Observational Signatures of Black Holes with Multiple Photon Spheres

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Motivation

- Recently, black holes with double photon spheres outside the event horizon have been found for dyonic black holes with a quasi-topological electromagnetic term and scalarized RN black holes.
- It is natural to check whether black holes with double photon spheres are physically viable, e.g., satisfying energy conditions.
- Due to strong gravitational lensing, photon spheres play a key role in imaging black holes. How does the existence of an extra photon sphere affect observational appearance of these black holes?

Photon Sphere

Schwarzschild Black Hole

- Consider a Schwarzschild black hole
$$ds^2 = -\left(1 - r_h/r\right) dt^2 + \left(1 - r_h/r\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$
where r_h is the horizon radius.
- The radial motion of null geodesics is described by an effective potential
$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\lambda}\frac{dx^\sigma}{d\lambda} = 0 \Rightarrow \left(\frac{dr}{d\lambda}\right)^2 + V_{\text{eff}}(r) = \frac{1}{b^2},$$
where the effective potential is
$$V_{\text{eff}}(r) = r^{-2}\left(1 - r_h/r\right).$$
- Unstable circular null geodesics at radius r_{ph} are determined by
$$V_{\text{eff}}\left(r_{\text{ph}}\right) = b_{\text{ph}}^{-2}, \quad V'_{\text{eff}}\left(r_{\text{ph}}\right) = 0, \quad V''_{\text{eff}}\left(r_{\text{ph}}\right) < 0 \Rightarrow r_{\text{ph}} = 3r_h/2.$$
- The unstable circular null geodesics constitute the photon sphere.

Photon Sphere
Schwarzschild Black Hole

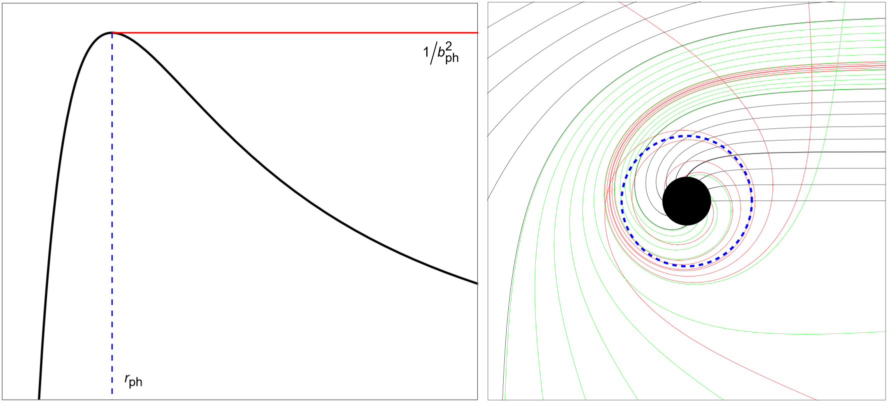


Figure: **Left**: The effective potential. **Right**: A selection of photon trajectories.

Photon Sphere

General Case

- For a general spherically-symmetric and static black hole solution

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{h(r)} + R(r)\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$

the Lagrangian governing null geodesics is

$$\mathcal{L} = \frac{1}{2}\left[-f(r)\,\dot{t}^2 + \frac{\dot{r}^2}{h(r)} + R(r)\left(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right)\right].$$

- The condition $\mathcal{L} = 0$ gives the radial component of the null geodesic equations,

$$\frac{f(r)}{h(r)}\frac{\dot{r}^2}{L^2} + V_{\text{eff}}(r) = \frac{1}{b^2},$$

where $b \equiv L/E$ is the impact parameter, and the effective potential is

$$V_{\text{eff}}(r) = \frac{f(r)}{R(r)}. \qquad (1)$$

- If the black hole is asymptotically flat, $V_{\text{eff}}(\infty) = 0 = V_{\text{eff}}(r_h)$ and $V_{\text{eff}}(r) > 0$ for $r > r_h$, which means that there must exist at least one photon sphere outside the event horizon, and the number of photon spheres is always one more than the number of anti-photon spheres.
- Using a topological argument, it was proved that a stationary, axisymmetric, asymptotically flat black hole spacetime admits at least one standard light ring outside the horizon.
- Until recently, asymptotically-flat black holes were supposed to possess a single photon sphere outside the event horizon, particularly in a physically reasonable model.

