Nonanalyticity and On-Shell Factorization of Inflation Correlators

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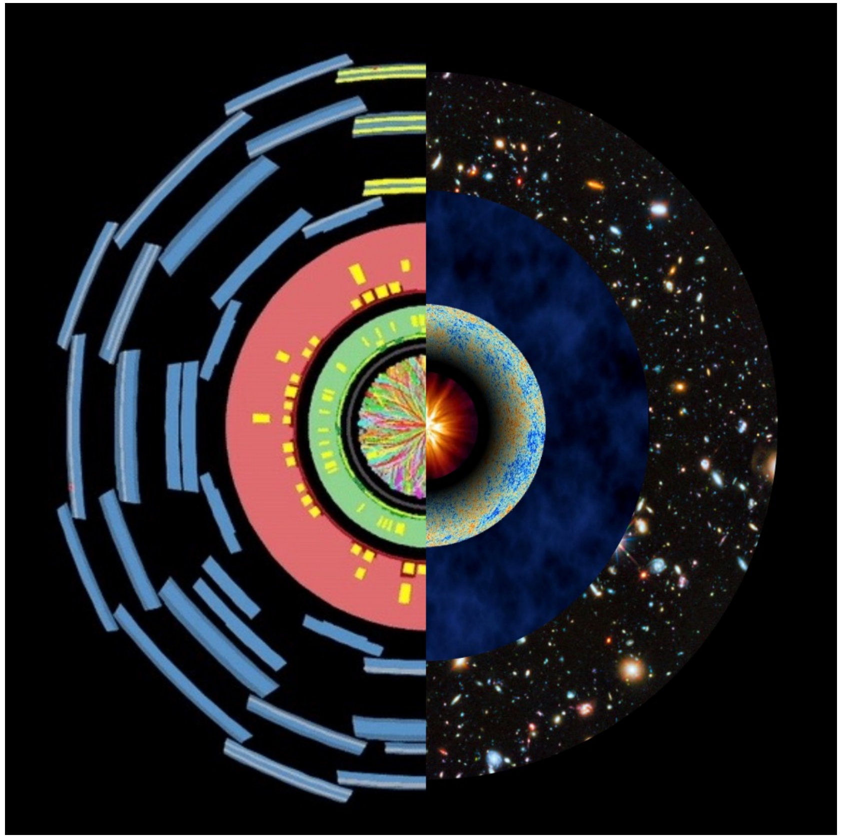
Based on 2208.13790, **2304.13295**, 2308.14802,

w/ Zhong-Zhi Xianyu

Motivation

Cosmological Collider Physics

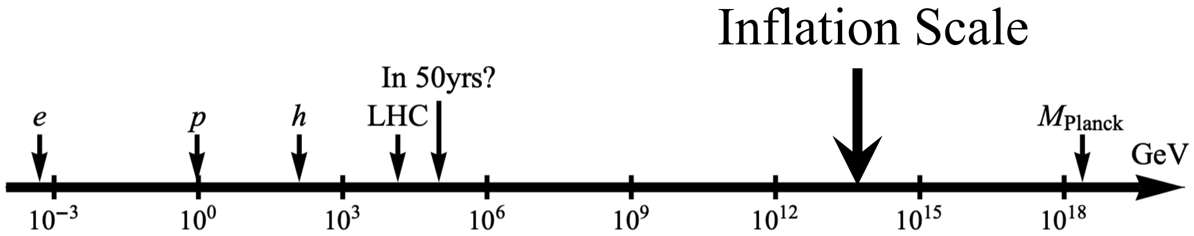
Inflation seems to be the highest energy observable process, with the energy scale characterized by the Hubble parameter, $H \sim 10^{14}$GeV.



We want to make use of the high energies of the early universe to study fundamental particle physics at high scale.

How? The "Cosmological Collider" signals provide us a unique window.

Arkani-Hamed, Maldacena, 1503.08043



Spacetime metric:

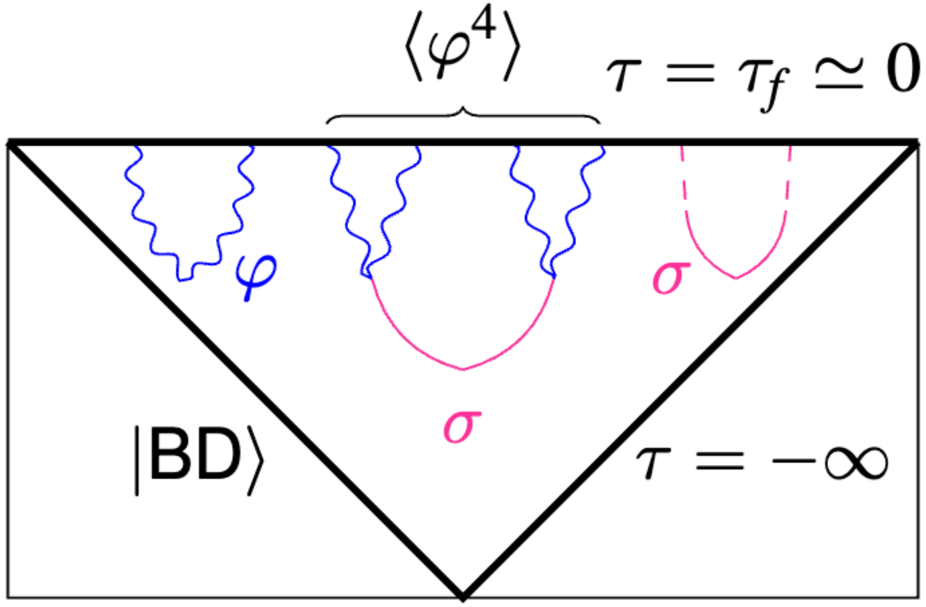
$$ds^2 = a^2(\tau)[-d\tau^2 + d\boldsymbol{x}^2], \qquad a(\tau) = -\frac{1}{H\tau}.$$

To drive the inflation, we need to introduce a scalar field named inflaton, which is nearly massless.

During the inflation, particles with mass $m \gtrsim H \equiv 1$ could be spontaneously produced due to the quantum fluctuations, and then leave imprints in the inflation correlators by interactions with inflaton.

Observables: Correlation functions of inflaton.

Such correlators are scalar perturbations and can be measured from CMB or LSS.

