The QCD phase transition intrigued by turbulence

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1. Introduction
2. Framework
3. Algorithm
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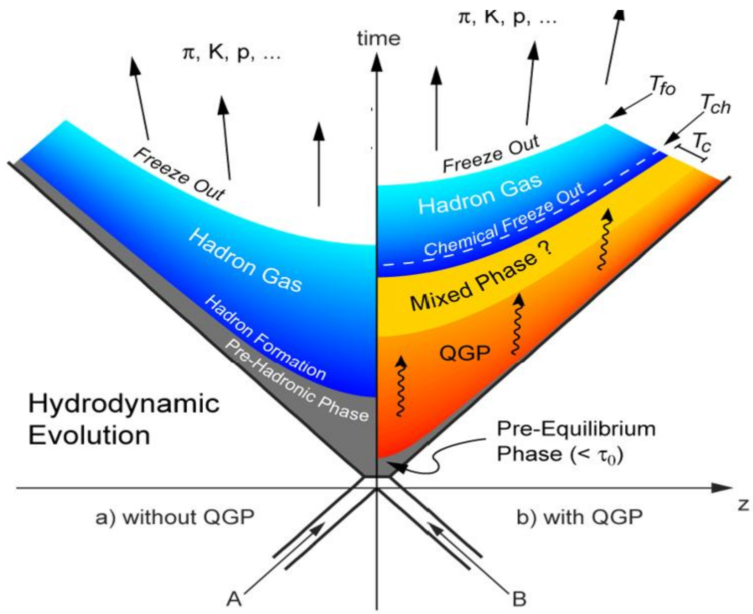
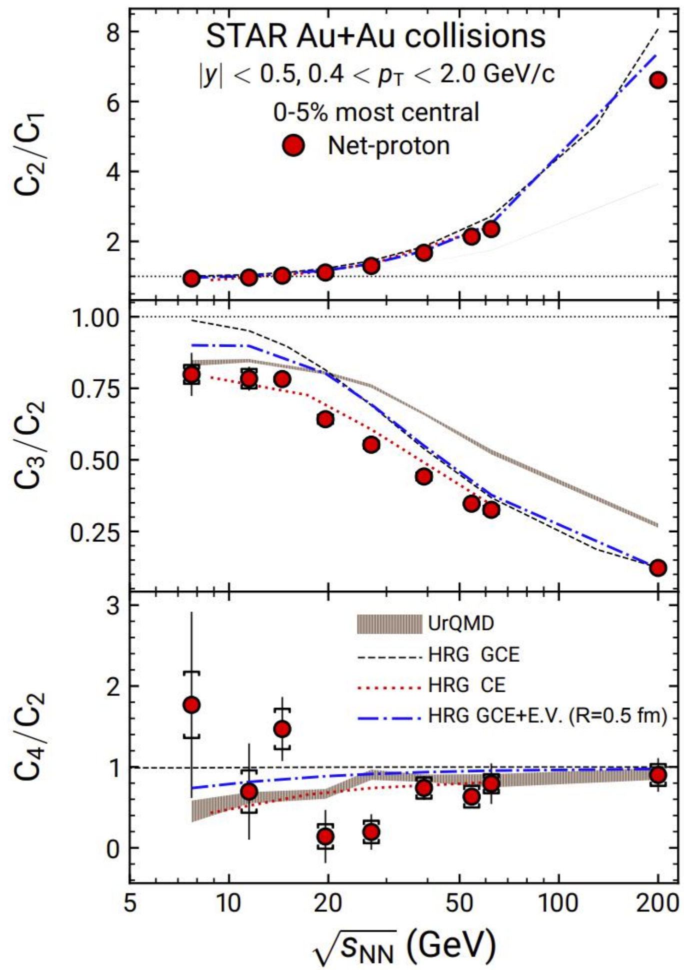
5. Conclusion and Future

Introduction



J. D. Bjorken Physical Review D 27 (1983) 140

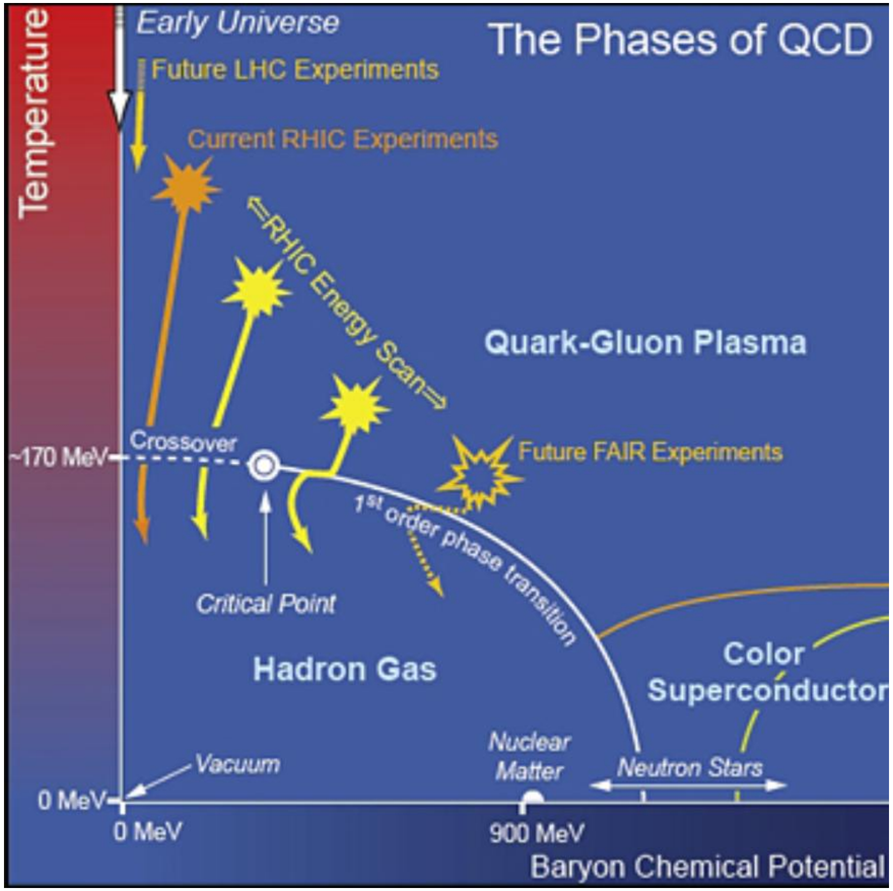
Introduction



$$\delta N = N - \langle N \rangle$$

$$\begin{aligned} C_1 &= \langle N \rangle, \\ C_2 &= \langle (\delta N)^2 \rangle = \mu_2, \\ C_3 &= \langle (\delta N)^3 \rangle = \mu_3, \\ C_4 &= \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2 \\ &= \mu_4 - 3\mu_2^2, \end{aligned}$$

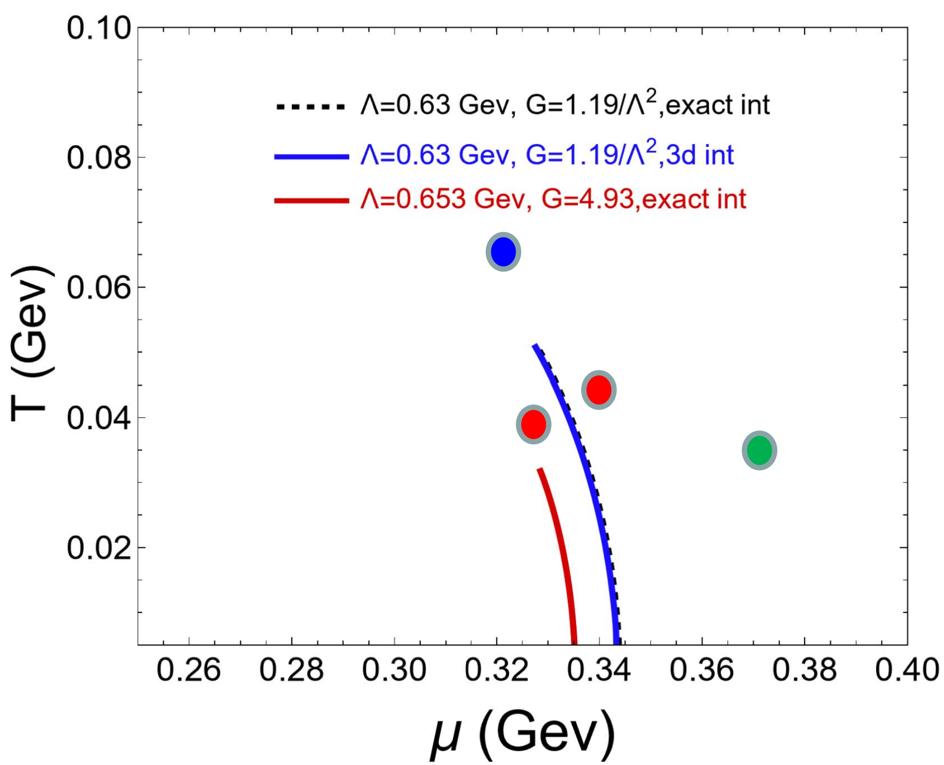
Introduction



USA-NSAC 2007 Long-range Plan

Introduction

$$\mathcal{L} = \bar{\psi}(i\hbar\gamma^{\mu}\partial_{\mu} - m_0)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2]$$



Our motivation is to study the evolution of the QCD system when there is a fluctuation around the phase transition line. For example, this fluctuation is caused by turbulence.

