Dynamical realization of the small field inflation in the post supercooled universe

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Collaborators: Hiroyuki Ishida, Shinya Matsuzaki

Based on: H.Ishida and S.Matsuzaki, Phys.Lett.B 804 (2020) 135390,
H.-X.Z., H.Ishida, and S.Matsuzaki, Phys.Lett.B 846 (2023) 138256

Outline

Introduction: Cosmology and inflation

Model: The walking dilaton inflation

Analyses & results: Dynamical trapping mechanism

Conclusions

Standard Big Bang Theory

Advantages:

- Hubble's Law
- Abundance of light elements
- CMB temperature

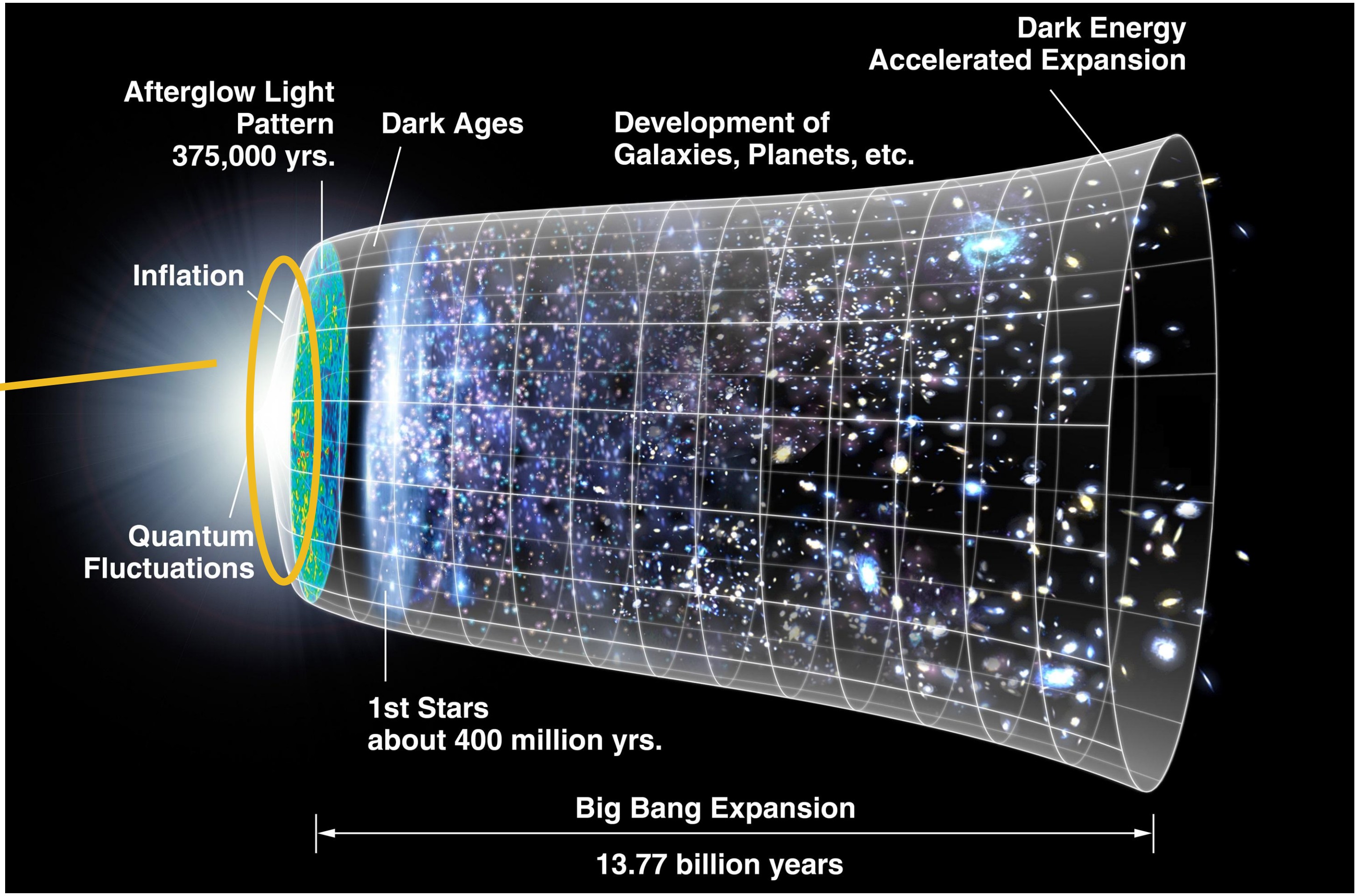
[XUANYU HAN/GETTY IMAGES]

Disadvantages:

- Horizon problem
- Flatness problem
- Singularities
- ......

Solution: Inflation

The universe experienced a period of exponential expansion after the Big Bang.



[NASA]

Then, what kind of dynamics is needed?

Small Field Inflation of CW type

The potential of inflaton:

[Coleman, Weinberg(1973)]

$$V(\phi) = \frac{\lambda\phi^4}{4}\left(\ln\frac{\phi^2}{v_\phi^2} - \frac{1}{2}\right) + V_0$$

Fine-tuning problem: [Iso, Kohri, Shimada(2016)]

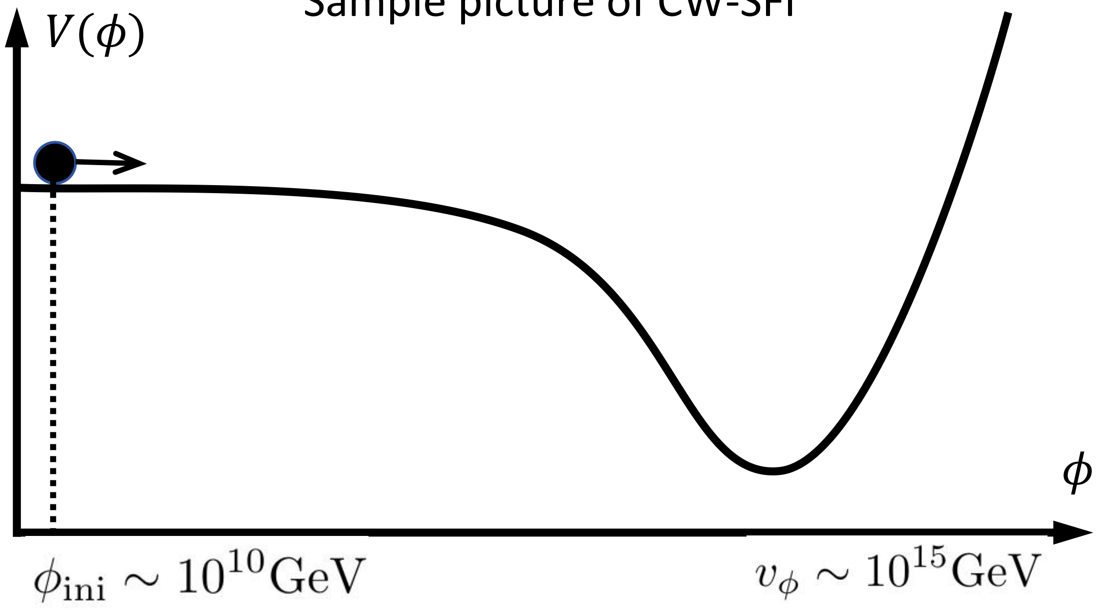
The inflation must start from the very small initial condition, essentially due to scale invariance

