地球物理与空间信息学院
School of Geophysics and Geomatics

带有薄吸积盘的高阶导数引力球对称黑洞的光学外观

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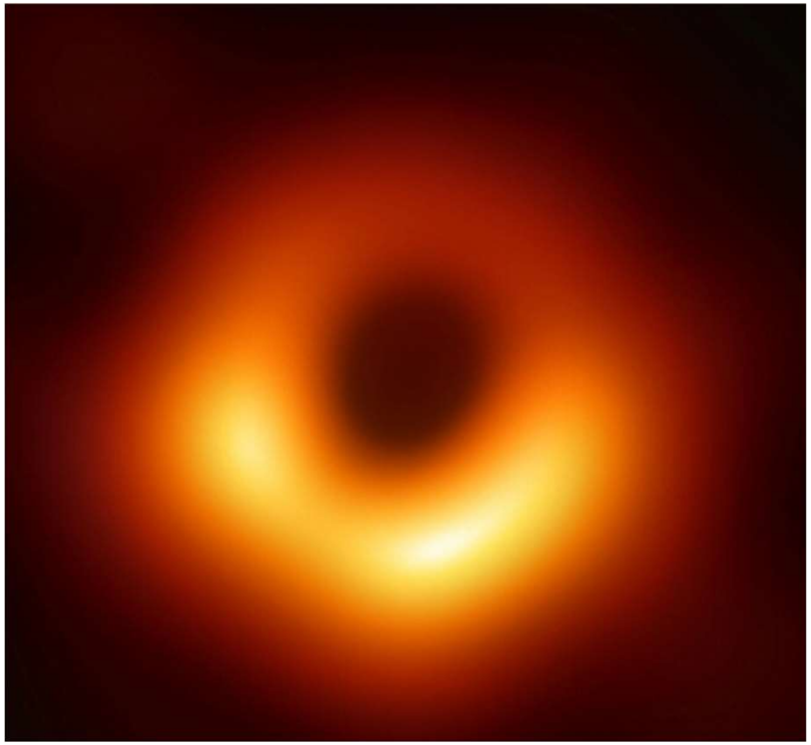
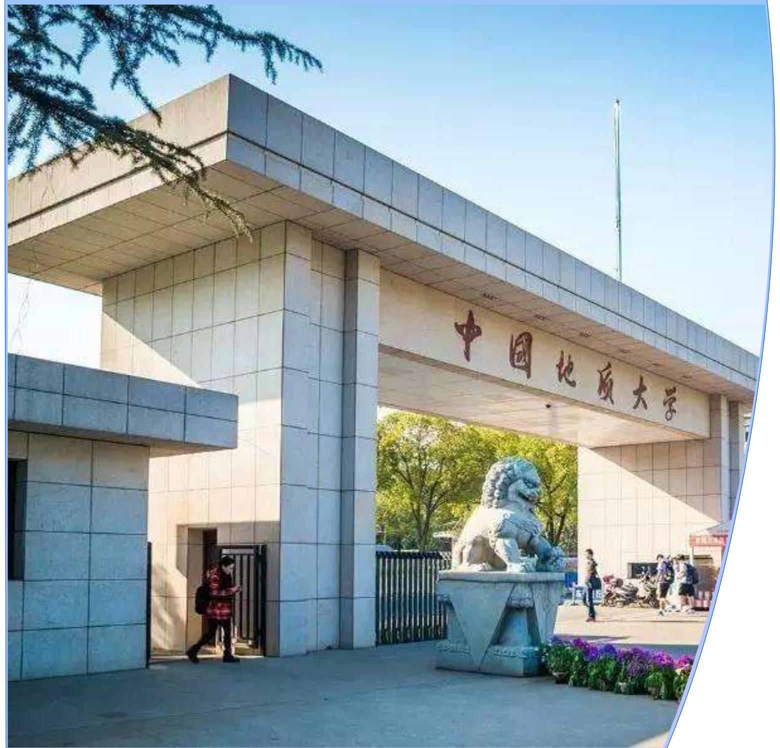
In 2019, the Event Horizon Telescope (EHT) collaboration reported the images of the supermassive BH at the center of the M87* galaxy





- 高阶导数引力理论下的球对称黑洞解
- 黑洞时空中的测地线
- 吸积盘的光线追踪与辐射通量
- 总结

二、高阶导数引力理论下的球对称黑洞解

The most general Lagrangean with quadratic curvature invariants

$$\mathcal{L} = \gamma R - \alpha C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \beta R^2$$

$$\beta = 0 \quad \gamma = 1$$

Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} - 4\alpha B_{\mu\nu} = 0$$

$$B_{\mu\nu} = \left(\nabla^\rho \nabla^\sigma + \frac{1}{2} R^{\rho\sigma} \right) C_{\mu\rho\nu\sigma}$$

H. Lu, A. Perkins, C.N. Pope, K.S. Stelle, Phys. Rev. Lett. ¨
114, 171601 (2015).

二、高阶导数引力理论下的球对称黑洞解

metric $$ds^2 = -h(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\varphi^2$$

Field equations

$$rh[rf'h' + 2f(rh'' + 2h')] + 4h^2(rf' + f - 1) - r^2 fh'^2 = 0$$

$$f'' + \frac{r^2 fh'^2 + 2fhh' + 4(f-1)h^2}{2rfh(rh' - 2h)}f' - \frac{3hf'^2}{4fh - 2rfh'} + \frac{r^3 fh' + (r^2 f - r^2)h}{\alpha r^2 f(rh' - 2h)} + \frac{r^3 fh'^3 - 3r^2 fhh'^2 - 8(f-1)h^3}{2r^2h^2(rh' - 2h)} = 0$$

consider a linear approximation for $f(r), h(r)$

$$f(r) = 1 - \frac{2M}{r} - e^{-\frac{r}{\sqrt{2\alpha}}}C_1\left(\frac{1}{8\alpha} + \frac{1}{4\sqrt{2\alpha}r}\right)$$

$$h(r) = h_c\left(1 - \frac{2M}{r} - e^{-\frac{r}{\sqrt{2\alpha}}}\left(\frac{C_1}{2\sqrt{2\alpha}r}\right)\right)$$