Quasi-topological Electromagnetism

- Consider the electromagnetic Lagrangian with a quasi-topological term in 4-dimensional space

$$\mathcal{L} = \sqrt{-g}\left\{-F^{\mu}_{\nu}F^{\nu}_{\mu}/4 - a\left[\left(F^{\mu}_{\nu}F^{\nu}_{\mu}\right)^2 - 2F^{\mu}_{\nu}F^{\nu}_{\rho}F^{\rho}_{\sigma}F^{\sigma}_{\mu}\right]\right\}.$$

- Depending on the black hole parameters, the asymptotically-flat dyonic black holes can have one or two photon spheres, which provides the "first such example in the literature." [1]
- The dyonic black holes with double photon spheres satisfy the dominant energy condition, but not the strong energy condition.

[1] *Sci. China Phys. Mech. Astron., 63:240411, 2020 [arXiv:1907.10876].*

Einstein-Maxwell-scalar Theory

- The scalar field is non-minimally coupled to the Maxwell field,

$$S \sim \int d^4x\sqrt{-g}\left[R - 2\left(\partial\phi\right)^2 - f\left(\phi\right)F_{\mu\nu}F^{\mu\nu}\right].$$

- The equations of motion are

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} &= 2T_{\mu\nu}, \\ \Box\phi &= f'\left(\phi\right)F_{\mu\nu}F^{\mu\nu}/4, \\ \partial_\mu\left(\sqrt{-g}f\left(\phi\right)F^{\mu\nu}\right) &= 0. \end{aligned}$$

- To accommodate RN black hole solutions with $\phi = 0$, the coupling function $f\left(\phi\right)$ must satisfy requires $f'\left(0\right) = 0$. In this talk, we focus on $f\left(\phi\right) = e^{\alpha\phi^2}$ with $\alpha > 0$.

- The linearized equation of motion for the scalar perturbation $\delta\phi$ in the scalar-free background is

$$\left(\Box - \mu_{\text{eff}}^2\right)\delta\phi = 0.$$

- If $\mu_{\text{eff}}^2 \equiv \alpha F_{\mu\nu}F^{\mu\nu}/2 < 0$, a tachyonic instability could drive the system away from the scalar-free solution.
- In RN black holes, the effective mass is $\mu_{\text{eff}}^2 = -\alpha Q^2/r^4 < 0$.

- The static, spherical black hole solutions:

$$ds^2 = -N(r)e^{-2\delta(r)}dt^2 + \frac{dr^2}{N(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right),$$
$$A_\mu dx^\mu = -\Phi(r)\, dt \text{ and } \phi = \phi(r).$$

- Imposing appropriate boundary conditions on the event horizon and the spatial infinity gives rise to a family of scalarized black hole solutions:

$$N(r_h) = 0, \delta(r_h) = \delta_0, \phi(r_h) = \phi_0, \Phi(r_h) = \Psi,$$
$$N(r\to\infty) = 1, \delta(\infty) = 0, \phi(\infty) = 0, \Phi(r\to\infty) = 0.$$

Scalarized RN Black Holes

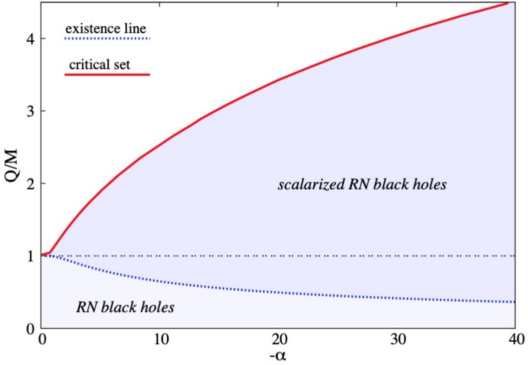


Figure: A scalarized BH bifurcates from a RN BH on the existence line, and the event horizon radius vanishes with the BH mass and charge remaining finite on the critical line[1].

