量子修正史瓦西黑洞吸积盘的光学外观

报告人：黄宇翔
合作者：郭森、崔宇昊、蒋青权、林恺

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Image of black holes shadow

- The image of the central black hole in the M87 galaxy, captured by the Event Vision Telescope, marks the first time humans have seen a black hole.

- Some questions:
  - Why are there bright rings ?
  - Why the brightness of bright ring is asymmetric ?

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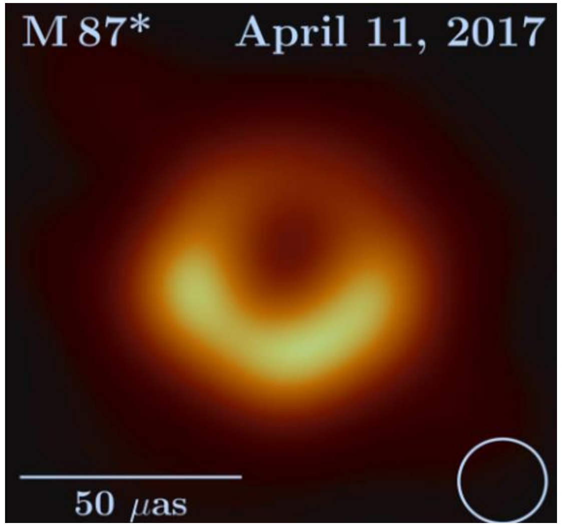
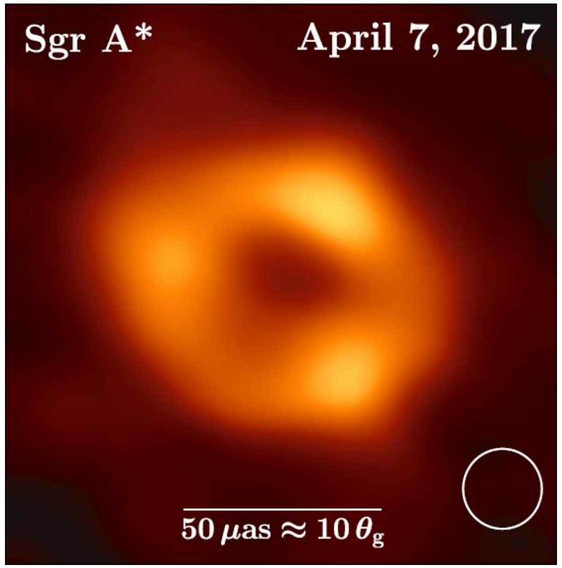
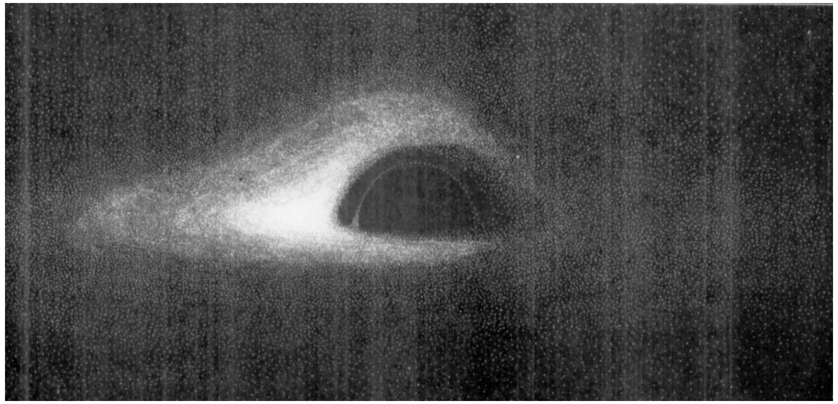




Quantum correction black holes

- The existence of black hole singularities leads to incomplete geodesics. Therefore, it is inevitable to consider the quantum effects around the singularity.



- The optical appearance of quantum corrected black holes seems to be significantly different from classical black holes.

J. -P. Luminet, Image of a Spherical Black Hole with Thin Accretion Disk. A&A, 1979.

D. I. Kazakov and S. N. Solodukhin, On Quantum deformation of the Schwarzschild solution, Nucl. Phys. B , 1994.

KS black hole effective potential

The geometry of four-dimensional spherically symmetric KS black hole is

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2,$$

with

$$f(r) = \frac{\sqrt{r^2 - a^2}}{r} - \frac{2M}{r},$$

where M is the mass of the black hole and a is a deformation parameter.