$$\phi_{\rm ini} \ll v_\phi$$

Incompatibility between N and n_s [specialized to CW]:

$$n_s \simeq 0.968 \Rightarrow N = \frac{3}{1-n_s} - \frac{3}{2} = 73.5$$

Sample picture of CW-SFI



Extremely tiny coupling, i.e., large hierarchy between mass and VEV:

$$\Delta_R^2 \simeq 2.137 \times 10^{-9} \Rightarrow \lambda = \left(\frac{m_\phi}{v_\phi}\right)^2 \sim 10^{-15}$$

Where the large hierarchy comes from?

Motivation

Can we dynamically solve the fine-tuning problem?

Actually, one proposal is already present: [Iso, Kohri, Shimada(2016)]

This mechanism to trap the inflaton around the false vacuum has been proposed, in which the trapping dynamically works due to the particle number density (like plasma or a medium) created by the preheating.

Now our purpose and goal are:

Propose alternative trapping mechanism by supercooling, and construct a model for small field inflation consistent with the observations by solving all 3 intrinsic problems.

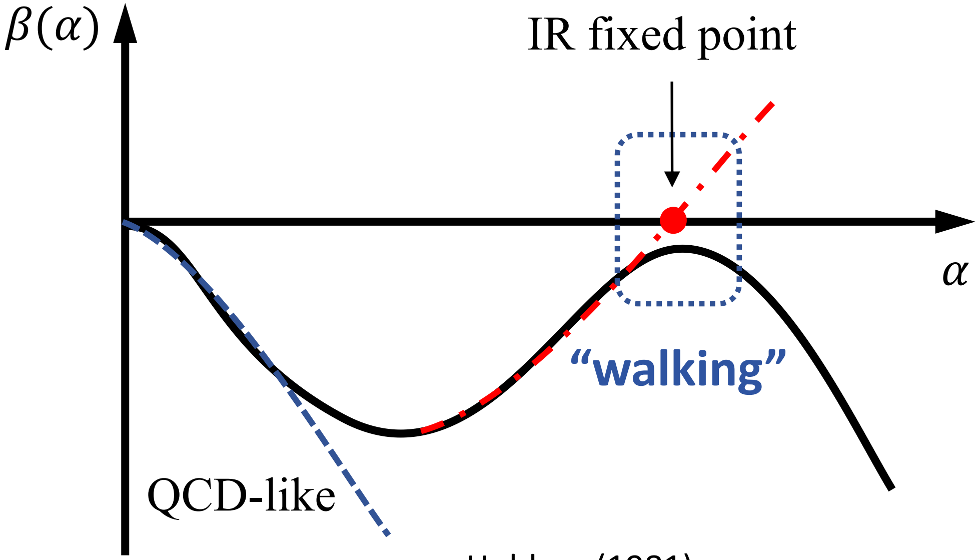
Many-flavor QCD

E.g., S.Matsuzaki and K.Yamawaki, JHEP 12, 053 (2015)

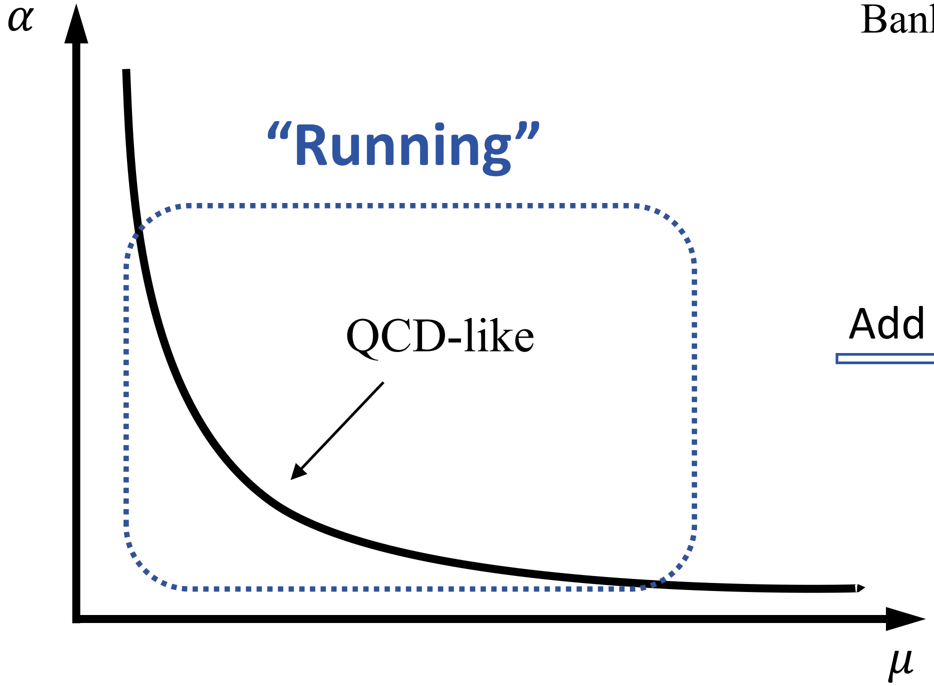
2-loop RGE: $$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\alpha(\mu) = -b\,\alpha^2(\mu) - c\,\alpha^3(\mu)$$

$(N_c = 3)$	$N_f < 8.05$	$8.05 < N_f < 16.5$	$16.5 < N_f$
$b = \frac{1}{64\pi}(33 - 2N_f)$	$+$	$+$	$-$
$c = \frac{1}{12\pi^2}(153 - 19N_f)$	$+$	$-$	$-$

