Probe ν Electromagnetic Properties with Atomic Radiative Emission of Neutrino Pair

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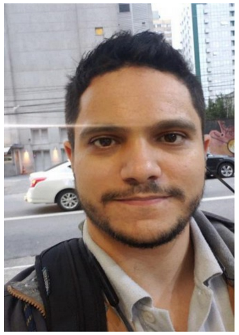

Pedro Pasquini

SFG & Pedro Pasquini, Eur.Phys.J.C 82 (2022) 3, 208 [2110.03510]

SFG & Pedro Pasquini, Phys.Lett.B 841 (2023) 137911 [2206.11717]

SFG & Pedro Pasquini, JHEP 12 (2023) 083 [2306.12953]

上海交通大學 SHANGHAI JIAO TONG UNIVERSITY

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李政道研究所 Tsung-Dao Lee Institute

Outline

1) **Neutrino Electromagnetic Properties**

2) **RENP with Light Mediator**

SFG & Pedro Pasquini, Eur.Phys.J.C 82 (2022) 3, 208 [2110.03510]

3) **RENP for Neutrino Electromagetic Properties**

SFG & Pedro Pasquini, Phys.Lett.B 841 (2023) 137911 [2206.11717]

SFG & Pedro Pasquini, JHEP 12 (2023) 083 [2306.12953]

Minimal Neutrinos

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Georg G. Raffelt

Stars as Laboratories for Fundamental Physics

The Astrophysics of Neutrinos, Axions, and Other Weakly Interacting Particles

In the standard model, neutrinos have been assigned the most minimal properties compatible with experimental data: zero mass, zero charge, zero dipole moments, zero decay rate, zero almost everything.

Pauli's Proposal of Neutrino

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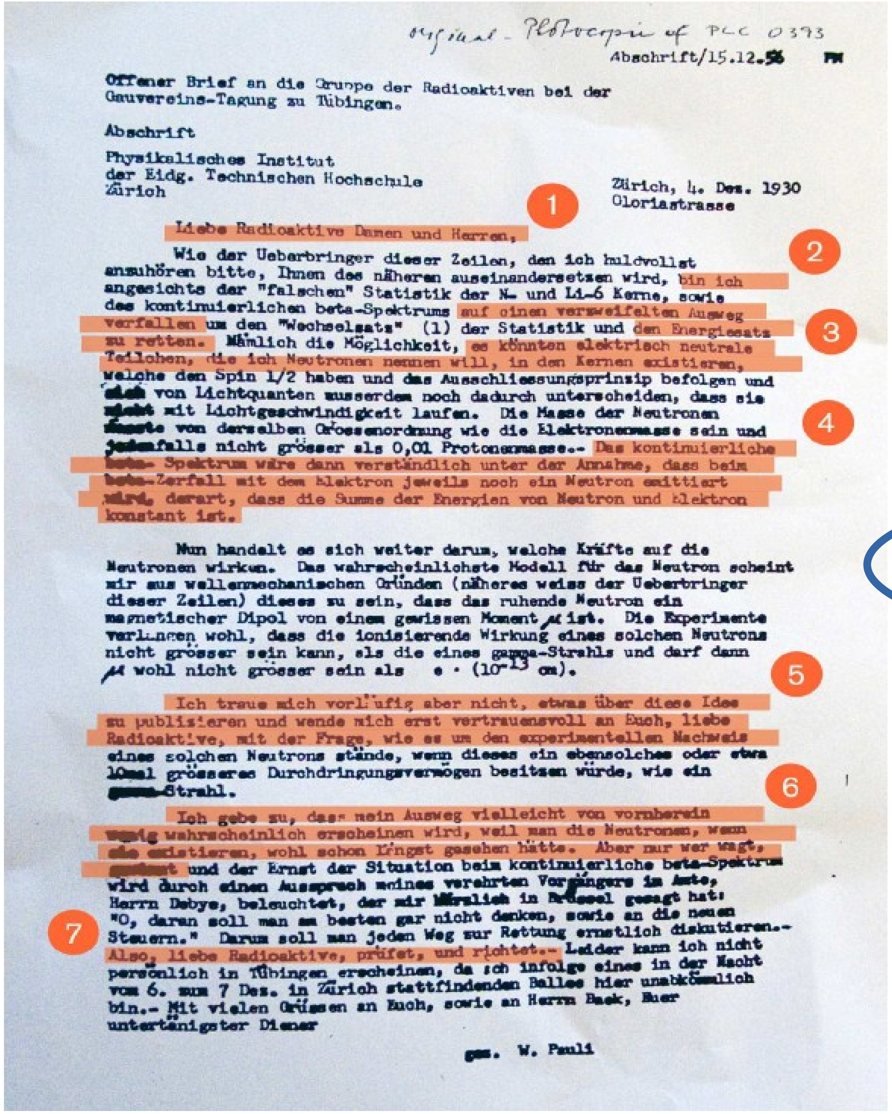

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Abschrift/15.12.56 PM

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

1 Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist. 2 3 4

Nun handelt es sich weiter darum, welche Kräfte auf die Neutronen wirken. Das wahrscheinlichste Modell für das Neutron scheint mir aus wellenmechanischen Gründen (näheres weiss der Ueberbringer dieser Zeilen) dieses zu sein, dass das ruhende Neutron ein magnetischer Dipol von einem gewissen Moment μ ist. Die Experimente verlangen wohl, dass die ionisierende Wirkung eines solchen Neutrons nicht grösser sein kann, als die eines gamma-Strahls und darf dann μ wohl nicht grösser sein als e · (10^-13 cm).

5 Ich traue mich vorläufig aber nicht, etwas über diese Idee zu publizieren und wende mich erst vertrauensvoll an Euch, liebe Radioaktive, mit der Frage, wie es um den experimentellen Nachweis eines solchen Neutrons stände, wenn dieses ein ebensolches oder etwa 10mal grösseres Durchdringungsvermögen besitzen würde, wie ein gamma-Strahl.

6 Ich gebe zu, dass mein Ausweg vielleicht von vornherein wenig wahrscheinlich erscheinen wird, weil man die Neutronen, wenn sie existieren, wohl schon längst gesehen hätte. Aber nur wer wagt, gewinnt und der Ernst der Situation beim kontinuierliche beta-Spektrum wird durch einen Ausspruch meines verehrten Vorgängers im Amte, Herrn Debye, beleuchtet, der mir kürzlich in Brüssel gesagt hat: "O, daran soll man am besten gar nicht denken, sowie an die neuen Steuern." Darum soll man jeden Weg zur Rettung ernstlich diskutieren.- 7 Also, liebe Radioaktive, prüfet, und richtet.- Leider kann ich nicht persönlich in Tübingen erscheinen, da ich infolge eines in der Nacht vom 6. zum 7 Dez. in Zürich stattfindenden Balles hier unabkömmlich bin.- Mit vielen Grüssen an Euch, sowie an Herrn Back, Euer untertänigster Diener

gez. W. Pauli

“ 亲爱的放射性女士先生们：

关于氮（N）和锂（Li） 6（译注： 6 是原子核的质量数，现在我们知道是质子数加上中子数，中子与质子质量差不多，当时只知道是质子质量的倍数）原子核的“错误”的统计定律和连续 β 谱，**我想到一个绝望的补救方法**，能够挽救统计定律以及能量守恒定律。这就是，**原子核中可能存在一个电中性的粒子，我称它为中子**，它的自旋是 1/2，满足不相容原理，而且与光量子不同，它不以光速运动。中子的质量应该与电子同一数量级，而且肯定不大于质子质量的 0.01 倍。假设 β 衰变时， 1 个中子和 1 个电子同时放出，它们的能量之和是常数，这就可以理解连续 β 谱。

还有一个问题，中子受到什么力？在波动力学基础上……**中子最可能的模型似乎是，静止的中子是个磁矩为 μ 的磁偶极矩**。实验似乎要求中子的电离效应不能大于 γ 射线，因此 μ 不能大于 e×10-13cm 。

目前我不敢就此想法发表任何东西，所以私下询问你们，亲爱的放射性朋友，如何在实验上证实这个中子，如果它的穿透能力等于或者十倍于 γ 射线。

我承认从先验的角度，我的方案不大可能，因为如果它们存在，早就应该被看到。但是敢拼才能赢，我的前任得拜 (P. Debye) 先生最近在布鲁塞尔向我表达了连续谱问题的严重性：‘哦，就像新的税一样，最好完全不去想它。’因此需要认真讨论每条拯救的道路。——因此，亲爱的放射性朋友，请检查并判断。——**不幸的是，我不能亲临图宾根，因为 12 月 6 日晚至 7 日有个舞会，我必须参加**……你们谦卑的侍者，沃尔夫冈 · 泡利。”

Neutrino Magnetic Properties

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- **Four types**

CPV

$$H_{EM} = \overline{\nu}\left[-i\sigma_{\mu\nu}q^{\nu}\left(\mu_\nu + i\epsilon_\nu\gamma_5\right) + \left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2}\right)q^2\left(\frac{\langle r_\nu^2\rangle}{6} + a_\nu\gamma_5\right)\right]\nu A^\mu(q),$$

MDM **EDM** **Charge Radius** **Anapole**

- **Flavor Structures**

$$j_\mu^{(\nu)} \equiv \bar{u}(p_i)\Lambda_\mu^{ij}(q)u(p_j)$$

$$\Lambda_\mu^{ij}(q) \equiv \left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2}\right)\left[f_Q^{ij}(q^2) + f_A^{ij}(q^2)q^2\gamma_5\right] - f_M^{ij}(q^2)i\sigma_{\mu\nu}q^\nu + f_E^{ij}(q^2)\sigma_{\mu\nu}q^\nu\gamma_5$$

$$\langle r_\nu^2\rangle \equiv 6\, df_Q^{ij}/dq^2\Big|_{q^2=0} \quad a_\nu^{ij} \equiv f_A^{ij}(0) \quad \mu_\nu^{ij} \equiv f_M^{ij}(0) \quad \epsilon_\nu^{ij} \equiv f_E^{ij}(0)$$

- **Dirac vs Majorana**

$$(q_\nu^{ij})^T = -q_\nu^{ij}\,, \quad (\mu_\nu^{ij})^T = -\mu_\nu^{ij}\,, \quad (\epsilon_\nu^{ij})^T = -\epsilon_\nu^{ij}\,, \quad \text{and} \quad (a_\nu^{ij})^T = a_\nu^{ij}$$

Neutrino Electron Scattering

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- **MDM & EDM**

$$\left(\frac{d\sigma_{\nu_\alpha e\to Xe}}{dT_r}\right)_{\mu,\epsilon} = \frac{\pi\alpha^2}{m_e^2}\left(\frac{1}{T_r} - \frac{1}{E_{\nu_\alpha}}\right)|\mu_\alpha^{\rm eff}|^2$$

$$|\mu_\alpha^{\rm eff}|^2 \equiv \sum_{ij}\left\{U_{\alpha i}(\mu_\nu^2 + \epsilon_\nu^2)_{ij}U_{\alpha j}^* + 2\,{\rm Im}[U_{\alpha i}(\mu_\nu\epsilon_\nu)_{ij}U_{\alpha j}^*]\right\}$$

