Soft photon theorem in QCD with massless quarks

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第十七届TeV工作组学术研讨会

Outline

1. Infrared singularities in wide-angle scattering

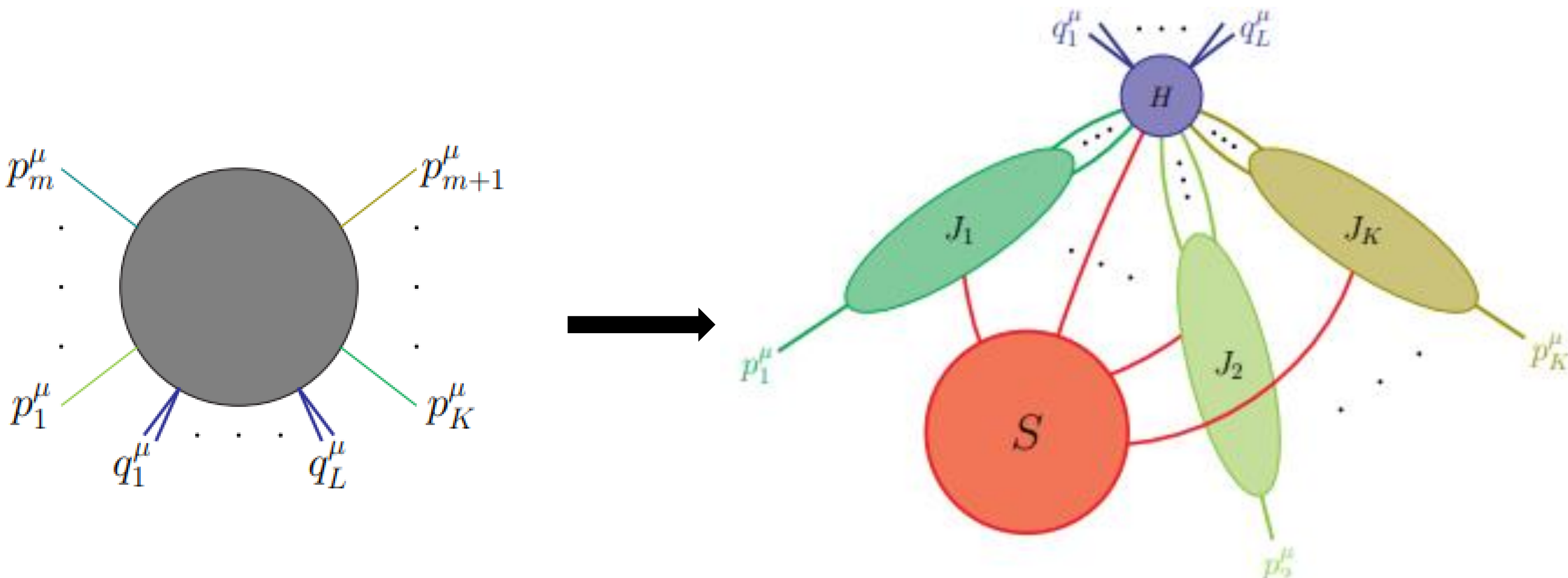
2. The soft photon theorem
   - The Low-Burnett-Kroll-Del Duca (LBKD) theorem
   - Our statement

3. Why?
   - The lowest-order result
   - QCD webs

4. Conclusion

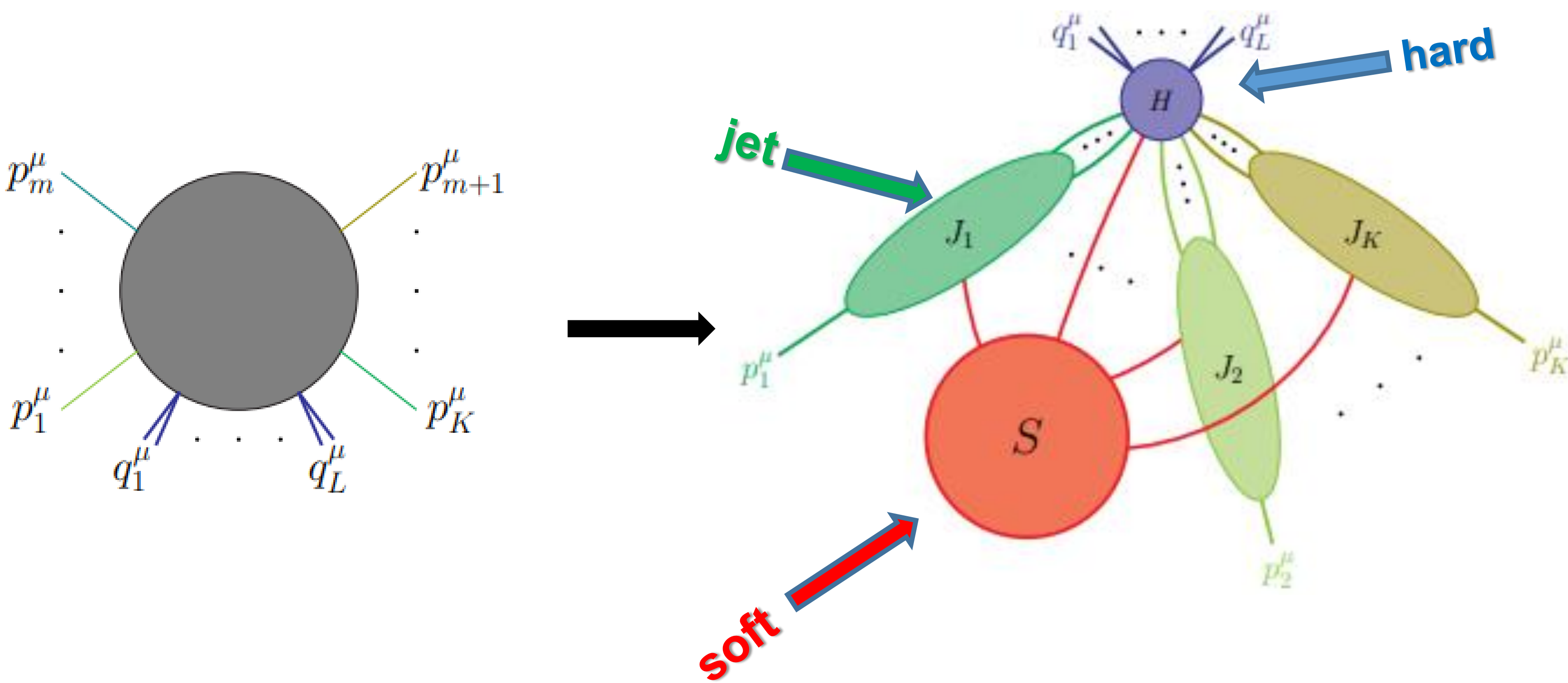
Infrared singularities of wide-angle scattering

General configurations of infrared singularities (pinch surfaces):