With the Euler-Lagrange equation

$$\frac{d}{d\lambda}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}\right) = \frac{\partial \mathcal{L}}{\partial x^\mu},$$

KS black hole effective potential

where Lagrangian is

$$\mathcal{L} = -\frac{1}{2} g_{\alpha\beta} \frac{\mathrm{d}x^{\alpha}}{\mathrm{d}\lambda} \frac{\mathrm{d}x^{\beta}}{\mathrm{d}\lambda} = \frac{1}{2}\left(f(r)\dot{t}^2 - \frac{\dot{r}^2}{f(r)} - r^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right)\right)$$

in which λ the affine parameter, x^{α} the four-velocity of the light ray, we can get

$$p_{\mathrm{t}} = g_{\mathrm{tt}}\dot{t} = -E, \qquad p_{\mathrm{r}} = g_{\mathrm{rr}}\dot{r}, \qquad p_{\phi} = g_{\phi\phi}\dot{\phi} = -L, \qquad p_{\theta} = g_{\theta\theta}\dot{\theta}.$$

we further can get the motion equation of the null geodesic

$$\frac{p_{\mathrm{t}}^2}{f(r)} - \frac{p_{\phi}^2}{r^2} = \frac{(p^{\mathrm{r}})^2}{f(r)}.$$

the radial component of the null geodesic is derived

$$p^{\mathrm{r}} = \pm E\sqrt{1 - \frac{b^2}{r^2} f(r)},$$

where $b = \frac{L}{E}$ is the impact parameter.

KS black hole effective potential

The effective potential is

$$V_{\text{eff}} = r^4\left(\frac{1}{b^2} - \frac{f(r)}{r^2}\right)$$

At the photon sphere, the motion equations of the light ray satisfy $\dot{r} = 0$ and $\ddot{r} = 0$, which also means

$$V_{\text{eff}} = \frac{1}{b_c^2} \qquad V'_{\text{eff}} = 0$$

where the prime denotes the first derivative with respect to the radial coordinate r.

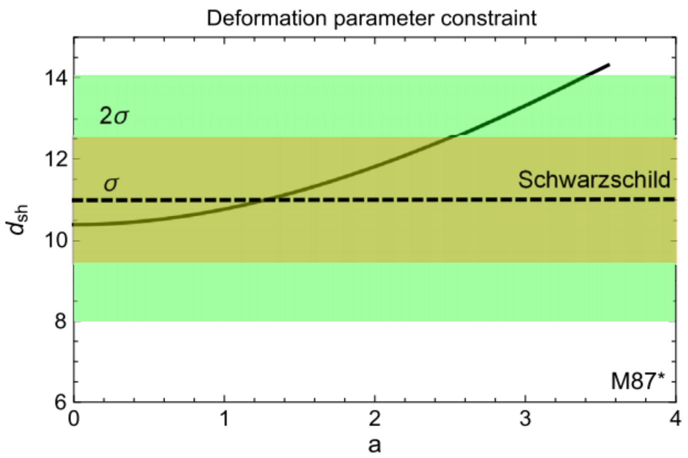
a	r_0	r_{ph}	r_{isco}	b_c
0	2	3	6	5.19615
0.1	2.0025	3.00333	6.0075	5.20048
0.5	2.06155	3.08193	6.18467	5.30259
1	2.23607	3.31284	6.7086	5.60259

The KS BH event horizon r_0 , shadow radius r_{ph} , and critical impact parameter b_c for different a values in case of a dimensionless BH mass of M=1.

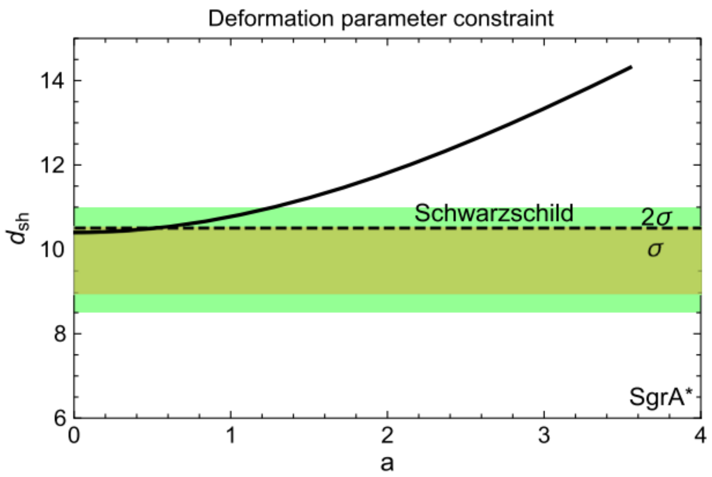
J. Peng, M. Y. Guo, and X. H. Feng, Influence of quantum correction on the black hole shadows, photon rings and lensing rings, Chin. Phys. C

Observation limitations

$1\sigma: \ a \leq 2.51$
$2\sigma: \ a \leq 3.39$



$$\delta = (42 \pm 3)\mu as$$
$$D = 16.8^{+0.8}_{-0.7} Mpc$$
$$M = (6.5 \pm 0.9) \times 10^9 M_\odot$$



$1\sigma: \ a \leq 0.55$
$2\sigma: \ a \leq 1.27$

$$\delta = (48.7 \pm 7)\mu as$$
$$D = (8.15 \pm 0.15) kpc$$
$$M = (4.0^{+1.1}_{-0.6}) \times 10^6 M_\odot$$

Shadow diameter of the KS BH as a function of a. The diameter of the $M87^*$ and Sgr A^* estimated with the EHT observations and its uncertainties in 1σ (2σ) confidence levels are marked with a horizonal dashed line and dark (or light) green shaded regions.

