The Effective Operator Basis of The Higgs Effective Field Theory

The Applications of The Algebraic Methods

孙浩

合作者：肖明磊，王一宁，于江浩

基于工作：*JHEP 05 (2023) 043*，*JHEP 04 (2023) 086*，*2211.11598 [hep-ph]*

中国科学院理论物理研究所

2023 年 12 月 15 日

Electroweak Chiral Theory

The Higgs effective field theory (HEFT) is also known as the electroweak chiral theory, which is an EFT of broken phase. Similar to The chiral perturbation theory (χPT),

χPT:	Quark Condensation $\langle \bar{q}q \rangle \neq 0$	$\rightarrow$	SSB: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$	$\rightarrow$	3 Goldstone π^+, π^0, π^-	
	Strong interaction or weal interaction?	$\leftarrow$	SSB: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$	$\leftarrow$	3 Goldstone eaten by the gauge bosons	:HEFT

The necessary existence of zero-mass Goldstone bosons suggests the presence of strong forces and that, in turn, leads to a natural guess for the mass scale of the physical spectrum in the Higgs sector. [1]

[1] Phys.Rev.D 22 (1980) 200

The Effective Lagrangian

The Lagrangian with the least number of derivatives:[2]

$$\begin{aligned}\mathcal{L}_2 = & -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{2}\langle W_{\mu\nu}W^{\mu\nu}\rangle - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{g_s^2}{16\pi^2}\theta_s G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \\ & + \frac{1}{2}\partial_\mu h\partial^\mu h - V(h) + \frac{v^2}{4}\langle \mathbf{V}_\mu \mathbf{V}^\mu\rangle \mathcal{F}_C(h) + \frac{v^2}{4}\langle \mathbf{T}\mathbf{V}_\mu\rangle\langle \mathbf{T}\mathbf{V}^\mu\rangle \mathcal{F}_T(h) \\ & + i\bar{Q}_L \not{D} Q_L + i\bar{Q}_R \not{D} Q_R + i\bar{L}_L \not{D} L_L + i\bar{L}_R \not{D} L_R \\ & - \frac{v}{\sqrt{2}}(\bar{Q}_L \mathbf{U}\mathcal{Y}_Q(h)Q_R) + h.c.) - \frac{v}{\sqrt{2}}(\bar{L}_L \mathbf{U}\mathcal{Y}_L(h)L_R + h.c.)\,, \end{aligned} \tag{1}$$

where $\mathbf{V}_\mu$ is the Golstone, $\mathbf{T}$ is the spurion characterizing the custodial symmetry breaking.

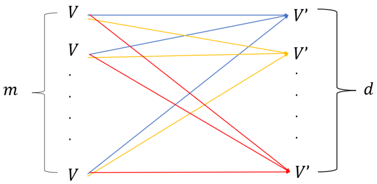
[2] JHEP 05 (2023) 043

Schur-Weyl Duality

For each irreducible space under $SU(N)$, V with dimension d, the tensor product space $V^{\otimes m}$ can be also decomposed in terms of the irreducible space V' of the symmetric group S_m, and its dimension is the multiplicity of V, m, its multiplicity is the dimenion of V, d, $V^{\otimes m} = V'^{\otimes d}$. Converse is also true.

A consequence is every irreducible representation (irrep) of $SU(N)$ can be related to an irrep of S_n, or a Young diagram.

$SU(2)$		$SU(3)$	
$\mathbf{1}$	□ □	$\mathbf{1}$	□ □ □
$\mathbf{2}$	□	$\mathbf{3}$	□
$\bar{\mathbf{2}}$	□	$\bar{\mathbf{3}}$	□ □
$\mathbf{3}$	□□	$\mathbf{8}$	□□ □



Littlewood-Richardson Rule

Furthermore, the decomposition of the tensor product of the $SU(N)$ irreps can be performed directly via the corresponding Young diagrams, called the outer product, which respects the Littlewood-Richardson rule.