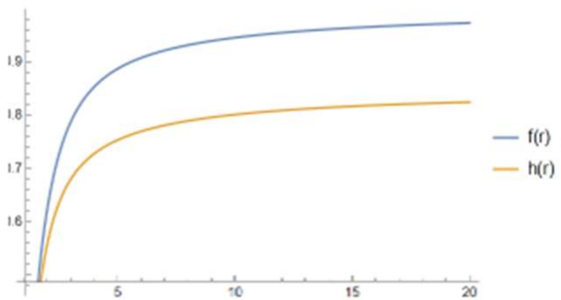
二、高阶导数引力理论下的球对称黑洞解

suppose that the spacetime has only one horizon at r_0

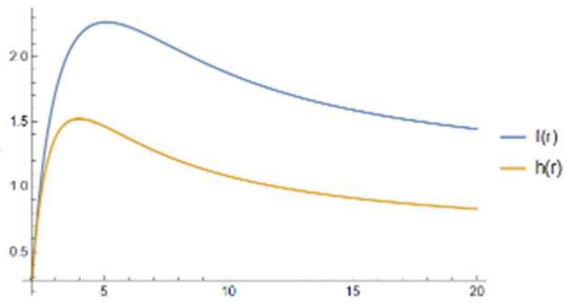
$$h(r) = h_1(r - r_0) + h_2(r - r_0)^2 + h_3(r - r_0)^3 + \cdots$$

$$f(r) = f_1(r - r_0) + f_2(r - r_0)^2 + f_3(r - r_0)^3 + \cdots$$

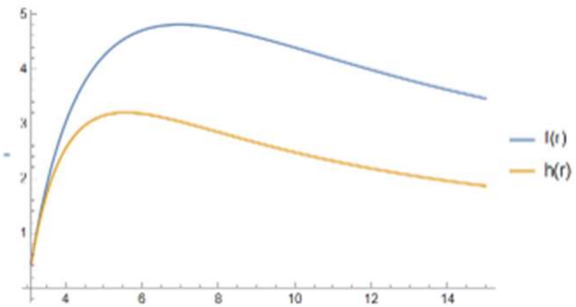
Set $h_1 = f_1$ ， $\alpha = 1/2$， all h_j and f_j with j ≥ 2 can be calculated from f_1.



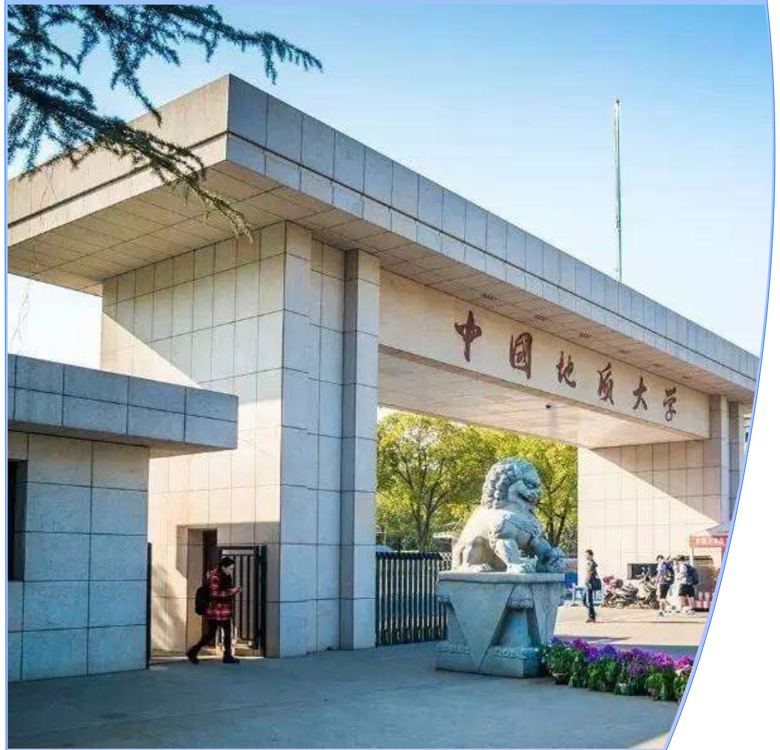
$r_0 = 1$



$r_0 = 2$



$r_0 = 3$



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二、黑洞时空中的测地线

Geodesic equation $-\kappa = g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$

$$-\kappa = -\frac{E^2}{h} + \frac{\dot{r}^2}{f} + \frac{L^2}{r^2} \quad E = h(r)\dot{t} \qquad L = r^2\dot{\varphi}$$

For time-like geodesics $\kappa = 1$

$$\dot{r}^2 = f(r)(\frac{E^2}{h(r)} - \frac{L^2}{r^2} - 1) \equiv V(r) \qquad \ddot{r} = \frac{1}{2}V'(r)$$

circular orbit $\dot{r} = 0 \quad \ddot{r} = 0$

$$E^2 = \frac{2h(r)^2}{2h(r) - rh'(r)} \qquad L^2 = \frac{r^3 h'(r)}{2h(r) - rh'(r)}$$

