第五届粒子物理天问论坛

正负电子湮灭到 $K\bar{K}\pi$ 过程及其对谬子反常磁矩的贡献

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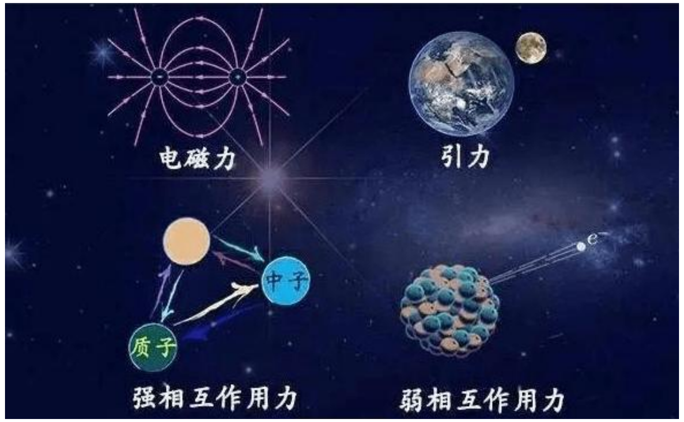
1. Background

In the current study, the fundamental particles that make up matter can be described by the "Standard Model (SM)". The elementary particles of SM are Quarks, Leptons, Gauge bosons and Higgs boson.

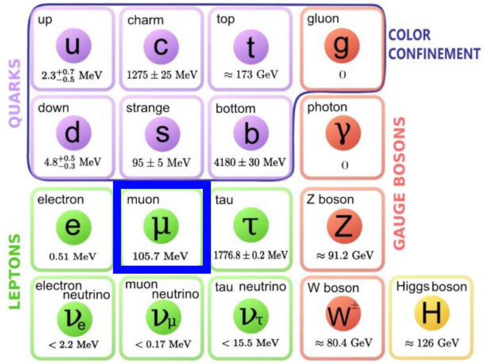


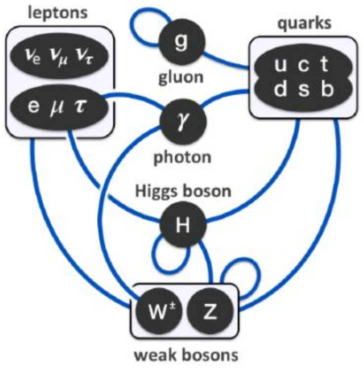
1. Quarks form various particle bound states
   p: $|uud\rangle$ n: $|udd\rangle$ J/ψ: $|c\bar{c}\rangle$

2. Leptons are often used as probes to detect the structure of other particles

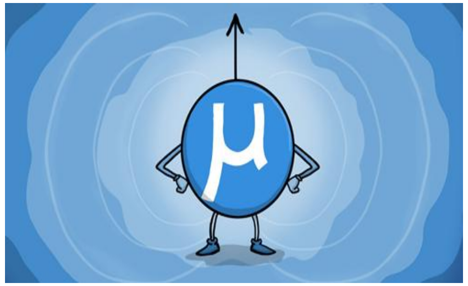
3. Gauge bosons transfer interactions

4. Higgs boson explains why particles have mass





Muon can be treat as the "heavy electron" because its properties are very similar to electron except for the mass, which is 200 times that of electron. The charged particles are subjected to magnetic moment in the magnetic field. According to the quantum field theory, the magnetic moment of the muon is

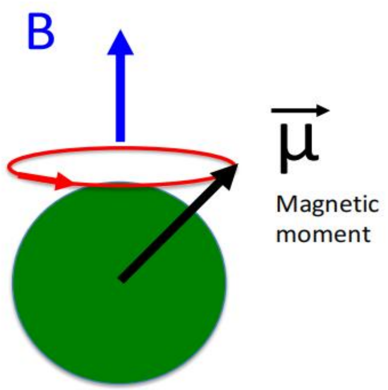


$$\vec{\mu}_\mu = g_\mu \left(\frac{q}{2m_\mu}\right)\vec{s}$$

g_μ is the spin-rotation magnetic ratio, also known as $Land\acute{e}\ \ g$ factor. From the **Dirac equation** $(i\gamma^\mu\partial_\mu - m)\psi = 0$, we can calculate $g_\mu = 2$. However, there is a small deviation between the experimental value g_μ^{exp} and **2**, and the difference between them called the anomalous magnetic moment. That is, the anomalous magnetic moment of moun is defined

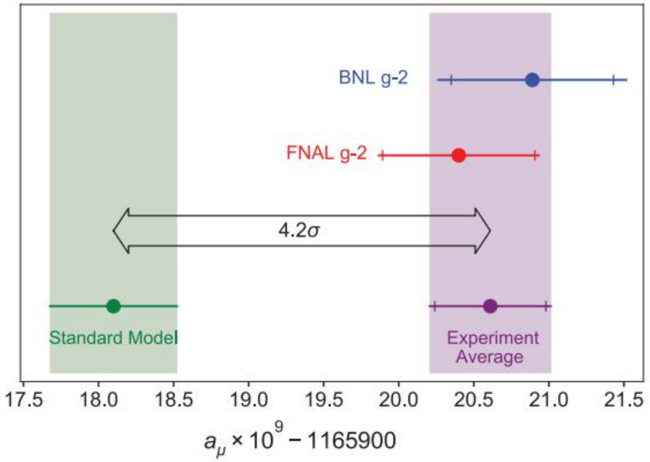
$$a_\mu = \frac{(g-2)_\mu}{2}$$

Theoretically, we can calculate the loop diagram to obtain g factor as precise as possible.



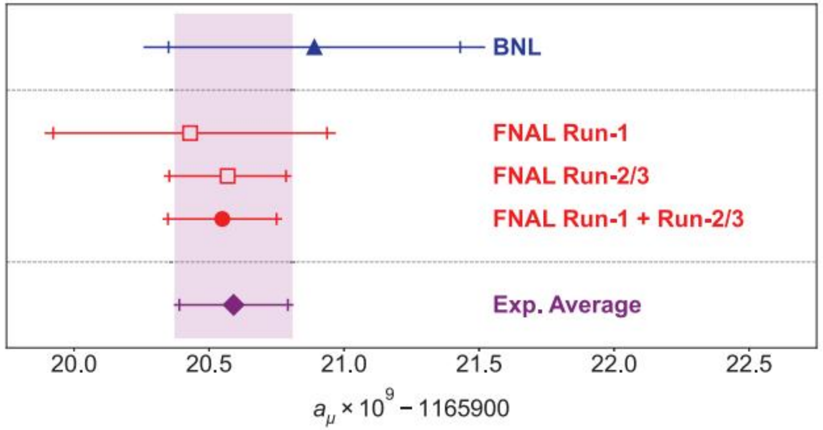
Magnetic moment (spin) processes under magnetic field.

In April 2021, Fermilab (FNAL) released the first measurement of the muon anomalous magnetic moment, with an averaged result of $4.2\ \sigma$ from SM.



PhysRevLett.126.141801

And then, on August 2023, the Fermilab has once again released the latest result. The result showed that the deviation of the experimental value from the SM reached $5.1\ \sigma$. This seems to be a signal that new physics exists?



PhysRevLett.131.161802

$$a_\mu(\text{Exp}) = 116592059(22) \times 10^{-11}$$
$$a_\mu(\text{SM}) = 116591810(43) \times 10^{-11}$$
$$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (249 \pm 48) \times 10^{-11}$$

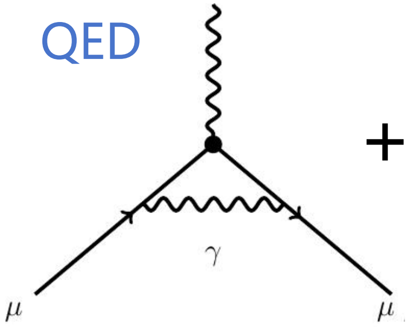
Precisely calculating the anomalous magnetic moment of muon can become a signal for searching for new physics.

$$a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLBL}}$$

→ from strong interactions.