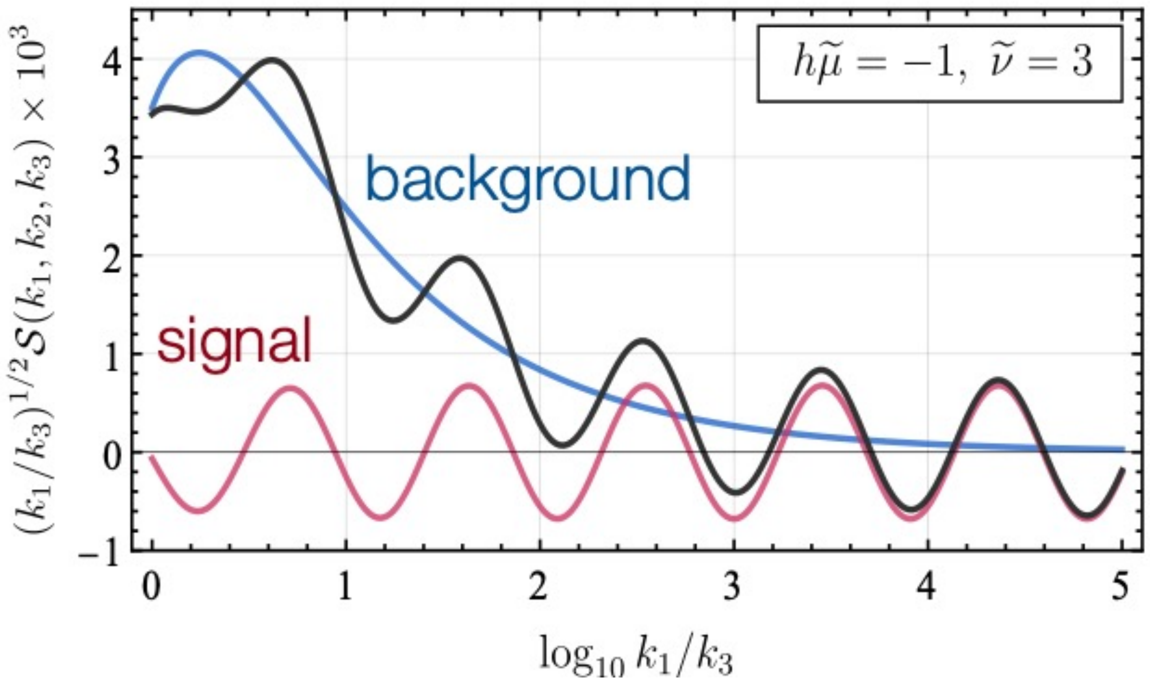


Particle production in dS. Here σ denotes the heavy particle and φ denotes the inflaton.

Cosmological Collider Physics

The inflation correlators have characteristic oscillatory patterns in the logarithm of momentum ratios, which is called a Cosmological Collider (CC) signal.

The frequency of this oscillation is connected with the mass of the heavy particle. So by measuring the frequency, we can recover the mass of the immediate particle.



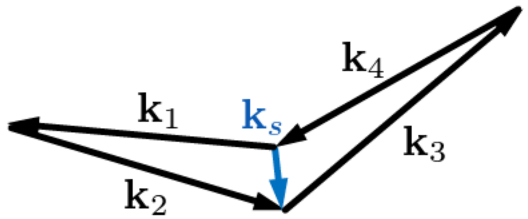
An example of a 3pt correlator (bispectrum).

Signals versus Background

In general the 4pt function has the form:

$$\langle \varphi_{\boldsymbol{k}_1}\varphi_{\boldsymbol{k}_2}\varphi_{\boldsymbol{k}_3}\varphi_{\boldsymbol{k}_4}\rangle' \sim K(\theta_i)\times J(k_1,k_2,k_3,k_4,k_s,k_t).$$

Here the factor $K(\theta_i)$ is purely kinematic.



In the squeezed limit $k_s \to 0$, the dynamic piece can be divided into three part:

$$\lim_{k_s\to 0} J = J_{\mathrm{EFT}} + J_{\mathrm{L}} + J_{\mathrm{NL}}.$$

$$r_1 \equiv \frac{k_s}{k_{12}}, \quad r_2 \equiv \frac{k_s}{k_{34}}.$$

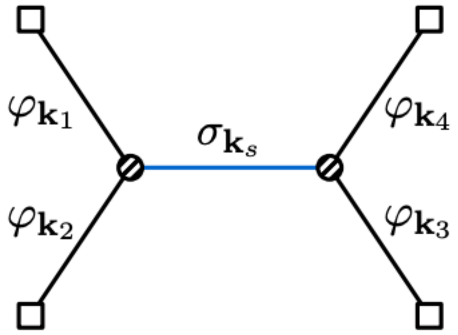
- J_{EFT}: EFT term, or the background piece. Fully analytic in both r_1 and r_2.
- J_{L}: Local signal, proportional to $(r_1/r_2)^{\pm i\omega} = (k_{34}/k_{12})^{\pm i\omega}$. Analytic in k_s.
- J_{NL}: Nonlocal signal, proportional to $(r_1 r_2)^{\pm i\omega} \propto k_s^{\pm 2i\omega}$. Nonanalytic in k_s.

Oscillatory pattern:

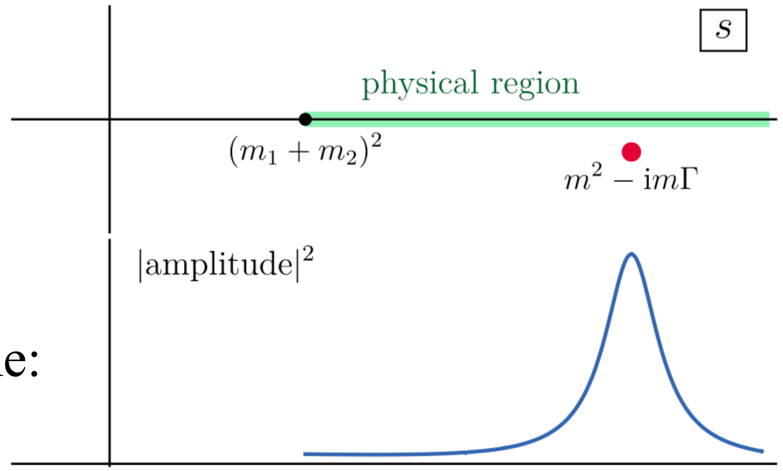
$$A\,(r_1 r_2)^{i\omega} + \mathrm{c.c.} = 2|A|\cos[\omega \log(r_1 r_2) + \vartheta].$$

Analytical Structure

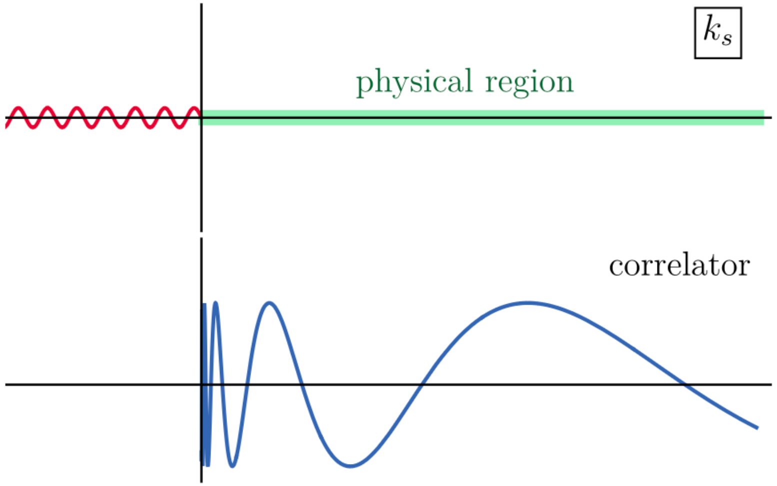
Singularity structures of amplitudes depend on the spacetime manifolds, and are closely related to characteristic patterns of observables.



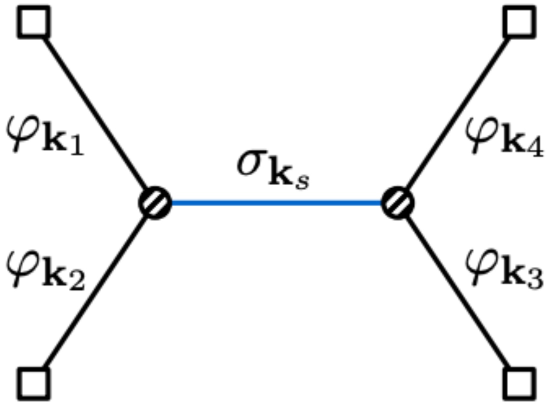
- Scattering amplitude in Minkowski spacetime:



- Boundary correlator in dS spacetime:



Calculation is Difficult



$\sim -\frac{\lambda^2}{E_s^2-k_s^2-m^2}$ in Minkowski.

$$= -\frac{\lambda_T^2}{16k_1\cdots k_4}\sum_{\mathsf{a},\mathsf{b}}\mathsf{ab}\int_{-\infty}^{0}\mathrm{d}\tau_1\mathrm{d}\tau_2\, e^{\mathrm{i}\mathsf{a}k_{12}\tau_1+\mathrm{i}\mathsf{b}k_{34}\tau_2}D_{\mathsf{ab}}(k_s;\tau_1,\tau_2).$$ in de Sitter.