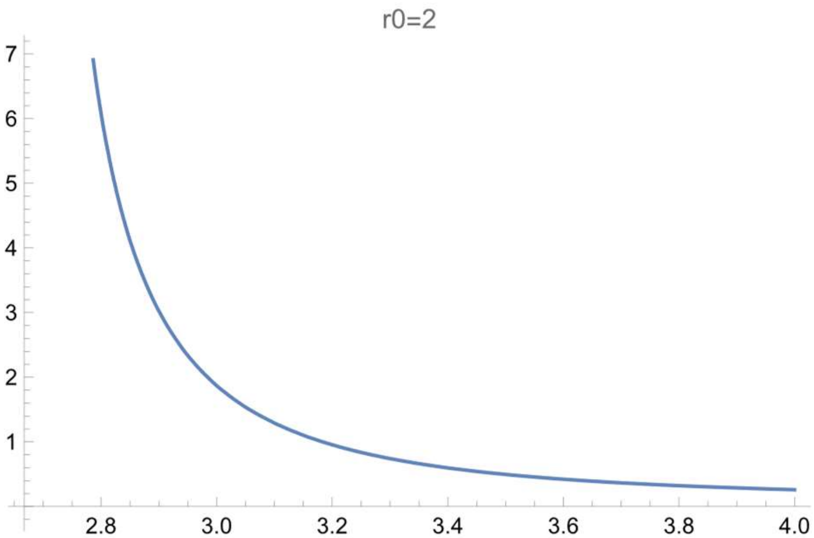
$$2h - rh' > 0 \quad h' > 0$$

$$r > 1.438 \qquad 2.666 < r < 3.996 \qquad 3.959 < r < 4.572$$

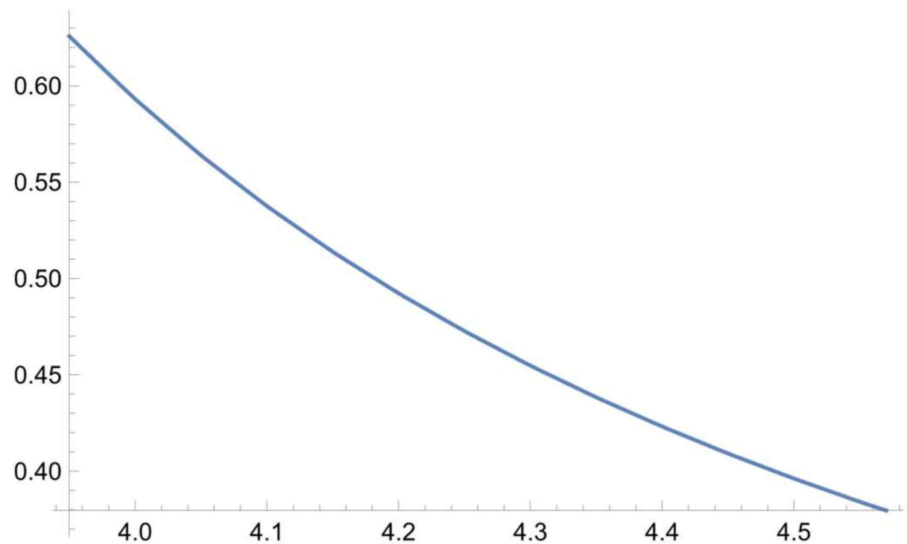
二、黑洞时空中的测地线

The stability of the orbit requires $V''(r) < 0$

$$r_0 = 1 \qquad r_{isco} = 3.50201$$



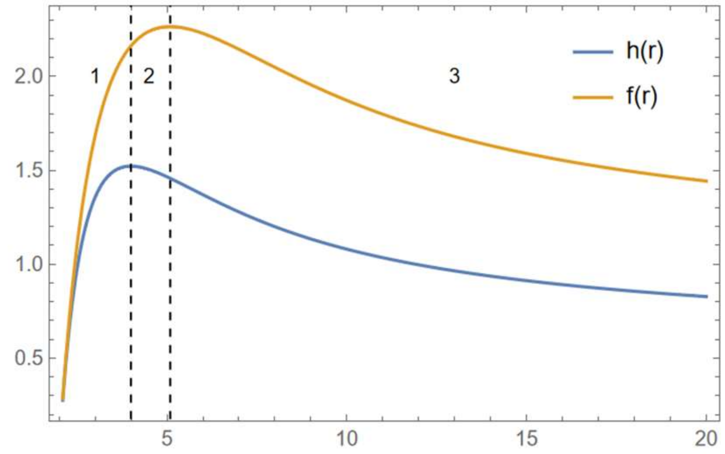
$r_0 = 2$



$r_0 = 3$

二、黑洞时空中的测地线

$$\ddot{r} = \frac{1}{2}\left[\left(\frac{E^2}{h} - \frac{L^2}{r^2} - 1\right) f' + f\left(\frac{2L^2}{r^3} - \frac{E^2 h'}{h^2}\right)\right]$$