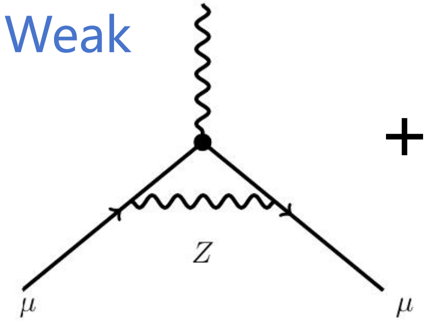
Main sources of the uncertainties and we hope to calculate the values more accurately and reduce the errors.

QED



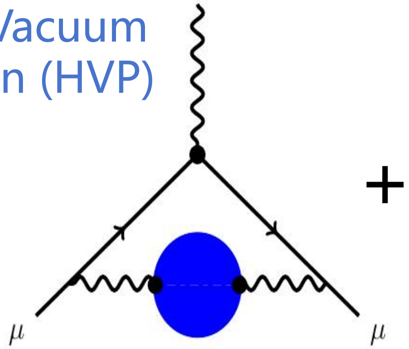
$+ \cdots$

Weak



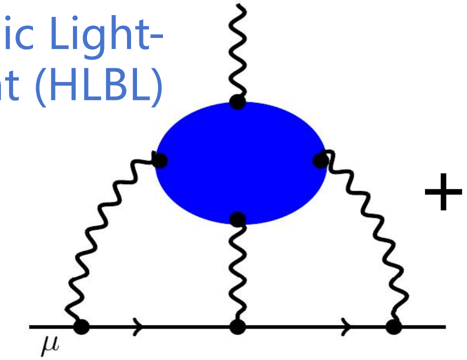
$+ \cdots$

Hadronic Vacuum Polarization (HVP)



$+ \cdots$

Hadronic Light-by-Light (HLBL)



$+ \cdots$

	values ($\times 10^{-11}$)
QED	116584718.931(104)
EW	153.6(1.0)
HVP	6845(40)
HLBL	92(18)
SM	116591810(43)
exp.(BNL)	116592089(63)
exp.(FNAL)	116592040(54)
exp.(avg.)	116592061(41)
a_μ^{SM}-a_μ^{exp}	251(59)

PhysRevLett.126.141801
PhysRevD.73.072003
Physics Reports 887 (2020) 1–166

HVP: $e^+e^- \to \pi^+\pi^-$

HLBL: $\eta' \to \pi^+\pi^-\gamma$

2. Theoretical framework

Strong interaction

↓ Basic theory

Quantum Chromodynamics (QCD)

↓ Basic parameter

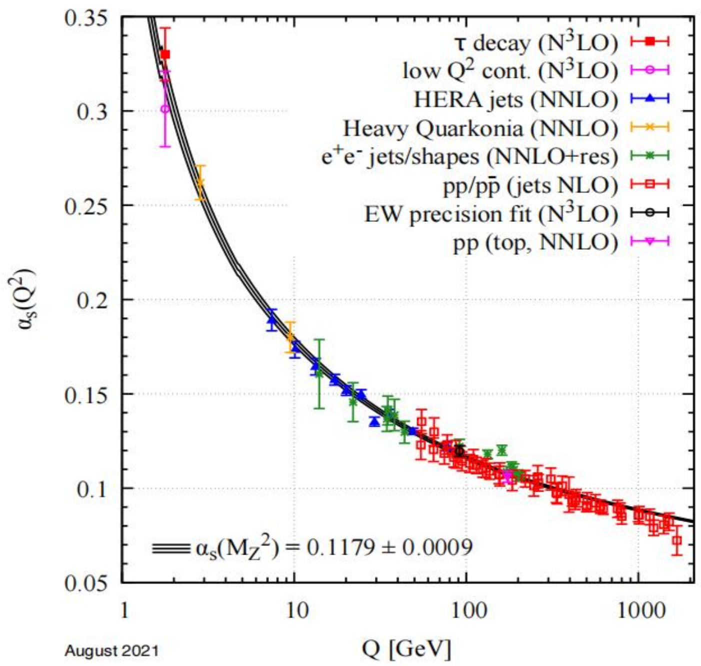
Strength: coupling constant α_s

↓ Property

Asymptotic freedom

Low energy region	High energy region
↓	↓
Effective field theory (EFT)	Perturbative QCD (pQCD)

Q ~ 2 GeV



χPT and RχT

Effective field theory (EFT)

- **Chiral Perturbation Theory (χPT)** $E_{\mathrm{cm}} \ll M_\rho$
- **Resonance Chiral Theory (RχT)** $M_\rho \leq E_{\mathrm{cm}} \leq 2\ \mathrm{GeV}$

In the regions where resonance states have not yet appeared, i.e., $E_{\mathrm{cm}} \ll M_\rho$ [M_ρ is the mass of $\rho(770)$], **χPT** provides a reliable set of theoretical tool for describing the pseudoscalar mesons which generate due to chiral symmetry breaking of the Goldstone bosons.

Nevertheless, in the intermediate energy regions [$M_\rho \leq E_{\mathrm{cm}} \leq 2\ \mathrm{GeV}$] where the resonance, like vector and tensor mesons, have appeared, the **χPT** not working. Instead of **RχT**, which expands the working states of **χPT** by including resonances as new degrees of freedom.

Some important processes contributing to $a_\mu^{\rm HVP}$	
$\pi^+\pi^-$ / K^+K^- / $K_S^0K_L^0$ / $\pi^0\gamma$ / $\eta\gamma$	JHEP07(2023)037
$\pi^+\pi^-\pi^0$ / $\pi^+\pi^-\eta$	JHEP03(2021)092 / PhysRevD.88.056001
$\boldsymbol{K\bar{K}\pi}$	**In contrast to the previous works, we add tensor mesons for the first time**
$\pi^+\pi^-\pi^0\pi^0$ / $\pi^+\pi^-\pi^+\pi^-$	
$K\bar{K}2\pi$	
……	
$R_{\rm QCD}[1.8-3.7\ {\rm GeV}]_{uds}$	JHEP07(2023)109
$R_{\rm QCD}[5.0-9.3\ {\rm GeV}]_{udsc}$	
$R_{\rm QCD}[9.3-12.0\ {\rm GeV}]_{udscb}$	
……	

χPT
RχT

pQCD

$$a_\mu^{\rm HVP}(e^+e^-\to K\bar{K}\pi)\propto\sigma(e^+e^-\to K\bar{K}\pi)\propto\mathcal{M}(e^+e^-\to K\bar{K}\pi)\propto F_V^{K\bar{K}\pi}(q^2,s,t)$$

Chiral Lagrangian

Pseudoscalar meson matrix $\mathbf{\Phi}$

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_0}{\sqrt{3}} \end{pmatrix}$$

Vector meson matrix $\mathbf{V}$

$$V_{\mu\nu} = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\phi \end{pmatrix}_{\mu\nu}$$

Tensor meson matrix $\mathbf{T}$

$$T_{\mu\nu} = \begin{pmatrix} \frac{a_2^0}{\sqrt{2}} + \frac{f_2}{\sqrt{2}} & a_2^+ & K_2^{*+} \\ a_2^- & -\frac{a_2^0}{\sqrt{2}} + \frac{f_2}{\sqrt{2}} & K_2^{*0} \\ K_2^{*-} & \bar{K}_2^{*0} & -f_2' \end{pmatrix}_{\mu\nu}$$

$$\eta_8 = \eta \cos\theta_P + \eta' \sin\theta_P \quad , \quad \eta_0 = -\eta \sin\theta_P + \eta' \cos\theta_P$$

$$K_S^0 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) \quad , \quad K_L^0 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0)$$

The Φ-matrix is a single angle arbitrary mixing, V-matrix and T-matrix are ideal mixing.