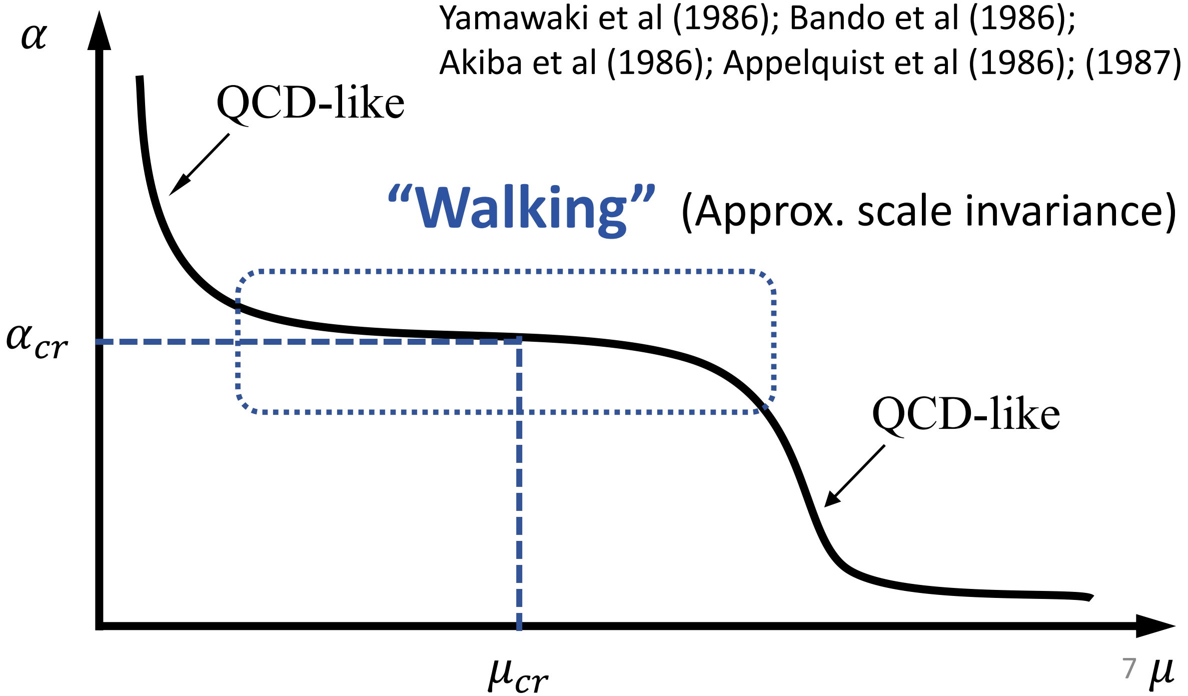
Caswell(1974)
Banks, Zaks(1982)



Holdom (1981);
Yamawaki et al (1986); Bando et al (1986);
Akiba et al (1986); Appelquist et al (1986); (1987)



Add many Fermions ⟹



Many-flavor QCD

Miransky scaling: [Miransky(1985)]

$$m_F \sim \Lambda_{\rm UV}\, e^{-\frac{\pi}{\sqrt{\alpha/\alpha_{\rm cr}-1}}} \quad \text{for} \quad \alpha > \alpha_{\rm cr}$$

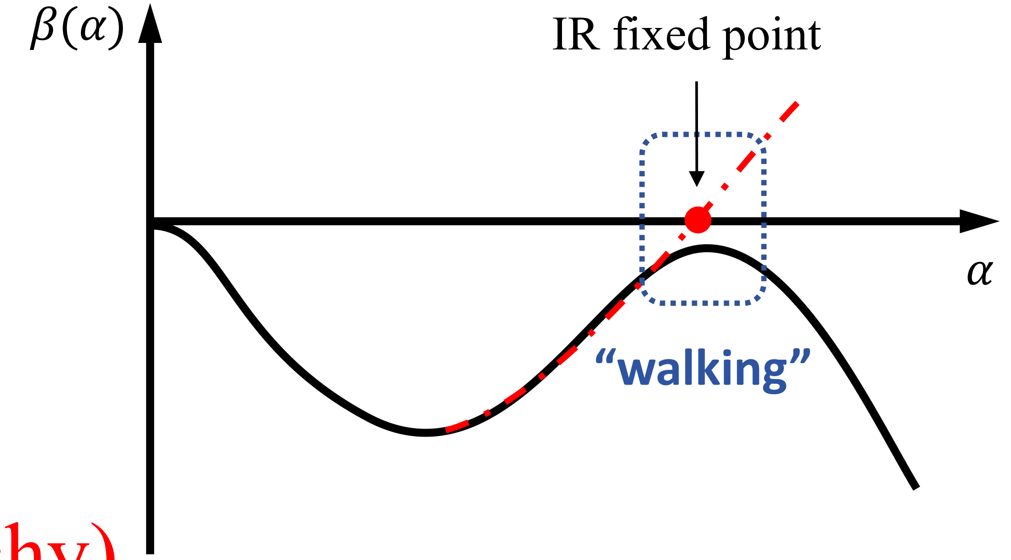
- m_F - dynamical mass of the fermion
- $\Lambda_{\rm UV}$ - the UV scale
- α_{cr} - the critical coupling

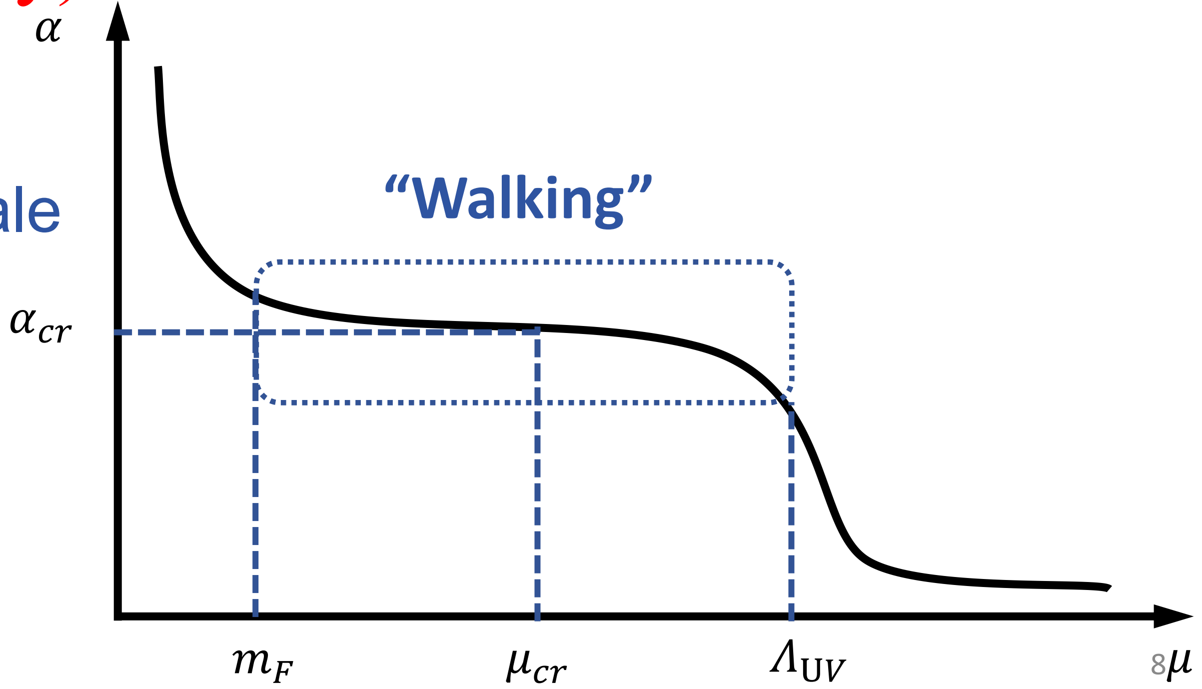
$\rightarrow m_F \ll \Lambda_{UV}$ when $\alpha \approx \alpha_{cr}$ (large hierarchy)