Introduction

The isotropic turbulence can be given by the following

$$u^i = \sum_n a(n_1, n_2, n_3) \sin 2\pi(n_1 x + n_2 y + n_3 z + \epsilon),$$

Where $u^\mu = (u_0, u_x, u_y, u_z)$, $u^i = (u_x, u_y, u_z)$, $u^0 = \sqrt{1+(u^i)^2}$, and $a = \sqrt{C^2 k^2 e^{-2(k/k_0)^2}}$, $k = 2\pi\, n$ is wave vector and $k_0 = 10$, $k = |\mathbf{k}|$, ϵ is the random number in $[0, 1]$ for each combination of $n = n_1, n_2, n_3$, $n_i = 0, \pm 1, \pm 2, \cdots$, $i = 1, 2, 3$.

1. A. Sakurai and F. Takayama. Molecular kinetic approach to the problem of compressible turbulence.
2. V.V. Aristov and O.I. Rovenskaya. *Application of the Boltzmann kinetic equation to the eddy problems.*

Introduction

$$\frac{\partial f}{\partial t}+\xi\frac{\partial f}{\partial \mathbf{x}}=J(f,f)=\int\int_0^{2\pi}\int_0^{b_m}(f'f'_*-f\,f_*)gbdbd\varepsilon d\boldsymbol{\xi}_*=-\nu(f)f+N(f,f),$$

The initial condition with isotropic turbulence

$$f_M(t,\mathbf{x},\xi)=\frac{\rho_0}{(2\pi T_0)^{3/2}}\exp\left[-\frac{\mathbf{c}^2}{2T_0}\right],\quad \mathbf{c}=\xi-\mathbf{V}_0,$$
$$\mathbf{V}_0=(u_0,v_0,w_0),$$

$$u^i=\sum_n a(n_1,n_2,n_3)\sin 2\pi(n_1x+n_2y+n_3z+\epsilon),$$

1. A. Sakurai and F. Takayama. Molecular kinetic approach to the problem of compressible turbulence.
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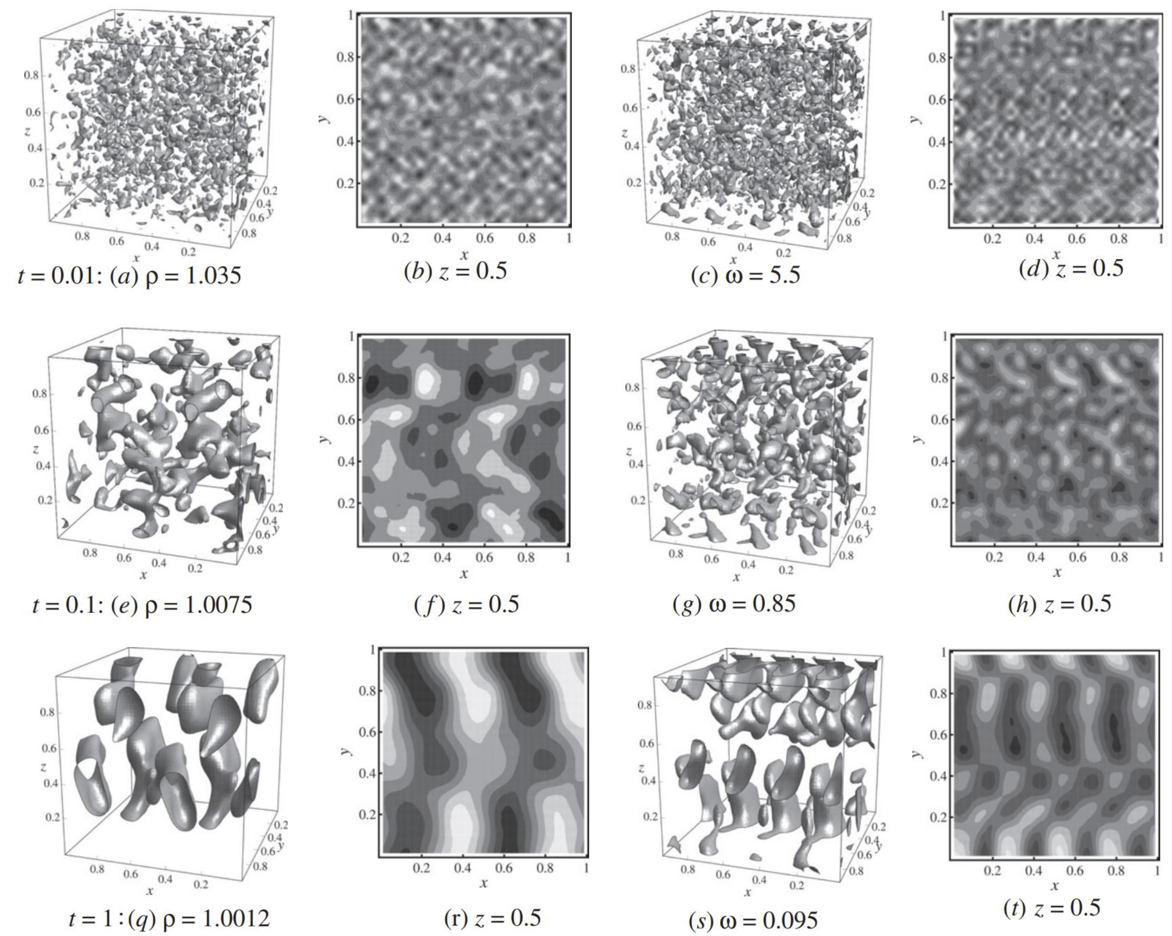
Introduction

The evolution of classical system intrigued by turbulence

$\rho(x, y, z)$ $\rho(x, y, z = 0.5)$ $\omega(x, y, z)$ $\omega(x, y, z = 0.5)$



1. A. Sakurai and F. Takayama. Molecular kinetic approach to the problem of compressible turbulence.
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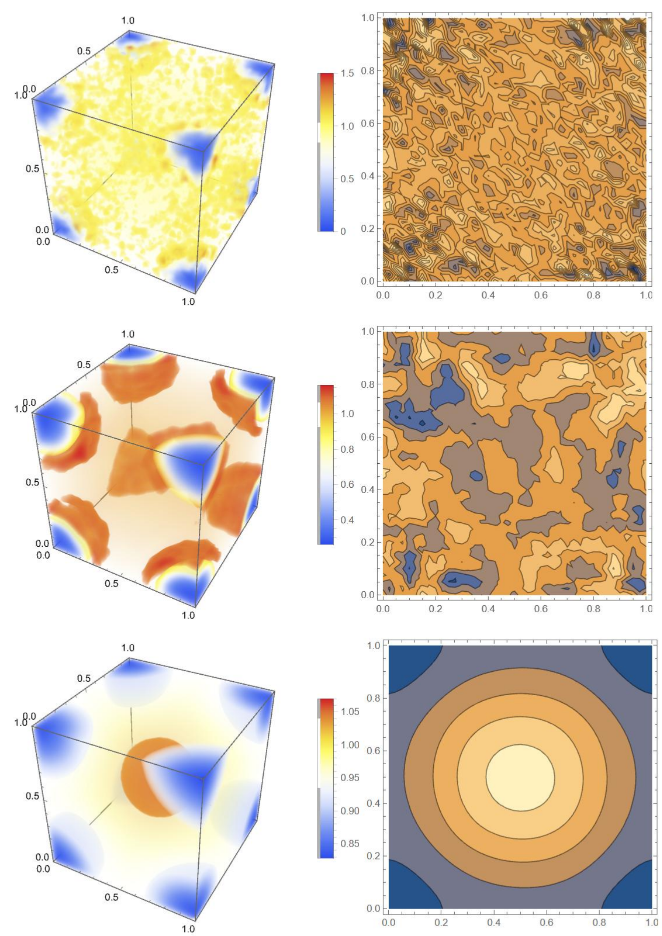
Introduction

