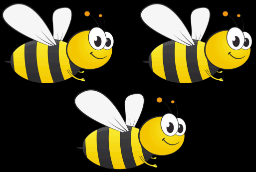
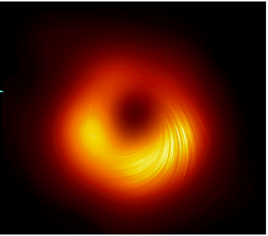


2023年黑洞图像学术研讨会

北京大学物理学院·高能物理研究中心

2023年12月



北京大学
PEKING UNIVERSITY

Bumblebee BHs in Light of EHT Images

Kavli Institute for Astronomy and Astrophysics

Speaker: Lijing Shao（邵立晶）

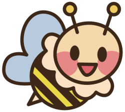
中国·北京

This talk is based on...

- **Collaborators:** Rui Xu, Dicong Liang, Zhan-Feng Mai, Zexin Hu
  - **BH:** Xu, Liang, Shao 2023, PRD [2209.02209]
  - **EHT:** Xu, Liang, Shao 2023, ApJ [2302.05671]
  - **EMRI:** Liang, Xu, Mai, Shao 2023, PRD [2212.09346]
  - **Pulsar:** Hu, Shao, Xu, Liang, Mai 2024





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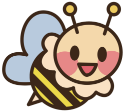
  - **EMRI:** Liang, Xu, Mai, Shao 2023, PRD [2212.09346]
  - **Pulsar:** Hu, Shao, Xu, Liang, Mai 2024
- Other work on bumblebee gravity ***not*** covered here
  - **GW:** Liang, Xu, Lu, Shao 2022, PRD [2207.14423]
  - **Thermodynamics:** Mai, Xu, Liang, Shao 2023, PRD [2304.08030]








Bumblebee gravity model

$$S = \int \sqrt{-g}\, \mathrm{d}^4 x \left[\frac{1}{2\kappa} \left(R + \xi B^{\mu} B^{\nu} R_{\mu\nu} \right) - \frac{1}{4} \alpha B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} \beta \left(D_{\mu} B_{\nu} \right) \left(D^{\mu} B^{\nu} \right) - V\left(B^{\mu} \right) \right] + S_{\mathrm{m}}$$

- As an illustrative example in **Standard Model Extension**

Nambu 1968; Will & Nordtvedt 1972; Hellings & Nordtvedt 1973; Kostelecký & Samuel 1989; Jacobson & Mattingly 2001
Kostelecký 2004 [hep-th/0312310]; Bluhm & Kostelecký 2005 [hep-th/0412320]; Bailey & Kostelecký 2006 [gr-qc/0603030]

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- As an illustrative example in **Standard Model Extension**
  - Gravity-bumblebee coupling $\Rightarrow$ correspondence with $s^{\mu\nu}$ & u

Nambu 1968; Will & Nordtvedt 1972; Hellings & Nordtvedt 1973; Kostelecký & Samuel 1989; Jacobson & Mattingly 2001

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Bumblebee gravity model

$$S=\int\sqrt{-g}\,\mathrm{d}^4x\left[\frac{1}{2\kappa}\left(R+\xi B^{\mu}B^{\nu}R_{\mu\nu}\right)-\frac{1}{4}\alpha B^{\mu\nu}B_{\mu\nu}\right.$$
$$\left.+\frac{1}{2}\beta\left(D_{\mu}B_{\nu}\right)\left(D^{\mu}B^{\nu}\right)-V\left(B^{\mu}\right)\right]+S_{\mathrm{m}}$$

- As an illustrative example in **Standard Model Extension**
  - Gravity-bumblebee coupling $\Rightarrow$ correspondence with $s^{\mu\nu}$ & u
  - Stückelberg ghost in Minkowski spacetime and $V=0$

Nambu 1968; Will & Nordtvedt 1972; Hellings & Nordtvedt 1973; Kostelecký & Samuel 1989; Jacobson & Mattingly 2001

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$$S = \int \sqrt{-g}\, \mathrm{d}^4 x \left[\frac{1}{2\kappa} \left(R + \xi B^\mu B^\nu R_{\mu\nu} \right) - \frac{1}{4} \alpha B^{\mu\nu} B_{\mu\nu} + \frac{1}{2} \beta \left(D_\mu B_\nu \right) \left(D^\mu B^\nu \right) - V\left(B^\mu \right) \right] + S_\mathrm{m}$$

- As an illustrative example in **Standard Model Extension**
  - Gravity-bumblebee coupling $\Rightarrow$ correspondence with $s^{\mu\nu}$ & u
  - Stückelberg ghost in Minkowski spacetime and $V = 0$
  - "Lorentz-violating" potential $V(B^\mu) = V(B^\mu B_\mu \pm b^2)$

Nambu 1968; Will & Nordtvedt 1972; Hellings & Nordtvedt 1973; Kostelecký & Samuel 1989; Jacobson & Mattingly 2001
Kostelecký 2004 [hep-th/0312310]; Bluhm & Kostelecký 2005 [hep-th/0412320]; Bailey & Kostelecký 2006 [gr-qc/0603030]