1 *Phys. Rev. Lett., 121(10):101102, 2018 [arXiv:1806.05190].*

Energy Conditions of Scalarized RN Black Holes

- The energy-momentum tensor is

$$
\begin{aligned}
T_t^t &= -2N(r)\left[\phi'(r)\right]^2 - 2f(\phi)\,e^{2\delta(r)}\left[\Phi'(r)\right]^2 \equiv -\rho, \\
T_r^r &= 2N(r)\left[\phi'(r)\right]^2 - 2f(\phi)\,e^{2\delta(r)}\left[\Phi'(r)\right]^2 \equiv p_1, \\
T_\theta^\theta &= T_\varphi^\varphi = -2N(r)\left[\phi'(r)\right]^2 + 2f(\phi)\,e^{2\delta(r)}\left[\Phi'(r)\right]^2 \equiv p_2 \equiv p_3.
\end{aligned}
$$

- We find that

$$
\begin{gathered}
\rho + p_1 > 0, \rho + p_2 > 0, \rho + p_3 > 0 \Longrightarrow \text{NEC is respected}, \\
\rho + p_1 > 0, \rho + p_2 > 0, \rho + p_3 > 0 \text{ and } \rho > 0 \Longrightarrow \text{WEC is respected}, \\
\rho > |p_1|, \rho > |p_2|, \rho > |p_3| \text{ and } \rho > 0 \Longrightarrow \text{DEC is respected}, \\
\rho + p_1 + p_2 + p_3 > 0 \Longrightarrow \text{SEC is respected}.
\end{gathered}
$$

Double Photon Spheres

Scalarized RN Black Holes

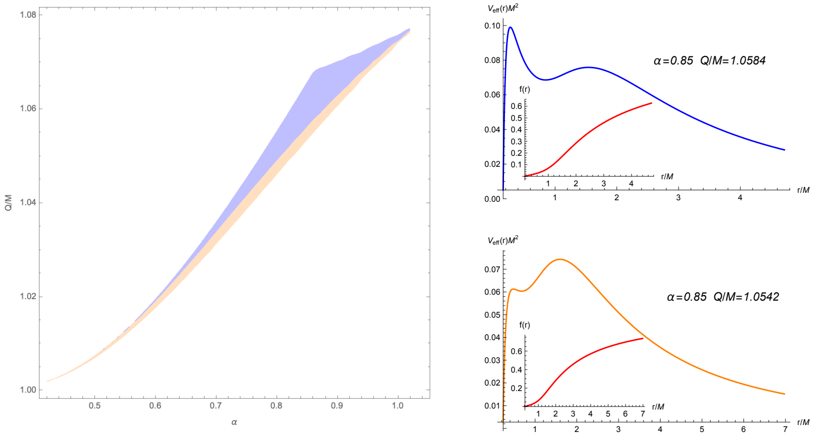
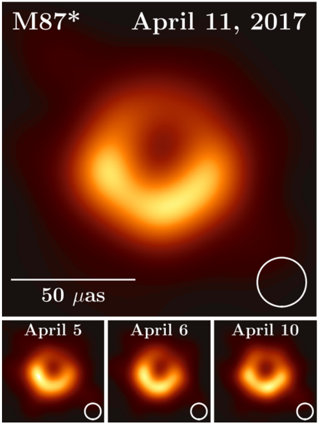


Figure: **Left**: In the blue/orange region, the effective potential at the inner photon sphere is higher/lower than that at the outer one. **Right**: It shows that $f(r)$ is monotonically increasing, which is a necessary condition for the validity of SEC.

Black Hole Image Illuminated by Accretion Disk



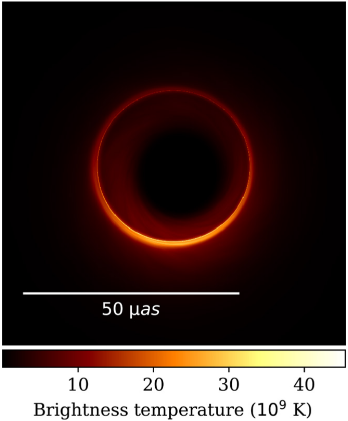


Figure: In 2019 April 10, the first black hole image of M87* was announced by Event Horizon Telescope, which is in good agreement with the predictions of the spacetime geometry of Kerr black holes.