- **Charge Radius & Anapole**

$$\frac{d\sigma_{\nu_\alpha e\to Xe}}{dT_r} = \frac{G_F^2 m_e}{2\pi}\left\{|\tilde{v}_{\alpha\alpha} \mp a_{\alpha\alpha}|^2 + |\tilde{v}_{\alpha\alpha} \pm a_{\alpha\alpha}|^2\left(1 - \frac{T_r}{E_\nu}\right)^2 - \left(|\tilde{v}_{\alpha\alpha}|^2 - |a_{\alpha\alpha}|^2\right)\frac{m_e T_r}{4E_\nu^2}\right\}$$
$$+ \frac{\alpha m_e}{144}\sum_{\beta\neq\alpha}|\langle r_\nu^2\rangle_{\alpha\beta}^{\rm eff}|^2\left[1 + \left(1 - \frac{T_r}{E_\nu}\right)^2 - \frac{m_e T_r}{4E_\nu^2}\right]. \qquad (2.10)$$

$$\langle r^2\rangle_{\alpha\beta}^{\rm eff} \equiv \sum_{ij} U_{\alpha i}U_{\beta j}^*\left[\langle r_\nu^2\rangle_{ij} - 6(a_\nu)_{ij}\right]$$

Combinations

Solar & Nuclei Scattering

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- **Solar Neutrinos @ Electron Scattering**

$$|\mu_\alpha^{\rm eff}|^2 \Rightarrow (\mu_\nu^\odot)^2 \equiv \sum_{ij} |\widetilde{U}_{ej}|^2 |(\mu_\nu)_{ij} - i(\epsilon_\nu)_{ij}|^2$$

$$\langle r^2 \rangle_{\alpha\beta}^{\rm eff} \Rightarrow \langle r_\nu^2 \rangle^\odot \equiv \sum_{ij} |\widetilde{U}_{ej}|^2 |\langle r_\nu^2 \rangle_{ij} - 6(a_\nu)_{ij}|$$

Combinations

- **Nuclei Scattering**

$$\left(\frac{d\sigma}{dT_r}\right)_{\rm CE\nu NS} = \frac{G_F^2 M}{2\pi}\left(1 - \frac{M T_r}{2E_\nu^2}\right)\left[\widetilde{v}_{\alpha\alpha}^{\,p} Z F_p + \widetilde{v}_{\alpha\alpha}^{\,n} N F_n\right]^2 + \frac{\alpha Z^2 F_p^2}{144} \sum_{\beta\neq\alpha} |\langle r_\nu^2 \rangle_{\alpha\beta}^{\rm eff}|^2 \left(1 - \frac{M T_r}{2E_\nu^2}\right) + Z^2 F_p^2 \frac{\pi\alpha^2}{m_e^2}\left(\frac{1}{T_r} - \frac{1}{E_{\nu_\alpha}} + \frac{T_r}{4E_{\nu_\alpha}^2}\right)|\mu_\alpha^{\rm eff}|^2. \quad (2.16)$$

Combinations

Celestial Constraints

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- **Neutrino Decay**

$$\Gamma(\nu_i \to \nu_j + \gamma) = \frac{1}{8\pi}\left(\frac{m_i^2 - m_j^2}{m_i}\right)^3 \left[|(\mu_\nu)_{ij}|^2 + |(\epsilon_\nu)_{ij}|^2\right]$$

Combinations

- **Stellar Cooling**

$$(\mu_\nu^{\circledast})^2 \equiv \sum_{ij}\left\{2m_{\gamma^*}^2\left[\frac{|\langle r_\nu^2\rangle_{ij}|^2}{36} + |(a_\nu)_{ij}|^2\right] + |(\mu_\nu)_{ij}|^2 + |(\epsilon_\nu)_{ij}|^2\right\}$$

Combinations

All measurements & observations are a complex combination of various EM property elements & mixing matrix elements!

Atomic Neutrino Pair Emission

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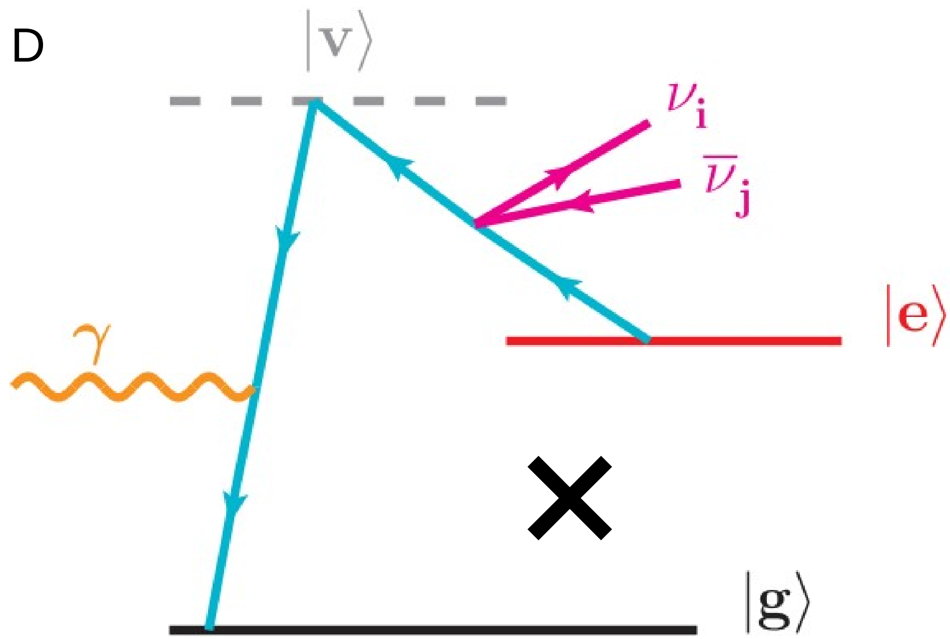
M. Yoshimura, Phys. Rev. D 75, 113007 (2007)



$$|e\rangle \to |v\rangle + \nu\bar{\nu}$$

$$|v\rangle \to |g\rangle + \gamma$$

$$\mathrm{E1} \times \mathrm{M1}$$

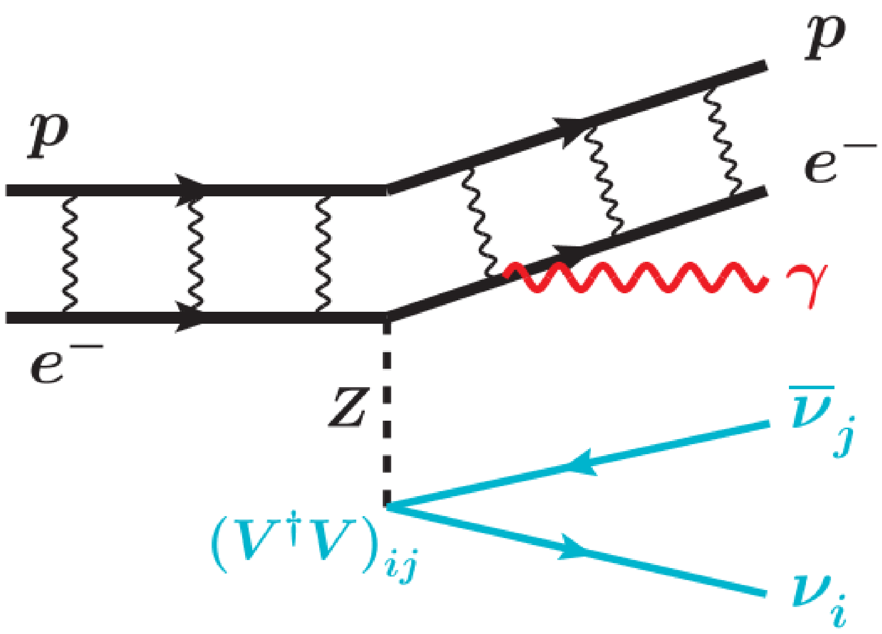
Jue Zhang & Shun Zhou, Phys.Rev.D 93 (2016) 11, 113020 [1604.08008]

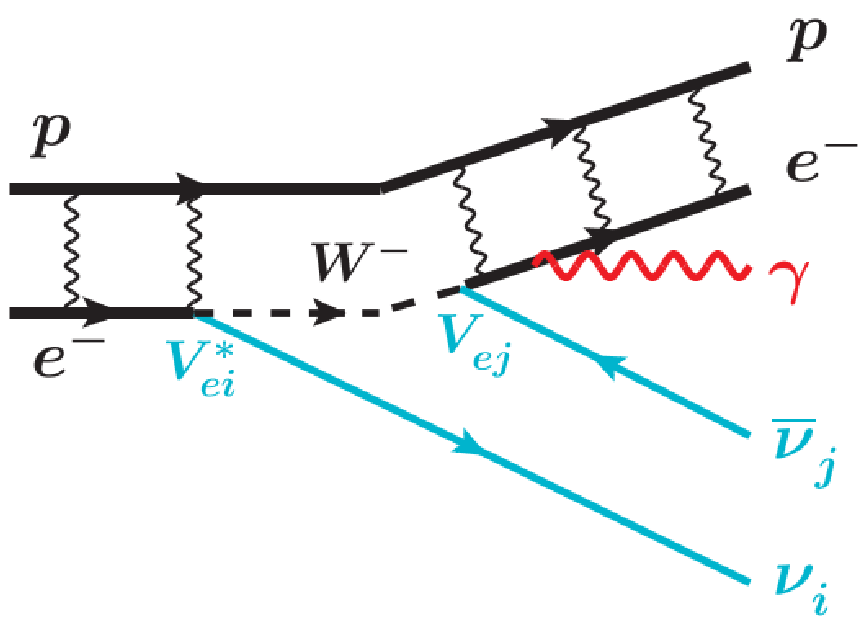
$$D_\gamma \equiv \pm \frac{e}{\omega} e^{\pm i \vec{k} \cdot \vec{x}} \vec{E}_\gamma \cdot \left[\vec{X}, H_0 \right]$$

$$H = H_0 + e^{\mp i \omega t} (D_\gamma + M_\gamma)$$

$$M_\gamma \equiv - \frac{e}{m} e^{\pm i \vec{k} \cdot \vec{x}} \vec{B}_\gamma \cdot \vec{S}$$

