北京航空航天大學
BEIHANG UNIVERSITY

Testing Bell inequalities in W boson pair production at future e+e− colliders

Qi Bi（毕琪）

Beihang University

In collaboration with Qing-Hong Cao, Kun Cheng and Hao Zhang

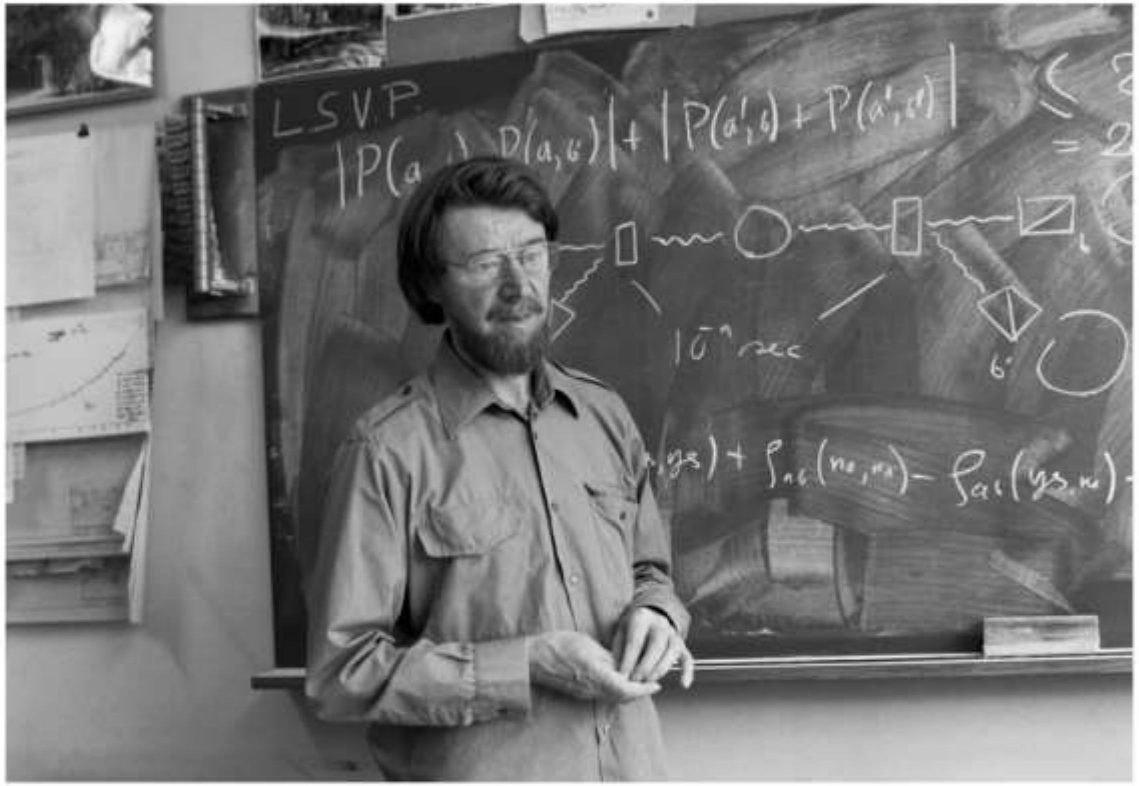
Based on arXiv:2307.14895[hep-ph]