Bumblebee gravity model

Field Equations

$$G_{\mu\nu} = \kappa (T_{\mathrm{m}})_{\mu\nu} + \kappa (T_B)_{\mu\nu}$$

$$D^\mu B_{\mu\nu} - 2B_\nu \frac{\mathrm{d}V}{\mathrm{d}\left(B^\lambda B_\lambda\right)} + \frac{\xi}{\kappa} B^\mu R_{\mu\nu} = 0$$

where

$$\begin{aligned}(T_B)_{\mu\nu} = \frac{\xi}{2\kappa} \Big[& g_{\mu\nu} B^\alpha B^\beta R_{\alpha\beta} - 2B_\mu B_\lambda R_\nu{}^\lambda - 2B_\nu B_\lambda R_\mu{}^\lambda - \Box_g \left(B_\mu B_\nu\right) \\ & - g_{\mu\nu} D_\alpha D_\beta \left(B^\alpha B^\beta\right) + D_\kappa D_\mu \left(B^\kappa B_\nu\right) + D_\kappa D_\nu \left(B_\mu B^\kappa\right) \Big] \\ & + B_{\mu\lambda} B_\nu{}^\lambda - g_{\mu\nu} \left(\frac{1}{4} B^{\alpha\beta} B_{\alpha\beta} + V\right) + 2B_\mu B_\nu \frac{\mathrm{d}V}{\mathrm{d}\left(B^\lambda B_\lambda\right)}\end{aligned}$$

Kostelecký 2004 [hep-th/0312310]; Bailey & Kostelecký 2006 [gr-qc/0603030]; Xu, Liang, Shao 2023 [2209.02209]

Simple bumblebee BHs

$$S = \int \sqrt{-g}\, \mathrm{d}^4 x \left[\frac{1}{2\kappa} \left(R + \xi B^{\mu} B^{\nu} R_{\mu\nu} \right) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - V\left(B^{\mu} \right) \right] + S_{\mathrm{m}}$$

Casana, Cavalcante, Poulis, Santos 2018 [1711.02273]

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- Casana *et al.* (2018) found an exact Schwarzschild-like solution in bumblebee gravity model

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + (1+\ell)\left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

where $\ell = \xi b^2$ with b^μ having only a nonvanishing *radial* component

Casana, Cavalcante, Poulis, Santos 2018 [1711.02273]

Extending bumblebee BHs

$$S = \int \sqrt{-g}\, \mathrm{d}^4 x \left[\frac{1}{2\kappa} \left(R + \xi B^{\mu} B^{\nu} R_{\mu\nu} \right) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - V\left(B^{\mu}\right) \right] + S_{\mathrm{m}}$$

- **Ansatz**

$$\mathrm{d}s^2 = -e^{2\nu}\mathrm{d}t^2 + e^{2\mu}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

$$b_\lambda = (b_t, b_r, 0, 0)$$

where μ, ν, b_t, and b_r are functions of r

Xu, Liang, Shao 2023 [2209.02209]

Extended analytical solution

$$\nu = \frac{1}{2}\ln\left(1 - \frac{2M}{r}\right), \qquad \mu = \mu_0 - \frac{1}{2}\ln\left(1 - \frac{2M}{r}\right), \qquad b_t = \lambda_0 + \frac{\lambda_1}{r}$$

$$b_r^2 = e^{2\mu_0}\left[\frac{1}{\xi}\frac{\left(e^{2\mu_0} - 1\right)r}{r - 2M} - \frac{\kappa\lambda_1^2}{3\xi M(r - 2M)} + \frac{\lambda_1^2(2r - M) + 6\lambda_0\lambda_1 Mr + 6\lambda_0^2 M^2 r}{3M(r - 2M)^2}\right]$$

- **Four integral constants:** $M, \mu_0, \lambda_0, \lambda_1$

Xu 2023 [2301.12666]; Xu, Liang, Shao 2023 [2209.02209]

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- $\lambda_0 = \lambda_1 = 0$ $\Longrightarrow$ Casana et al. 2018 [1711.02273]

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- $\lambda_0 = \lambda_1 = 0$ $\Rightarrow$ Casana et al. 2018 [1711.02273]
- $\mu_0 = 0$: Schwarzschild with a nontrivial b_λ $\Rightarrow$ Fan 2018 [1709.04392]

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- $\lambda_0 = \lambda_1 = 0$ $\Rightarrow$ Casana et al. 2018 [1711.02273]
- $\mu_0 = 0$: Schwarzschild with a nontrivial b_λ $\Rightarrow$ Fan 2018 [1709.04392]
- $\mu_0 = M = 0$: Minkowski metric, with a nontrivial b_λ

Xu 2023 [2301.12666]; Xu, Liang, Shao 2023 [2209.02209]