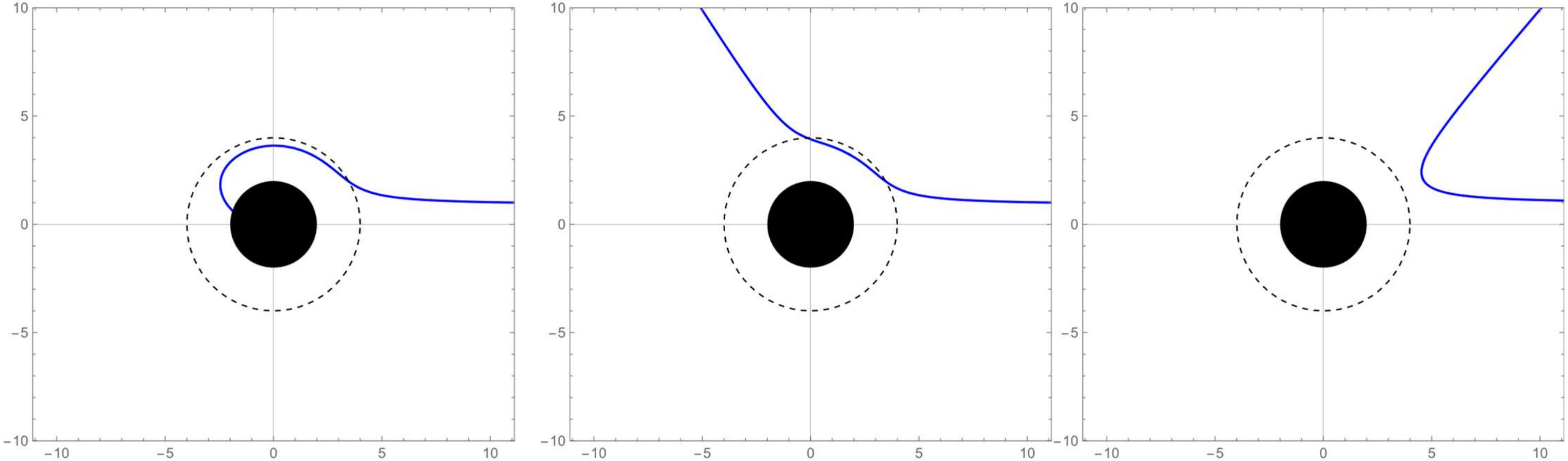


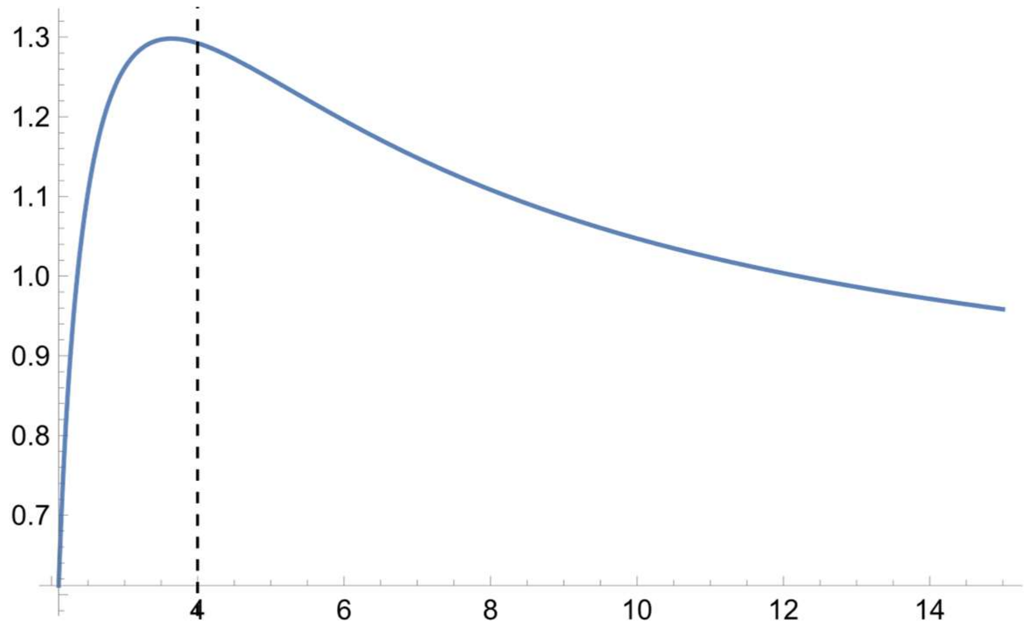
For any E,L

$$\ddot{r} = \frac{2L^2 f}{r^3} - \frac{L^2 f'}{r^2} - f' + E^2\left(\frac{f'}{h} - \frac{f h'}{h^2}\right)$$

$$hf' - fh' > 0$$

二、黑洞时空中的测地线





L=1.25

二、黑洞时空中的测地线

Null geodesic $$0 = g_{\mu\nu}\frac{dx^{\mu}}{d\lambda}\frac{dx^{\nu}}{d\lambda}$$

Orbital equation

$$\left(\frac{dr}{d\varphi}\right)^2 = r^4 f(r)\left(\frac{1}{b^2 h(r)} - \frac{1}{r^2}\right) \qquad b = \frac{L}{E}$$

