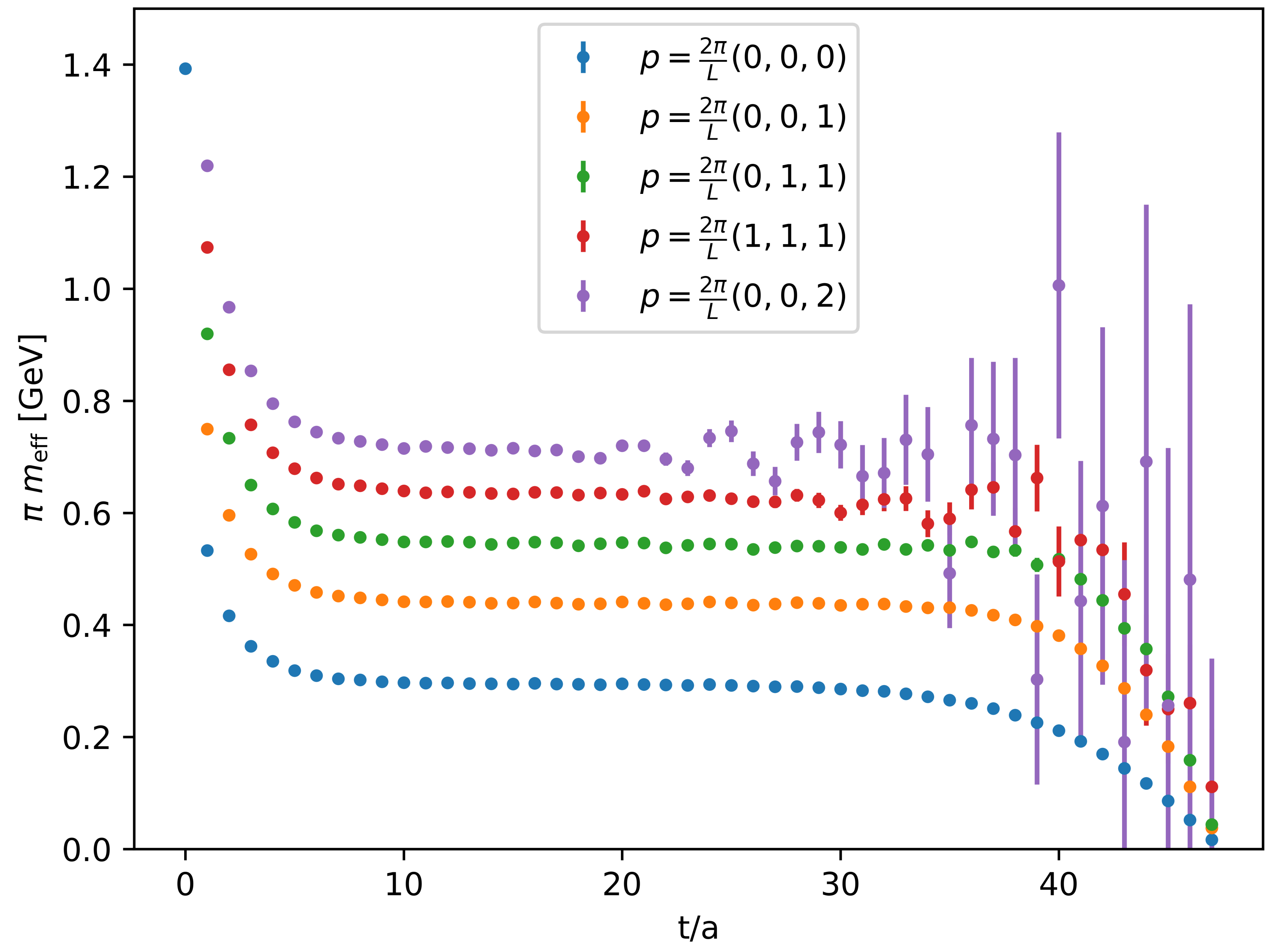
Two point correlation function

$$\left\langle \mathcal{O}_M(x)\mathcal{O}_M^\dagger(0)\right\rangle$$

$$= \left\langle \bar{\psi}_\alpha^{f_1,a}(x)\Gamma_{\alpha\beta}\psi_\beta^{f_2,a}(x)\bar{\psi}_{\alpha'}^{f_2,b}(0)\Gamma_{\alpha'\beta'}\psi_{\beta'}^{f_1,b}(0)\right\rangle$$

$$= -\Gamma_{\alpha\beta}\Gamma_{\alpha'\beta'}G_{\beta'\alpha}^{f_1,ba}(0,x)G_{\beta'\alpha'}^{f_2,ab}(x,0)$$

$$= -\operatorname{tr}\left[\Gamma G^{f_2}(x,0)\Gamma G^{f_1}(0,x)\right]$$



$$\left\langle \mathcal{O}\left(\vec{x},t\right)\mathcal{O}^\dagger(0)\right\rangle = \sum_{n=0}^{\infty}\frac{\langle 0|\mathcal{O}|n\rangle\langle n|\mathcal{O}^\dagger|0\rangle}{2E_n}e^{-E_n t} = c_0e^{-E_0t}+c_1e^{-E_1t}+\cdots$$

Excited states are suppressed at large t

$\longrightarrow$ $\left\langle \mathcal{O}\left(\vec{x},t\right)\mathcal{O}^\dagger(0)\right\rangle \approx c_0e^{-E_0t}$ for one state fit

$c_0e^{-E_0t}+c_1e^{-E_1t}$ for two state fit

Simulation in Lattice QCD

Determination of LECs on Lattice

Operators

↓

Energy levels

↓

Phase shift

↓

Scattering length

Four point correlation function

- Solve generalized eigenvalue problem (GEVP) M. Luscher and U. Wolff, Nucl. Phys. B 339, 222 (1990).

$$C_{ij}(t) = \sum_{t_{src}} \left\langle \mathcal{O}_i \left(t + t_{src}\right) \mathcal{O}_j^\dagger \left(t_{src}\right) \right\rangle \qquad C(t)v^n(t) = \lambda^n(t) C\left(t_0\right) v^n(t)$$

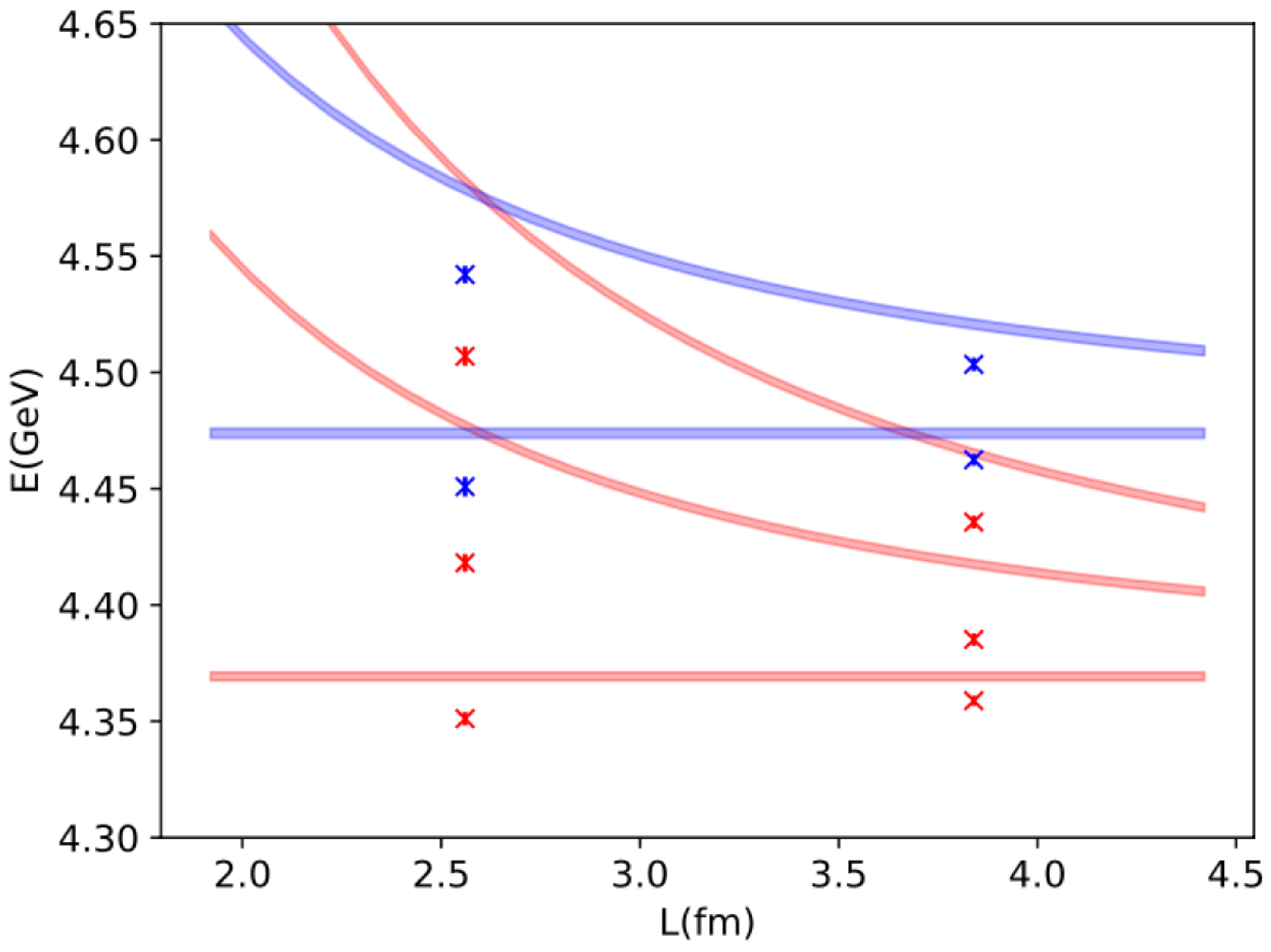
- Extract effective mass by two-exponential fit

$$\lambda^n(t) = \left(1 - A_n\right) e^{-E_n\left(t - t_0\right)} + A_n e^{-E_n'\left(t - t_0\right)}$$

- Compare with non-interacting energy

$$E^{\text{free}} = \sqrt{m_1^2 + \mathbf{p}_1^2} + \sqrt{m_2^2 + \mathbf{p}_2^2}$$

⟶ Energy shift ΔE



H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).