Light deflection in the KS black hole

By deformation of the null geodesic equation

$$\Omega(u) \equiv \left(\frac{\mathrm{d}u}{\mathrm{d}\phi}\right)^2 = 2Mu^3 + \frac{1}{b^2} - u^2\sqrt{1-a^2u^2},$$

where $u = \frac{1}{r}$. By Cardano formula, we can get

$$2Mu^3 + \frac{1}{b^2} - u^2\sqrt{1-a^2u^2} \equiv 2MG(u)$$
$$= 2M(u-u_1)(u-u_2)(u-u_3).$$

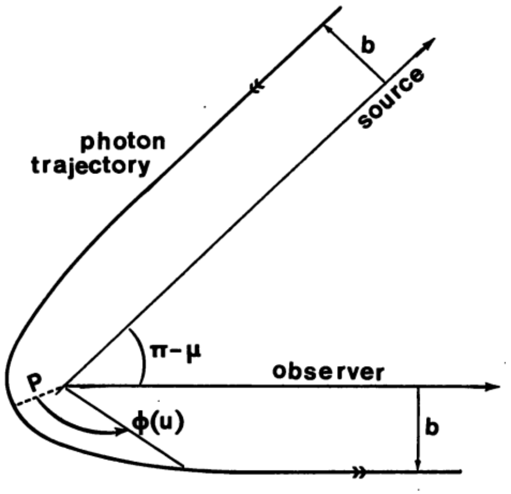
$b > b_c$: $u_1 \leq u_2 < u_3$

$b = b_c$: $u_1 = -\frac{1}{6M}$ $u_2 = u_3 = \frac{1}{3M}$

$b < b_c$: $u_1 \leq 0,$ u_2 and u_3 complex conjugate.

where $u_1 = \frac{P-2M-Q}{4MP}, \quad u_2 = \frac{1}{P}, \quad u_3 = \frac{P-2M+Q}{4MP}$

$$Q^2 \equiv (P-2M)(P+6M)$$



Trajectory of photons

Light deflection in the KS black hole

The bending angle of the light ray is

$$\psi(u) = \sqrt{\frac{2}{M}} \int_0^{u_2} \frac{\mathrm{d}u}{\sqrt{(u-u_1)(u-u_2)(u-u_3)}} - \pi.$$

Convert the above equation to elliptic integral

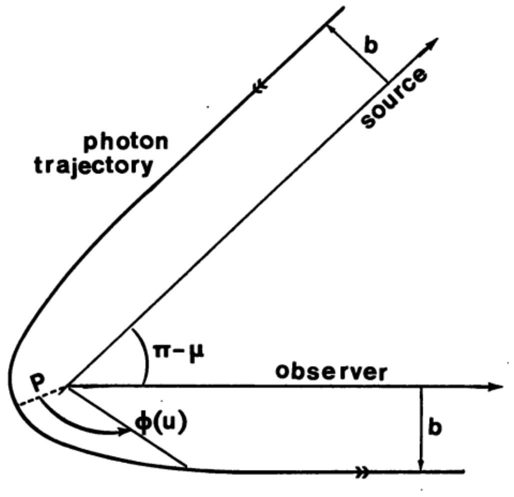
$$\psi(u) = \sqrt{\frac{2}{M}} \left(\frac{2F(\Psi_1, k)}{\sqrt{u_3 - u_1}} - \frac{2F(\Psi_2, k)}{\sqrt{u_3 - u_1}} \right) - \pi$$

where $\Psi_1 = \frac{\pi}{2}$, $\Psi_2 = \sin^{-1}\sqrt{\frac{-u_1}{u_2 - u_1}}$, $k^2 = \frac{u_2 - u_1}{u_3 - u_1}$.

The total change of bending angle is

$$\psi(u) = 2\sqrt{\frac{P}{Q}}(K(k) - F(\Psi_2, k)) - \pi$$

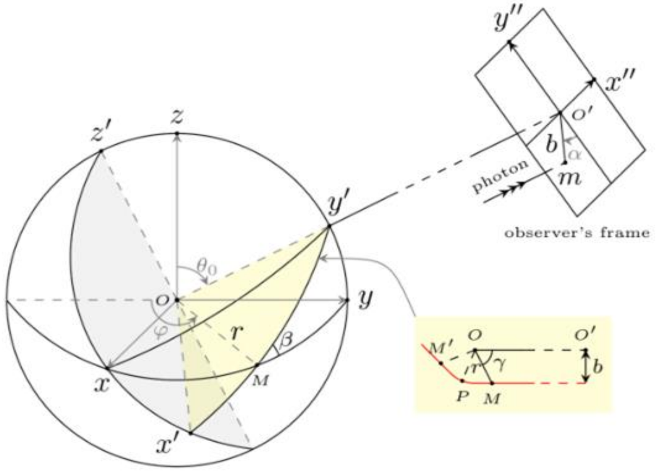
where *K(k)* is the complete elliptic integrals of the first kind.