The circular orbit of photons

$$r_s = 1.437 \qquad b_s = 2.357$$

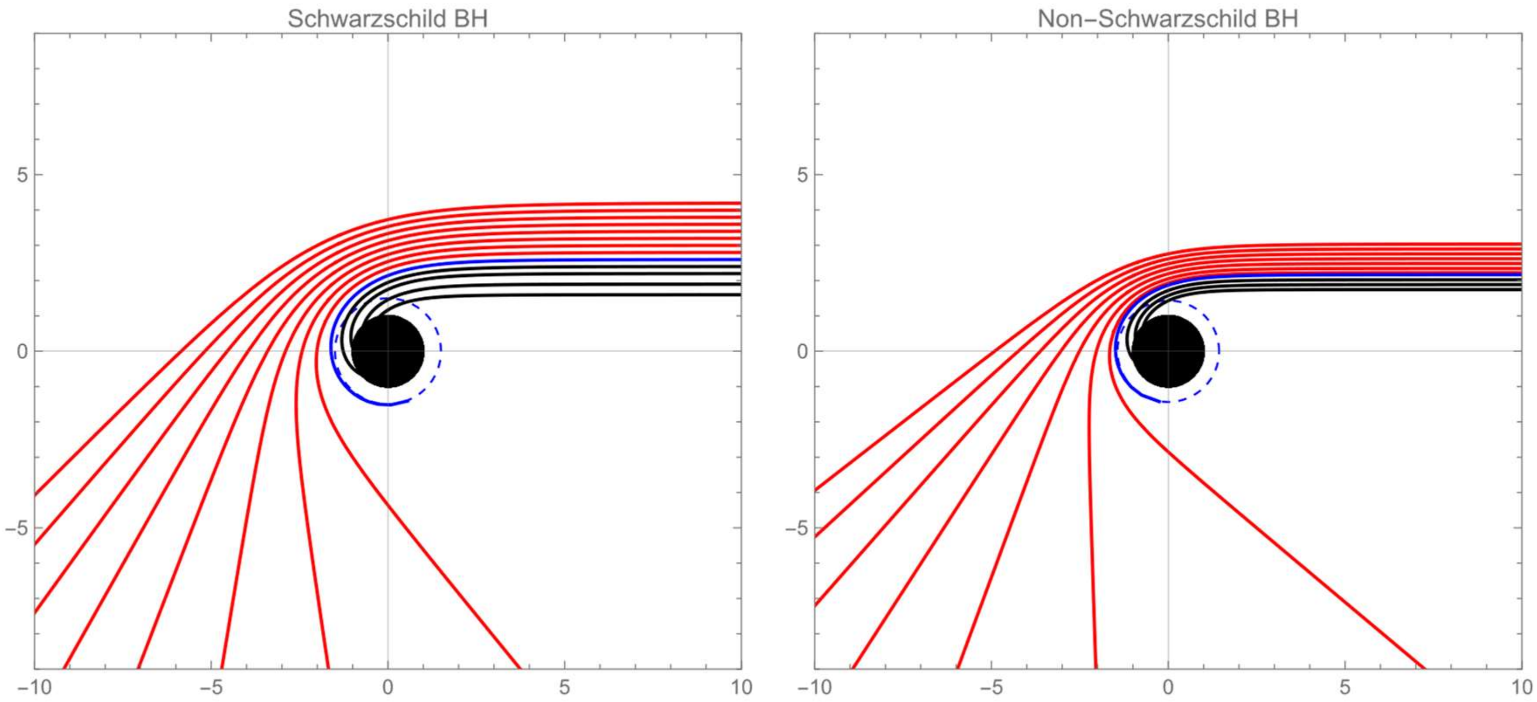
The deflection angle of photons

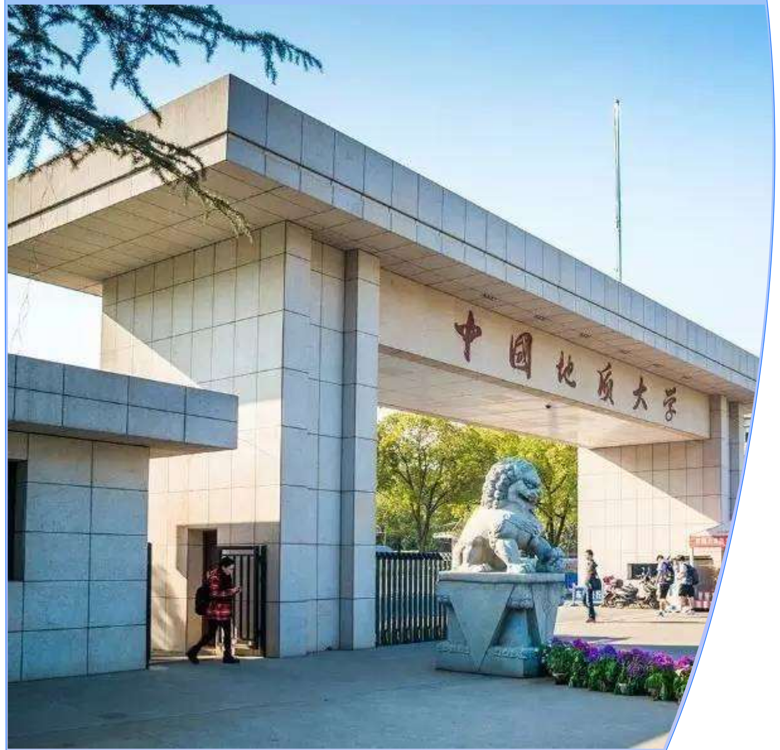
$$\gamma = \int_{r_{sourse}}^{\infty} \frac{dr}{\sqrt{r^4 f(r)\left(\frac{1}{b^2 h(r)} - \frac{1}{r^2}\right)}}$$

二、黑洞时空中的测地线

The deflection angle of photons passing through perihelion

$$\gamma_1 = 2\int_{r_p}^{\infty} \frac{dr}{\sqrt{r^4 f(r)\left(\frac{1}{b^2 h(r)} - \frac{1}{r^2}\right)}} - \gamma$$





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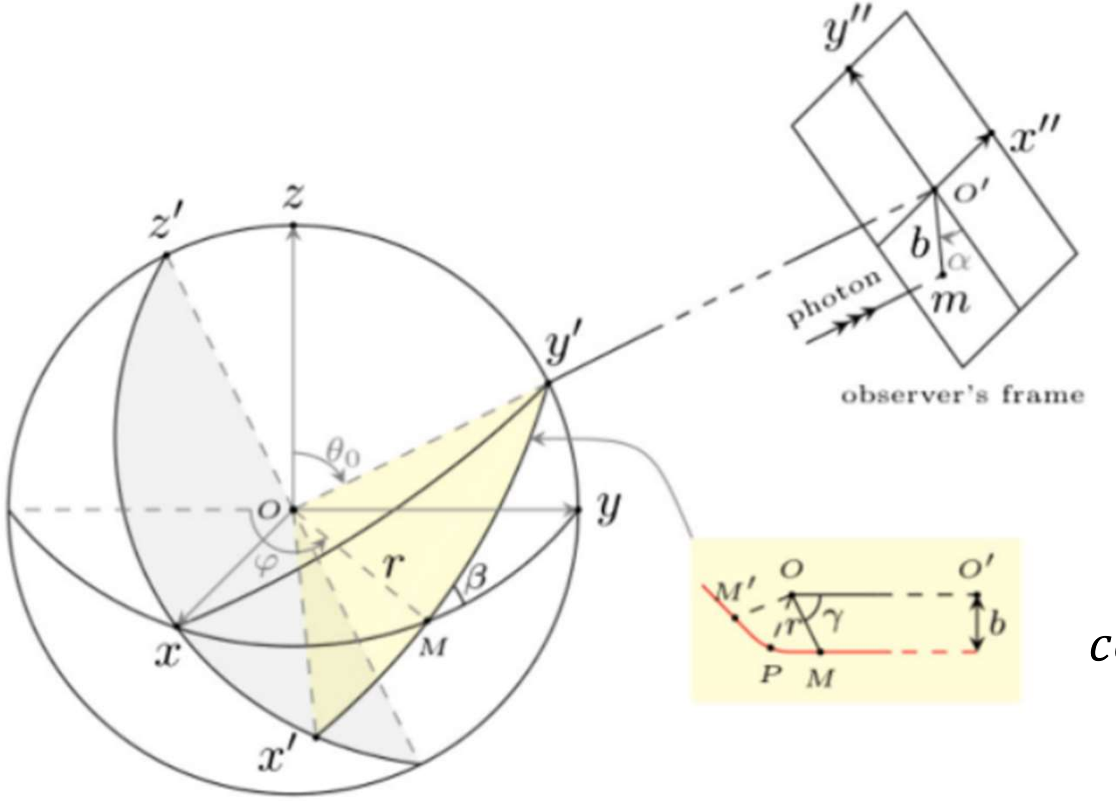


Sourse (r,φ)