The evolution of classical system intrigued by turbulence

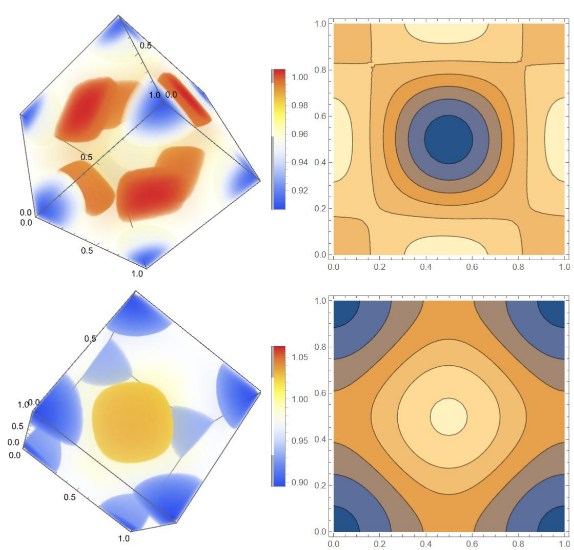
$\rho(x, y, z)$ $\rho(x, y, z = 0.5)$

t

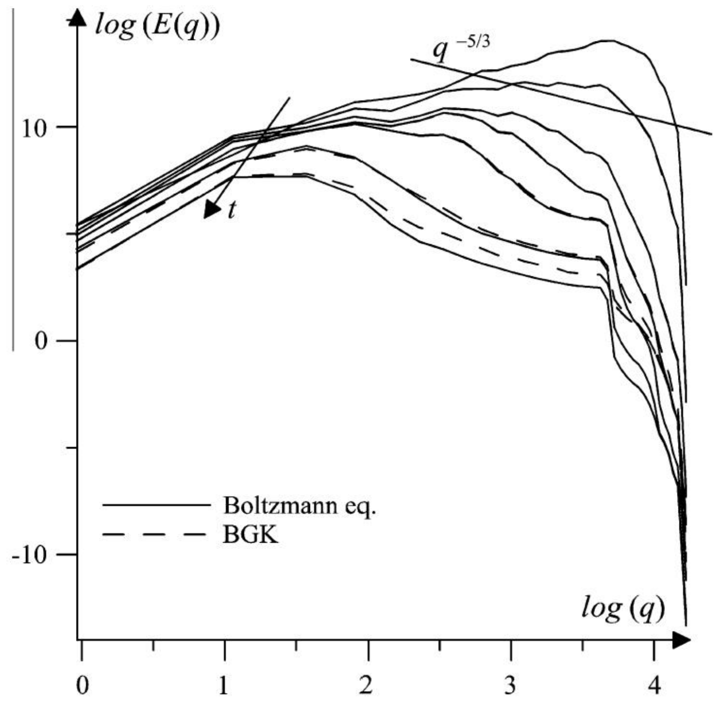


We recalculated it to a long time.

$\rho(x, y, z)$ $\rho(x, y, z = 0.5)$



Introduction



BGK:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = -\frac{\mathrm{f} - f_M}{\tau}$$

Boltzmann eq:

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} = C[f]$$

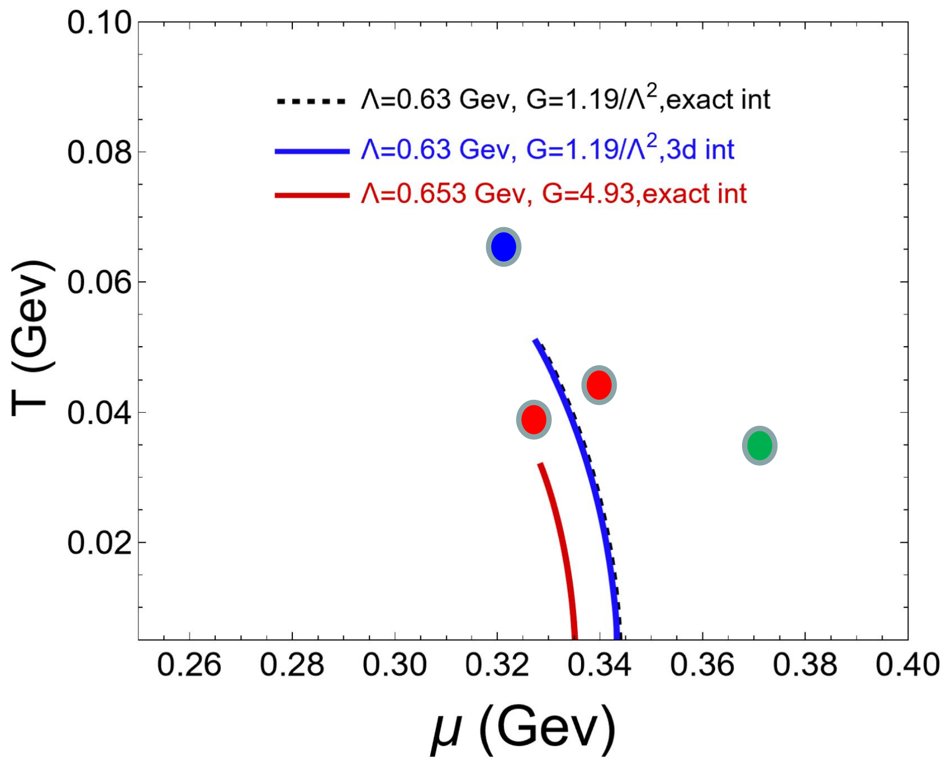
$$E_{nml} = (u_{x,nml}^2 + u_{y,nml}^2 + u_{z,nml}^2)/2.$$

$$q = \sqrt{n^2 + m^2 + l^2},$$

There are number of points lay in the lane with inclination "-5/3", as expected for the inertial interval of 3D isotropic turbulence.

A.N. Kolmogorov, *J. Fluid Mech.*, **13**, 82-85, (1962).

Introduction



$$u^i = \sum_n a(n_1, n_2, n_3) \sin 2\pi(n_1 x + n_2 y + n_3 z + \epsilon),$$

Let's explore how the QCD system evolves in the presence of fluctuations around the phase transition line, which are caused by turbulence.

Framework

$$\mathcal{L} = \bar{\psi}(i\hbar\gamma^{\mu}\partial_{\mu} - m_0)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2]$$

$$\mathbf{v}_q = \mathbf{p}/E_p, \text{ and } E_p = \sqrt{\mathbf{p}^2 + m(x)^2}$$

transport equation: $$\partial_t f_q + \mathbf{v}_q \cdot \nabla_{\mathbf{x}} f_q - \nabla_{\mathbf{x}} E_p \cdot \nabla_{\mathbf{p}} f_q = C[f_q].$$

Gap equation: $$m(x) = m_0 + 4GN_cN_f m(x) \int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} (1 - f_q(x,p) - f_{\bar{q}}(x,p))$$

1. *Che Ming Ko, et al, Nucl.Phys.A 928 (2014) 234-246.*
2. P. Rehberg, et al, *Nucl.Phys.A 653 (1999) 415-435.*
3. WeiMin Zhang, et al, Phys.Rev.C 45 (1992) 1900-1917.
4. Pengfei Zhuang, et al, *Quantum transport theory and nonequilibrium chiral condensates.*
5. Abdellatif Abada, Chiral Phase Transition in an Expanding Quark-Antiquark Plasma.

Framework

For simplicity, we chose the collision term with the relaxation time model.

$$C[f_q] = -\frac{p \cdot u}{E_p} \frac{f_q - f_q^{eq}}{\tau}$$

τ is the relaxation time, which is chosen as $\tau = 0.1$fm or use the relaxation time formula.

$$f_q^{eq} = \frac{1}{e^{(p \cdot u - \mu(x))/T(x)} + 1}$$

herein, the temperature and chemical potential are fixed by the matching conditions for number density and energy at every space-time point, i.e

$$n_{eq}(x) = n(x)$$
$$e_{eq}(x) = e(x)$$

$\Longrightarrow$ $\mu(x),\ T(x)$

Framework

The number density $n_{eq}(x)$ and energy density $n_{eq}(x)$are all defined at the local rest frame of the fluid.

$$n_{eq}(x) = \int \frac{d^3p}{(2\pi)^3}(f_q^{eq} - f_{\bar{q}}^{eq}) = \int \frac{d^3p}{(2\pi)^3}(\frac{1}{e^{(p\cdot u-\mu)/T}+1} - \frac{1}{e^{(p\cdot u+\mu)/T}+1}),$$

$$e_{eq}(x) = \int \frac{d^3p}{(2\pi)^3}E_p(f_q^{eq} + f_{\bar{q}}^{eq}) = \int \frac{d^3p}{(2\pi)^3}E_p(\frac{1}{e^{(p\cdot u-\mu)/T}+1} + \frac{1}{e^{(p\cdot u+\mu)/T}+1})$$

Where $p\cdot u = p_0 = E_p$, $u^\mu = (1, \mathbf{0})$

While the number density n(x) and energy density e(x) are defined by the following

$$n(x) = J^\mu u_\mu, \quad e(x) = T^{\mu\nu}u_\mu u_\nu.$$

Framework

The current density $J^{\mu}(x)$ and stress tensor density $T^{\mu\nu}(x)$ are defined by the following,

$$J^{\mu} = \int \frac{d^3p}{(2\pi)^3} \frac{p^{\mu}}{E_p} (f_q - f_{\bar{q}}), \quad T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{p^{\mu}p^{\nu}}{E_p} (f_q + f_{\bar{q}})$$

Framework

$$n(x) = J^\mu u_\mu, \quad e(x) = T^{\mu\nu} u_\mu u_\nu.$$

The fluid velocity $u^\mu = \gamma(1, v_x, v_y, v_z)$ can be obtained by the ernergy flow, $T^{\mu\nu} u_\nu = e u^\mu$, then we can get

$$\frac{T^{00} - T^{0i} v^i}{T^{0i} - T^{ij} v^j} = 1/v^i,$$

$$\Rightarrow T^{0i} - T^{ij} v^j = T^{00} v^i - T^{0j} v^i v^j.$$

Where $v^i = (v^x, v^y, v^z)$ for i=1,2,3.