Simulation in Lattice QCD

Determination of LECs on Lattice

Operators → **Energy levels** → **Phase shift** → **Scattering length**

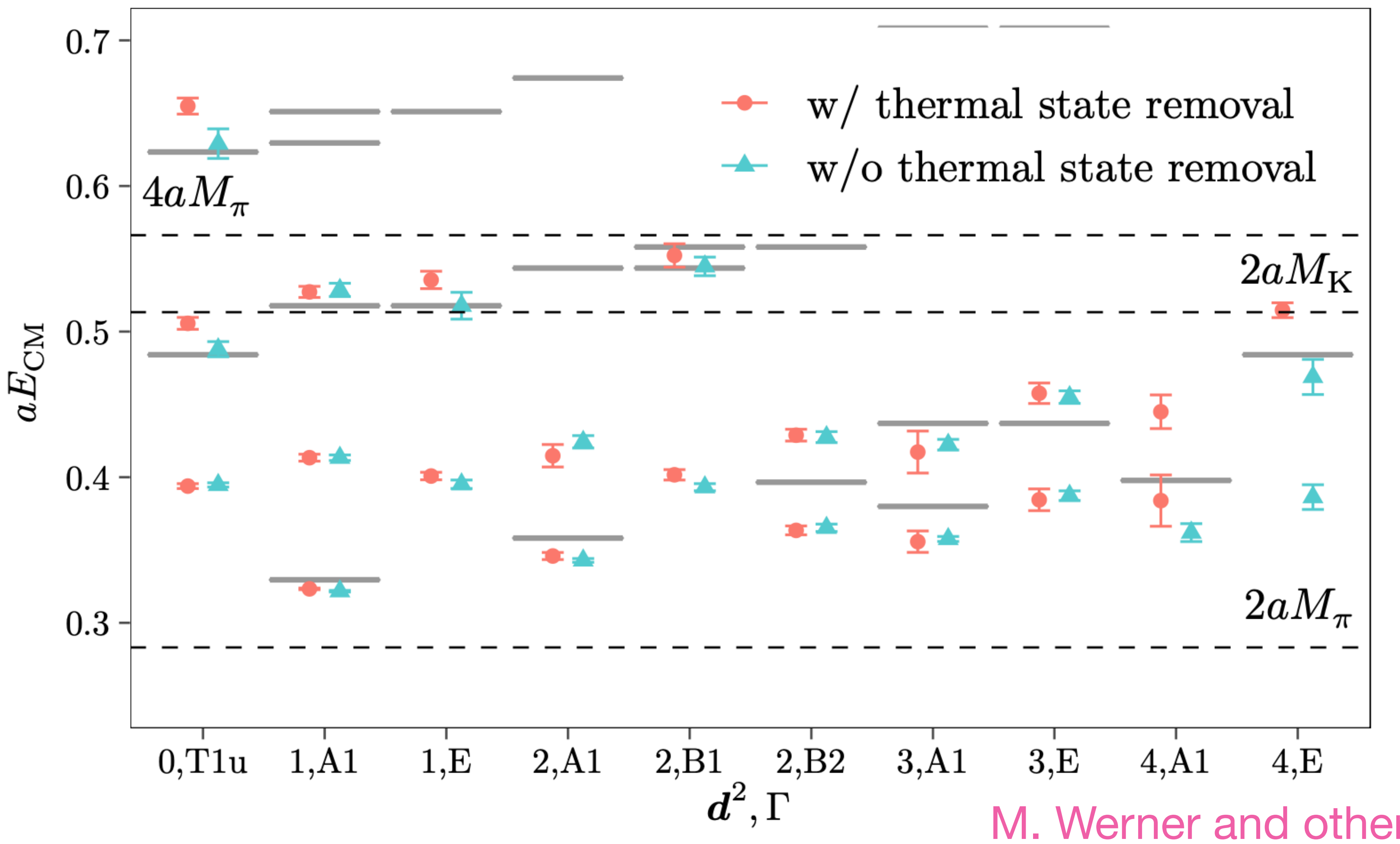
Luscher's formula for two-body scattering M. Luscher, Nucl. Phys. B 354, 531 (1991).

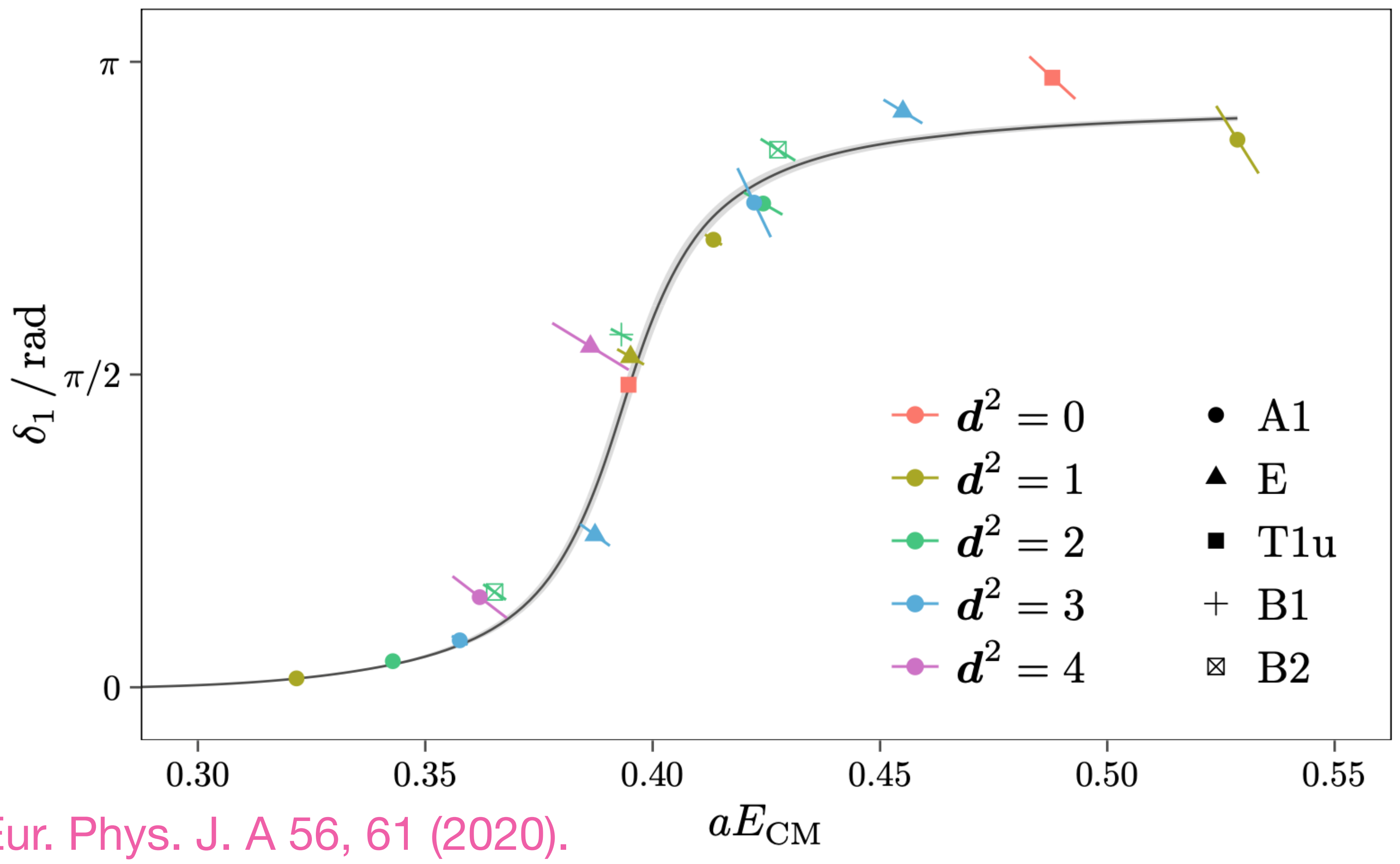
$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (1 + i\mathbf{M})] = 0$$

For single channel s-wave scattering

$$p \cot \delta_0(p) = \frac{2}{L\sqrt{\pi}} \mathscr{Z}_{00}\left(1; q^2\right)$$

$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} \qquad q = \frac{pL}{2\pi}$$

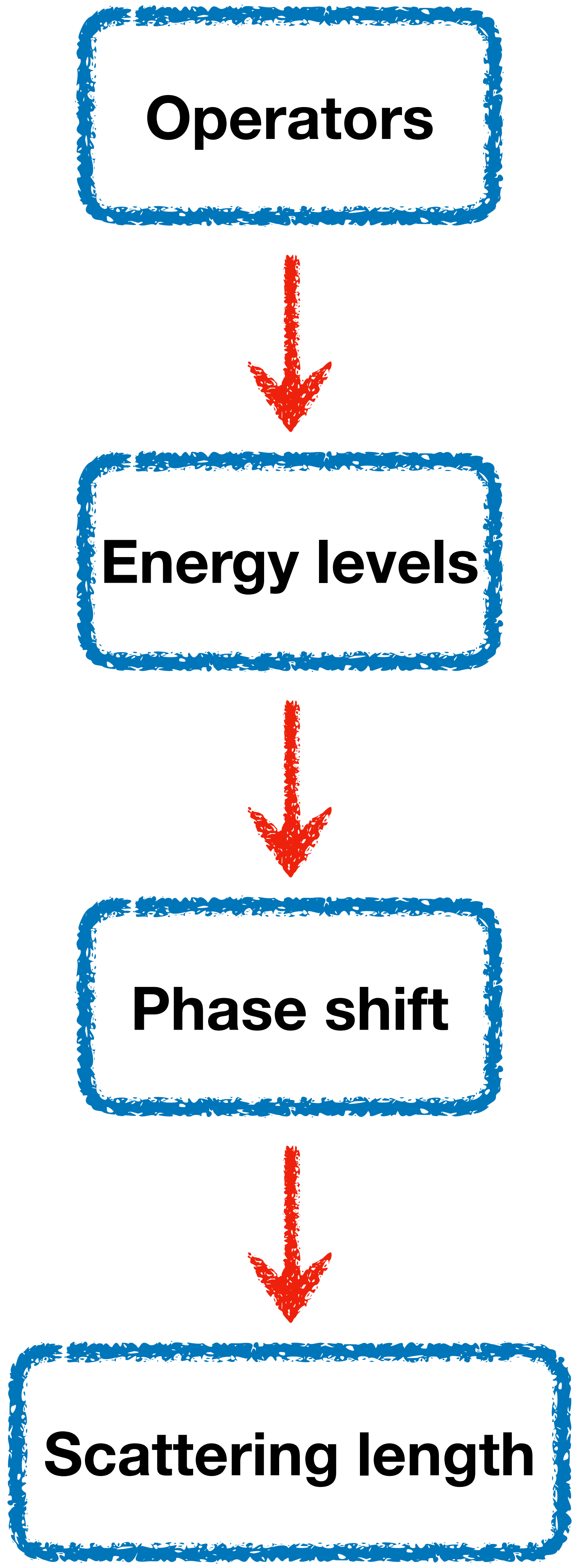




M. Werner and others, Eur. Phys. J. A 56, 61 (2020).

Simulation in Lattice QCD

Determination of LECs on Lattice



Effective range expansion

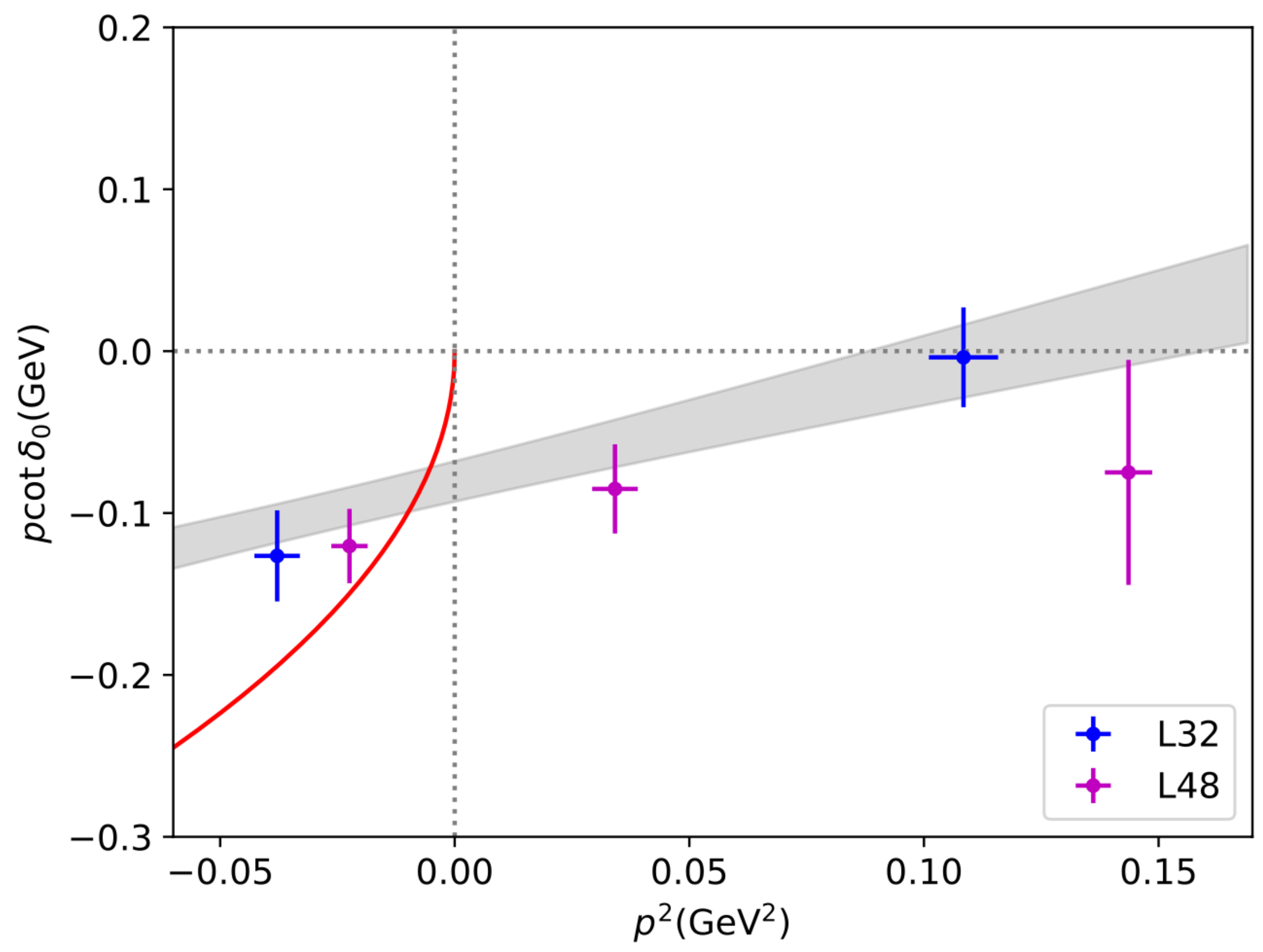
$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$

