Elastic (anti-)neutrino-nucleon scattering in covariant baryon chiral perturbation theory

Jin-Man Chen(陈金满)

In collaboration with: Ze-Rui Liang(梁泽锐)、De-Liang Yao(姚德良)

Hunan University

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Outline

- **Neutrino nucleon(νN) scattering** is an important way to understand the properties of neutrinos.

- There have been large scientific installations at home and abroad to research the properties of neutrinos such as:

  1. The Daya Bay Neutrino experiment in China [F. P. An, et al, PRL 108 (2012), 171803]
  2. The Jiangmen Neutrino experiment [F. An, et al, JPG 43 (2016) 3, 030401]
  3. The Super Kamiokande Neutrino experiment in Japan [Y. Fukuda, 10.1142/9789812704948_0002]

- The study of neutrinos interactions with matter can provide an important input for the accurate determination of neutrino oscillation parameters. See review [Z. Z. Xing and Z. h. Zhao, RPP 84 (2021) 6, 066201] .

Earlier studies

- Early νN scattering was mainly extracted from neutrino nuclear quasielastic scattering experiments [L.A. Ahrens, et al, PRD 35 (1987), 785]

- In 2010 and 2015, the MiniBooNE published $\nu(\bar{\nu})$ -induced neutral current(NC) differential cross section data, concentrated in energy intervals below 2 GeV. [MiniBooNE collaboration, PRD 82 (2010) 092005] [MiniBooNE collaboration, PRD 91 (2015) 012004]

- D. Pelevalov gave experimental data near the threshold [D. Perevalov, FERMILAB-THESIS-2009-47 (2009)] .
  - In the low energy region, the data of νN quasielastic scattering have a complicated varying behavior, which has not been explained in theory.

- G.T. Garvey et al. calculated the strange quark axial vector form factor of the nucleon by analyzing the ratio of proton-to-neutron neutrino-induced [G.T. Garvey et al, PRC C 48 (1993), 1919-1925] , and calculated the strange quark form factor and the axial vector dipole mass of the proton by refitting BNL experimental data on neutrino-proton elastic scattering. [G.T. Garvey et al, PRC 48 (1993), 761-765]

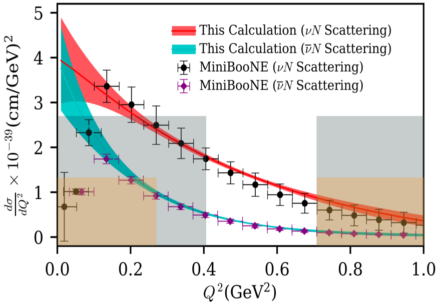
- R.S. Sufian et al. determined the NC axial vector form factor $G_A(Q^2)$, and calculated the differential cross section $\frac{d\sigma_{\nu N\to\nu N}}{dQ^2}$ for NC near $Q^2 = 0$ [R. S. Sufian et al, JHEP 01 (2020), 136] .

- Dipole parametrization
- In $Q^2 \lesssim 0.15\,GeV^2$, the differential cross section predictions of $\nu(\bar{\nu})N$ scattering starts to deviate from MiniBooNE result

  The possible reason:

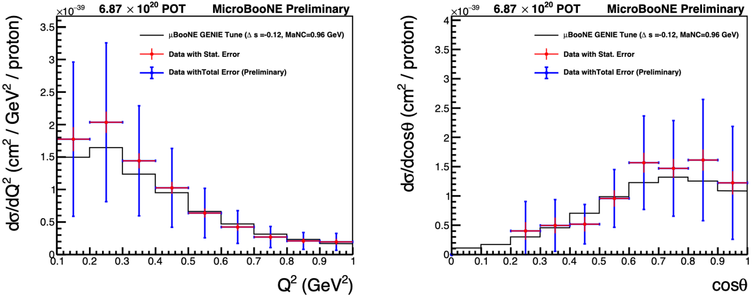
  1、Pauli blocking effect

  2、Nuclear shadowing



R. S. Sufian et al, JHEP 01 (2020), 136.

Figure 1: Final differential cross sections $d\sigma/dQ^2$ (left) and $d\sigma/d\cos\theta$ (right) with both statistical and systematic uncertainties.



[Ren Lu, JPS Conf.Proc. 37 (2022) 020309.]