Let use the perturbation method to solve velocity, first let the velocity expanded order by order like as

$$v^i = v_1^i + v_2^i + v_3^i + \cdots$$

$$v_1^i \sim \mathbf{O}(1), v_2^i \sim \mathbf{O}(2), v_3^i \sim \mathbf{O}(3)$$

Framework

Let assume

$$v_1^i \sim \mathbf{O}(1), v_2^i \sim \mathbf{O}(2), v_3^i \sim \mathbf{O}(3)$$

$$\hat{T}^{ij} \sim T^{0i} \sim T^{i0} \sim \mathbf{O}(1)$$

Where $\widehat{T}^{ij} = T^{ij} - T^{ii}$, Then we can now solve the velocity order by order.

$$T^{0i} - T^{ij}v^j = T^{00}v^i - T^{0j}v^iv^j$$

$$\Downarrow$$

the first order

$$v_1^i = \frac{T^{i0}}{T^{00} + T^{ii}}$$

the second order

$$v_2^i = -\frac{\hat{T}^{ij}v_0^j}{T^{00} + T^{ii}}$$

the third order

$$v_3^i = -\frac{\hat{T}^{ij}v_2^j}{T^{00} + T^{ii}} + \frac{T^{0j}v_1^iv_1^j}{T^{00} + T^{ii}}$$

Framework

Finally, we will sum all of these above results as the velocity, $v^i = v_1^i + v_2^i + v_3^i$. The corresponding fluid velocity $u^\mu = (u_0, u^i)$ can be constructed by $u_0 = \gamma = \frac{1}{\sqrt{1-v^2}}$, $u^i = u_0 * v^i$, herein, $v^2 = \sum_i (v^i)^2$.

There is another alternative method to obtain the fluid velocity named the Eckart frame. In Eckart's frame, the fluid velocity can be more easily get by the following relations,

$$v^i = \frac{J^i}{J^0}$$

Framework

$$\mathrm{C}[\mathrm{f}_q] = -\frac{\mathrm{p}\cdot u}{E_p}\frac{f_q - f_q^{eq}}{\tau}$$

$$n_{eq}(x) = n(x)$$
$$e_{eq}(x) = e(x)$$

$$\xrightarrow{\mu(x),\ T(x)}$$

$$f_q^{eq} = \frac{1}{e^{(p\cdot u - \mu(x))/T(x)} + 1}$$

Algorithm

$$\partial_t f_q + \mathbf{v}_q \cdot \nabla_{\mathbf{x}} f_q - \nabla_{\mathbf{x}} E_p \cdot \nabla_{\mathbf{p}} f_q = C[f_q].$$

$$+$$

$$m(x) = m_0 + 4GN_cN_f m(x) \int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} \frac{1}{E_p} (1 - f_q(x,p) - f_{\bar{q}}(x,p))$$

The transport equation will be split into two steps.

1. The first step is to describe the free-streaming of particles.

$$\partial_t f_q + \mathbf{v}_q \cdot \nabla_{\mathbf{x}} f_q - \nabla_{\mathbf{x}} E_p \cdot \nabla_{\mathbf{p}} f_q = 0.$$

To solve this part, we can use the QUICK algorithm or WENO method.

2. The second step is collision term

$$\partial_t f_q = C[f_{eq}].$$

Algorithm

We use the boundary conditions as follows

$$f(x+1, y+1, z+1, \vec{p}) = f(x, y, z, \vec{p})$$

The grid of phase space

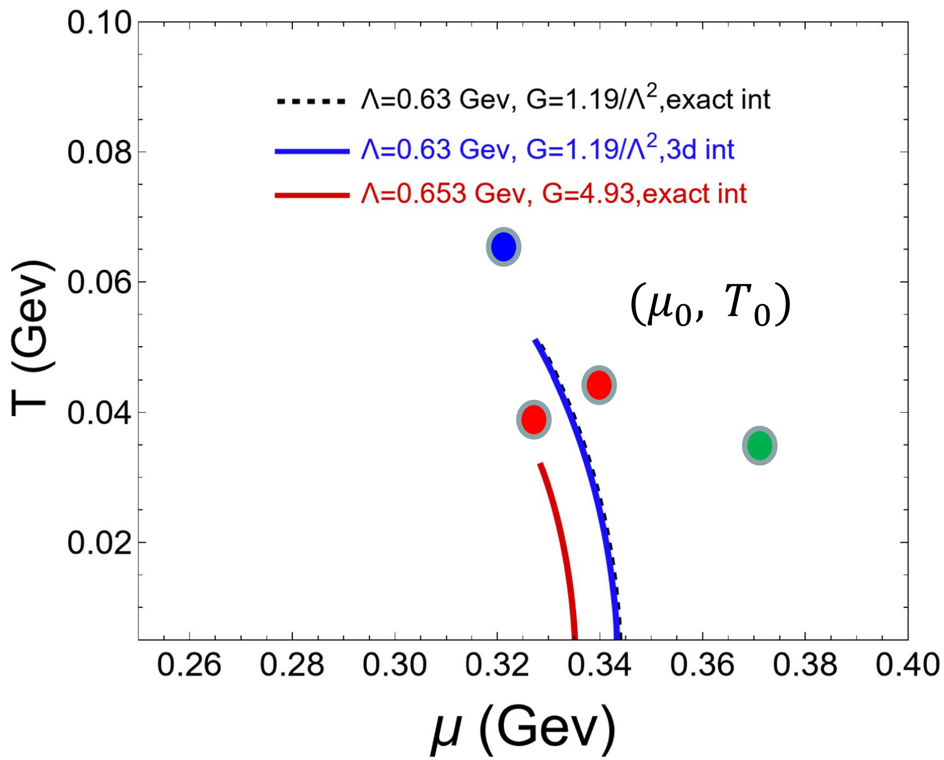
$$x, y,\ z\ are\ interval\ on\ [-1,1]fm$$