The corresponding Young diagrams of the decomposed irreps of $SU(N)$ can be obtained by the outer product of the original diagrams. The semi-standard Young tableaux of the Young diagrams form a basis of the corresponding $SU(N)$ irreps.

$$\begin{gathered} \mathbf{2}\times\bar{\mathbf{2}}=\mathbf{3}+\mathbf{1}\,,\quad \mathbf{3}\times\bar{\mathbf{3}}=\mathbf{8}+\mathbf{1} \\ \yng(1)\times\yng(1)=\yng(2)+\yng(1,1)\,,\quad \yng(1)\times\yng(2)=\yng(2,1)+\yng(1,1,1) \end{gathered} \tag{2}$$

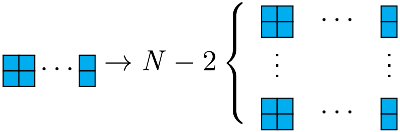
Primary Young Diagrams(1)

Lorentz group algebra $\mathfrak{so}(3,1) \cong \mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R$, which means a Lorentz invariant corresponds to the Young diagram looks like

$$\mathfrak{su}(2)_R \Leftarrow$$

$$\Rightarrow \mathfrak{su}(2)_L\,. \qquad (3)$$

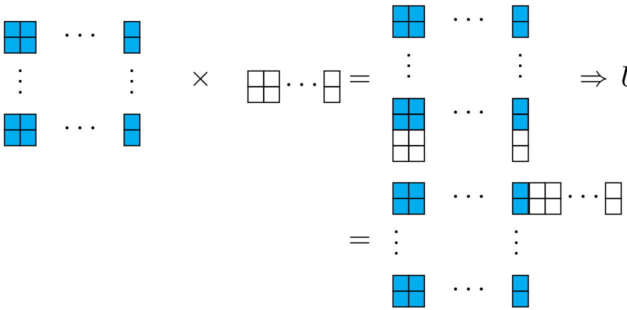
Considering the derivatives, it arises the redundancy by the integration-by-part (IBP), can be eliminated by an extra symmetry $U(N)$, where N is the number of particles. The Lorentz structures are not only $\mathfrak{su}(2)_L \times \mathfrak{su}(2)_R$ invariants but also $U(N)$ invariants.[3]



(4)

[3] Phys.Rev.D 100 (2019) 1, 016015, Phys.Rev.D 104 (2021) 1, 015026

Primary Young Diagrams(2)



$\Rightarrow U(N) - invariant$

$\Rightarrow \mathfrak{so}(3,1) - invariant$

$\Rightarrow$ primary Young diagram (5)

Group Tensors

The $SU(2)$- and $SU(3)$-invariants corresponding to the Young diagrams of the form

$$SU(2) \sim \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline \end{array} \cdots \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}, \quad SU(3) \sim \begin{array}{|c|c|}\hline & \\ \hline & \\ \hline & \\ \hline \end{array} \cdots \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array}. \tag{6}$$

Let $SU(2)$ as an example,

$$\begin{aligned} \mathbf{2} &\sim \boxed{i} \sim \phi_i \\ \bar{\mathbf{2}} &\sim \boxed{j} \sim \epsilon^{jl}\phi_l \\ \mathbf{3} &\sim \begin{array}{|c|c|}\hline i & j \\ \hline \end{array} \sim \phi^I \tau^{I\,l}{}_i \epsilon_{lj}\,. \end{aligned} \tag{7}$$

The only invariant composed by ϕ^j and ϕ_i is

$$\mathbf{2} \times \bar{\mathbf{2}} \to \mathbf{1} \quad \Rightarrow \boxed{i} \times \boxed{j} \to \begin{array}{|c|}\hline i \\ \hline j \\ \hline \end{array}$$

$$\phi_i \times \phi^l \epsilon_{jl} \to \phi_i \times \phi^l \epsilon_{jl} \times \epsilon^{ij} = \phi_i \phi^i \tag{8}$$

Soft Golstones

As mentioned before, the Golstones satisfy Adler's zero condition, which means the corresponding Lorentz structures vanish when the Golstones' momentum becomes soft,

$$\lim_{p\to 0}\mathcal{B}(p)=0\,, \tag{9}$$

which further constrains the Lorentz structures. The soft Lorentz structures can be extracted by a set of linear equations about the original ones[4]

$$\sum_j c_{ij}\mathcal{B}_j(p)=0\,, \tag{10}$$

of which the solution space is the structures of the one satisfying the Adler's zero condition.