Weak Interactions





Huang, Sasao, Xing & Yoshimura [1904.10366]

$$\sqrt{2} G_F \left(v_{ij} J^\mu_V - a_{ij} J^\mu_A \right) \bar{\nu}_{iL} \gamma_\mu \nu_{jL}$$

$$J^\mu_{V(A)} \equiv \bar{e} \gamma^\mu (\gamma_5) e$$

$$v_{ij} \equiv U_{ei} U^*_{ej} - \delta_{ij} \left(\frac{1}{2} - 2 s^2_w \right)$$

$$a_{ij} \equiv U_{ei} U^*_{ej} - \frac{1}{2} \delta_{ij}$$

Atomic Radiance

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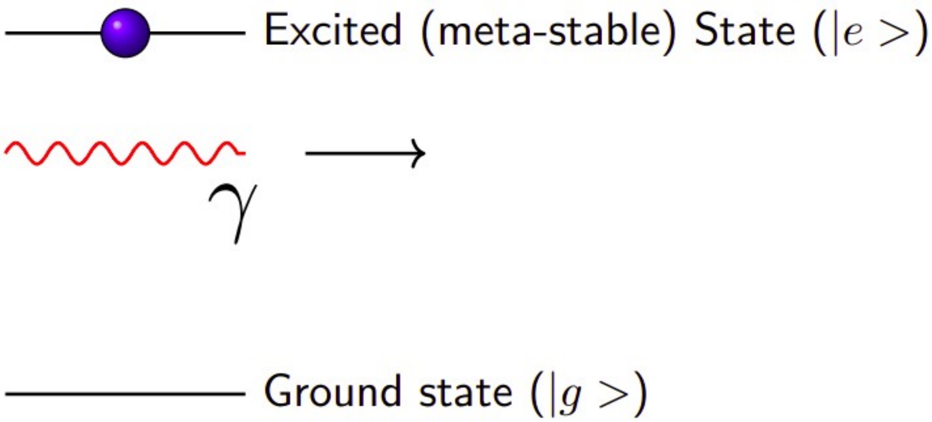
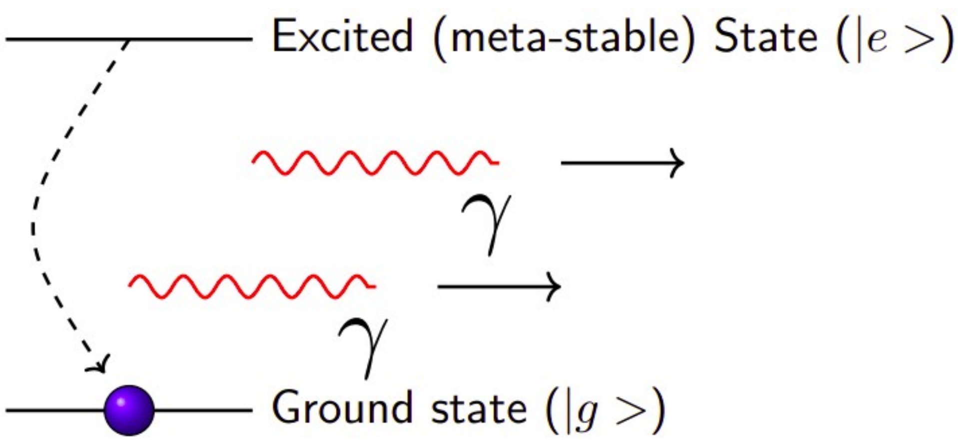
- **Spontaneous radiance**

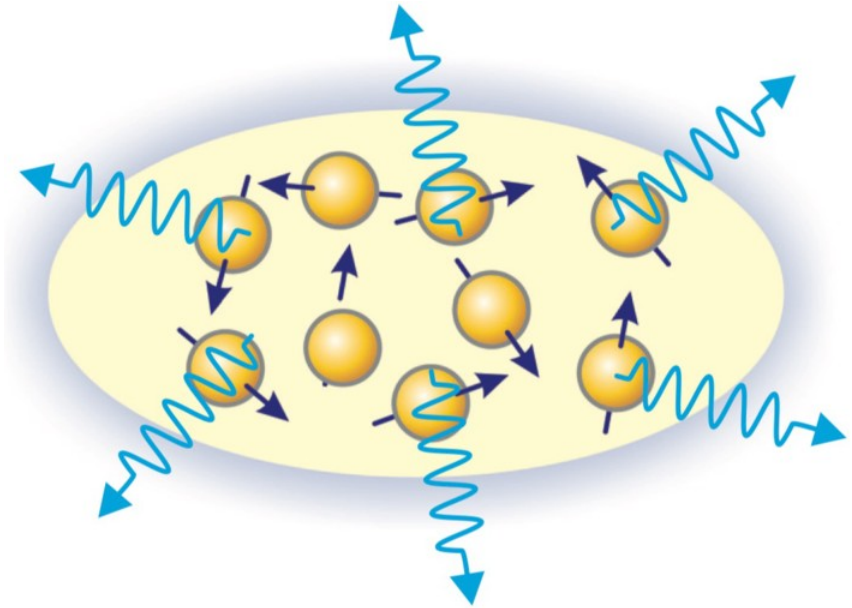


- **Stimulated radiance**



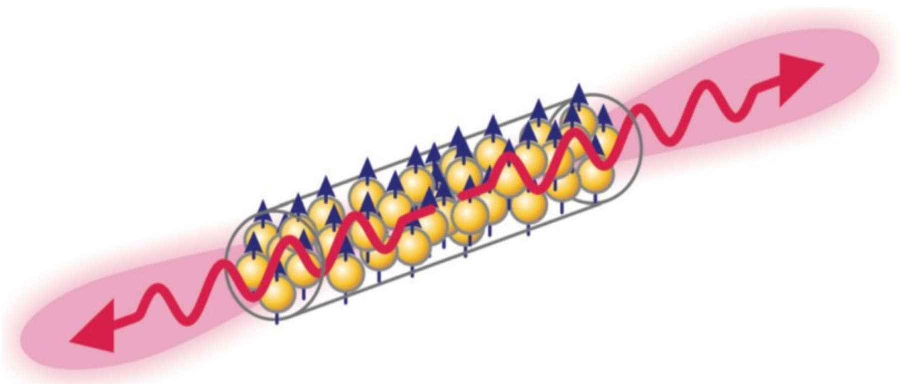
Macroscopic Coherence

$$\mathcal{T}_{\rm tot} = \sum_{\vec{x}_i} N_i \langle g_i, N_k^i + 1_{k'} | T | e_i, N_k^i \rangle$$



spontaneous emission
from a body of target atoms

$$\Gamma \propto n_a^2$$



explosive PSR

Fukumi et al, Prog. Theor. Exp. Phys. 2012, 04D002 [1211.4904]

Spectral Function

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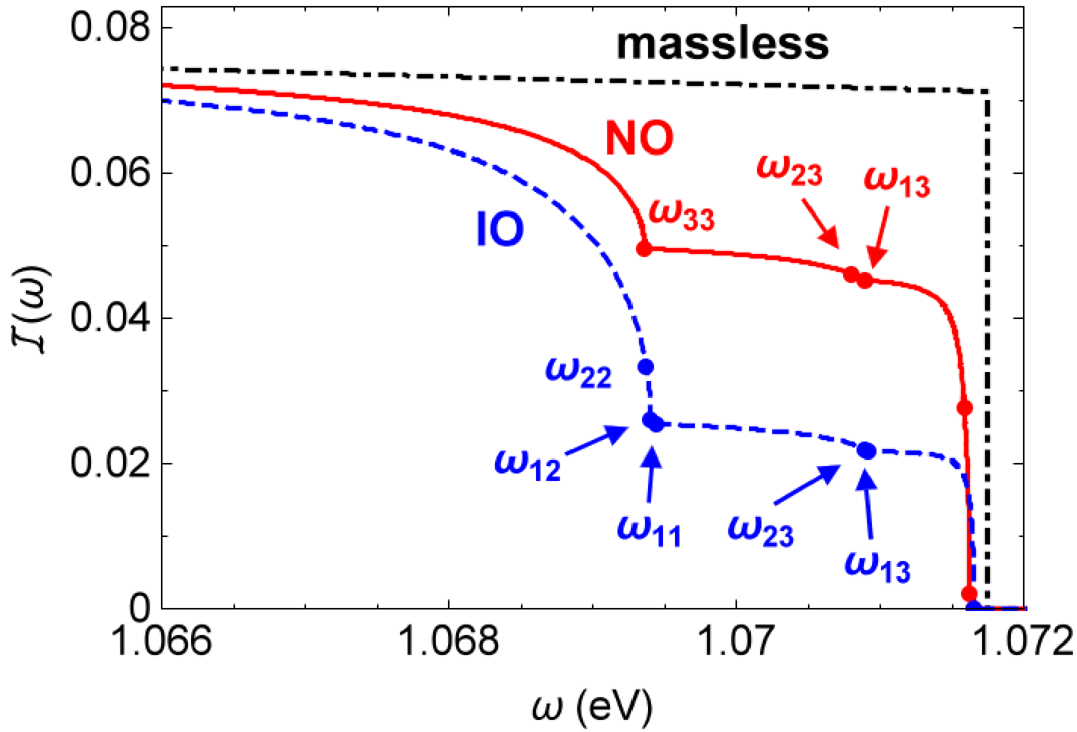

$$\Gamma \equiv \Gamma_0 \mathcal{I}(\omega)$$

$$\Gamma_0 \approx 0.002\,\mathrm{s}^{-1} \frac{n_a^2 n_\gamma}{(10^{21}\mathrm{cm}^{-3})^3} \left(\frac{V}{10^2 \mathrm{cm}^3} \right)$$

$$\omega_{ij}^{\max} \equiv \frac{E_e - E_g}{2} - \frac{1}{2} \frac{(m_i + m_j)^2}{(E_e - E_g)}$$

$$\mathcal{I}(\omega) \equiv \sum_{ij} \frac{\Delta_{ij}(\omega)}{(E_{vg} - \omega)^2} \Theta(\omega - \omega_{ij}^{\max}) \left[|a_{ij}|^2 I_{ij}^{(D)} - \delta_M \mathrm{Re}[a_{ij}^2] I_{ij}^{(M)} \right]$$

$$I_{ij}^{(D)} \equiv \frac{1}{3} \left\{ E_{eg}(E_{eg} - 2\omega) - \frac{1}{2}(m_i^2 + m_j^2) + \frac{\omega^2}{2} \left[1 - \frac{1}{3} \Delta_{ij}(\omega)^2 \right] - \frac{(E_{eg} - \omega)^2 (\Delta m_{ij}^2)^2}{2 E_{eg}^2 (E_{eg} - 2\omega)^2} \right\}$$

Majorana Neutrinos: $I_{ij}^{(M)} \equiv m_i m_j$

Measurable Properties

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1) **Mass**

Song et al [1510.00421]; Zhang & Zhou [1604.08008]

2) **CP phase**

Yoshimura & Sasao [1506.08003]

3) **Dirac vs Majorana**

Dinh, Petcov, Sasao, Tanaka & Yoshimura [1209.4808]

4) **Unitarity**

Huang, Sasao, Xing & Yoshimura [1904.10366]

5) **Sterile neutrino**

Dinh & Petcov [1411.7459]

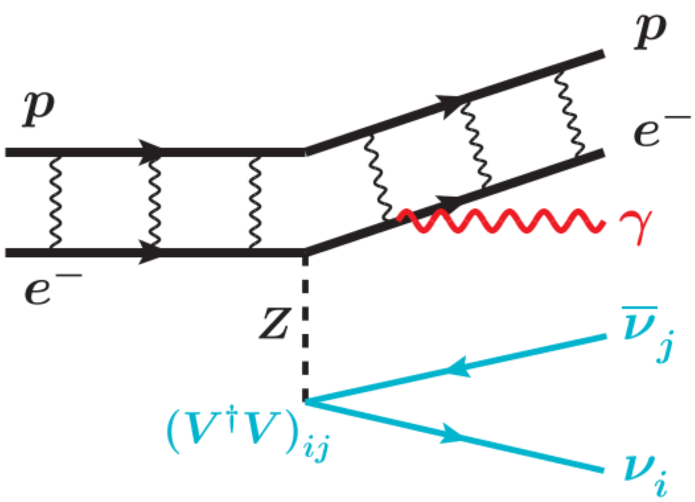
6) **Axion**

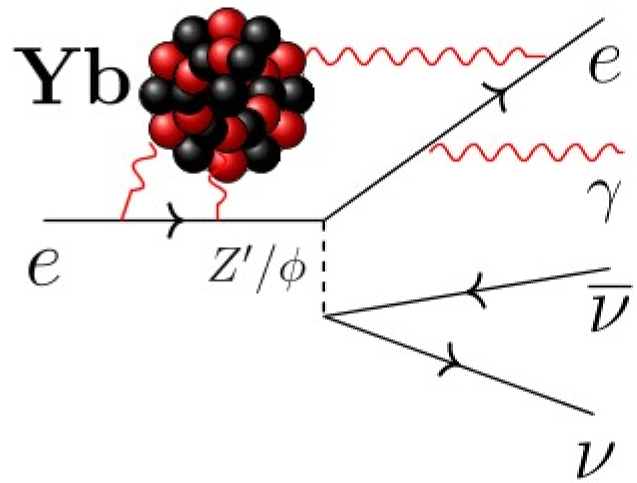
Huang & Zhou [1905.00367]