Approximate scale invariance $\beta(\alpha_{\rm cr}) \approx 0$

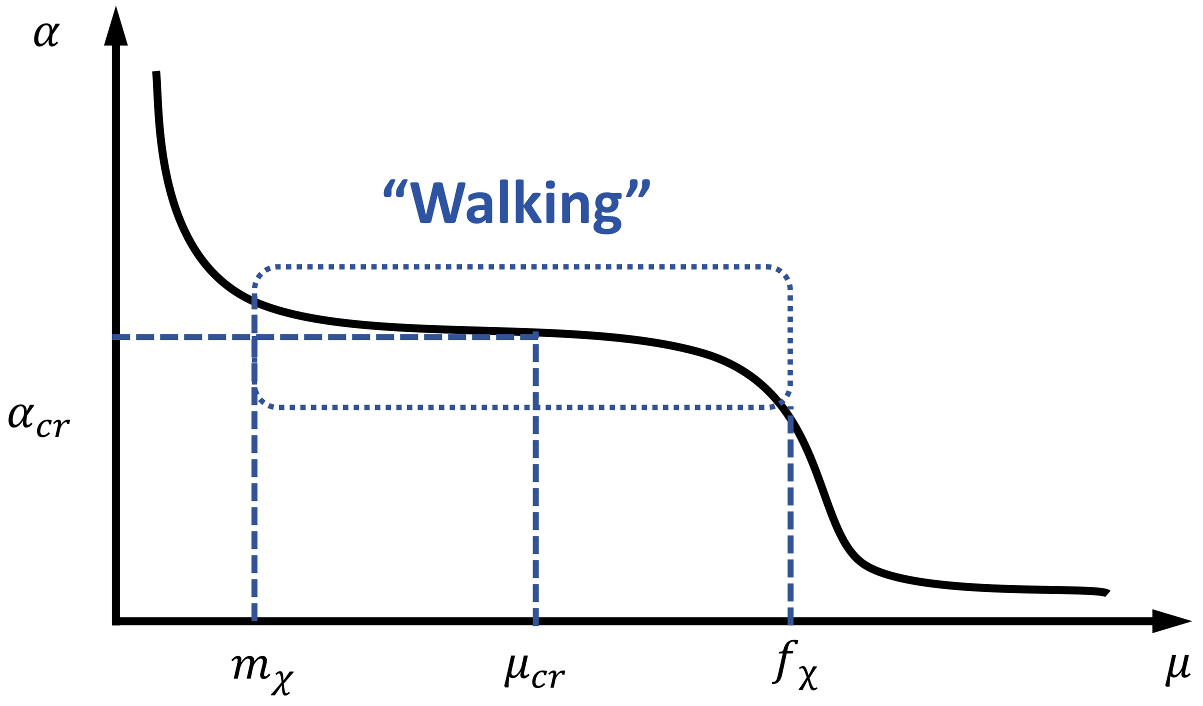
Spontaneously breaking of (approximate) scale symmetry → "pseudo-walking **dilaton**"

Dilaton gets mass due to the explict breaking induced by the scale anomaly.





Many-flavor QCD



Dilaton mass (Milansky scaling):

$$m_\chi \sim f_\chi \, e^{-\frac{\pi}{\sqrt{\alpha/\alpha_{\rm cr}-1}}}$$

f_χ: scale symmetry breaking scale

In our model, we regard this dilaton as the inflaton.

Model: The walking dilaton inflation

The walking dilaton potential: [Ishida, Matsuzaki(2020)]

$$V(\chi) = \underbrace{-\frac{C}{2N_f}\chi \,\mathrm{Tr}\left[\langle U\rangle + \langle U^\dagger\rangle\right]}_{\text{“tadpole”}} + \underbrace{\frac{\lambda_\chi}{4}\chi^4\left(\ln\frac{\chi}{v_\chi} + A\right)}_{\text{“CW-type potential”}} + V_0$$

χ : dilaton v_χ : VEV of dilaton
N_f : number of fermions
V_0 : vacuum energy with $V(v_\chi)$=0
A : fixed by $V'(v_\chi) = 0$

$$U = \mathrm{e}^{2\mathrm{i}\pi/f_\pi}$$

$$C = N_f\frac{m_\pi^2 f_\pi^2}{2v_\chi}$$

☆ Tadpole by explicit chiral symmetry breaking

☆ CW-type potential by explicit scale anomaly

Cosmological parameters

slow-roll parameters:
$$\begin{cases} \epsilon & = \frac{M_{\rm pl}^2}{2}\left(\frac{V'(\chi)}{V(\chi)}\right)^2 \\ \eta & = M_{\rm pl}^2\left(\frac{V''(\chi)}{V(\chi)}\right) \end{cases}$$

e-folding number: $N = \frac{1}{M_{\rm pl}^2}\int_{\chi_{\rm end}}^{\chi_{\rm ini}} d\chi \left(\frac{V(\chi)}{V'(\chi)}\right)$

scalar perturbation: $\Delta_R^2 = \frac{V(\chi)}{24\pi^2 M_{\rm pl}^4 \epsilon(\chi)}$

spectral index $n_s = 1 - 6\epsilon + 2\eta \simeq 1 + 2\eta$

Cosmological parameters in the present model

$$\eta \simeq 24 \frac{M_{\rm pl}^2}{v_\chi^2} \frac{\chi^2}{v_\chi^2} \ln \frac{\chi^2}{v_\chi^2} ,$$

$$\epsilon \simeq \frac{\pi^4}{2} \left(\frac{M_{\rm pl}}{v_\chi} \right)^2 \left(\frac{m_\pi}{m_F} \right)^4 ,$$

$$\Delta_R^2 \simeq \frac{2}{\pi^{10}} \left(\frac{m_F}{v_\chi} \right)^4 \cdot \left(\frac{v_\chi}{M_{\rm pl}} \right)^6 \left(\frac{m_F}{m_\pi} \right)^4 ,$$

$$\underline{N} \simeq \frac{(\chi_{\rm end} - \chi_{\rm ini})}{\sqrt{2\epsilon} M_{\rm pl}} \simeq \frac{(\chi_{\rm end} - \chi_{\rm ini}) v_\chi}{6\pi^2 M_{\rm pl}^2} \left(\frac{m_F}{m_\pi} \right)^2$$

Constraints:

Planck Collaboration(2018):

$$\Delta_R^2 \simeq 2.137 \times 10^{-9}$$

$$n_s \simeq 0.968$$

Incompatibility between N and n_s resolved!