Scattering amplitude

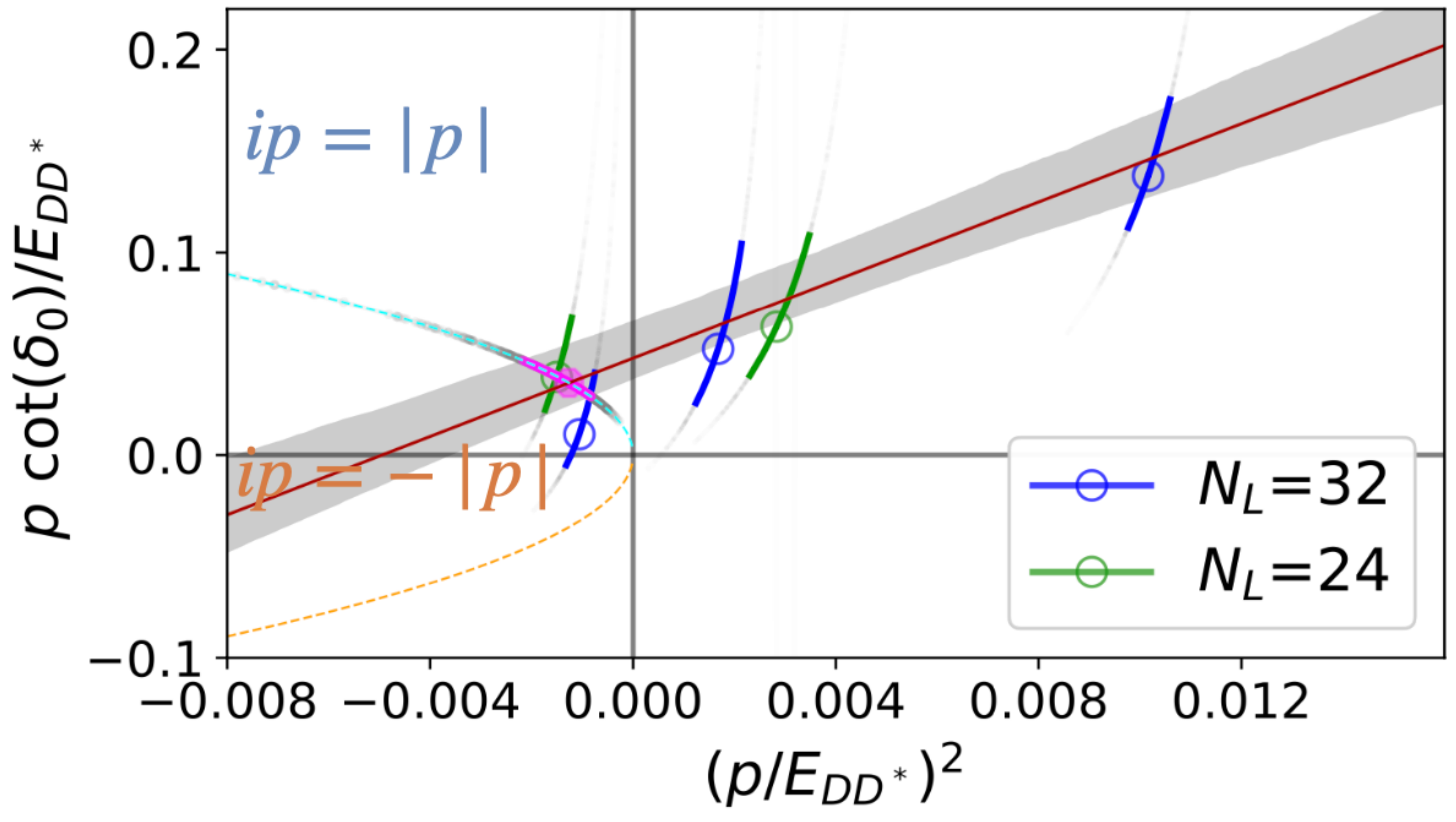
$$t_l^{(J)} \sim \frac{1}{p\cot\delta_l^{(J)} - ip}$$

For partial wave l and total angular momentum J

- L. Liu, K. Orginos, F.-K. Guo, C. Hanhart, and U.-G. Meissner, Phys. Rev. D 87, 014508 (2013).
- Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda, and J. Meng, arXiv:2302.04505 [hep-lat] (2023).
- H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).
- M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129, 032002 (2022).
- ……



H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).



M. Padmanath and S. Prelovsek, Phys. Rev. Lett. 129, 032002 (2022).

Lattice setup

Gauge configurations from CLQCD

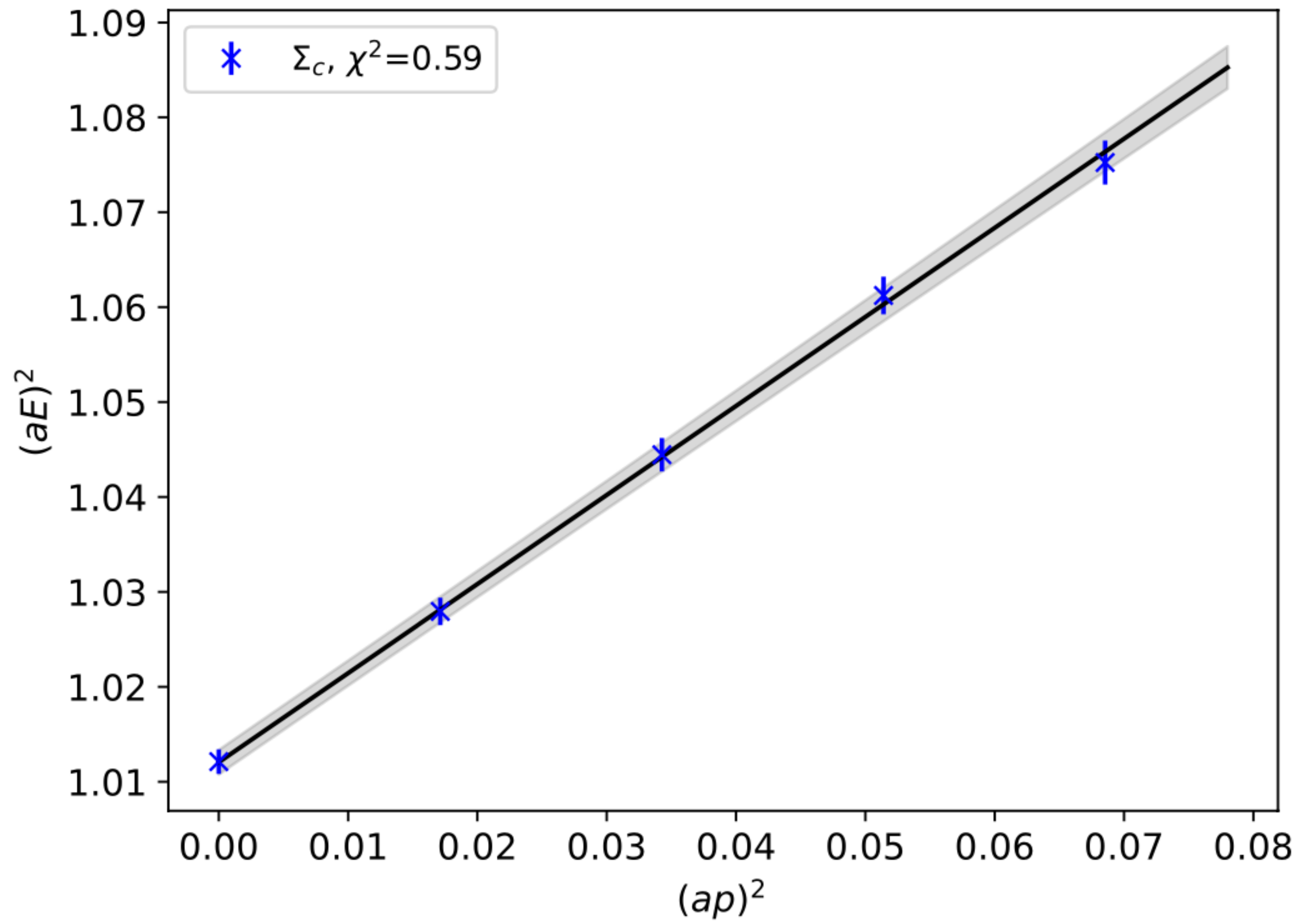
Ensembles	β	$L^3 \times T$	lattice space(fm)	am_l	am_s	$N_{\text{conf.}}$
1	6.2	$24^3 \times 72$	0.108	−0.2770	−0.2400	
2	6.2	$32^3 \times 64$	0.108	−0.2770	−0.2400	
3	6.2	$32^3 \times 64$	0.108	−0.2790	−0.2400	
4	6.2	$48^3 \times 96$	0.108	−0.2790	−0.2400	
5	6.41	$32^3 \times 96$	0.080	−0.2295	−0.2050	371
6	6.41	$48^3 \times 96$	0.080	−0.2295	−0.2050	201
7	6.41	$32^3 \times 64$	0.080	−0.2320	−0.2050	
8	6.41	$48^3 \times 96$	0.080	−0.2320	−0.2050	
9	6.72	$48^3 \times 144$	0.050	−0.1850	−0.1700	

The results presented in this talk are based on

- Two ensembles of gauge configurations with 2+1 dynamical quark flavors.
- Pion mass ~ 295 MeV.
- tree-level Symanzik-improved gauge action.
- Shekholeslami-Wohlert fermion action with tree-level tadpole improvement
- Distillation quark smearing

Charm quark mass

- Determined by physical spin-averaged mass of η_c and J/Ψ
- DCB mass deviate from continuum dispersion relation due to lattice artifacts



H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).

Lattice setup

Smearing

- Signal-to-noise ratio
- Quark smearing method: Jacobi smearing, distillation smearing, ……

Distillation quark smearing method

$$-\nabla^2_{xy}(t) = 6\delta_{xy} - \sum_{j=1}^{3}\left(\tilde{U}_j(x,t)\delta_{x+\hat{j},y} + \tilde{U}_j^{\dagger}(x-\hat{j},t)\delta_{x-\hat{j},y}\right) \qquad \square(t) = V(t)V^{\dagger}(t) \longrightarrow \square_{xy}(t) = \sum_{k=1}^{N_{ev}} \boldsymbol{v}_x^{(k)}(t)\boldsymbol{v}_y^{(k)\dagger}(t)$$

$$N_t \times N_x^3 \times N_{color} \times N_{spin} \longrightarrow N_t \times N_{spin} \times N_{ev}$$

- M. Peardon, J. Bulava, J. Foley, C. Morningstar, J. Dudek, R. G. Edwards, B. Joo, H.-W. Lin, D. G. Richards, and K. J. Juge, Phys. Rev. D 80, 054506 (2009).
- R. G. Edwards, J. J. Dudek, D. G. Richards, and S. J. Wallace, Phys. Rev. D 84, 074508 (2011).
- C. Egerer, D. Richards, and F. Winter, Phys. Rev. D 99, 034506 (2019).
- H. Xing, J. Liang, L. Liu, P. Sun, and Y.-B. Yang, arXiv:2210.08555 [hep-lat] (2022).
- ……

- Perambulators and eigenvectors are generated from CLQCD ensembles.
- The number of eigenvectors is set to $N_{ev} = 100$.
- Contraction done by opt_einsum package in Python.

Results

BChPT scattering length results Z.-R. Liang, P.-C. Qiu, and D.-L. Yao, JHEP 07, 124 (2023), L. Meng and S.-L. Zhu, Phys. Rev. D 100, 014006 (2019).