7) **Dark photon**

Bhoonah, Bramante & Song [1909.07387]

Light Mediators





$$|\mathcal{M}|^2 \sim \frac{g^4}{(q^2+m^2)^2} \approx \begin{cases} \dfrac{g^4}{q^4} & m^2 \ll q^2 \\ \dfrac{g^4}{m^4} & m^2 \gg q^2 \end{cases}$$

Vector Light Mediator

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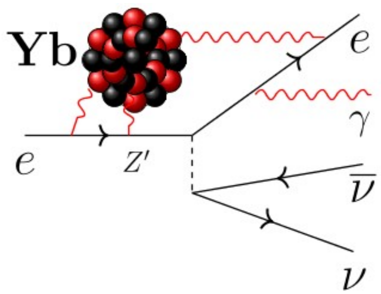
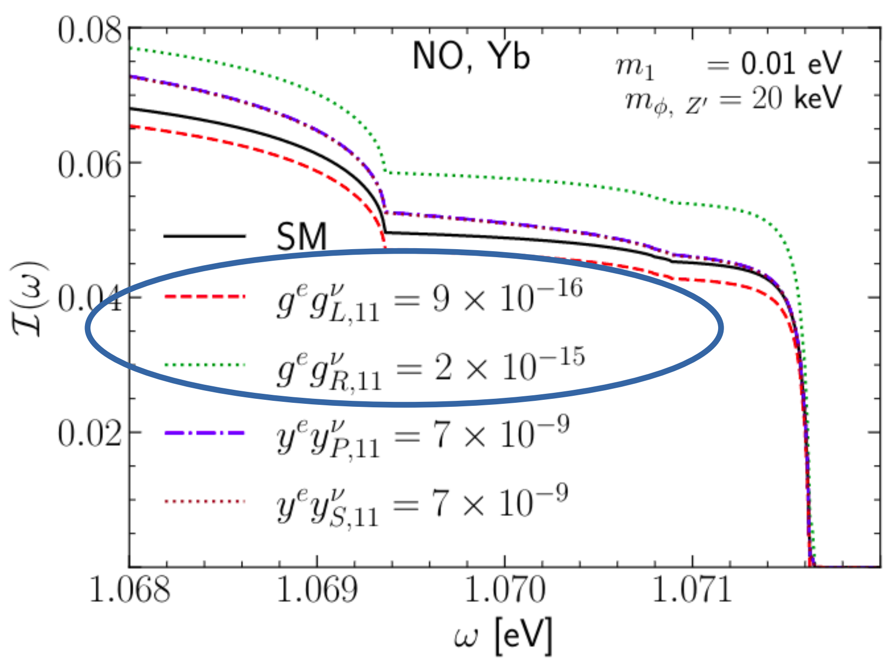
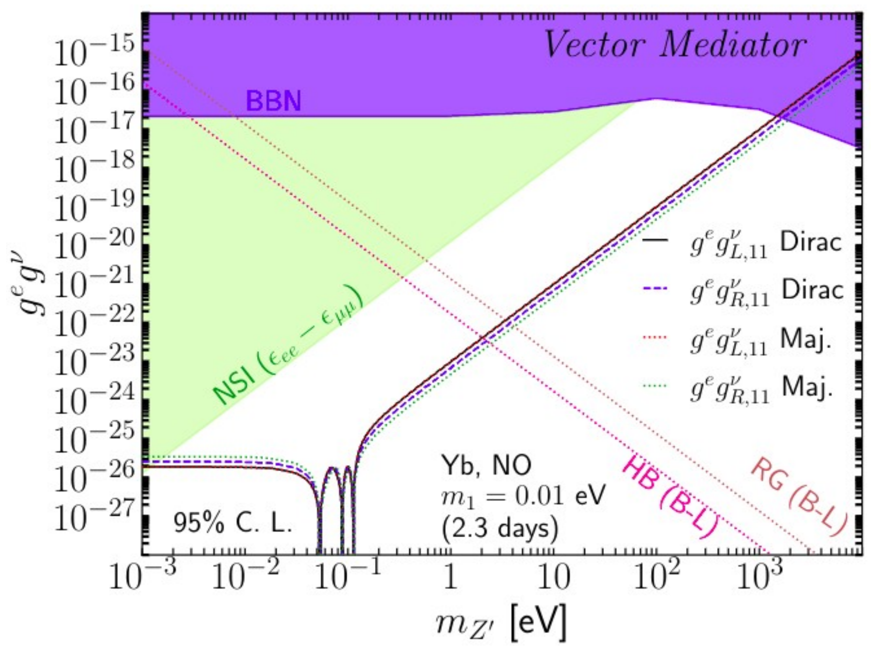


$$g^e \bar{e} \gamma^\mu \gamma_5 e Z'_\mu + \bar{\nu}_i \gamma^\mu (g^\nu_{L,ij} P_L + g^\nu_{R,ij} P_R) \nu_j Z'_\mu$$

$$a^L_{ij} \equiv a_{ij} + \frac{g^e g^\nu_{L,ij}}{\sqrt{2} G_F (q^2 - m^2_{Z'})} \delta_{ij} \qquad a^R_{ij} \equiv \frac{g^e g^\nu_{R,ij}}{\sqrt{2} G_F (q^2 - m^2_{Z'})} \delta_{ij}$$





$$\mathcal{I}_{Z'} \equiv \sum_{ij} \frac{\Delta_{ij}(\omega)\Theta(\omega - \omega^{\max}_{ij})}{(E_{vg} - \omega)^2} \left[\left(|a^L_{ij}|^2 + |a^R_{ij}|^2 - 2\delta_M \mathrm{Re}[a^L_{ij} a^R_{ij}] \right) I^{(D)}_{ij} + \left(\delta_M \mathrm{Re}\left[(a^L_{ij})^2 + (a^R_{ij})^2 \right] - 2\mathrm{Re}\left[a^{L*}_{ij} a^R_{ij} \right] \right) m_i m_j \right]$$

Scalar Light Mediator

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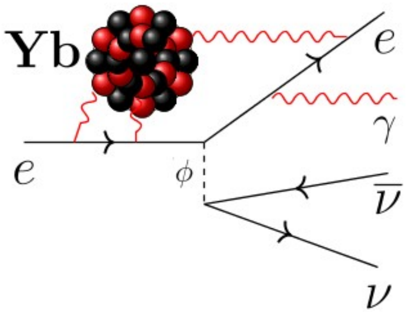
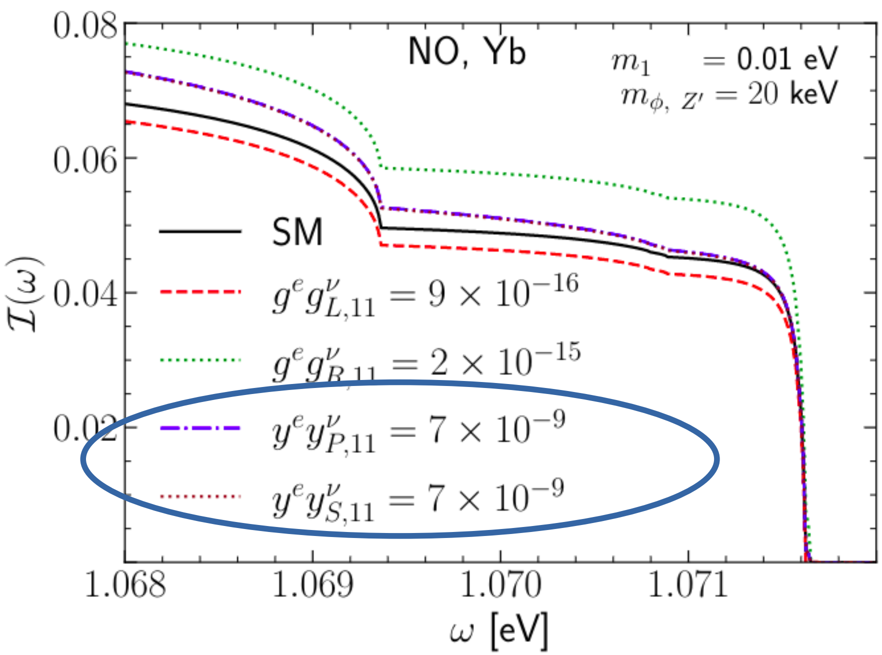
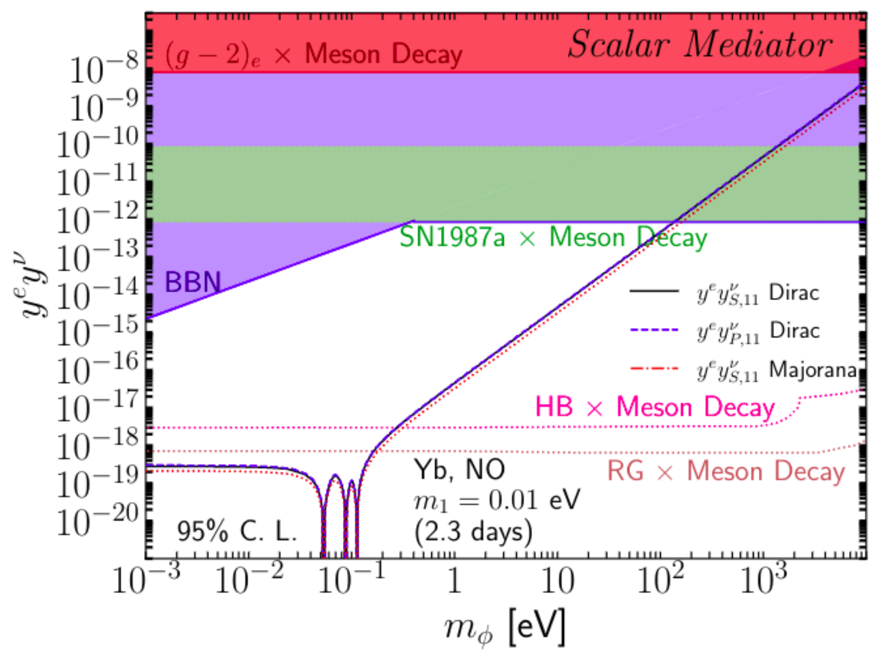


$$i y_P^e \bar{e}\gamma_5 e \phi + \bar{\nu}_i (y_{S,ij}^\nu + i\gamma_5 y_{P,ij}^\nu)\nu_j \phi + h.c.$$

$$\mathcal{I}_\phi = \sum_{ij} \frac{\Delta_{ij}(\omega)}{(E_{vg}-\omega)^2}\Theta(\omega - \omega_{ij}^{\max})\left[I_{ij}^{\rm SM}(\omega) + \delta I_{ij}(\omega)\right]$$