Cosmological parameters

At this bechmark point: [Ishida, Matsuzaki(2020)]

$$v_\chi \simeq 1.7 \times 10^{15}\mathrm{GeV}, \quad m_\chi \simeq 3.8 \times 10^{8}\mathrm{GeV}$$
$$m_F \simeq 4.1 \times 10^{11}\mathrm{GeV}, \quad m_\pi \simeq 6.7 \times 10^{4}\mathrm{GeV}$$

which is in agreement with the observations.

So far, we have solved

☑ Extremely tiny CW coupling

☑ Incompatibility between N and n_s

But, the fine-tuning problem is still there...

$$\chi_{\mathrm{ini}} \simeq 6.7 \times 10^{9}\mathrm{GeV} \ll v_\chi .$$

Model in the finite temperature

χ : SU(N_f) singlet dilaton
s^i : SU(N_f) adjoint scalar mesons
Note $N_f^2 - 1$ pions (π^i) : negligible couplings, due to lightness enough

The effective potential with thermal corrections: [**H.-X.Z.**, H.Ishida, and S.Matsuzaki]

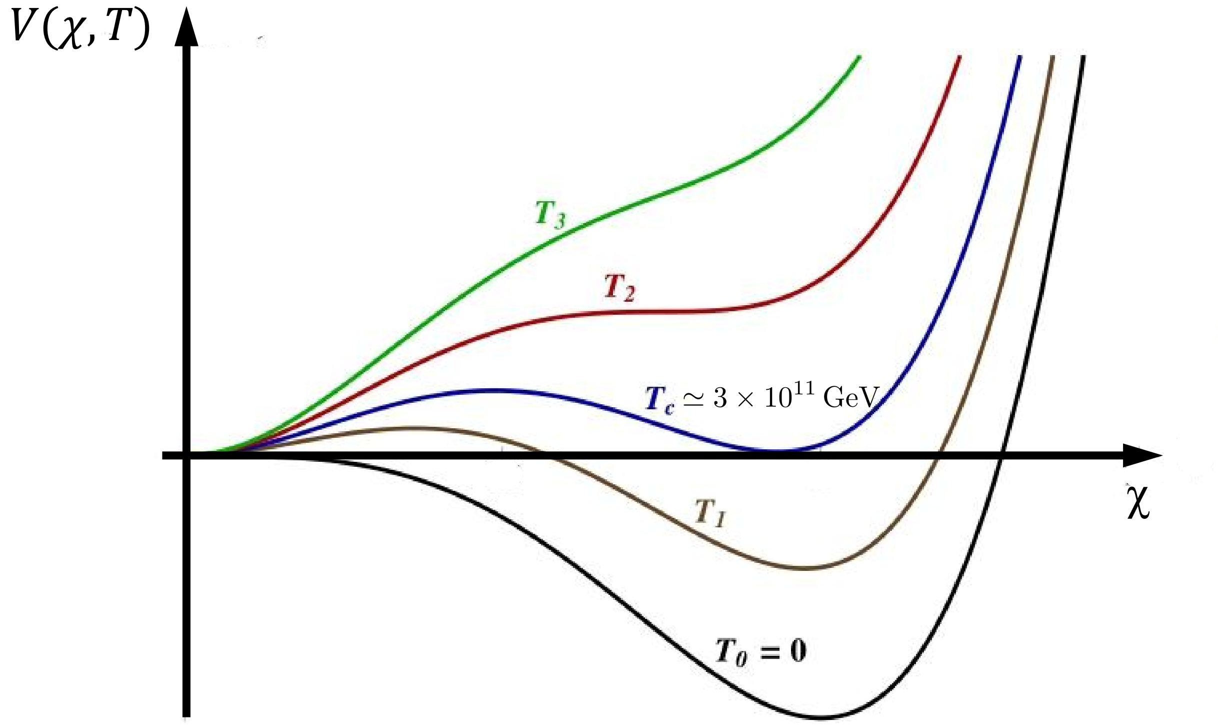
$$V_{\rm eff}(\chi,T) = -C\,\chi + \frac{N_f^2-1}{64\pi^2}\mathcal{M}_{s^i}^4(\chi,T)\left(\ln\frac{\mathcal{M}_{s^i}^2(\chi,T)}{\mu_{GW}^2} - \frac{3}{2}\right) + \frac{T^4}{2\pi^2}(N_f^2-1)J_B\left(\mathcal{M}_{s^i}^2(\chi,T)/T^2\right) + V_0\,,$$

where

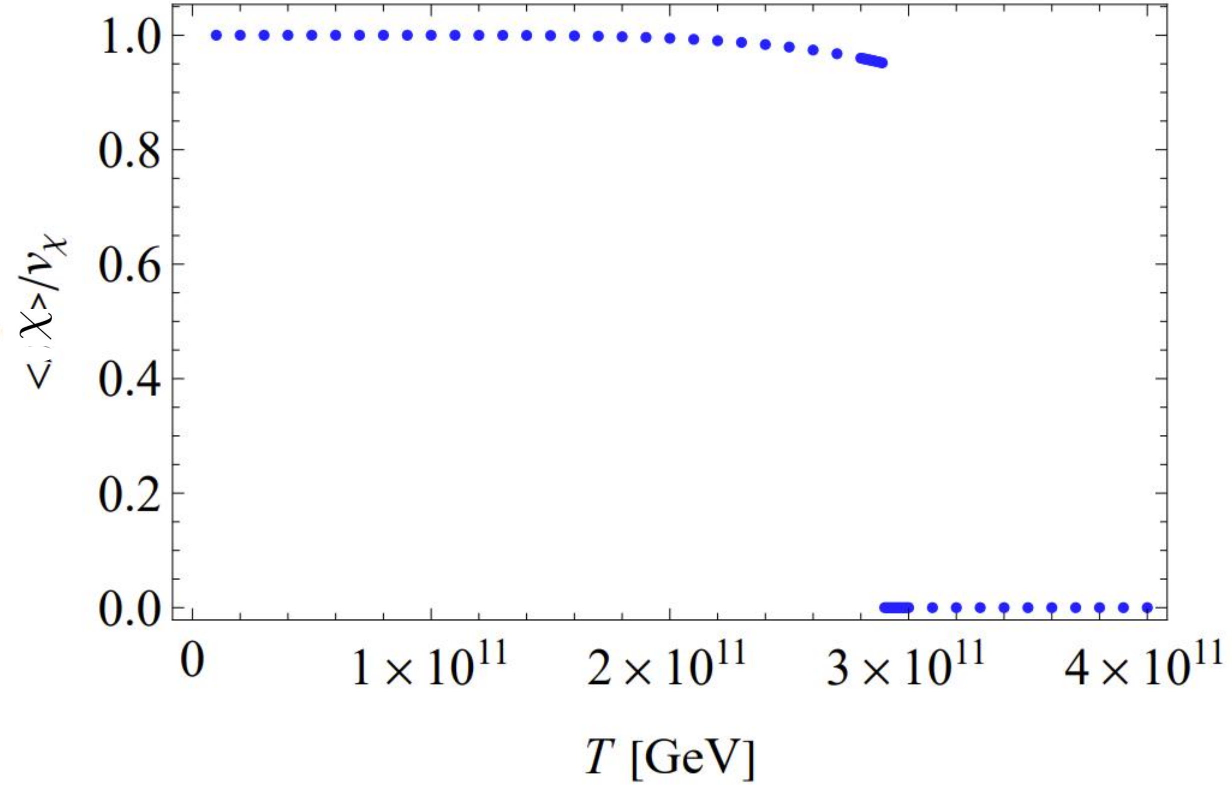
$$J_B(X^2) \equiv \sum_{a=0}^{N_f^2-1}\int_0^\infty x^2\ln\left(1-e^{-\sqrt{x^2+X^2}}\right)\mathrm{d}x,$$