The chiral **Lagrangians** are constructed in terms of pseudoscalar, vector and tensor mesons as degrees of freedom with the requirements of being invariant under the parity (P), charge conjugation (C) and Hermiticity (h.c.) transformations.

operator	$\mathcal{O}(p^n)$	P	C	h.c.
u_μ	1	$-u^\mu$	$(u_\mu)^T$	u_μ
$h_{\mu\nu}$	2	$-h^{\mu\nu}$	$(h_{\mu\nu})^T$	$h_{\mu\nu}$
$\chi_\pm$	2	$\pm\chi_\pm$	$(\chi_\pm)^T$	$\pm\chi_\pm$
$f_\pm^{\mu\nu}$	2	$\pm f_{\pm\mu\nu}$	$\mp(f_\pm^{\mu\nu})^T$	$f_\pm^{\mu\nu}$
$V_{\mu\nu}$	0	$V^{\mu\nu}$	$-(V_{\mu\nu})^T$	$V_{\mu\nu}$
$T_{\mu\nu}$	0	$T^{\mu\nu}$	$(T_{\mu\nu})^T$	$T_{\mu\nu}$
$\varepsilon_{\mu\nu\rho\sigma}$	0	$-\varepsilon^{\mu\nu\rho\sigma}$	$\varepsilon_{\mu\nu\rho\sigma}$	$\varepsilon_{\mu\nu\rho\sigma}$

P, C and hermiticity properties of operators contained in chiral lagrangeans.

Equation of motions (EOMs) : $\nabla^\mu u_\mu = \frac{i}{2}\left(\chi_- - \frac{1}{N_f}\langle\chi_-\rangle\right)$

Schouten identity : $g_{\alpha\lambda}\varepsilon_{\mu\nu\rho\sigma} = -g_{\alpha\mu}\varepsilon_{\nu\rho\sigma\lambda} - g_{\alpha\nu}\varepsilon_{\rho\sigma\lambda\mu} - g_{\alpha\rho}\varepsilon_{\sigma\lambda\mu\nu} - g_{\alpha\sigma}\varepsilon_{\lambda\mu\nu\rho}$

Partial integration : $\nabla(ABC) = \nabla ABC + A\nabla BC + AB\nabla C = 0$

Constraints: selecting the linearly independent terms.

contain pseudoscalar only — include pseudoscalar, vector and tensor

$$\mathcal{L}_{R\chi T} = \mathcal{L}_{JPPP} + \mathcal{L}_{VJ} + \mathcal{L}_{VPP} + \mathcal{L}_{VPPP} + \mathcal{L}_{VJP} + \mathcal{L}_{VVP} + \mathcal{L}_{TPP} + \mathcal{L}_{TJP} + \mathcal{L}_{TVP}$$

where J is the electromagnetic current.

$$\mathcal{L}_{\rm JPPP} = i\frac{N_C\sqrt{2}}{12\pi^2F^3}\varepsilon_{\mu\nu\rho\sigma}\langle\partial^\mu\Phi\partial^\nu\Phi\partial^\rho\Phi v^\sigma\rangle \quad , \quad \mathcal{L}_{\rm VJ} = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu}f_+^{\mu\nu}\rangle \quad , \quad \mathcal{L}_{\rm VPP} = i\frac{G_V}{\sqrt{2}}\langle V_{\mu\nu}u^\mu u^\nu\rangle$$

$$\begin{aligned}\mathcal{L}_{\rm VPPP} =& \frac{ig_1}{M_V}\varepsilon_{\mu\nu\alpha\beta}\langle V^{\mu\nu}(h^{\alpha\gamma}u_\gamma u^\beta - u^\beta u_\gamma h^{\alpha\gamma})\rangle + \frac{ig_2}{M_V}\varepsilon_{\mu\nu\alpha\beta}\langle V^{\mu\nu}(h^{\alpha\gamma}u^\beta u_\gamma - u_\gamma u^\beta h^{\alpha\gamma})\rangle \\ &+ \frac{ig_3}{M_V}\varepsilon_{\mu\nu\alpha\beta}\langle V^{\mu\nu}(u_\gamma h^{\alpha\gamma}u^\beta - u^\beta h^{\alpha\gamma}u_\gamma)\rangle + \frac{g_4}{M_V}\varepsilon_{\mu\nu\alpha\beta}\langle\{V^{\mu\nu}, u^\alpha u^\beta\}\chi_-\rangle + \frac{g_5}{M_V}\varepsilon_{\mu\nu\alpha\beta}\langle u^\alpha V^{\mu\nu}u^\beta\chi_-\rangle\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\rm VJP} =& \frac{c_1}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \frac{c_2}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\alpha}, f_+^{\rho\sigma}\}\nabla_\alpha u^\nu\rangle + \frac{ic_3}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\sigma}\}\chi_-\rangle \\ &+ \frac{ic_4}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle V^{\mu\nu}[f_-^{\rho\sigma}, \chi_+]\rangle + \frac{c_5}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, f_+^{\rho\alpha}\}u^\sigma\rangle + \frac{c_6}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\alpha}, f_+^{\rho\sigma}\}u^\nu\rangle \\ &+ \frac{c_7}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, f_+^{\rho\alpha}\}u_\alpha\rangle\end{aligned}$$

$$\mathcal{L}_{\rm VVP} = d_1\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + id_2\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\sigma}\}\chi_-\rangle + d_3\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla_\alpha V^{\mu\nu}, V^{\rho\alpha}\}u^\sigma\rangle + d_4\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\}u_\alpha\rangle$$

PhysRevD.88.056001

Chiral Lagrangian related to the tensor

$$\mathcal{L}_{\mathrm{TPP}} = g_T \langle T_{\mu\nu}\{u^\mu, u^\nu\}\rangle$$ → Eur. Phys. J. C 52, 315–323 (2007)

$$\mathcal{L}_{\mathrm{TJP}} = ic_{T1}\varepsilon_{\mu\nu\rho\sigma}\langle[T^{\mu\alpha}, f_+^{\rho\sigma}]\nabla_\alpha u^\nu\rangle + ic_{T2}\varepsilon_{\mu\nu\rho\sigma}\langle[\nabla^\nu T^\mu_\alpha, f_+^{\rho\sigma}]u^\alpha\rangle + ic_{T3}\varepsilon_{\mu\nu\rho\sigma}\langle[\nabla^\nu T^\mu_\alpha, f_+^{\rho\alpha}]u^\sigma\rangle$$

$$\mathcal{L}_{\mathrm{TVP}} = id_{T1}\varepsilon_{\mu\nu\rho\sigma}\langle[T^{\mu\alpha}, V^{\rho\sigma}]\nabla_\alpha u^\nu\rangle + id_{T2}\varepsilon_{\mu\nu\rho\sigma}\langle[\nabla^\nu T^\mu_\alpha, V^{\rho\sigma}]u^\alpha\rangle + id_{T3}\varepsilon_{\mu\nu\rho\sigma}\langle[\nabla^\nu T^\mu_\alpha, V^{\rho\alpha}]u^\sigma\rangle$$