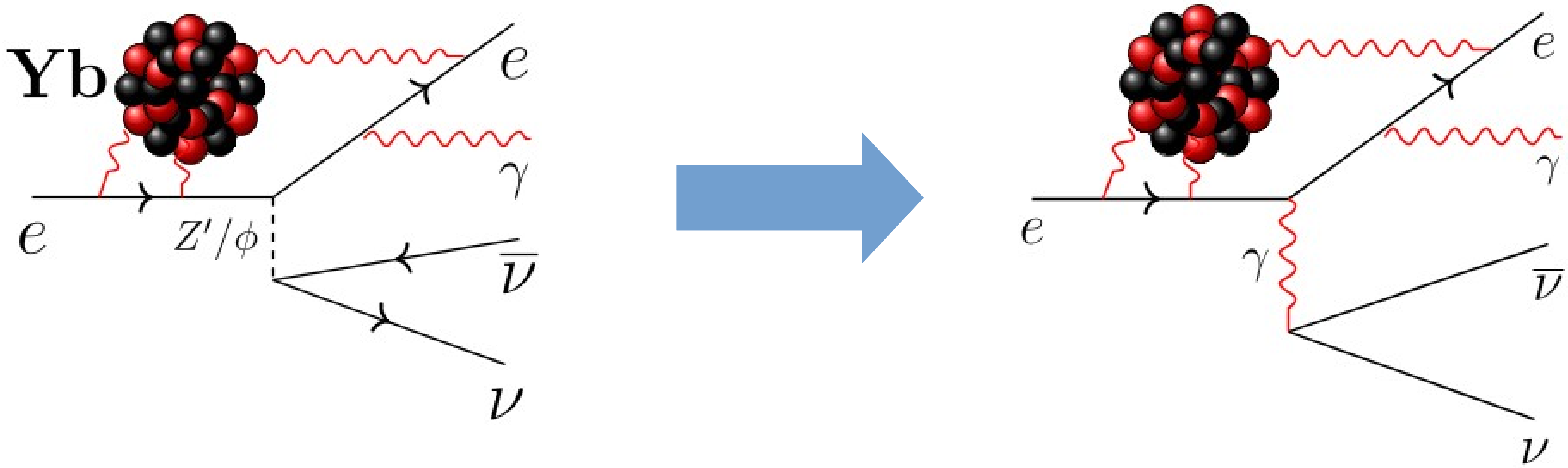




$$\delta I_{ij} \equiv \frac{|y^e|^2\omega^2}{m_e^2 G_F^2}\frac{\left[|y_{S,ij}^\nu|^2 + (1-\delta_M)|y_{P,ij}^\nu|^2\right]E_{eg}(E_{eg}-2\omega)}{24[E_{eg}(E_{eg}-2\omega)-m_\phi^2]^2} - \frac{|y^e|^2\omega^2}{m_e^2 G_F^2}\frac{|y_{S,ij}^\nu|^2(m_i+m_j)^2 + (1-\delta_M)|y_{P,ij}^\nu|^2(m_i-m_j)^2}{24[E_{eg}(E_{eg}-2\omega)-m_\phi^2]^2}$$

$$+\frac{y^e\omega^2}{6\sqrt{2}G_F}\frac{{\rm Re}\left[a_{ij}y_{S,ij}^\nu\right](m_i-m_j)\left[1-\frac{(m_i+m_j)^2}{E_{eg}(E_{eg}-2\omega)}\right]}{m_e[E_{eg}(E_{eg}-2\omega)-m_\phi^2]} - (1-\delta_M)\frac{{\rm Im}\left[a_{ij}y_{P,ij}^\nu\right](m_i+m_j)\left[1-\frac{(m_i-m_j)^2}{E_{eg}(E_{eg}-2\omega)}\right]}{m_e[E_{eg}(E_{eg}-2\omega)-m_\phi^2]}$$

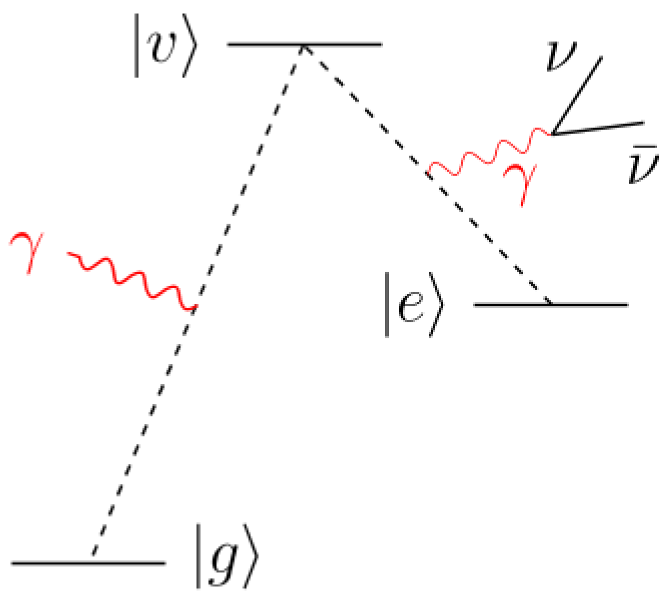
Light mediator → Massless Photon?



- Massless photon is also a light mediator!

- RENP can also be sensitivity to EM interactions

M1 vs Electron Magnetic Moment



$$\mathrm{E1} \times \mathrm{M1}$$

- **Can electron side contribute M1?**
- **Electron has both charge & MM**

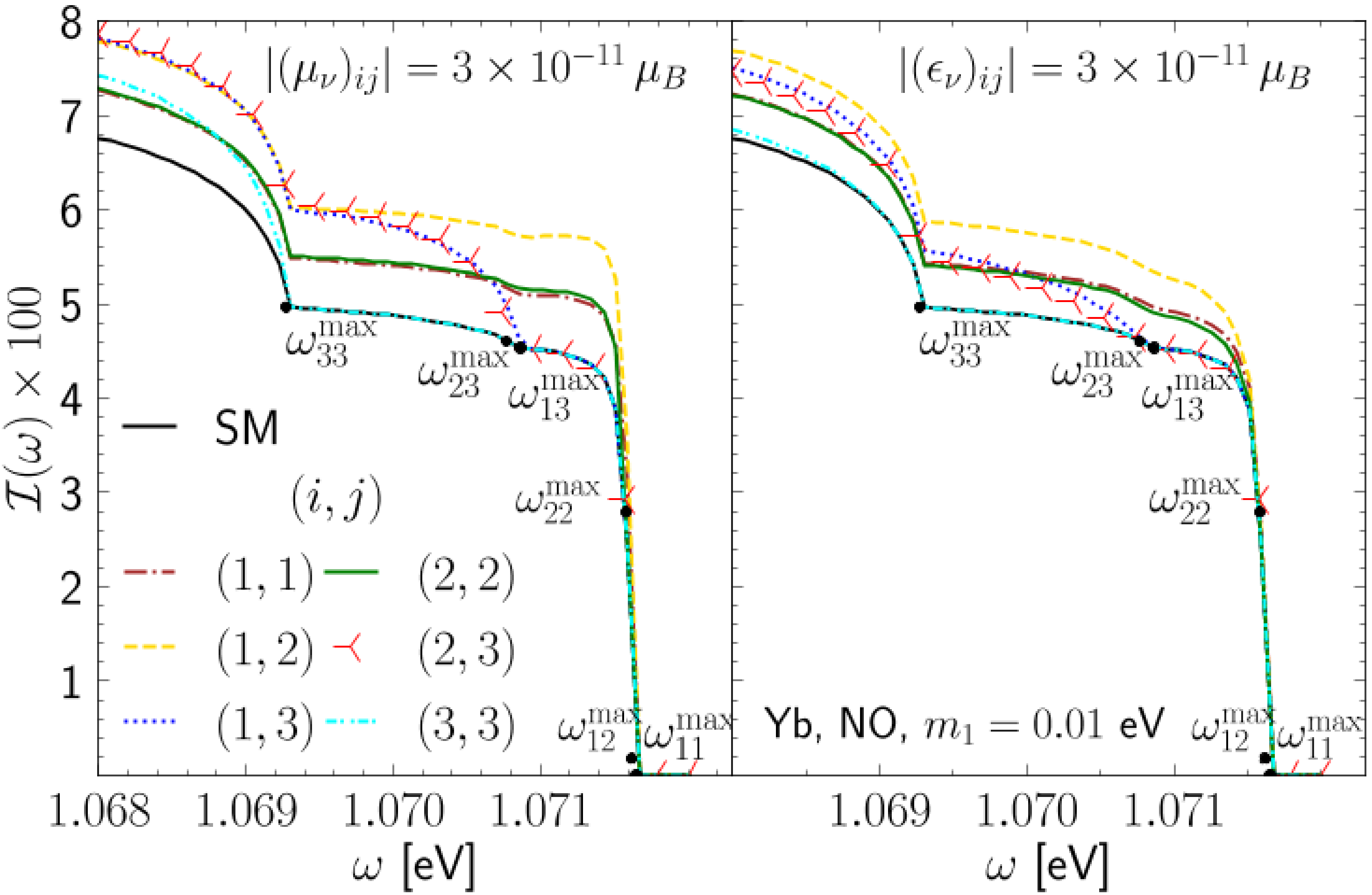
$$i\gamma^\nu \partial_\nu \psi - m\psi = 0$$

$$eA_\mu \overline{\psi} \gamma^\mu \psi \quad \longrightarrow \quad \frac{e}{4m} \overline{\psi} \sigma^{\mu\nu} \psi F_{\mu\nu}$$

$$\langle v | \overline{e} \sigma^{ij} e | e \rangle = -2\epsilon_{ijk} \mathbf{S}^k_{ve}$$

