

黑洞阴影：从几何光学、变换光学到波动光学

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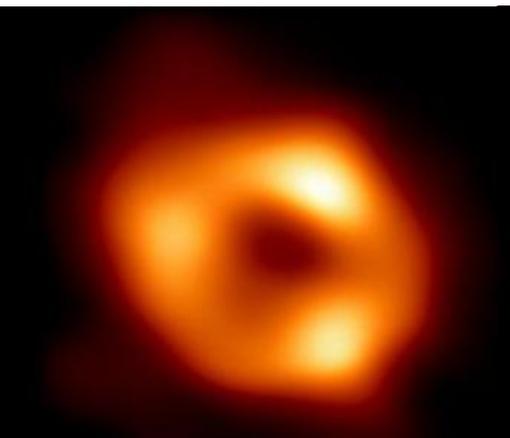
1. 阴影与黑洞

1.1: 黑洞阴影



生活中常见的阴影

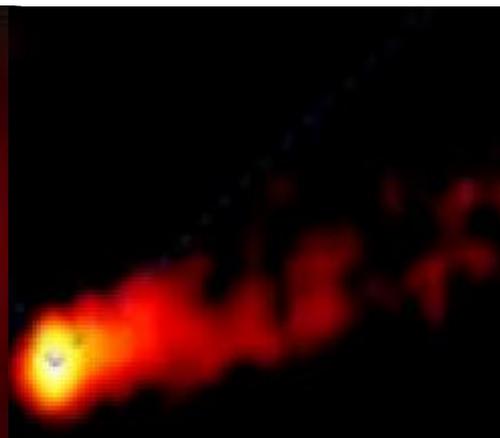
阴影：光线在同种均匀介质中沿直线传播，不能穿过不透明物体而形成的较暗区域



黑洞阴影:人马座A*



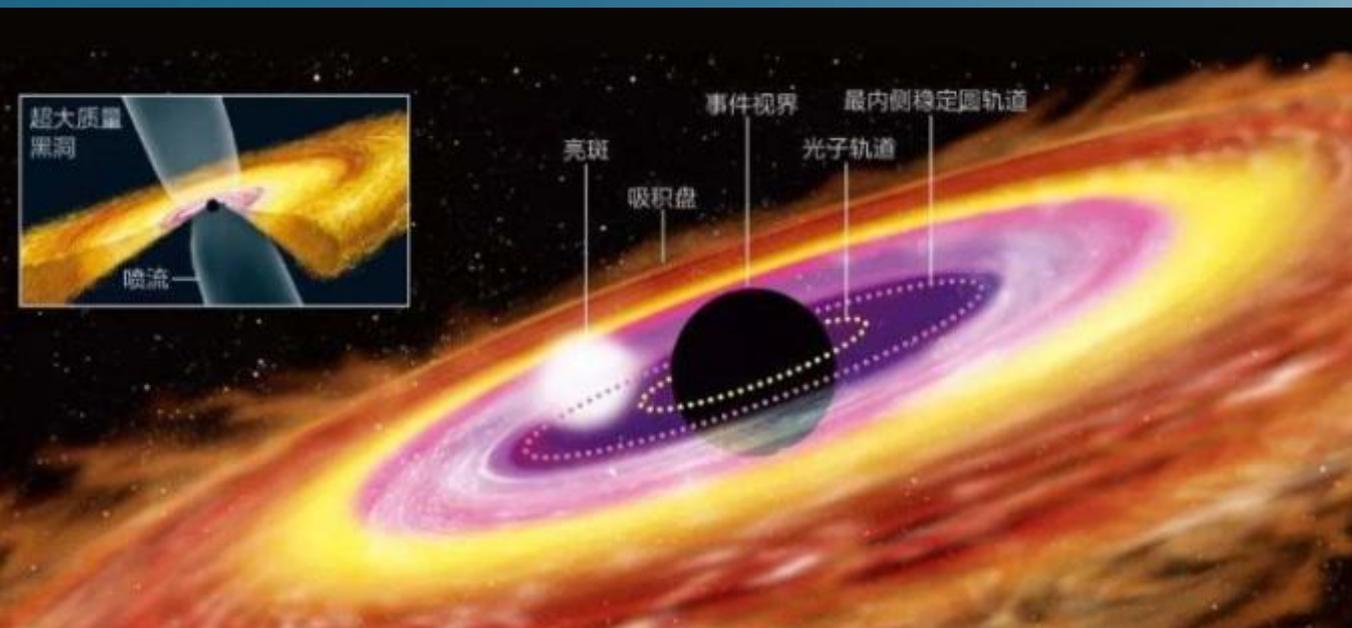
黑洞阴影: M87



M87黑洞与喷流阴影

黑洞是时空曲率大到光都无法从其事件视界逃脱的天体

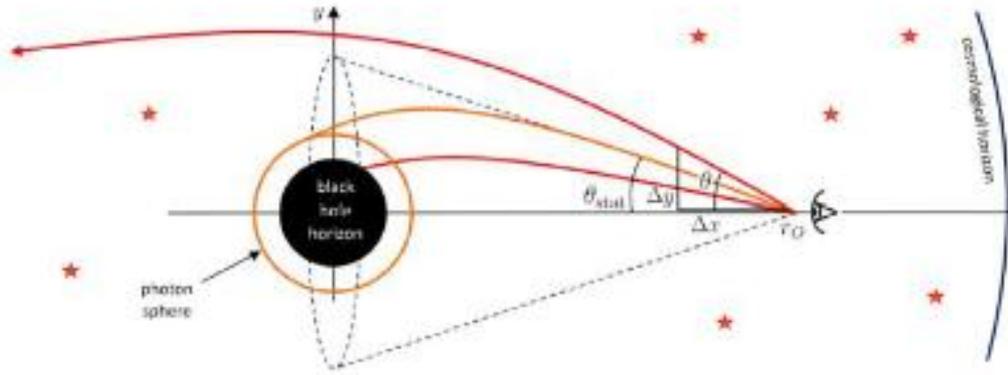
1. 阴影与黑洞



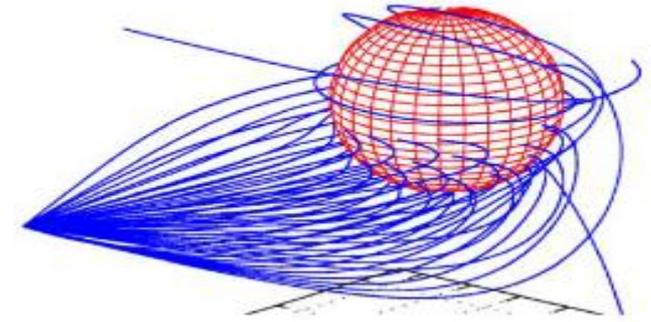
黑洞喷流
与吸积物
质



2.几何光学



光线偏折



光线追踪方案

Xiao-Xiong Zeng, Hai-Qing Zhang, Hongbao Zhang, Eur.Phys.J.C 80 (2020) 9, 872