17th Workshop of TeV Physics, Nanjing, Dec 16th, 2023

Bell inequality

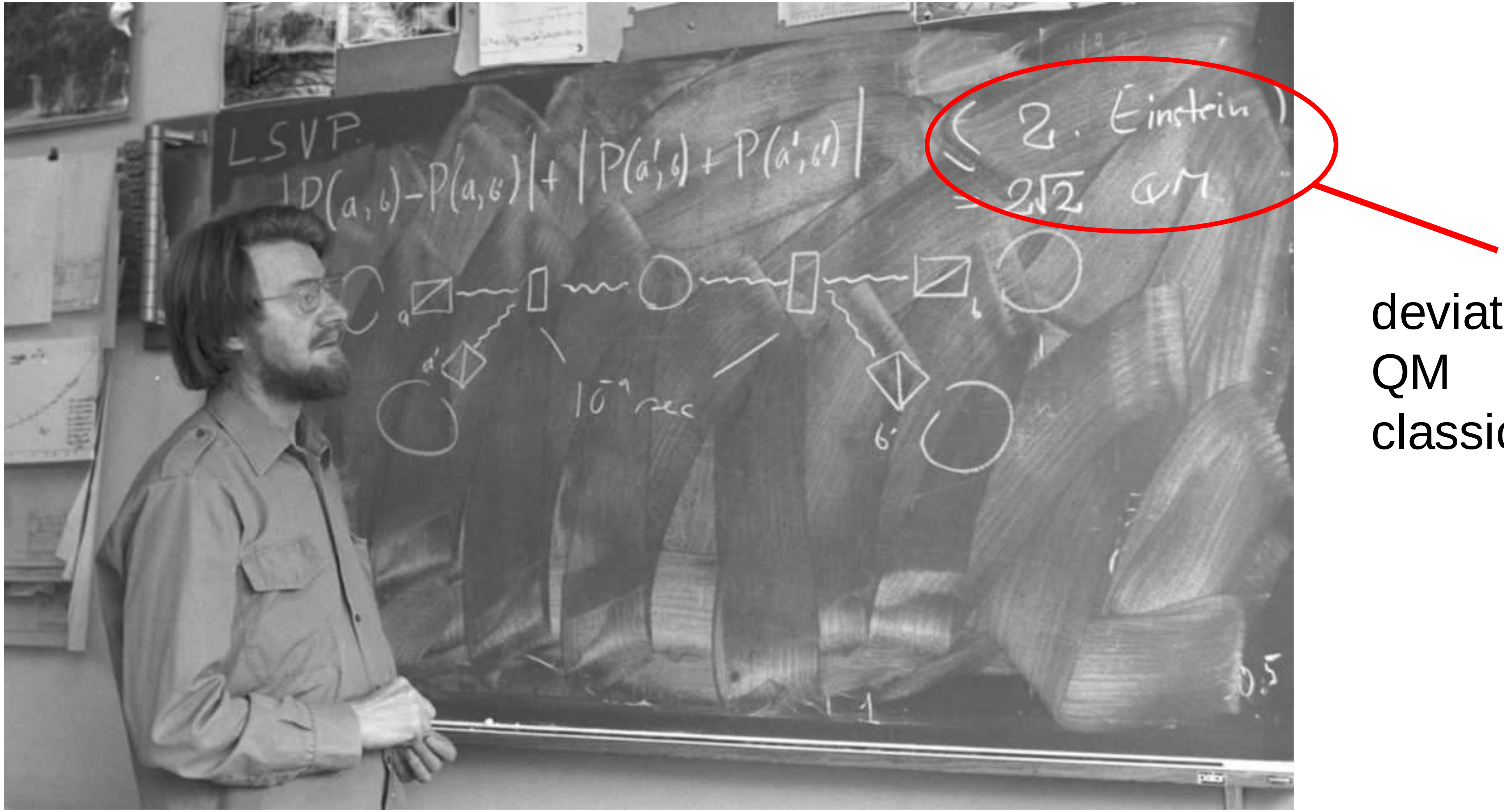
- *Quantum Theory* or *Realistic Local Theory* ?