Numerical bumblebee BHs

$$\nu = \sum_{n=1}^{\infty} \frac{\nu_n}{r^n}, \qquad \mu = \sum_{n=0}^{\infty} \frac{\mu_n}{r^n}, \qquad b_t = \sum_{n=0}^{\infty} \frac{\lambda_n}{r^n}$$

- Field equations give recurrence relations of ν_n, μ_n, and λ_n

Xu, Liang, Shao 2023 [2209.02209]

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- **Two families of horizon solutions** (g_{rr} diverging at r_h)

Xu, Liang, Shao 2023 [2209.02209]

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  1. a vanishing radial component of the vector field ($b_r = 0$)

     $\Rightarrow$ **3** free parameters $\{\mu_1, \lambda_0, \lambda_1\}$

Xu, Liang, Shao 2023 [2209.02209]

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- Field equations give recurrence relations of ν_n, μ_n, and λ_n
- **Two families of horizon solutions** (g_{rr} diverging at r_h)
  1. a vanishing radial component of the vector field ($b_r = 0$)
     $\Rightarrow$ **3** free parameters $\{\mu_1, \lambda_0, \lambda_1\}$
  2. a vanishing radial component of the Ricci tensor ($R_{rr} = 0$)
     $\Rightarrow$ **5** free parameters $\{\mu_0, \mu_1, \mu_2, \lambda_0, \lambda_1\}$

Xu, Liang, Shao 2023 [2209.02209]

Conserved quantities

Komar Mass

Conserved current $J_M^\mu \equiv K_\nu R^{\mu\nu} = D_\nu D^\mu K^\nu$, with $K^\mu = (1, 0, 0, 0)$

$$M_{\rm K} \equiv -\frac{1}{4\pi} \int \mathrm{d}^3 x \sqrt{-g}\, J_M^t = \mathrm{e}^{-\mu_0} \mu_1$$

Xu, Liang, Shao 2023 [2209.02209]

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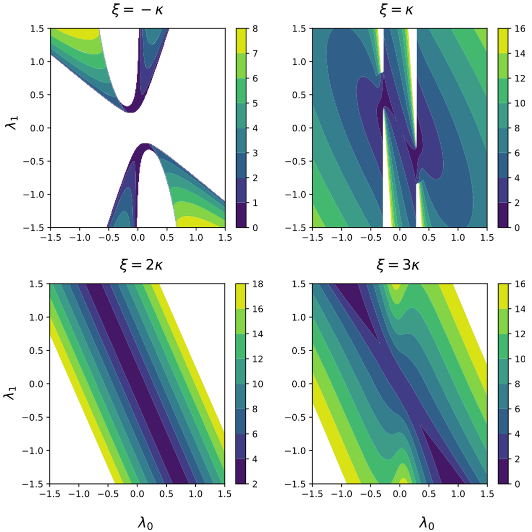
Bumblebee Charge

Conserved current $J_Q^\mu \equiv \xi b_\nu R^{\mu\nu}/\kappa = -D_\nu b^{\nu\mu}$

$$Q \equiv -\frac{1}{4\pi} \sqrt{\frac{\kappa}{2}} \int \mathrm{d}^3 x \sqrt{-g}\, J_Q^t = \sqrt{\frac{\kappa}{2}}\, \mathrm{e}^{-\mu_0} \lambda_1$$

Xu, Liang, Shao 2023 [2209.02209]

Horizon r_h in family $b_r = 0$



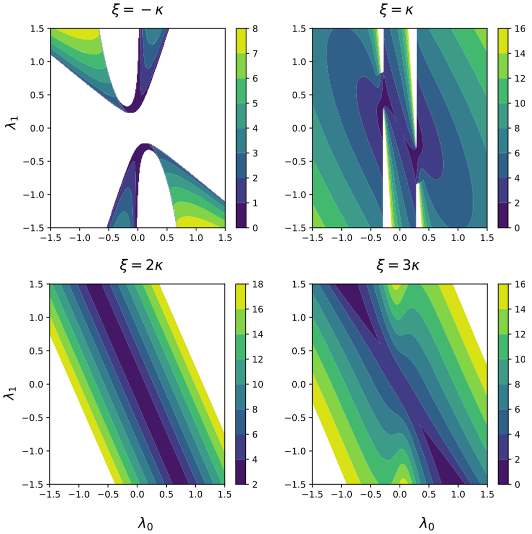
Family $b_r = 0$

Parameters: $\mu_1, \lambda_0, \lambda_1$

r_h in unit of μ_1 = ADM mass = Komar mass in family $b_r = 0$

Xu, Liang, Shao 2023 [2209.02209]