Xiao-Xiong Zeng, Ke-Jian He, Guo-Ping Li, Sci.China Phys.Mech.Astron. 65 (2022) 9, 290411

U. Leonhardt, Science 312, 1777 (2006)

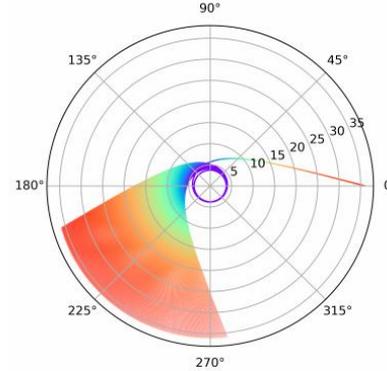
3.1 基于折射率模拟 arXiv:2206.07500

$$n_f = \frac{1}{\sqrt{g_{00}}} \left(\frac{dl_c}{dl_f} \right)$$

$$\rho = \text{const.} \exp\left(\int \frac{1}{\sqrt{r^2 f(r)}}\right)$$

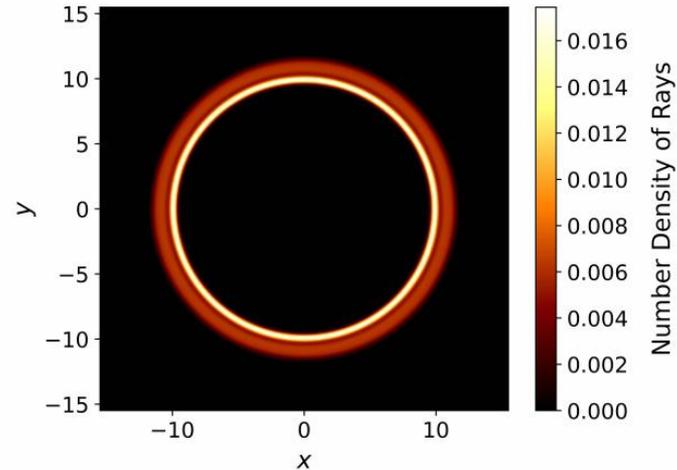
$$\frac{d\rho}{d\phi} = \rho \sqrt{\frac{\rho^2 n^2}{b^2} - 1}$$

$$n_{Sch} = \frac{\left(1 + \frac{M}{2\rho}\right)^3}{\left(1 - \frac{M}{2\rho}\right)}$$