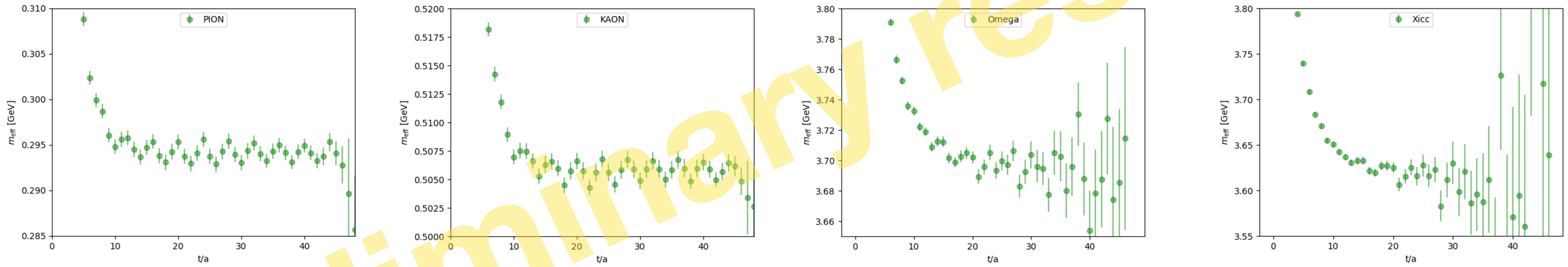
(S, I)	Processes	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$ Tree	$\mathcal{O}(p^3)$ Loop	EOMS	HB
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	-0.27	0.29	-0.11	-0.001	$-0.09^{+0.12}_{-0.13}$	$-0.20(1)$
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	-0.27	0.27	-0.13	-0.47	-0.60 ± 0.13	$-0.25(1)$
$(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	0.27	0.34	0.13	0.30	1.03 ± 0.19	$0.92(2)$
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	-0.12	0.04	-0.01	-0.06	-0.16 ± 0.02	$-0.10(2)$
$(-1, 0)$	$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$	0.54	0.24	0.25	0.16	$1.19^{+0.22}_{-0.21}$	$2.15(11)$
	$\Omega_{cc}\eta \to \Omega_{cc}\eta$	-0.001	0.37	0.0	$0.05 + 0.55i$	$0.42^{+0.18}_{-0.19} + 0.55i$	$0.57(3) + 0.21i$
$(-1, 1)$	$\Omega_{cc}\pi \to \Omega_{cc}\pi$	0.0	0.04	0.0	-0.04	-0.01 ± 0.02	$-0.002(1)$
	$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$	0.0	0.31	0.0	$-0.04 + 0.10i$	$0.27^{+0.13}_{-0.13} + 0.10i$	$0.26(1) + 0.19i$
$(0, \frac{1}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	0.25	0.04	0.01	0.04	0.34 ± 0.02	$0.36(1)$
	$\Xi_{cc}\eta \to \Xi_{cc}\eta$	-0.001	0.32	0.0	-0.26	$0.06^{+0.14}_{-0.15}$	$0.34(1) + 0.10i$
	$\Omega_{cc}K \to \Omega_{cc}K$	0.27	0.29	0.11	$-0.01 + 0.55i$	$0.66^{+0.13}_{-0.13} + 0.55i$	$1.18(6) + 0.29i$

Results

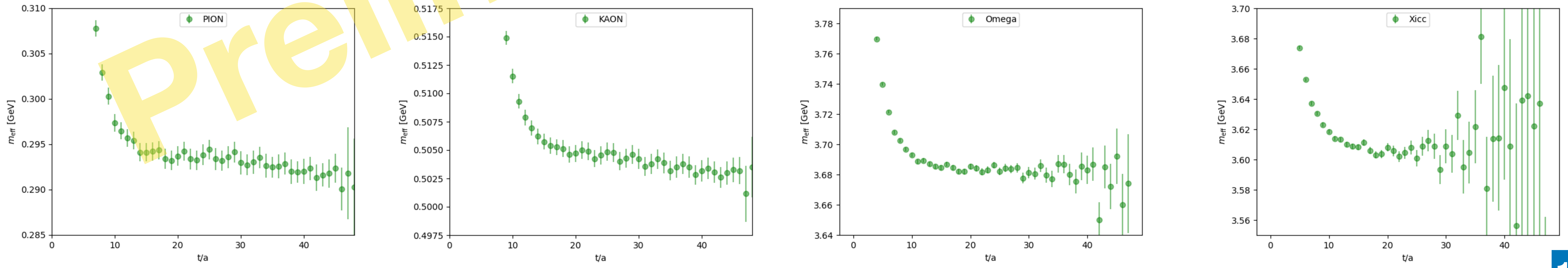
Single particle effective mass

	π	K	Ξ_{cc}	Ω_{cc}
L32	0.2942(9)	0.5052(6)	3.6053(15)	3.6847(9)
L48	0.2942(5)	0.5058(3)	3.6235(10)	3.6996(9)

L48



L32

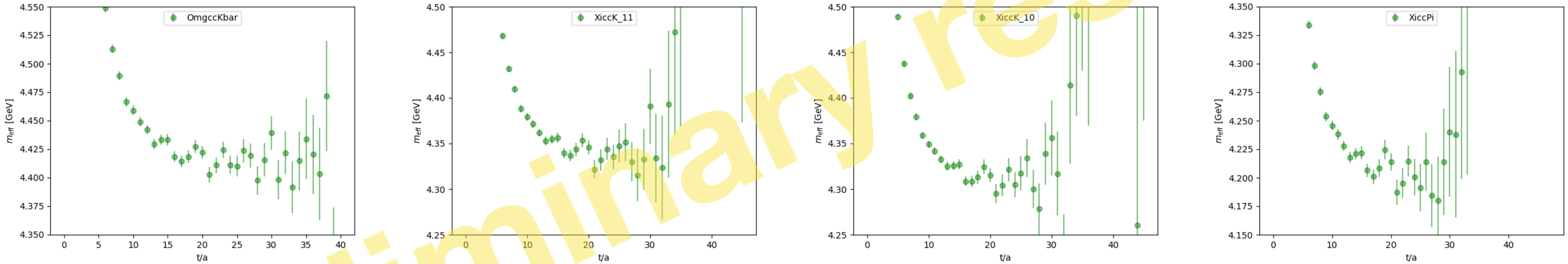


Results

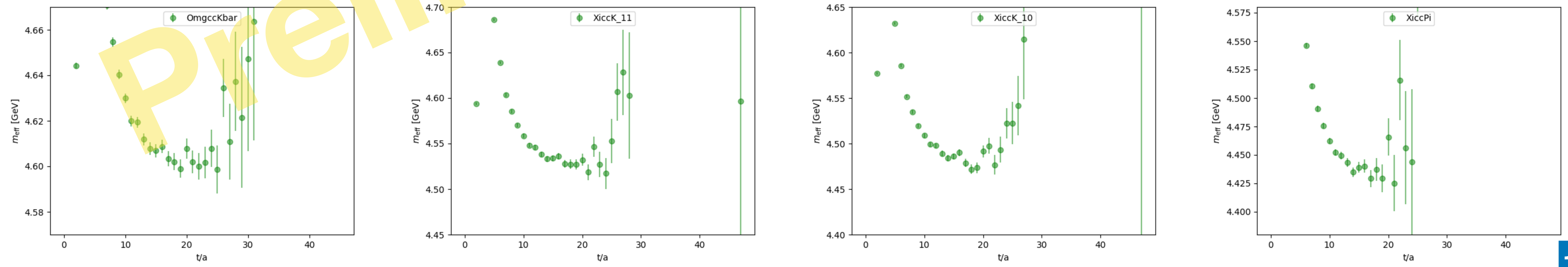
Two particle effective mass

	$\Omega_{cc}\bar{K}(-2,1/2)$	$\Xi_{cc}K(1,1)$	$\Xi_{cc}K(1,0)$	$\Xi_{cc}\pi(0,3/2)$
L32	4.1955(12)	4.1171(17)	4.0971(19)	3.9073(19)
L48	4.2091(12)	4.1340(12)	4.1276(12)	3.9215(14)