EM Moments Scan

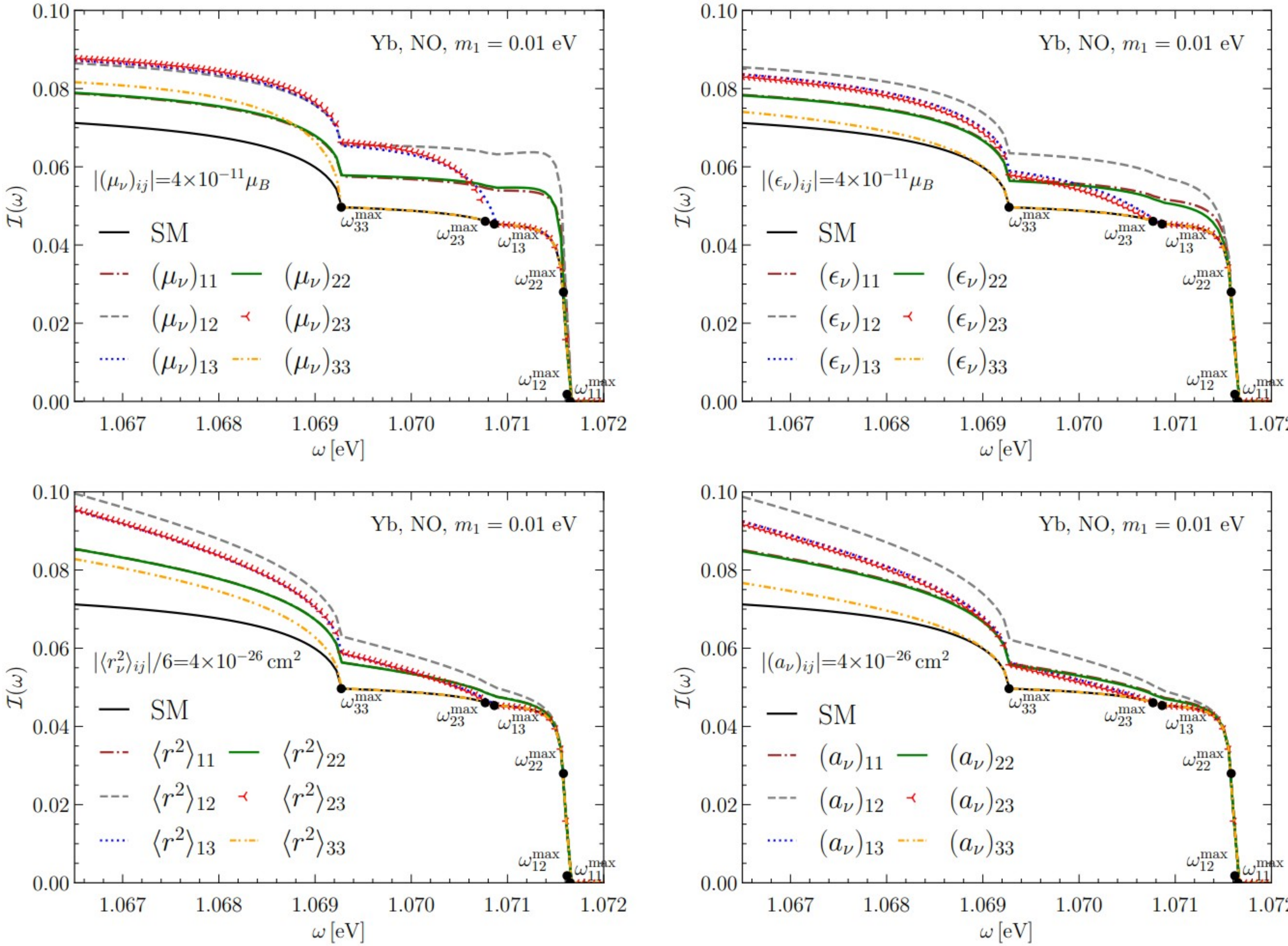
李政道研究所 Tsung-Dao Lee Institute



Tune trigger laser frequencies to scan multiple points.

EM Moments Scan

李政道研究所
Tsung-Dao Lee Institute



Tune trigger laser frequencies to scan multiple points.

EM Moments Scan

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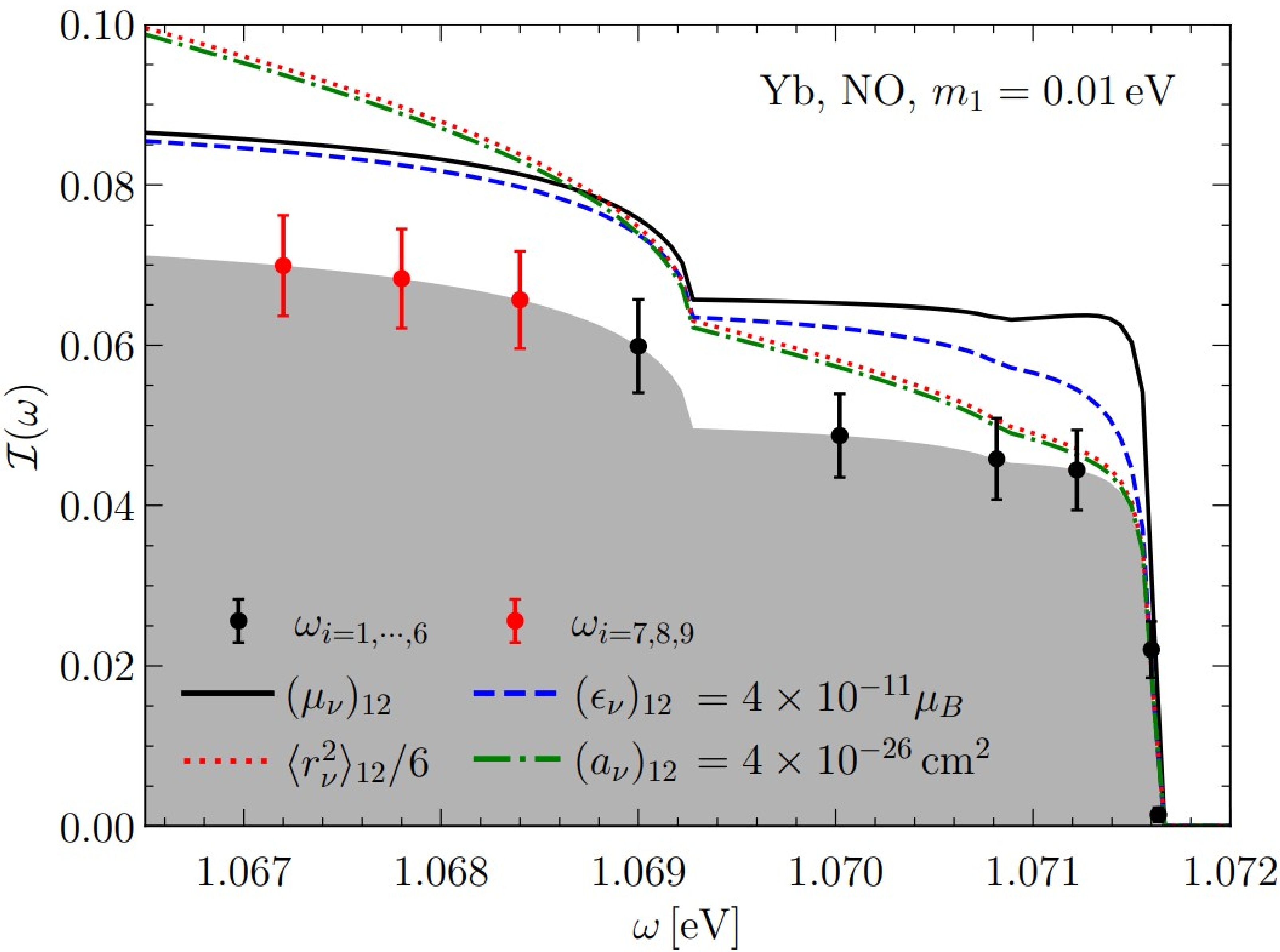

Tune trigger laser frequencies to scan multiple points.

Distinguishing EM Types

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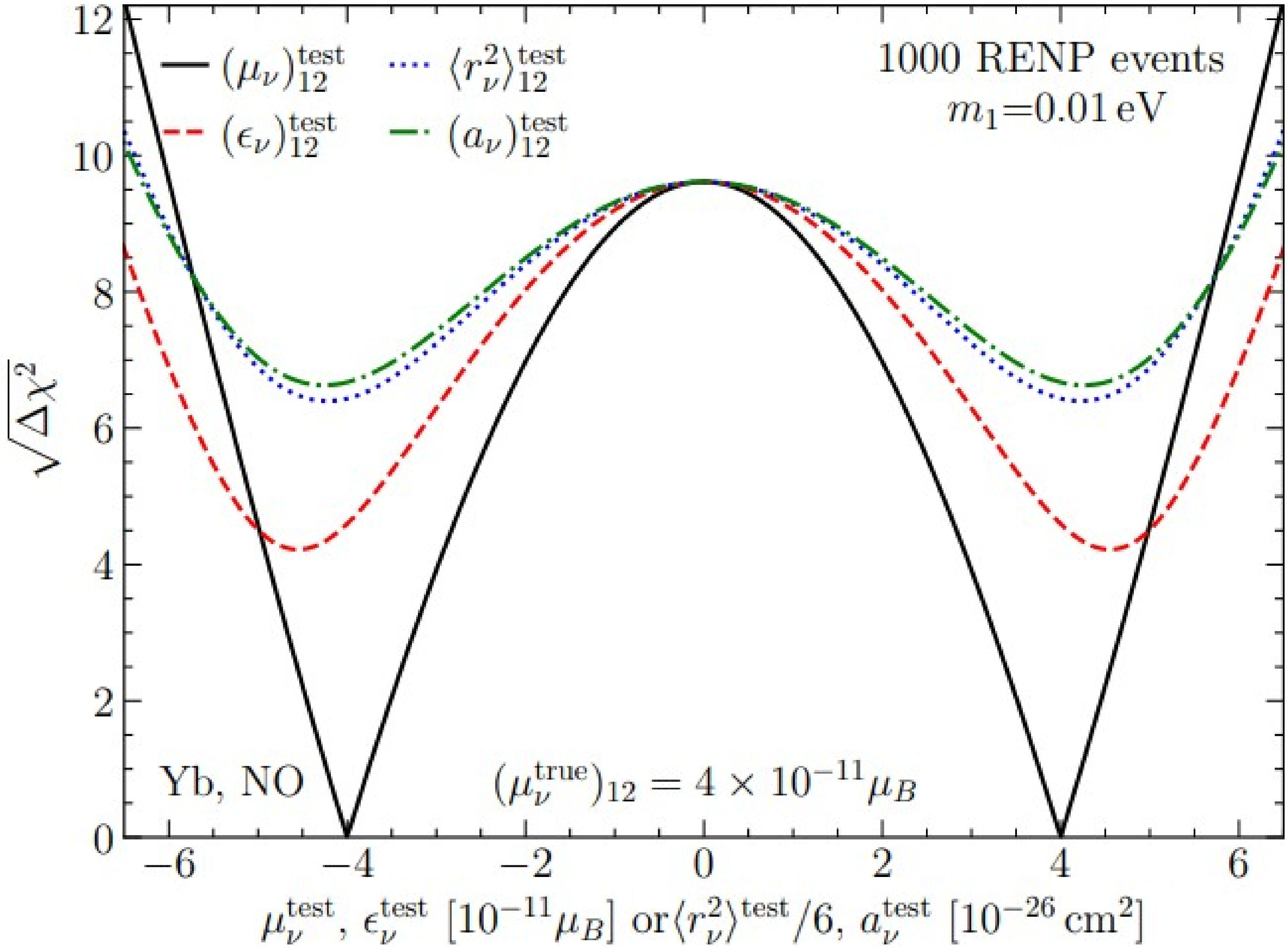