[4] JHEP 05 (2023) 043

Suprions

The spurion $\mathbf{T}$ is treated as vacuum expectation values of a dynamical degree of freedom and thus do not enter the Lorentz sector. In the $SU(2)$ sector, the self-contractions among the spurions are redundant, which means the spurions should be symmetric, thus the corresponding Young diagram composed of j spurion is of the form

$$\overbrace{\square \ldots \square}^{2j}, \tag{11}$$

and the complete $SU(2)$ tensor can be factorized as[5]

$$\begin{matrix}\square\\\square\end{matrix} \ldots \begin{matrix}\square\\\square\end{matrix}\,\overbrace{\square \ldots \square}^{2j} \times \overbrace{\square \ldots \square}^{2j} \rightarrow \begin{matrix}\square\square\\\square\square\end{matrix} \ldots \begin{matrix}\square\\\square\end{matrix} \tag{12}$$

[5] JHEP 04 (2023) 086

NLO Operators (Overview)

(From JHEP 05 (2023) 043)

Classes	$\mathcal{N}_{\text{type}}$	$\mathcal{N}_{\text{term}}$	$\mathcal{N}_{\text{operator}}$
UhD^4	$3+6+0+0$	15	15
X^2Uh	$6+4+0+0$	10	10
$XUhD^2$	$2+6+0+0$	8	8
X^3	$4+2+0+0$	6	6
ψ^2UhD	$4+8+0+0$	13(16)	$13n_f{}^2 \quad (16n_f{}^2)$
ψ^2UhD^2	$6+10+0+0$	60(80)	$60n_f{}^2 \quad (80n_f{}^2)$
ψ^2UhX	$7+7+0+0$	22(28)	$22n_f{}^2 \quad (28n_f{}^2)$
ψ^4	$12+24+4+8$	117(160)	$\frac{1}{4}n_f{}^2(31-6n_f+335n_f{}^2) \quad (n_f{}^2(9-2n_f+125n_f{}^2))$
Total	123	261(313)	$\frac{335n_f{}^4}{4}-\frac{3n_f{}^3}{2}+\frac{411n_f{}^2}{4}+39 \quad (39+133n_f{}^2-2n_f{}^2-2n_f{}^3+125n_f{}^4)$ $\mathcal{N}_{\text{operatrs}}(n_f=1)=224(295), \quad \mathcal{N}_{\text{operatrs}}(n_f=3)=7704(11307)$

Table 1: We present the complete statistics of the NLO operators. The types of operators are separated into four categories (C-B, $\not{C}$-B, B-$\not{C}$, $\not{C}$-$\not{B}$). The numbers of terms and operators are also listed for the SM without (with) the right-handed sterile neutrinos.

NLO Operators (4-Fermion Sector)

ψ^4	sub-types	Number	$n_f=1$	$n_f=3$
$(\overline{L}L)^2$	$L_L^{\dagger 2} L_R^2 + h.c.$: $n_f^2(n_f^2+1)$ $L_L^2 L_L^{\dagger 2}$: $\frac{1}{2}n_f^2(3n_f^2+2n_f+1)$ $L_R^2 L_R^{\dagger 2}$: $\frac{1}{4}n_f^2(n_f^2+2n_f+1)$ $L_L L_L^\dagger L_R L_R^\dagger$: $2n_f^4$	$\frac{1}{4}n_f^2(19n_f^2+6n_f+7)$	8	441
$(\overline{L}L)(\overline{Q}Q)$	$Q_L^\dagger Q_R L_L^\dagger L_R + h.c.$: $12n_f^4$ $Q_L^\dagger Q_L L_L^\dagger L_L$: $6n_f^4$ $Q_R^\dagger Q_R L_L^\dagger L_L$: $6n_f^4$ $Q_L^\dagger Q_L L_R^\dagger L_R$: $2n_f^4$ $Q_R^\dagger Q_R L_R^\dagger L_R$: $2n_f^4$ $Q_L^\dagger Q_R L_L^\dagger L_R + h.c.$: $6n_f^4$	$34n_f^4$	34	2754
$(\overline{Q}Q)^2$	$Q_L^{\dagger 2} Q_R^2 + h.c.$: $4n_f^2(3n_f^2+1)$ $Q_L^{\dagger 2} Q_L^2$: $n_f^2(3n_f+1)$ $Q_R^{\dagger 2} Q_R^2$: $n_f^2(3n_f+1)$ $Q_L^\dagger Q_L Q_R^\dagger Q_R$: $12n_f^4$	$n_f^2(30n_f^2+6)$	36	2484
Q^3L	$L_L Q_L^3 + h.c.$: $4n_f^4$ $L_L Q_L Q_R^2 + h.c.$: $2n_f^3(3n_f-1)$ $L_R Q_R Q_L^2 + h.c.$: $n_f^3(3n_f-1)$ $L_R Q_R^3 + h.c.$: $2n_f^4$	$n_f^2(15n_f^2-3n_f)$	12	1134