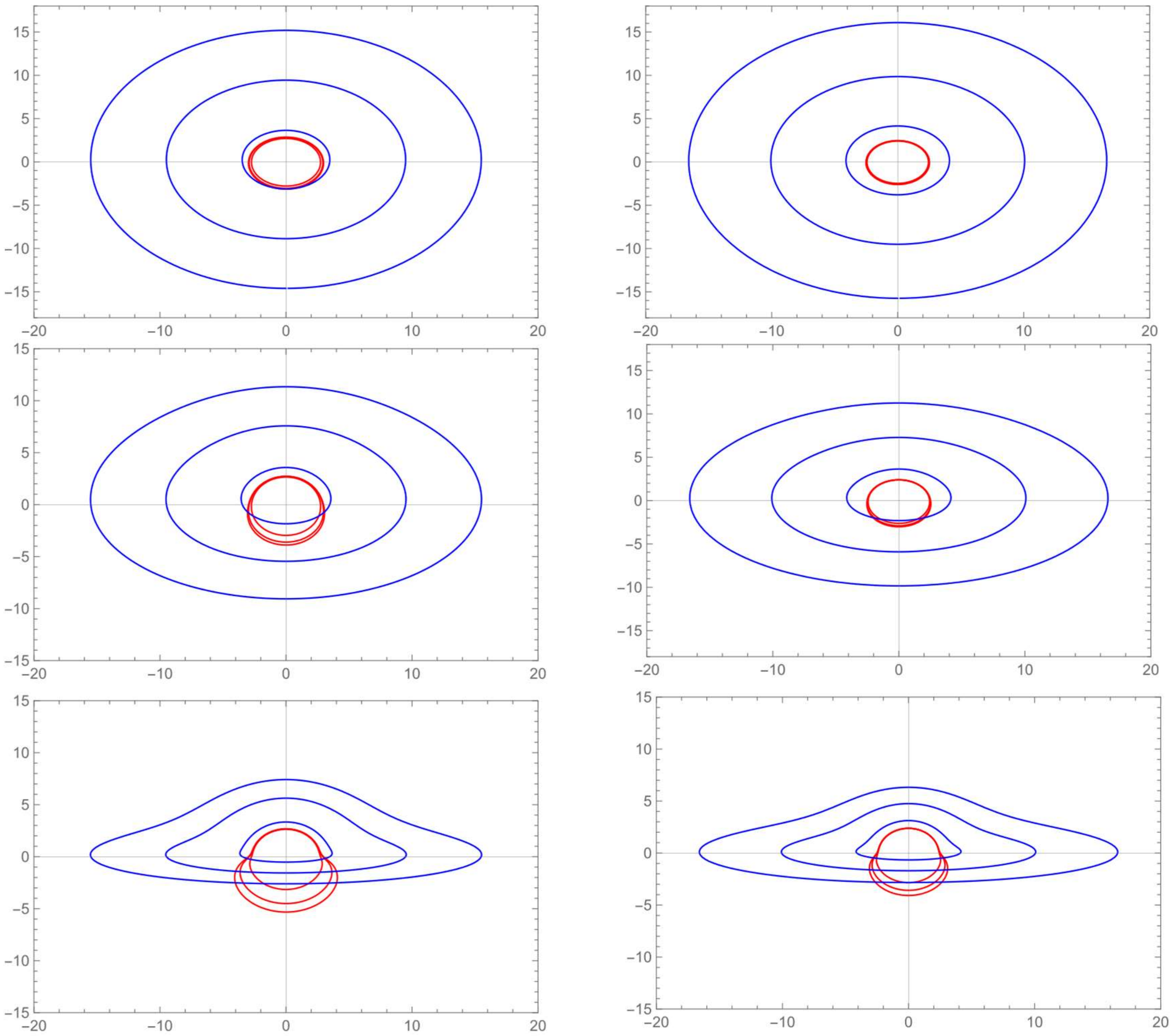
Direct image (b,α)

Secondary image $(b,\alpha+\pi)$

$$cos\alpha = cos\gamma\sqrt{\cos^2\alpha+\cos^2\theta_0}$$

J.-P. Luminet, Image of a spherical black hole with thin accretion disk, Astron. Astrophys. 75, 1 (1979).

二、黑洞时空中的测地线

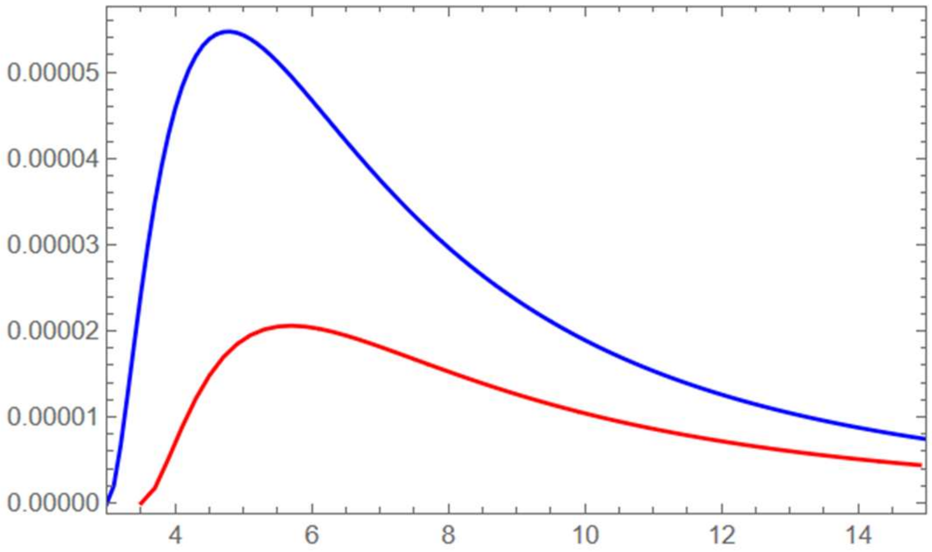


三、吸积盘的光线追踪与辐射通量

The expression of radiation flux of a Novikov-Thorne thin accretion disk

$$F = -\frac{\dot{M}}{2\pi\sqrt{-g}}\frac{\Omega_{,r}}{(E-\Omega L)^2}\int_{r_{in}}^{r}(E-\Omega L)L_{,r}dr$$

$$E = -\frac{g_{tt}}{\sqrt{-g_{tt}-g_{\varphi\varphi}\Omega^2}} \quad L = -\frac{g_{\varphi\varphi}\Omega}{\sqrt{-g_{tt}-g_{\varphi\varphi}\Omega^2}} \quad \Omega = \frac{d\varphi}{dt} = -\sqrt{-\frac{g_{tt,r}}{g_{\varphi\varphi,r}}}$$