Black Hole Image Illuminated by Accretion Disk

Single Photon Sphere

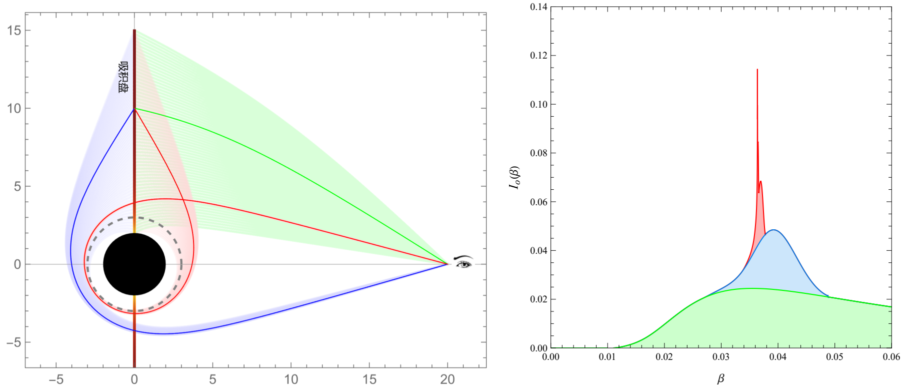
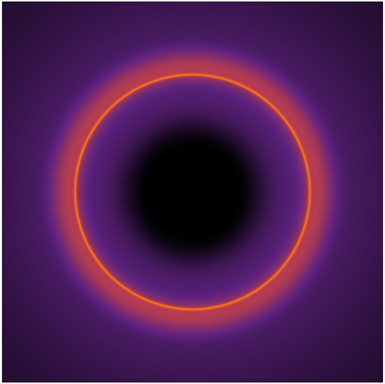


Figure: Schwarzschild black holes are illuminated by accretion disks extending to the horizon. The direct emission, lensing ring and photon ring correspond to $n = 0$ (green), $n = 1$ (blue), and $n \geq 2$ (red), respectively[1].

[1] *Phys.Rev.D 100 (2019) 2, 024018 [arXiv:1906.00873].*

Black Hole Image Illuminated by Accretion Disk

Single Photon Sphere



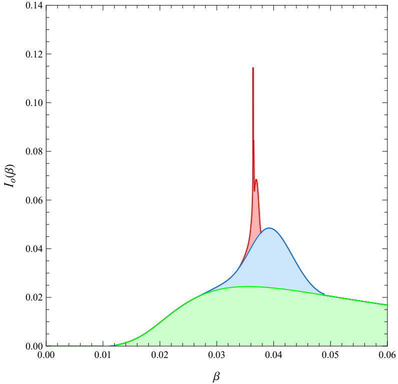


Figure: The lensing ring superimposed upon the direct emission produces a thin ring, while the photon ring makes negligible contributions to the total observed brightness due to its exponential narrowness.

Black Hole Image Illuminated by Accretion Disk

Double Photon Spheres

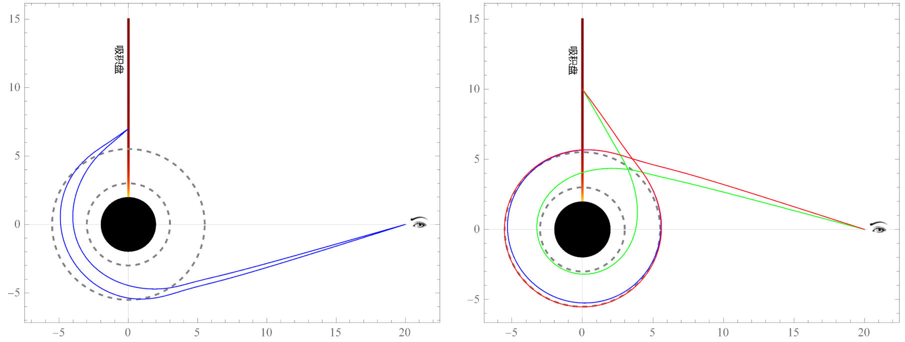
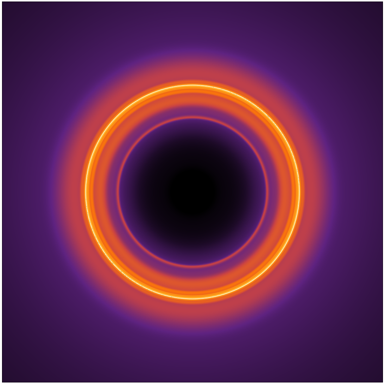


Figure: Two $n = 1$ light trajectories (**Left**) and Three $n = 2$ light trajectories (**Right**).

Black Hole Image Illuminated by Accretion Disk

Double Photon Spheres



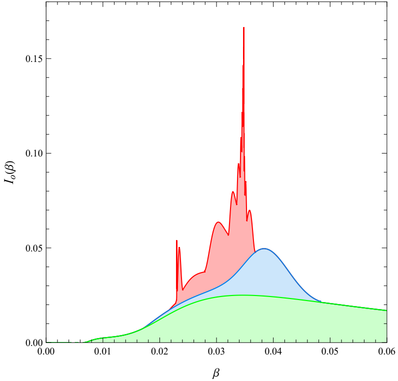


Figure: The photon ring becomes significantly wide, leading to a sizable contribution to the total flux. The internal structure of the photon ring can be observed.