- First measurement of muon neutrino neutral current elastic(NCE) scattering from protons ($0.1 < Q^2 < 1 GeV^2$)
- Plan to extract the strange quark contribution to the axial form factor

Our work

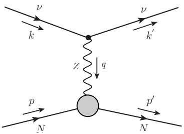
- Neutrino nucleon scattering is the most basic process to explore the interactions between neutrinos and matter.
- For neutrino nucleon scattering process, QCD can no longer be used directly for perturbation calculation at low energies due to the color confinement effect.
- Chiral perturbation theory (ChPT) is an effective field theory of QCD at low energy based on chiral symmetry at hadronic level.

Our aim

- In ChPT, calculate the 4 form factors(FFs) of the neutral current(NC) and charged current(CC)
- We will combine the 4 form factors and the differential cross sections to carry out numerical calculation and compare with experimental results
- Try to explain the discrepancy between theoretical and experimental data in the low energy region

Outline

- The process of neutrino nucleon elastic scattering:
  $\nu(k) + N(p) \to \nu(k') + N(p')$



Only consider single boson exchange !

- The differential cross section:

$$\frac{d\sigma}{dQ^2} = \frac{|\overline{\mathcal{M}}|^2}{64\pi m_N^2 E_\nu^2}$$

- The scattering amplitude

$$\mathcal{M} = i\frac{G_F}{\sqrt{2}} L_\mu H^\mu$$

$$L_\mu \equiv \bar{\nu}(k')\gamma_\mu(1-\gamma_5)\nu(k)$$

$$H^\mu \equiv \left\langle N(p') \middle| \mathcal{J}_Z^\mu \middle| N(p) \right\rangle$$

- The amplitude squared:

$$|\overline{\mathcal{M}}|^2 = \frac{G_F^2}{2} L_{\mu\nu} H^{\mu\nu}$$

- The leptonic and hadronic tensors:

$$L_{\mu\nu} = 8\left[k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k\cdot k' \pm i\epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta\right]$$

$$H_{\mu\nu} = \frac{1}{2} Tr\left[\left(\not{p} + m_N\right)\widetilde{F}_\mu\left(\not{p}' + m_N\right)F_\nu\right]$$

with $\widetilde{F}_\mu = \gamma_0 F_\mu^\dagger \gamma_0$
where

$$H^\mu = \left\langle N(p') \middle| \mathcal{J}^\mu \middle| N(p) \right\rangle = \bar{u}(\not{p}')F^\mu u(p)$$

The Form Factors and Differential Cross Section

- Ignore the second class currents

$$F^{\mu} = \gamma^{\mu} F_1 + \frac{i}{2m_N}\sigma^{\mu\nu} q_{\nu} F_2 - \gamma^{\mu}\gamma_5 G_A - \frac{q^{\mu}}{m_N}\gamma_5 G_P$$

where F_1, F_2 , G_A and G_P are the nucleon weak NC Dirac, Pauli, axial and induced pseudoscalar form factors.

- The differential cross section(NC)

$$\frac{d\sigma}{\mathrm{d}Q^2} = \frac{G_F^2 M^2}{8\pi E_{\nu}^2}\left[A(Q^2) \pm \frac{(s-u)}{M^2}B(Q^2) + \frac{(s-u)^2}{M^4}C(Q^2)\right]$$

with

$$A(Q^2) \equiv \frac{Q^2}{M^2}\left[G_A^2(1+\eta) + 4\eta F_1 F_2 - (1-\eta)\left(F_1^2 - \eta F_2^2\right)\right]$$

$$B(Q^2) \equiv \frac{Q^2}{M^2} G_A\left(F_1 + F_2\right)$$

$$C(Q^2) \equiv \frac{1}{4}\left[G_A^2 + F_1^2 + \eta F_2^2\right]$$

where $(s-u) = 4ME_{\nu} - Q^2$, $\eta = Q^2/4M^2$

- Next we are going to calculate the form factors

- Vector current is conserved:

$$F_i = \left(\frac{1}{2} - \sin^2\theta_W\right)\left[F_i^{EM,p} - F_i^{EM,n}\right]\tau_3$$
$$- \sin^2\theta_W\left[F_i^{EM,p} + F_i^{EM,n}\right] - \frac{1}{2}F_i^{(s)}, \quad i = 1, 2,$$

