On-Shell Construction of Effective Field Theories

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Outline

1. The On-Shell Correspondence
2. The Basis of Independent Operators
3. Eigenbasis Of Symmetry Algebra
4. Conclusion

Why On-Shell?

Effective Field Theory is a future key tool in the TeV phenomenology

$$\mathcal{L}_{eff} = \mathcal{L}_{\rm SM} + \sum_{d>4} \frac{c_{d,i}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)} \ , \quad \Lambda \sim \text{TeV}.$$

The language of effective operators has multiple drawbacks:

- A lot of redundancy relations: EOM, IBP, group identities,. . .
- Subtle to connect with physical observables.
- Description of physics is basis dependent.
- Computation of scattering amplitudes are involved.

Amplitude-Operator Correspondence T.Ma, J.Shu, M.-L.Xiao [1902.06752]

Consider the **leading contact amplitude** that an operator produces

$$\mathcal{O} \sim \mathcal{M}(p_1, \ldots, p_N; \psi_1, \ldots, \psi_N)$$

Spinor-Helicity Variables

The on-shellness unites the kinematic variables p_i and the wave functions ψ_i

	$p^2 = 0$	$p^2 = M^2$
p_i^μ	$\langle i\vert\sigma^\mu\vert i]$	$\epsilon_{IJ}\langle i^I\vert\sigma^\mu\vert i^J]$
$\psi_i^{(-)/(s)}$	$\vert i\rangle$	$\vert i\rangle^I\delta_I^s$
$\epsilon_i^{\mu,(-)/(s)}$	$\frac{\langle i\vert\sigma^\mu\vert\xi]}{\sqrt{2}[i\xi]}$	$\frac{\langle i^I\vert\sigma^\mu\vert i^J]}{\sqrt{2}M}\tau_{IJ}^s$

The correspondence is one-to-one:

- Every operator has a unique corresponding (combination of) amplitude(s).
- Every on-shell contact amplitude has a unique corresponding operator.
- They live in isomorphic linear spaces $[\mathcal{O}] \simeq [\mathcal{M}]$.

Outline

Young Tensor Method

The on-shell amplitudes still suffer from

1. Momentum conservation $\sum_i p_i = 0$ (equivalent to IBP of operators).
2. Schouten identities (equivalent to all kinds of $D = 4$ Lorentz group identities).

$$\langle ij\rangle\langle kl\rangle + \langle ik\rangle\langle lj\rangle + \langle il\rangle\langle jk\rangle = 0$$

3. Spin-statistics (when there are repeated fields).

The Magical Game of Young Tensor Method

- Draw the primary Young diagram $\{N, n, \tilde{n}\}$
- Construct the array of numbers to fill in $\#i = \tilde{n} - 2h_i$
- Fill in the numbers to form Semi-Standard Young Tableau (SSYT)
- Translate the SSYT into amplitudes or operators (y-basis)

H.-L.Li, Z.Ren, J.Shu, M.-L.Xiao, J.-H.Yu, Y.-H.Zheng (2020-2023)

1	1	1	2
2	2	3	3
4	5		

$$\Rightarrow [53][34]\langle 13\rangle\langle 23\rangle$$

$$\Rightarrow (\psi_1\sigma^\mu\bar{\psi}_4)D_\mu D_\nu\phi_3 \times(\psi_2\sigma^\nu\bar{\psi}_5)$$

Dealing With Repeated Fields

Suppose $\{1, 2\}$ are fermions of the same species

$$\mathcal{M}(1, 2, \dots) \mp \mathcal{M}(2, 1, \dots)$$

Dealing With Repeated Fields

Suppose $\{1, 2\}$ are fermions of the same species

$$
\begin{aligned}
\mathcal{M}(1^p, 2^r, \dots) \mp \mathcal{M}(2^r, 1^p, \dots) \quad &= \quad \begin{array}{|c|c|}\hline p & r \\ \hline \end{array} \quad \oplus \quad \begin{array}{|c|}\hline p \\ \hline r \\ \hline \end{array} \\
n_f^2 \quad &= \quad \frac{n_f(n_f+1)}{2} \quad + \quad \frac{n_f(n_f-1)}{2}
\end{aligned}
$$

Dealing With Repeated Fields

Suppose $\{1,2\}$ are fermions of the same species

$$\begin{aligned} \mathcal{M}(1^p,2^r,\ldots)\mp\mathcal{M}(2^r,1^p,\ldots) &= \young(pr) \oplus \young(p,r) \\ n_f^2 &= \frac{n_f(n_f+1)}{2} + \frac{n_f(n_f-1)}{2} \end{aligned}$$

Likewise, $\mathcal{O}^{prs} = \young(prs) \oplus \young(pr,s) \oplus \young(ps,r) \oplus \young(p,r,s)$

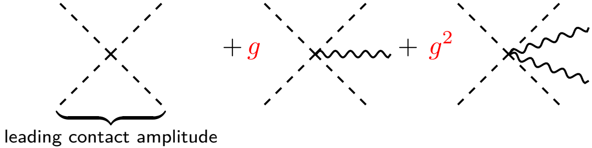
1. Permute
2. Reduce to the y-basis

- Operators as flavor tensors (p-basis) $\Leftrightarrow$ Standard Young Tableau.
- Operators as flavor components (f-basis) $\Leftrightarrow$ Semi-Standard Young Tableau.
- Some of the irreducible tensors may be killed $\Rightarrow$ flavor relations.