Horizon r_h in family $b_r = 0$



Family $b_r = 0$

Parameters: $\mu_1, \lambda_0, \lambda_1$

When $\xi = 2\kappa$,

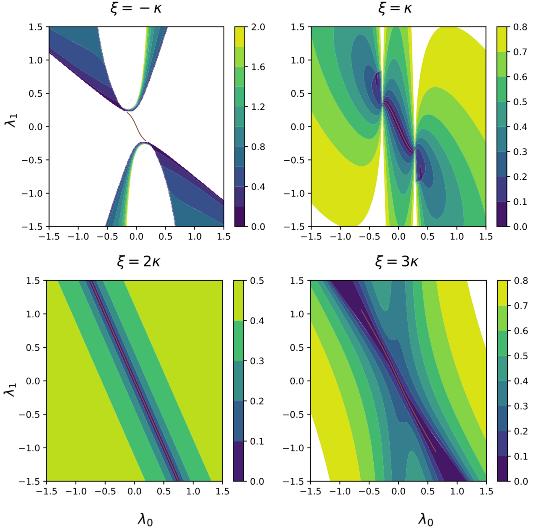
$\lambda_1 = -2\lambda_0$ gives

Schwarzschild metric

r_h in unit of μ_1 = ADM mass = Komar mass in family $b_r = 0$

Xu, Liang, Shao 2023 [2209.02209]

BHs in family $b_r = 0$



Family $b_r = 0$

Parameters: $\mu_1, \lambda_0, \lambda_1$

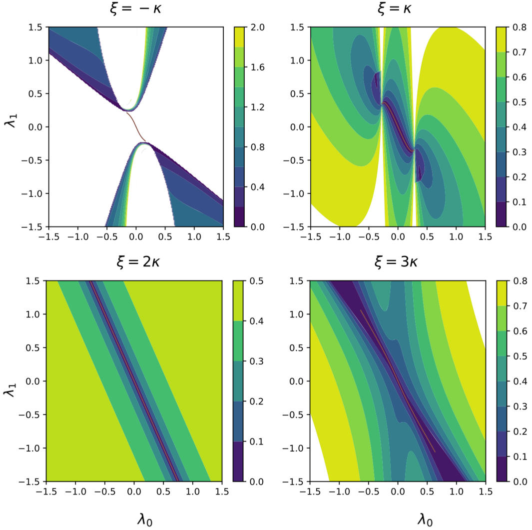
Demanding $g_{tt}\big|_{r_h} = 0$ gives BH solutions

(brown curves in λ_1-λ_0 plane)

r_h in unit of μ_1 = ADM mass = Komar mass in family $b_r = 0$

Xu, Liang, Shao 2023 [2209.02209]

BHs in family $b_r = 0$



Family $b_r = 0$

Parameters: $\mu_1, \lambda_0, \lambda_1$

Demanding $g_{tt}\big|_{r_h} = 0$ gives BH solutions

(brown curves in λ_1-λ_0 plane)

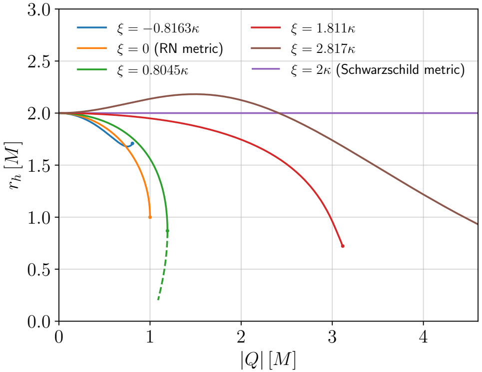
Mass $M_{\mathrm{K}} = \mu_1$

Charge $Q = \sqrt{\kappa/2}\lambda_1$

r_h in unit of μ_1 = ADM mass = Komar mass in family $b_r = 0$

Xu, Liang, Shao 2023 [2209.02209]

BHs in family $b_r = 0$



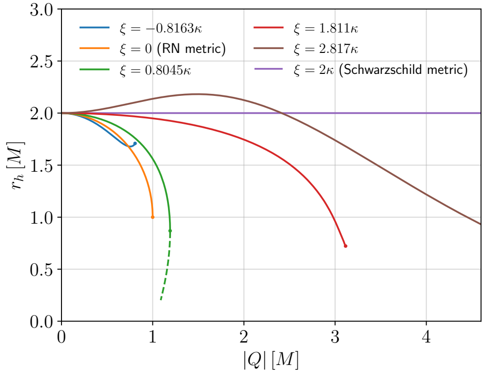
$\xi < 0$: $Q^{(\max)} < M$

$\xi = 0$: Reissner-Nordström

$\xi > 0$: $Q^{(\max)} > M$

Xu, Liang, Shao 2023, PRD [2209.02209]; Xu, Liang, Shao 2023, ApJ [2302.05671]

BHs in family $b_r = 0$



$\xi < 0$: $Q^{(\max)} < M$

$\xi = 0$: Reissner-Nordström

$\xi > 0$: $Q^{(\max)} > M$

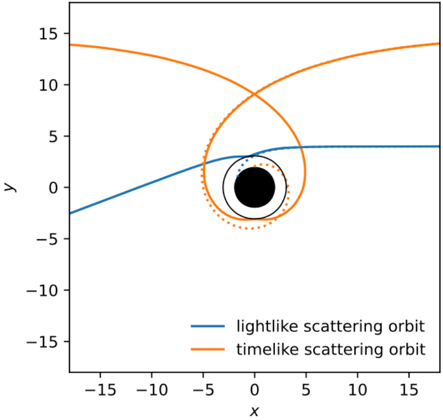
$\xi < 2\kappa$: $r_h^{(\max)} = 2M$

$\xi = 2\kappa$: stealth Schwarzschild

$\xi > 2\kappa$: $r_h^{(\max)} > 2M$

Xu, Liang, Shao 2023, PRD [2209.02209]; Xu, Liang, Shao 2023, ApJ [2302.05671]