- The electric and magnetic Sachs form factors:

$$G_E(Q^2) = F_1^{EM}(Q^2) - \frac{Q^2}{4m_N^2}F_2^{EM}(Q^2),$$
$$G_M(Q^2) = F_1^{EM}(Q^2) + F_2^{EM}(Q^2),$$

- The weak axial vector form factor:

$$G_A(Q^2) = \frac{\tau_3}{2}G_A^v(Q^2) - \frac{1}{2}G_A^{(s)}(Q^2).$$

e.g. dipole parameterization:

$$G_A^v(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}, \quad G_A^{(s)}(Q^2) = \frac{\Delta s}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

- We want to calculate the form factors in ChPT.

- The effective Lagrangian

$$\mathcal{L}_{eff} = \sum_{i=1}^{2} \mathcal{L}_{\pi\pi}^{(2i)} + \sum_{j=1}^{3} \mathcal{L}_{\pi N}^{(j)}$$

- Meson field is contained in U :

$$U = u^2 = exp\left(i\frac{\Phi}{F}\right)$$

where $\Phi = \vec{\phi} \cdot \vec{\tau} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix}$

- πN Lagrangian

$$\begin{aligned}
\mathcal{L}_{\pi N}^{(1)} =& \bar{\Psi}_N \{ i\not{D} - m + \frac{1}{2} g \not{u} \gamma_5 \} \Psi_N \\
\mathcal{L}_{\pi N}^{(2)} =& \frac{1}{8m} c_6 \bar{\Psi}_N \Big\{ F_{\mu\nu}^{+} \sigma^{\mu\nu} \Big\} \Psi_N + \frac{1}{8m} c_7 \bar{\Psi}_N \Big\{ \langle F_{\mu\nu}^{+} \rangle \sigma^{\mu\nu} \Big\} \Psi_N + \cdots \\
\mathcal{L}_{\pi N}^{(3)} =& \frac{i}{2m} d_6 \bar{\Psi}_N [D^\mu, f_{\mu\nu}^{+}] D^\nu \Psi_N + h.c. + \frac{2i}{m} d_7 \bar{\Psi}_N \big(\partial^\mu \hat{v}_{\mu\nu}^{(s)} \big) D^\nu \Psi_N + h.c. \\
& + \frac{1}{2} d_{16} \gamma^\mu \gamma_5 \bar{\Psi}_N \langle \chi_+ \rangle u_\mu \Psi_N + \frac{i}{2} d_{18} \gamma^\mu \gamma_5 \bar{\Psi}_N [D_\mu, \chi_-] \Psi_N \\
& + \frac{1}{2} d_{22} \gamma^\mu \gamma_5 \bar{\Psi}_N [D^\nu, f_{\mu\nu}^{-}] \Psi_N + \cdots
\end{aligned}$$

- $\pi\pi$ Lagrangian

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+ \rangle$$
$$\mathcal{L}_{\pi\pi}^{(4)} = \frac{1}{8}\ell_4 \langle u_\mu u^\mu \rangle \langle \chi_+ \rangle + \frac{1}{16}(\ell_3 + \ell_4)\langle \chi_+ \rangle^2 + \cdots$$

- The chiral blocks

$$f_{\mu\nu}^{\pm} = u f_{\mu\nu}^{L} u^\dagger \pm u^\dagger f_{\mu\nu}^{R} u$$
$$f_{\mu\nu}^{L} = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu]$$
$$f_{\mu\nu}^{R} = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu]$$
$$\hat{v}_{\mu\nu}^{(s)} = \partial_\mu \hat{v}_\nu^{(s)} - \partial_\nu \hat{v}_\mu^{(s)}$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u$$
$$\chi = diag(M^2, M^2)$$

- The covariant derivative:

$$D_\mu = \partial_\mu + \Gamma_\mu - i\hat{v}_\mu^s$$

where

$$u_\mu = i\left\{u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\right\}$$
$$\Gamma_\mu = \frac{1}{2}\left\{u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - i\ell_\mu)u^\dagger\right\}$$

- Introduce the $\mathcal{Z}$ boson

$$\ell_\mu = \left(\frac{g_W}{2\cos\theta_W}\right)\left(-2cos^2\theta_W\right)\mathcal{Z}_\mu \frac{\tau_3}{2}$$
$$r_\mu = \left(\frac{g_W}{2\cos\theta_W}\right)\left(2sin^2\theta_W\right)\mathcal{Z}_\mu \frac{\tau_3}{2}$$
$$\hat{v}_\mu^{(s)} = \left(\frac{g_W}{2\cos\theta_W}\right)\left(2sin^2\theta_W\right)\mathcal{Z}_\mu \frac{\tau_0}{2}$$

- The hadronic part has a more complicated case due to the strong interactions inside the nucleon.

