Quark masses and low energy constants in the continuum from CLQCD ensembles

Yi-Bo Yang

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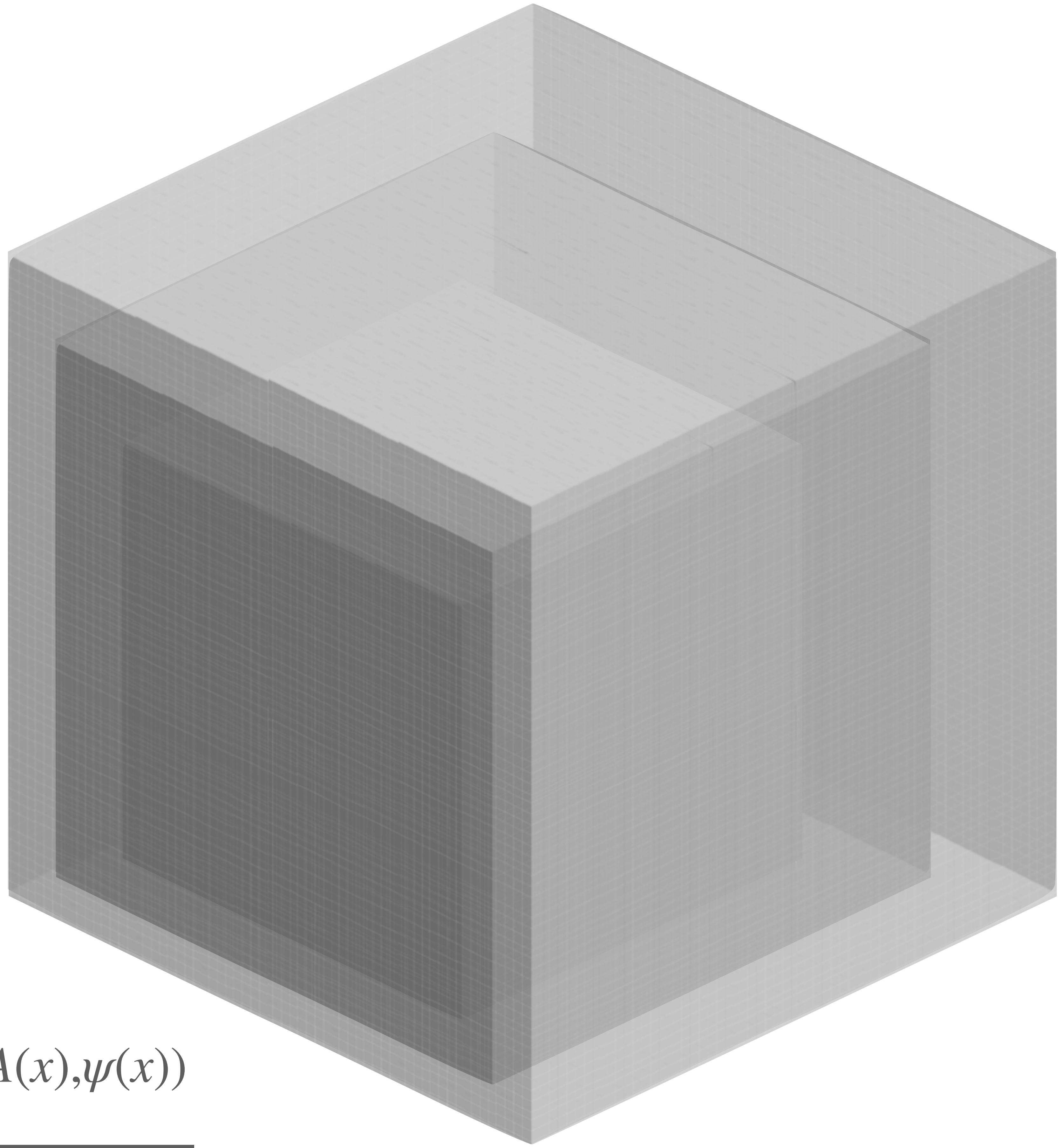
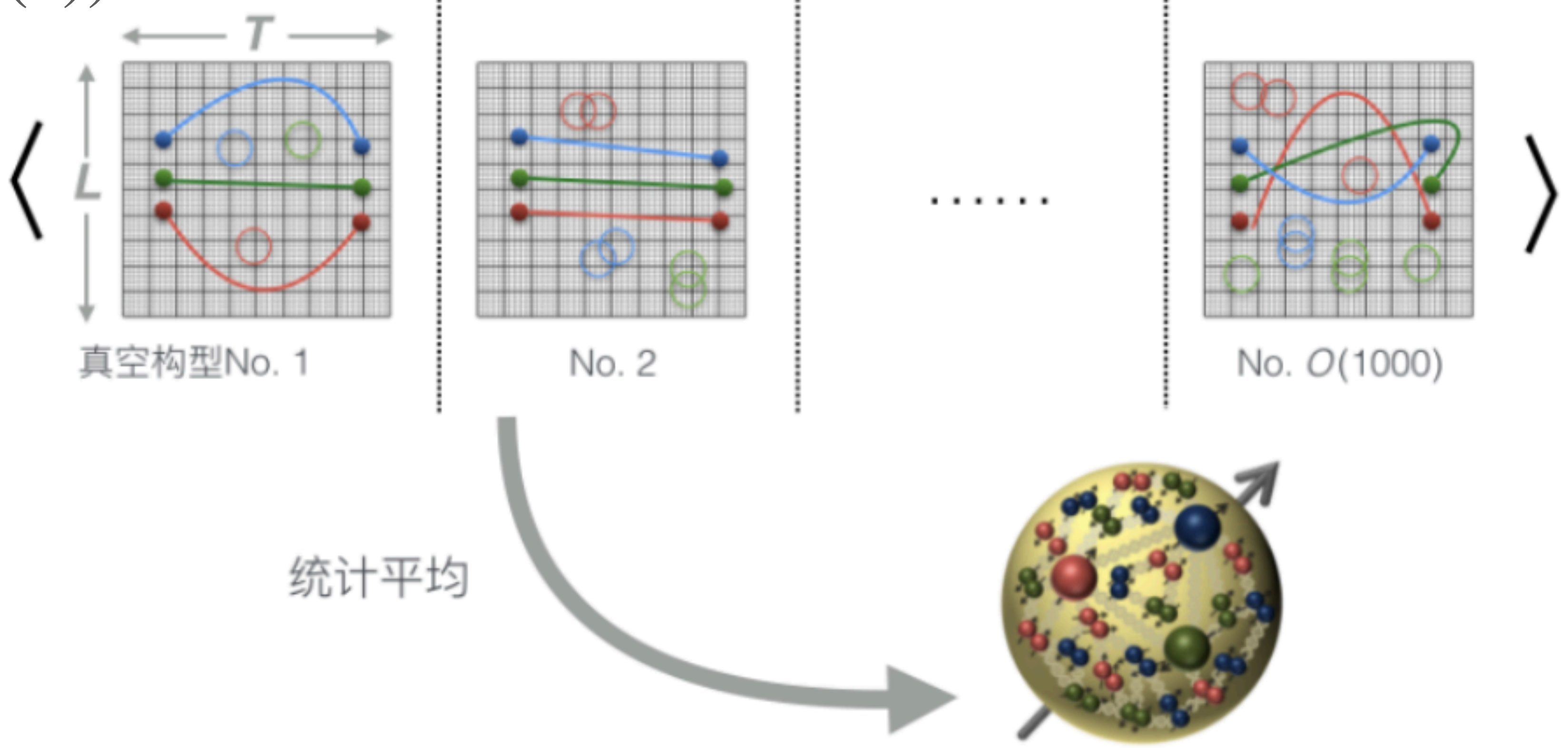