Infrared singularities of wide-angle scattering

General configurations of infrared singularities (pinch surfaces):



Infrared singularities of wide-angle scattering

- Infrared singularities are characterized by the **Landau equations**

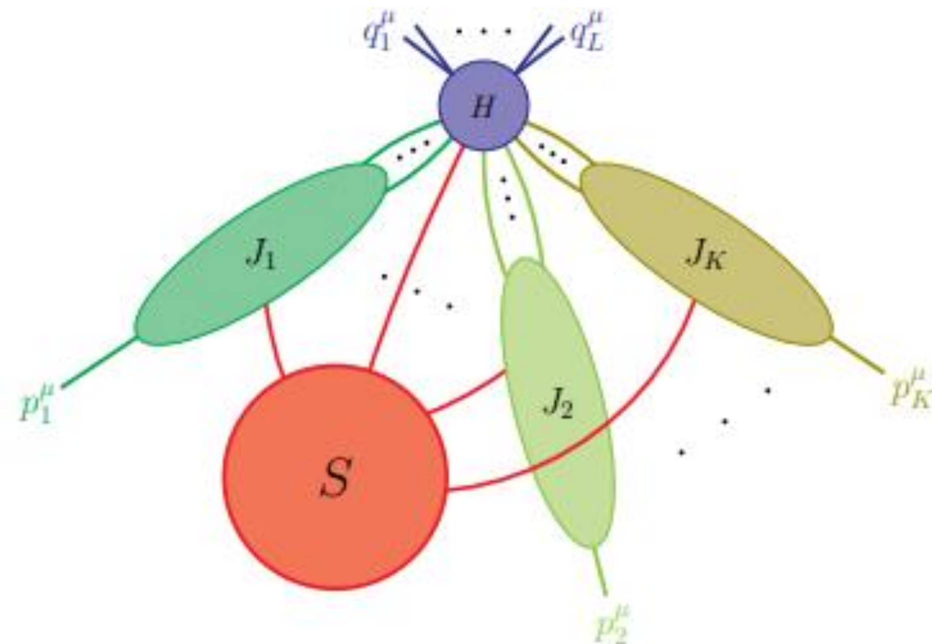
$$\alpha_e l_e^2(k,p,q) = 0 \qquad \forall e \in G$$
$$\frac{\partial}{\partial k_a}\mathcal{D}\,(k,p,q;\alpha) = 0 \qquad \forall a \in \{1,\ldots,L\}\,.$$

(necessary conditions for infrared singularities)

- For each solution of the Landau equations, the integration contour is "**pinched**" by poles of the integrand.
- In order that the integral is infrared divergent, one also needs to perform the **power counting**.
- Power counting result: infrared singularities in the wide-angle scattering are at most **logarithmic**.

Infrared singularities of wide-angle scattering

Constraints



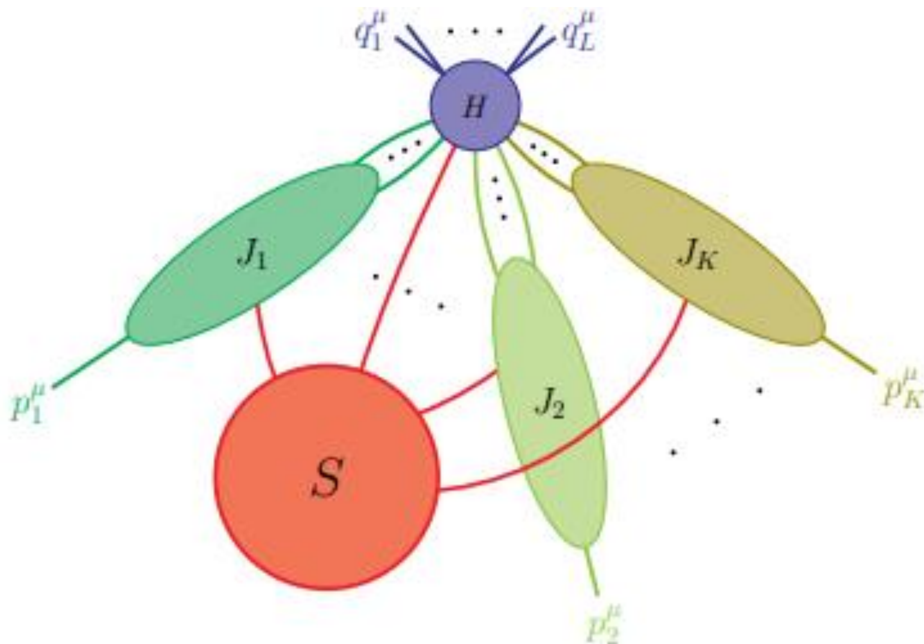
1, soft propagators cannot attach to H directly;

2, soft scalars or fermions cannot attach to the jets directly;

3, for each jet, the set of its propagators connecting H consists of a single physical parton and arbitrary number of scalar-polarized gauge bosons.

Infrared singularities of wide-angle scattering

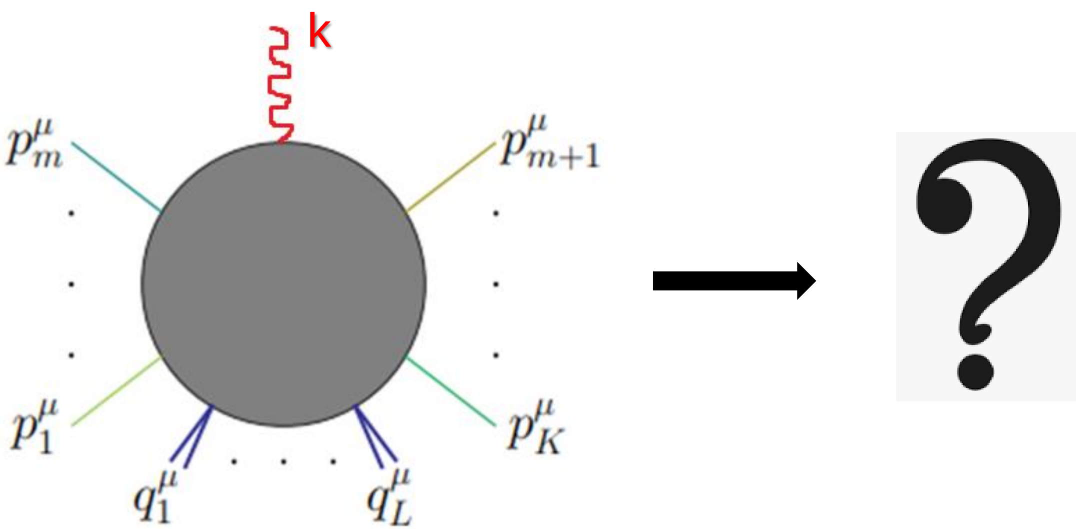
Constraints



These constraints are essential for the hard-collinear-soft factorization, where the subgraphs H, J, and S can be "decoupled" from each other.