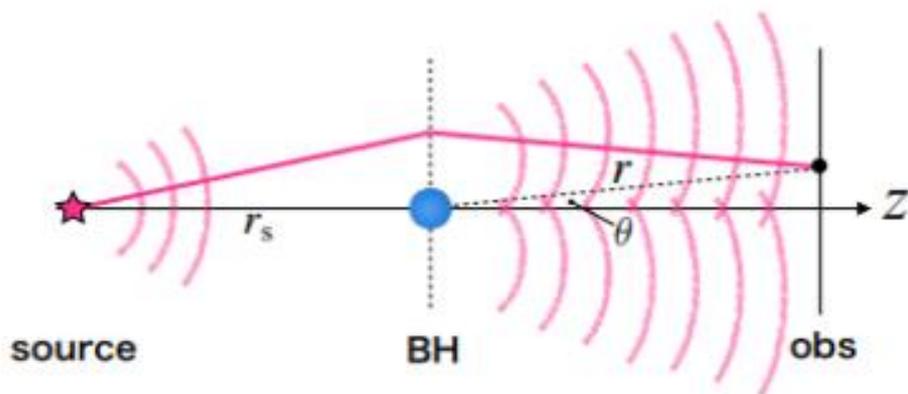
史瓦西黑洞对应的超材料光子运动轨迹

球对称时空中各向同性坐标系下的零测地线方程与平坦时空中各向同性介质中的光线轨迹方程相同



光子环

4.1: 总体思路

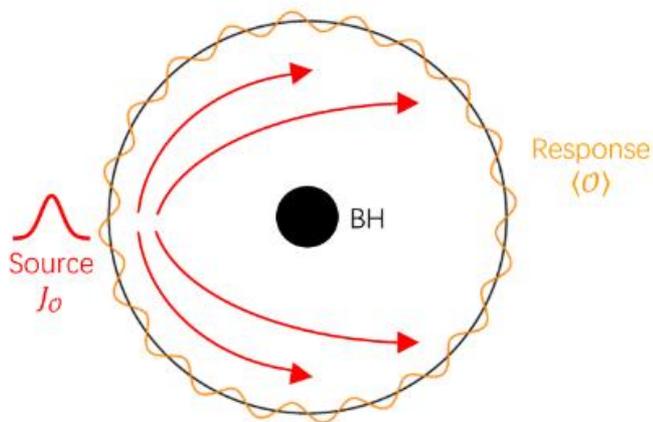


arXiv:1502.05468

在波的几何光学极限下，透镜系统成像是由透镜方程确定每条光线的路径而得到的。由于光线是无质量检验粒子（光子）的轨迹，所以用波来表示图像就是用波来表示粒子的运动，本质上，这与在几何光学中研究光子的运动是一致的。

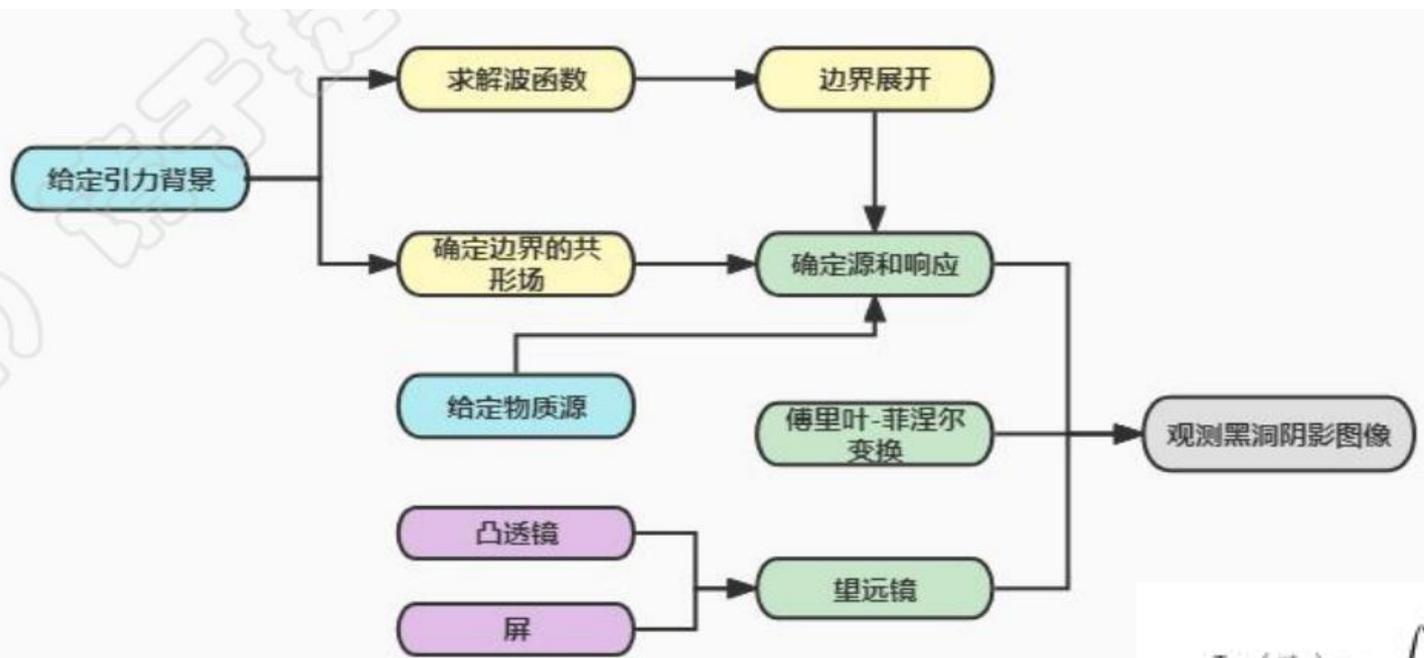
基于波动光学，在得到波源被黑洞散射后的散射波后，人们发现散射波包含黑洞阴影信息，这一图像能否在全息的框架下实现？

Phys. Rev. Lett. 123 , 031602 (2019)



波源与响应函数

JHEP 10 (2022) 189 2201.03161 [hep-th]

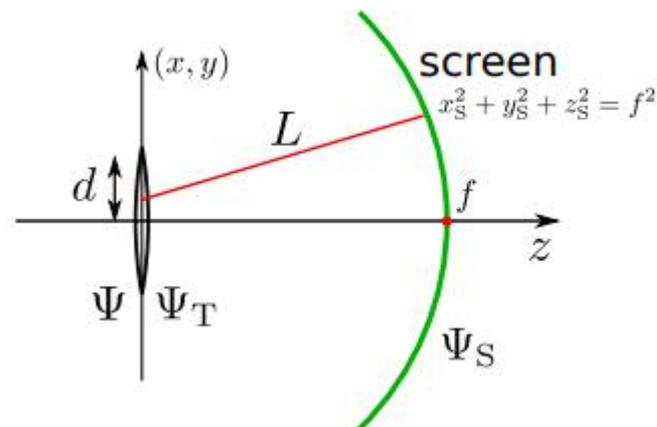


$$\Psi_S(\vec{x}_S) = \int_{|\vec{x}| < d} d^2x \langle \mathcal{O}(\vec{x}) \rangle e^{-\frac{i\omega}{T} \vec{x} \cdot \vec{x}_S},$$