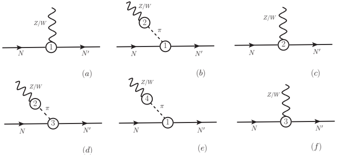


Figure 2: The tree-level diagrams of $\nu(\bar{\nu})N$ elastic scattering

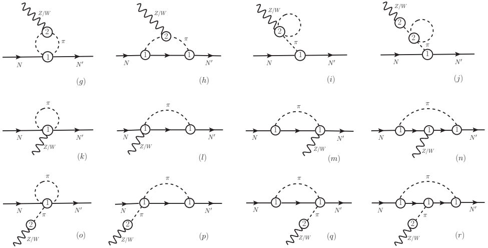


Figure 3: The one-loop diagrams of $\nu(\bar{\nu})N$ elastic scattering

- The pion exchange in hadron vertices mainly contributes to G_P, and G_P is entered into the CC scattering process.

Amplitudes of NC

- The amplitude: $\mathcal{M} = i\frac{G_F}{\sqrt{2}} L_\mu H^\mu$

$$
\begin{aligned}
H^\mu =& H^\mu_{tree} + H^\mu_{loop} = H^\mu_a + H^\mu_b + H^\mu_c + H^\mu_d + H^\mu_e + H^\mu_f + H^\mu_{loop} \\
H^\mu_a =& \bar{u}(p')\Big\{\big(\cos 2\theta_W \frac{\tau_3}{2} - \sin^2\theta_W\big)\gamma^\mu - g_A \frac{\tau_3}{2}\gamma^\mu\gamma_5\Big\}u(p) \\
H^\mu_b =& \bar{u}(p')\Big\{\frac{g_A m_N}{t - m_\pi^2}\tau_3 q^\mu \gamma_5\Big\}u(p) \\
H^\mu_c =& \bar{u}(p')\Big\{\big[c_6 \cos 2\theta_W \frac{\tau_3}{2} - (c_6 + 2c_7)\sin^2\theta_W\big]\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}\Big\}u(p) \\
H^\mu_d =& \bar{u}(p')\Big\{\frac{2m_N m_\pi^2(2d_{16} - d_{18})}{t - m_\pi^2}\tau_3 q^\mu\gamma_5\Big\}u(p) \\
H^\mu_e =& \bar{u}(p')\Big\{\frac{2g_A m_N m_\pi^2 \ell_4}{F^2(t - m_\pi^2)}\tau_3 q^\mu\gamma_5\Big\}u(p) \\
H^\mu_f =& \bar{u}(p')\Big\{\big(4d_7 t\sin^2\theta_W - d_6 t\cos 2\theta_W\tau_3\big)\gamma^\mu + \big(d_6 t\cos 2\theta_W\tau_3 - 4d_7 t\sin^2\theta_W\big)\frac{i\sigma^{\mu\nu}q_\nu}{2m_N} \\
& - \big(4d_{16}m_\pi^2 + d_{22}t\big)\frac{\tau_3}{2}\gamma^\mu\gamma_5 + d_{22}m_N\tau_3 q^\mu\gamma_5\Big\}u(p) \\
H^\mu_{loop} =& \cdots
\end{aligned}
$$

- The constants marked in red are the low energy constants(LECs).

Renormalization and Parameter Values of NC

- The ultraviolet(UV) divergences $\Longrightarrow$ $\overline{MS}-1$(or $\widetilde{MS}$) scheme.
  The UV divergences from loops are subtracted by the infinite parts in the bare paramaters g, c_6 ,c_7, d_6, d_7, d_{16}, d_{18}, d_{22}, ℓ_4, respectively.