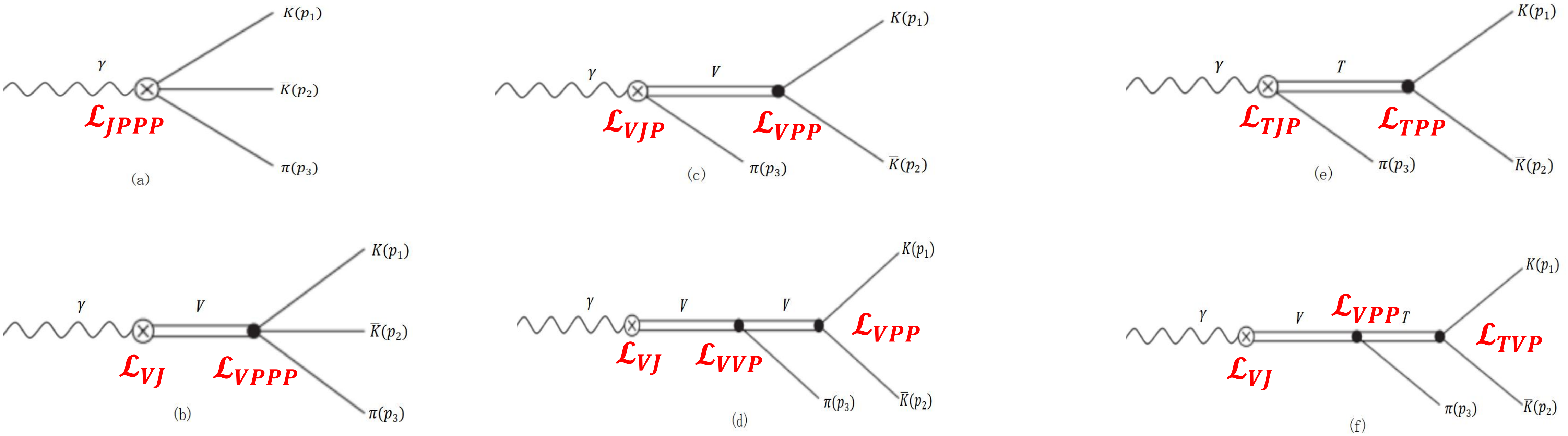
$$u = \exp\left\{\frac{i}{\sqrt{2}F}\Phi\right\} = 1 + \frac{i}{\sqrt{2}F}\Phi + \mathcal{O}\left(\Phi^2\right)$$

$$u_\mu = i\left\{u^\dagger\left(\partial_\mu - ir_\mu\right)u - u\left(\partial_\mu - il_\mu\right)u^\dagger\right\}$$
$$\chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u$$
$$f_\pm^{\mu\nu} = uF_L^{\mu\nu}u^\dagger \pm u^\dagger F_R^{\mu\nu}u$$
$$h_{\mu\nu} = \nabla_\mu u_\nu + \nabla_\nu u_\mu$$

$$r_\mu = v_\mu + a_\mu \ , \ l_\mu = v_\mu - a_\mu$$
$$F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i\left[r^\mu, r^\nu\right] \ , \ F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i\left[l^\mu, l^\nu\right]$$
$$\nabla_\mu X = \partial_\mu X + \left[\Gamma_\mu, X\right] \ , \ \Gamma_\mu = \frac{1}{2}\left\{u^\dagger\left(\partial_\mu - ir_\mu\right)u - u\left(\partial_\mu - il_\mu\right)u^\dagger\right\}$$

Feynman diagrams for the $K\bar{K}\pi$ processes

$$e^+(q_1)e^-(q_2) \to K(p_1)\bar{K}(p_2)\pi(p_3)\text{:}\ e^+e^- \to K^+K^-\pi^0\text{、}e^+e^- \to K_S^0K_L^0\pi^0\text{、}e^+e^- \to K_S^0K^\pm\pi^\mp$$



$$\mathcal{M}^{K\bar{K}\pi} = -\frac{4\pi\alpha}{q^2} i \left[F_V^{K\bar{K}\pi}(q^2, s, t) \right] \varepsilon_{\mu\nu\alpha\beta} p_1^\nu p_2^\alpha p_3^\beta \bar{v}(q_1)\gamma^\mu u(q_2)$$

Form factors: $F_V^{K\bar{K}\pi}(q^2, s, t) = F_a + F_b + F_c + F_d + F_e + F_f$

3. Results

Form factors of $e^+e^- \to K^+(p_1)K^-(p_2)\pi^0(p_3)$

$$F_a = -\frac{N_C}{12\pi^2 F^3}$$

$$F_b = \frac{2\sqrt{2}F_V}{F^3 M_V (M_\rho^2 - q^2)} GR_1(q^2,s) + \frac{2\sqrt{2}F_V}{3F^3 M_V (M_\omega^2 - q^2)} GR_1(q^2,s) - \frac{4\sqrt{2}F_V}{3F^3 M_V (M_\phi^2 - q^2)} GR_2(q^2,s)$$

$$\begin{aligned} F_c = &- \frac{2\sqrt{2}G_V}{3F^3 M_V (M_\rho^2 - s)} CR_1(q^2,s,m_\pi^2) - \frac{2\sqrt{2}G_V}{F^3 M_V (M_\omega^2 - s)} CR_1(q^2,s,m_\pi^2) \\ &- \frac{2\sqrt{2}G_V}{3F^3 M_V (M_{K^*}^2 - t)} CR_2(q^2,t) - \frac{2\sqrt{2}G_V}{3F^3 M_V (M_{K^*}^2 - u)} CR_2(q^2,u) \end{aligned}$$

$$\begin{aligned} F_f = &- \frac{F_V g_T}{\sqrt{2}F^3 M_{K_2^*}^2 (M_\rho^2 - q^2)(M_{K_2^*}^2 - t)} DT(q^2,s,t) - \frac{F_V g_T}{3\sqrt{2}F^3 M_{K_2^*}^2 (M_\omega^2 - q^2)(M_{K_2^*}^2 - t)} DT(q^2,s,t) \\ &- \frac{\sqrt{2}F_V g_T}{3F^3 M_{K_2^*}^2 (M_\phi^2 - q^2)(M_{K_2^*}^2 - t)} DT(q^2,s,t) \\ &- \frac{F_V g_T}{\sqrt{2}F^3 M_{K_2^*}^2 (M_\rho^2 - q^2)(M_{K_2^*}^2 - u)} DT(q^2,s,u) - \frac{F_V g_T}{3\sqrt{2}F^3 M_{K_2^*}^2 (M_\omega^2 - q^2)(M_{K_2^*}^2 - u)} DT(q^2,s,u) \\ &- \frac{\sqrt{2}F_V g_T}{3F^3 M_{K_2^*}^2 (M_\phi^2 - q^2)(M_{K_2^*}^2 - u)} DT(q^2,s,u) \end{aligned}$$

$$\begin{aligned} F_d = &\frac{4F_V G_V}{F^3 (M_\rho^2 - q^2)(M_\omega^2 - s)} DR(q^2,s,m_\pi^2) + \frac{4F_V G_V}{3F^3 (M_\omega^2 - q^2)(M_\rho^2 - s)} DR(q^2,s,m_\pi^2) \\ &+ \frac{2F_V G_V}{F^3 (M_\rho^2 - q^2)(M_{K^*}^2 - t)} DR(q^2,t,m_K^2) + \frac{2F_V G_V}{3F^3 (M_\omega^2 - q^2)(M_{K^*}^2 - t)} DR(q^2,t,m_K^2) \\ &- \frac{4F_V G_V}{3F^3 (M_\phi^2 - q^2)(M_{K^*}^2 - t)} DR(q^2,t,m_K^2) \\ &+ \frac{2F_V G_V}{F^3 (M_\rho^2 - q^2)(M_{K^*}^2 - u)} DR(q^2,u,m_K^2) + \frac{2F_V G_V}{3F^3 (M_\omega^2 - q^2)(M_{K^*}^2 - u)} DR(q^2,u,m_K^2) \\ &- \frac{4F_V G_V}{3F^3 (M_\phi^2 - q^2)(M_{K^*}^2 - u)} DR(q^2,u,m_K^2) \end{aligned}$$

$$F_e = \frac{2g_T}{F^3 M_{K_2^*}^2 (M_{K_2^*}^2 - t)} CT(q^2,s,t) + \frac{2g_T}{F^3 M_{K_2^*}^2 (M_{K_2^*}^2 - u)} CT(q^2,s,u)$$