“Compact Hills” in family $b_r = 0$

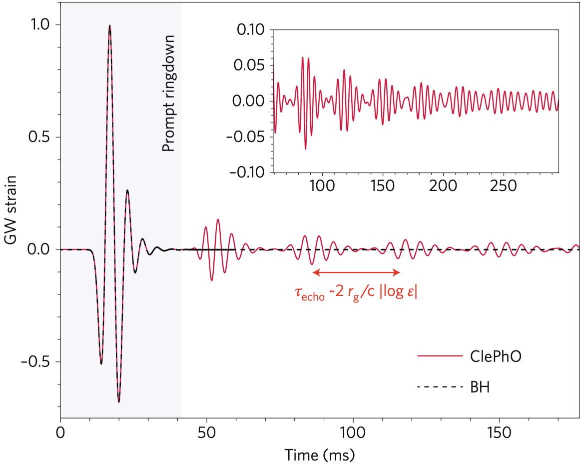


Solutions that $g_{rr} = \infty$ but $g_{tt}\big|_{r_h} \neq 0$ have bouncing geodesics

- **black circle:** r_h of CHs
- **black disk:** Schwarzschild radius
- **solid:** geodesic of CHs
- **dashed:** geodesic of BHs

Xu, Liang, Shao 2023, PRD [2209.02209]

Gravitational-wave echo?



Cardoso & Pani 2017, Nat. Astron. [1709.01525]

Perihelion Advance

Family $b_r = 0$

$$\delta\varphi \approx \pi\mu_1 \left(3 - \frac{2\xi\lambda_0^2 + 2\xi\lambda_0\lambda_1/\mu_1 + (\xi-\kappa)\lambda_1^2/\mu_1^2}{4\xi\left(1-\xi/2\kappa\right)\lambda_0^2 - 4} \right) \frac{r_{\rm min} + r_{\rm max}}{r_{\rm min} r_{\rm max}}$$

depending on λ_0 and λ_1 $\Leftarrow$ note: $b_t = \sum_{n=0}^{\infty} \frac{\lambda_n}{r^n}$

Xu, Liang, Shao 2023, PRD [2209.02209]

Perihelion Advance

Family $b_r = 0$

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depending on λ_0 and λ_1 $\Leftarrow$ note: $b_t = \sum_{n=0}^{\infty} \frac{\lambda_n}{r^n}$

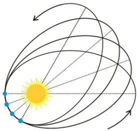
Family $R_{rr} = 0$

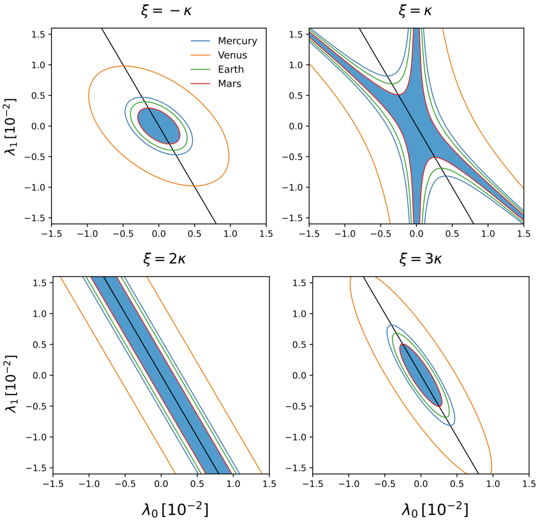
$$\delta\varphi \approx 2\pi\left(e^{\mu_0} - 1\right) + e^{\mu_0}\left(7 + 2\frac{\mu_2}{\mu_1^2}\right) \frac{\pi\mu_1\left(r_{\rm min} + r_{\rm max}\right)}{3 r_{\rm min} r_{\rm max}}$$

depending on μ_0 and μ_2 $\Leftarrow$ note: $\mu = \sum_{n=0}^{\infty} \frac{\mu_n}{r^n}$

Xu, Liang, Shao 2023, PRD [2209.02209]