- The power counting breaking(PCB) terms $\Longrightarrow$ the EOMS scheme.

	Power Counting	Relativistic	Analyticity
$\overline{MS}-1$	×	✓	✓
HB	✓	×	×
IR	✓	✓	×
EOMS	✓	✓	✓

- The extended-on-mass-shell (EOMS) ✓
  The PCB terms in loops can be properly canceled by the LECs in the chiral Lagrangians.

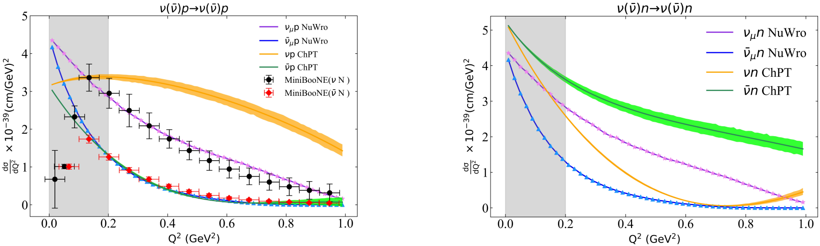
- The parameter values

m_N	m_π	m_μ	$\mathring{g}_A$	F_π	E_ν	$E_{\bar{\nu}}$	c_6
0.939	0.138	0.10565	1.13	0.0924	0.80	0.65	1.35 ± 0.04

c_7	d_6	d_7	d_{16}	d_{18}	d_{22}	l_3
-2.68 ± 0.08	0	-0.49	-0.83 ± 0.03	-0.20 ± 0.08	0.96 ± 0.03	0

[D. L. Yao et al, PRD 98 (2018) 7, 076004] [D. L. Yao et al, PLB 794 (2019), 109–113]

Preliminary Numerical Results of NC



- Preliminary numerical results of NC. Left: $\nu(\bar{\nu})p$ -NC ; Right: $\nu(\bar{\nu})n$ -NC

- The figures above show the preliminary numerical results in the NC, the values of those paratemers are in need of further improvement
- The shaded gray indidates the ChPT valid region
- The $d\sigma/dQ^2$ for $\bar{\nu}p$ scattering agree well with the MiniBooNE
- The $d\sigma/dQ^2$ for νp scattering is far different from the MiniBooNE
- Our preliminary results still don't agree with the MiniBooNE data in $Q^2 \lesssim 0.15 GeV^2$

Outline

The Relationship between NC and CC

- The Lagrangian of electromagnetic and weak interaction in SM

$$\mathcal{L}_{EW} = -e\mathcal{J}^{\mu}_{EM}A_{\mu} - \frac{g}{2\cos\theta_W}\mathcal{J}^{\mu}_{NC}\mathcal{Z}_{\mu} - \frac{g}{2\sqrt{2}}\mathcal{J}^{\mu}_{CC}W^{\dagger}_{\mu} + h.c.$$

where $\mathcal{J}^{\mu}_{EM}$、$\mathcal{J}^{\mu}_{NC}$ and $\mathcal{J}^{\mu}_{CC}$ are the electromagnetic (EM) current, weak neutral current (NC) and charged current (CC), respectively.

- In the quark sector (only consider u and d) :

$$\mathcal{J}^{\mu}_{EM} = \frac{1}{3}V^{\mu}_{0} + V^{\mu}_{3}$$

$$\mathcal{J}^{\mu}_{NC} = \left(1 - 2\sin^2\theta_W\right)V^{\mu}_{3} - A^{\mu}_{3} - \frac{2}{3}\sin^2\theta_W V^{\mu}_{0}$$

$$\mathcal{J}^{\mu}_{CC} = \cos\theta_C\left[(V^{\mu}_{1} - A^{\mu}_{1}) + i(V^{\mu}_{2} - A^{\mu}_{2})\right]$$

and $\mathcal{J}^{\mu}_{CC}W^{\dagger}_{\mu} + h.c. = \sum_{a=1}^{2}\mathcal{J}^{\mu}_{CC,a}W^{a}_{\mu}$
where