Single-soft photon emission

With the emission of an additional soft photon (momentum: k)



The soft external photon is decoupled from the rest of the graph.

Single-soft photon emission

- **The Low-Burnett-Kroll-Del Duca (LBKD) theorem:**

For $\omega \ll m \ll \sqrt{s}$,

$$M_3(\{p_i\}, k, \epsilon(k)) = \sum_i \delta_i \, e_i \, \frac{p_i^\mu}{p_i \cdot k} \big[\underbrace{\epsilon_\mu(k)}_{\text{leading}} - \underbrace{(k_\mu \epsilon^\nu(k) - \epsilon_\mu(k) k^\nu) \, O_\nu \, (p_i, k)}_{\text{next-to-leading}} \big] \, M_2(\{p_i\})$$

- Low 1958: next-to-leading contribution, for $\omega \ll m^2/\sqrt{s}$
- Gribov 1967: leading contribution, for $\omega \ll m$
- Burnett & Kroll 1968: including particle spins
- Del Duca 1990: next-to-leading contribution, for $\omega \ll m$
- ….

Single-soft photon emission

- We are interested in the kinematic limit where the photon energy is much lower than that of external partons, but much larger than the masses of some quark:

$$m \ll \omega \ll \sqrt{s}$$

The soft photon theorem then receives corrections at leading power!

$$M_3^{(f)}(\{p_i\}, k, \epsilon(k)) = \left[e_f + \Gamma_{\mathrm{EM}}^{(f)}(\alpha_s) \sum_{j=1}^{n_0} e_j \right] \left[\frac{p_1 \cdot \epsilon(k)}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon(k)}{p_2 \cdot k} \right] M_2^{(f)}(\{p_i\}) \,.$$

new

Single-soft photon emission

Comments on the correction term $\Gamma_{\mathrm{EM}}^{(f)}(\alpha_S)\sum_{j=1}^{n_0} e_j$

- It is independent of the charges of the external particles, but sensitive to their color representations.

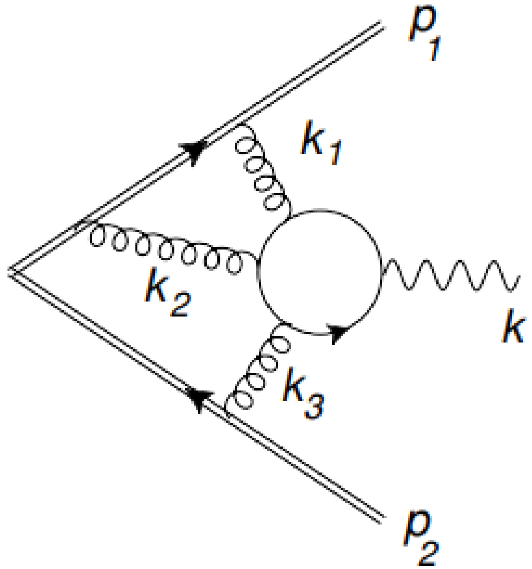
- e_j represents the charges of the internal massless quarks.

- The constant $\Gamma_{\mathrm{EM}}^{(f)}(\alpha_S)$ starts from $\mathcal{O}(\alpha_S^3)$.

- The constant $\Gamma_{\mathrm{EM}}^{(f)}(\alpha_S)$ is **finite** and **real** to all orders.

Single-soft photon emission

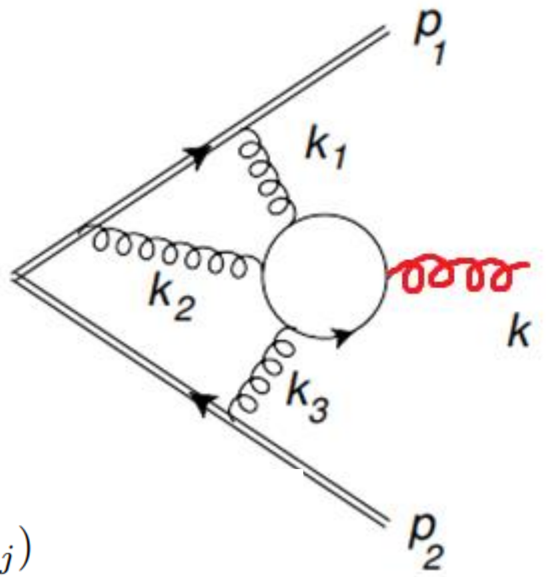
The term $\Gamma^{(f)}_{\mathrm{EM}}(\alpha_S)\sum_{j=1}^{n_0} e_j$ comes from the following graphs:



This $\mathcal{O}(\alpha_S^3)$ result can be directly obtained from the computation of the **three-loop single-soft current** or the four-loop cusp anomalous dimension.

Chen, Luo, Yang, Zhu '23
Herzog, YM, Mistlberger, Suresh '23

Using the soft-gluon current



$$\lim_{q\to 0}\mathcal{A}_{p_1 p_2 \dots p_n q} = \mathbf{J}(q)\mathcal{A}_{p_1 p_2 \dots p_n}.$$

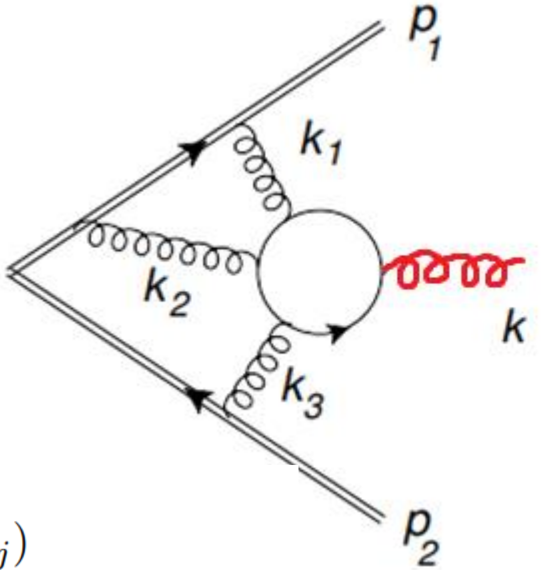
$$\mathbf{J}(q) = \frac{i g_S}{C_A}\epsilon^a_\mu(q)\sum_{i\neq j}\left(\frac{p_i^\mu}{p_i\cdot q}-\frac{p_j^\mu}{p_j\cdot q}\right)\left[f^{abc}\mathbf{T}_i^b\mathbf{T}_j^c K_2(q,p_i,p_j)\right.$$
$$\left. + iC_A\left(d_{4A}^{abcd}K_{4A}(q,p_i,p_j) + \underbrace{n_f d_{4F}^{abcd}K_{4F}(q,p_i,p_j)}_{\text{the associated term}}\right)\left\{\mathbf{T}_i^b,\mathbf{T}_i^c\right\}\mathbf{T}_j^d\right].$$

$$d_R^{abcd} = \frac{1}{4!}\left[\mathrm{Tr}\left(T_R^a T_R^b T_R^c T_R^d\right) + \text{symmetric permutations}\right].$$

$$K_X(q,p_i,p_j) = \sum_{o=0}^{\infty} a_S^o\left(\frac{(-2qp_i - i0)(-2qp_j - i0)}{(-2p_i p_j - i0)\mu^2}\right)^{-o\epsilon} K_X^{(o)}.$$

$$a_S = \frac{\alpha_S^0}{\pi}\left(\frac{4\pi}{\mu^2}\right)^{\epsilon} e^{-\gamma_E\epsilon}, \qquad \gamma_E = 0.577216\dots.$$