Table 2: The numbers of the independent operators in class ψ^4 without right-handed neutrino in detail, where $(\overline{L}L)^2$ is the pure lepton sector, $(\overline{L}L)(\overline{Q}Q)$ is the mixed quark-lepton sector, $(\overline{Q}Q)^2$ is the pure quark sector, and Q^3L is the baryon-number-violating sector.

NLO Operators (Summary)

For the first time, we could obtain the 224 (7704) operators for one (three) generation fermions, 295 (11307) with right-handed neutrinos, and find there were 8 (11) terms of operators missing and many redundant operators can be removed in the effective theory without (with) right-handed neutrinos.

NNLO Operators

In JHEP 04 (2023) 086, we obtain that there are 11506 (1927574) NNLO operators with one (three) generations of the SM fermions assuming right-handed neutrino exists.

Class	$\mathcal{N}_{\text{operator}}$
UhD^6	114
X^2UhD^2	130
$XUhD^4$	164
ψ^2XUhD	$156{n_f}^2$
ψ^2UhX^2	$156{n_f}^2$
ψ^4UhD	$2{n_f}^2(-1+2n_f+259{n_f}^2)$
ψ^4XUh	${n_f}^2(-16+9n_f+727{n_f}^2)$
ψ^2UhD^3	$224{n_f}^2$
ψ^2XUhD^2	$816{n_f}^2$
ψ^4UhD^2	$\frac{1}{2}{n_f}^2(91+56n_f+6059{n_f}^2)$
ψ^6Uh	$\frac{1}{9}{n_f}^2(28-24n_f+73{n_f}^2+15nf^3+10006{n_f}^4)$
ψ^2UhD^4	$834{n_f}^2$
Total	$\frac{1}{18}(7452+39899{n_f}^2+690{n_f}^3+77087{n_f}^4+30{n_f}^5+20012{n_f}^6)$ $n_f=1$: 8065, $n_f=3$: 1179181

Orthogonality

For a compact group G, its finite-dimension representations are complete-reducible, whose the characters of different classes orthogonal,

$$\int d\mu_G(z)\chi_i(z)^*\chi_j(z) = \delta_{ij}\,, \quad z \text{ is the maximal ring variable.} \tag{13}$$

The decomposition of the tensor product of the compact group G can be done similarly to the discrete group,

$$n_j = \int d\mu_G(z)\chi_j^*(z)\left(\chi_{\mathbf{r}_1}(z)\chi_{\mathbf{r}_2}(z)\ldots\chi_{\mathbf{r}_3}(z)\right)\,, \tag{14}$$

where n_j is the multiplicity of the irrep j in the decomposition of the tensor product $\mathbf{r}_1 \times \mathbf{r}_2 \times \cdots \times \mathbf{r}_3$.

Hilbert Series

The Hilbert series is a generating function of the group invariants, which takes the form that

$$HS = 1 + n_1 Q + n_2 Q^2 + n_3 Q^3 + \dots\,, \tag{15}$$

where Q is a field of some representation of a group, and n_i is the number of the independent invariants composed of i Q's. Thus the Hilbert series counts the number of independent operators. For the compact group the coefficient n_i can be obtained by the orthogonal relation

$$n_m = \int d\mu_G(z)\chi_1^*(z)\text{Sym}\left(\underbrace{\chi_r(z)\chi_r(z)\dots\chi_r(z)}_{m}\right)\,, \tag{16}$$

where $\chi_1(z) = 1$ is the character for the trivial representation.

Conformal Algebra

$$\text{conformal group} = \mathfrak{so}(3,1) + \{D\,, K\} \cong \mathfrak{so}(6)\,. \tag{17}$$

K is the special conformal transformation generator

D is the dilation generator .