全息框架下构建黑洞阴影的研究思路

4. 波动光学

$$\Psi_T(\vec{x}) = e^{-i\omega \frac{|\vec{x}|^2}{2f}} \Psi(\vec{x})$$



望远镜系统

$$\Psi_S(\vec{x}_S) = \int_{|\vec{x}| \leq d} d^2x \Psi_T(\vec{x}) e^{i\omega L}$$

$$\Psi_S(\vec{x}_S) \propto \int_{|\vec{x}| < d} d^2x \Psi(\vec{x}) e^{-\frac{i\omega}{f} \vec{x} \cdot \vec{x}_S}$$

$$\begin{aligned} L &= \sqrt{(x_S - x)^2 + (y_S - y)^2 + z_S^2} \\ &= \sqrt{f^2 - 2\vec{x}_S \cdot \vec{x} + |\vec{x}|^2} \simeq f - \frac{\vec{x}_S \cdot \vec{x}}{f} + \frac{|\vec{x}|^2}{2f} \end{aligned}$$

$$\Psi_S(\vec{x}_S) = \int_{|\vec{x}| < d} d^2x \langle O(\vec{x}) \rangle e^{-\frac{i\omega}{f} \vec{x} \cdot \vec{x}_S}$$

e-Print: 2309.07404 [gr-qc]

引力背景

$$ds^2 = -F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\xi^2),$$

$$F(r) = 1 - \frac{2\mathcal{M}_n}{r} + \frac{r^2}{l^2} = 1 - \frac{2M}{r} + \frac{8\sqrt{n}M}{\sqrt{\pi}r^2} + \frac{r^2}{l^2} + \mathcal{O}(n^{3/2}).$$

$$M = \frac{\sqrt{\pi} \left(\frac{1}{z_h^2} + 1 \right)}{2z_h^2 \left(\frac{\sqrt{\pi}}{z_h} - 4\sqrt{n} \right)}$$

$$T = \frac{\sqrt{\pi} (z_h^2 + 3) - 8\sqrt{n}z_h (z_h^2 + 2)}{4\pi z_h (\sqrt{\pi} - 4\sqrt{n}z_h)}$$

为了更好的分析波的转播，引入乌龟坐标

$$v_e = t + z_* = t - \int \frac{1}{F(z)} dz$$

引力背景变为

$$ds^2 = \frac{1}{z^2} [-F(z)dv_e^2 - 2dzdv_e + d\Omega^2],$$

$$D_b D^b \Phi - \mathcal{M}^2 \Phi = 0$$

引入 $\Phi = z\phi$ 在这一引力背景下，根据全息重整化，边界处的波函数可表示成

$$\phi(v_e, z, \theta, \xi) = J_{\mathcal{O}}(v_e, \theta, \xi) + z\langle \mathcal{O} \rangle + O(z^2)$$

把标量、场波源和响应函数进行波函数分解

$$\phi(v_e, z, \theta) = e^{-i\omega v_e} \sum_{l=0}^{\infty} c_{l0} \mathcal{Z}_l(z) Y_{l0}(\theta)$$

$$J_{\mathcal{O}}(v, \theta) = e^{-i\omega v} \frac{1}{2\pi\sigma^2} \exp\left[-\frac{(\pi - \theta)^2}{2\sigma^2}\right] = e^{-i\omega v} \sum_{l=0}^{\infty} c_{l0} Y_{l0}(\theta),$$

$$\langle \mathcal{O} \rangle_{J_{\mathcal{O}}} = e^{-i\omega v_e} \sum_{l=0}^{\infty} c_{l0} \langle \mathcal{O} \rangle_{J_{\mathcal{O}l}} Y_{l0}(\theta),$$

$$c_{l0} = (-1)^l \sqrt{\frac{l+1/2}{2\pi}} \exp\left[-\frac{1}{2}(l+1/2)^2 \sigma^2\right]$$

能得到径向的运动方程

$$z^2 F \mathcal{Z}_l'' + z^2 [F' + 2i\omega] \mathcal{Z}_l' + [(2 - 2F) + zF' - z^2 l(l+1)] \mathcal{Z}_l = 0,$$

$$\mathcal{Z}_l = 1 + z \langle \mathcal{O} \rangle_l + O(z^2)$$

$$\langle \mathcal{O} \rangle_{J_{\mathcal{O}l}} = \langle \mathcal{O} \rangle_l + \omega$$

边界条件:

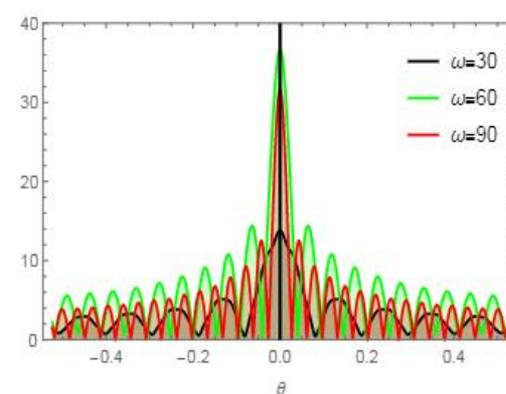
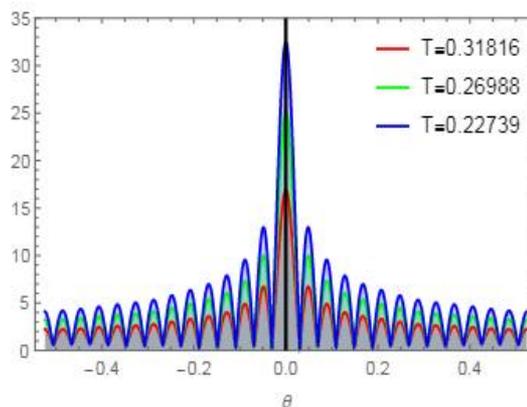
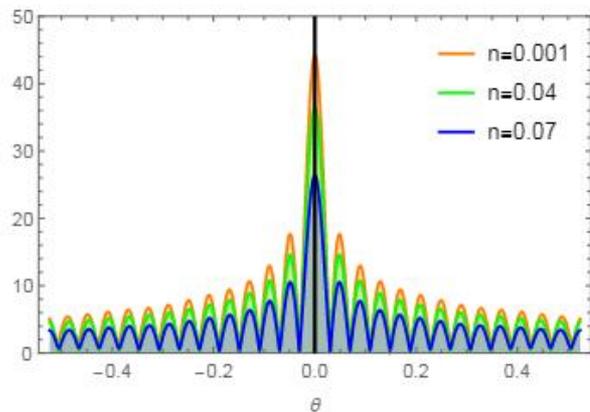
边界处

$$Z_l(0) = 1.$$

视界处

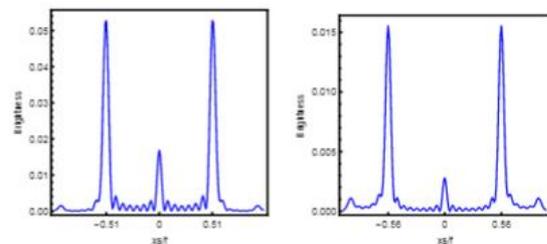
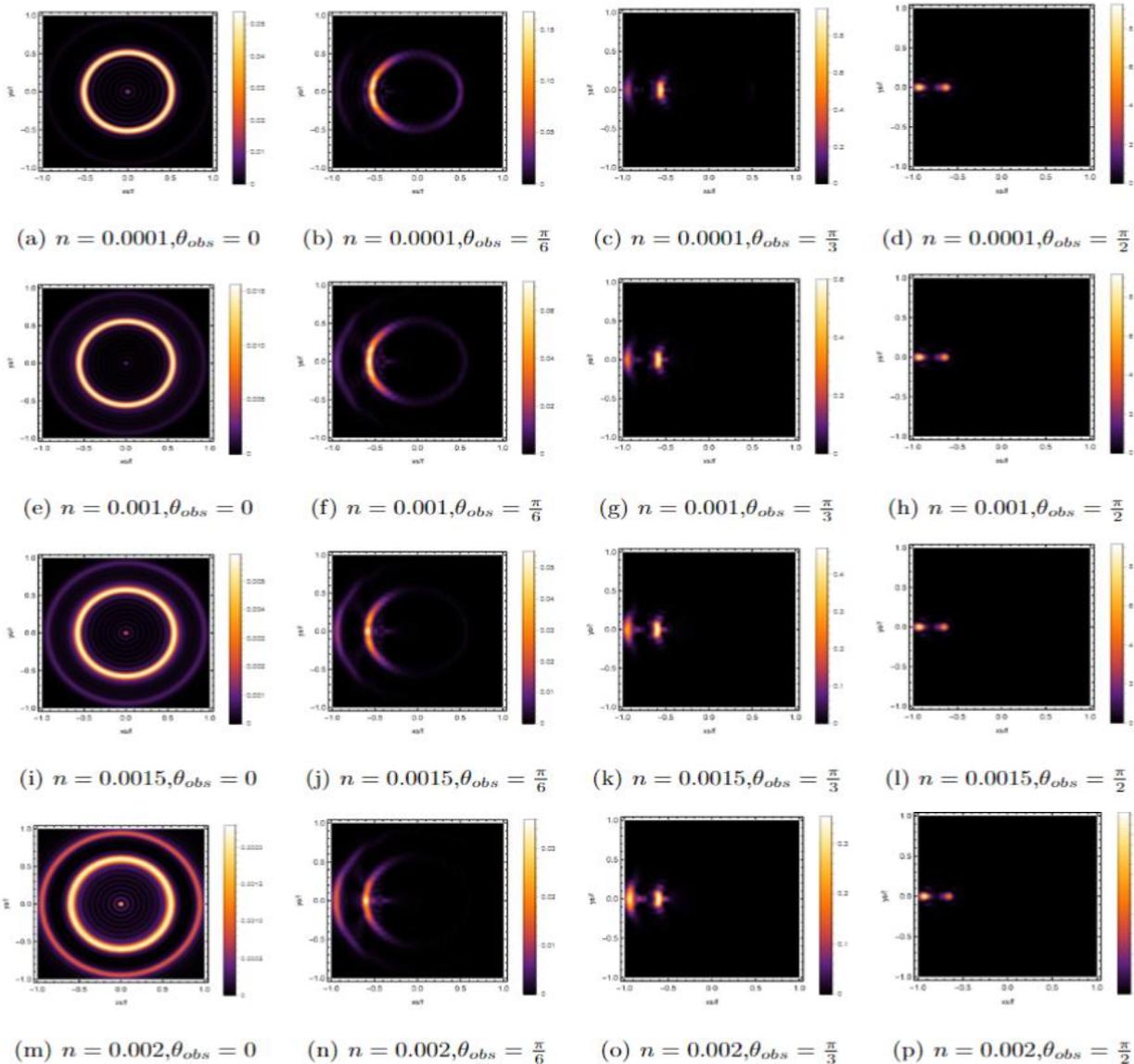
运动方程中 $F=0$