(See Chen, Wang, Xianyu: 1703.10166 for SK formalism.)

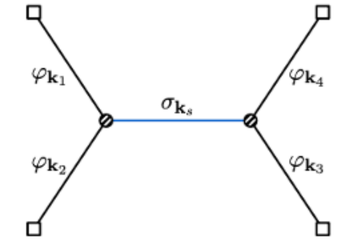
$$D_{-+}(k_s;\tau_1,\tau_2) = \frac{\pi}{4}e^{-\pi\tilde{\nu}}(\tau_1\tau_2)^{3/2}\mathrm{H}^{(1)}_{\mathrm{i}\tilde{\nu}}(-k_s\tau_1)\mathrm{H}^{(2)}_{-\mathrm{i}\tilde{\nu}}(-k_s\tau_2)$$

$$D_{+-}(k_s;\tau_1,\tau_2) = D^*_{-+}(k_s;\tau_1,\tau_2),$$

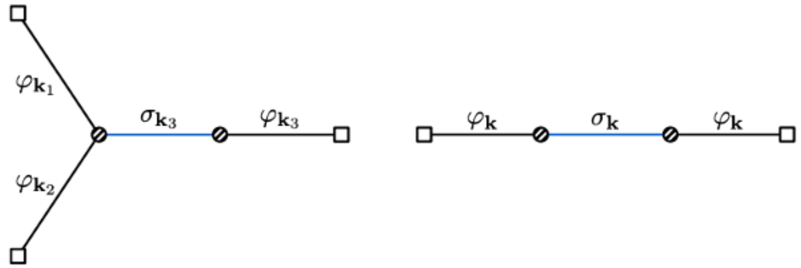
$$D_{++}(k_s;\tau_1,\tau_2) = D_{-+}(k_s;\tau_1,\tau_2)\theta(\tau_1-\tau_2)+D_{+-}(k_s;\tau_1,\tau_2)\theta(\tau_2-\tau_1),$$

$$D_{--}(k_s;\tau_1,\tau_2) = D_{+-}(k_s;\tau_1,\tau_2)\theta(\tau_1-\tau_2)+D_{-+}(k_s;\tau_1,\tau_2)\theta(\tau_2-\tau_1).$$

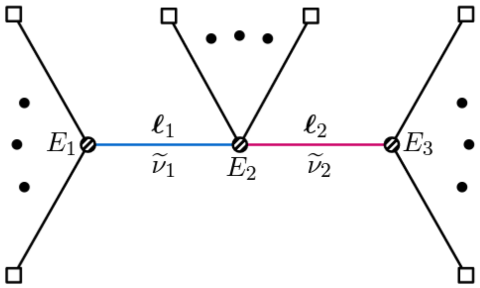
Current Stage



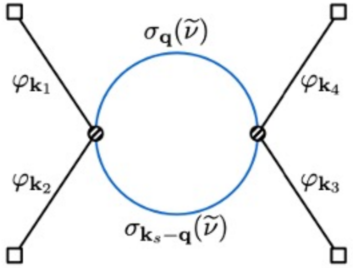
ZQ, Xianyu,
2208.13790



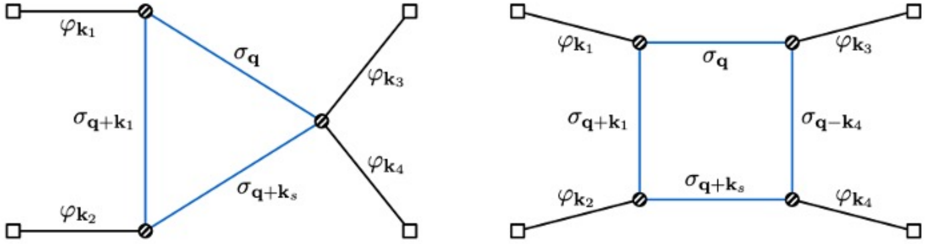
ZQ, Xianyu,
2301.07047



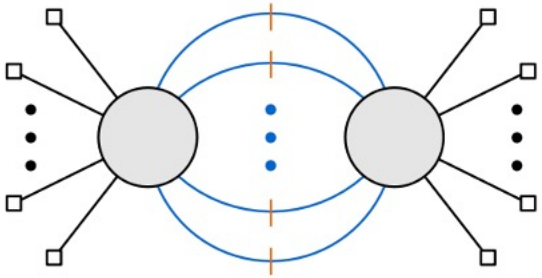
Xianyu, Zang,
2309.10849



Full result,
Xianyu and Zhang,
2309.10849



Nonlocal signal,
ZQ, Xianyu,
2304.13295



Factorization and Cutting rule,
ZQ, Xianyu,
2308.14802

Useful Tool: PMB

Partial Mellin-Barnes Representation

- For QFT in flat spacetime, the Fourier transform is useful, since its kernel is the eigenmode of translation.

- In dS, there is no more time translation, but we have dilation symmetry. The corresponding integral transform is the so-called Mellin transform:

$$F(s) = \int_0^\infty dz\, z^{s-1} f(z), \qquad f(z) = \int_{c-i\infty}^{c+i\infty} \frac{ds}{2\pi i}\, z^{-s} F(s).$$

- Partial MB Rep: Only perform the inverse Mellin transform to the internal modes.

Partial Mellin-Barnes Representation

- PMB of scalar propagators: **ZQ**, Xianyu: 2208.13790.