Black Hole Image Illuminated by Accretion Disk

Scalarized RN Black Hole

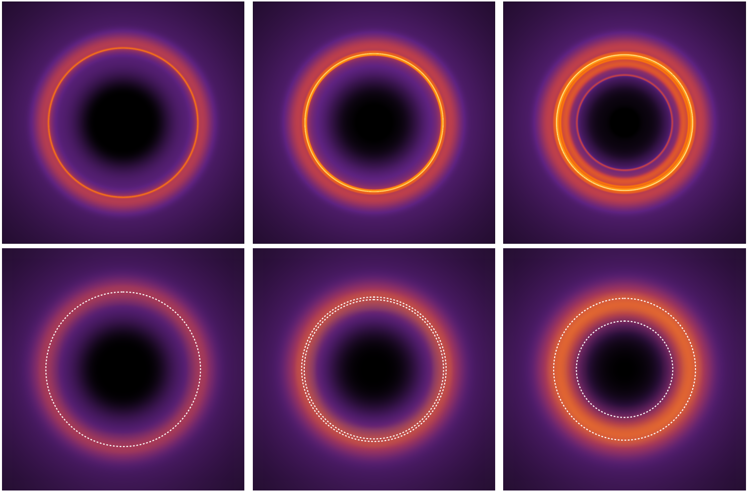


Figure: The high resolution images are blurred to correspond roughly to the EHT resolution.

Visibility of Black Hole Image

- A complex visibility is a Fourier component of the observed intensity $I_o(\mathbf{x})$,

$$V\left(\mathbf{u}\right)=\int I_o\left(\mathbf{x}\right)e^{-2\pi i\mathbf{u}\cdot\mathbf{x}}\mathrm{d}^2\mathbf{x},$$

  where $\mathbf{u}$ is the dimensionless baseline vector projected orthogonal to the z-axis of the spacetime and measured in units of the observation wavelength.

- It showed that the complex visibility of the photon ring can provide a pronounced, dominant signal on long baselines. These signatures offer the possibility of precise measurements of black hole mass and spin, as well as tests of general relativity, using only a sparse interferometric array[1].

[1] Sci.Adv. 6 (2020) 12, eaaz1310 *[arXiv:1907.04329]*.

Visibility of Black Hole Image

- **Single photon sphere**: If the photon ring is approximated as an infinitesimally thin ring with intensity $I_o(\beta) \sim \delta(\beta - \beta_{\text{ph}})$, the complex visibility is

$$V(u) \sim \frac{\cos(2\pi\beta_{\text{ph}} u - \pi/4)}{\sqrt{\beta_{\text{ph}} u}},$$

describing a weakly damped oscillation with a period $1/\beta_{\text{ph}}$.

- **Double photon sphere**: If the photon ring is approximated as two infinitesimally thin rings with intensity $I_o(\beta) \sim \delta(\beta - \beta_{\text{in}}) + \delta(\beta - \beta_{\text{out}})$, the complex visibility is,

$$V(u) \sim \frac{\cos(2\pi\beta_{\text{in}} u - \pi/4)}{\sqrt{\beta_{\text{in}} u}} + \frac{\cos(2\pi\beta_{\text{out}} u - \pi/4)}{\sqrt{\beta_{\text{out}} u}},$$

indicating a beat signal with a period $\Delta u = 1/(\beta_{\text{out}} - \beta_{\text{in}})$.