$$px, py,\ pz\ is\ interval\ on\ [-1.5\Lambda\ , 1.5\Lambda]$$

$Npx, Npy,\ Npz$=26

Nx,Ny,Nz=21 or 31

Algorithm



$m_0 = 0$

$\tau_{rel} = 0.1$fm/c

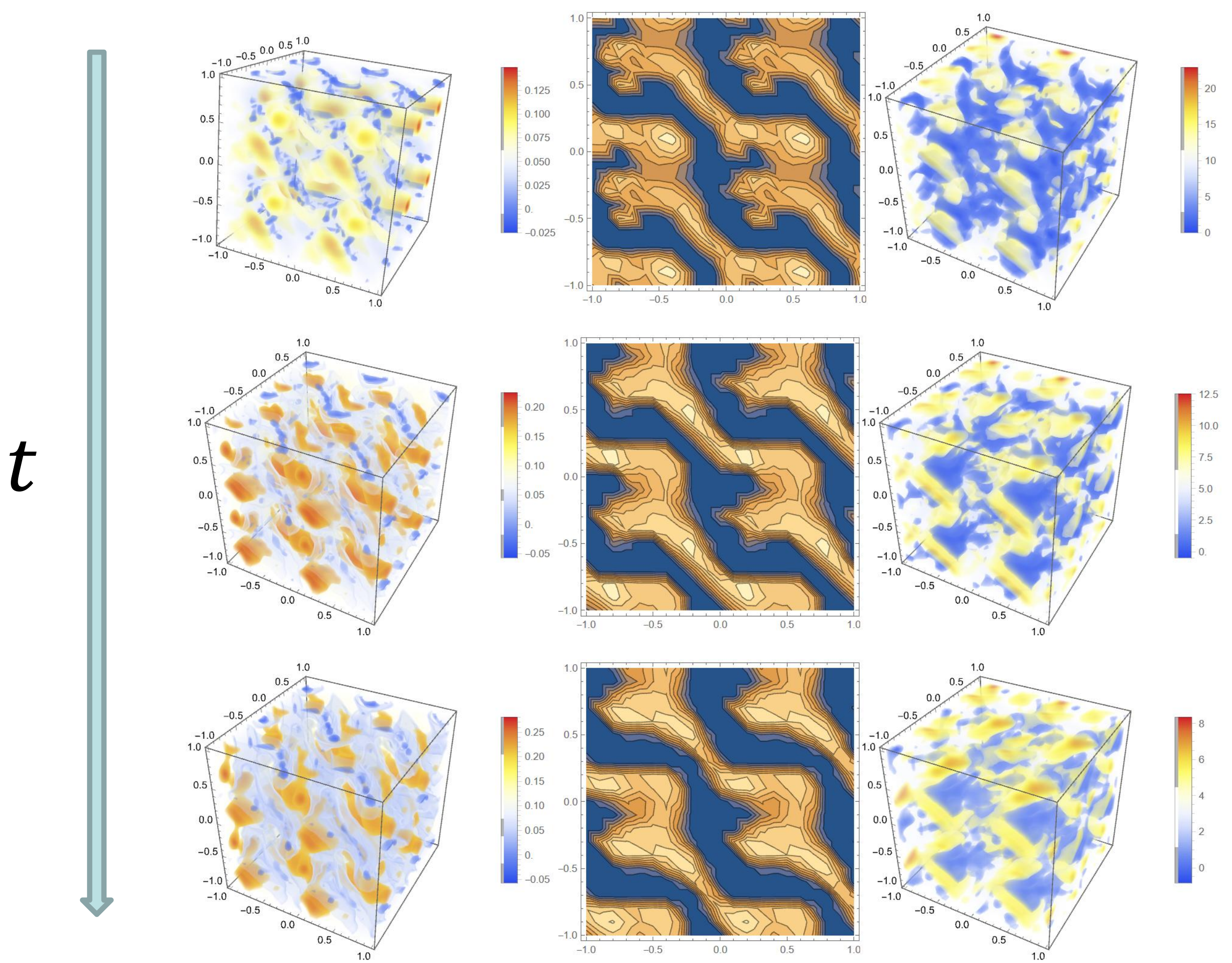
$$u^i = \sum_n a(n_1, n_2, n_3) \sin 2\pi(n_1 x + n_2 y + n_3 z + \epsilon),$$

Results

1. The evolution of number density and vorticity

$$\omega = \left(\omega_x^2 + \omega_y^2 + \omega_y^2\right)^{1/2}$$

$n(x, y, z)$ $\quad$ $n(x, y, z = 0.0)$ $\quad$ $\omega(x, y, z)$

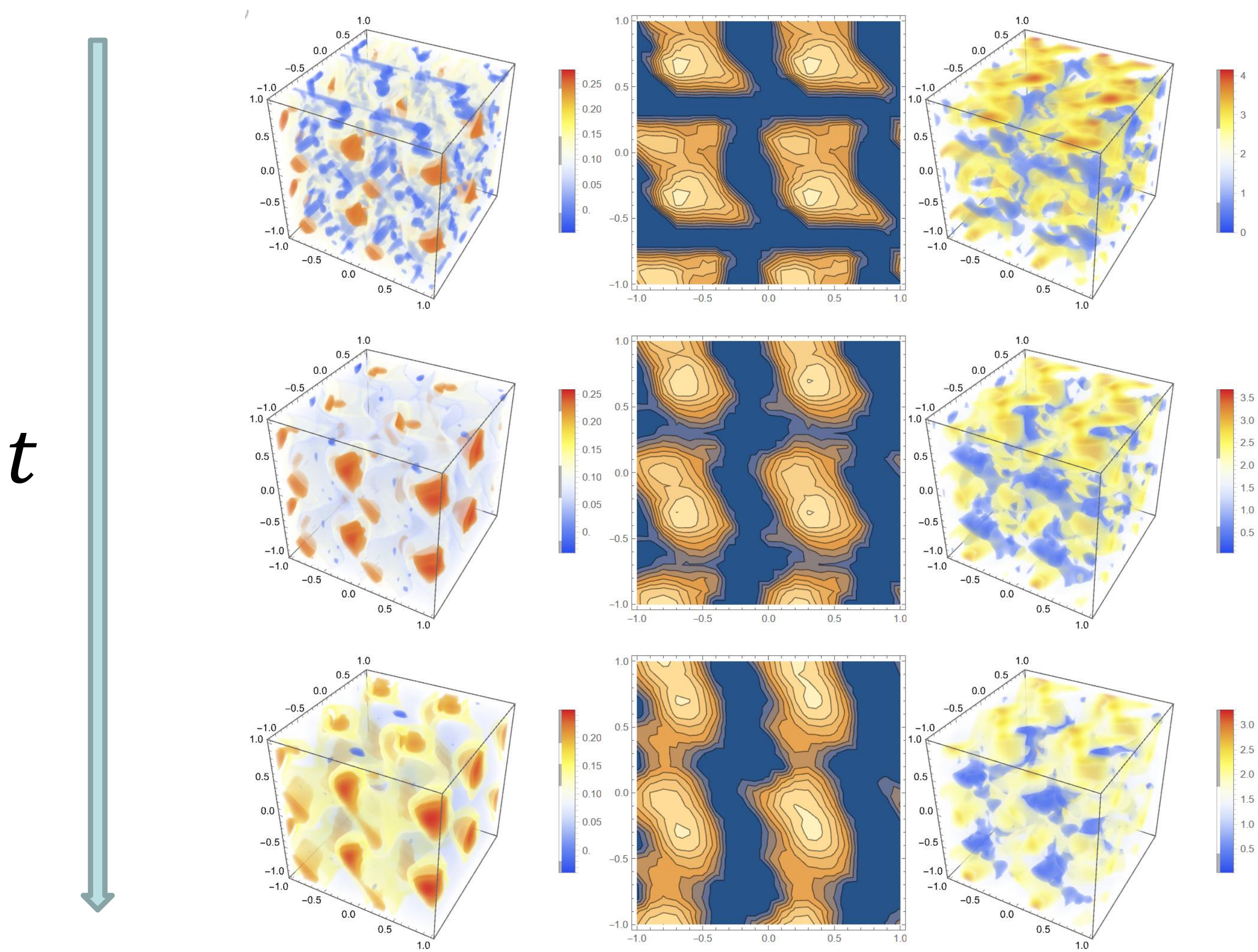


$$\boxed{\omega = \nabla \times \mathbf{v}}$$

Results

1.The evolution of number density and vorticity

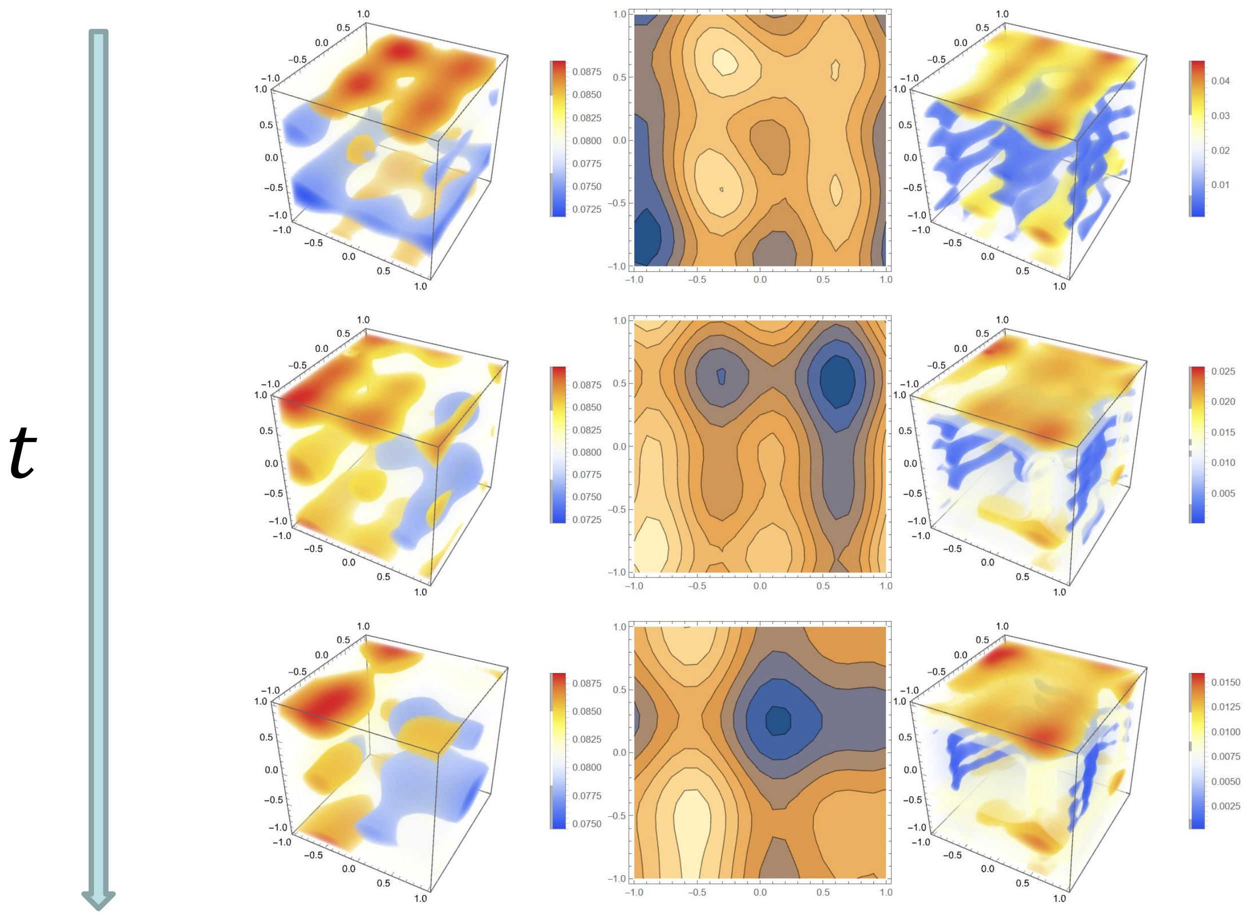
$n(x, y, z)$ $\quad$ $n(x, y, z = 0.0)$ $\quad$ $\omega(x, y, z)$