Trajectory of photons

Thickness disk accretion

Novikov-Thorne disk model



The coordinate system

Direct image with coordinate $(b^{(d)}, \alpha)$ and the secondary image with coordinate $(b^{(s)}, \alpha + \pi)$

Assuming that the deflection angle from M to the observer is γ, and the observers inclination angle is θ_0 , we can get

$$\cos\alpha = \cos\gamma\sqrt{\cos^2\alpha + \cot^2\theta_0}.$$

Thickness disk accretion

For the direct image of the accretion disk

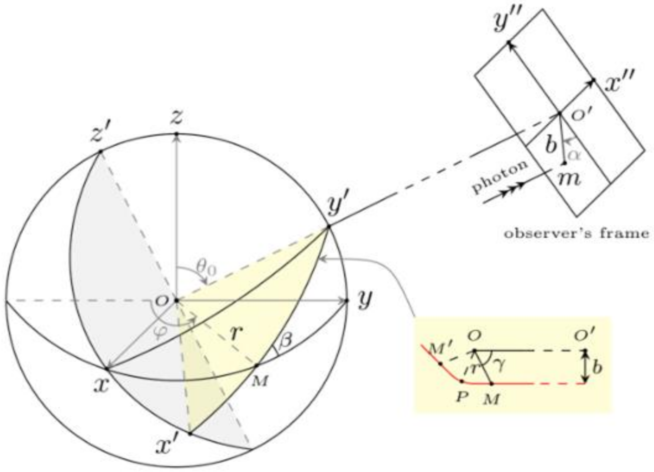
$$\gamma = \frac{1}{\sqrt{2M}} \int_0^{1/r} \frac{\mathrm{d}u}{\sqrt{G(u)}} = 2\sqrt{\frac{P}{Q}} \left(F(\zeta_r, k) - F(\zeta_\infty, k) \right)$$

where

$$k^2 = \frac{Q - R + 6M}{2Q},$$

$$sin^2\zeta_\infty = \frac{Q - R + 2M}{Q - R + 6M},$$

$$sin^2\zeta_r = \frac{Q - R + 2M + 4MP/r}{Q - R + 6M},$$



The coordinate system

The radius r as a function of the α and P is obtained, we have

$$\frac{1}{r} = \frac{P - 2M - Q}{4MP} + \frac{Q - P + 6M}{4MP} sn^2 \left(\frac{\gamma}{2}\sqrt{\frac{Q}{P}} + F(\zeta_\infty, k) \right)$$

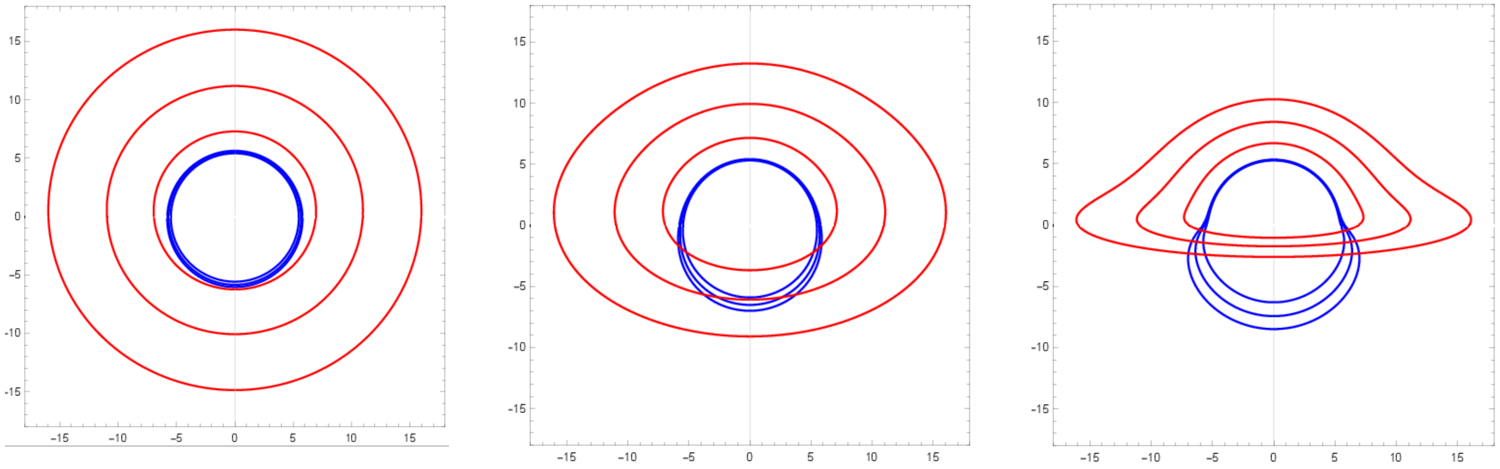
where sn is the Jacobi elliptic function.

Thickness disk accretion

For the (1 + n)th order image of the accretion disk is

$$2n\pi - \gamma = 2\sqrt{\frac{P}{Q}}\left(2K(k) - F(\zeta_r, k) - F(\zeta_\infty, k)\right)$$

in which *K(k)* is the complete elliptic integral.