$$\mathcal{M}_{s^i}^2(\chi,T) = m_{s^i}^2(\chi) + \frac{T^2}{6}\left((N_f^2+1)\lambda_1 + 2N_f\lambda_2\right)\Big|_{\lambda_1=-\lambda_2/N_f}$$

Thermal phase transition



C.f. Fig. Schematic potential deformation for 1st order phase transition



Ultra-supercooling 1st order phase transition

$$\frac{v_\chi(T_c)}{T_c} \simeq 5400 \gg 1$$

Trapping mechanism

The inflaton keeps being trapped but gets shifted with the false vacuum, depending on T !

The probability of bubble nucleation rate per unit volume per unit time is:

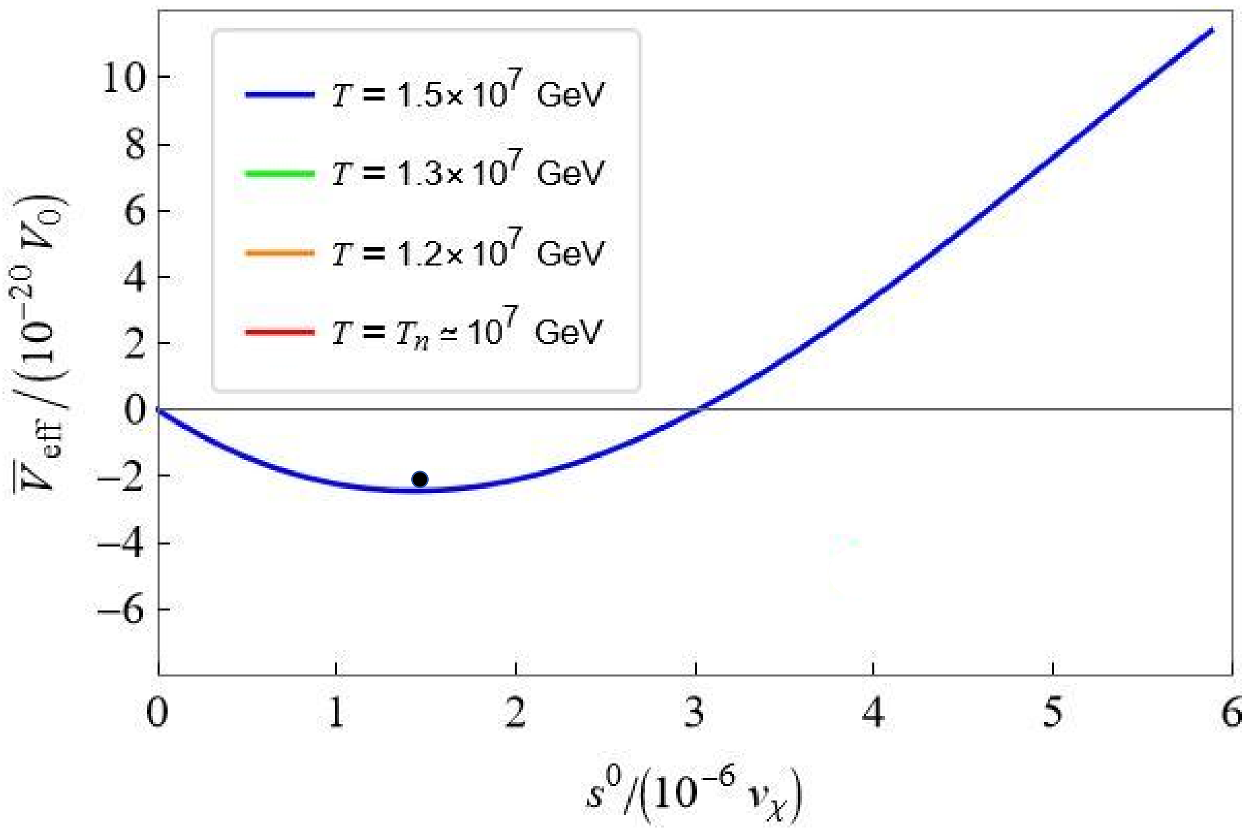
$$\Gamma(T) \simeq T^4 \left(\frac{S_3/T}{2\pi}\right)^{3/2} \exp\left(-\frac{S_3(T)}{T}\right)$$

The nucleation temperature is defined as:

$$\frac{\Gamma(T_n)}{H(T_n)^4} \sim 1 \Rightarrow \frac{S_3(T_n)}{T_n} \sim 100$$

Analytic approximations for bubble action:

$$\frac{S_3(T_n)}{T_n} \simeq \frac{37.794\pi^2}{\sqrt{6}} \frac{N_f^{3/2}}{\lambda_2^{3/2}(N_f^2-1)^{1/2}} \frac{1}{\ln(\mu_{\rm GW}/T_n)}$$



So, the nucleation temperature is exponentially suppressed by the extremely tiny coupling. Further numerical analysis shows that

the tunneling never happens and the inflaton *keeps being trapped* until the barrier almost vanishes!