Results

1.The evolution of number density and vorticity

$n(x,y,z)$ $\quad$ $n(x,y,z=0.0)$ $\quad$ $\omega(x,y,z)$

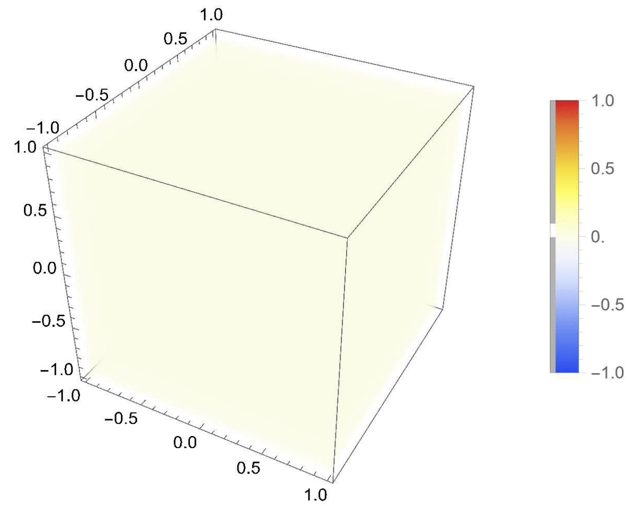


The evolution of the system tends to a stable state.

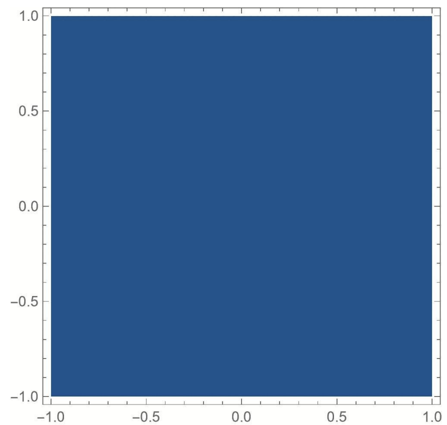
Results

1.The animation of number density and vorticity

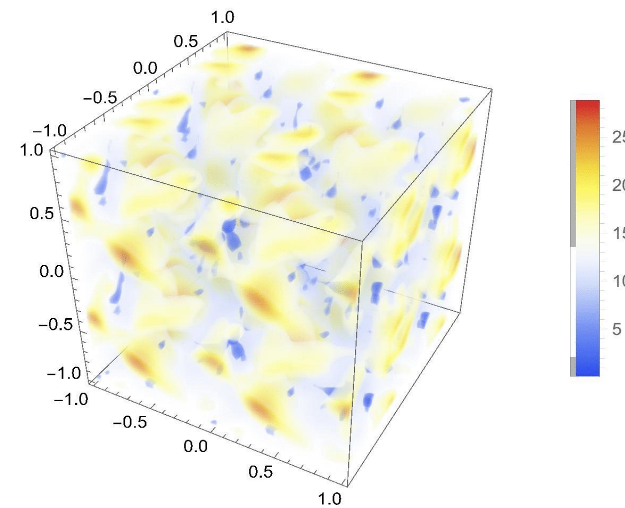
$n(x, y, z)$



$n(x, y, z = 0.0)$

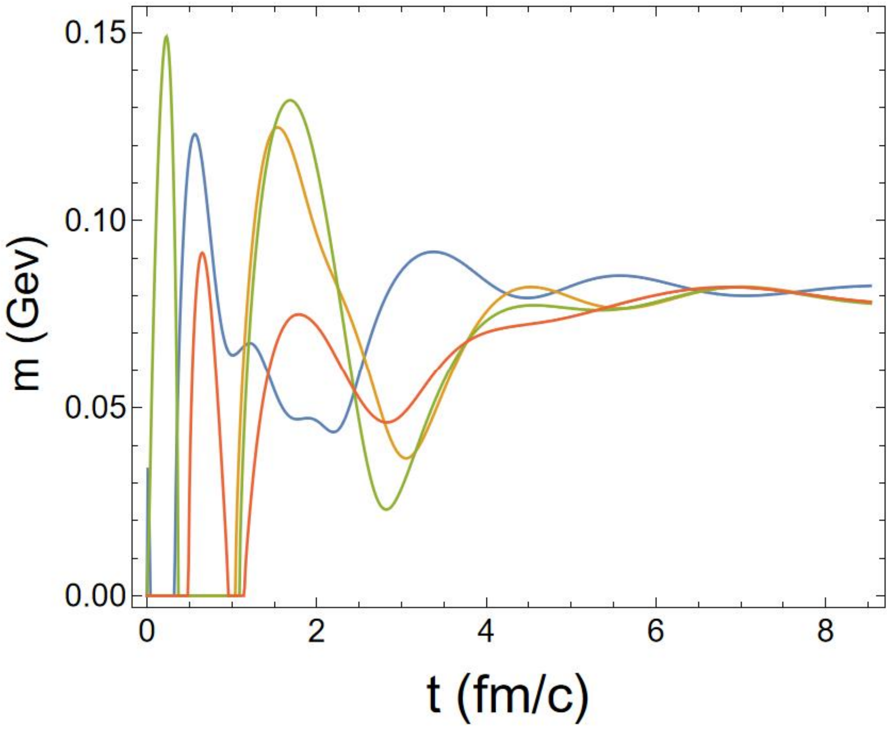


$\omega(x, y, z)$



Results

2.The evolution of mass at different points



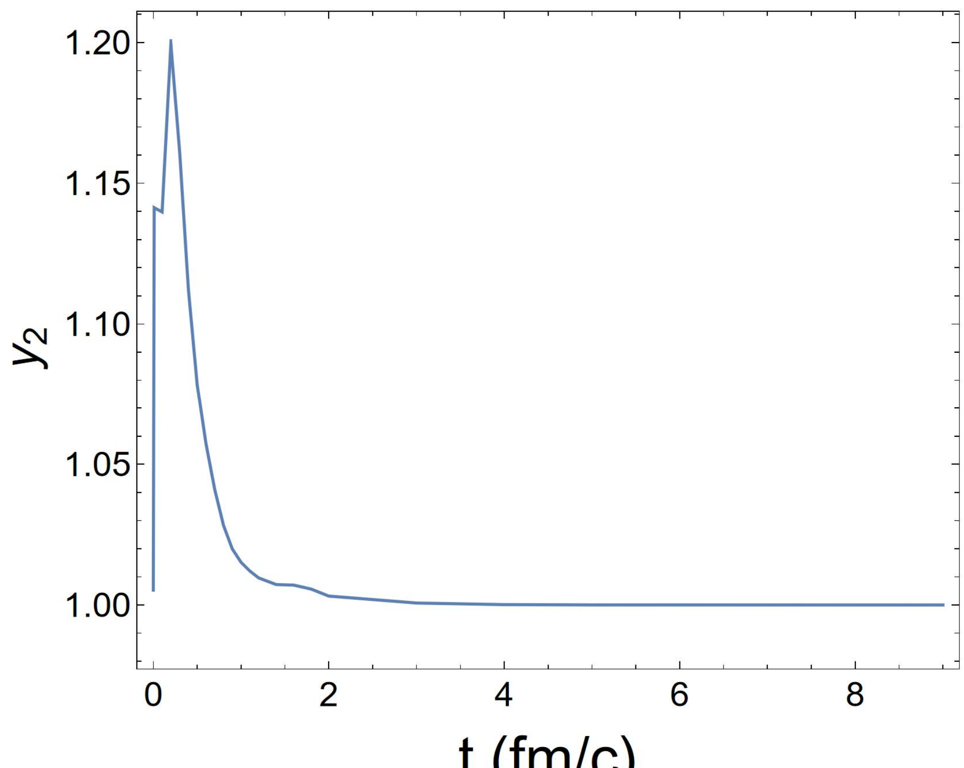
The evolution of the system tends to a stable state.