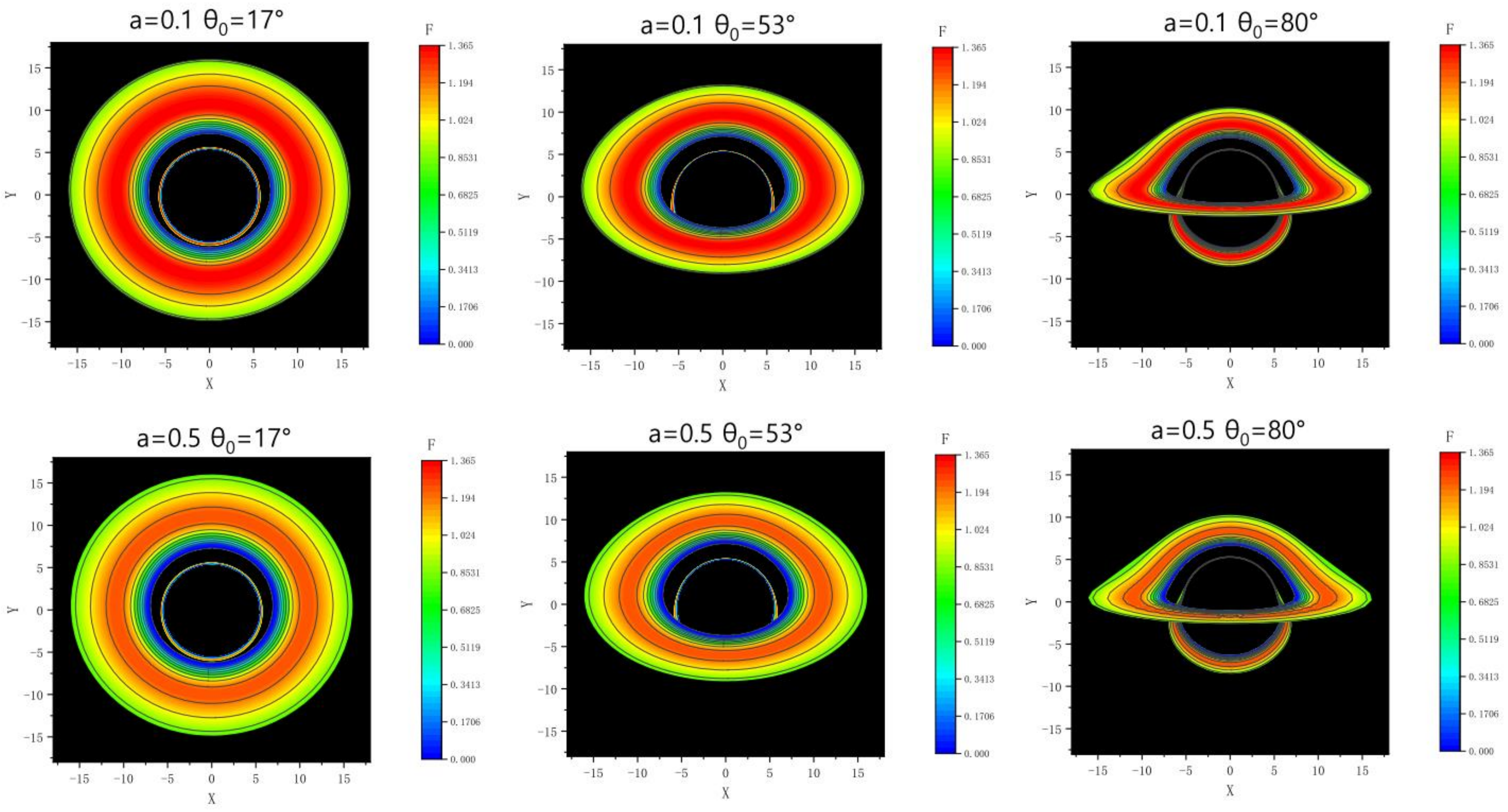


Direct and secondary images of the accretion disk around KS BH

Thickness disk accretion

The radial dependence of energy flux radiated by a thin accretion disk around a BH is

$$F=-\frac{\dot{M}}{4\pi\sqrt{-g}}\frac{\Omega_{,\mathrm{r}}}{(E-\Omega L)^2}\int_{r_{\mathrm{in}}}^{r}(E-\Omega L)L_{,\mathrm{r}}\mathrm{d}r$$



$$\Omega=\frac{\mathrm{d}\phi}{\mathrm{d}t}=\sqrt{-\frac{g_{\mathrm{tt,r}}}{g_{\phi\phi,\mathrm{r}}}}.$$