Mandelstam variables

$$s = p_{12}^2 = (p_1 + p_2)^2$$
$$t = p_{23}^2 = (p_2 + p_3)^2$$
$$u = p_{13}^2 = (p_1 + p_3)^2 = q^2 + 2m_K^2 + m_\pi^2 - s - t$$

$$F_V^{K^+K^-\pi^0}(q^2,s,t) = F_a + F_b + F_c + F_d + F_e + F_f$$

Form factors of $e^+e^- \to K_S^0(p_1)K_L^0(p_2)\pi^0(p_3)$

$$F_a = \frac{N_C}{12\pi^2 F^3}$$

$$F_b = -\frac{2\sqrt{2}F_V}{F^3 M_V(M_\rho^2 - q^2)}GR_1(q^2,s) + \frac{2\sqrt{2}F_V}{3F^3 M_V(M_\omega^2 - q^2)}GR_1(q^2,s) - \frac{4\sqrt{2}F_V}{3F^3 M_V(M_\phi^2 - q^2)}GR_2(q^2,s)$$

$$\begin{aligned} F_c = &-\frac{2\sqrt{2}G_V}{3F^3 M_V(M_\rho^2 - s)}CR_1(q^2,s,m_\pi^2) + \frac{2\sqrt{2}G_V}{F^3 M_V(M_\omega^2 - s)}CR_1(q^2,s,m_\pi^2) \\ &+ \frac{4\sqrt{2}G_V}{3F^3 M_V(M_{K^*}^2 - t)}CR_1(q^2,t,m_K^2) + \frac{4\sqrt{2}G_V}{3F^3 M_V(M_{K^*}^2 - u)}CR_1(q^2,u,m_K^2) \end{aligned}$$

$$\begin{aligned} F_d = &-\frac{4F_V G_V}{F^3(M_\rho^2 - q^2)(M_\omega^2 - s)}DR(q^2,s,m_\pi^2) + \frac{4F_V G_V}{3F^3(M_\omega^2 - q^2)(M_\rho^2 - s)}DR(q^2,s,m_\pi^2) \\ &- \frac{2F_V G_V}{F^3(M_\rho^2 - q^2)(M_{K^*}^2 - t)}DR(q^2,t,m_K^2) + \frac{2F_V G_V}{3F^3(M_\omega^2 - q^2)(M_{K^*}^2 - t)}DR(q^2,t,m_K^2) \\ &- \frac{4F_V G_V}{3F^3(M_\phi^2 - q^2)(M_{K^*}^2 - t)}DR(q^2,t,m_K^2) \\ &- \frac{2F_V G_V}{F^3(M_\rho^2 - q^2)(M_{K^*}^2 - u)}DR(q^2,u,m_K^2) + \frac{2F_V G_V}{3F^3(M_\omega^2 - q^2)(M_{K^*}^2 - u)}DR(q^2,u,m_K^2) \\ &- \frac{4F_V G_V}{3F^3(M_\phi^2 - q^2)(M_{K^*}^2 - u)}DR(q^2,u,m_K^2) \end{aligned}$$

$$F_e = 0$$

$$\begin{aligned} F_f = &\frac{F_V g_T}{\sqrt{2}F^3 M_{K_2^*}^2(M_\rho^2 - q^2)(M_{K_2^*}^2 - t)}DT(q^2,s,t) - \frac{F_V g_T}{3\sqrt{2}F^3 M_{K_2^*}^2(M_\omega^2 - q^2)(M_{K_2^*}^2 - t)}DT(q^2,s,t) \\ &- \frac{\sqrt{2}F_V g_T}{3F^3 M_{K_2^*}^2(M_\phi^2 - q^2)(M_{K_2^*}^2 - t)}DT(q^2,s,t) \\ &+ \frac{F_V g_T}{\sqrt{2}F^3 M_{K_2^*}^2(M_\rho^2 - q^2)(M_{K_2^*}^2 - u)}DT(q^2,s,u) - \frac{F_V g_T}{3\sqrt{2}F^3 M_{K_2^*}^2(M_\omega^2 - q^2)(M_{K_2^*}^2 - u)}DT(q^2,s,u) \\ &- \frac{\sqrt{2}F_V g_T}{3F^3 M_{K_2^*}^2(M_\phi^2 - q^2)(M_{K_2^*}^2 - u)}DT(q^2,s,u) \end{aligned}$$

$$F_V^{K_S^0 K_L^0 \pi^0}(q^2,s,t) = F_a + F_b + F_c + F_d + F_e + F_f$$

Form factors of $e^+e^- \to K_S^0(p_1)K^+(p_2)\pi^-(p_3)$

$$F_a = 0$$

$$F_b = \frac{2\sqrt{2}F_V}{F^3 M_V (M_\rho^2 - q^2)}[G_{123}(t-u)] - \frac{2\sqrt{2}F_V}{3F^3 M_V (M_\omega^2 - q^2)} GR_1(q^2,s) + \frac{4\sqrt{2}F_V}{3F^3 M_V (M_\phi^2 - q^2)} GR_2(q^2,s)$$

$$F_c = \frac{2\sqrt{2}G_V}{3F^3 M_V (M_\rho^2 - s)} CR_1(q^2,s,m_\pi^2) - \frac{4\sqrt{2}G_V}{3F^3 M_V (M_{K^*}^2 - t)} CR_1(q^2,t,m_K^2) + \frac{2\sqrt{2}G_V}{3F^3 M_V (M_{K^*}^2 - u)} CR_2(q^2,u)$$

$$\begin{aligned} F_d = &- \frac{4F_V G_V}{3F^3 (M_\omega^2 - q^2)(M_\rho^2 - s)} DR(q^2,s,m_\pi^2) \\ &+ \frac{2F_V G_V}{F^3 (M_\rho^2 - q^2)(M_{K^*}^2 - t)} DR(q^2,t,m_K^2) - \frac{2F_V G_V}{3F^3 (M_\omega^2 - q^2)(M_{K^*}^2 - t)} DR(q^2,t,m_K^2) \\ &+ \frac{4F_V G_V}{3F^3 (M_\phi^2 - q^2)(M_{K^*}^2 - t)} DR(q^2,t,m_K^2) \\ &- \frac{2F_V G_V}{F^3 (M_\rho^2 - q^2)(M_{K^*}^2 - u)} DR(q^2,u,m_K^2) - \frac{2F_V G_V}{3F^3 (M_\omega^2 - q^2)(M_{K^*}^2 - u)} DR(q^2,u,m_K^2) \\ &+ \frac{4F_V G_V}{3F^3 (M_\phi^2 - q^2)(M_{K^*}^2 - u)} DR(q^2,u,m_K^2) \end{aligned}$$

$$F_e = -\frac{2g_T}{F^3 (M_{a_2}^2 - s)}[C_{T123}(t-u)] - \frac{2g_T}{F^3 M_{K_2^*}^2 (M_{K_2^*}^2 - u)} CT(q^2,s,u)$$

$$\begin{aligned} F_f = &\frac{\sqrt{2}F_V g_T}{F^3 (M_\rho^2 - q^2)(M_{a_2}^2 - s)}[D_{T123}(t-u)] \\ &- \frac{F_V g_T}{\sqrt{2}F^3 M_{K_2^*}^2 (M_\rho^2 - q^2)(M_{K_2^*}^2 - t)} DT(q^2,s,t) + \frac{F_V g_T}{3\sqrt{2}F^3 M_{K_2^*}^2 (M_\omega^2 - q^2)(M_{K_2^*}^2 - t)} DT(q^2,s,t) \\ &+ \frac{\sqrt{2}F_V g_T}{3F^3 M_{K_2^*}^2 (M_\phi^2 - q^2)(M_{K_2^*}^2 - t)} DT(q^2,s,t) \\ &+ \frac{F_V g_T}{\sqrt{2}F^3 M_{K_2^*}^2 (M_\rho^2 - q^2)(M_{K_2^*}^2 - u)} DT(q^2,s,u) + \frac{F_V g_T}{3\sqrt{2}F^3 M_{K_2^*}^2 (M_\omega^2 - q^2)(M_{K_2^*}^2 - u)} DT(q^2,s,u) \\ &+ \frac{\sqrt{2}F_V g_T}{3F^3 M_{K_2^*}^2 (M_\phi^2 - q^2)(M_{K_2^*}^2 - u)} DT(q^2,s,u) \end{aligned}$$

$$F_V^{K_S^0 K^+ \pi^-}(q^2,s,t) = F_a + F_b + F_c + F_d + F_e + F_f$$

The form factors of $e^+e^- \to K_S^0(p_1)K^-(p_2)\pi^+(p_3)$ differs by one phase from $e^+e^- \to K_S^0(p_1)K^+(p_2)\pi^-(p_3)$