  ➡ Bell inequality



1964: John Bell at CERN sharpened the formulation of the Einstein Podolsky Rosen experiment in terms of Bell inequalities

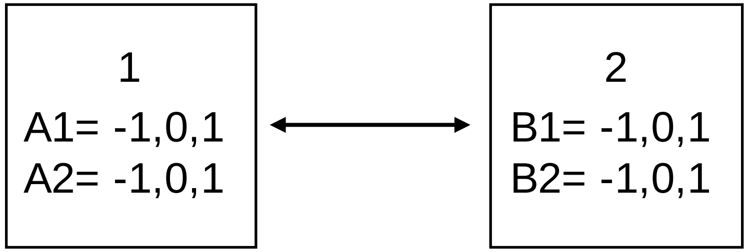
Bell inequality



deviations of
QM
classical physics

Bell inequality

- The initial state is a mixed state
  ➡ (Generalized) Bell inequality



$$\begin{aligned}\mathcal{I}_3 \equiv &+ \left[P(A_1 = B_1) + P(B_1 = A_2 + 1)\right. \\ &\left.+ P(A_2 = B_2) + P(B_2 = A_1)\right] \\ &- \left[P(A_1 = B_1 - 1) + P(B_1 = A_2)\right. \\ &\left.+ P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)\right]\end{aligned}$$

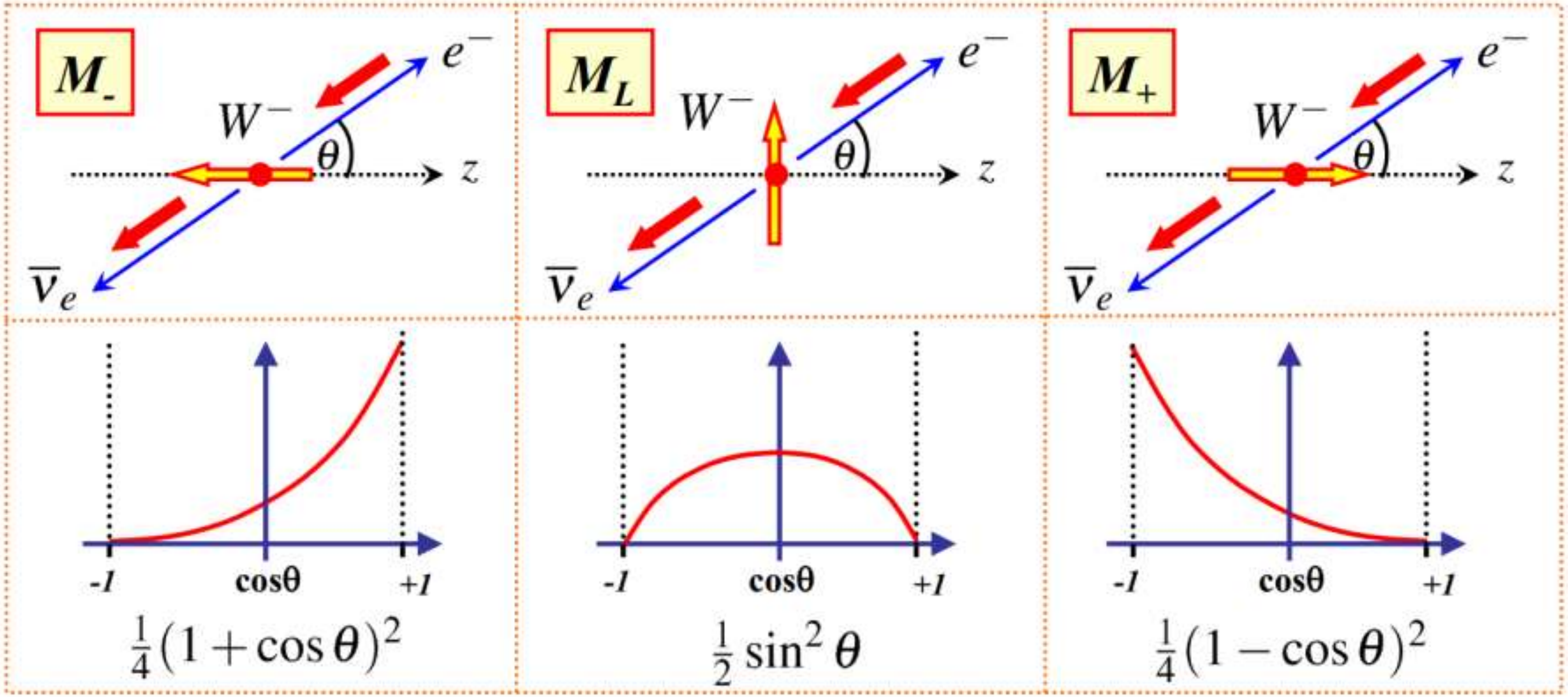
$$\max_{\hat{A}_1, \hat{A}_2, \hat{B}_1, \hat{B}_2} \mathcal{I}_3(\hat{A}_1, \hat{A}_2; \hat{B}_1, \hat{B}_2) > 2$$