Visibility of Black Hole Image

Single Photon Sphere

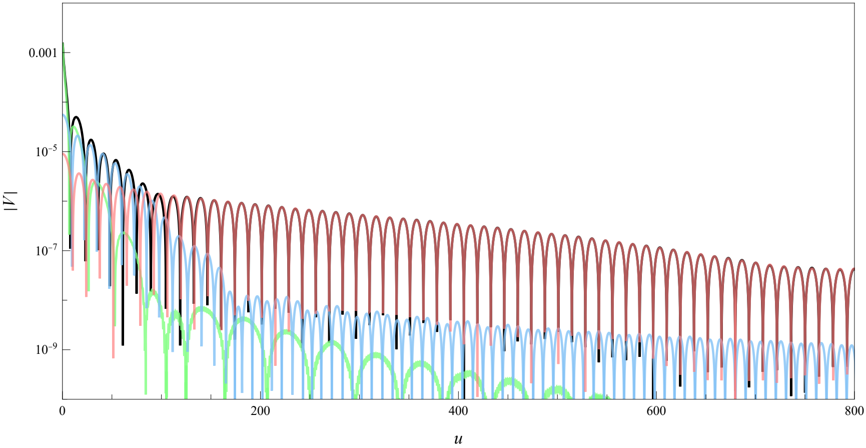


Figure: The visibility amplitudes $|V_n(u)|$ of the direct, lensing and photon rings are colored green, blue and red, respectively.

Visibility of Black Hole Image

Double Photon Spheres

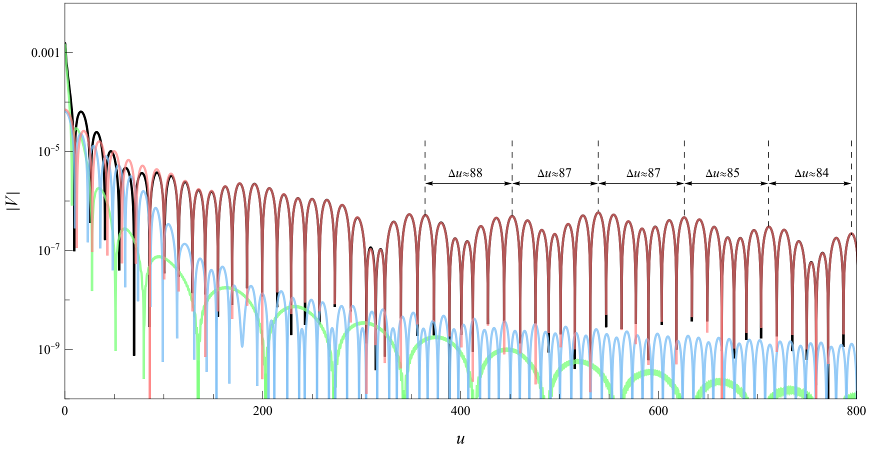


Figure: The period of the beat signal can be estimated as $\Delta u = 1/\left(\beta_{\text{out}} - \beta_{\text{in}}\right) = 83.333$.

Hot Spot

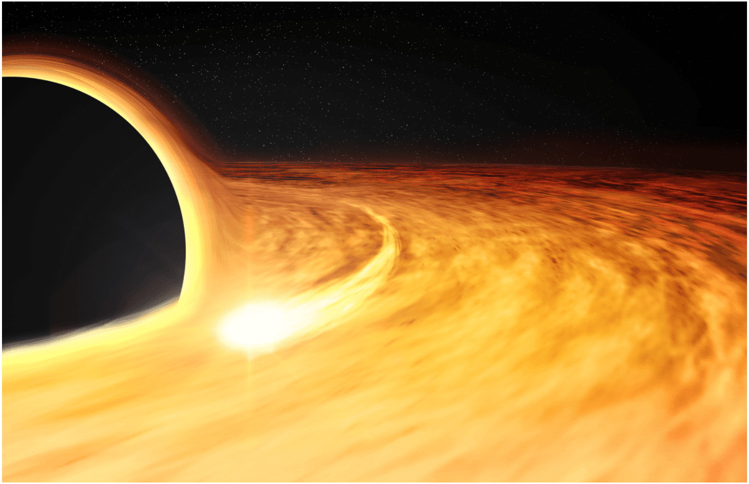


Figure: Bright spot: artist's impression showing a disc of hot gas orbiting a rapidly-spinning black hole. The elongated spot depicts an X-ray-bright region in the disc. (Courtesy: NASA/CXC/M Weiss)

Hot Spot

Toy Model

- The hot spot orbits counterclockwise along a circular geodesic at $r_e = r_{\mathrm{ISCO}}$.
- We employ a grid of 1000×1000 pixels for each snapshot and generate 500 snapshots.
- At a specific time t_k, each pixel within the image plane is assigned a specific intensity I_{klm}. which collectively forms the lensed image of the hot spot.
- Subsequently, the analysis focuses on the following image properties
  - Time integrated image:
$$\langle I \rangle_{lm} = \sum_k I_{klm}.$$
  - Total temporal flux:
$$F_k = \sum_{lm} \Delta\Omega I_{klm}.$$
  - Temporal magnitude:
$$m_k = -2.5 \lg \left[F_k / \min\left(F_k \right) \right].$$