$$\mathcal{J}^{\mu}_{CC,a} = \sqrt{2}\cos\theta_C(V^{\mu}_{a} - A^{\mu}_{a})$$

- The vector and axial-vector currents

$$V^{\mu}_{a} \equiv \bar{q}\gamma^{\mu}\frac{\tau_a}{2}q\,, \quad a = 0,1,2,3$$

$$A^{\mu}_{a} \equiv \bar{q}\gamma^{\mu}\gamma_5\frac{\tau_a}{2}q\,, \quad a = 1,2,3$$

where $q = (q_u, q_d)^T$

- Isospin decomposition:

$$H_\mu = \langle N'(p')|\mathcal{J}_\mu|N(p)\rangle = \chi_f^\dagger \left[\frac{\tau^a}{2}\mathcal{H}_\mu^V + \frac{\tau^0}{2}\mathcal{H}_\mu^S\right]\chi_i \ , \quad a=1,2,3$$

- NC(a=3): $\mathcal{H}_\mu^V = (1-2\sin^2\theta_W)\mathcal{V}_\mu^V - \mathcal{A}_\mu^V \ , \quad \mathcal{H}_\mu^S = -2\sin^2\theta_W \mathcal{V}_\mu^S$
- CC(a=1,2): $\mathcal{H}_\mu^V = \cos\theta_C(\mathcal{V}_\mu^V - \mathcal{A}_\mu^V) \ , \quad \mathcal{H}_\mu^S = 0$

$$\begin{aligned}
V_\mu^a =& \bar{u}(p')\left[\gamma^\mu \mathcal{F}_1^V + \frac{i}{2m}\sigma^{\mu\nu}q_\nu \mathcal{F}_2^V\right]\frac{\tau^a}{2}u(p) = \mathcal{V}_\mu^V \chi_f^\dagger \frac{\tau^a}{2}\chi_i \ , \quad a=1,2,3 \\
A_\mu^a =& \bar{u}(p')\left[\gamma^\mu\gamma_5 \mathcal{G}_A^V + \frac{q^\mu}{m}\gamma_5 \mathcal{G}_P^V\right]\frac{\tau^a}{2}u(p) = \mathcal{A}_\mu^V \chi_f^\dagger \frac{\tau^a}{2}\chi_i \ , \quad a=1,2,3 \\
V_\mu^0 =& \bar{u}(p')\left[\gamma^\mu \mathcal{F}_1^S + \frac{i}{2m}\sigma^{\mu\nu}q_\nu \mathcal{F}_2^S\right]\frac{\tau^0}{2}u(p) = \mathcal{V}_\mu^S \chi_f^\dagger \frac{\tau^0}{2}\chi_i \\
\mathcal{J}_\mu^{NC} =& (1-2\sin^2\theta_W)\, V_\mu^3 - A_\mu^3 - 2\sin^2\theta_W V_\mu^0 \\
=& \chi_f^\dagger\left[\left(\cos 2\theta_W \mathcal{V}_\mu^V - \mathcal{A}_\mu^V\right)\frac{\tau^3}{2} - 2\sin^2\theta_W \mathcal{V}_\mu^0 \frac{\tau^0}{2}\right]\chi_i \\
\mathcal{J}_\mu^{CC} =& \cos\theta_C\left[(V_\mu^1 - A_\mu^1) + i(V_\mu^2 - A_\mu^2)\right] = \cos\theta_C\left(\mathcal{V}_\mu^V - \mathcal{A}_\mu^V\right)\chi_f^\dagger \frac{\tau^1 + i\tau^2}{2}\chi_i
\end{aligned}$$

There are 6 unkown form factors $(\mathcal{F}_{1,2}^V \ , \ \mathcal{F}_{1,2}^S \ , \ \mathcal{G}_{A,P})$

- The relationship between NC and CC:
  NC and CC have the FFs $(\mathcal{F}_i^V, \mathcal{G}_i^V)$ in common!