The irreps can be labeled by (j_L, j_R, Δ), where (j_L, j_R) determines a Lorentz group irrep and Δ determines the canonical dimension.

$$\begin{aligned} &R_{(0,0,1)} = \phi\,, \quad R_{(0,0,2)} = D_\mu\phi\,, \\ &R_{(\frac{1}{2},0,\frac{3}{2})} = \psi_L\,, \quad R_{(\frac{1}{2},0,\frac{5}{2})} = D_\mu\psi_L \oplus \sigma_\mu D^\mu\psi_L\,, \end{aligned} \tag{18}$$

where the subspace $\sigma_\mu D^\mu \psi_L$ is redundant because of the equation of motion (EOM), and should be eliminated[6].

[6] JHEP 09 (2019) 019 (erratum)

Single Particle Module

Taking a (reducible) representation of the conformal group

$$R_\phi = \begin{pmatrix} \phi \\ D_\mu \phi \\ D_{(\mu} D_{\nu)} \phi \\ \vdots \end{pmatrix}, \quad \phi \in (j_L, j_R) \tag{19}$$

called single particle module (SPM), where $D_{(\mu}D_{\nu)}$ means the EOM redundancy has been eliminated.

Similarly the effective operators also form a module

$$R_{\mathcal{O}} \begin{pmatrix} \mathcal{O} \\ D_\mu \mathcal{O} \\ D_{(\mu} D_{\nu)} \mathcal{O} \\ \vdots \end{pmatrix}, \quad \mathcal{O} \in (0,0)\,, \tag{20}$$

of which only the first sub-module should be kept due to IBP.

Conformal Characters

For a general irrep $D_{(\mu_1} \dots D_{\mu_n)}\phi$, where $\phi \in (j_L, j_R)$, its character takes the form

$$\chi_{(\Delta, j_L, j_R)}(z) = D^n \text{Sym}_n(\chi_{(\mathbf{2},\mathbf{2})}(z))\chi_{(j_L,j_R)}(z)\,, \tag{21}$$

thus the character of the single-particle module is

$$\begin{aligned}\chi_\phi(z) &= \left(D^n \sum_{n=0}^{\infty} \text{Sym}_n(\chi_{(\mathbf{2},\mathbf{2})}(z))\right) \times \chi_{(j_L,j_R)}(z) \\ &= P(D,z)\chi_{(j_L,j_R)}(z)\,,\end{aligned} \tag{22}$$

where $P(D,z) = D^n \sum_{n=0}^{\infty} \text{Sym}_n(\chi_{(\mathbf{2},\mathbf{2})}(z))$.
In particular, the character of the effective operators without IBP redundancy is still 1.[7]

[7] JHEP 10 (2017) 199

Master Formula

$$HS = \int d\mu_G(z) \prod_i \mathrm{PE}_f(\psi_i, \chi_{\psi_i}(z)) \prod_j \mathrm{PE}(\phi_j, \chi_{\phi_j}(z)) \,, \qquad (23)$$

where

$$\begin{aligned} \chi_\phi(z) &= \chi_{\text{Conformal}}(z)\chi_{\text{Internal}}(z)\,, \\ d\mu_G(z) &= d\mu_{\text{Internal}}(z)\, d\mu'_{\text{Conformal}}(z)\,, \\ d\mu'_{\text{Conformal}}(z) &= \frac{1}{P(D,z)} d\mu_{SO(3,1)}(z)\,. \end{aligned} \qquad (24)$$

The number of independent operators composed by n_i ϕ_i, n_j ψ_j, and k derivatives D can be extracted by

$$n(\ldots, n_i, \ldots, n_j, \ldots, k) = \prod_i \frac{d}{d\phi_i} \prod_j \frac{d}{d\phi_j} HS \Big|_{\phi_i=\psi_j=D=1} \,. \qquad (25)$$

Plethystic Exponential

$$Q \text{ is boson: } \mathrm{PE}(Q,\chi_r(z)) = \exp\left\{\sum_{k\geq 1}\frac{Q^k\chi_r(z^k)}{k}\right\}$$

$$\to \mathrm{Sym}\left(\underbrace{\chi_r(z)\chi_r(z)\ldots\chi_r(z)}_{m}\right) = \frac{d^m}{dQ^m}\mathrm{PE}(Q,\chi_r(z))\bigg|_{Q=0} \tag{26}$$

$$Q \text{ is fermion: } \mathrm{PE}_f(Q,\chi_r(z)) = \exp\left\{\sum_{k\geq 1}\frac{(-1)^{k-1}Q^k\chi_r(z^k)}{k}\right\}$$

$$\to \mathrm{Asym}\left(\underbrace{\chi_r(z)\chi_r(z)\ldots\chi_r(z)}_{m}\right) = \frac{d^m}{dQ^m}\mathrm{PE}_f(Q,\chi_r(z))\bigg|_{Q=0} \tag{27}$$