$$E=-\frac{g_{\mathrm{tt}}}{\sqrt{-g_{\mathrm{tt}}-g_{\phi\phi}\Omega^2}},$$

$$L=\frac{g_{\phi\phi}\Omega}{\sqrt{-g_{\mathrm{tt}}-g_{\phi\phi}\Omega^2}},$$

Direct images of the accretion disk around KS BH

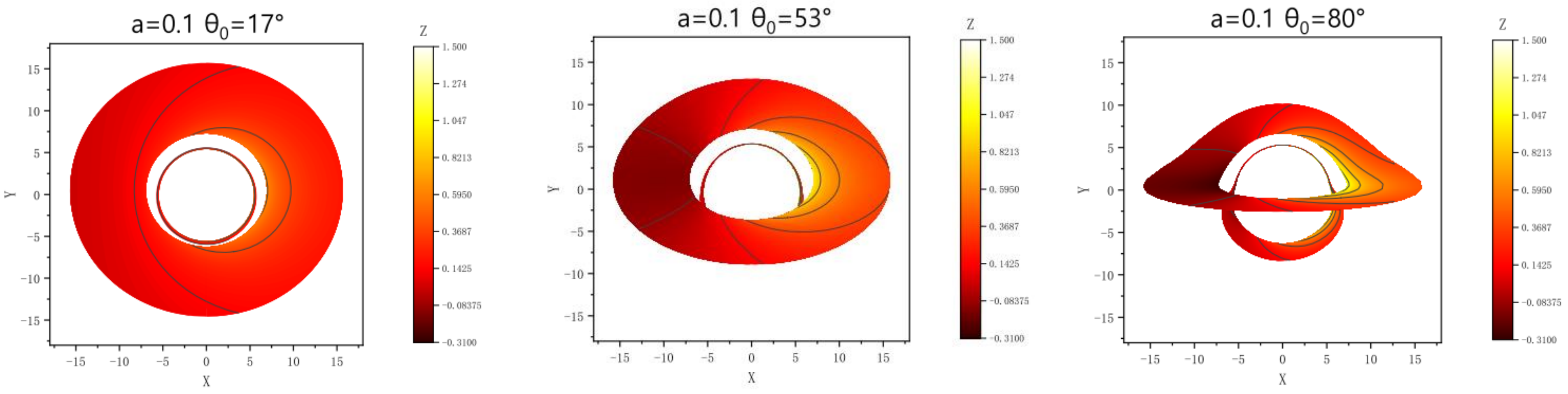
Thickness disk accretion

The observed flux F_{obs} is different from the source F duo to the redshift