$$F_{max} = 5.471\times 10^{-5}\cdot \dot{M}$$

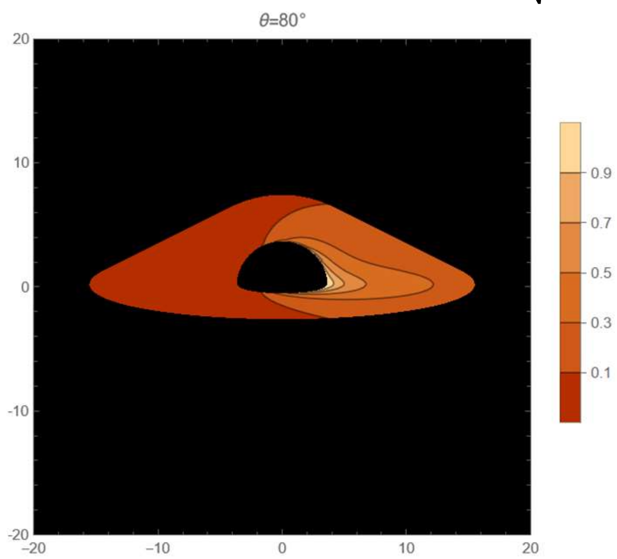
三、吸积盘的光线追踪与辐射通量

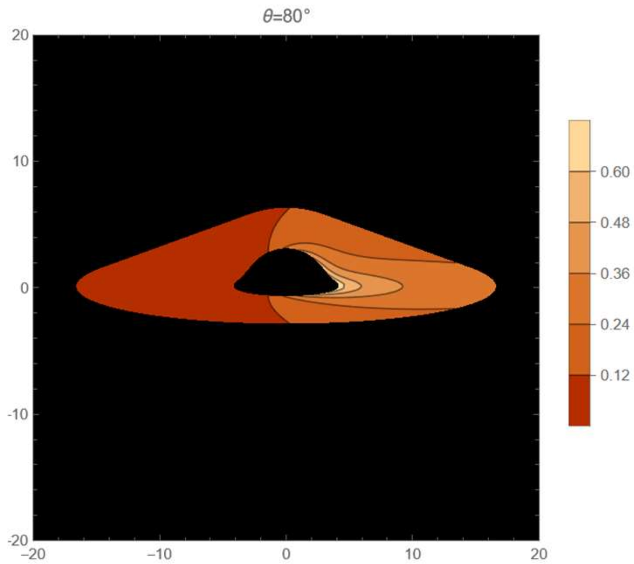
Gravitational redshift and Doppler shift can affect the radiative flux

$$F_{obs} = \frac{F}{(z+1)^4}$$

$$E_{em} = p_t \mu^t + p_\varphi \mu^\varphi = p_t \mu^t (1 + \Omega \frac{p_\varphi}{p_t})$$

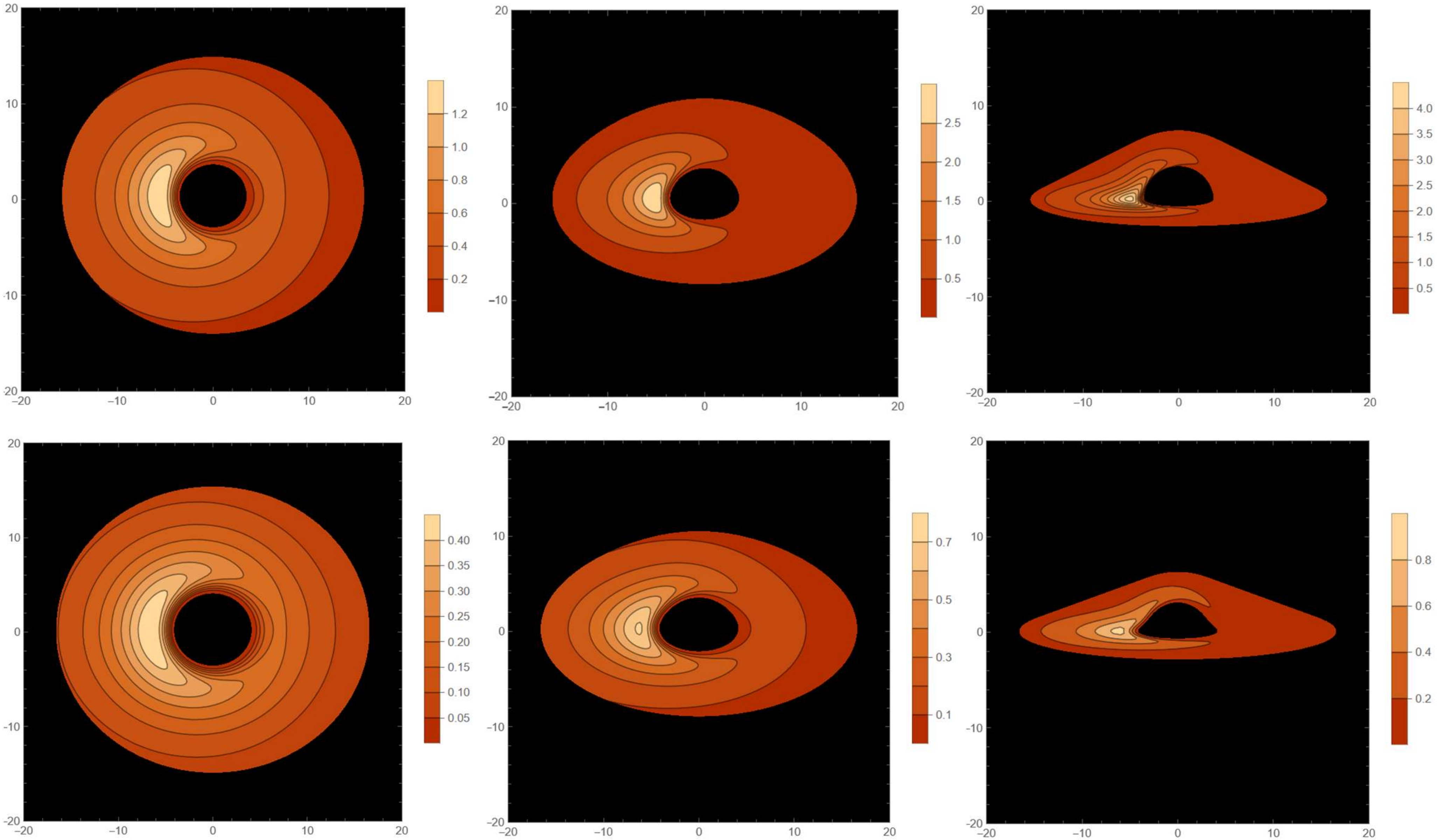
$$1 + z = \frac{E_{em}}{E_{obs}} = \frac{(1 + b\Omega \sin\theta_0 \sin\alpha)}{\sqrt{-g_{tt} - \Omega^2 g_{\varphi\varphi}}}$$