数值方法可以采用多种思路，我们采用伪谱方法，其关键在与选取节点，Chebyshev 多项式和构建Chebyshev 矩阵元，然后将相应的运动方程矩阵化。

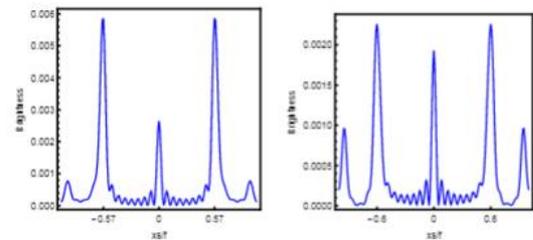


响应函数的幅度随不同参数的变化

4. 波动光学

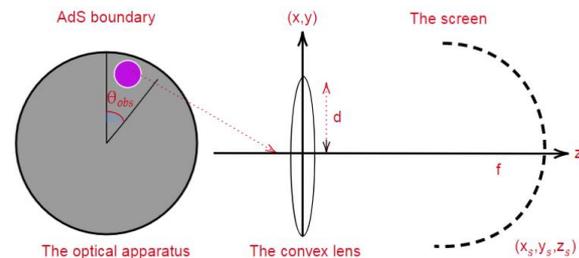


(a) $n = 0.0001$ (b) $n = 0.001$



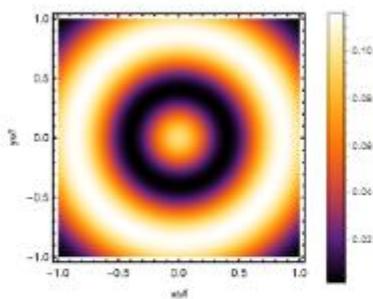
(c) $n = 0.0015$ (d) $n = 0.002$

不同非对易参数对应亮度曲线

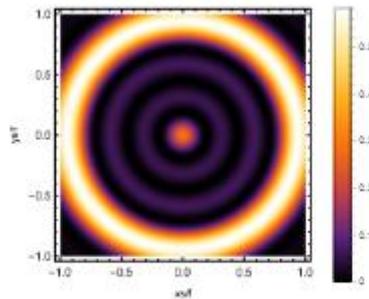


观者在不同观测角时，不同非对易参数对应的爱因斯坦环

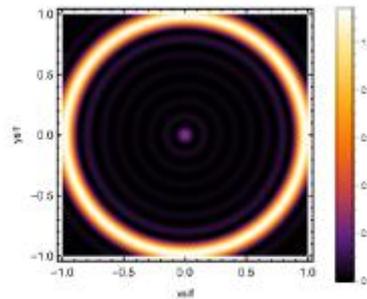
观测系统



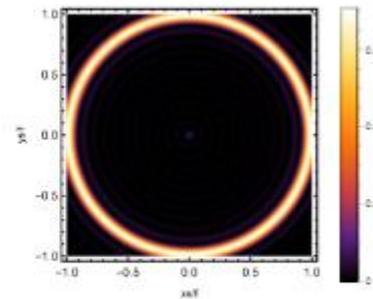
(a) $\omega = 11$



(b) $\omega = 21$

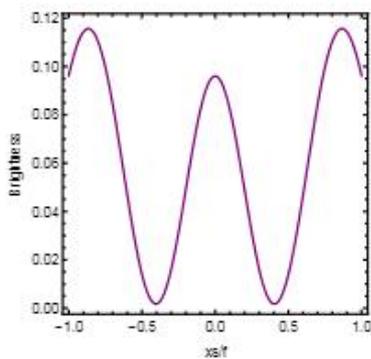


(c) $\omega = 41$

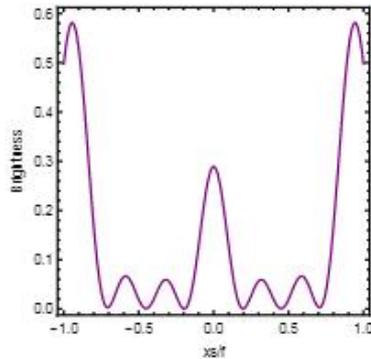


(d) $\omega = 61$

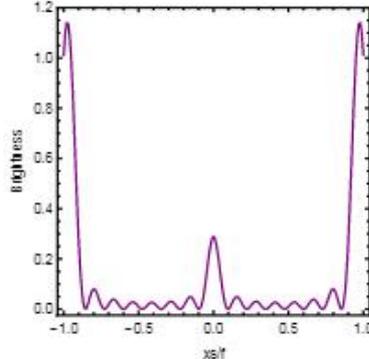
不同频率对应的爱因斯坦环



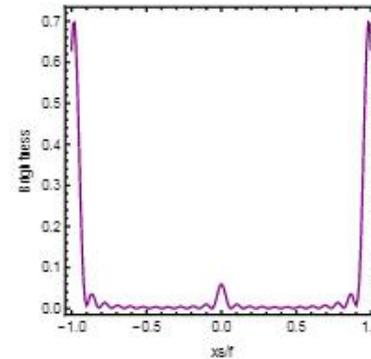
(a) $\omega = 11$



(b) $\omega = 21$