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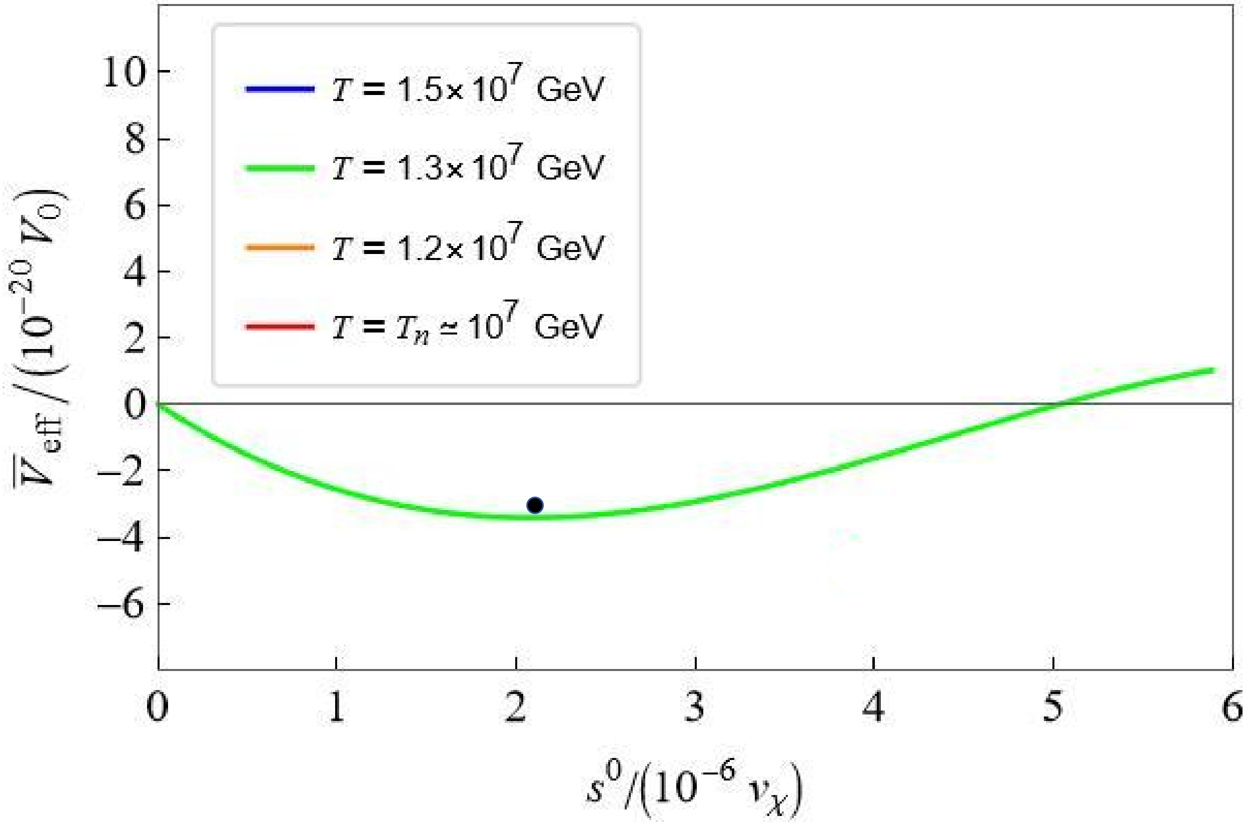

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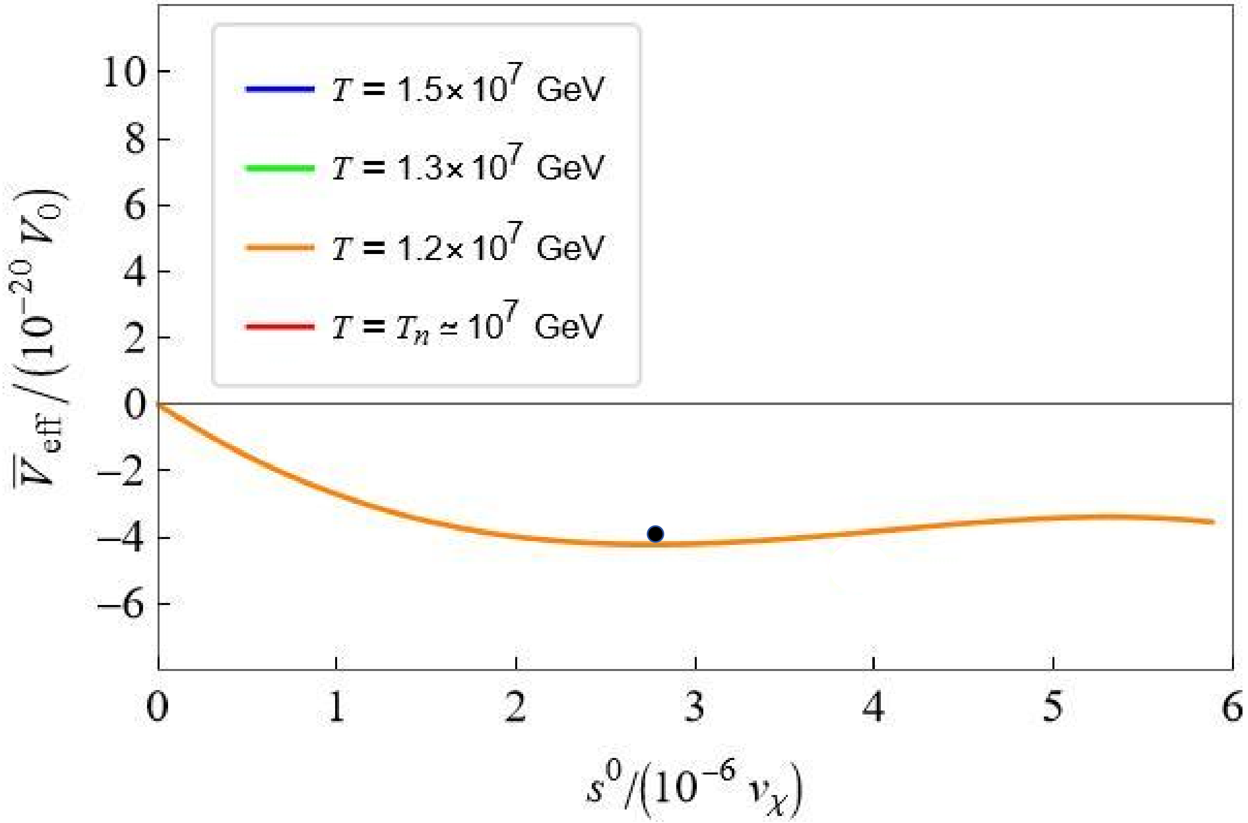