Results

3.The fluctuation of the number density

2004.0575454

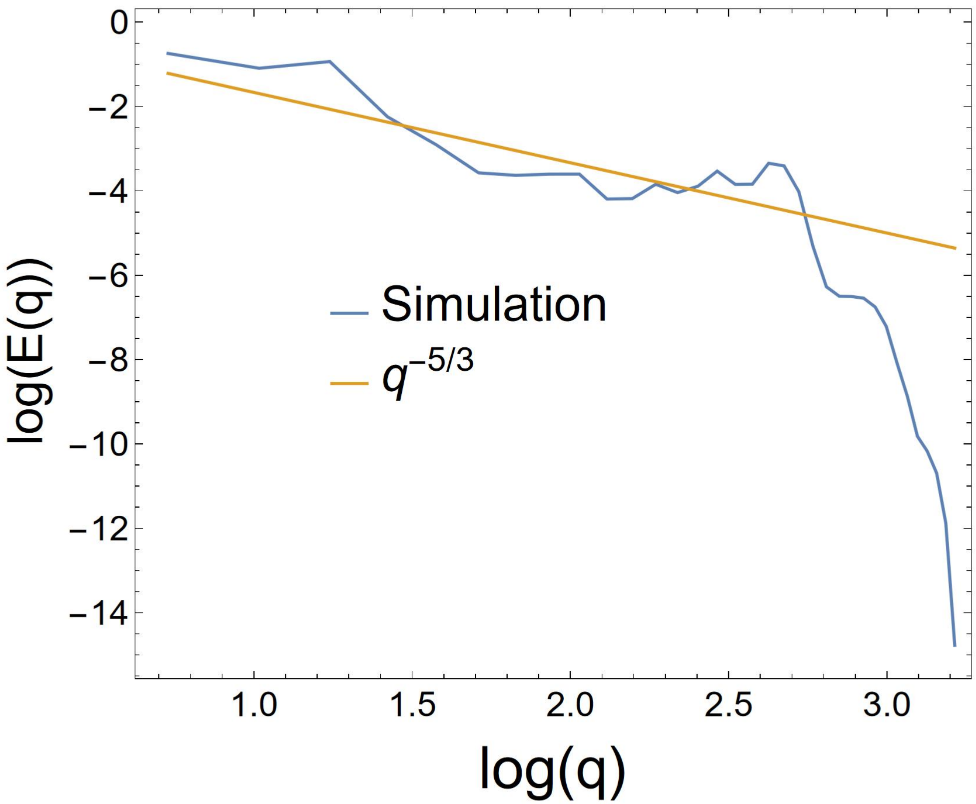
$$y_2 = \frac{\int \rho(x)\,dx \int \rho^3(x)\,dx}{\left[\int \rho^2(x)\,dx\right]^2}$$



This means that there is a phase transition under the evolution of the system.

Results

4.The turbulence properties



$$u(t,\mathbf{x}_{ijk}) = \frac{1}{V}\sum_{n,m,l=0}^{N_p-1} u(t,\mathbf{n})\,\cos(\frac{2\pi}{L_x}nx_i)\cos(\frac{2\pi}{L_y}my_j)\cos(\frac{2\pi}{L_z}lz_k),$$

$$u(t,\mathbf{n}) = dxdydz\sum_{i,j,k=0}^{N-1} u(t,\mathbf{x}_{ijk})\,\cos(\frac{2\pi}{L_x}nx_i)\cos(\frac{2\pi}{L_y}my_j)\cos(\frac{2\pi}{L_z}lz_k).$$

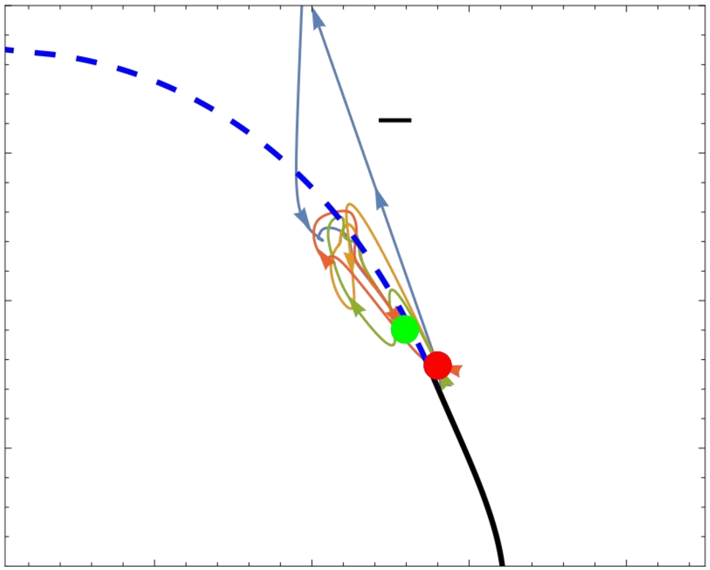
$$E_{nml} = (u_{x,nml}^2 + u_{y,nml}^2 + u_{z,nml}^2)/2.$$

$$q = \sqrt{n^2+m^2+l^2},$$

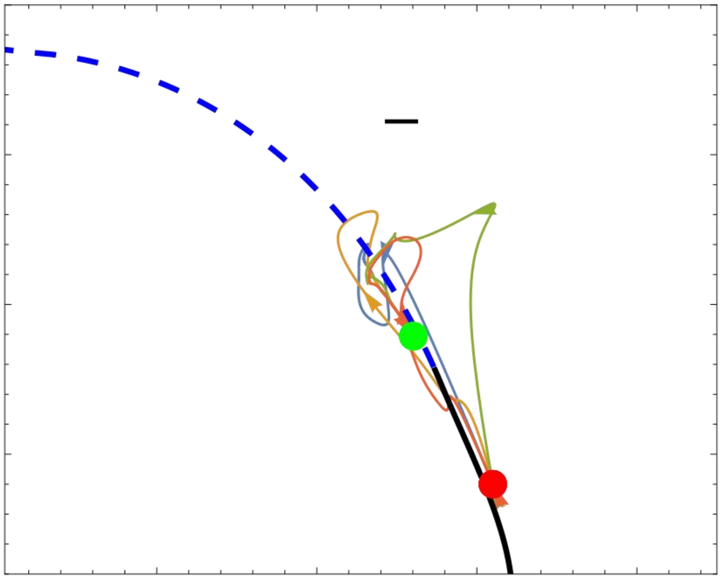
Results

5.The plot of (T-μ)

near the CEP at right-hand side



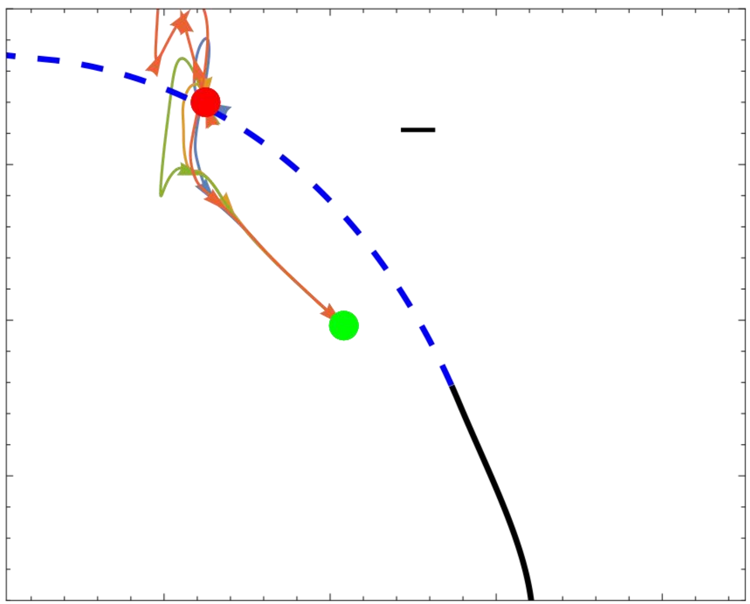
far from the CEP at right-hand side



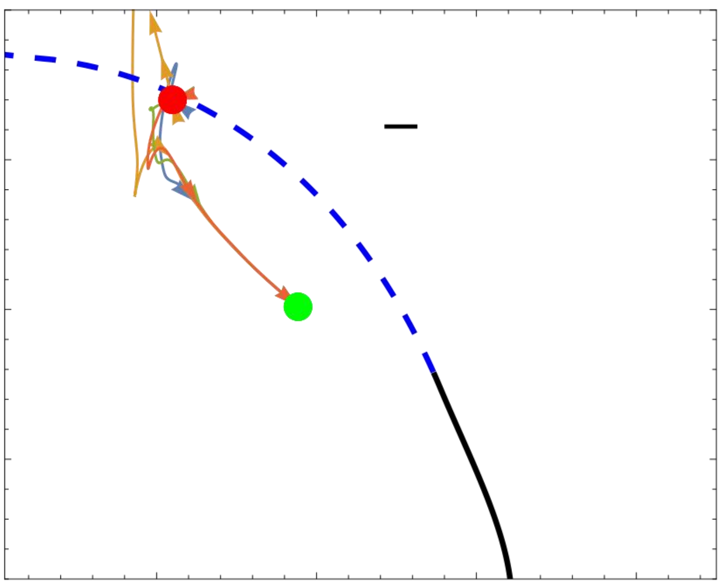
Results

5.The plot of (T-μ)