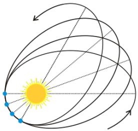
Perihelion Advance: family $b_r = 0$

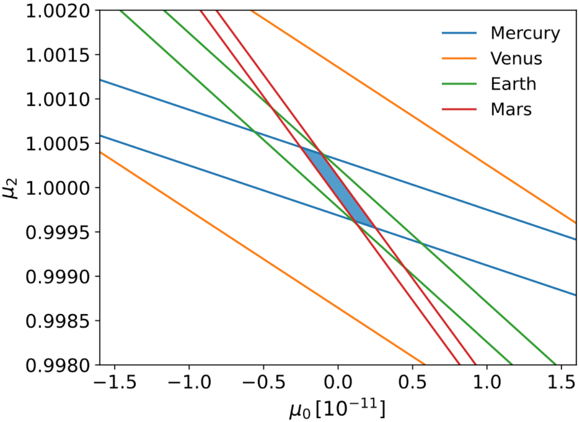




Xu, Liang, Shao 2023, PRD [2209.02209]

Perihelion Advance: family $R_{rr} = 0$





Xu, Liang, Shao 2023, PRD [2209.02209]

BH Shadows: family $b_r = 0$

- Angular diameter of a **BH shadow**

$$d = 2\theta_g \left(\sigma_{\rm LR}/M\right)$$

where $\sigma_{\rm LR}$ is the **critical impact parameter** for photons, and $\theta_g \equiv M/D$ with D the distance to BH

Xu, Liang, Shao 2023, PRD [2209.02209]

BH Shadows: family $b_r = 0$

- Angular diameter of a **BH shadow**

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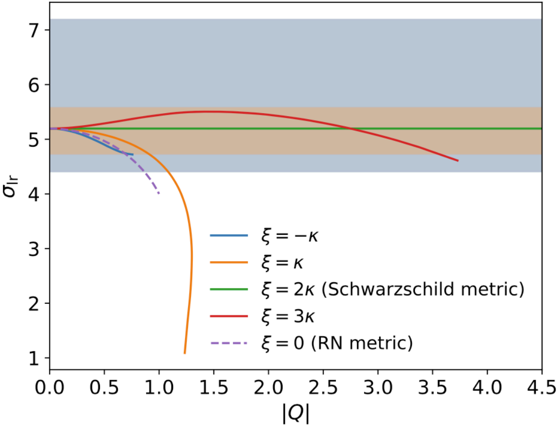
- **Event Horizon Telescope** results

M87* (EHT 2019)	Sgr A* (EHT 2022)
Akiyama et al. 2019, ApJL 875:L1	Akiyama et al. 2022, ApJL 930:L12
$d = 42 \pm 3\,\mu$as	$d = 51.8 \pm 2.3\,\mu$as
$\theta_g = 3.62 \pm 0.60\,\mu$as	$\theta_g = 5.02 \pm 0.20\,\mu$as

Xu, Liang, Shao 2023, PRD [2209.02209]

Bounds from BH Shadows

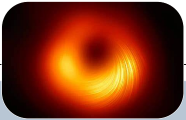


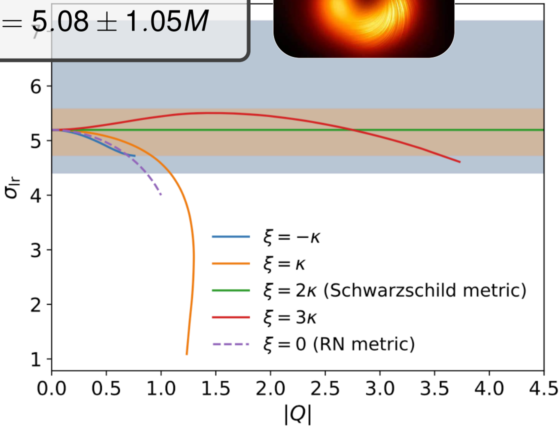
Xu, Liang, Shao 2023, ApJ [2302.05671]

Bounds from BH Shadows

M87* (EHT 2019)

$$\sigma_{\rm LR} = 5.08 \pm 1.05M$$





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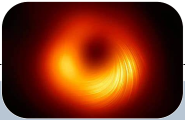
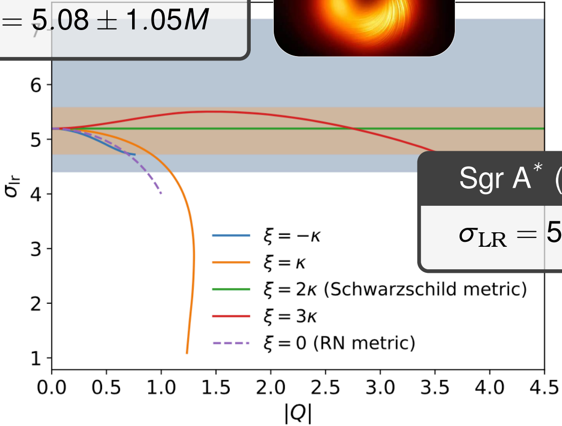
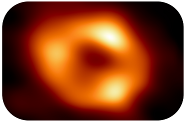