Using the soft-gluon current



$$\lim_{q\to 0} \mathcal{A}_{p_1 p_2 \ldots p_n q} = \mathbf{J}(q) \mathcal{A}_{p_1 p_2 \ldots p_n}.$$

$$\begin{aligned}\mathbf{J}(q) = \frac{i g_S}{C_A} \epsilon^a_\mu(q) \sum_{i\neq j} \left(\frac{p_i^\mu}{p_i \cdot q} - \frac{p_j^\mu}{p_j \cdot q} \right) \Big[& f^{abc} \mathbf{T}_i^b \mathbf{T}_j^c K_2(q, p_i, p_j) \\ & + i C_A \left(d_{4A}^{abcd} K_{4A}(q, p_i, p_j) + \underbrace{n_f d_{4F}^{abcd} K_{4F}(q, p_i, p_j)}_{\text{the associated term}} \right) \left\{ \mathbf{T}_i^b, \mathbf{T}_i^c \right\} \mathbf{T}_j^d \Big].\end{aligned}$$

$$K_{4F}^{(0)} = K_{4F}^{(1)} = K_{4F}^{(2)} = 0. \tag{2.14}$$

$$\begin{aligned}K_{4F}^{(3)} = & -\frac{\zeta_2}{2} + \frac{\zeta_3}{6} + \frac{5\zeta_5}{6} + \left(\frac{7\zeta_3^2}{3} - \frac{47\zeta_3}{36} - \frac{15\zeta_2}{4} + \frac{13\zeta_4}{8} + \frac{25\zeta_5}{18} + \frac{5\zeta_6}{2} \right) \epsilon \\ & \left(\frac{35\zeta_3^2}{9} + \frac{19\zeta_2\zeta_3}{4} + 7\zeta_4\zeta_3 - \frac{1471\zeta_3}{108} - \frac{239\zeta_2}{12} - \frac{589\zeta_4}{24} + \frac{5\zeta_2\zeta_5}{4} + \frac{1423\zeta_5}{108} \right. \\ & \left. + \frac{25\zeta_6}{6} + \frac{155\zeta_7}{4} \right) \epsilon^2 + \mathcal{O}(\epsilon^3).\end{aligned} \tag{2.15}$$

$\Rightarrow$ **No epsilon poles!**

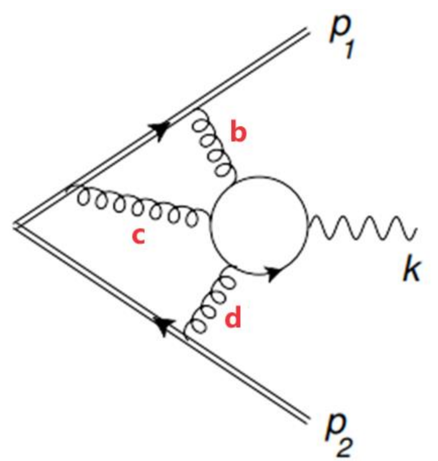
Using the soft-gluon current

In the context of our study,

$$M_3^{(f)}(\{p_i\}, k, \epsilon(k)) = \left[e_f + \Gamma_{\rm EM}^{(f)}(\alpha_s) \sum_{j=1}^{n_0} e_j \right] \left[\frac{p_1 \cdot \epsilon(k)}{p_1 \cdot k} - \frac{p_2 \cdot \epsilon(k)}{p_2 \cdot k} \right] M_2^{(f)}(\{p_i\}) \, .$$

We have: $$\Gamma_{\rm EM}^{(F)} = - \left(\frac{\alpha_s}{\pi} \right)^3 C_F^{(3)} \left(-\frac{\zeta_2}{2} + \frac{\zeta_3}{6} + \frac{5\zeta_5}{6} \right) ,$$

$$C_F^{(3)} = d^{bcd} T_F^b T_F^c T_F^d \qquad d^{bcd} \propto {\rm Tr} \left(T_F^b T_F^c T_F^d + {\rm permutations} \right)$$



QCD webs

The constant $\Gamma^{(f)}_{\rm EM}(\alpha_S)$ is **finite** and **real** to **all orders**.

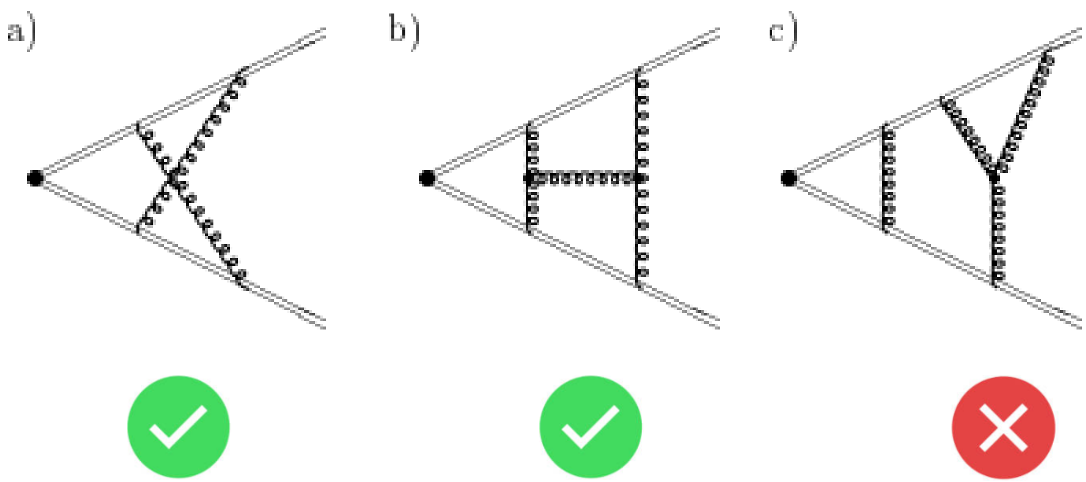
Why?

To see this, we need the concept of QCD webs.

QCD webs

The constant $\Gamma^{(f)}_{\mathrm{EM}}(\alpha_S)$ is **finite** and **real** to **all orders**.

- A **web** is a set of gluon lines which cannot be partitioned without cutting at least one of its lines.