L48

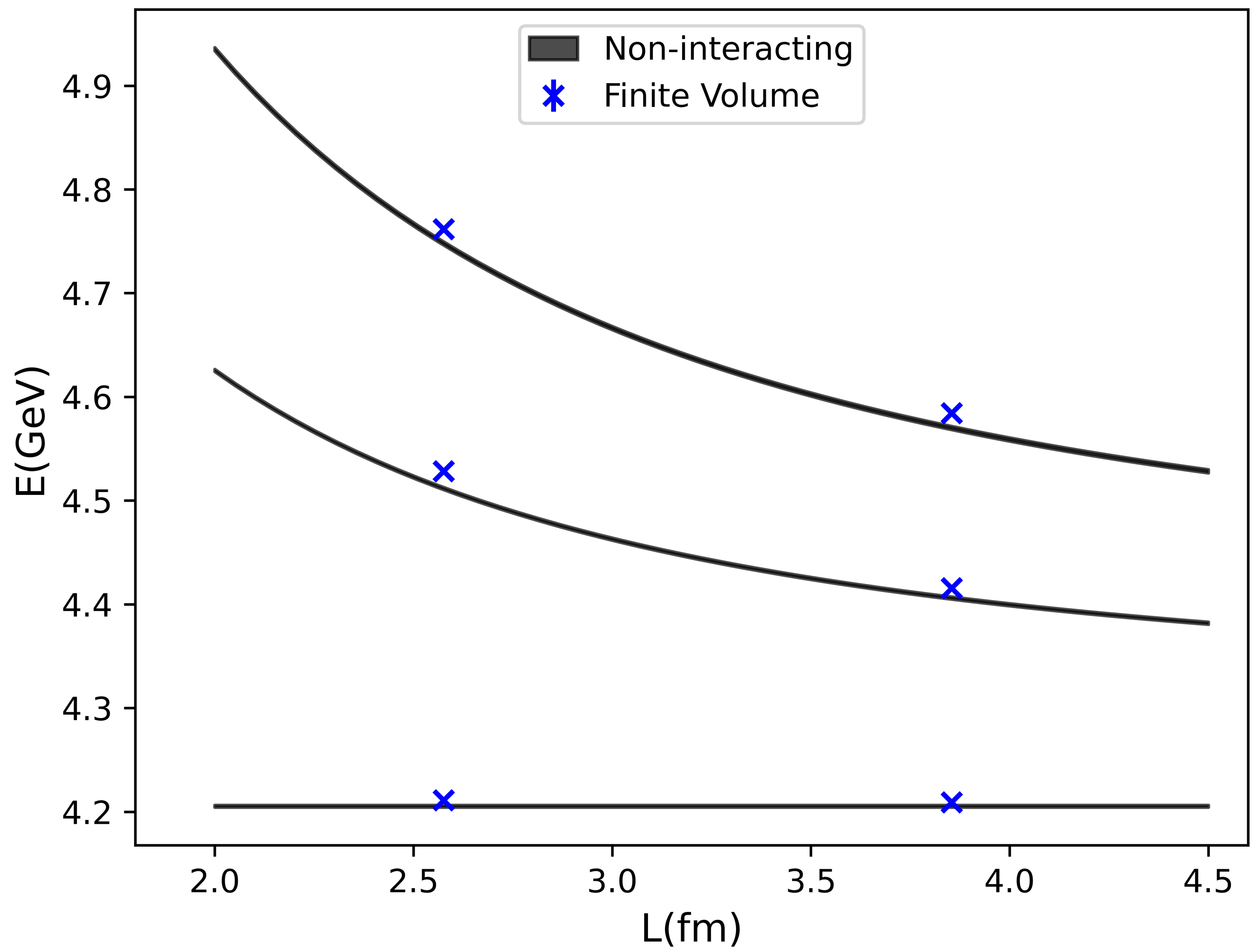


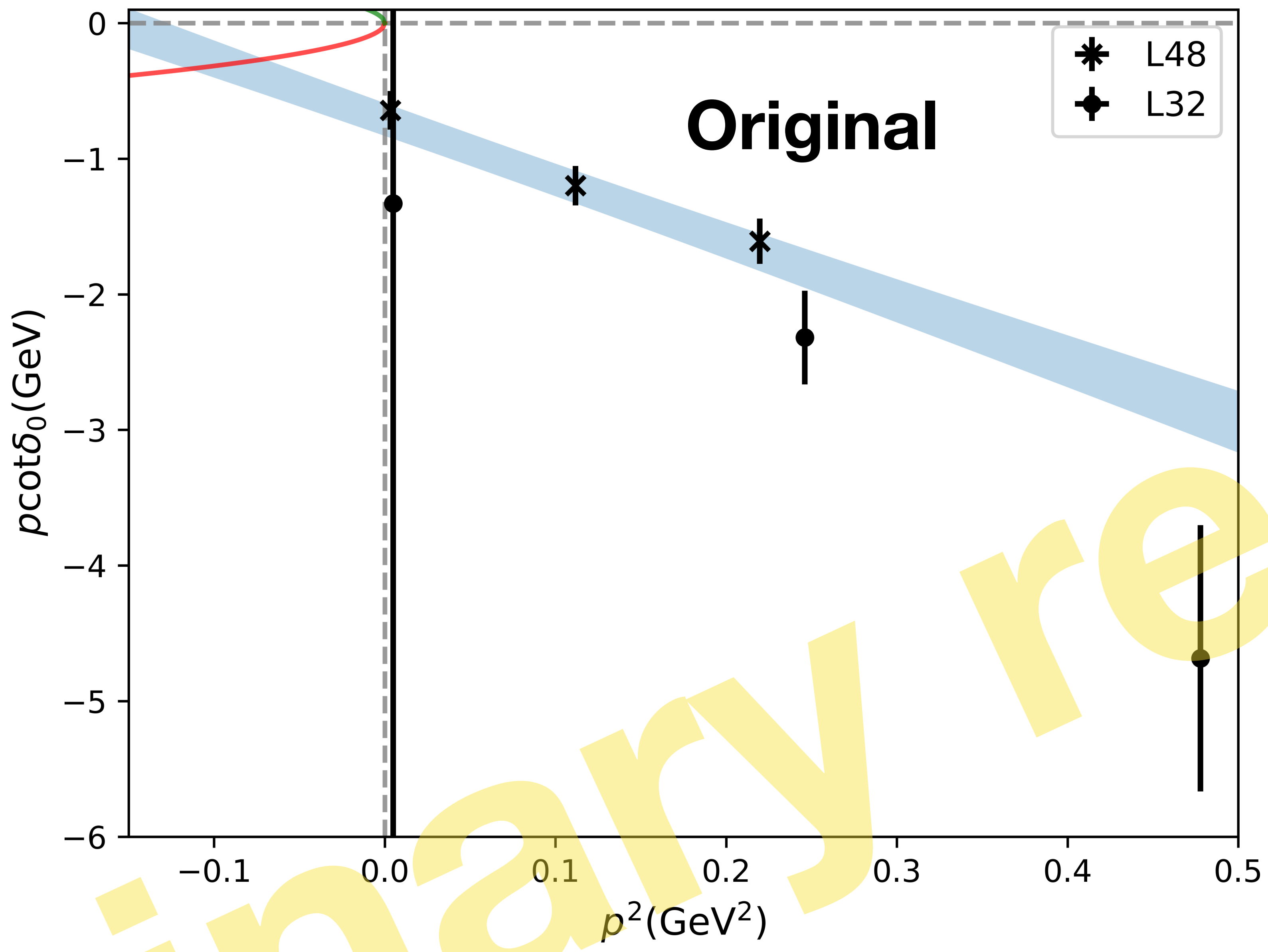
L32

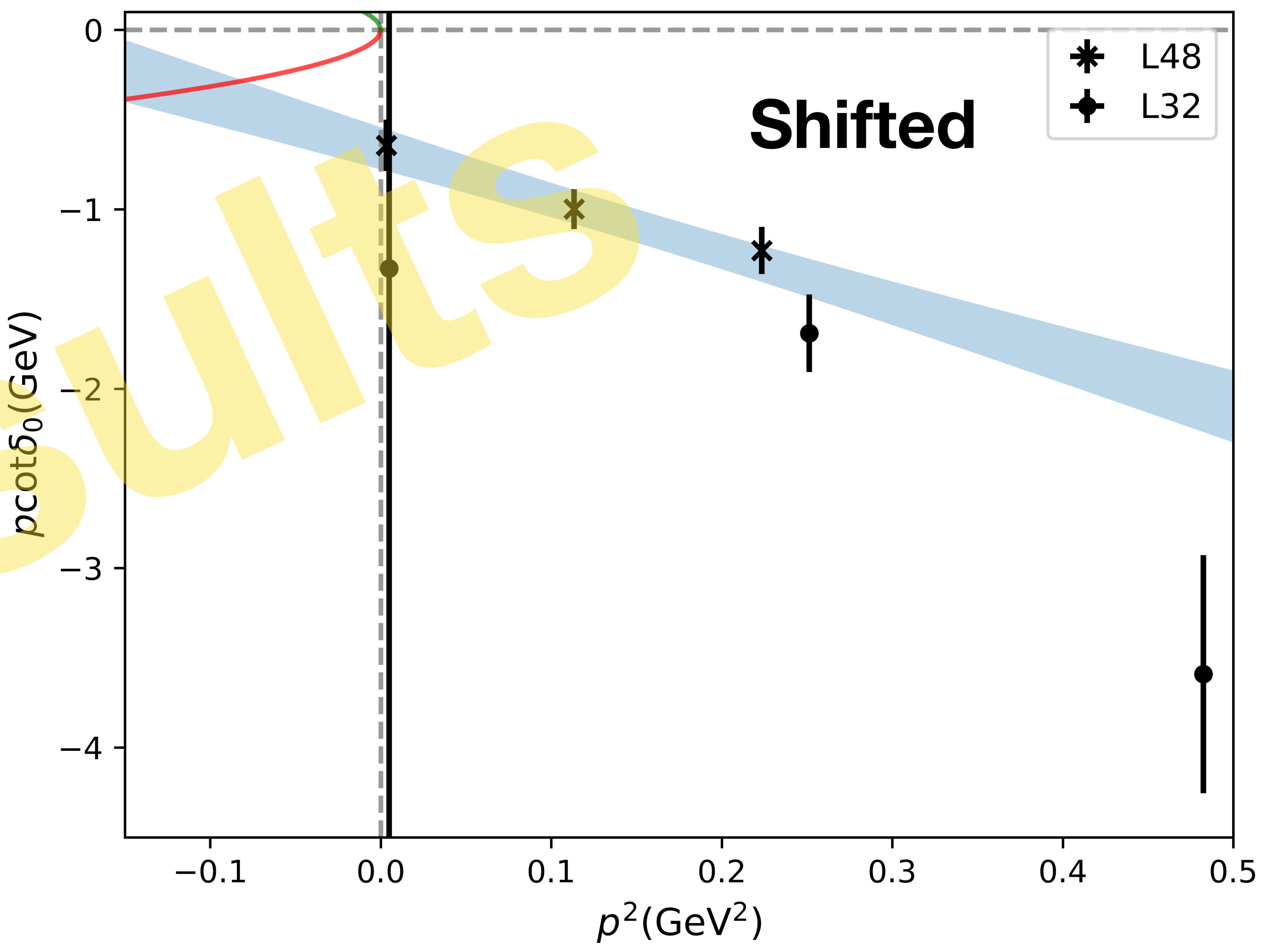


Results

$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}\ (-2,\ 1/2)$







$$E'_p = E_p + \left(M^{cont.}_{\phi(p)} + M^{cont.}_{B_{cc}(p)}\right) - \left(M^{lat.}_{\phi(p)} + M^{lat.}_{B_{cc}(p)}\right)$$

Original

Chisq/dof: 1.56
Scattering length: -0.278(47)
Effective range: -1.76(17)

Shifted

Chisq/dof: 1.18
Scattering length: -0.299(53)
Effective range: -1.14(20)

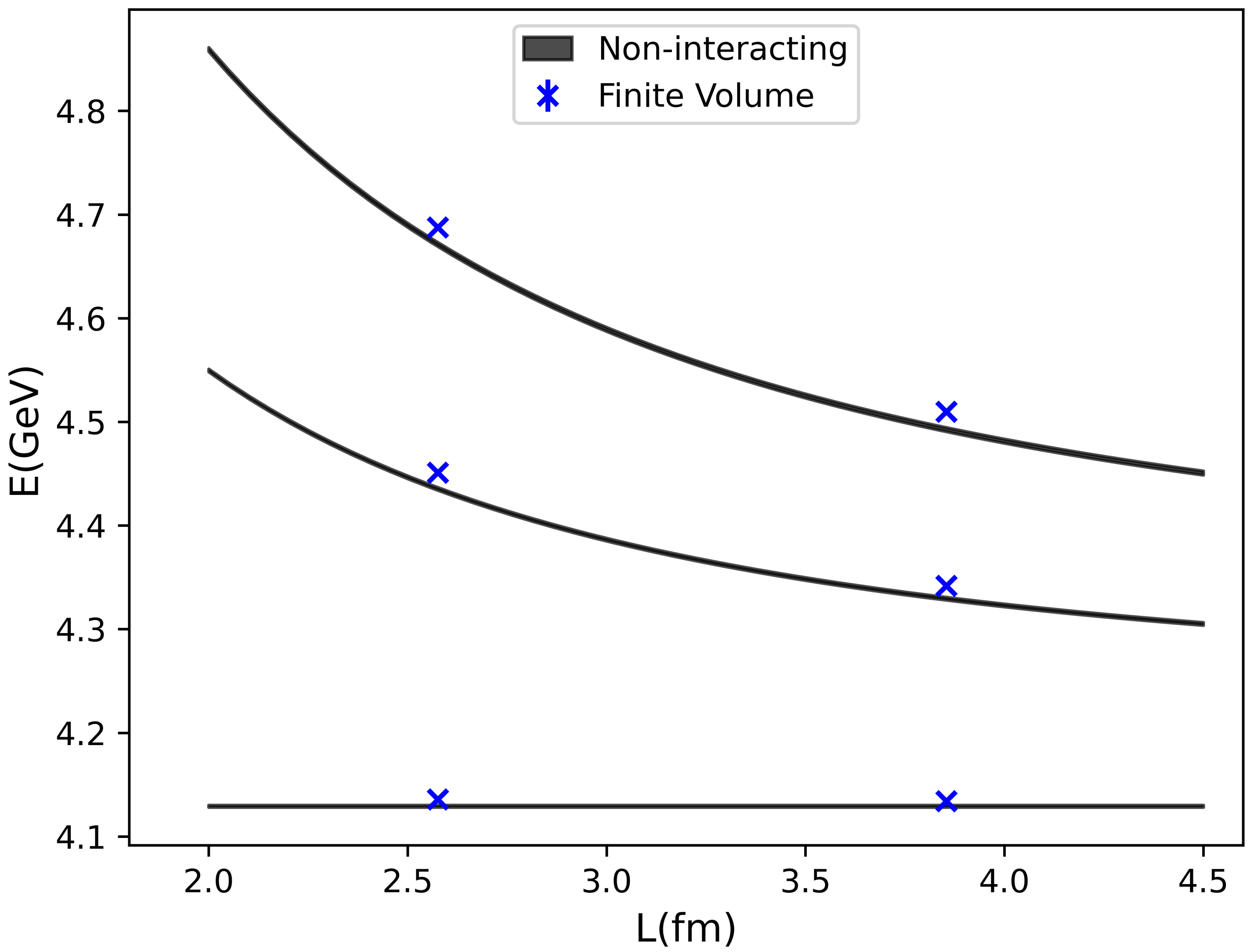
(S,I)	Processes	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$ Tree	$\mathcal{O}(p^3)$ Loop	EOMS	HB
$(-2,\frac{1}{2})$	$\Omega_{cc}\bar{K}\to\Omega_{cc}\bar{K}$	-0.27	0.29	-0.11	-0.001	$-0.09^{+0.12}_{-0.13}$	$-0.20(1)$
$(1,1)$	$\Xi_{cc}K\to\Xi_{cc}K$	-0.27	0.27	-0.13	-0.47	-0.60 ± 0.13	$-0.25(1)$
$(1,0)$	$\Xi_{cc}K\to\Xi_{cc}K$	0.27	0.34	0.13	0.30	1.03 ± 0.19	$0.92(2)$
$(0,\frac{3}{2})$	$\Xi_{cc}\pi\to\Xi_{cc}\pi$	-0.12	0.04	-0.01	-0.06	-0.16 ± 0.02	$-0.10(2)$