三、吸积盘的光线追踪与辐射通量

观测通量分布



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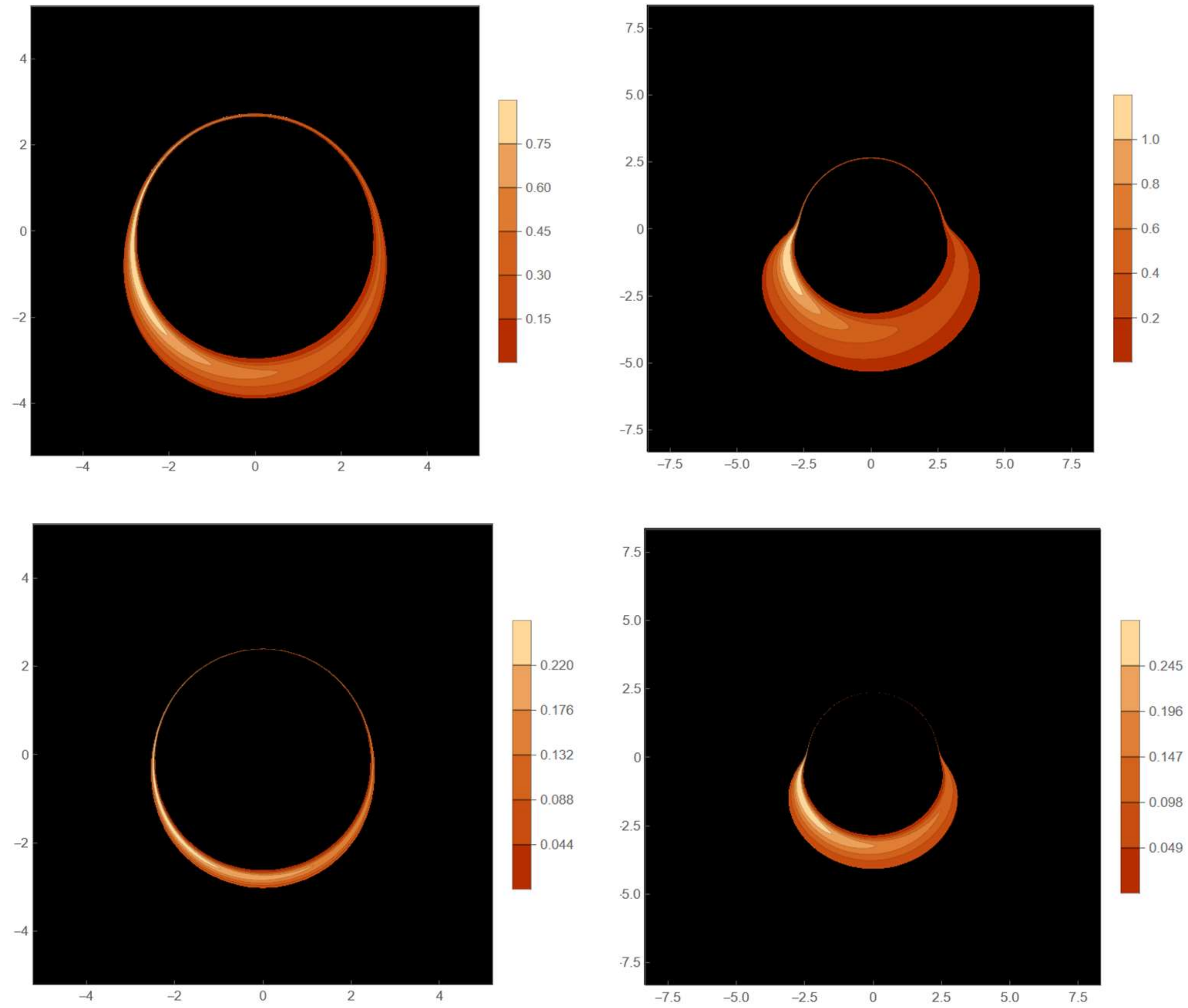

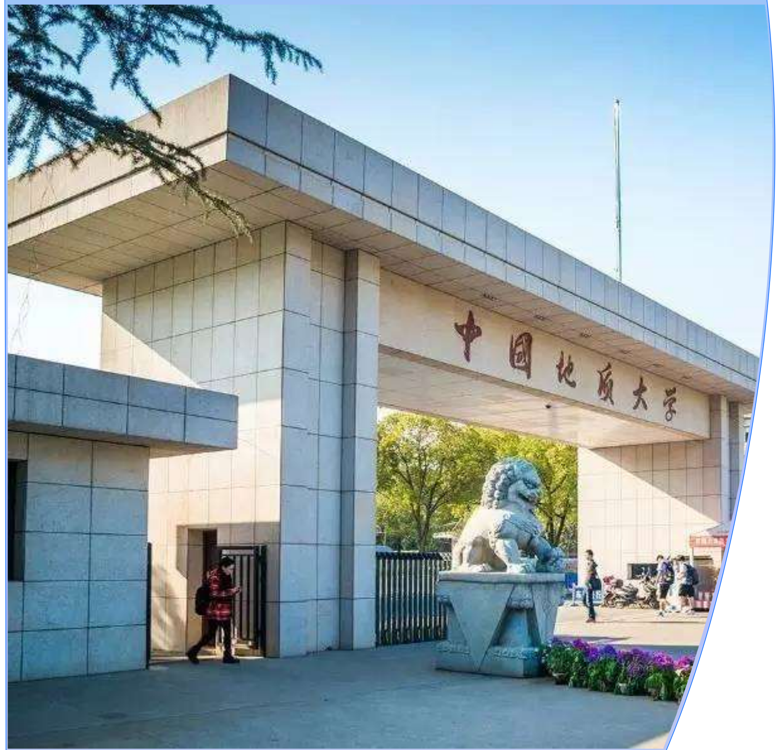

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四、总结

1、高阶导数引力下视界为2、3的非史瓦西黑洞没有稳定的圆轨道，另外这两种时空下在 $h'(r) < 0$ 时，黑洞可能表现为排斥粒子。

2、视界为1时，非史瓦西黑洞薄吸积盘的主要图像与史瓦西黑洞相差不大，非史瓦西黑洞的次级图像比史瓦西黑洞更加紧凑。

3、非史瓦西黑洞的本征辐射通量，红移分布，与观测通量分布整体小于史瓦西黑洞，但分布规律与史瓦西黑洞相差无几，观测倾角越大，红移现象越明显。

主要参考文献

[1] H. Lu, A. Perkins, C.N. Pope, K.S. Stelle, Phys. Rev. Lett. ¨ 114, 171601 (2015).

[2] J.-P. Luminet, Image of a spherical black hole with thin accretion disk, Astron. Astrophys. 75, 1 (1979).

[3] K. Akiyama et al. [Event Horizon Telescope Collab- oration], Astrophys. J. L1-L6: 875 (2019);

[4] R. Abbott et al, [LIGO Scientific Collaboration, Virgo Collaboration, and KAGRA Collaboration], Observation of Gravitational Waves from a Binary Black Hole Merger, Phys. Rev. Lett. 116: 061102, (2016).

谢谢