- Properties of webs:

- exponentiation (Sterman ’81, Gatheral ’83, Frenkel, Taylor ’84)

- cancellation of infrared subdivergences (Frenkel, Gatheral, Taylor ’84)

Summary

We studied the radiation of a photon, whose energy is much lower than that of the external partons but much larger than the masses of some quarks.

Beyond two loops, the conventional soft photon theorem receives corrections at leading power in the photon energy, associated with soft virtual loops of massless fermions.

These corrections are shown to be finite and real to all orders, conforming with the soft-gluon current results.

Summary

We studied the radiation of a photon, whose energy is much lower than that of the external partons but much larger than the masses of some quarks.

Beyond two loops, the conventional soft photon theorem receives corrections at leading power in the photon energy, associated with soft virtual loops of massless fermions.

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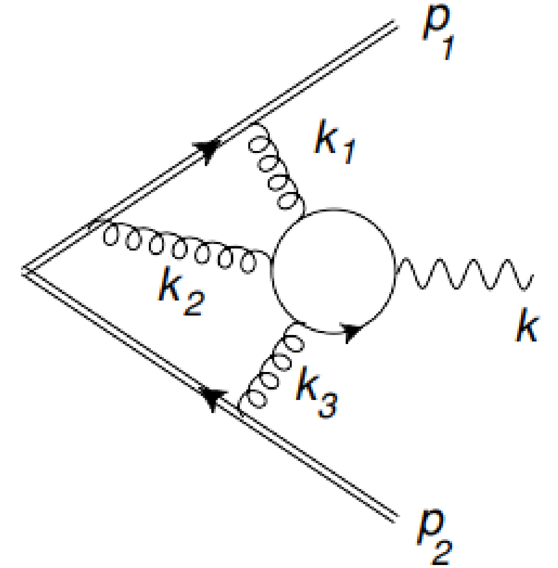
THANK YOU!

Backup slides

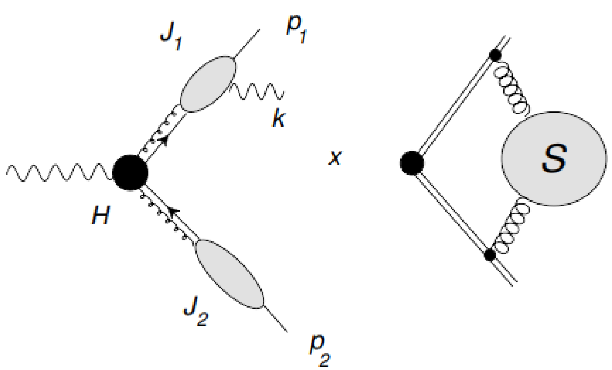
Single-soft photon emission



- A closer look at the graph:

- In general, the leading pinch surfaces feature factorization properties:

  (soft photon attached to J)



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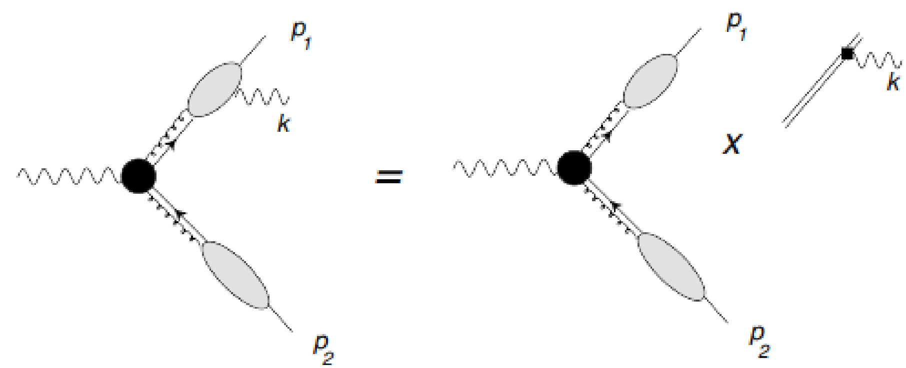
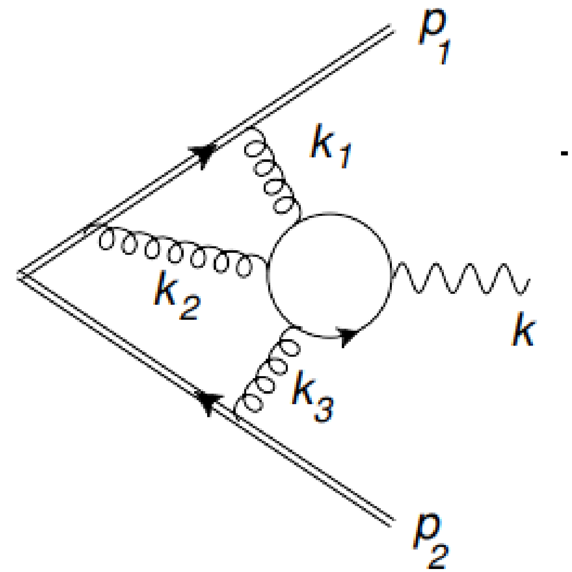


Single-soft photon emission



- A closer look at the graph:

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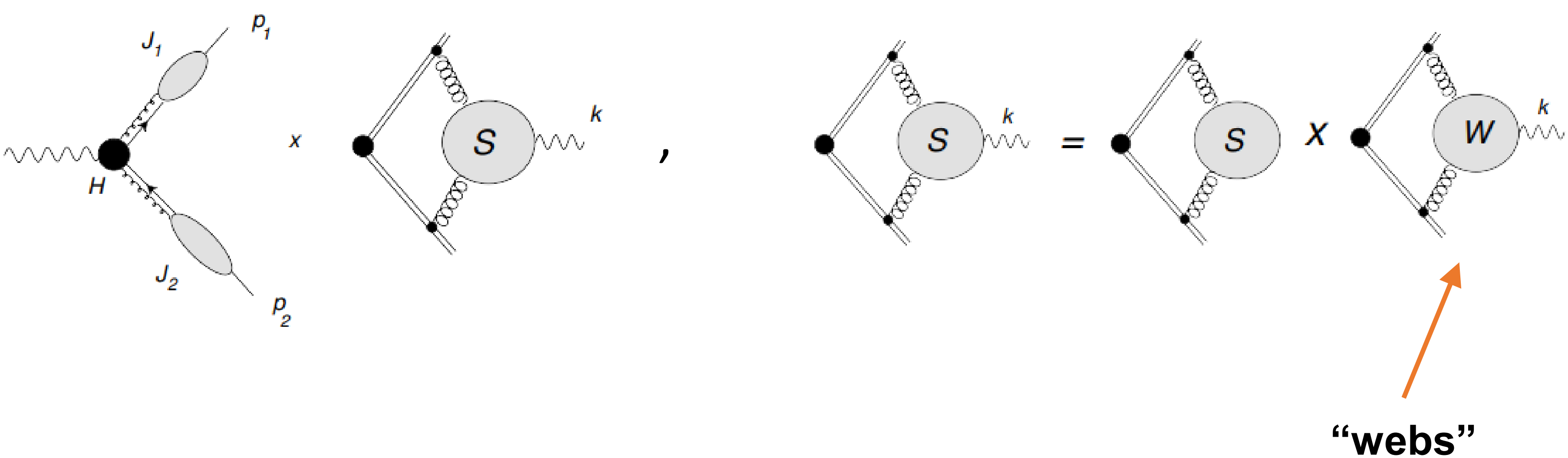
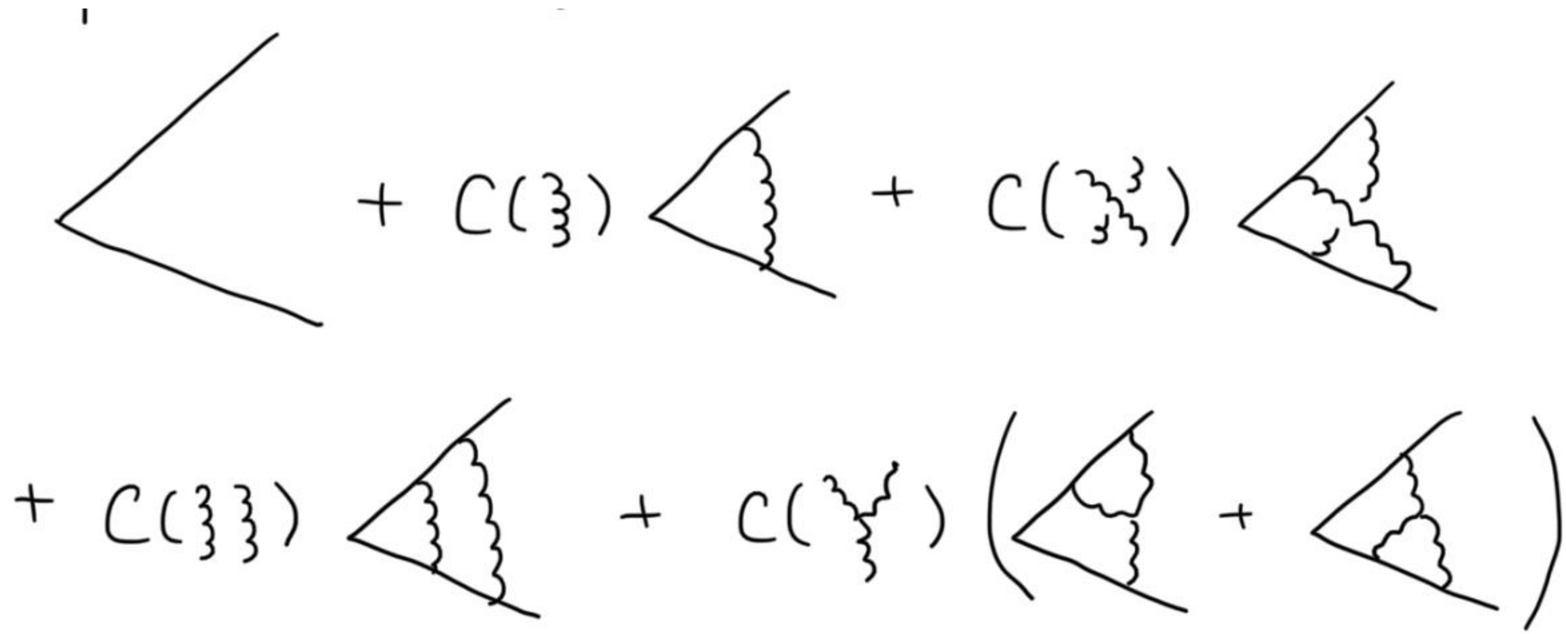


QCD webs

- **Exponentiation** (Sterman ’81, Gatheral ’83, Frenkel&Taylor ’84)



$$+\ \mathcal{O}(\alpha_s^3)$$

QCD webs

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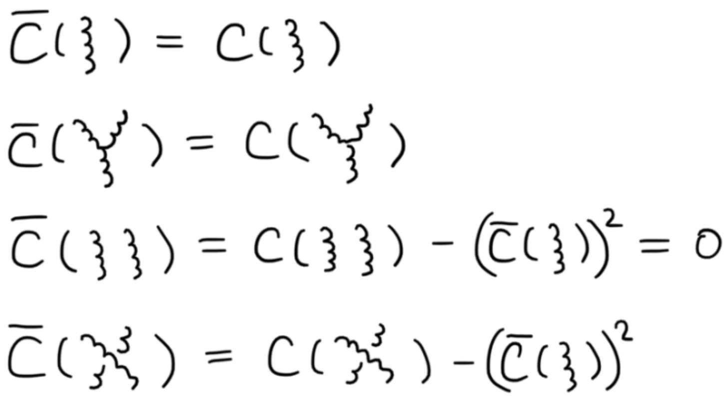

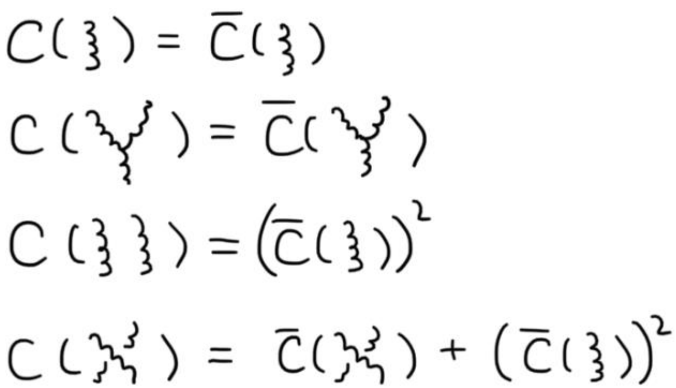

QCD webs

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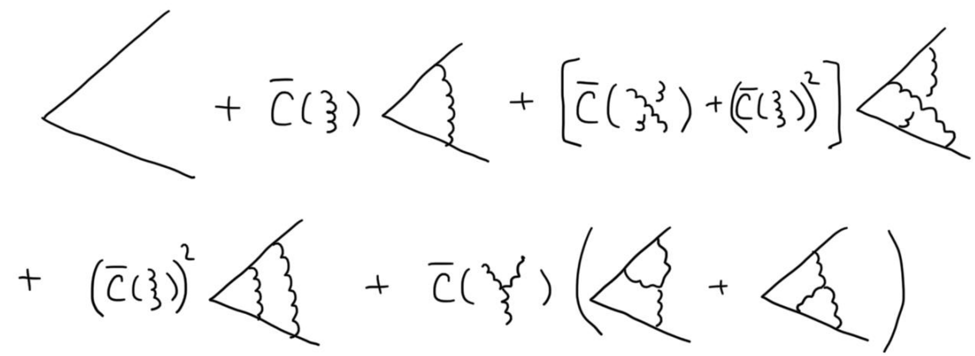
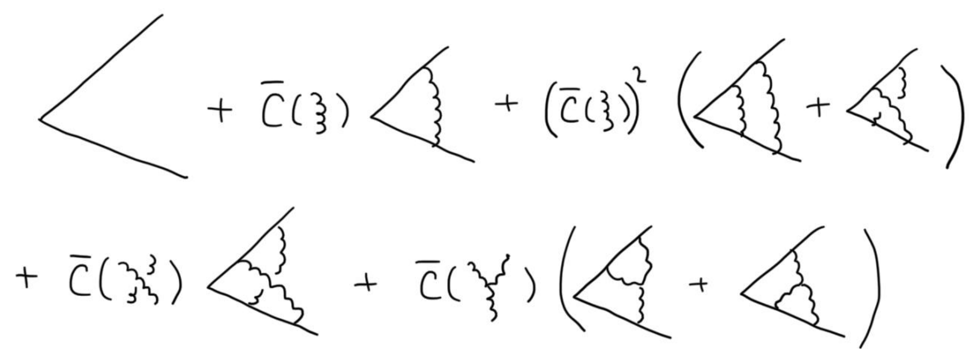
=