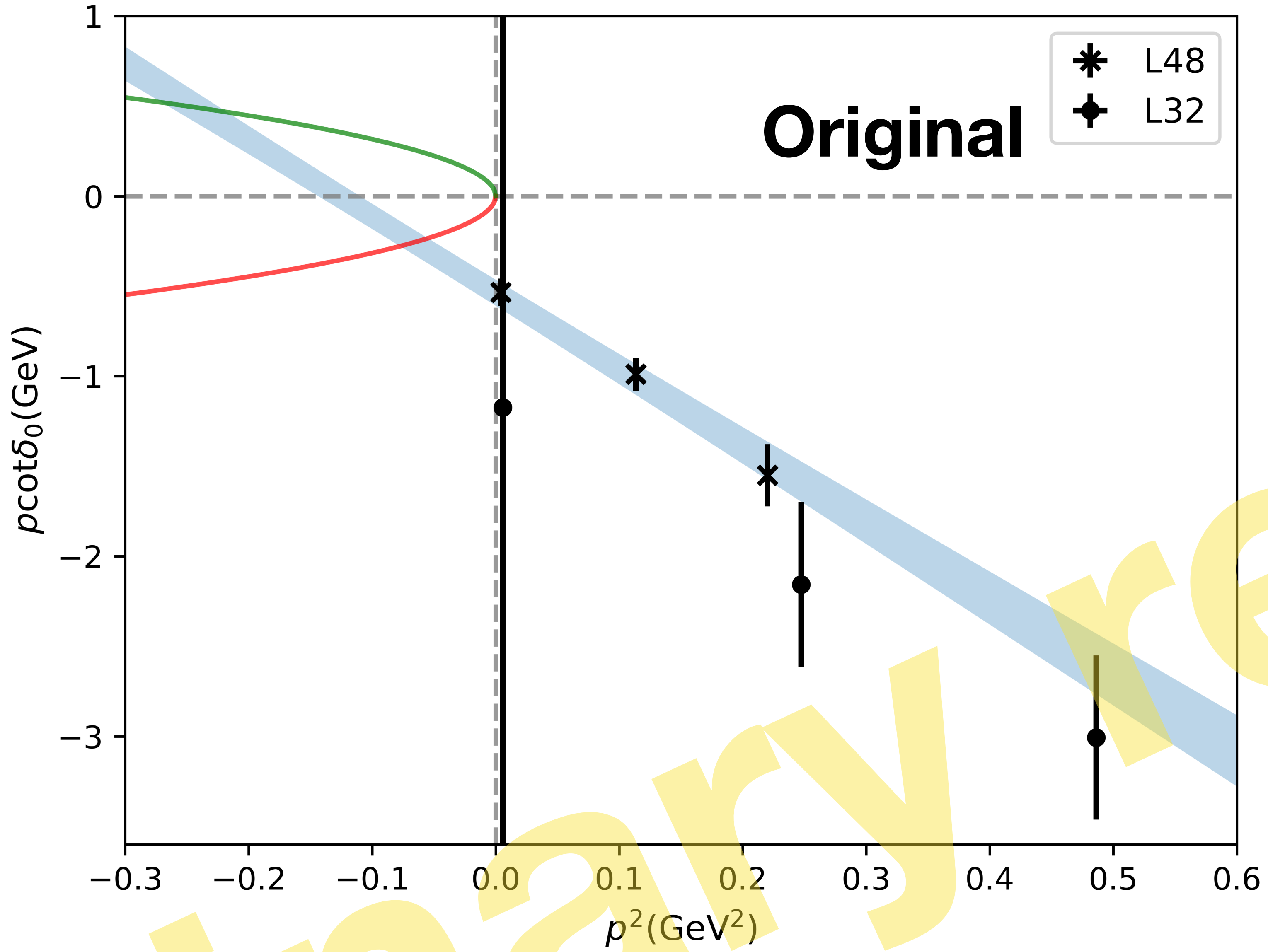
$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$

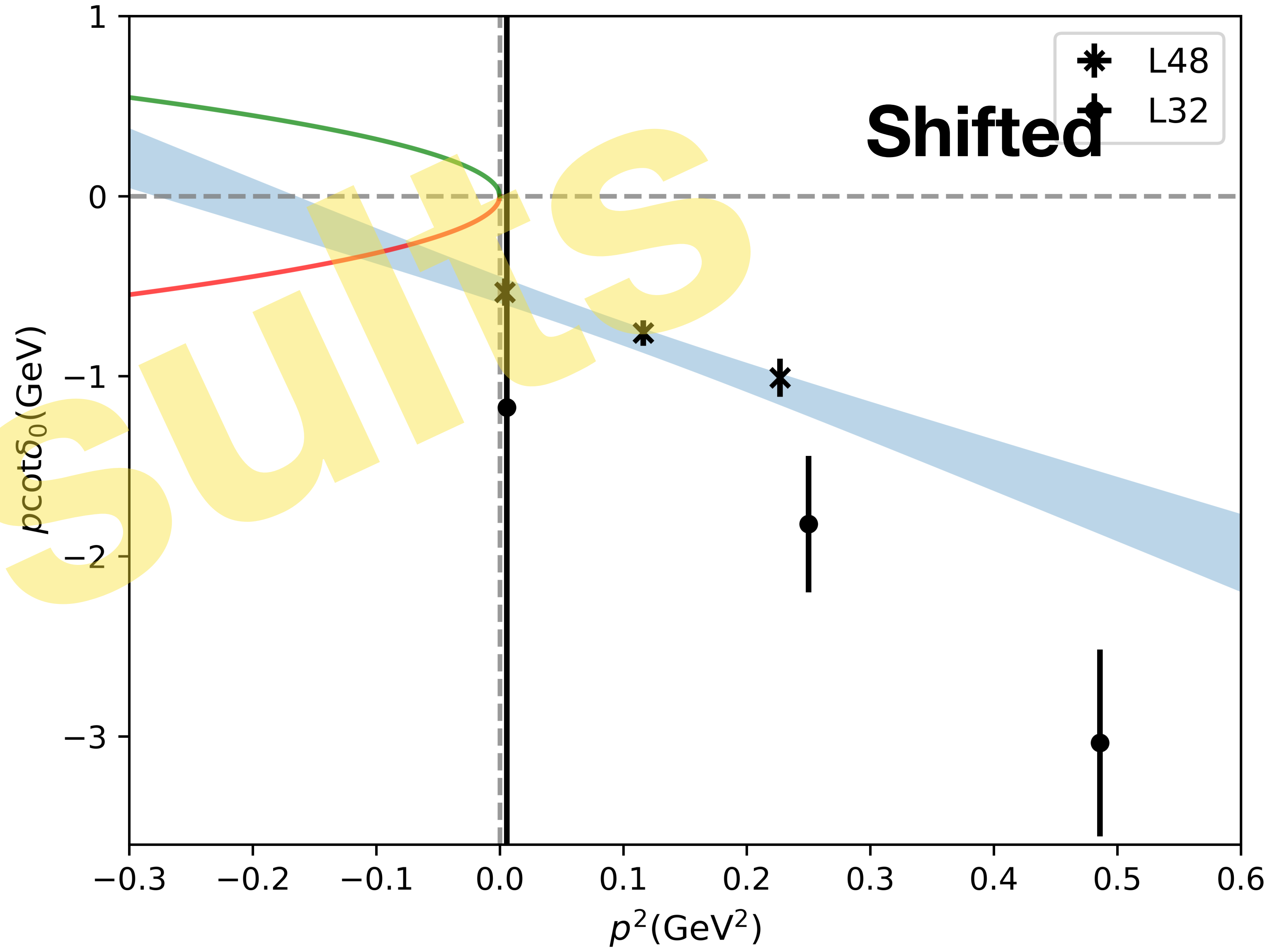
$$t_l^{(J)} \sim \frac{1}{p\cot\delta_l^{(J)} - ip}$$

Results

$\Xi_{cc}K \to \Xi_{cc}K\ (1,\ 1)$








```
Chisq/dof: 0.42
Scattering length: -0.368(50)
Effective range: -1.67(11)
```

```
Chisq/dof: 1.27
Scattering length: -0.379(54)
Effective range: -0.96(16)
```

$$E'_p = E_p + \left(M^{cont.}_{\phi(p)} + M^{cont.}_{B_{cc}(p)}\right) - \left(M^{lat.}_{\phi(p)} + M^{lat.}_{B_{cc}(p)}\right)$$

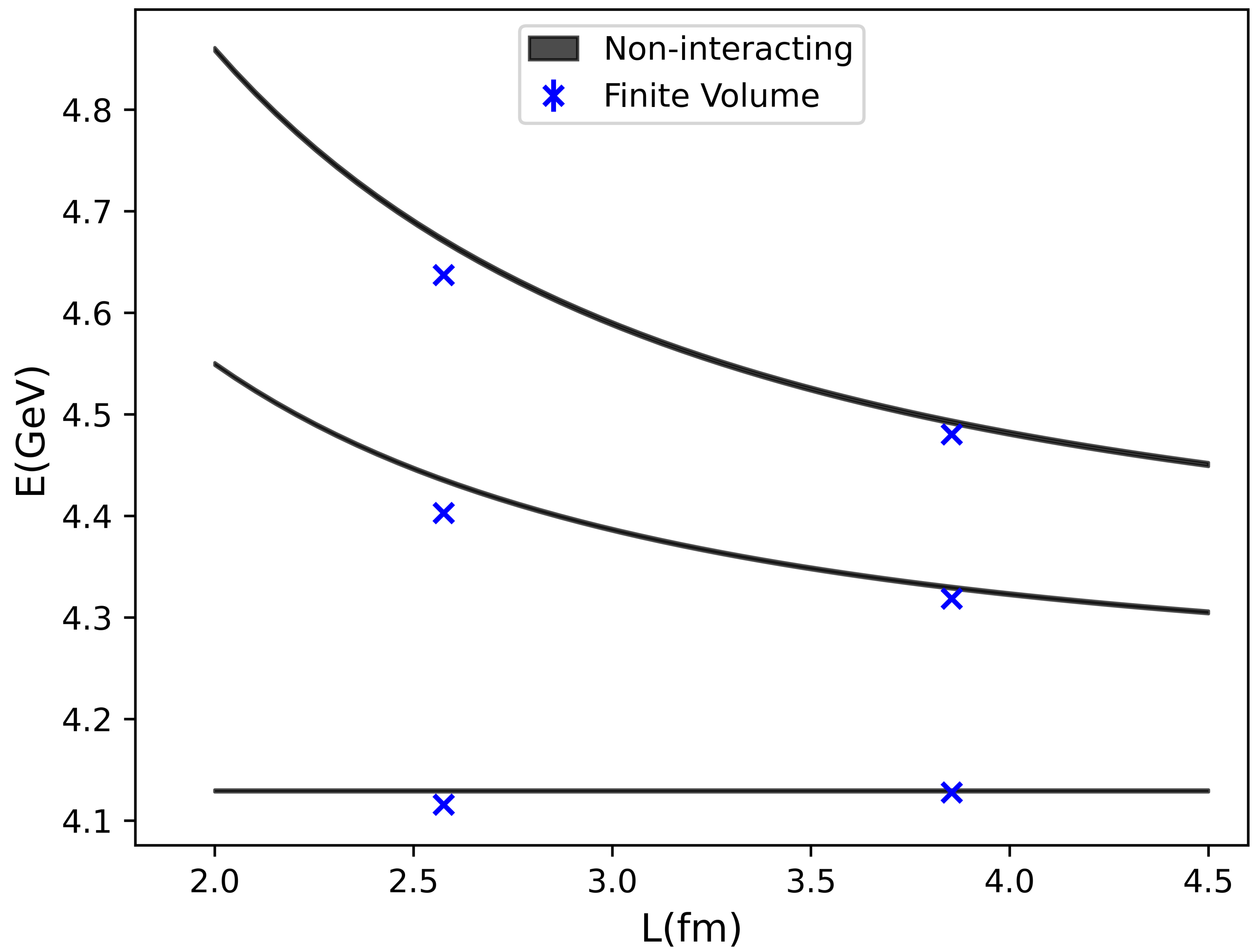
(S, I)	Processes	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$ Tree	$\mathcal{O}(p^3)$ Loop	EOMS	HB
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	-0.27	0.29	-0.11	-0.001	$-0.09^{+0.12}_{-0.13}$	$-0.20(1)$
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	-0.27	0.27	-0.13	-0.47	-0.60 ± 0.13	$-0.25(1)$
$(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	0.27	0.34	0.13	0.30	1.03 ± 0.19	$0.92(2)$
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	-0.12	0.04	-0.01	-0.06	-0.16 ± 0.02	$-0.10(2)$