Collins-Gisin-Linden-Massar-Popescu (CGLMP) inequality

D. Collins, N. Gisin, N. Linden, S. Massar, S. Popescu, Phys. Rev. Lett. 88, 040404 (2002)

The Verification in EW scale

- EW interactions allow for spin reconstruction from decay



The Verification in EW scale

- The density matrix

$$\hat{\rho}_W = \frac{1}{3}\hat{I}_3 + d^i \hat{S}_i + q^{ij}\hat{S}_{\{ij\}}, \;\; i,j = 1,2,3$$

$$\begin{aligned}\hat{\rho}_{WW} \;=\; & \frac{1}{9}\hat{I}_9 + \frac{1}{3}d^i_+\hat{S}^+_i \otimes \hat{I}_3 + \frac{1}{3}d^i_-\hat{I}_3 \otimes \hat{S}^-_i \\ & + \frac{1}{3}q^{ij}_+\hat{S}^+_{\{ij\}} \otimes \hat{I}_3 + \frac{1}{3}q^{ij}_-\hat{I}_3 \otimes \hat{S}^-_{\{ij\}} \\ & + C^{ij}_d\hat{S}^+_i \otimes \hat{S}^-_j + C^{i,jk}_{d,q}\hat{S}^+_i \otimes \hat{S}^-_{\{jk\}} \\ & + C^{ij,k}_{q,d}\hat{S}^+_{\{ij\}} \otimes \hat{S}^-_k + C^{ij,k\ell}_q\hat{S}^+_{\{ij\}} \otimes \hat{S}^-_{\{k\ell\}}\end{aligned}$$