$+ \mathcal{O}(\alpha_s^3)$

=



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QCD webs

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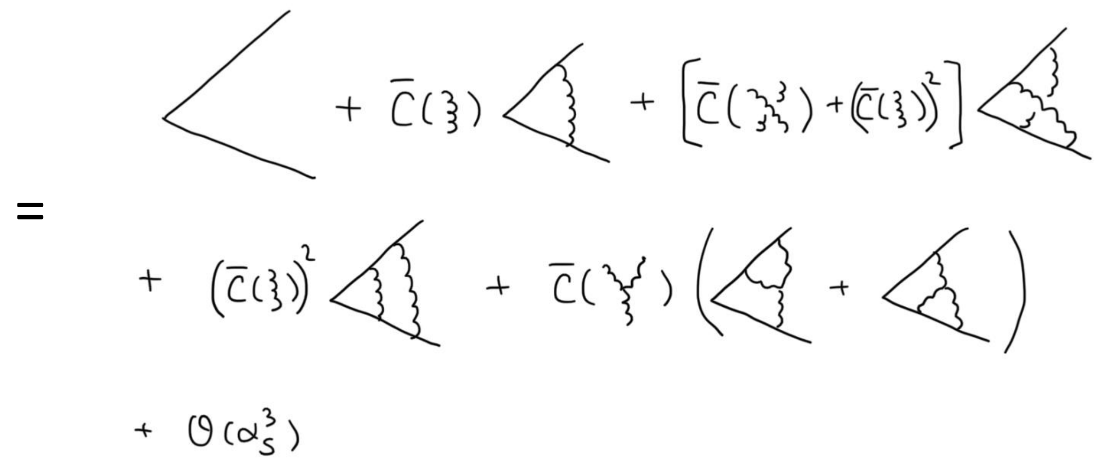
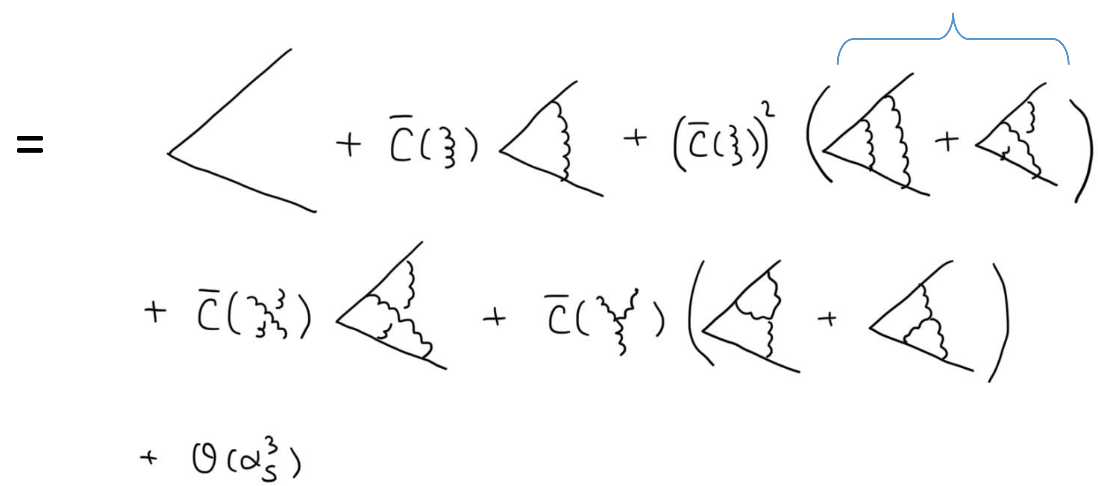


Use the “eikonal identity”



QCD webs

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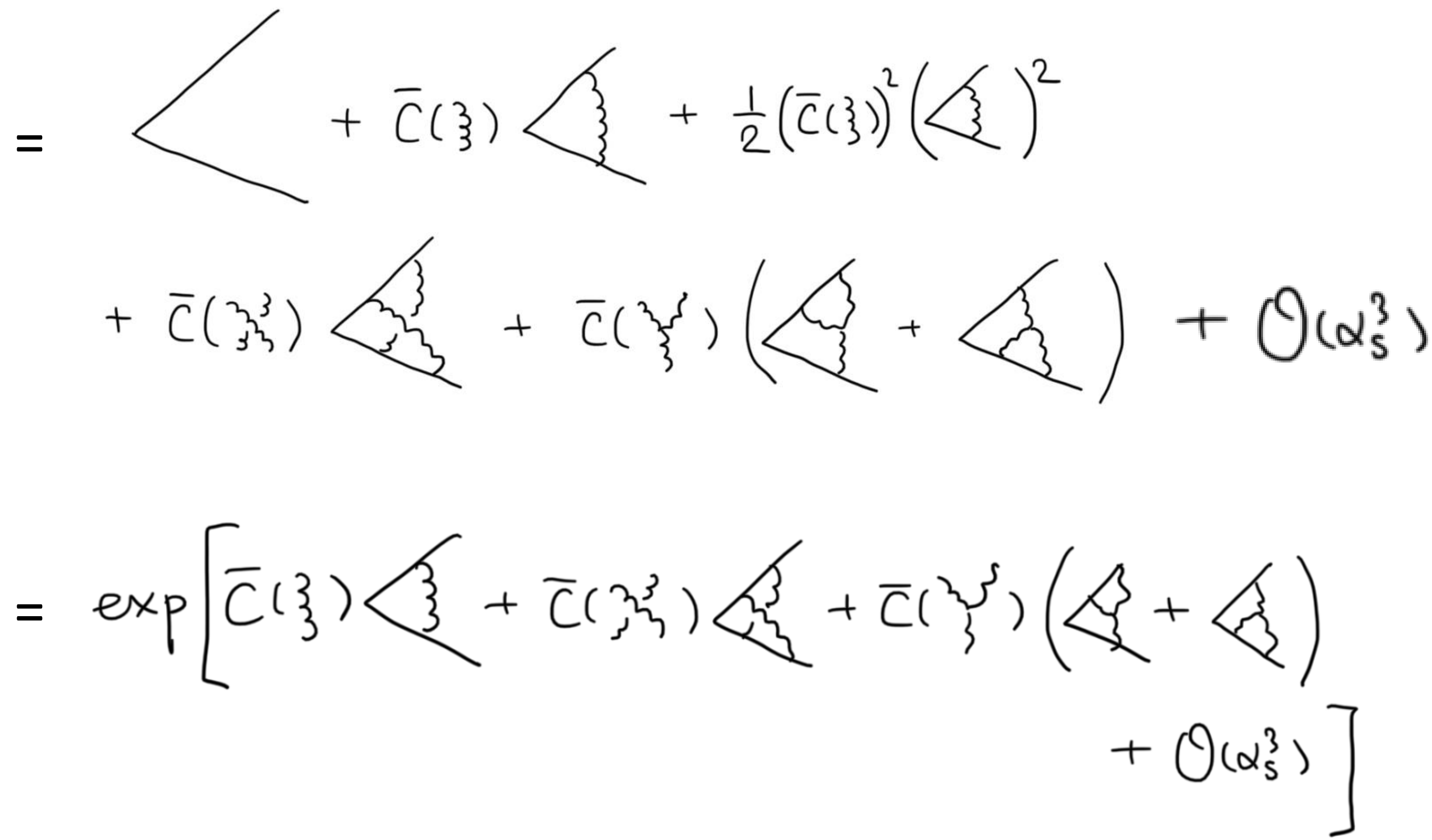