- Definition of FFs

$$H^{\mu} = \left\langle N(p') \middle| \mathcal{J}^{\mu} \middle| N(p) \right\rangle = \bar{u}(p')\Gamma^{\mu} u(p)$$

$$\Gamma^{\mu} = \gamma^{\mu} F_1 + \frac{i}{2m_N}\sigma^{\mu\nu} q_{\nu} F_2 - \gamma^{\mu}\gamma_5 G_A - \frac{q^{\mu}}{m_N}\gamma_5 G_P$$

- Physical amplitudes in NC:

$$H^{\mu}(\nu N \to \nu N') = \chi^{\dagger}_{N'} H^{\mu}_{NC} \chi_N$$

$$F_i\,(\nu N \to \nu N') = \chi^{\dagger}_{N'}\left[(1 - 2\sin^2\theta_W)\mathcal{F}_i^V \frac{\tau_3}{2} - 2\sin^2\theta_W \mathcal{F}_i^S \frac{\tau_0}{2}\right]\chi_N\ , \quad i = 1, 2$$

$$G_i\,(\nu N \to \nu N') = \chi^{\dagger}_{N'}\left[\mathcal{G}_i^V \frac{\tau_3}{2}\right]\chi_N\ , \quad i = A, P$$

- Physical amplitudes in CC:

$$H^{\mu}(\nu_\ell n \to \ell^- p) = \chi^{\dagger}_{p} H^{\mu}_{CC} \chi_n$$

$$F_i\,(\nu_\ell n \to \ell^- p) = \chi^{\dagger}_{p}\left[\cos\theta_C \cdot \mathcal{F}_i^V \frac{\tau_i}{2}\right]\chi_n\ , \quad i = 1, 2$$

$$G_i\,(\nu_\ell n \to \ell^- p) = \chi^{\dagger}_{p}\left[\cos\theta_C \cdot \mathcal{G}_i^V \frac{\tau_i}{2}\right]\chi_n\ , \quad i = A, P$$

- The differential cross section(CC)

$$\frac{d\sigma}{\mathrm{d}Q^2} = \frac{G_F^2 M^2 |V_{ud}|^2}{8\pi E_\nu^2}\left[A(Q^2) \pm \frac{(s-u)}{M^2} B(Q^2) + \frac{(s-u)^2}{M^4} C(Q^2)\right]$$

with

$$A(Q^2) \equiv \frac{(m^2+Q^2)}{M^2}\Bigg\{(1+\eta)\,G_A^2 + 4\eta F_1 F_2 - (1-\eta)\left(F_1^2 - \eta F_2^2\right)$$
$$-\frac{m^2}{4M^2}\left[\left(F_1+F_2\right)^2 + \left(G_A + 2G_P\right)^2 - \left(\frac{Q^2}{M^2}+4\right) G_P^2\right]\Bigg\}$$
$$B(Q^2) \equiv \frac{Q^2}{M^2} G_A\left(F_1+F_2\right)$$
$$C(Q^2) \equiv \frac{1}{4}\left[G_A^2 + F_1^2 + \eta F_2^2\right]$$

where $(s-u) = 4ME_\nu - Q^2 - m^2$, $\eta = Q^2/4M^2$, M and m are the nucleon and the lepton mass, respectively.

Preliminary Numrical Results of CC

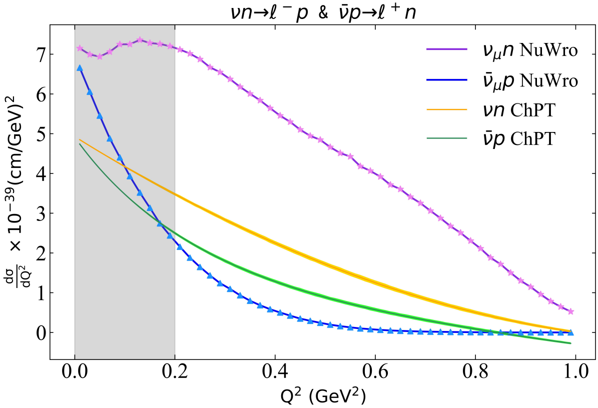


Figure 4: Preliminary numerical results of CC

- The shaded gray indidates the ChPT valid region
- Detailed results of CC are ongoing!