The Verification in EW scale

- The probability in Bell inequality

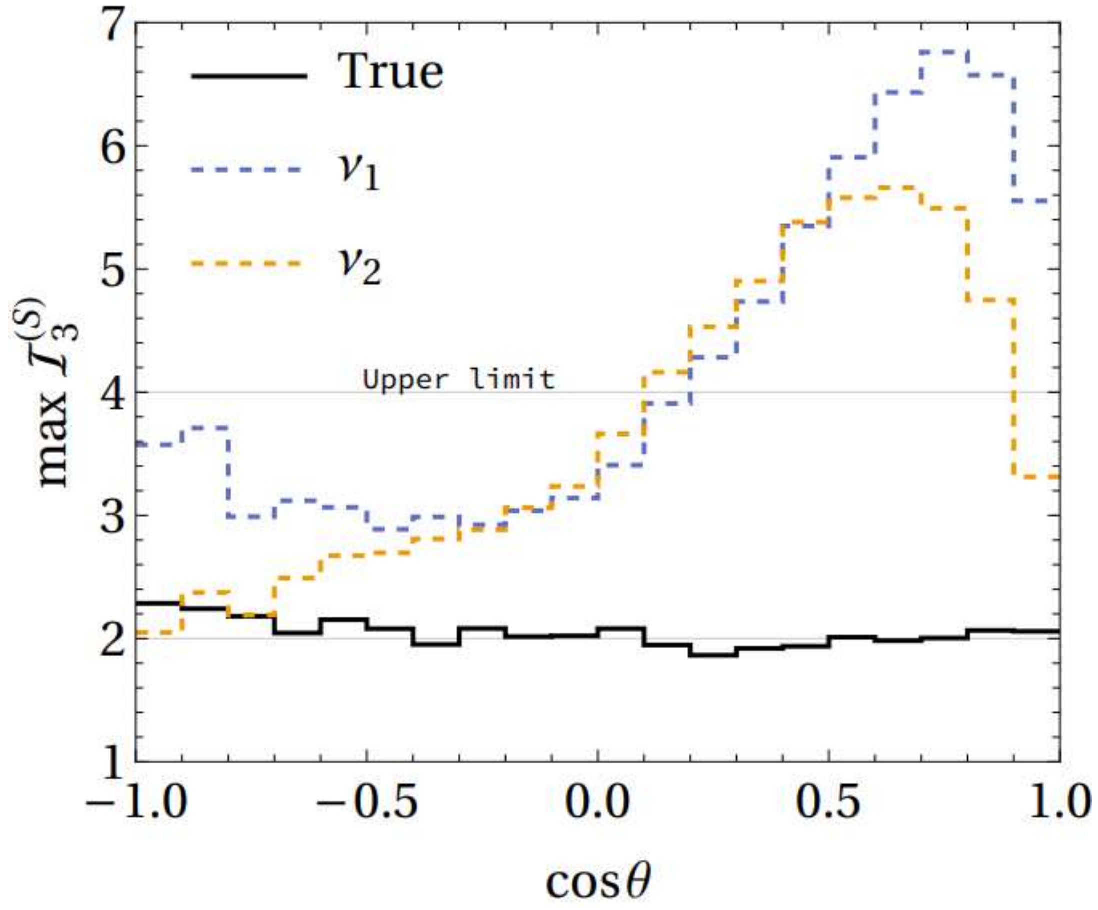
$$P(\vec{n}, \vec{n}'; \rho_{WW}) = \mathrm{Tr}\left[\hat{\rho}_{WW} \hat{\Pi}_{\mathbf{n}} \otimes \hat{\Pi}_{\mathbf{n}'}\right]$$

- Projection operators of the spin eigenstates:

$$\hat{\Pi}_{\mathbf{n}} = \frac{1}{2}(\hat{S}_{\mathbf{n}} + \hat{S}^2_{\mathbf{n}})$$