Black Hole Image Illuminated by Hot Spot

Single Photon Sphere

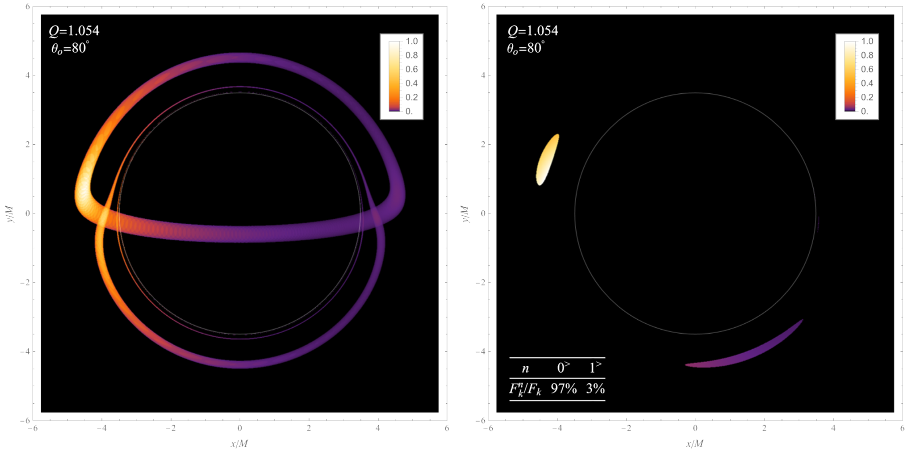


Figure: **Left**: Time integrated images for a complete orbit of the hot spot. **Right**: Snapshot when the temporal magnitude reaches the highest peak.

Black Hole Image Illuminated by Hot Spot

Single Photon Sphere

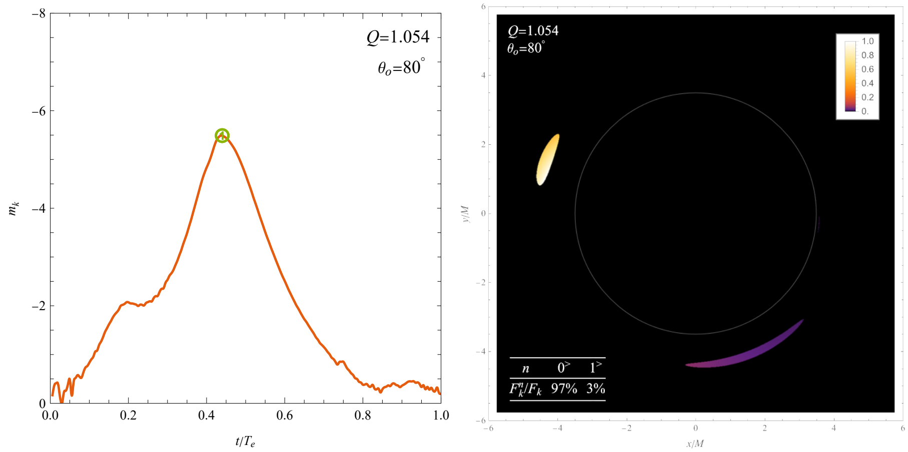
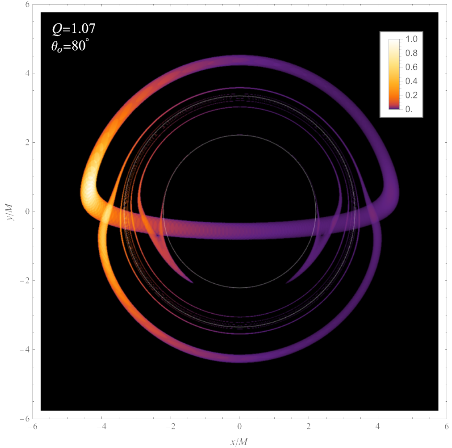
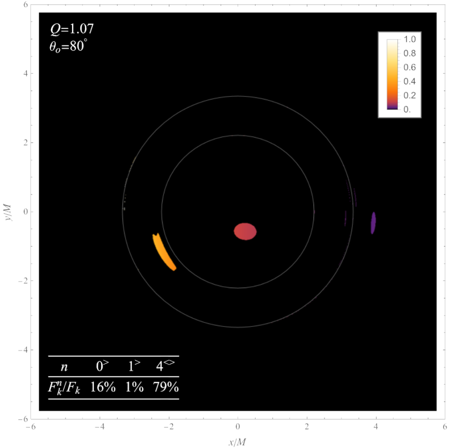


Figure: **Left**: Temporal magnitude m_k a function of t/T_e. **Right**: Snapshot when the temporal magnitude reaches the highest peak.