Sensitivity on Individual Elements

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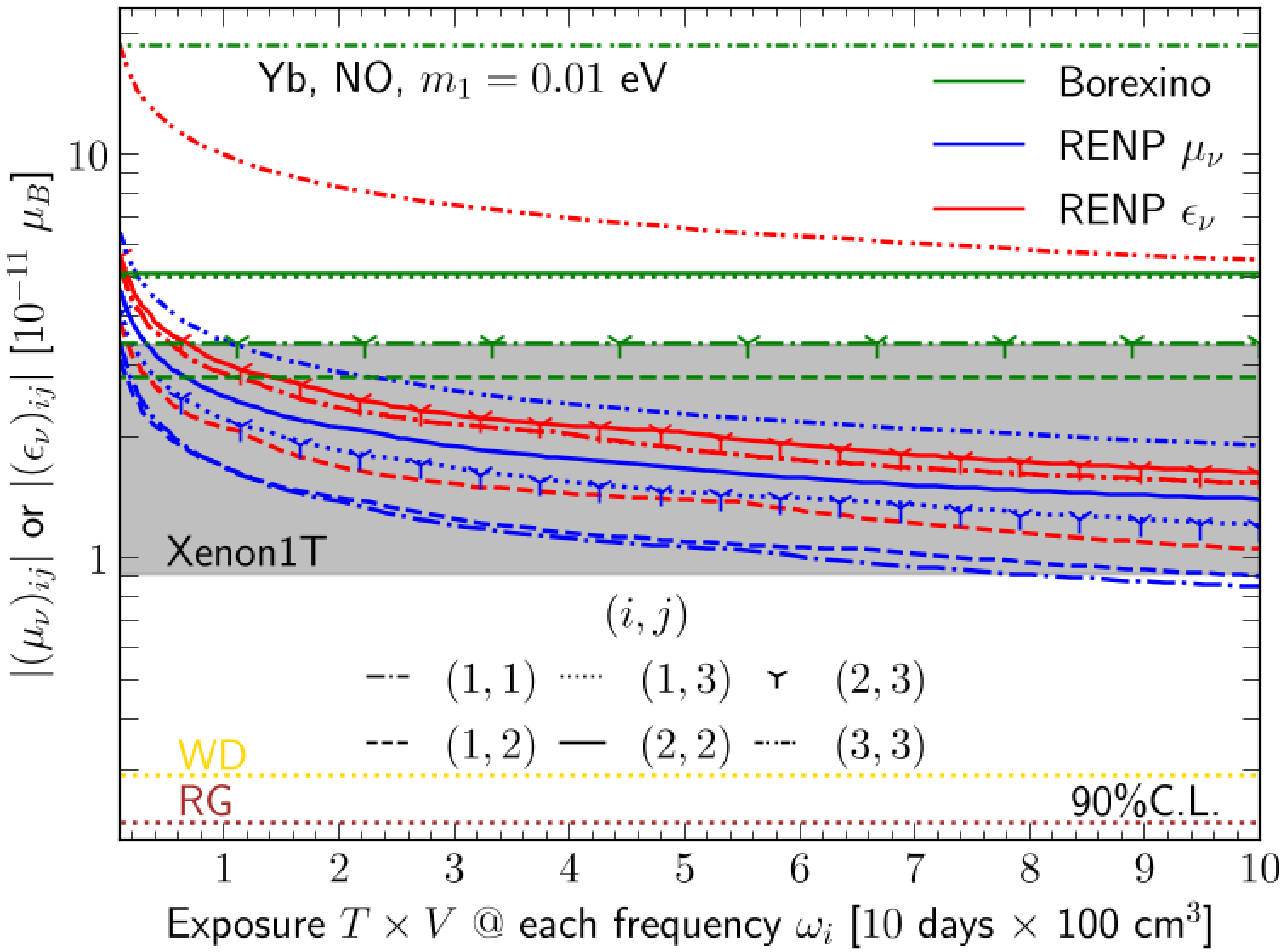

Summary

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1) **Advantages of RENP**

2) **NSI with light mediator**

- **Typical momentum transfer @ eV**
- **Sensitivity improved by 2~3 orders**

3) **Neutrino EM moments**

- **Disentangle ν EM moments**
- **Identify individual elements**

Thank You

Neutrino Oscillation

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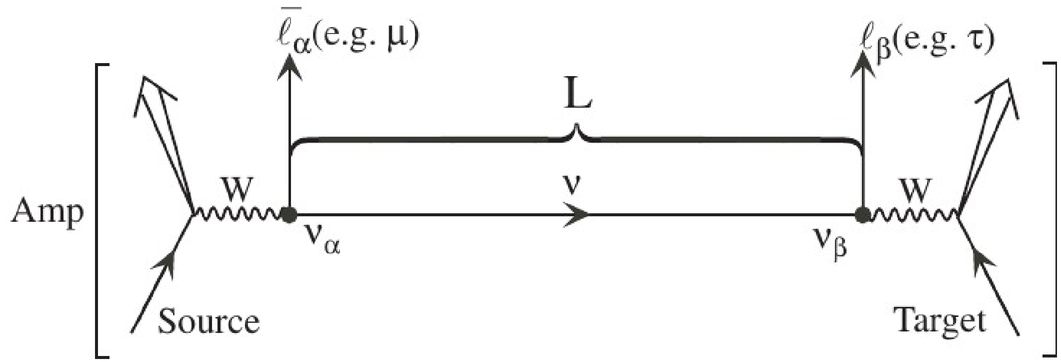

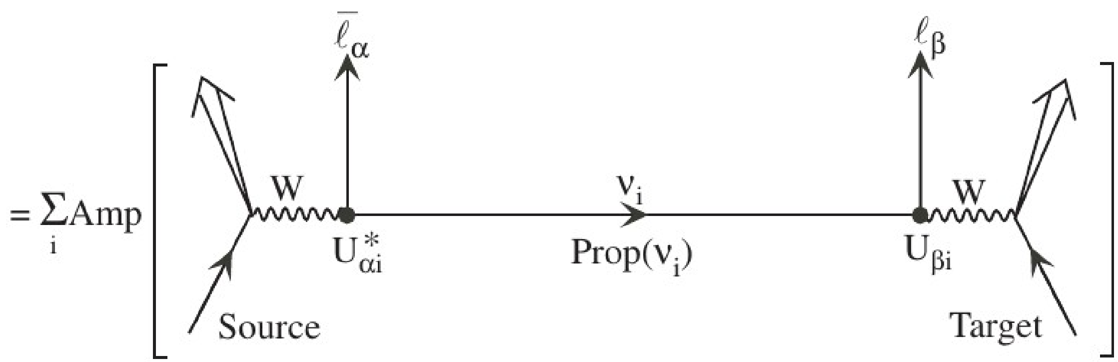

[Kayser, https://arxiv.org/abs/hep-ph/0506165]

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \rightarrow \boxed{\sum_i U_{\alpha i} e^{i(E_i t - \vec{P}_i \cdot \vec{x})} \nu_i} = \boxed{\sum_i U_{\alpha i} P_i U^\dagger_{\beta i} \nu_\beta} \equiv \sum_\beta A_{\alpha\beta} \nu_\beta$$

3-Neutrino Oscillation

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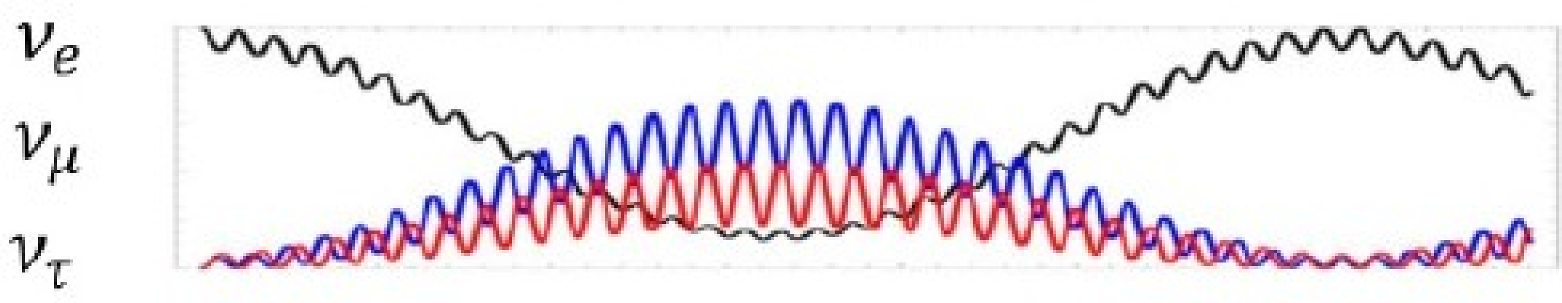
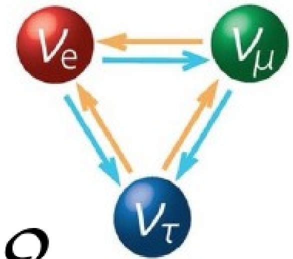
Mass difference + Mixing → Neutrino Oscillation



- **PMNS Matrix**



$$U_{\rm PMNS} = \mathcal{P} \begin{pmatrix} c_s c_r & s_s c_r & s_r e^{-i\delta_{\rm D}} \\ -s_s c_a - c_s s_a s_r e^{i\delta_{\rm D}} & +c_s c_a - s_s s_a s_r e^{i\delta_{\rm D}} & s_a c_r \\ +s_s s_a - c_s c_a s_r e^{i\delta_{\rm D}} & -c_s s_a - s_s c_a s_r e^{i\delta_{\rm D}} & c_a c_r \end{pmatrix} \mathcal{Q}$$

with $\mathcal{P} \equiv \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ & $\mathcal{Q} \equiv \mathrm{diag}(e^{i\delta_{\rm M1}}, 1, e^{i\delta_{\rm M3}})$

[$(s, a, r) \equiv (12, 23, 13)$ for (**solar, atmospheric, reactor**) angles]

Neutrino Properties

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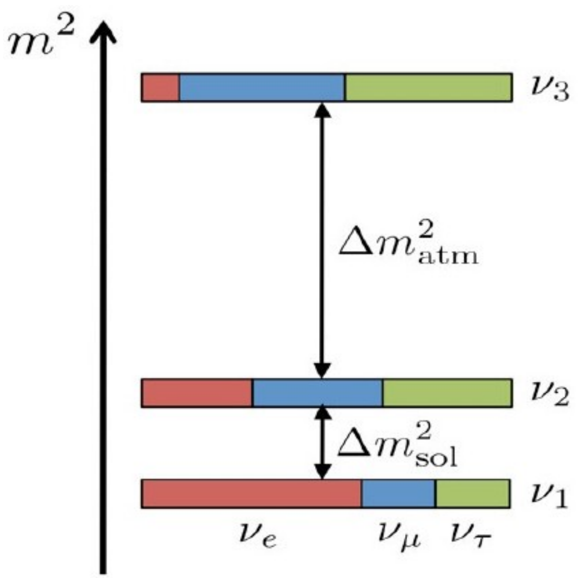
(for **NO**)	-1σ	Best Value	$+1\sigma$
$\Delta m_s^2 \equiv \Delta m_{21}^2\ (10^{-5}\mathrm{eV}^2)$	7.30	7.50	7.72
$\left\|\Delta m_a^2 \equiv \Delta m_{31}^2\right\|\ (10^{-3}\mathrm{eV}^2)$	2.52	2.56	2.59
$\sin^2\theta_s\ (\theta_s \equiv \theta_{12})$	0.302 (33.3°)	0.318 (34.3°)	0.334 (35.3°)
$\sin^2\theta_a\ (\theta_a \equiv \theta_{23})$	0.544 (47.54°)	0.566 (48.79°)	0.582 (49.72°)
$\sin^2\theta_r\ (\theta_r \equiv \theta_{13})$	0.02147 (8.43°)	0.02225 (8.58°)	0.02280 (8.69°)
δ_D	191°	216°	257°
δ_{Mi}	??	??	??