Group Extension

The symmetry group is extended by two discrete groups[8],

$$SO(3,1)\times G_{\text{Internal}} \to (SO(3,1)\rtimes\{1,\mathcal{P}\})\times(G_{\text{Internal}}\rtimes\{1,\mathcal{C}\}) \\ = SO(3,1)^{+}\sqcup SO(3,1)^{+}\sqcup G^{+}_{\text{Internal}}\sqcup G^{-}_{\text{Internal}}\,, \tag{28}$$

The corresponding characters:

$$\chi_\phi(z,x)\to\chi^{+}_{\phi}(z,x)\chi^{+}_{\text{Internal}}(x)+\chi^{+}_{\phi}(z,x)\chi^{-}_{\text{Internal}}(x) \\ +\chi^{-}_{\phi}(z,x)\chi^{+}_{\text{Internal}}(x)+\chi^{-}_{\phi}(z,x)\chi^{-}_{\text{Internal}}(x)\,. \tag{29}$$

The orthogonality relation becomes

$$\frac{1}{4}\int d\mu\chi^{*}_{\phi}\chi_{\phi}=1\,. \tag{30}$$

[8] 2211.11598 [hep-ph]

CP-even Operarors

The complete Hilbert series can be divided into 4 disadjoint sectors[9],

$$HS = \frac{1}{4}(HS^{++} + HS^{+-} + HS^{-+} + HS^{--})\,, \tag{31}$$

where

$$\begin{aligned} HS^{++} &= \int d\mu_G^{++}(z) \prod_i \mathrm{PE}_f(\psi_i, \chi_{\psi_i}^{++}(z)) \prod_j \mathrm{PE}(\phi_j, \chi_{\phi_j}^{++}(z))\,, \\ HS^{+-} &= \int d\mu_G^{+-}(z) \prod_i \mathrm{PE}_f(\psi_i, \chi_{\psi_i}^{+-}(z)) \prod_j \mathrm{PE}(\phi_j, \chi_{\phi_j}^{+-}(z))\,, \\ HS^{-+} &= \int d\mu_G^{-+}(z) \prod_i \mathrm{PE}_f(\psi_i, \chi_{\psi_i}^{-+}(z)) \prod_j \mathrm{PE}(\phi_j, \chi_{\phi_j}^{-+}(z))\,, \\ HS^{--} &= \int d\mu_G^{--}(z) \prod_i \mathrm{PE}_f(\psi_i, \chi_{\psi_i}^{--}(z)) \prod_j \mathrm{PE}(\phi_j, \chi_{\phi_j}^{--}(z))\,. \end{aligned} \tag{32}$$

[9] JHEP 01 (2021) 142

Nonlinear Symmetry

The nonlinear symmetry of the HEFT implies the Goldstones are soft, characterized by $\mathbf{V}_\mu$. Its SPM is

$$R_{\mathbf{V}} = \begin{pmatrix} \mathbf{V}_\mu \\ D_{(\nu}\mathbf{V}_{\mu)} \\ D_{(\rho}D_\nu\mathbf{V}_{\mu)} \\ \vdots \end{pmatrix} \cong \begin{pmatrix} D_\mu\phi \\ D_{(\nu}D_{\mu)}\phi \\ D_{(\rho}D_\nu D_{\mu)}\phi \\ \vdots \end{pmatrix} \cong R_\phi/\phi\,. \tag{33}$$

The character takes the form that[10]

$$\chi_{\mathbf{V}} = \chi_\phi - 1\,. \tag{34}$$

[10] 2211.11598 [hep-ph], JHEP 02 (2023) 064

Spurions

The spurions can not be applied by the derivatives, which means its SPM is truncated, $R_{\mathbf{T}} = \mathbf{T}$.
Besides, the self-contractions of the spurions are redundant, which present an equivalence relation of the effective operators,

$$\mathcal{O}_1 \sim \mathcal{O}_2 \Leftrightarrow \mathcal{O}_1 \propto \mathcal{O}_2 \times [\mathbf{T}] \,, \tag{35}$$

where $[\mathbf{T}]$ is some invariant composed solely of the spurion.
In the Hilbert series method, these redundant contributions can be eliminated by the quotient of two Hilbert series[11],

$$HS = \frac{HS(\phi, \mathbf{T})}{HS(\mathbf{T})} \,, \tag{36}$$

where $HS(\phi, \mathbf{T})$ is the Hilbert series treating the spurion as normal scalar fields, while $HS(\mathbf{T})$ is the Hilbert series of the spurion.