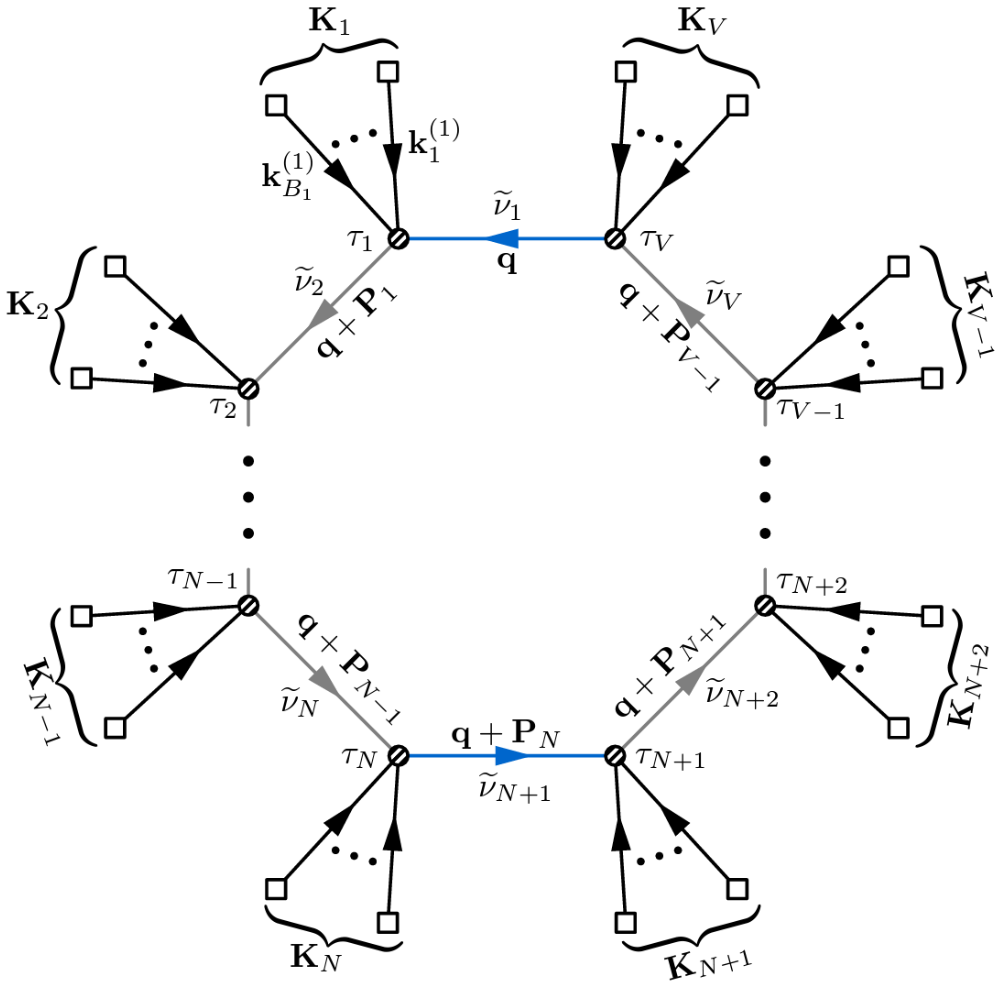
$$D_{\pm\mp}(k;\tau_1,\tau_2)=\frac{1}{4\pi}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}\,e^{\mp\mathrm{i}\pi(s_1-s_2)}\left(\frac{k}{2}\right)^{-2s_{12}}(-\tau_1)^{-2s_1+3/2}(-\tau_2)^{-2s_2+3/2}$$
$$\times\,\Gamma\left[s_1-\frac{\mathrm{i}\widetilde{\nu}}{2},s_1+\frac{\mathrm{i}\widetilde{\nu}}{2},s_2-\frac{\mathrm{i}\widetilde{\nu}}{2},s_2+\frac{\mathrm{i}\widetilde{\nu}}{2}\right],$$
$$D_{\pm\pm}(k;\tau_1,\tau_2)=D_{\mp\pm}(k;\tau_1,\tau_2)\theta(\tau_1-\tau_2)+D_{\pm\mp}(k;\tau_1,\tau_2)\theta(\tau_2-\tau_1).$$

There are IR poles in the Γ-functions, corresponding to the late-time expansion of the propagators.

- The time integrals and the loop integrals are factorized and simplified. The price paid is the integral of the Mellin variables.

- After the time and loop integrals, we finish the Mellin integral using the residue theorem.

1-Loop Configuration



Energy inflow at vertex i: $E_i = \sum_j |\vec{k}_j^{(i)}|$.

Momentum inflow at vertex i: $\vec{K}_i = \sum_j \vec{k}_j^{(i)}$.

Momentum partial sum: $\vec{P}_n = \sum_{j=1}^{n} \vec{K}_j$.

Question: What is the nonanalytic behavior in the squeezed limit when a partial sum of momentum inflow goes to zero?

No degeneracy: Assume the energies and the soft momentum transfer are independent.

1-Loop Correlator with PMB

- Dependence of energy: Time integral.
- Dependence of momentum: Loop integral, independent of SK indices.
- Nonanalyticities of energy are all encoded in the time integral.
- Nonanalyticities of momentum transfer are all encoded in the loop integral.

1-Loop Correlator with PMB

$$\mathcal{T}(\{\mathbf{k}\}) = \sum_{\mathsf{a}_1,\cdots,\mathsf{a}_V=\pm} \int_{-\infty}^{0} \prod_{\ell=0}^{V} \left[\mathrm{d}\tau_\ell \, (\mathrm{i}\mathsf{a}_\ell)(-\tau_\ell)^{p_\ell} \prod_{i=1}^{B_\ell} C_{\mathsf{a}_\ell}\left(k_i^{(\ell)};\tau_\ell\right)\right] \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} D^{(\widetilde{\nu}_1)}_{\mathsf{a}_V\mathsf{a}_1}\left(q;\tau_V,\tau_1\right)$$
$$\times D^{(\widetilde{\nu}_2)}_{\mathsf{a}_1\mathsf{a}_2}\left(|\mathbf{q}+\mathbf{P}_1|;\tau_1,\tau_2\right)\cdots D^{(\widetilde{\nu}_V)}_{\mathsf{a}_{V-1}\mathsf{a}_V}\left(|\mathbf{q}+\mathbf{P}_{V-1}|;\tau_{V-1},\tau_V\right).$$

With PMB, the time integral and loop momentum integral are factorized and simplified:

Time integral:

$$\int_{-\infty}^{0} \mathrm{d}\tau_1\cdots\mathrm{d}\tau_V\,(-\tau_1)^{\widehat{p}_1-2s_{1\bar{1}}}\cdots(-\tau_V)^{\widehat{p}_V-2s_{V\bar{V}}}e^{\mathrm{i}(\mathsf{a}_1E_1\tau_1+\cdots+\mathsf{a}_VE_V\tau_V)}$$
$$\times\mathcal{N}(\tau_1,\tau_2)\mathcal{N}(\tau_2,\tau_3)\cdots\mathcal{N}(\tau_V-1,\tau_V)\mathcal{N}(\tau_V,\tau_1),$$

Loop integral:

$$\mathcal{L} = \int\frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\mathcal{P};\qquad \mathcal{P}\equiv|\mathbf{q}|^{-2s_{V\bar{1}}}|\mathbf{q}+\mathbf{P}_1|^{-2s_{1\bar{2}}}|\mathbf{q}+\mathbf{P}_2|^{-2s_{2\bar{3}}}\cdots|\mathbf{q}+\mathbf{P}_{V-1}|^{-2s_{(V-1)\bar{V}}}.$$

Loop Integral Analysis

- Only the soft region of integral contributes to nonlocal signal:

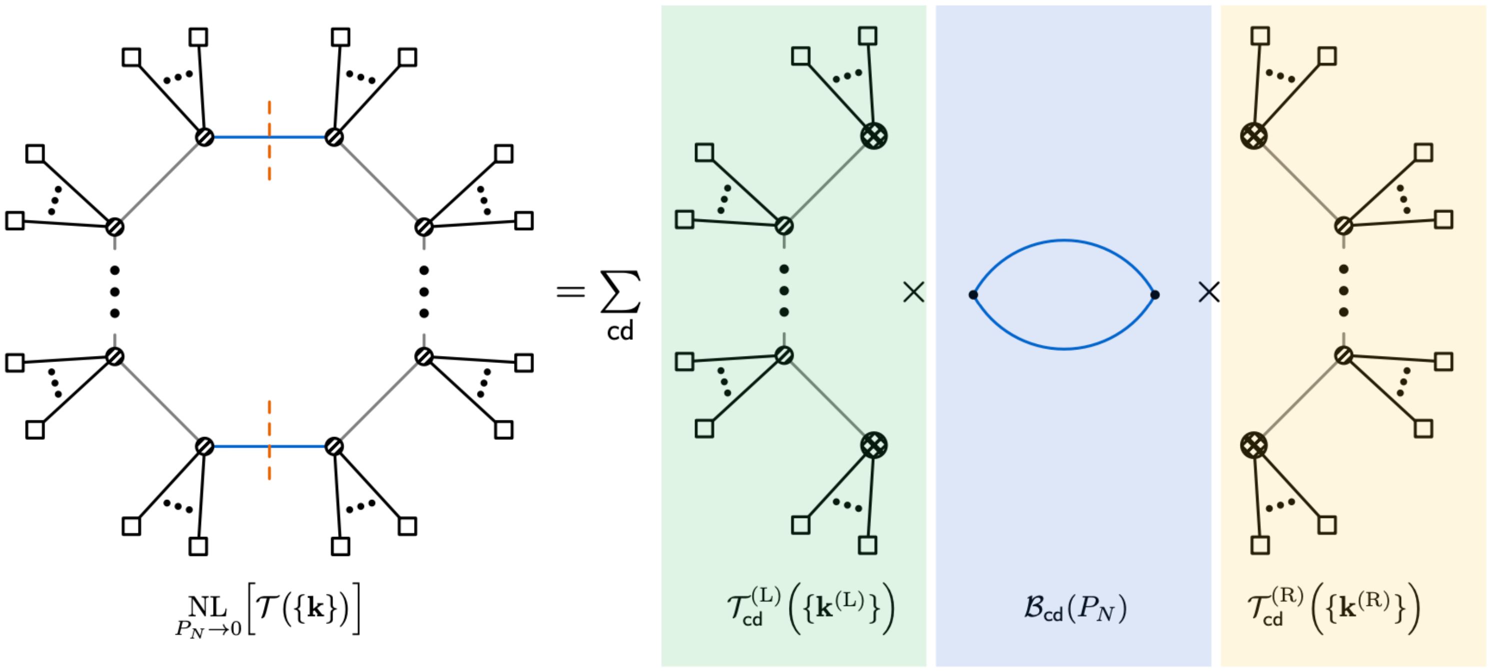
$$\mathcal{L} = \frac{1}{(4\pi)^{3/2}}\Gamma\left[\begin{matrix} s_{V\bar{1}N(\overline{N+1})} - \frac{3}{2}, \frac{3}{2} - s_{V\bar{1}}, \frac{3}{2} - s_{N(\overline{N+1})} \\ 3 - s_{V\bar{1}N(\overline{N+1})}, s_{V\bar{1}}, s_{N(\overline{N+1})} \end{matrix}\right] P_N^{3-2s_{V\bar{1}N(\overline{N+1})}}\left[1 + \mathcal{O}(P_N)\right]$$
$$\times \left(P_1^{-2s_{1\bar{2}}} \cdots P_{N-1}^{-2s_{(N-1)\bar{N}}}\right)\left(P_{N+1}^{-2s_{(N+1)(\overline{N+2})}} \cdots P_{V-1}^{-2s_{(V-1)\bar{V}}}\right) + \text{terms analytic in } P_N$$

- Since $P_N \to 0$, we should take left poles of $s_V, \bar{s}_1, s_N, \bar{s}_{N+1}$.

- Time integral: only right poles. Loop integral: UV poles, but analytic.

- The only choice is the IR nonlocal poles. Cutting rule: $D_{ab} \to \mathrm{Re}[D]$.

1-Loop Factorization Theorem



$$\lim_{P_N\to 0}\mathcal{T}(\{\mathbf{k}\}) = \sum_{\mathrm{c,d}=\pm}\mathcal{T}^{(\mathrm{L})}_{\mathrm{cd}}\left(\{\mathbf{k}^{(\mathrm{L})}\}\right)\mathcal{T}^{(\mathrm{R})}_{\mathrm{cd}}\left(\{\mathbf{k}^{(\mathrm{R})}\}\right)\mathcal{B}_{\mathrm{cd}}(P_N) + \text{analytic}$$

Left subgraph

Right subgraph

Bubble signal

- Focusing on the nonlocal signal, the 1-loop graph is factorized into three parts.
- The singularity is fully encoded in the bubble signal.
- The graph is singular when all cut (blue) lines become simultaneously soft.
- Every cut line can be replaced by its real part and thus the SK indices are automatically stripped off.

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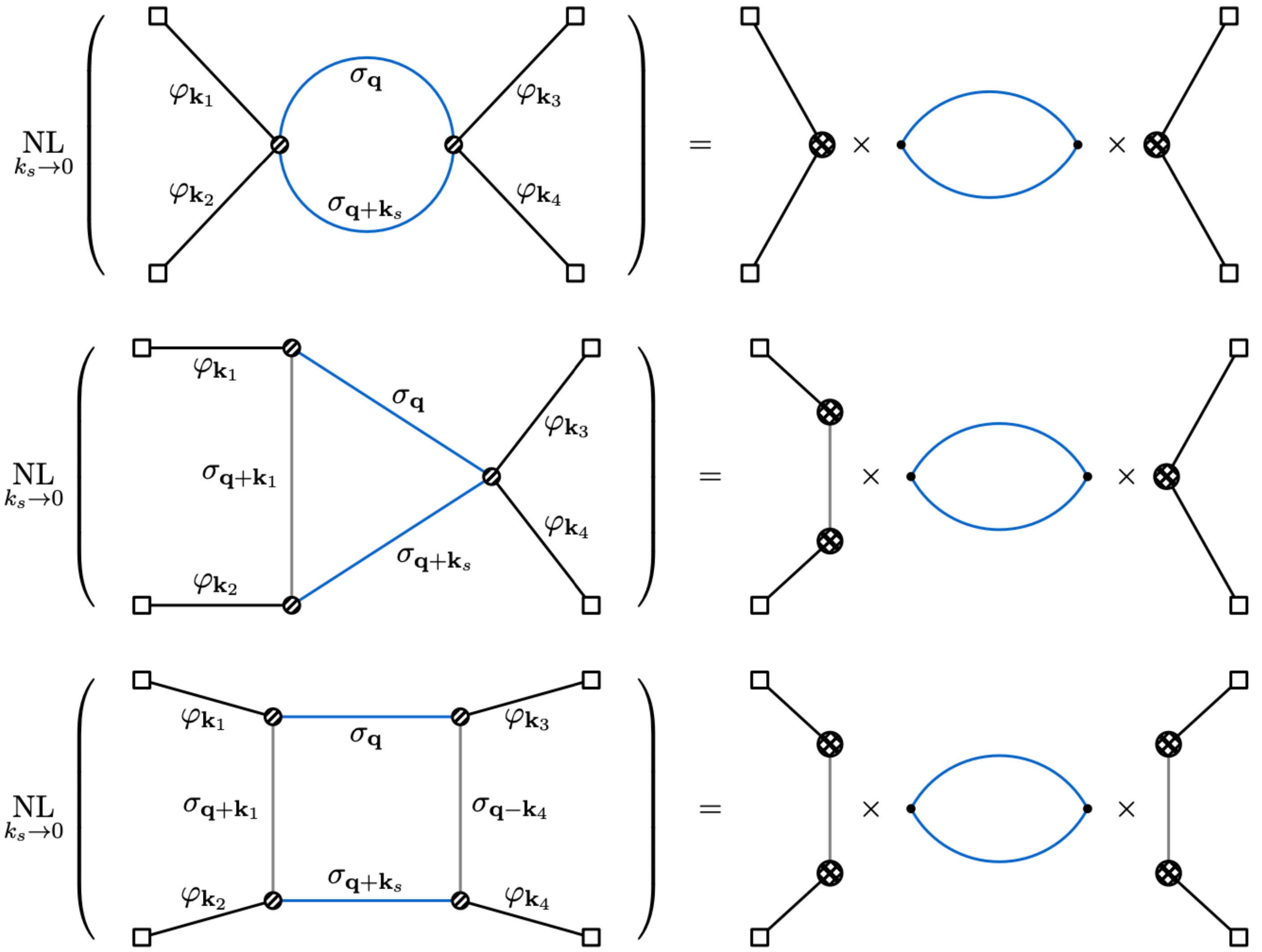
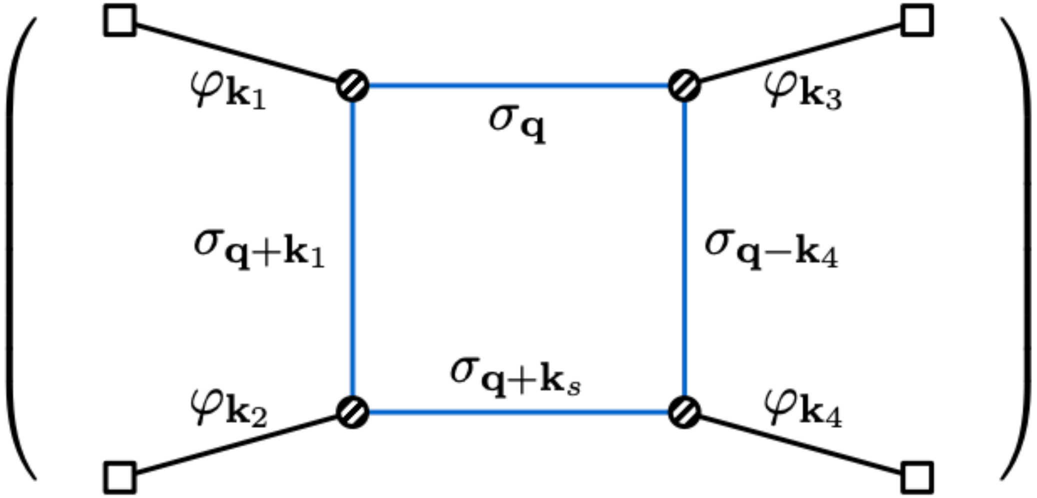
Applications