- Considered the Δ-resonance: Δ(1232), a state of spin-$\frac{3}{2}$
  The chiral effective Lagrangian

$$\mathcal{L}_{eff} = \sum_{i=1}^{2} \mathcal{L}_{\pi\pi}^{(2i)} + \sum_{j=1}^{3} \mathcal{L}_{\pi N}^{(j)} + \sum_{k=1}^{2} \left[\mathcal{L}_{\pi\Delta}^{(k)} + \mathcal{L}_{\pi N\Delta}^{(k)} \right]$$

- The effective Lagrangian for pion-Δ and pion-nucleon-Δ interactions

$$\mathcal{L}_{\pi\Delta}^{(1)} = \bar{\Psi}^{i,\mu} \xi_{ij}^{\frac{3}{2}} \left(i\gamma_{\mu\nu\alpha} D^{\alpha,jk} - m_\Delta \gamma_{\mu\nu} \delta^{jk} \right) \xi_{kl}^{\frac{3}{2}} \Psi^{l,\nu}$$
$$\mathcal{L}_{\pi\Delta}^{(2)} = \bar{\Psi}^{i,\mu} \xi_{ij}^{\frac{3}{2}} \left(a_1 \, Tr[\chi_+] \delta^{jk} g_{\mu\nu} \right) \xi_{kl}^{\frac{3}{2}} \Psi^{l,\nu}$$

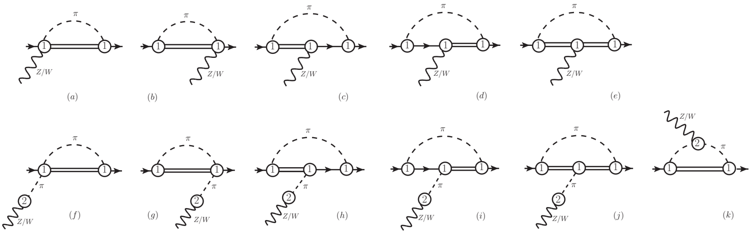
$$\mathcal{L}_{\pi N\Delta}^{(1)} = h_A \bar{\Psi}^{i,\alpha} \xi_{ij}^{\frac{3}{2}} \omega_\alpha^j \Psi_N + H.c.$$
$$\mathcal{L}_{\pi N\Delta}^{(2)} = \bar{\Psi}^{i,\alpha} \xi_{ij}^{\frac{3}{2}} \Big\{ - i\frac{b_1}{2} F_{\alpha\beta}^{+,j} \gamma_5 \gamma^\beta + ib_2 F_{\alpha\beta}^{-,j} \gamma^\beta + ib_3 \omega_{\alpha\beta}^j \gamma^\beta + i\frac{b_7}{m} F_{\alpha\beta}^{-,j} iD^\beta + i\frac{b_8}{m} \omega_{\alpha\beta}^j iD^\beta \Big\} \Psi_N + H.c.$$

- The covariant derivative: $\mathcal{D}_{\mu,ij} = (\partial_\mu + \Gamma_\mu)\delta_{ij} - i\epsilon_{ijk} Tr[\tau^k \Gamma_\mu]$

Interactions with Δ

- The projection operator: $\xi^{\frac{3}{2}}_{ij} = \delta_{ij} - \frac{1}{3}\tau_i\tau_j$
- The Dirac matrices: $\gamma_{\mu\nu\alpha} = \frac{1}{4}\left\{\left[\gamma_\mu, \gamma_\nu\right], \gamma_\alpha\right\}$, $\gamma_{\mu\nu} = \frac{1}{2}\left[\gamma_\mu, \gamma_\nu\right]$
- The chiral blocks:

$$F^{\pm,i}_{\mu\nu} = \frac{1}{2}Tr[\tau^i F^{\pm}_{\mu\nu}] \ , \ \omega^i_\mu = \frac{1}{2}Tr[\tau^i u_\mu] \ , \ \omega^i_{\mu\nu} = \frac{1}{2}Tr[\tau^i [D_\mu, u_\nu]]$$



- The calculation of the Feynman digrams with Δ-resonance is on-going .

Outline

Summary

- In ChPT, we calculated the scattering amplitudes up to $\mathcal{O}(p^3)$ in NC (tree diagrams and loop diagrams) and extracted 4 FFs.
- Obtained the relationship: NC and CC have the FFs $(\mathcal{F}_i^v, \mathcal{G}_i^v)$ in common
- We have obtained the preliminary numerical results of the NC and CC.

Next step

- We will continue to try to explain the complicated varying behavior of the MiniBooNE in the low energy region. If it's still unexplained, then Pauli blocking and nuclear shadowing effect have a great influence.
- Because our partial preliminary numerical results of NC are not consistent with MiniBooNE data, we will consider the Δ-resonance and the contribution of strange quark in later work.

Thank you very much for your patience!