Notation

$$GR_1(q^2,s) = G_{123}(q^2+s-4m_K^2-m_\pi^2) + 4G_{45}m_K^2 - 4g_2(q^2-2m_K^2-m_\pi^2) - 4g_4(m_K^2-m_\pi^2)$$

$$GR_2(q^2,s) = G_{123}(q^2-s-m_\pi^2) + 2G_{45}m_\pi^2 - 2g_2(q^2-2m_K^2-m_\pi^2) + 4g_4(m_K^2-m_\pi^2)$$

$$CR_1(q^2,x,m^2) = C_{125}q^2 - C_{1256}x + C_{1235}m^2$$

$$CR_2(q^2,x) = C_{125}q^2 - C_{1256}x + C_{1235}m_K^2 + 24c_4(m_K^2-m_\pi^2)$$

$$DR(q^2,x,m^2) = D_{123}m^2 + d_3(q^2+x)$$

$$CT(q^2,s,x) = C_{T123}\left[M_{K_2^*}^2(q^2-2s-x+3m_K^2) + (q^2-x-m_K^2)(m_K^2-m_\pi^2)\right] + 2c_{T3}(M_{K_2^*}^2-x)(m_K^2-m_\pi^2)$$

$$C_{T123} = 2c_{T1} - 2c_{T2} - c_{T3}$$

$$DT(q^2,s,x) = D_{T123}\left[M_{K_2^*}^2(q^2-2s-x+3m_K^2) + (q^2-x-m_K^2)(m_K^2-m_\pi^2)\right] + 2d_{T3}(M_{K_2^*}^2-x)(m_K^2-m_\pi^2)$$

$$D_{T123} = 2d_{T1} - 2d_{T2} - d_{T3}$$

High-energy behavior: $Q \to \infty$, $F_V^{K\bar{K}\pi}(q^2,s,t) = 0$

Chiral limit: $m_\pi = m_K = 0$, $M_\rho = M_\omega = M_\phi = M_{K^*} = M_V$

$$G_{123} = g_1 + 2g_2 - g_3 = 0$$

$$C_{125} = c_1 - c_2 + c_5 = 0$$

$$C_{1235} = c_1 + c_2 + 8c_3 - c_5 = 0$$

$$g_2 = \frac{N_C M_V}{192\sqrt{2}\pi^2 F_V}$$

$$C_{1256} = c_1 - c_2 - c_5 + 2c_6 = -\frac{N_C M_V}{96\sqrt{2}\pi^2 G_V}$$

$$d_3 = -\frac{N_C M_V^2}{192\pi^2 F_V G_V}$$

$$G_V = \frac{F^2}{F_V}$$

$$D_{123} = d_1 + 8d_2 - d_3 = \frac{F^2}{8F_V^2}$$

Revise the form factors

$$\frac{1}{M_R^2 - x} \longrightarrow \frac{1}{M_R^2 - x} + \frac{\beta'}{M_{R'}^2 - x} + \frac{\beta''}{M_{R''}^2 - x} \longrightarrow \frac{1}{M_R^2 - x - iM_R\Gamma_R(x)} + \frac{\beta'}{M_{R'}^2 - x - iM_{R'}\Gamma_{R'}(x)} + \frac{\beta''}{M_{R''}^2 - x - iM_{R''}\Gamma_{R''}(x)}$$

The original momentum structure

In order to expand the energy regions to 2.3 GeV

Add widths

$$R = \{\rho(770)\ ,\ \omega(782)\ ,\ \phi(1020)\ ,\ K^*(892)\ ,\ a_2(1320)\ ,\ K_2^*(1430)\}$$

$$R' = \{\rho(1450)\ ,\ \omega(1420)\ ,\ \phi(1680)\ ,\ K^*(1410)\ ,\ a_2(1700)\ ,\ K_2^*(1980)\}$$

$$R'' = \{\rho(1700)\ ,\ \omega(1650)\ ,\ \phi(2170)\ ,\ K^*(1680)\}$$

The unknown parameters are $F_V, g_4, c_4, G_{45}, g_T, c_{T3}, d_{T3}, C_{T123}, D_{T123}, \beta', \beta''$ and the mass and widths of the resonance states.

Decay widths related to the $K\bar{K}\pi$ processes

$$\Gamma(K^* \to K\pi) = \frac{G_V^2}{64\pi F^4}\frac{\lambda^{3/2}(M_{K^*}^2, m_K^2, m_\pi^2)}{M_{K^*}^3}$$

$$\Gamma(K^{*0} \to K^0\gamma) = \frac{\alpha(M_{K^*}^2 - m_K^2)^3}{24M_{K^*}^5}\left\{\frac{4\sqrt{2}}{3FM_V}\left[C_{1235}m_K^2 - C_{1256}M_{K^*}^2\right] - \frac{2F_V}{F}\left(\frac{1}{M_\rho^2} - \frac{1}{3M_\omega^2} + \frac{2}{3M_\phi^2}\right)\left[D_{123}m_K^2 + d_3 M_{K^*}^2\right]\right\}^2$$

$$\Gamma(K^{*\pm} \to K^\pm\gamma) = \frac{\alpha(M_{K^*}^2 - m_K^2)^3}{24M_{K^*}^5}\left\{\frac{2\sqrt{2}}{3FM_V}\left[C_{1235}m_K^2 - C_{1256}M_{K^*}^2 + 24c_4(m_K^2 - m_\pi^2)\right] - \frac{2F_V}{F}\left(\frac{1}{M_\rho^2} + \frac{1}{3M_\omega^2} - \frac{2}{3M_\phi^2}\right)\left[D_{123}m_K^2 + d_3 M_{K^*}^2\right]\right\}^2$$

$$\Gamma(a_2 \to \eta\pi) = \frac{g_T^2}{360\pi F^4}\frac{\lambda^{5/2}(M_{a_2}^2, m_\eta^2, m_\pi^2)}{M_{a_2}^7}\left\{-2\sqrt{2}\sin(2\theta_P) - \cos(2\theta_P) + 3\right\}$$

$$\Gamma(a_2 \to K\bar{K}) = \frac{g_T^2}{120\pi F^4}\frac{(M_{a_2}^2 - 4m_K^2)^{5/2}}{M_{a_2}^2}$$

$$\Gamma(a_2 \to \eta'\pi) = \frac{g_T^2}{360\pi F^4}\frac{\lambda^{5/2}(M_{a_2}^2, m_{\eta'}^2, m_\pi^2)}{M_{a_2}^7}\left\{2\sqrt{2}\sin(2\theta_P) + \cos(2\theta_P) + 3\right\}$$

$$\Gamma(a_2 \to \pi\gamma) = \frac{\alpha}{80}\left(\frac{M_{a_2}^2 - m_\pi^2}{M_{a_2}}\right)^5\left\{-\frac{\sqrt{2}}{F}C_{T123} + \frac{F_V}{FM_\rho^2}D_{T123}\right\}^2$$

$$\Gamma(K_2^* \to K\pi) = \frac{g_T^2}{80\pi F^4}\frac{\lambda^{5/2}(M_{K_2^*}^2, m_K^2, m_\pi^2)}{M_{K_2^*}^7}$$

$$\Gamma(K_2^* \to K^*\pi) = \frac{3D_{T123}^2}{1280\pi F^2 M_{K^*}^2}\frac{\lambda^{5/2}(M_{K_2^*}^2, M_{K^*}^2, m_\pi^2)}{M_{K_2^*}^5}$$

$$\Gamma(K_2^* \to \rho K) = \frac{3D_{T123}^2}{1280\pi F^2 M_\rho^2}\frac{\lambda^{5/2}(M_{K_2^*}^2, M_\rho^2, m_K^2)}{M_{K_2^*}^5}$$

$$\Gamma(K_2^* \to \omega K) = \frac{D_{T123}^2}{1280\pi F^2 M_\omega^2}\frac{\lambda^{5/2}(M_{K_2^*}^2, M_\omega^2, m_K^2)}{M_{K_2^*}^5}$$

$$\Gamma(K_2^{*+} \to K^+\gamma) = \frac{\alpha}{80}\left(\frac{M_{K_2^*}^2 - m_K^2}{M_{K_2^*}}\right)^5\left\{-\frac{\sqrt{2}}{F}C_{T123} + \frac{F_V}{2F}\left(\frac{1}{M_\rho^2} + \frac{1}{3M_\omega^2} + \frac{2}{3M_\phi^2}\right)D_{T123}\right\}^2$$