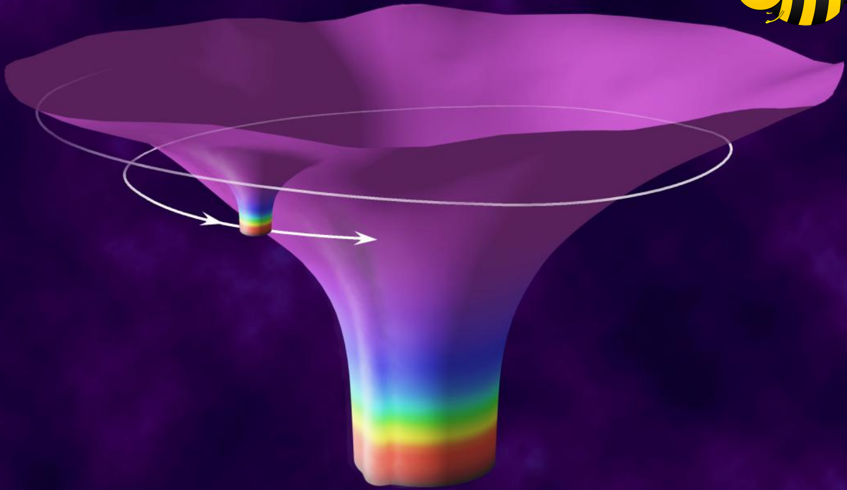


Sgr A* (EHT 2022)

$\sigma_{\rm LR} = 5.16 \pm 0.31M$



Xu, Liang, Shao 2023, ApJ [2302.05671]



Extreme mass ratio inspirals (EMRIs)

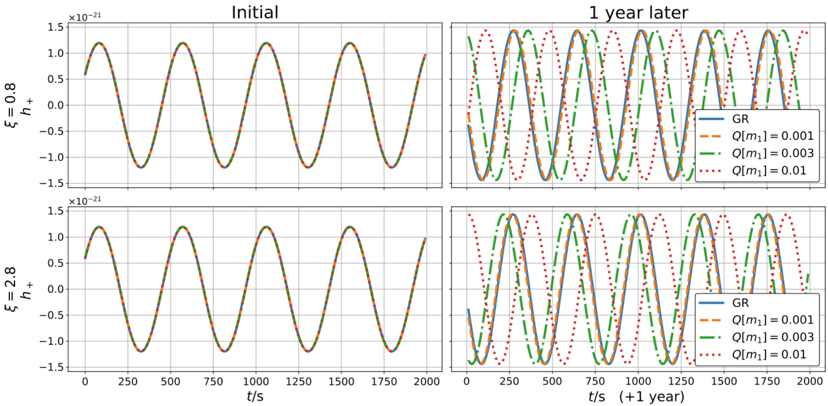
Bumblebee EMRIs

- **Geodesics (+energy loss) change** *w.r.t.* GR EMRIs

Liang, Xu, Mai, Shao 2023 [2212.09346]

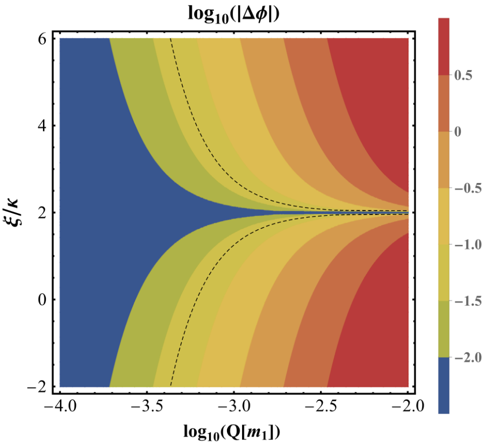
Bumblebee EMRIs

- **Geodesics (+energy loss) change** *w.r.t.* GR EMRIs
- e.g., a $(10^6, 10)\,M_\odot$ EMRI at $D_L = 100\,\mathrm{Mpc}$, starting at $5R_S$



Liang, Xu, Mai, Shao 2023 [2212.09346]

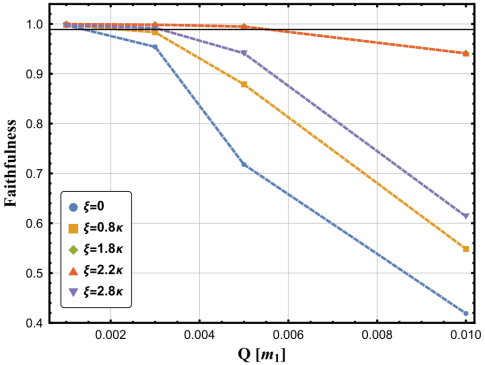
Bumblebee EMRIs



Phase difference for $(10^6, 10)\, M_\odot$ EMRIs starting at $5R_S$, after 1 year

Liang, Xu, Mai, Shao 2023 [2212.09346]