All 1PI 1-loop 4pt nonlocal signals.
ZQ, Xianyu: 2304.13295.

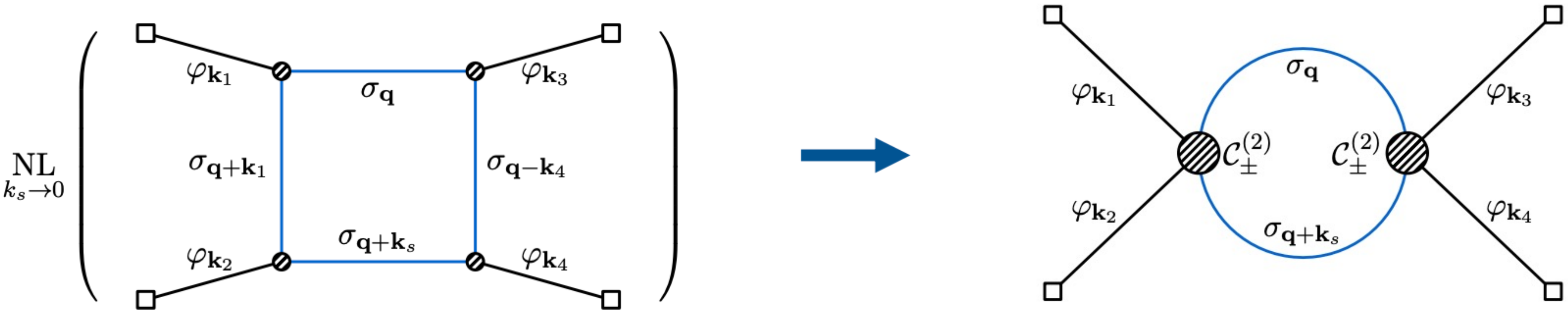
The closed-form of subgraphs (2pt tree with time insertions) are derived by our improved bootstrap method in 2301.07047 w/ Xianyu.



$\underset{k_s \to 0}{\mathrm{NL}}$



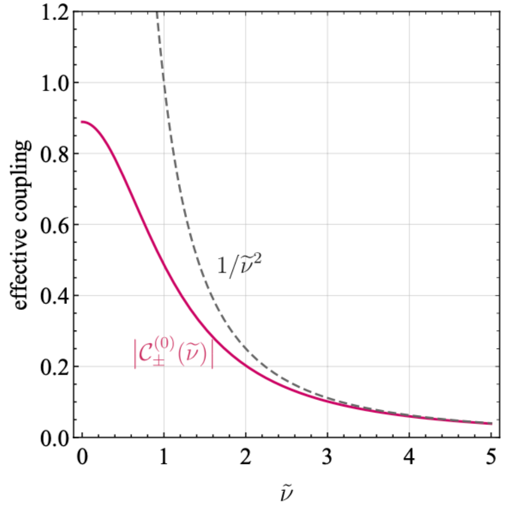
$$\lim_{k_s\to 0}\Big[\mathcal{T}_{\rm box}(\{\mathbf{k}\})\Big]_{\rm NL} = -\frac{k_s^3}{2(4\pi)^{7/2}k_1k_2k_3k_4k_{12}^4k_{34}^4}\left(\frac{k_s^2}{4k_{12}k_{34}}\right)^{2\mathrm{i}\widetilde{\nu}}\frac{(2+\mathrm{i}\widetilde{\nu})^4}{(3+2\mathrm{i}\widetilde{\nu})^2}\sinh^2(\pi\widetilde{\nu})$$
$$\times\,\Gamma\left[3+2\mathrm{i}\widetilde{\nu}, -\frac{3}{2}-2\mathrm{i}\widetilde{\nu}\right]\Gamma^2\left[\frac{3}{2}+\mathrm{i}\widetilde{\nu}, -2-\mathrm{i}\widetilde{\nu}\right]+\text{c.c.}.$$



There is a boundary OPE version for understanding:

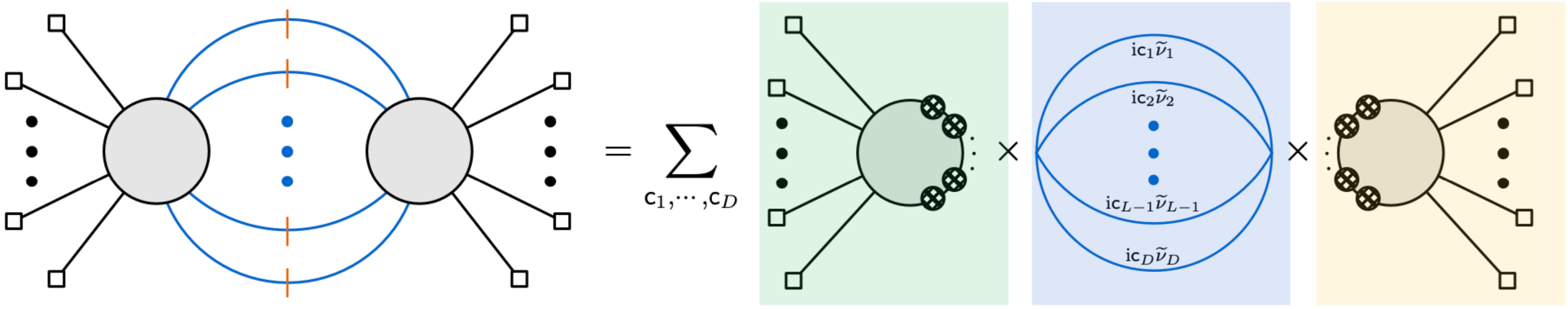
Each cut line is soft and thus the two endpoints are far away from each other. Therefore the left (right) points are close and can be pinched together (OPE) as an effective coupling.