$$F_{\text{obs}} = \frac{F}{(1+z)^4}.$$

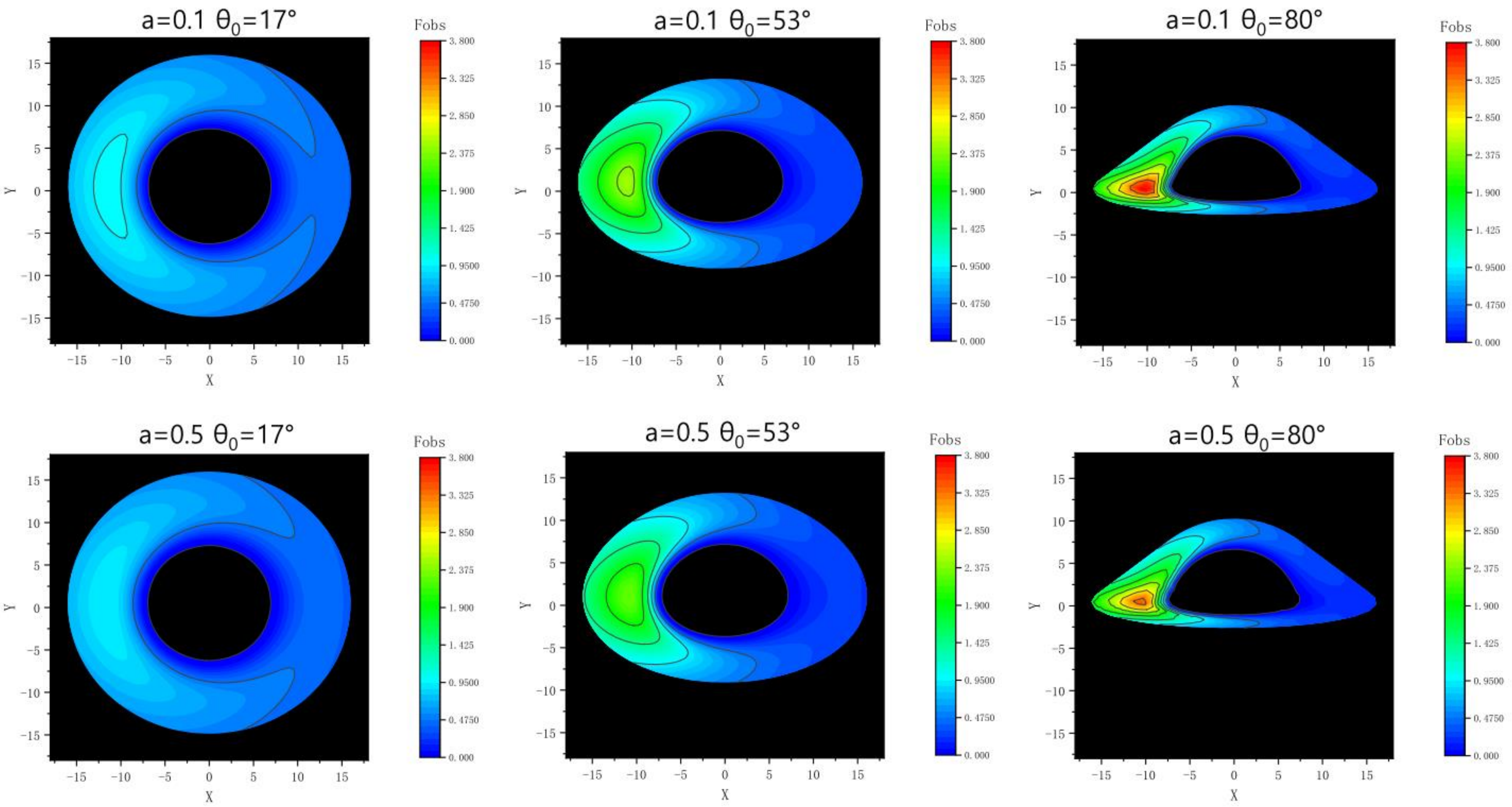
The redshift factor is

$$1 + z = \frac{E_{\text{em}}}{E_{\text{obs}}} = \frac{1 + b\Omega \cos\beta}{\sqrt{-g_{\text{tt}} - 2g_{\text{t}\phi} - g_{\phi\phi}}}.$$



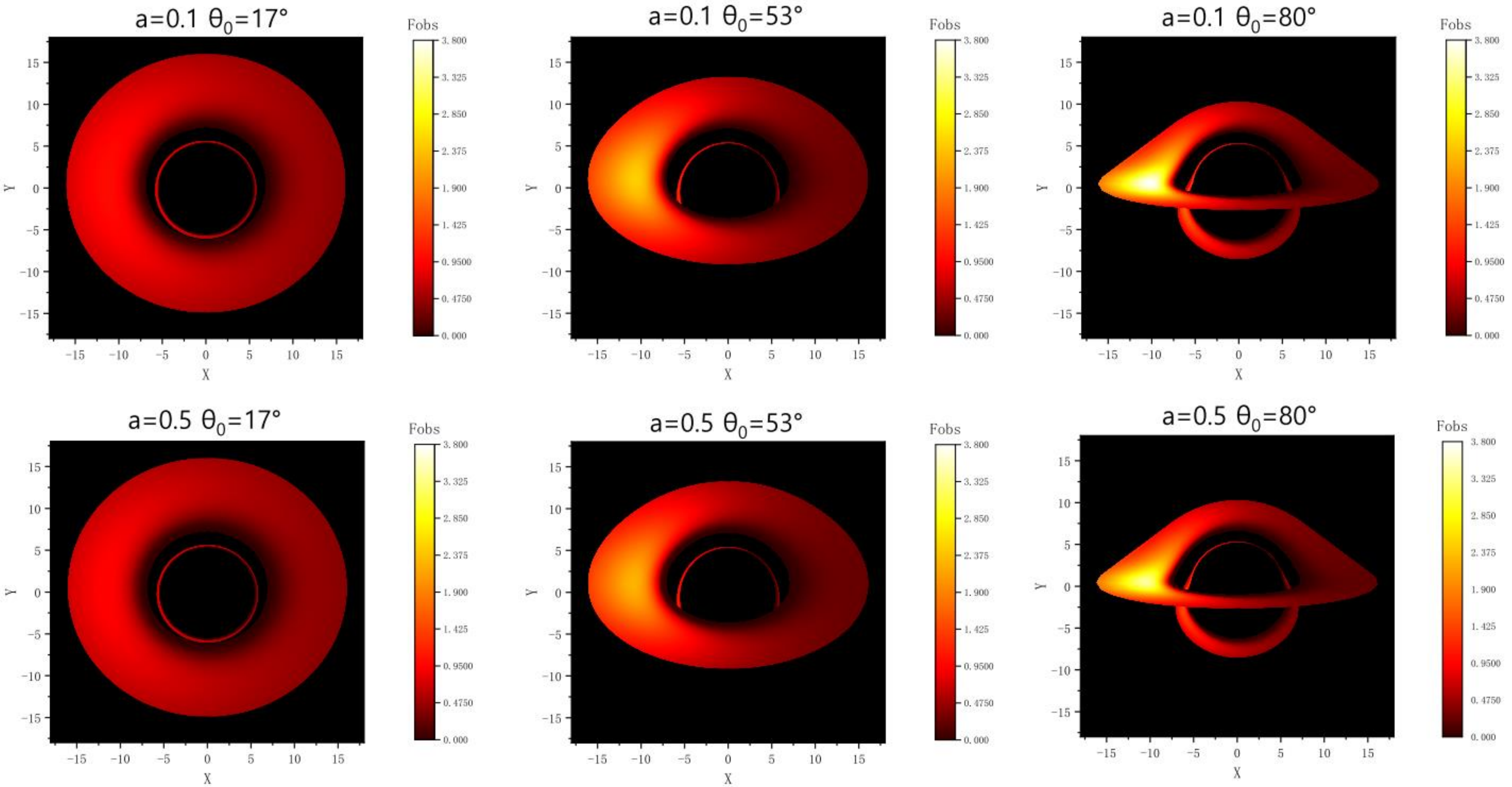
Redshift distribution (curves of constant redshift z) of the accretion disk around the KS BH