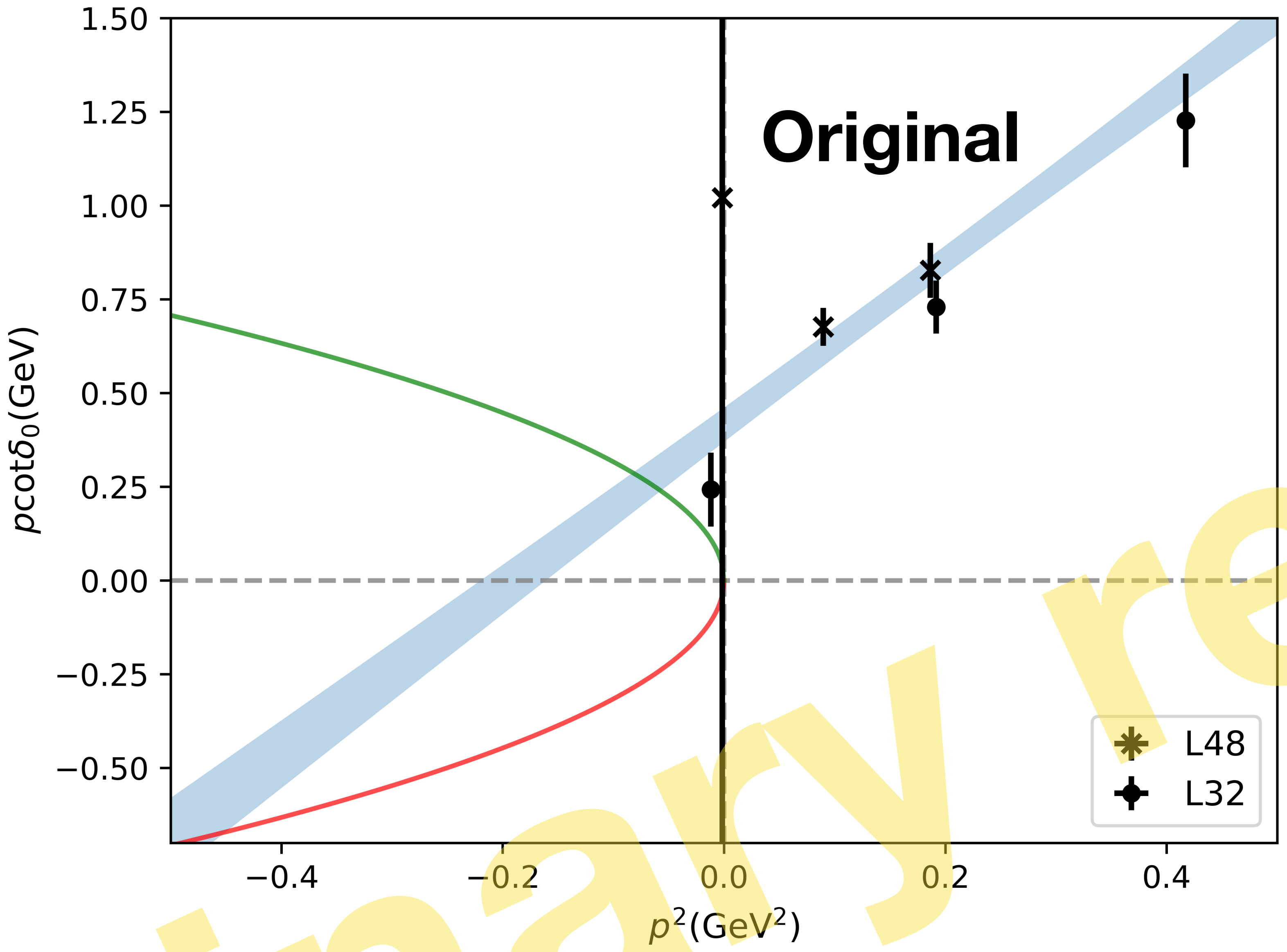
$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$

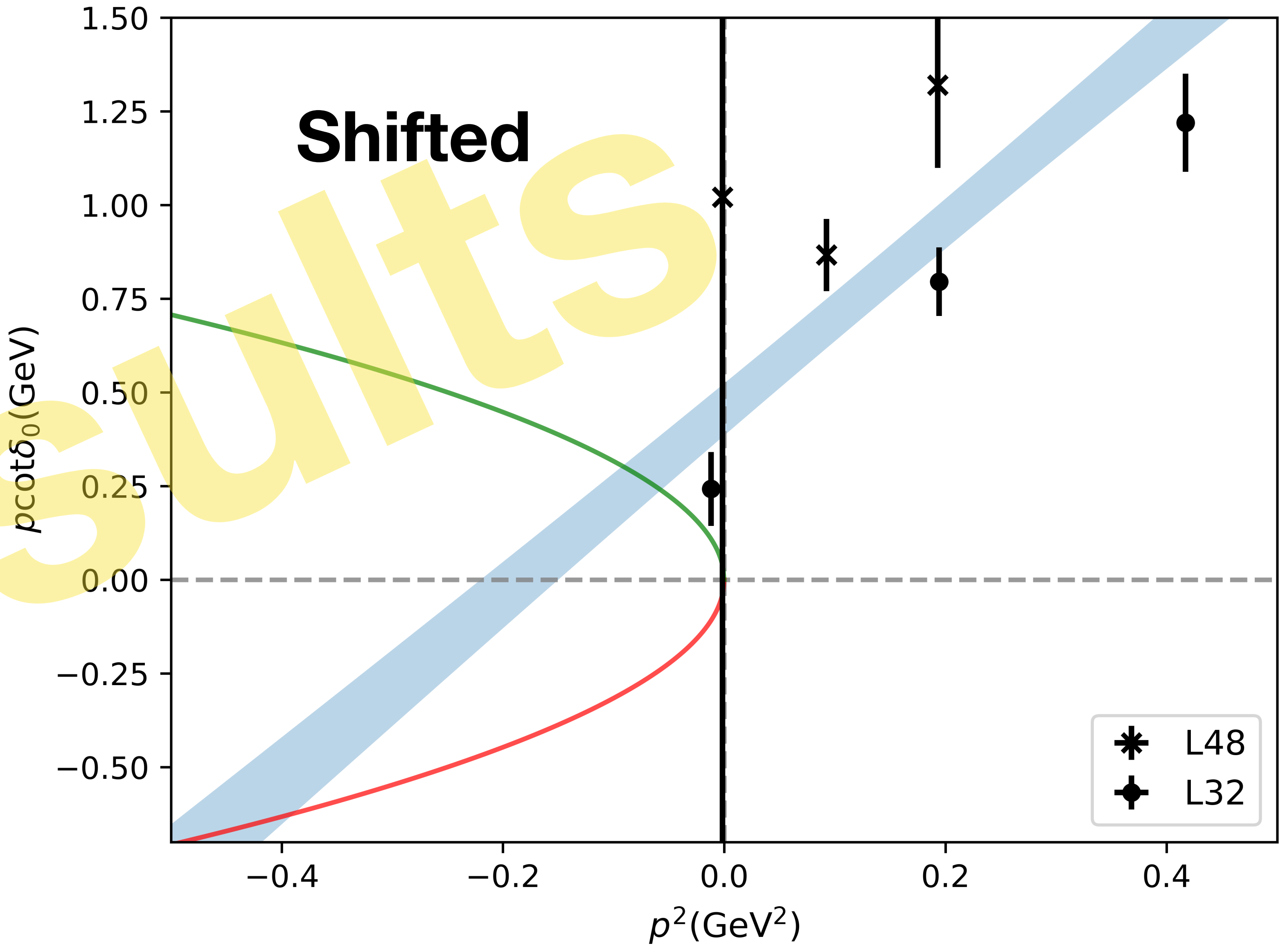
$$t_l^{(J)} \sim \frac{1}{p\cot\delta_l^{(J)} - ip}$$

Results

$\Xi_{cc}K \to \Xi_{cc}K\ (1,\ 0)$







Chisq/dof: 0.87
Scattering length: 0.475(53)
Effective range: 0.867(56)

Chisq/dof: 1.57
Scattering length: 0.434(66)
Effective range: 0.978(72)

$$E'_p = E_p + \left(M^{cont.}_{\phi(p)} + M^{cont.}_{B_{cc}(p)}\right) - \left(M^{lat.}_{\phi(p)} + M^{lat.}_{B_{cc}(p)}\right)$$

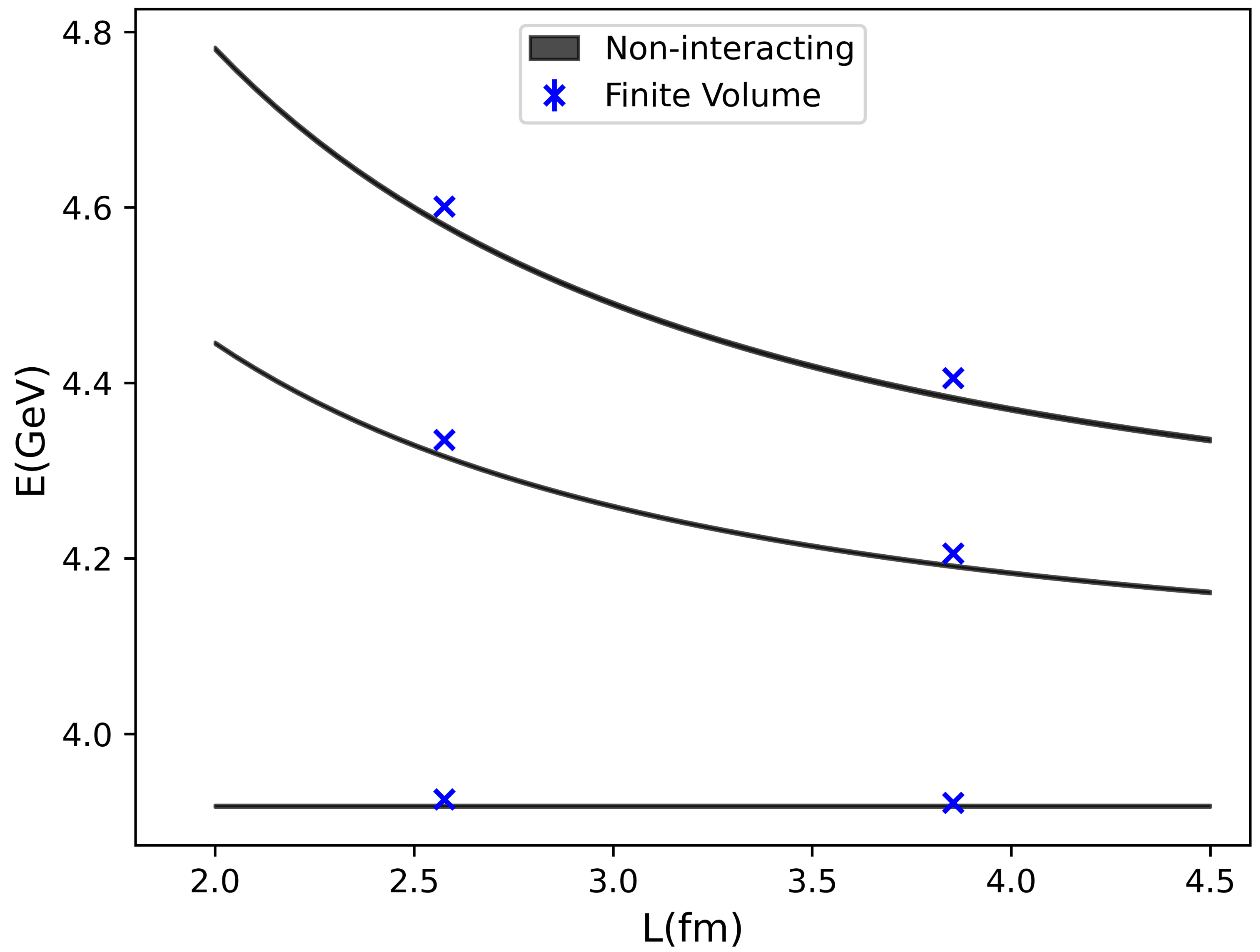
(S,I)	Processes	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$ Tree	$\mathcal{O}(p^3)$ Loop	EOMS	HB
$(-2,\frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	-0.27	0.29	-0.11	-0.001	$-0.09^{+0.12}_{-0.13}$	$-0.20(1)$
$(1,1)$	$\Xi_{cc}K \to \Xi_{cc}K$	-0.27	0.27	-0.13	-0.47	-0.60 ± 0.13	$-0.25(1)$
$(1,0)$	$\Xi_{cc}K \to \Xi_{cc}K$	0.27	0.34	0.13	0.30	1.03 ± 0.19	$0.92(2)$
$(0,\frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	-0.12	0.04	-0.01	-0.06	-0.16 ± 0.02	$-0.10(2)$