Basic idea of Lattice QCD



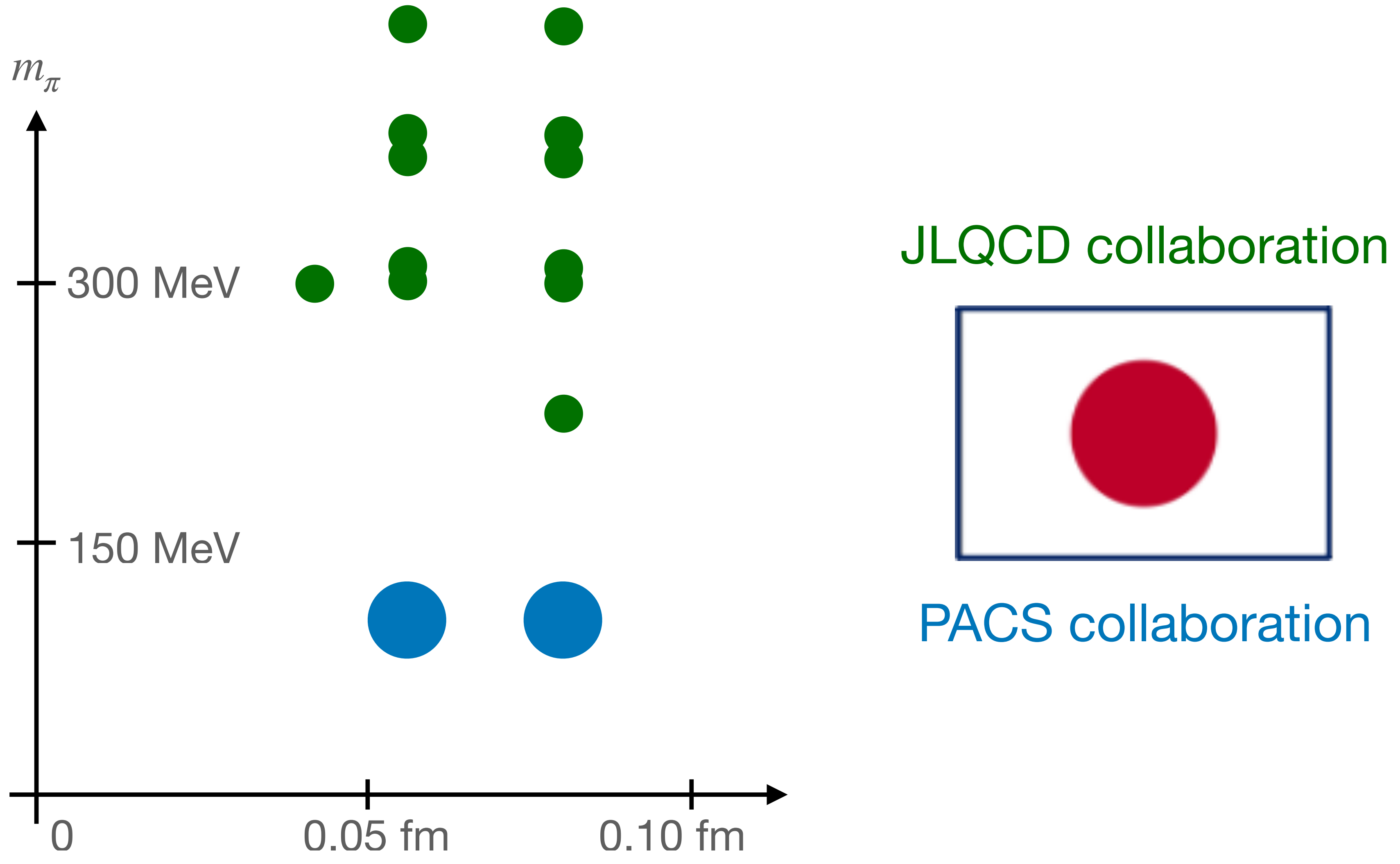
- Discretize the Euclidean space-time into a 4-D lattice with finite size and lattice spacing；
- Sample the QCD path integral with the weights from the QCD action；