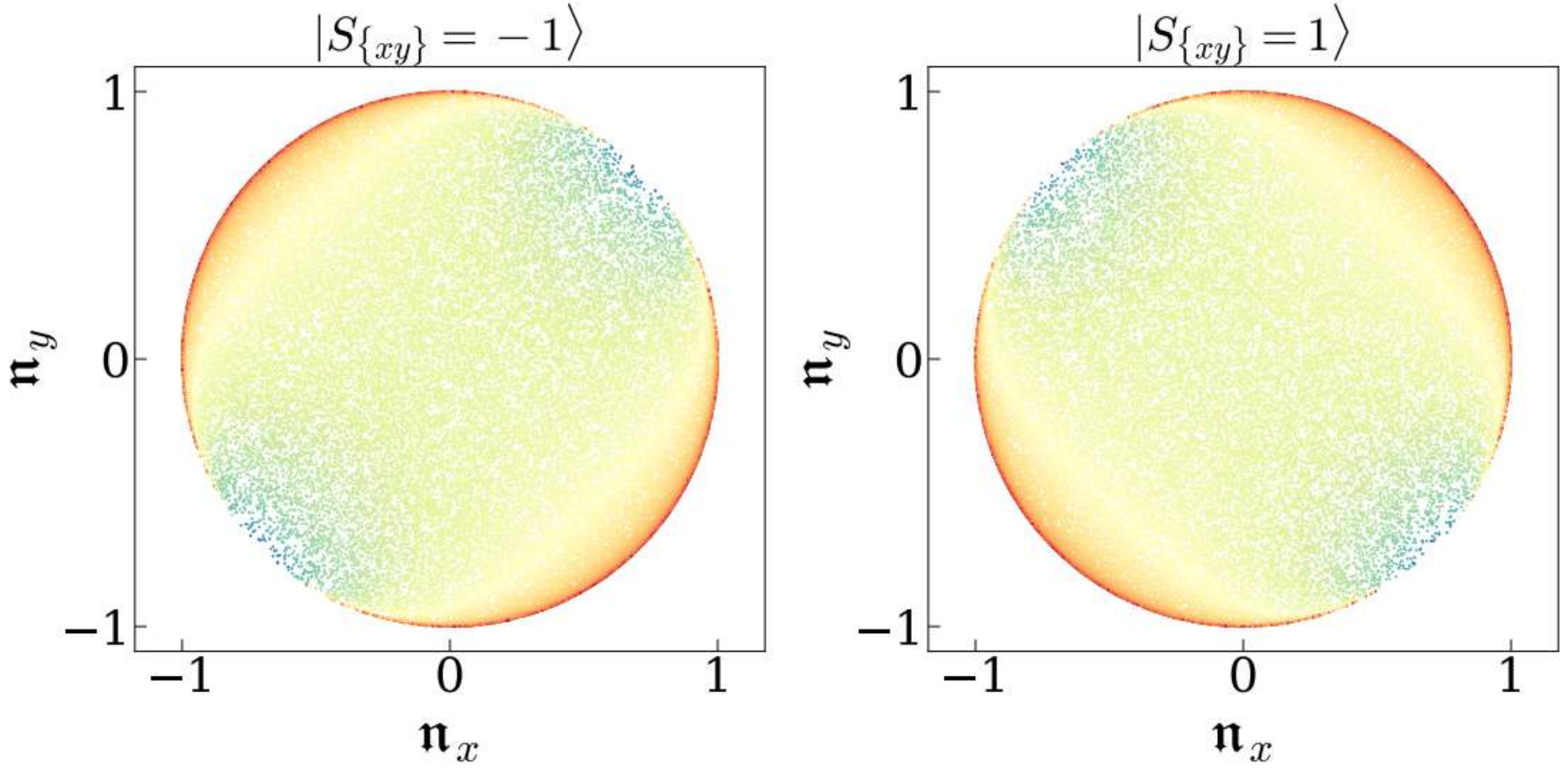
WW production at Higgs factory

- Collider phenomenology
  - di-lepton decay mode $\rightarrow$ twofold discrete ambiguity