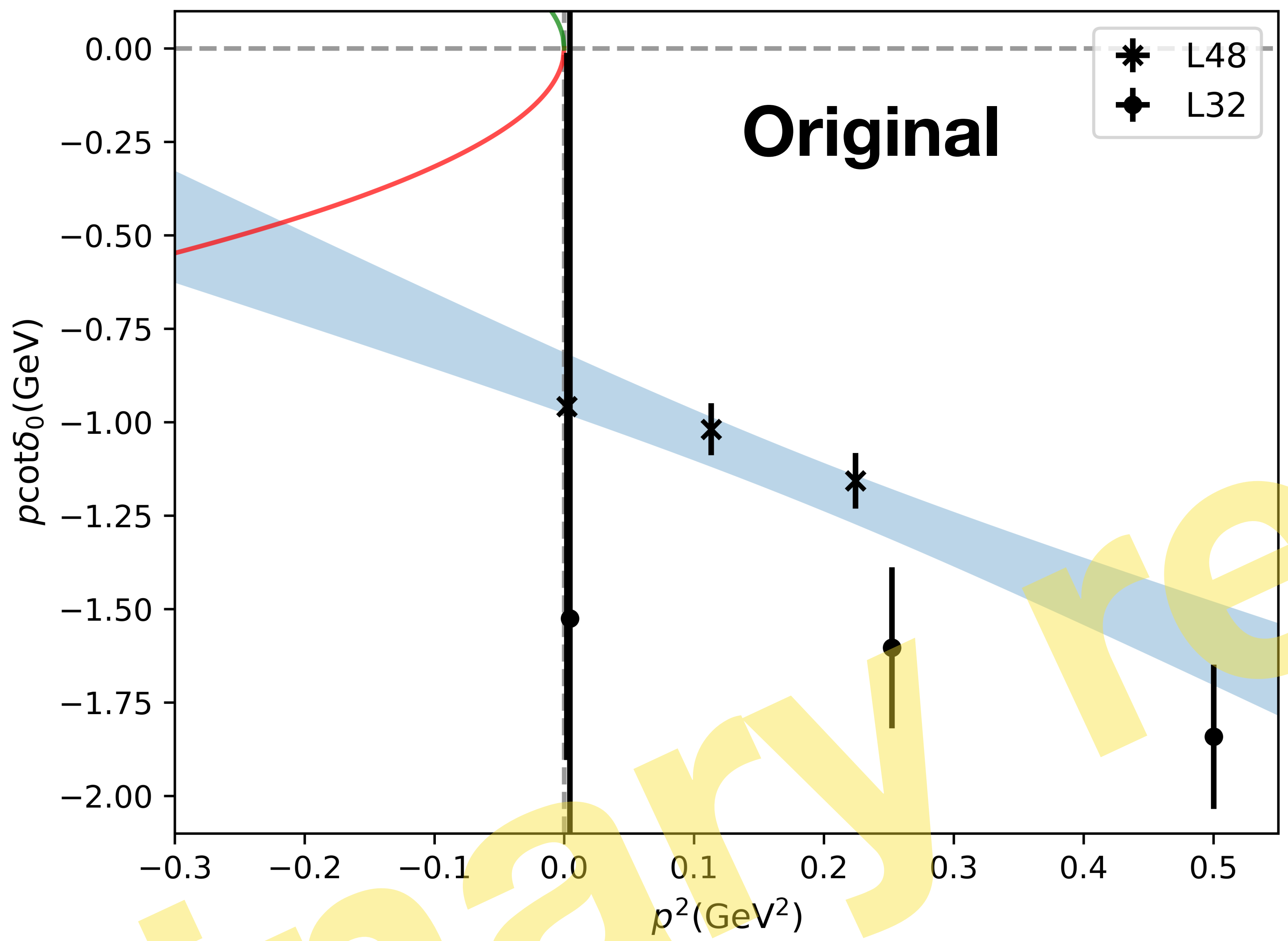
$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$

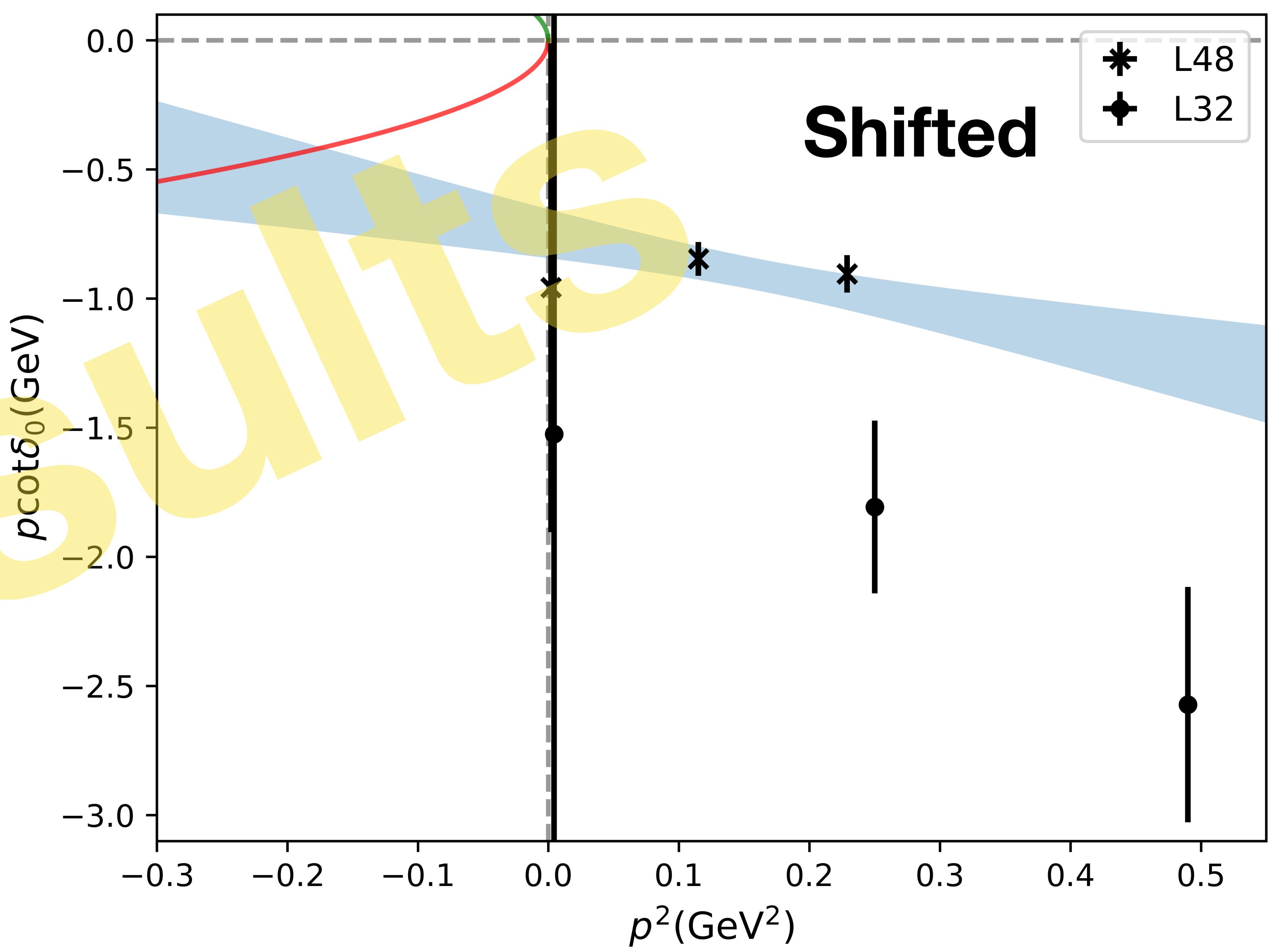
$$t_l^{(J)} \sim \frac{1}{p\cot\delta_l^{(J)} - ip}$$

Results

$\Xi_{cc}\pi \to \Xi_{cc}\pi$ (0, 3/2)







$$E'_p = E_p + \left(M^{cont.}_{\phi(p)} + M^{cont.}_{B_{cc}(p)}\right) - \left(M^{lat.}_{\phi(p)} + M^{lat.}_{B_{cc}(p)}\right)$$

Original:

```
Chisq/dof: 0.55
Scattering length:  -0.220(20)
Effective range:  -0.55(11)
```

Shifted:

```
Chisq/dof: 1.85
Scattering length:  -0.263(33)
Effective range:  -0.39(18)
```

(S, I)	Processes	$\mathcal{O}(p^1)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$ Tree	$\mathcal{O}(p^3)$ Loop	EOMS	HB
$(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	-0.27	0.29	-0.11	-0.001	$-0.09^{+0.12}_{-0.13}$	$-0.20(1)$
$(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	-0.27	0.27	-0.13	-0.47	-0.60 ± 0.13	$-0.25(1)$
$(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	0.27	0.34	0.13	0.30	1.03 ± 0.19	$0.92(2)$
$(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	-0.12	0.04	-0.01	-0.06	-0.16 ± 0.02	$-0.10(2)$

$$p\cot\delta_0 = \frac{1}{a_0} + \frac{1}{2}r_0 p^2$$

$$t_l^{(J)} \sim \frac{1}{p\cot\delta_l^{(J)} - ip}$$

Summary and outlook

Summary

- Better understanding of hadron spectroscopy.
- Determination of LECs in BChPT.
- Examination of the validity of HDAS.
- A virtual state in $(S, I) = (1,0)$ channel.

Outlook

- More interpolators.
- Coupled channels.
- Lighter pion mass ~ 220 MeV.
- Chiral extrapolation.

(S, I)	Scattering channel	Connected(C)/ Disconnected(D)	(S, I)	Relation to $D\phi$ scattering
✓ $(-2, \frac{1}{2})$	$\Omega_{cc}\bar{K} \to \Omega_{cc}\bar{K}$	C	$(2, \frac{1}{2})$	$D_sK \to D_sK$
$(-1, 1)$	$\Omega_{cc}\pi \to \Omega_{cc}\pi$	C	$(1, 1)$	$D_s\pi \to D_s\pi$
	$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$			$DK \to DK$
	$\Omega_{cc}\pi \to \Xi_{cc}\bar{K}$			$D_s\pi \to DK$
$(-1, 0)$	$\Xi_{cc}\bar{K} \to \Xi_{cc}\bar{K}$	D	$(1, 0)$	$DK \to DK$
	$\Omega_{cc}\eta \to \Omega_{cc}\eta$			$D_s\eta \to D_s\eta$
	$\Xi_{cc}\bar{K} \to \Omega_{cc}\eta$			$DK \to D_s\eta$
✓ $(1, 0)$	$\Xi_{cc}K \to \Xi_{cc}K$	C	$(-1, 0)$	$D\bar{K} \to D\bar{K}$
✓ $(1, 1)$	$\Xi_{cc}K \to \Xi_{cc}K$	C	$(-1, 1)$	$D\bar{K} \to D\bar{K}$
$(0, \frac{1}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	D	$(0, \frac{1}{2})$	$D\pi \to D\pi$
	$\Xi_{cc}\eta \to \Xi_{cc}\eta$			$D\eta \to D\eta$
	$\Omega_{cc}K \to \Omega_{cc}K$			$D_s\bar{K} \to D_s\bar{K}$
	$\Xi_{cc}\pi \to \Xi_{cc}\eta$			$D\pi \to D\eta$
	$\Xi_{cc}\pi \to \Omega_{cc}K$			$D\pi \to D_s\bar{K}$
	$\Xi_{cc}\eta \to \Omega_{cc}K$			$D\eta \to D_s\bar{K}$
✓ $(0, \frac{3}{2})$	$\Xi_{cc}\pi \to \Xi_{cc}\pi$	C	$(0, \frac{3}{2})$	$D\pi \to D\pi$