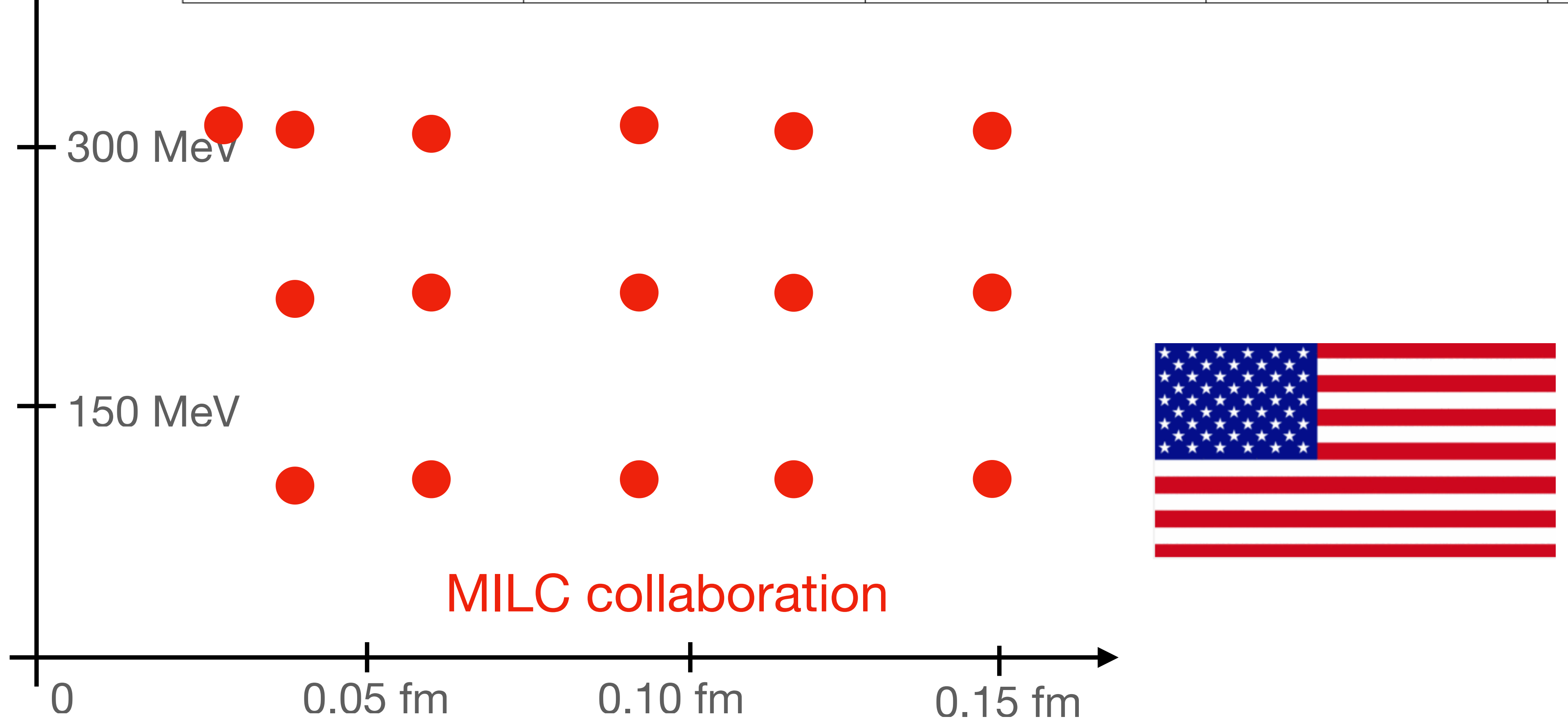
$$\langle \mathcal{O} \rangle = \frac{\int [\mathrm{d}A\mathrm{d}\psi]\mathcal{O}(A,\psi)e^{-\int \mathrm{d}^4x\mathscr{L}(A(x),\psi(x))}}{\int [\mathrm{d}A\mathrm{d}\psi]e^{-\int \mathrm{d}^4x\mathscr{L}(A(x),\psi(x))}}$$

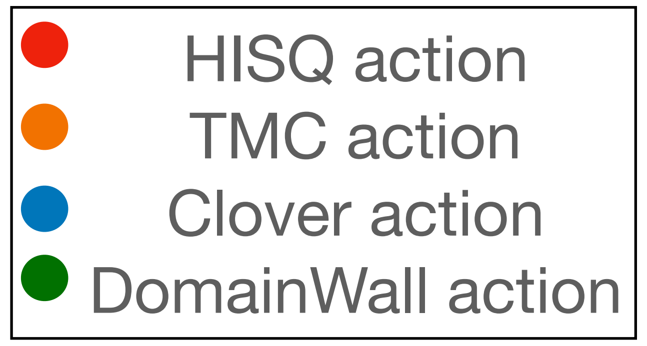
- Repeat the calculation at different lattice spacing and volume, and then obtain the result in the continuum/infinite-volume limits.