WW production at Higgs factory

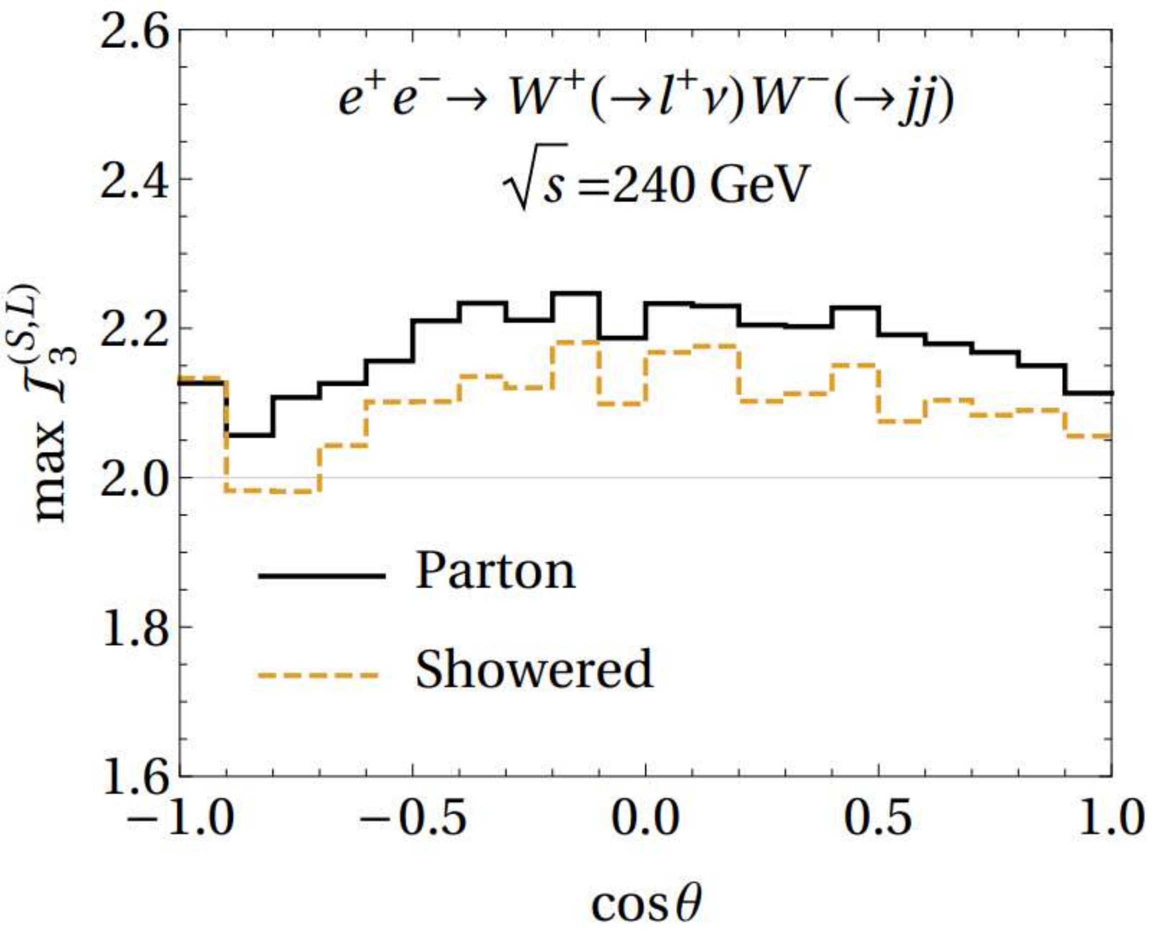
- Collider phenomenology: semi-leptonic decay mode
- Circular polarization → linear polarization



$$\hat{\Pi}_{\mathbf{n}} = \hat{I}_3 - \hat{S}_{\mathbf{n}}^2$$

WW production at Higgs factory

- Collider phenomenology: semi-leptonic decay mode



At 240GeV e+e− collider, one can verify the violation of the Bell inequality at 5.0σ significance with 300 fb-1 integrated luminosity.

Summary

- We provide an approach to test Bell inequalities in W pair systems using a new set of Bell observables based on measuring the linear polarization of W bosons.
- Our observables depend on only part of the density matrix that can be correctly measured in the **semi-leptonic decay** mode of W.
- It is still an open question to testing the entanglement in an essential QFT system (beyond quantum mechanism).

Thank you ！

Backup

Parameter reconstruction

$$\left\langle \mathfrak{n}_i^{\pm} \right\rangle = d_i^{\pm},$$

$$\left\langle \mathfrak{q}_{ij}^{\pm} \right\rangle = \frac{2}{5} q_{ij}^{\pm},$$

$$\left\langle \mathfrak{n}_i^{+} \mathfrak{n}_j^{-} \right\rangle = C_{ij}^{d},$$

$$\left\langle \mathfrak{q}_{ij}^{+} \mathfrak{q}_{kl}^{-} \right\rangle = \frac{4}{25} C_{ij,kl}^{q},$$

$$\left\langle \mathfrak{n}_i^{+} \mathfrak{q}_{jk}^{-} \right\rangle = \frac{2}{5} C_{i,jk}^{dq},$$

$$\left\langle \mathfrak{q}_{ij}^{+} \mathfrak{n}_k^{-} \right\rangle = \frac{2}{5} C_{ij,k}^{qd}.$$