Black Hole Image Illuminated by Hot Spot

Double Photon Spheres





n	$0^{>}$	$1^{>}$	$4^{<>}$
F_k^n/F_k	16%	1%	79%

Figure: **Left**: Time integrated images for a complete orbit of the hot spot. **Right**: Snapshot when the temporal magnitude reaches the second highest peak.

Black Hole Image Illuminated by Hot Spot

Double Photon Spheres

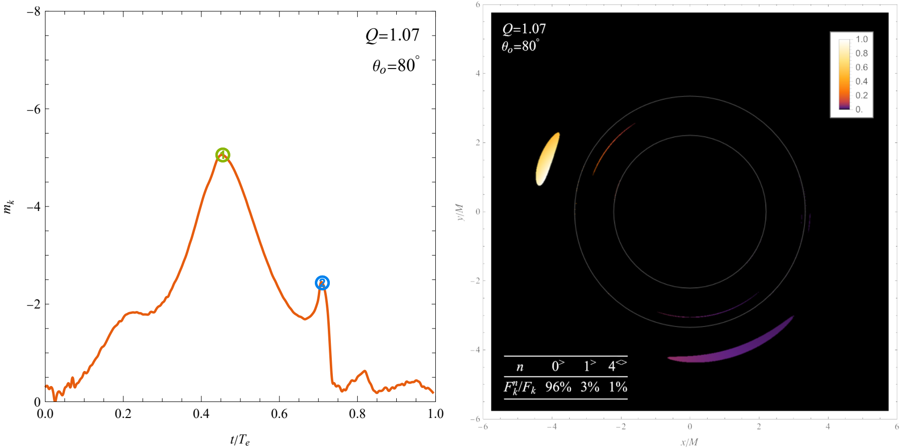


Figure: **Left**: Temporal magnitude m_k a function of t/T_e. **Right**: Snapshot when the temporal magnitude reaches the highest peak.

Black Hole Image Illuminated by Hot Spot

Double Photon Spheres

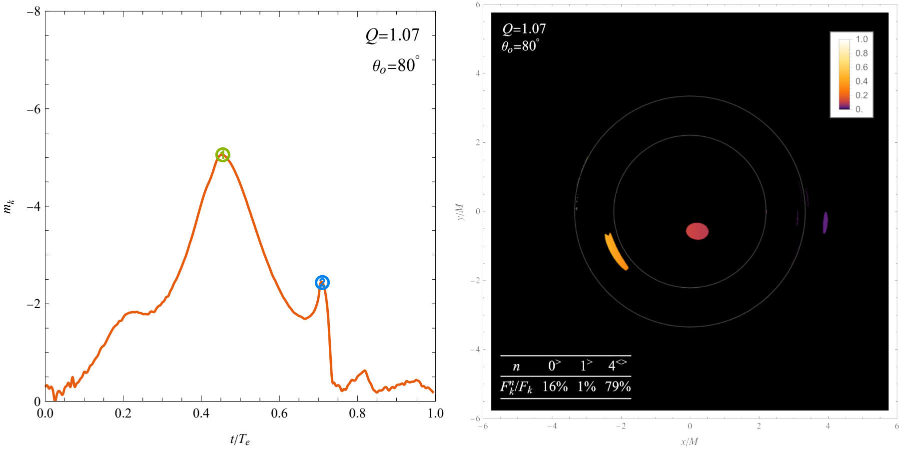


Figure: **Left**: Temporal magnitude m_k a function of t/T_e. **Right**: Snapshot when the temporal magnitude reaches the second highest peak.

Summary

- In Einstein's gravity, black holes with double photon spheres may not be common, but definitely not exotic.
- The existence of double photon spheres outside the event horizon leads to distinctive observational signatures.
    - Significantly increase the intensity flux of accretion disk images.
    - Result in a beat signal in the corresponding complex visibility.
    - Produce a more pronounced second-highest peak in temporal magnitudes of hot spot images.
- It will be of great interest if our analysis can be generalized to more astrophysically realistic models.