Example: $\mathcal{O}^{prs} \sim Q^pQ^rQ^sL = \underbrace{\young(prs) \oplus \young(pr,s) \oplus \young(p,r,s)}_{10+8+1=19} \Leftrightarrow \underbrace{\mathcal{O}^{prs}-\mathcal{O}^{rps}+\mathcal{O}^{srp}-\mathcal{O}^{spr}=0}_{3^3-19=8\ \text{constraints}}$

More About Amplitude-Operator Correspondence

Why only leading order contact amplitudes?

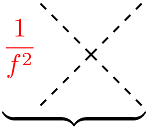
$H^4D^2 \supset$ 

- Not satisfying Ward Identity individually – invalid on-shell amplitudes for gauge bosons.
- Not parametrically independent – can be determined by gauge symmetry.
  (recursion relation)

More About Amplitude-Operator Correspondence

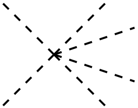
Why only leading order contact amplitudes?

$$\frac{1}{2}\frac{\partial^\mu \pi^a \partial_\mu \pi^a}{1-\pi^a\pi^a/f^2} \supset \frac{1}{f^2}$$



leading contact amplitude

$$+\ \frac{1}{f^4}$$



$$+\ \cdots$$

- Not satisfying Adler's Zero condition individually – invalid on-shell amplitudes for Goldstones.
- Not parametrically independent – can be determined by non-linear symmetry.
  (**soft** recursion relation)

Can be applied to chiral effective field Theories (ChPT, HEFT, . . .)

I.Low, J.Shu, M.-L.Xiao, Y.-H.Zheng [2209.00198]

H.Sun, M.-L.Xiao, J.-H.Yu [2206.07722], [2210,14939]

Outline

Casimir Acting On Amplitudes

Consider the scattering state under some particular representation $\mathbf{R}$

$$\mathbb{C}_{\mathcal{I}}\mathcal{M}(\mathcal{I}\to\bar{\mathcal{I}})=\sum_{i\in\mathcal{I}}\mathbb{C}_i\mathcal{M}=C(\mathbf{R})\mathcal{M}$$

Example: Total Angular Momentum J from Poincaré Casimir W^2

$W_{\mathcal{I}}^2\mathcal{M}=-s_{\mathcal{I}}J(J+1)\mathcal{M}$, where $W_\mu=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}P^\nu J^{\rho\sigma}$

$$J_i^{\mu\nu}=p_i^\mu\frac{\partial}{\partial p_{i\nu}}-p_i^\nu\frac{\partial}{\partial p_{i\mu}}=\langle i|\sigma^{\mu\nu}|\partial_i\rangle+[i|\sigma^{\mu\nu}|\partial_i]$$

Diagonalizing $W_{\mathcal{I}}^2$ on the amplitude basis $\Rightarrow$ generalized partial wave basis!

J.Shu, M.-L.Xiao, Y.-H.Zheng [2111,08019]

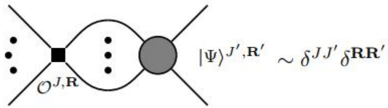
The Operator J-Basis

The representation is inherited by the corresponding operator!

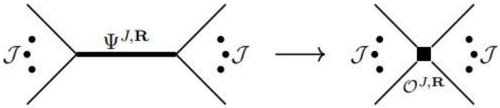
$$\mathcal{M}^{J,\mathbf{R}} \sim \mathcal{O}^{J,\mathbf{R}}\ , \quad \text{for specified channel } \mathcal{I}.$$

Applications:

- Selection Rules in Loop Integrals. M.Jiang, J.Shu, M.-L.Xiao, Y.-H.Zheng [2001.04481]



- UV Origin from Resonances. H.-L.Li, Y.-H.Ni, M.-L.Xiao, J.-H.Yu [2204.03660], [2309.15933]



Outline

Conclusion and Outlook

- **Systematic** way of analysing operator basis of EFT.
- **Novel way of thinking** about *operators v.s. scattering amplitudes*.
- **On-shell** calculations:
  1. anomalous dimension matrix (ADM) from **unitarity**.
  2. matching at tree/loop level.
- Focus on **observables**: S-matrix program, amplitude bootstrap.
- Extensions: non-linear symmetry, non-relativistic, extra dimension, non-perturbative effects. . .

Thank you for your attention!