Scattering cross section

$$\sigma^{K\bar{K}\pi}(q^2) = \frac{\alpha^2}{192\pi q^6}\int_{s_{\min}}^{s_{\max}} ds \int_{t_{\min}}^{t_{\max}} dt\ \phi(q^2,s,t)\left|F_V^{K\bar{K}\pi}(q^2,s,t)\right|^2$$

$$\begin{aligned}\phi(q^2,s,t) = & -m_{\bar{K}}^2(m_{\bar{K}}^2+q^2-s-t)^2+(-m_K^2-m_{\bar{K}}^2+s)(m_{\bar{K}}^2+m_\pi^2-t)(-m_{\bar{K}}^2-q^2+s+t)\\ & -m_\pi^2(m_K^2+m_{\bar{K}}^2-s)^2-m_K^2(m_{\bar{K}}^2+m_\pi^2-t)^2+4m_K^2m_{\bar{K}}^2m_\pi^2\end{aligned}$$

$$s_{\min} = (m_K+m_{\bar{K}})^2 \qquad s_{\max} = (\sqrt{q^2}-m_\pi)^2$$

$$t_{\min} = \frac{1}{4s}\left\{(q^2-m_K^2+m_{\bar{K}}^2-m_\pi^2)^2-\left[\lambda^{1/2}(q^2,s,m_\pi^2)+\lambda^{1/2}(s,m_K^2,m_{\bar{K}}^2)\right]^2\right\}$$

$$t_{\max} = \frac{1}{4s}\left\{(q^2-m_K^2+m_{\bar{K}}^2-m_\pi^2)^2-\left[\lambda^{1/2}(q^2,s,m_\pi^2)-\lambda^{1/2}(s,m_K^2,m_{\bar{K}}^2)\right]^2\right\}$$

To determine the unknown parameters, we fit them with the experimental data.

Least Squares

experimental data: x_i , $y_i \pm \sigma_i$

fitting function:

$$\chi^2 = \sum_i \frac{(\mu(x_i;\theta)-y_i)^2}{\sigma_i^2}$$

Invariant mass spectrum

$$\text{Events} \propto \frac{d\sigma}{d\sqrt{s}} \quad \text{or} \quad \text{Events} \propto \frac{d\sigma}{d\sqrt{t}}$$

Joint fitting:

$$\chi^2_{total} = \chi^2_{cross-sectiom} + \chi^2_{invariant-mass} + \chi^2_{decay\ widths}$$

Fitting results

TABLE III. Fitting results for parameters.

parameters	Fit	Ref [6]	Ref [5]	Ref [4]	PDG [18]
F_V	0.1386 ± 0.0002	0.138 ± 0.001	0.142 ± 0.001	0.148 ± 0.001	
g_4	-0.0306 ± 0.0009	–	–	–	
c_4	-0.0011 ± 0.0001	–	–	–	
G_{45}	-0.0265 ± 0.0020	–	-0.492 ± 0.002	-0.493 ± 0.003	
g_T	0.0237 ± 0.0003	–	–	–	
c_3^T	0.0648 ± 0.0321	–	–	–	
d_3^T	-0.7627 ± 0.1817	–	–	–	
C_{T123}	0.0405 ± 0.0015	–	–	–	
D_{T123}	0.6866 ± 0.0142	–	–	–	
$\theta_P(^\circ)$	-20.5000 ± 0.2364	-20.50 ± 0.30	-19.61 ± 0.10	21.37 ± 0.26	
$\beta'_{K^+K^-\pi^0}$	-0.3824 ± 0.0018	–	–	–	
$\beta''_{K^+K^-\pi^0}$	-0.0118 ± 0.0027	–	–		
$\beta'_{K_S^0K_L^0\pi^0}$	-0.5127 ± 0.0019	–	–		
$\beta''_{K_S^0K_L^0\pi^0}$	-0.0008 ± 0.0049	–	–		
$\beta'_{K_S^0K^\pm\pi^\mp}$	-0.5091 ± 0.0020	–	–		
$\beta''_{K_S^0K^\pm\pi^\mp}$	0.1066 ± 0.0051	–	–		
$M_{\rho'}$	1.5455 ± 0.0006	1.519 ± 0.001	1.519 ± 0.002	1.550 ± 0.012	1.465 ± 0.025
$\Gamma_{\rho'}$	0.2548 ± 0.0017	0.381 ± 0.003	0.340 ± 0.001	0.238 ± 0.018	0.400 ± 0.060
$M_{\omega'}$	1.4170 ± 0.0028	1.250 ± 0.003	1.253 ± 0.003	1.249 ± 0.003	1.410 ± 0.060
$\Gamma_{\omega'}$	0.1605 ± 0.0041	0.290 ± 0.002	0.310 ± 0.003	0.307 ± 0.007	0.290 ± 0.190
$M_{\phi'}$	1.6485 ± 0.0007	1.656 ± 0.003	1.640 ± 0.003	1.641 ± 0.005	1.680 ± 0.020
$\Gamma_{\phi'}$	0.1896 ± 0.0005	0.136 ± 0.001	0.090 ± 0.002	0.086 ± 0.007	0.150 ± 0.050
$M_{K^{*\prime}}$	1.3800 ± 0.0024	–	–	–	1.414 ± 0.015
$\Gamma_{K^{*\prime}}$	0.2100 ± 0.0017	–	–	–	0.232 ± 0.021
$M_{\rho''}$	1.7399 ± 0.0013	1.720 ± 0.001	1.720 ± 0.001	1.794 ± 0.012	1.720 ± 0.020
$\Gamma_{\rho''}$	0.2678 ± 0.0179	0.250 ± 0.001	0.150 ± 0.005	0.297 ± 0.033	0.250 ± 0.100
$M_{\omega''}$	1.6588 ± 0.0291	1.725 ± 0.002	1.725 ± 0.010	1.700 ± 0.011	1.670 ± 0.030
$\Gamma_{\omega''}$	0.3973 ± 0.0134	0.400 ± 0.001	0.400 ± 0.003	0.400 ± 0.013	0.315 ± 0.035
$M_{\phi''}$	2.2226 ± 0.0049	2.160 ± 0.001	2.126 ± 0.025	2.086 ± 0.022	2.162 ± 0.007
$\Gamma_{\phi''}$	0.1400 ± 0.0012	0.105 ± 0.010	0.100 ± 0.014	0.108 ± 0.017	$0.100^{+0.031}_{-0.023}$
$M_{K^{*\prime\prime}}$	1.7400 ± 0.0306	–	–	–	1.718 ± 0.018
$\Gamma_{K^{*\prime\prime}}$	0.2100 ± 0.1608	–	–	–	0.322 ± 0.110
N_1	24.5966 ± 0.8802	–	–	–	
N_2	3.4564 ± 0.0955	–	–	–	
N_3	2.1643 ± 0.0498	–	–	–	