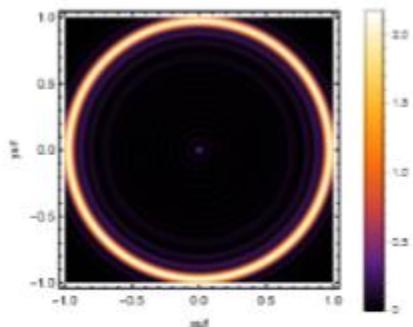


(c) $\omega = 41$

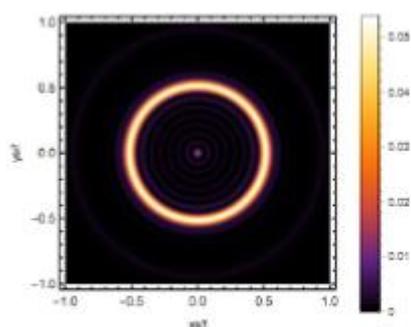


(d) $\omega = 61$

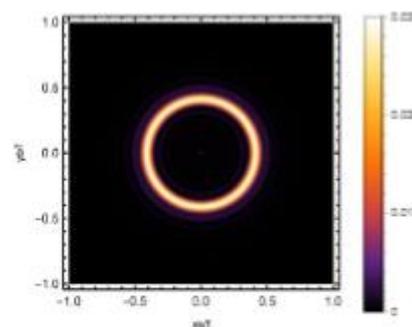
不同频率对应亮度曲线



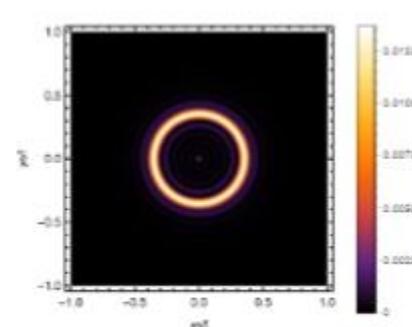
(a) $T = 0.299$



(b) $T = 0.393$

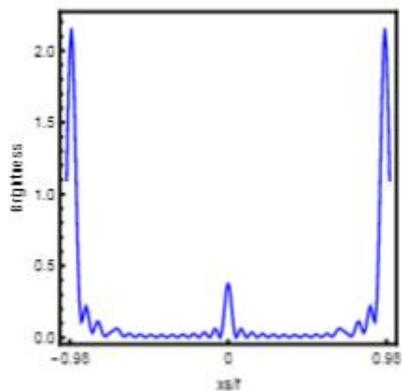


(c) $T = 0.485$

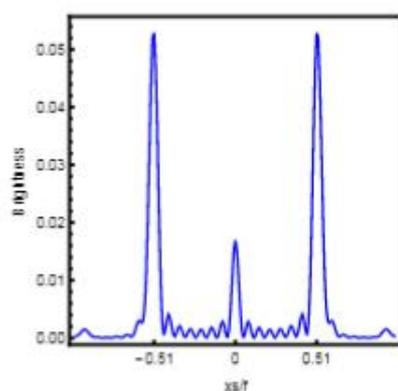


(d) $T = 0.558$

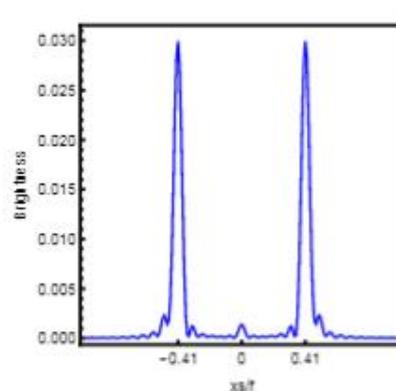
不同温度对对应的爱因斯坦环



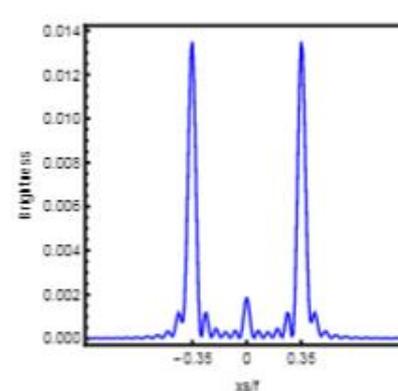
(a) $T = 0.299$



(b) $T = 0.393$



(c) $T = 0.485$



(d) $T = 0.558$

不同温度对应亮度曲线

几何光学分析

运动方程

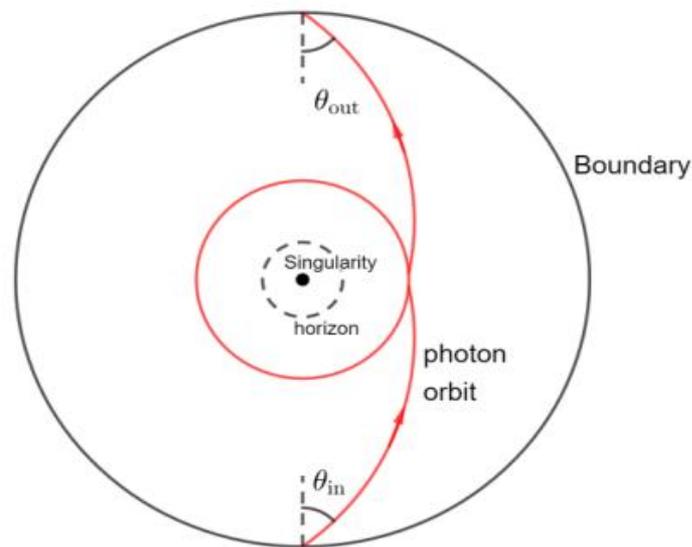
$$-F(r) \left(\frac{dt}{d\nu} \right)^2 + \frac{1}{F(r)} \left(\frac{dr}{d\nu} \right)^2 + r^2 \sin^2 \theta \left(\frac{d\phi}{d\nu} \right)^2 = 0,$$