Salas, Forero, Gariazzo, Martinez-Mirave, Mena, Ternes, Tortola & Valle, [arXiv:2006.11237]

Neutrino Masses

Normal Ordering (NO)



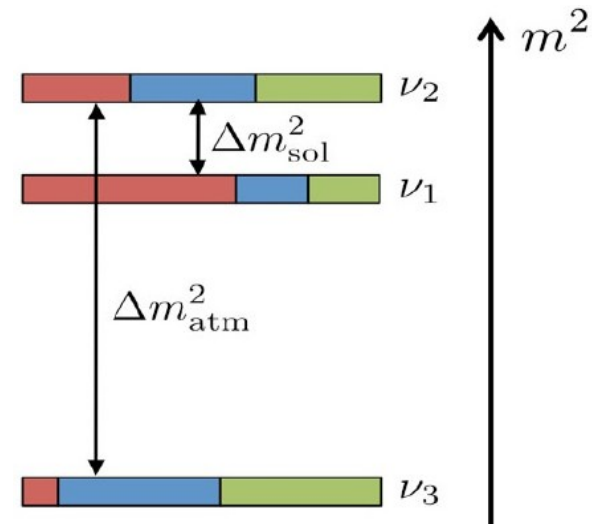
$$m_1 \lesssim m_2 < m_3$$

$$m_1$$

$$m_2 = \sqrt{m_1^2 + \Delta m_s^2}$$

$$m_3 = \sqrt{m_1^2 + \Delta m_a^2}$$

Inverted Ordering (IO)



$$m_3 < m_1 \lesssim m_2$$

$$m_1 = \sqrt{m_3^2 + \Delta m_a^2}$$

$$m_2 = \sqrt{m_3^2 + \Delta m_a^2 + \Delta m_s^2}$$

$$m_3$$

江门中微子实验 (JUNO)

All in One?

1) **Masses (δm^2_{ij}, ordering, absolute values)**
2) **Mixing Angles (θ_{12}, θ_{23}, θ_{13})**
3) **CP Phase**
4) **Dirac vs Majorana**
5) **Non-Standard Interactions**

Is it possible to measure all these in a single experiment?

- **RENP provides an independent measurement of these!**

Stimulated Radiance

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- **Identical bosonic particles**

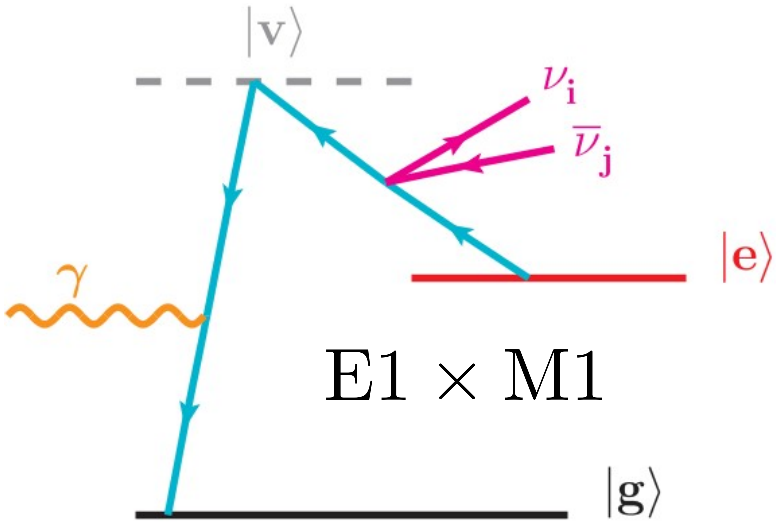
$$|N_k\rangle \equiv (2\omega_k)^{\frac{N_k}{2}} \frac{(a_k^\dagger)^{N_k}}{\sqrt{N_k!}} |0\rangle \qquad |N_k+1\rangle = \sqrt{2\omega_k}\frac{a_k^\dagger}{\sqrt{N_k+1}}|N_k\rangle$$

$$\langle N_k + 1_{k_1} | a_{k_2}^\dagger | N_k \rangle = \sqrt{2\omega_{k_1}} \delta^{(3)}_{k_1 k_2} \left[\delta^{(3)}_{k k_2} \sqrt{N_k+1} + (1 - \delta^{(3)}_{k k_2}) \right]$$

$$\langle g, N_k + 1_{k'} | T | e, N_k \rangle = \begin{cases} \sqrt{N_k+1} \frac{\mathcal{M}_{kk'} e^{i\vec{k}'\cdot\vec{x}}}{L^{3/2}} 2\pi\delta(E_e - E_g - \omega_{k'}) & \text{if} \quad \vec{k} = \vec{k}', \\ \frac{\mathcal{M}_{kk'} e^{i\vec{k}'\cdot\vec{x}}}{L^{3/2}} 2\pi\delta(E_e - E_g - \omega_{k'}) & \text{if} \quad \vec{k} \neq \vec{k}', \end{cases}$$

$$\mathcal{M}_{kk'} \equiv -i\omega_{k'} e \langle g | \vec{d} \cdot \vec{\epsilon}_{k'} | e \rangle \qquad \Gamma \propto n_\gamma$$

Selection Rules



$$\langle g|\vec{d}|p\rangle \cdot \vec{E} \frac{G_F \sum_{ij} a_{ij} \nu_j^\dagger \vec{\sigma} \nu_i}{\epsilon_{pg} - \omega} \cdot \langle p|\vec{S}_e|e\rangle$$

	E1	M1
ΔJ	$0, \pm 1$	$0, \pm 1$
ΔM_J	$0, \pm 1$	$0, \pm 1$
Parity	$\pi_i = -\pi_f$	$\pi_i = \pi_f$

Coupling	$\mathcal{L}_{\rm new}$	Non-Relativistic Transition	Type
scalar	$y_S^e \bar{e} e$	$\langle f|i\rangle$	E1
pseudo-scalar	$i y_P^e \bar{e} \gamma_5 e$	$\frac{\mathbf{q}}{2m_e} \cdot \langle f|\boldsymbol{\sigma}|i\rangle$	M1
vector	$g_V^e \bar{e} \gamma^\mu e$	$(\langle f|i\rangle, \frac{\mathbf{q}}{2m_e} \cdot \langle f|\boldsymbol{\sigma}\boldsymbol{\sigma}|i\rangle)$	E1
axial-vector	$g_A^e \bar{e} \gamma^\mu \gamma_\mu e$	$(\frac{\mathbf{q}}{2m_e} \cdot \langle f|\boldsymbol{\sigma}|i\rangle, \langle f|\boldsymbol{\sigma}|i\rangle)$	M1

Neutrino EM Dipole Moments

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$$H_M = \bar{\nu}\left[-f_M(q^2) i\sigma_{\mu\nu} q^\nu + f_E(q^2)\sigma_{\mu\nu} q^\nu \gamma_5\right] \nu A^\mu(q)$$

Magnetic Moment: $\mu_\nu \equiv f_M(0)$

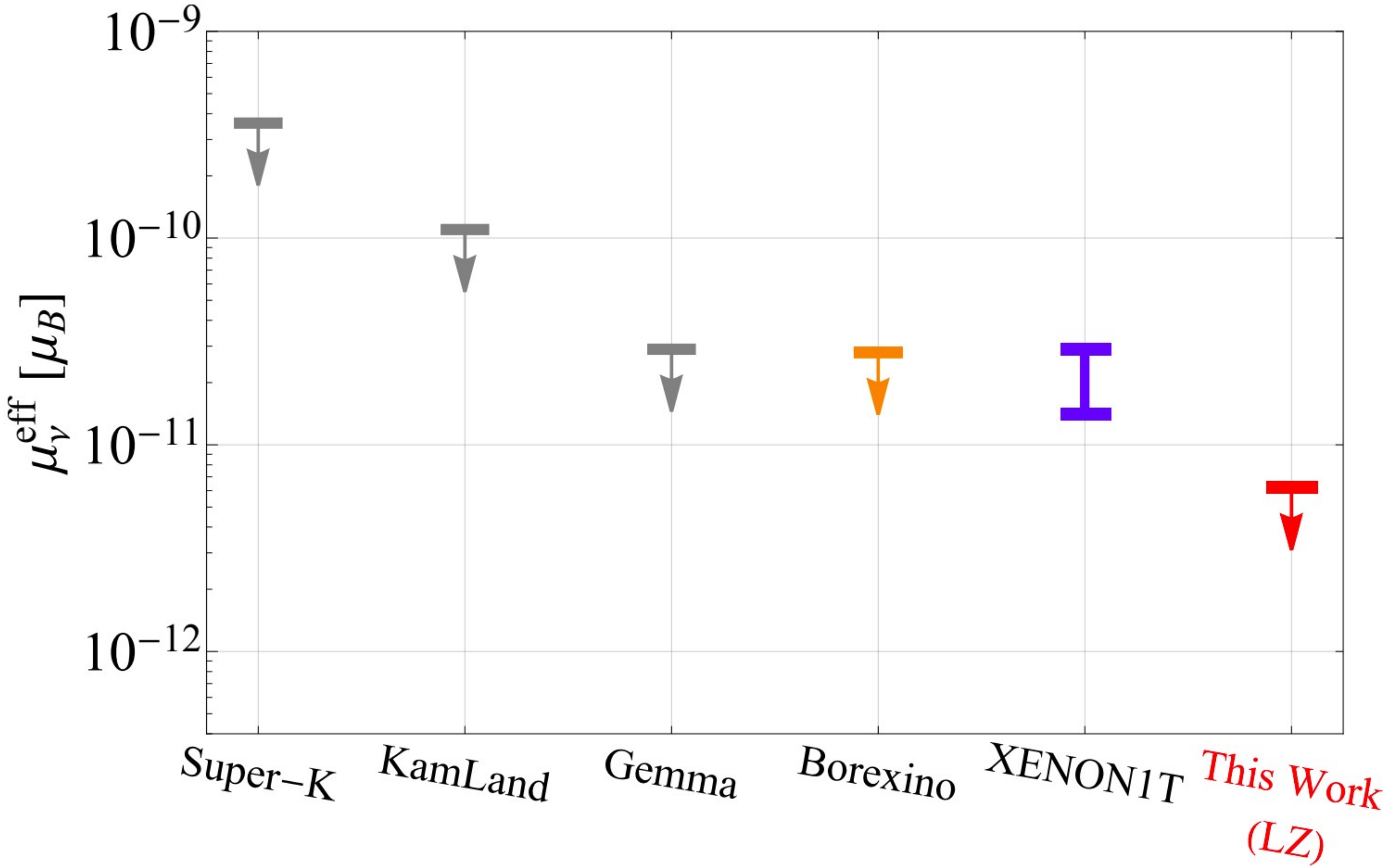
Electric Moment: $\epsilon_\nu \equiv f_E(0)$

Both are negligibly small in SM!

- **EDM violates CP!**
- **Majorana Neutrino**

$$(\mu_\nu)_{ij} = -(\mu_\nu)_{ji} \qquad (\epsilon_\nu)_{ij} = -(\epsilon_\nu)_{ji}$$

Existing Constraints



Corona, Bonivento, Cadeddu, Cargioli & Dordei [2207.05036]

Existing Measurements

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- **Electron recoil experiments**

$$(\mu_\alpha^{\text{eff}})^2 \equiv \sum_j \left| \sum_k U^*_{\alpha k} \left[(\mu_\nu)_{jk} - i (\epsilon_\nu)_{jk} \right] \right|^2$$

1) Recoil measurement is sensitive to not just MDM but also EDM

2) Not a single element, but a linear combination of multiple elements

3) Possible cancellations

- **Plasmon decay in stellar cooling** $\gamma^* \to \nu \bar{\nu}$

$$(\mu_\nu^{\odot})^2 \equiv \sum_{ij} |(\mu_\nu)_{ij}|^2 + |(\epsilon_\nu)_{ij}|^2$$

Suffers from various uncertainties

Giunt & Studenikin [1403.6344]