The results meet the physical intuition: The coupling scales as $1/m^2$ for large mass.



All Loop Factorization Theorem

The above arguments at 1-loop order can be generalized to all loop order, but with some more subtleties.
ZQ, Xianyu: 2308.14802.



$$\lim_{P\to0}\mathcal{T}(\{\mathbf{k}\}) \supset \sum_{\substack{\mathsf{c}_1,\cdots,\mathsf{c}_D=\pm\\ \text{nonlocal}}} \mathcal{G}^{(\mathrm{L})}_{\mathsf{c}_1\cdots\mathsf{c}_D}\left(\{\mathbf{k}^{(\mathrm{L})}\}\right)\mathcal{G}^{(\mathrm{R})}_{\mathsf{c}_1\cdots\mathsf{c}_D}\left(\{\mathbf{k}^{(\mathrm{R})}\}\right)\mathfrak{M}_{\mathsf{c}_1\cdots\mathsf{c}_D}(P) + (\text{subleading nonlocal signals}) + (\text{terms analytic in } K)$$

- Focusing on the leading nonlocal signal, the loop graph is factorized into three parts.

  The singularity is fully encoded in the melon signal.

  The graph is singular when all cut (blue) lines become simultaneously soft.

  Every cut line can be replaced by its real part and thus the SK indices are automatically stripped off.

Outlooks

Outlooks

- Calculations:
  - Position correlators?
  - Numerical calculations?
- General dS Amplitude structures:
  - Analysis of (all) other possible singularity structures.
  - Dispersion relation.
  - Sufficient use of boundary conformal symmetry?
- Applications in phenomenology:
  - SUSY/SUGRA signals?
  - String amplitudes in dS?

Thanks

Backup

Tree Result

The result is complicated:

$$\mathcal{T}_\varphi = \frac{\lambda^2}{16k_1k_2k_3k_4k_s^5}\Big[\mathcal{I}_{\mathrm{BG},>}(r_1,r_2)+\mathcal{I}_{\mathrm{L},>}(r_1,r_2)+\mathcal{I}_{\mathrm{NL}}(r_1,r_2)\Big]$$

$$\mathcal{I}_{\mathrm{BG},>} = \frac{1}{\mathrm{i}\widetilde{\nu}}r_1^5\sum_{n_1,n_2=0}^{\infty}\frac{(-1)^{n_{12}}}{n_1!n_2!}\left(\frac{r_1}{2}\right)^{2n_{12}}(-\mathrm{i}\widetilde{\nu})_{-n_1}(\mathrm{i}\widetilde{\nu})_{-n_2}\mathcal{F}\left[\begin{matrix}2n_2+\frac{5}{2}-\mathrm{i}\widetilde{\nu},2n_{12}+5\\2n_2+\frac{7}{2}-\mathrm{i}\widetilde{\nu}\end{matrix}\middle|-\frac{r_1}{r_2}\right]+\mathrm{c.c.}$$

$$\mathcal{I}_{\mathrm{L},>} = \frac{\sin \mathrm{i}\pi\widetilde{\nu}}{2\pi}\left(\frac{r_1}{r_2}\right)^{\mathrm{i}\widetilde{\nu}}\mathbf{F}_{\widetilde{\nu}}(r_1)\mathbf{F}_{-\widetilde{\nu}}(r_2)+\mathrm{c.c.},$$

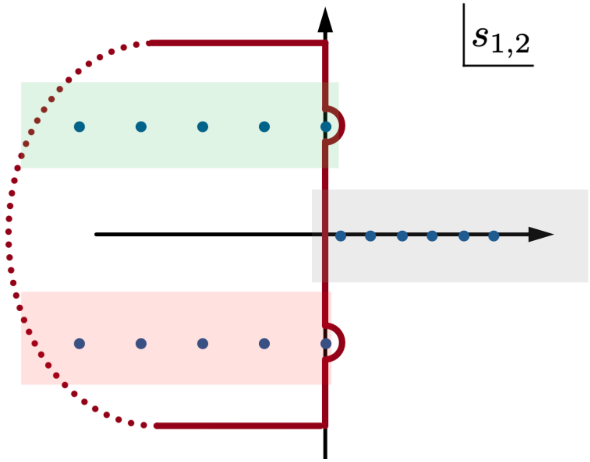
$$\mathcal{I}_{\mathrm{NL}} = \frac{\sin \mathrm{i}\pi\widetilde{\nu}}{2\pi}\left(\frac{r_1r_2}{4}\right)^{\mathrm{i}\widetilde{\nu}}\mathbf{F}_{\widetilde{\nu}}(r_1)\mathbf{F}_{\widetilde{\nu}}(r_2)+\mathrm{c.c.}.$$

$$\mathbf{F}_{\widetilde{\nu}}(r) \equiv r^{5/2}\Gamma\left[\frac{5}{2}+\mathrm{i}\widetilde{\nu},-\mathrm{i}\widetilde{\nu}\right]{}_2\mathrm{F}_1\left[\begin{matrix}\frac{5}{4}+\frac{\mathrm{i}\widetilde{\nu}}{2},\frac{7}{4}+\frac{\mathrm{i}\widetilde{\nu}}{2}\\1+\mathrm{i}\widetilde{\nu}\end{matrix}\middle|r^2\right] \qquad r_1\equiv\frac{k_s}{k_{12}},\quad r_2\equiv\frac{k_s}{k_{34}}.$$

Tree-Level Example

For the opposite-sign integral, the time integral is factorized and simple:

$$\mathcal{I}^{p_1p_2}_{\pm\mp}(r_1,r_2)=\frac{1}{4\pi}e^{\mp\mathrm{i}\pi(p_1-p_2)/2}r_1^{5/2+p_1}r_2^{5/2+p_2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}\left(\frac{r_1}{2}\right)^{-2s_1}\left(\frac{r_2}{2}\right)^{-2s_2}$$
$$\times\,\Gamma\left[p_1+\tfrac{5}{2}-2s_1,p_2+\tfrac{5}{2}-2s_2,s_1-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_1+\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2+\tfrac{\mathrm{i}\widetilde{\nu}}{2}\right].$$



$$s_1=-n_1\mp\frac{\mathrm{i}\widetilde{\nu}}{2},\quad s_2=-n_2\pm\frac{\mathrm{i}\widetilde{\nu}}{2}$$ => local signal

$$s_1=-n_1\mp\frac{\mathrm{i}\widetilde{\nu}}{2},\quad s_2=-n_2\mp\frac{\mathrm{i}\widetilde{\nu}}{2}$$ => nonlocal signal

Tree-Level Example

For the same-sign integral, we split it into two parts with the help of $\theta(\tau_1 - \tau_2) = 1 - \theta(\tau_2 - \tau_1)$, such that

$$D_{\pm\pm}(k;\tau_1,\tau_2) = D_{\gtrless}(k;\tau_1,\tau_2) + \Big[D_{\lessgtr}(k;\tau_1,\tau_2) - D_{\gtrless}(k;\tau_1,\tau_2)\Big]\theta(\tau_2-\tau_1).$$

Then:

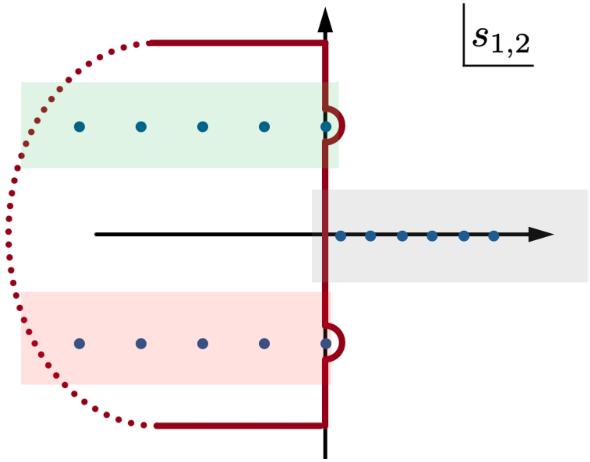
$$\mathcal{I}^{p_1p_2}_{\pm\pm}(r_1,r_2) = \mathcal{I}^{p_1p_2}_{\pm\pm,\mathrm{F},>}(r_1,r_2) + \mathcal{I}^{p_1p_2}_{\pm\pm,\mathrm{TO},>}(r_1,r_2). \qquad (r_1 < r_2)$$

$$\begin{aligned}
\mathcal{I}^{p_1p_2}_{\pm\pm,\mathrm{F},>}(r_1,r_2) \equiv & -k_s^{5+p_1+p_2}\int_{-\infty}^{0} \mathrm{d}\tau_1\mathrm{d}\tau_2\,(-\tau_1)^{p_1}(-\tau_2)^{p_2}e^{\pm\mathrm{i}(k_{12}\tau_1+k_{34}\tau_2)}D_{\gtrless}(k_s;\tau_1,\tau_2),\\
\mathcal{I}^{p_1p_2}_{\pm\pm,\mathrm{TO},>}(r_1,r_2) \equiv & -k_s^{5+p_1+p_2}\int_{-\infty}^{0}\mathrm{d}\tau_2\int_{-\infty}^{\tau_2}\mathrm{d}\tau_1\,(-\tau_1)^{p_1}(-\tau_2)^{p_2}e^{\pm\mathrm{i}(k_{12}\tau_1+k_{34}\tau_2)}\\
&\times\Big[D_{\lessgtr}(k;\tau_1,\tau_2) - D_{\gtrless}(k;\tau_1,\tau_2)\Big].
\end{aligned}$$

Tree-Level Example

The factorized part is similar to the opposite-sign integral:

$$\mathcal{I}^{p_1p_2}_{\pm\pm,\mathrm{F},>} = \frac{1}{4\pi}e^{\mp\mathrm{i}\pi(p_1+p_2)/2}r_1^{5/2+p_1}r_2^{5/2+p_2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}\,(\pm\mathrm{i}e^{\pm2\mathrm{i}\pi s_1})\left(\frac{r_1}{2}\right)^{-2s_1}\left(\frac{r_2}{2}\right)^{-2s_2}$$
$$\times\,\Gamma\left[p_1+\tfrac{5}{2}-2s_1,p_2+\tfrac{5}{2}-2s_2,s_1-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_1+\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2-\tfrac{\mathrm{i}\widetilde{\nu}}{2},s_2+\tfrac{\mathrm{i}\widetilde{\nu}}{2}\right],$$



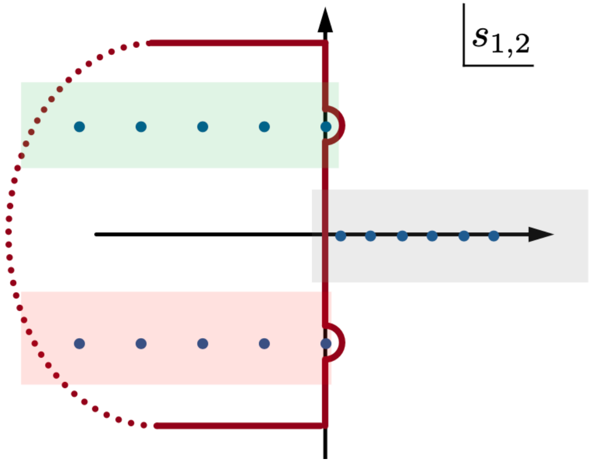
$$s_1=-n_1\mp\frac{\mathrm{i}\widetilde{\nu}}{2},\quad s_2=-n_2\pm\frac{\mathrm{i}\widetilde{\nu}}{2}$$ => local signal

$$s_1=-n_1\mp\frac{\mathrm{i}\widetilde{\nu}}{2},\quad s_2=-n_2\mp\frac{\mathrm{i}\widetilde{\nu}}{2}$$ => nonlocal signal

Tree-Level Example

The time-ordered part is more complicated:

$$\mathcal{I}^{p_1p_2}_{\pm\pm,\mathrm{TO},>} = \frac{1}{4\pi} e^{\mp \mathrm{i}\pi(p_1+p_2)/2} r_1^{5+p_1+p_2} \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{\mathrm{d}s_1}{2\pi\mathrm{i}} \frac{\mathrm{d}s_2}{2\pi\mathrm{i}} \left(\mp\mathrm{i}e^{\pm 2\mathrm{i}\pi s_1} \pm \mathrm{i}e^{\pm 2\mathrm{i}\pi s_2}\right)\left(\frac{r_1}{2}\right)^{-2s_{12}}$$
$$\times \Gamma\left[p_2+\tfrac{5}{2}-2s_2, p_1+p_2+5-2s_{12}, s_1-\tfrac{\mathrm{i}\widetilde{\nu}}{2}, s_1+\tfrac{\mathrm{i}\widetilde{\nu}}{2}, s_2-\tfrac{\mathrm{i}\widetilde{\nu}}{2}, s_2+\tfrac{\mathrm{i}\widetilde{\nu}}{2}\right]$$
$$\times\, {}_2\widetilde{\mathrm{F}}_1\left[\begin{matrix} p_2+\frac{5}{2}-2s_2, p_1+p_2+5-2s_{12} \\ p_2+\frac{7}{2}-2s_2 \end{matrix} \middle| -\frac{r_1}{r_2}\right].$$



$$s_1 = -n_1 \mp \frac{\mathrm{i}\widetilde{\nu}}{2}, \quad s_2 = -n_2 \pm \frac{\mathrm{i}\widetilde{\nu}}{2}$$ => background

$$s_1 = -n_1 \mp \frac{\mathrm{i}\widetilde{\nu}}{2}, \quad s_2 = -n_2 \mp \frac{\mathrm{i}\widetilde{\nu}}{2}$$ => 0

Tree-Level Cutting rule

To see this, recall:

$$D_{\pm\mp}(k;\tau_1,\tau_2)=\frac{1}{4\pi}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}s_1}{2\pi\mathrm{i}}\frac{\mathrm{d}s_2}{2\pi\mathrm{i}}\,e^{\mp\mathrm{i}\pi(s_1-s_2)}\left(\frac{k}{2}\right)^{-2s_{12}}(-\tau_1)^{-2s_1+3/2}(-\tau_2)^{-2s_2+3/2}$$
$$\times\,\Gamma\left[s_1-\frac{\mathrm{i}\widetilde{\nu}}{2},s_1+\frac{\mathrm{i}\widetilde{\nu}}{2},s_2-\frac{\mathrm{i}\widetilde{\nu}}{2},s_2+\frac{\mathrm{i}\widetilde{\nu}}{2}\right],$$

The phase factors $e^{\mp i\pi(s_1-s_2)}$ are the same on the nonlocal poles. So the four propagators are the same, and also equivalent to their real part. Furthermore, there is no step function in the real part, so the nonlocal signal is automatically factorized.

Tree-level cutting rule: For the nonlocal signal of a tree graph, cut an internal line, and replace D_{ab} by $\mathrm{Re}[D]$ for the cut line.