$$\mathcal{R} = F(r)/r^2, \quad \omega = F(r)\dot{t}, \quad \mathcal{L} = r^2\dot{\phi}$$

$$\dot{r}^2 = \omega^2 - \mathcal{L}^2 \mathcal{R}$$

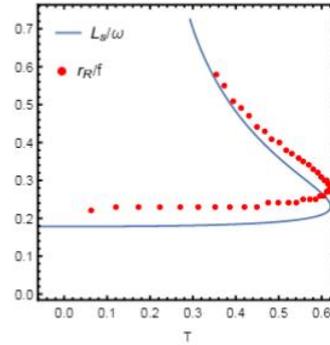
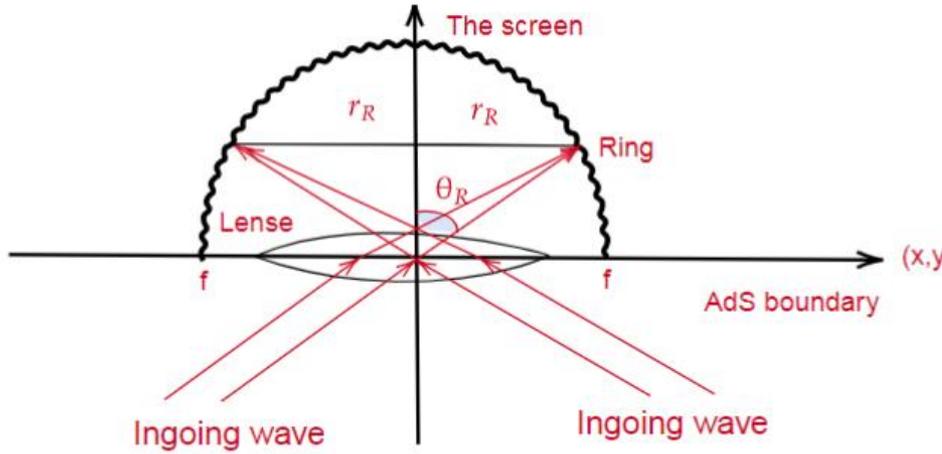
$$\begin{aligned} \cos \theta_{\text{in}} &= \frac{g_{jk} u^j n^k}{|u||n|} \Big|_{r=\infty} \\ &= \sqrt{\frac{\dot{r}^2/F}{2/F + L^2/r^2}} \Big|_{r=\infty} \end{aligned}$$

$$\begin{aligned} \sin^2 \theta_{\text{in}} &= 1 - \cos^2 \theta_{\text{in}} \\ &= \frac{\mathcal{L}^2 \mathcal{R}}{\dot{r}^2 + \mathcal{L}^2 \mathcal{R}} \Big|_{r=\infty} \\ &= \frac{\mathcal{L}^2}{\omega^2}. \end{aligned}$$

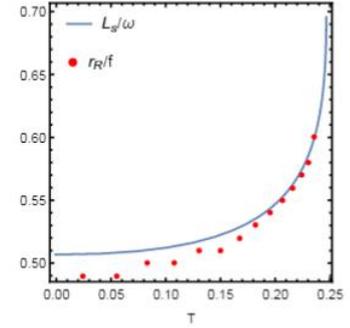


光子运动轨迹

4. 波动光学



(a) $n = 0.0001$



(b) $n = 0.001$

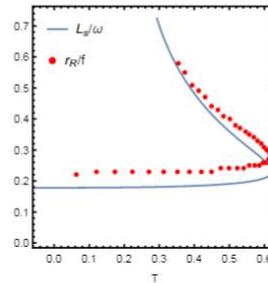
几何光学与波动光学的结果对比：不同非对易参数

环半径与环角的几何关系

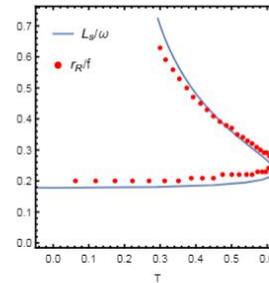
$$\sin \theta_R = \frac{r_R}{f}$$

$$\sin \theta_R = \sin \theta_{in}$$

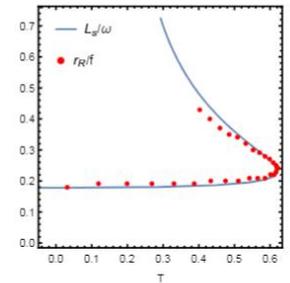
$$\frac{r_R}{f} = \frac{L_s}{\omega}$$



(a) $\omega = 80$



(b) $\omega = 120$



(c) $\omega = 160$

几何光学与波动光学的结果对比：不同波源频率

- 1: 在全息的框架下，成功地构建了黑洞阴影图像（爱因斯坦环）。
- 2: 为了得到阴影图像，须引入一个望远镜系统（透镜焦距、半径等不影响观测）。
- 3: 波动光学得结果与几何光学得到的结果完全一致。

谢谢！