Bumblebee EMRIs in LISA



Faithfulness

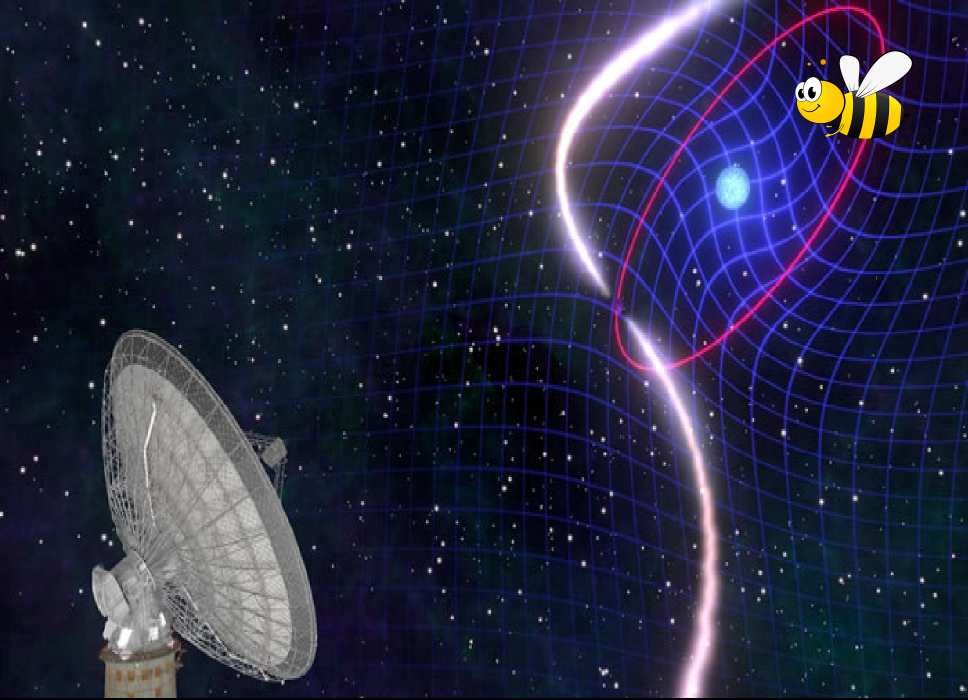
$$\mathcal{F}\left[s_1, s_2\right] = \max_{\{t_c, \varphi_c\}} \frac{\langle s_1 \mid s_2 \rangle}{\sqrt{\langle s_1 \mid s_1 \rangle \langle s_2 \mid s_2 \rangle}}$$

with inner product

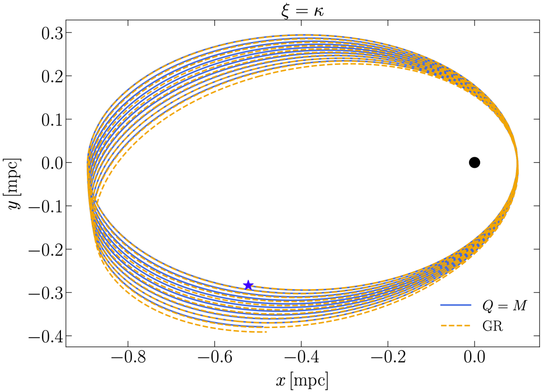
$$\langle s_1 \mid s_2 \rangle = 2 \int_{f_{\min}}^{f_{\max}} \frac{\tilde{s}_1(f)\tilde{s}_2^*(f) + \tilde{s}_1^*(f)\tilde{s}_2(f)}{S_n(f)} \mathrm{d}f$$

$\mathcal{F}_{\text{threshold}} = 0.989$ for SNR $\rho = 30$ $\Rightarrow Q \sim \mathcal{O}\left(10^{-3}\right)$

Liang, Xu, Mai, Shao 2023 [2212.09346]



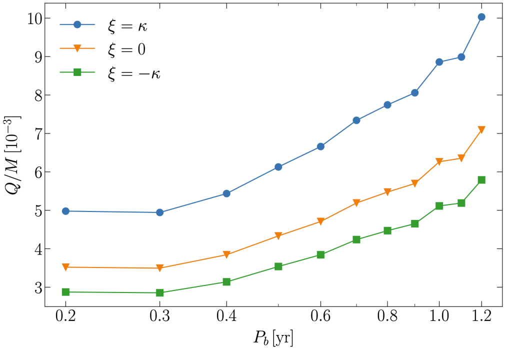
Pulsars around Bumblebee Sgr A*



Orbital Precession for a $P_{\rm b} = 0.5\,{\rm yr}$ pulsar around Sgr A*

Hu, Shao, Xu, Liang, Mai 2024

Pulsars around Bumblebee Sgr A*



Bumblebee Charge for pulsars around Sgr A*

Hu, Shao, Xu, Liang, Mai 2024

Summary

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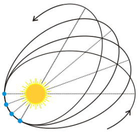

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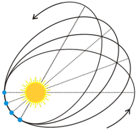

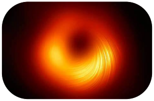

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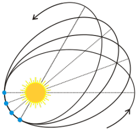
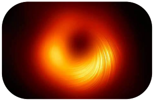
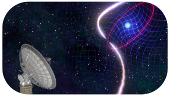
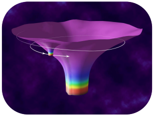

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