[11] 2211.11598 [hep-ph], JHEP 02 (2023) 064

Numbers of the Higher-Dimension Operators (1)

(From 2211.11598 [hep-ph])

d_χ	$\mathcal{N}_{\text{operator}}$	$n_f = 1$	$n_f = 3$
3	$16{n_f}^2$	16	144
4	$125{n_f}^4 - 2{n_f}^3 + 117{n_f}^2 + 33$	273	11157
5	$\frac{4}{3}{n_f}^2\left(575{n_f}^2 + 3n_f + 340\right)$	1224	66288
6	$\frac{1}{9}(17150{n_f}^6 - 24{n_f}^5 + 50015{n_f}^4 + 258{n_f}^3 + 21467{n_f}^2 + 3726)$	10288	1861292
7	$\frac{2}{9}{n_f}^2(94325{n_f}^4 + 372{n_f}^3 + 151379{n_f}^2 + 1104n_f + 46184)$	65192	18124552
8	$\frac{1}{18}(453789{n_f}^8 - 4116{n_f}^7 + 3445400{n_f}^6 + 12264{n_f}^5 + 3565703{n_f}^4 + 26124{n_f}^3 + 891320{n_f}^2 + 112986)$	472415	321147047
9	$\frac{4}{45}{n_f}^2(4602717{n_f}^6 - 87269{n_f}^5 + 16523580{n_f}^4 + 50830{n_f}^3 + 12294138{n_f}^2 + 86344n_f + 2417430)$	3190024	3829824336
10	$\frac{1}{225}(63392868{n_f}^{10} - 3810240{n_f}^9 + 1050088850{n_f}^8 - 22937520{n_f}^7 + 2305118919{n_f}^6 + 4351290{n_f}^5 + 1315821375{n_f}^4 + 8708370{n_f}^3 + 221625088{n_f}^2 + 21318300)$	22060788	54658127796

Table 5: The numbers of HEFT operators up to chiral dimension 10 of the case the spurion is of $d_\chi = 0$.

Numbers of the Higher-Dimension Operators (2)

The result of the Hilbert series is consistent to the Young tensor method and the result of other authors[12].

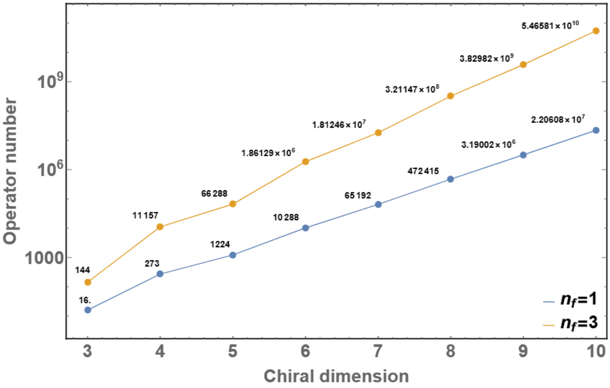


Figure 2: The growth of the number of the operators in the HEFT in the case of dimensionless spurion.

[12] JHEP 02 (2023) 064

- For the first time, we obtain the 224 (7704) operators for one (three) generation fermions, 295 (11307) with right-handed neutrinos, and find there were 8 (11) terms of operators missing and many redundant operators can be removed in the effective theory without (with) right-handed neutrinos.
- We obtain that there are 11506 (1927574) NNLO operators with one (three) generations of the SM fermions assuming right-handed neutrino exists.
- We extend the Hilbert series method to the discrete symmetry and count the higher-dimension operators of the HEFT. In particular these two methods are consistent.
- Both of the Young tensor and Hilbert series methods are general and can be applied to many other theories such as dark matter and axion theories[13].

[13] 2305.16770 [hep-ph], 2306.05999 [hep-ph], JHEP 11 (2023) 196

Thank you!