Explicit form of I3

$$
\begin{aligned}
&\mathcal{I}_3(\hat{S}_{\vec{a}_1}, \hat{S}_{\vec{a}_2}; \hat{S}_{\{x_3 y_3\}}, \hat{S}_{\{x_4 y_4\}}) \\
&= 2q^{-}_{ij}(\omega_{1i}\omega_{1j} + \omega_{2i}\omega_{2j} - 2\omega_{3i}\omega_{3j}) \\
&\quad + 2C^{dq}_{i,jk} a_{1i}(2\epsilon_{1j}\epsilon_{1k} - \epsilon_{2j}\epsilon_{2k} - \epsilon_{3j}\epsilon_{3k} + \omega_{1j}\omega_{1k} \\
&\qquad\quad - 2\omega_{2j}\omega_{2k} + \omega_{3j}\omega_{3k}) \\
&\quad + 2C^{dq}_{i,jk} a_{2i}(-2\epsilon_{1j}\epsilon_{1k} + \epsilon_{2j}\epsilon_{2k} + \epsilon_{3j}\epsilon_{3k} + 2\omega_{1j}\omega_{1k} \\
&\qquad\quad - \omega_{2j}\omega_{2k} - \omega_{3j}\omega_{3k}) \\
&\quad + 6C^{q}_{ij,kl} a_{1i}a_{1j}(-\epsilon_{2k}\epsilon_{2l} + \epsilon_{3k}\epsilon_{3l} - \omega_{1k}\omega_{1l} + \omega_{3k}\omega_{3l}) \\
&\quad + 6C^{q}_{ij,kl} a_{2i}a_{2j}(\epsilon_{2k}\epsilon_{2l} - \epsilon_{3k}\epsilon_{3l} - \omega_{2k}\omega_{2l} + \omega_{3k}\omega_{3l})
\end{aligned}
$$

Density matrix

- The density matrix

$$\hat{\rho}_{WW} \propto \mathcal{M}(e^+e^- \to W^+W^-)\hat{\rho}_{e^+e^-}\mathcal{M}(e^+e^- \to W^+W^-)^\dagger$$

9×4 matrix in spin space

spin density matrix of the initial state e+e–

$$\begin{aligned}\hat{\rho}_{WW} &= \frac{1}{9}\hat{I}_9 + \frac{1}{3}d_+^i \hat{S}_i^+ \otimes \hat{I}_3 + \frac{1}{3}d_-^i \hat{I}_3 \otimes \hat{S}_i^- \\ &+\frac{1}{3}q_+^{ij}\hat{S}_{\{ij\}}^+ \otimes \hat{I}_3 + \frac{1}{3}q_-^{ij}\hat{I}_3 \otimes \hat{S}_{\{ij\}}^- \\ &+C_d^{ij}\hat{S}_i^+ \otimes \hat{S}_j^- + C_{d,q}^{i,jk}\hat{S}_i^+ \otimes \hat{S}_{\{jk\}}^- \\ &+C_{q,d}^{ij,k}\hat{S}_{\{ij\}}^+ \otimes \hat{S}_k^- + C_q^{ij,k\ell}\hat{S}_{\{ij\}}^+ \otimes \hat{S}_{\{k\ell\}}^-\end{aligned}$$

$$\hat{\rho}_W = \frac{1}{3}\hat{I}_3 + d^i\hat{S}_i + q^{ij}\hat{S}_{\{ij\}}, \quad i,j = 1,2,3$$