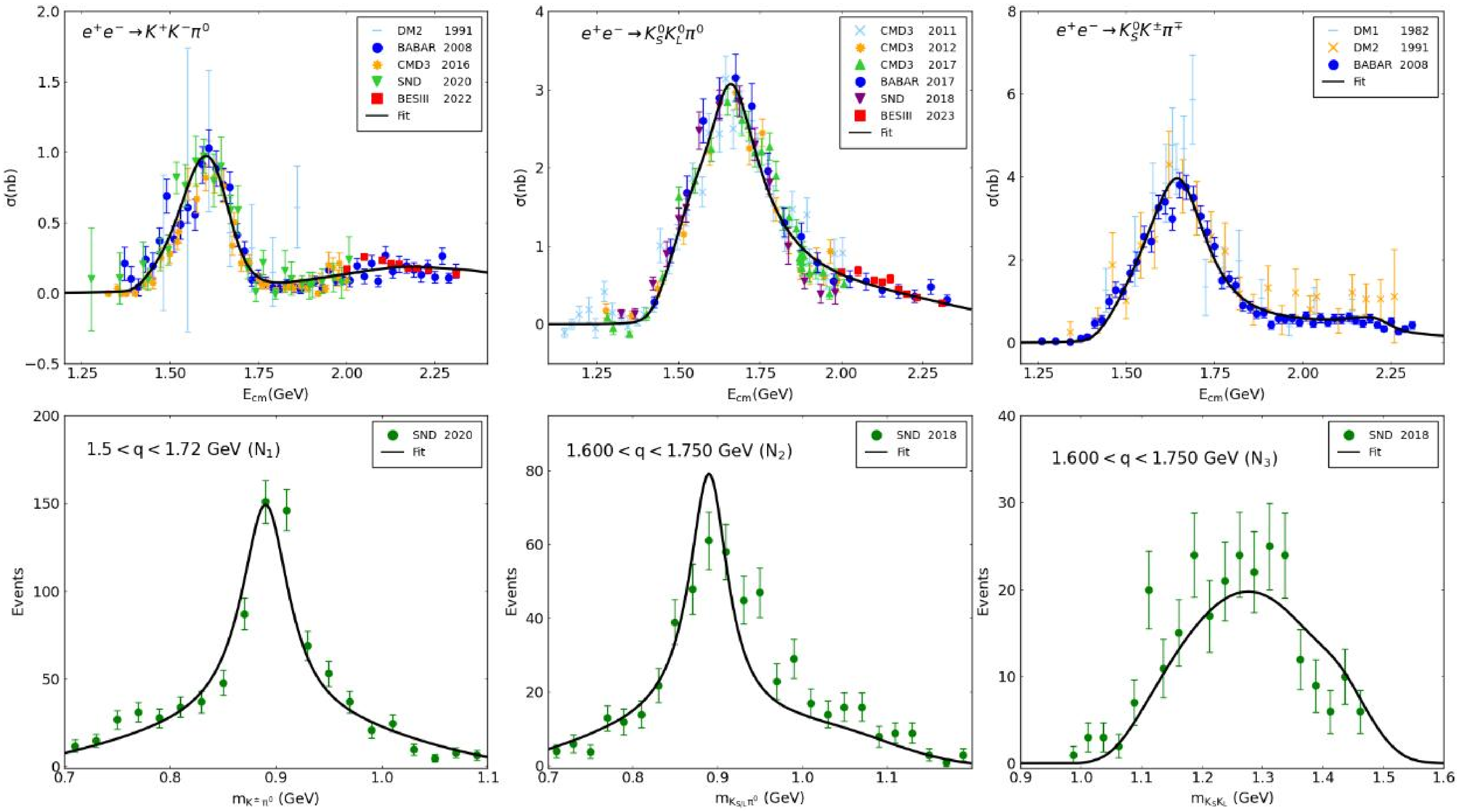
TABLE IV. Fitting results for the decay widths and PDG data are for 2022.

Width	Unit(GeV)	Fit	PDG [18]
$\Gamma(K^* \to K\pi)$	10^{-2}	5.047	4.845 ± 0.042
$\Gamma(K^{*0} \to K^0\gamma)$	10^{-4}	0.911	1.164 ± 0.100
$\Gamma(K^{*\pm} \to K^\pm\gamma)$	10^{-5}	3.043	5.040 ± 0.470
$\Gamma(a_2 \to \eta\pi)$	10^{-2}	2.274	1.552 ± 0.147
$\Gamma(a_2 \to \eta'\pi)$	10^{-3}	5.877	5.243 ± 0.890
$\Gamma(a_2 \to K\bar{K})$	10^{-4}	4.672	5.885 ± 1.002
$\Gamma(a_2^\pm \to \pi^\pm\gamma)$	10^{-4}	4.128	3.114 ± 0.323
$\Gamma(K_2^* \to K\pi)$	10^{-2}	4.453	5.057 ± 0.155
$\Gamma(K_2^* \to K^*\pi)$	10^{-2}	2.137	2.503 ± 0.159
$\Gamma(K_2^* \to \rho K)$	10^{-3}	7.539	8.818 ± 0.828
$\Gamma(K_2^* \to \omega K)$	10^{-3}	2.192	2.939 ± 0.813
$\Gamma(K_2^{*\pm} \to K^\pm\gamma)$	10^{-4}	2.073	2.400 ± 0.503

Fitting curve

Scattering cross section

Invariant mass spectrum



Calculate the contributions of $e^+e^- \to K\bar{K}\pi$ to $(g-2)_\mu$

Leading order (LO) HVP correction to
the muon anomalous magnetic moment:

$$a_\mu^{\text{HVP, LO}} = \frac{\alpha(0)^2}{3\pi^2}\int_{m_\pi^2}^{\infty}\frac{K(s)}{s}R(s)ds$$

where

$$K(s) = \frac{x^2}{2}(2-x^2) + \frac{(1+x^2)(1+x)^2}{x^2}\left(\ln(1+x) - x + \frac{x^2}{2}\right) + \frac{1+x}{1-x}x^2\ln x$$

$$x = \frac{1-\beta_\mu}{1+\beta_\mu}\ , \quad \beta_\mu = \sqrt{1-4m_\mu^2/s}$$

$$R(s) = \frac{3s}{4\pi\alpha(s)}\sigma(e^+e^- \to K\bar{K}\pi)$$

$$\begin{aligned}&\sigma(K\bar{K}\pi)\\ &= \sigma(K^+K^-\pi^0) + \sigma(K_SK_L\pi^0) + \sigma\left(K_SK^\pm\pi^\mp\right) + \sigma\left(K_LK^\pm\pi^\mp\right)\\ &\cong \sigma(K^+K^-\pi^0) + \sigma(K_SK_L\pi^0) + 2\sigma\left(K_SK^\pm\pi^\mp\right)\end{aligned}$$

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$a_\mu \times 10^{-10}$	Fit	KNT18	DHMZ19
$a_\mu^{K_S^0K^\pm\pi^\mp}[1.260 \le \sqrt{s} \le 1.937\ \text{GeV}]$	0.8938	0.88 ± 0.05	–
$a_\mu^{K^+K^-\pi^0}[1.370 \le \sqrt{s} \le 1.937\ \text{GeV}]$	0.1710	0.17 ± 0.01	–
$a_\mu^{K_S^0K_L^0\pi^0}[1.325 \le \sqrt{s} \le 1.937\ \text{GeV}]$	0.7675	0.79 ± 0.07	–
$a_\mu^{K\bar{K}\pi}[1.260 \le \sqrt{s} \le 1.937\ \text{GeV}]$	2.7270	2.71 ± 0.12	–
$a_\mu^{K\bar{K}\pi}[\text{threshold} \le \sqrt{s} \le 1.8\ \text{GeV}]$	2.4056	2.44 ± 0.11	2.45 ± 0.13
$a_\mu^{K\bar{K}\pi}[\text{threshold} \le \sqrt{s} \le 2\ \text{GeV}]$	2.8265	2.80 ± 0.12	–
$a_\mu^{K\bar{K}\pi}[\text{threshold} \le \sqrt{s} \le 2.3\ \text{GeV}]$	3.1783	–	–

4. Summary

(1) We use the effective field theory to calculate form factors and decay widths of $e^+e^- \to K\bar{K}\pi$.

(2) The scattering cross section, invariant mass spectrum and decay widths are fitted with experimental data and determining the unknown parameters.

(3) Finally, we calculate the contributions of $e^+e^- \to K\bar{K}\pi$ to $a_\mu^{\mathrm{HVP,\,LO}}$.

(4) Then, we will run error band and calculate the uncertainties.

Thank you for your attention!