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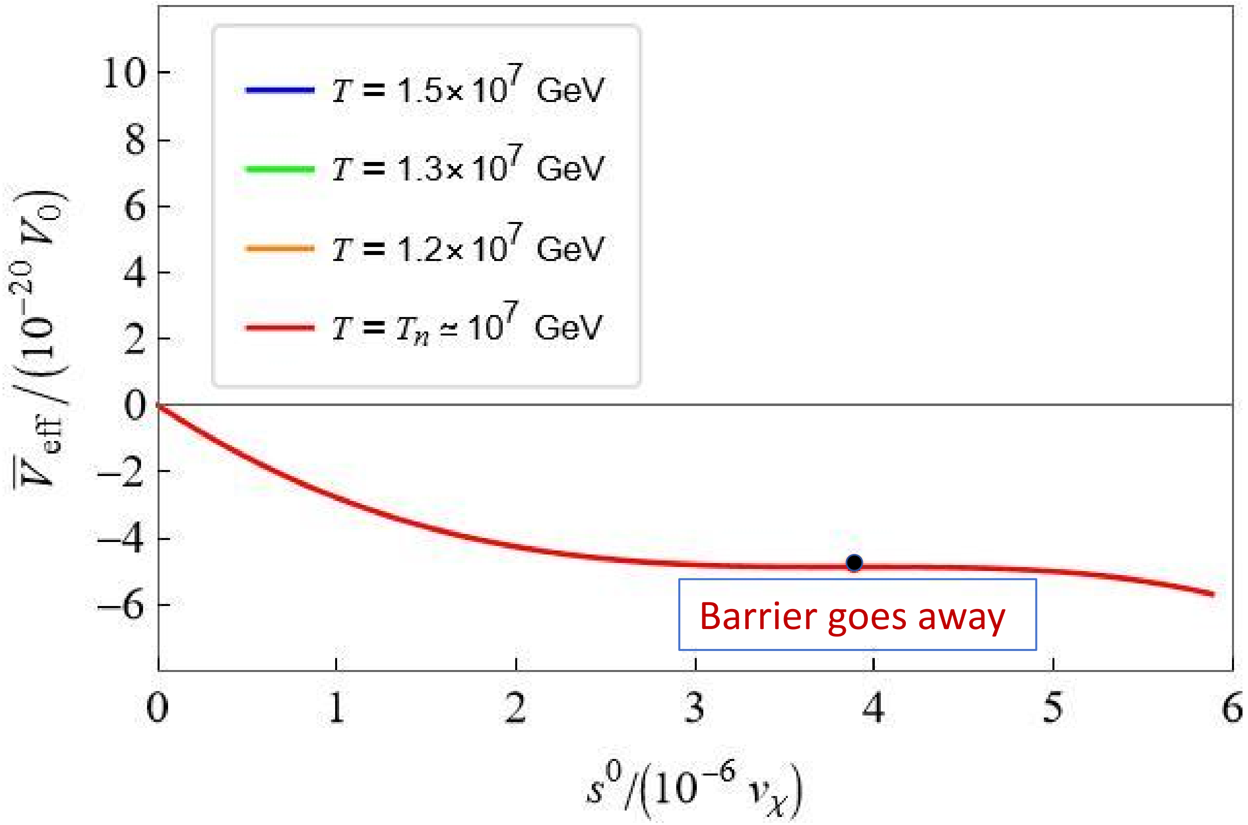

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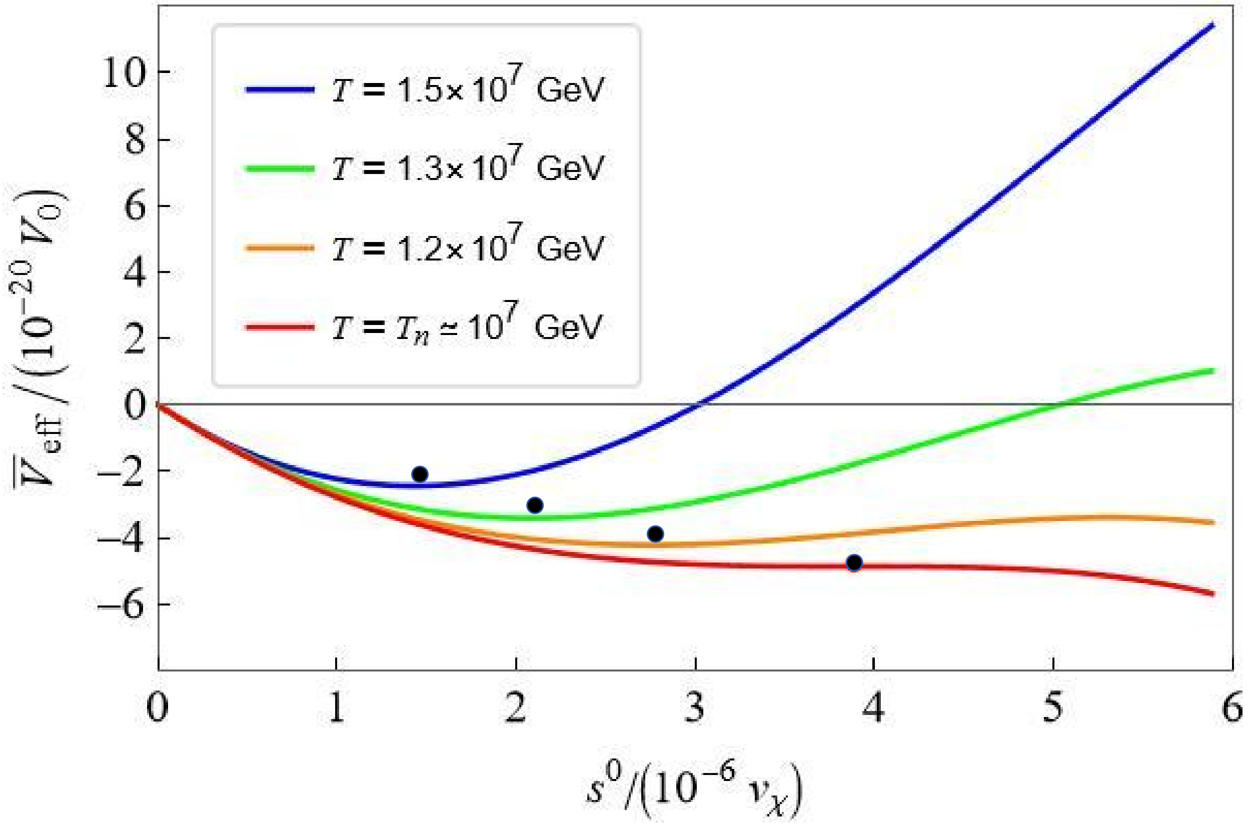

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Analyses & results

The inflationary history goes like:

- Initial thermal corrections traps dilaton in the false vacuum.
- When $T < T_c$, the universe undergoes a supercooling period until the barrier vanishes at $T = T_n$.
- The inflaton rolls down the potential and the universe gets into the slow-roll phase.

Based on this bechmark parameters (compatible with [Ishida, Matsuzaki(2020)]):

$$N_c = 3\,, \qquad N_f = 8\,, \qquad v_\chi = 1.7 \times 10^{15}\,\mathrm{GeV}\,,$$
$$m_F = 6.2 \times 10^{11}\,\mathrm{GeV}\,, \qquad m_\pi = 1.1 \times 10^{5}\,\mathrm{GeV}\,,$$

we *dynamically* fixed the initial conditions of the inflaton $\chi_{\rm ini} \sim 7 \times 10^{9}\mathrm{GeV}\,.$

Conclusions

- Large hierarchy is explained by the walking behavior in large N_F QCD.
- Dilaton arising from the spontaneous breaking of (approximate) scale symmetry is regarded as inflaton.
- Thermal corrections traps the inflaton in the false vacuum and thus dynamically solved the fine-tuning problem in small field inflation.
- We gave the bechmark parameters consistent with observation.
- Similar supercooling dynamical trapping mechanism could be applicable to other types of small field inflation or initial condition for preheating models.

Thanks for your attention!