Thickness disk accretion



Flux distribution in unit of F_{obs} of direct image for the KS BH

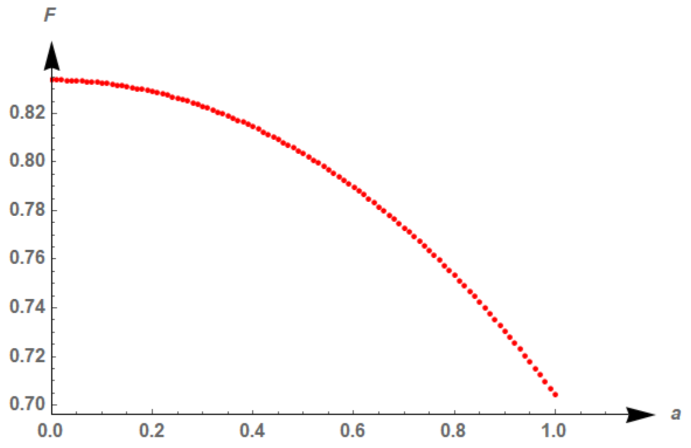
Thickness disk accretion



Flux distribution in unit of F_{obs} of direct image for the KS BH

Thickness disk accretion

$$F_{\text{obs}} = \frac{F}{(1+z)^4}.$$



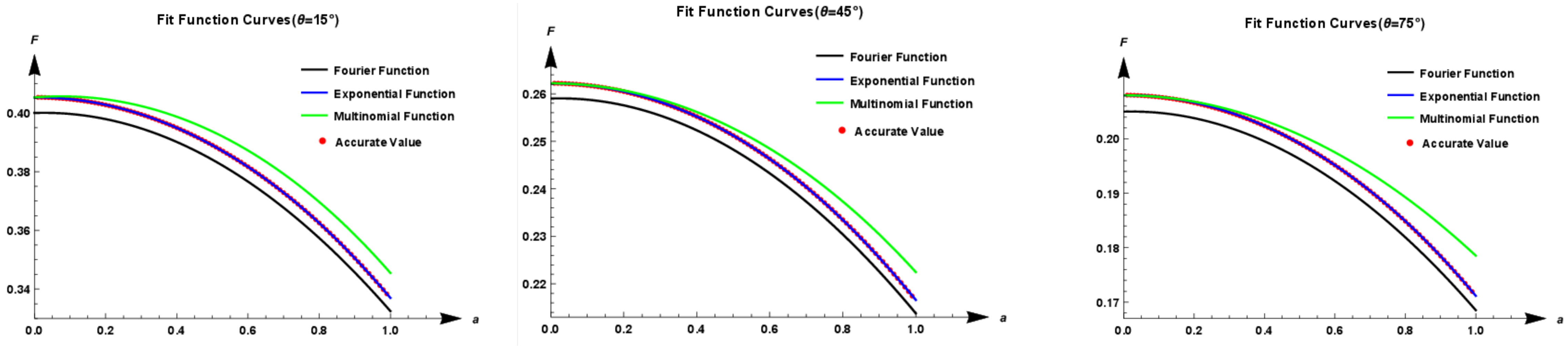
Flux distribution in unit of F_{obs} of direct image for the KS BH

Exponential Function: $F = a_1 e^{-\left(\frac{a-a_2}{a_3}\right)^2} + a_4 e^{-\left(\frac{a-a_5}{a_6}\right)^2}$

Fourier Function: $F = b_1 + b_2 \cos(ab_3) + b_4 \sin(ab_5)$

Multinomial Function: $F = c_1 a^6 + c_2 a^5 + c_3 a^4 + c_4 a^3 + c_5 a^2 + c_6 a + c_7$

Thickness disk accretion



Flux distribution in unit of F_{obs} of direct image for the KS BH

Function	F	$F_{\text{obs}}(\theta_0 = 17°)$	$F_{\text{obs}}(\theta_0 = 53°)$	$F_{\text{obs}}(\theta_0 = 80°)$
Fourier	6.1196×10^{-5}	2.46304×10^{-5}	8.91666×10^{-6}	8.25137×10^{-6}
Exponential	4.64208×10^{-9}	1.02525×10^{-9}	7.94073×10^{-10}	8.33254×10^{-10}
Multinomial	9.21466×10^{-5}	2.67287×10^{-5}	7.06738×10^{-6}	9.61621×10^{-6}

The mean squared error of Fourier functions, Exponential functions, and Multinomial functions with accurate values for $\alpha = \frac{\pi}{2}$ and $r = 15M$.

Summary

- The deformation parameter values of KS BH can be constrained through the limitations imposed by EHT observational data.

- The optical appearance of a black hole surrounded by a thick accretion disk depends on the observation angle.

- An elevation in deformation parameters results in a reduction of the extensive radiation flux range on the accretion disk.

- The relationship between radiation flux and deformation parameter values adheres to an exponential function pattern.

Thank you!