near the cross-over line at right-hand side



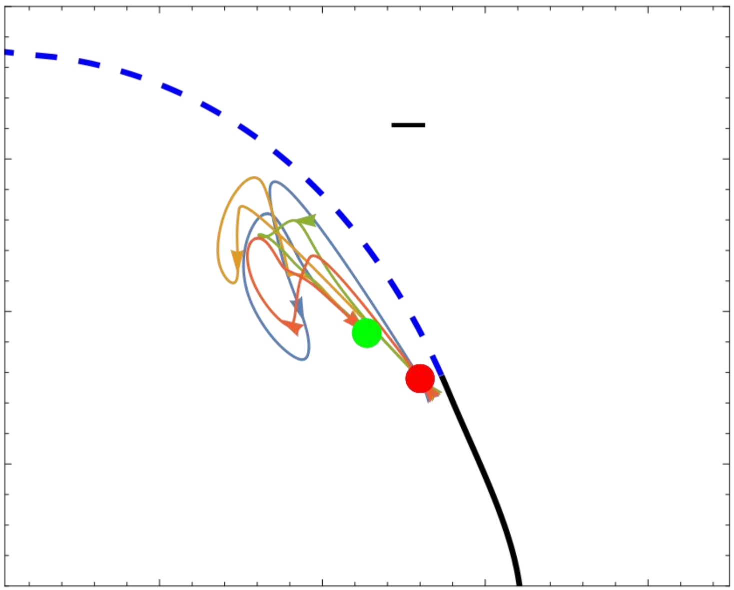
near the cross-over line at left-hand side



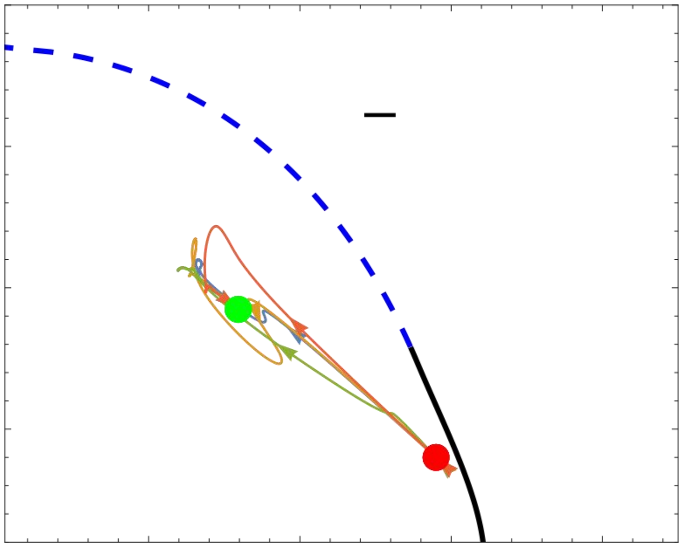
Results

5.The plot of (T-μ)

near the CEP at left-hand side



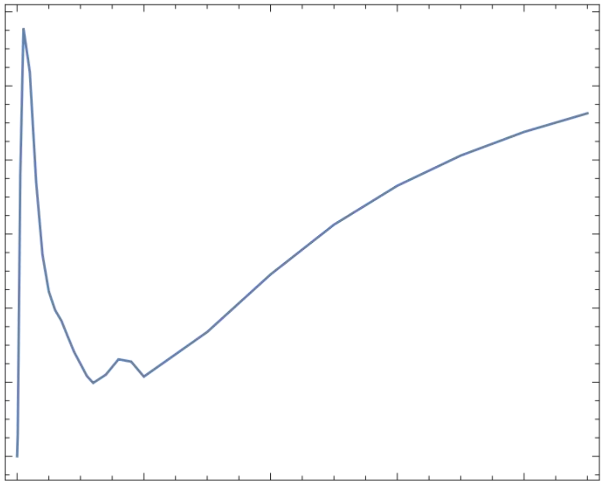
far from the CEP at left-hand side



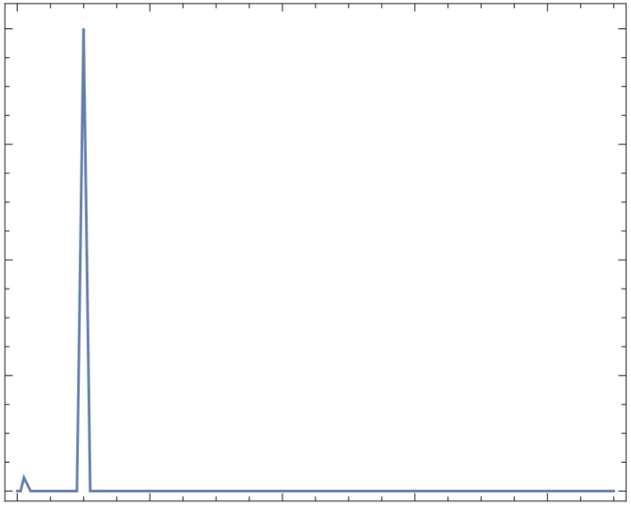
Results

6. (e-3P)/T^4

near the CEP at right-hand side



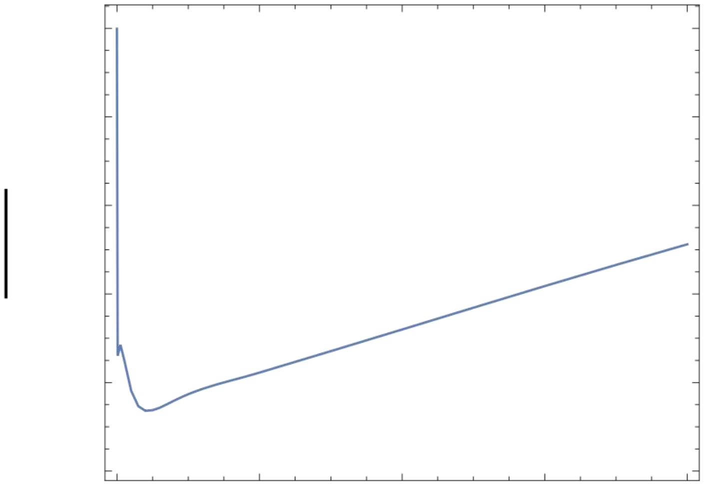
far from the CEP at right-hand side



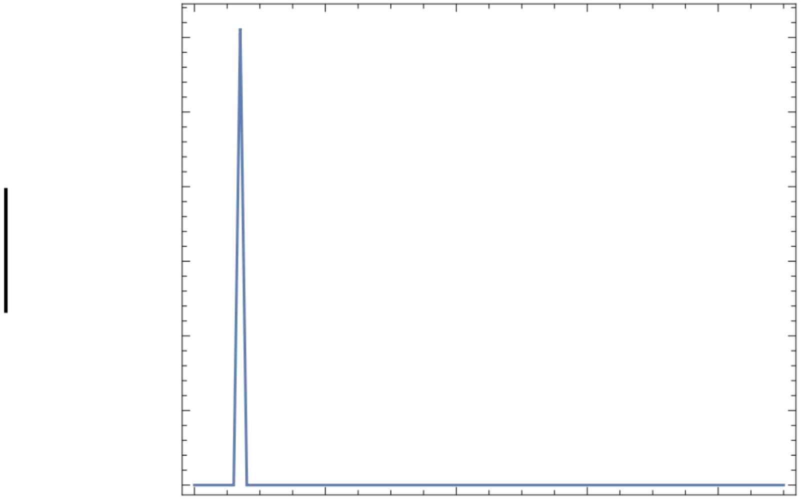
Results

6. (e-3P)/T^4

near the CEP at left-hand side



far from the CEP at left-hand side



Conclusion and Future

1. The phase transition is intrigued by turbulence.

2. The evolution of system is tending to a stable state.

Thank You!