QCD webs

- **Cancellation of subdivergences** (Frenkel&Gatheral&Taylor '84)

- **Statement: at each order, subdivergences are cancelled in the exponent.**

$$
\begin{aligned}
F^{(3)} = &\sum_{W^{(3)}} \bar{C}(W^{(3)})\mathcal{F}(W^{(3)}) \\
&+ \left(\sum_{W^{(2)}} \bar{C}(W^{(2)})\mathcal{F}(W^{(2)})\right)\cdot\left(\sum_{W^{(1)}} \bar{C}(W^{(1)})\mathcal{F}(W^{(1)})\right) \\
&+ \frac{1}{3!}\left(\sum_{W^{(1)}} \bar{C}(W^{(1)})\mathcal{F}(W^{(1)})\right)^3
\end{aligned}
$$

QCD webs

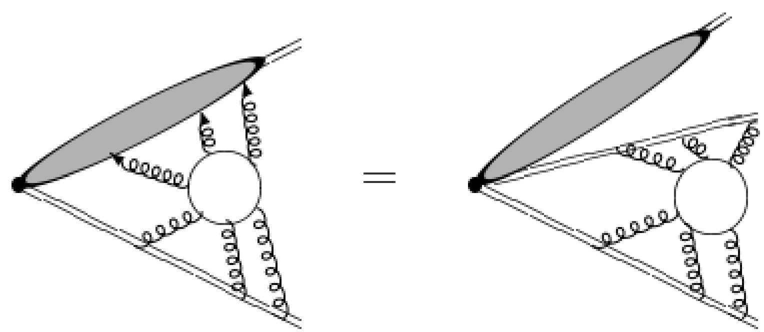
- **Cancellation of subdivergences** (Frenkel&Gatheral&Taylor ’84)

- **Statement: at each order, subdivergences are cancelled in the exponent.**

$$\sum_{W^{(3)}} \overline{C}(W^{(3)})\mathcal{F}(W^{(3)}) = F^{(3)} - \frac{1}{3!}\left(\sum_{W^{(1)}} \overline{C}(W^{(1)})\mathcal{F}(W^{(1)})\right)^3$$

$$-\left(\sum_{W^{(2)}} \overline{C}(W^{(2)})\mathcal{F}(W^{(2)})\right)\cdot\left(\sum_{W^{(1)}} \overline{C}(W^{(1)})\mathcal{F}(W^{(1)})\right)$$

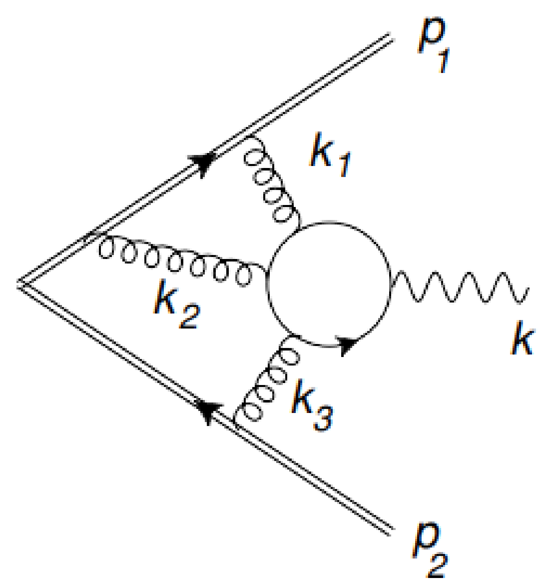
- Subdivergences of $F^{(3)}$:



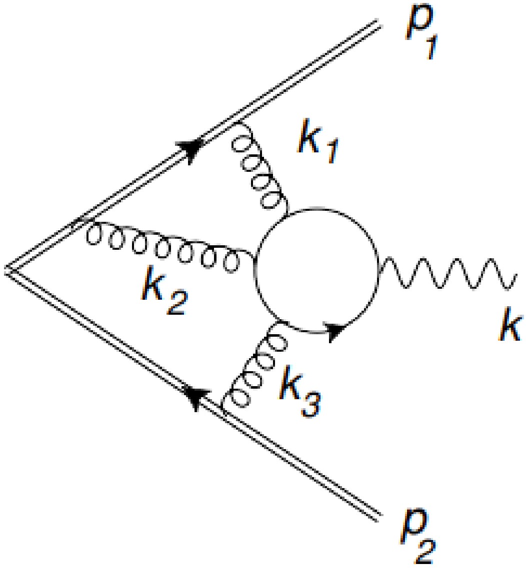
QCD webs

- We have stated:
  - Webs play an important role in the context of our study.
  - Webs are in the exponents of the complete sum over eikonal graphs.
  - **There are no subdivergences in the sum over webs order by order.**

Let's see it in the lowest-order example:



Lowest-order analysis



- 3-soft divergence ✓
- 2-soft-1-collinear divergence? ✗
- 1-soft-2-collinear divergence? ✗
  - Collinear to p1 or p2
  - Collinear to k
- 3-collinear divergence? ✗
- Actually, we can obtain the analytic expression from the soft current of the emission of a single gluon!

(based on Chen&Luo&Yang&Zhu'23, Herzog&Ma&Mistlberger&Suresh '23)