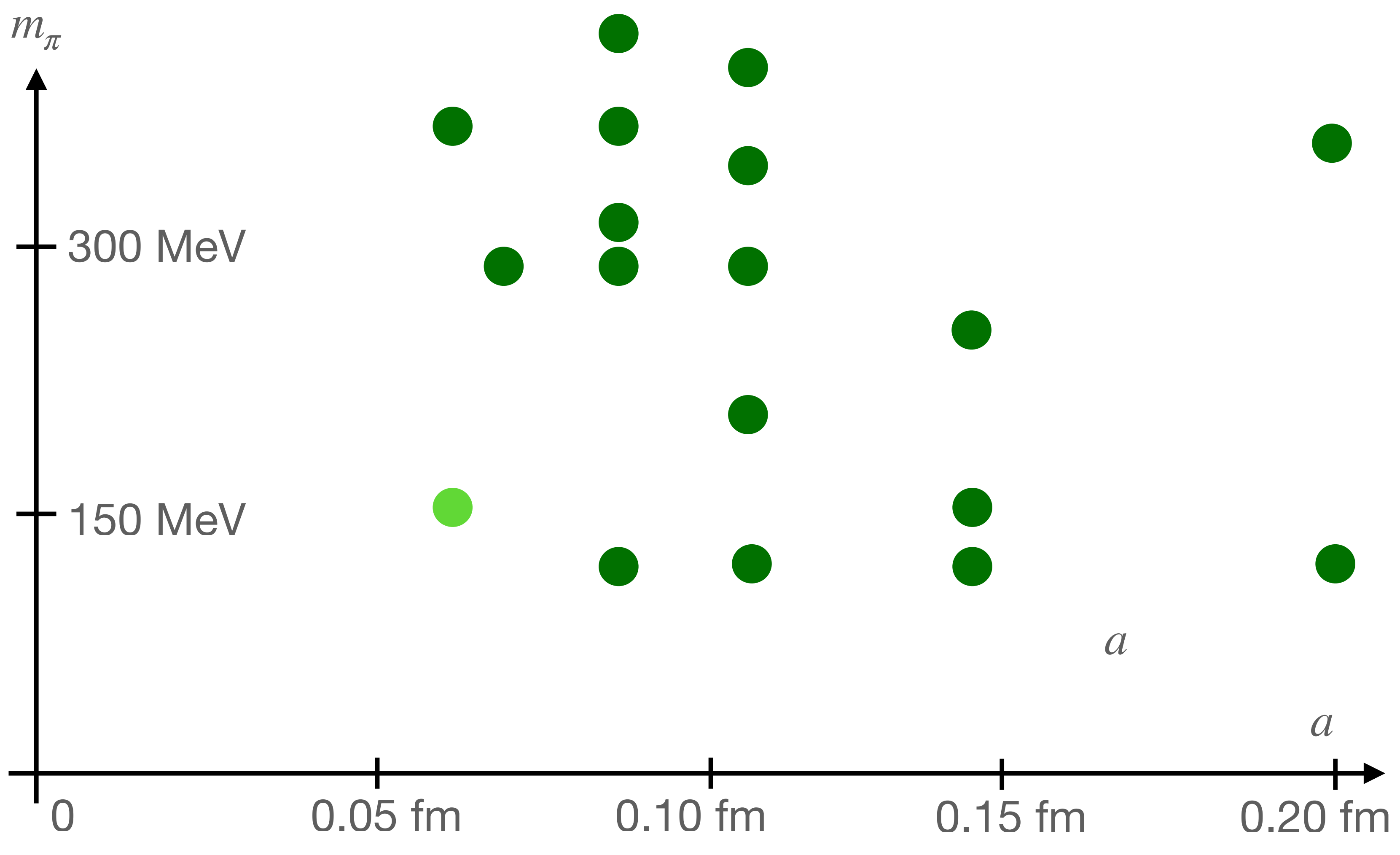


	Naive	Staggered/HISQ	Wilson/Clover	Twisted-mass	Overlap/Domain wall
Form	$D^{\text{naive}} = \gamma_\mu(\delta_{x,x+\mu} - \delta_{x,x-\mu})$	$D^{\text{st}} = \gamma_\mu^{\text{st}}(x)(\delta_{x,x+\mu} - \delta_{x,x-\mu})$	$D^{\text{clv}} = D + aD^2 + ac_{sw}F_{\mu\nu}\sigma^{\mu\nu}$	$D^{\text{tm}} = D^{\text{clv}} + i\tau_3 m$	$D^{\text{ov}} = [1+\gamma_5 D(-\rho)/\sqrt{D^\dagger(-\rho)D(-\rho)}]/\rho$
Fermion copies	16	4	1	1	1
Chiral symmetry breaking	N/A	$\mathcal{O}(a^4)$	$\mathcal{O}(\alpha_s/a)$	$\mathcal{O}(\alpha_s)$	N/A
Cost	1	~1/4	~1.1	~1.1	~10-100

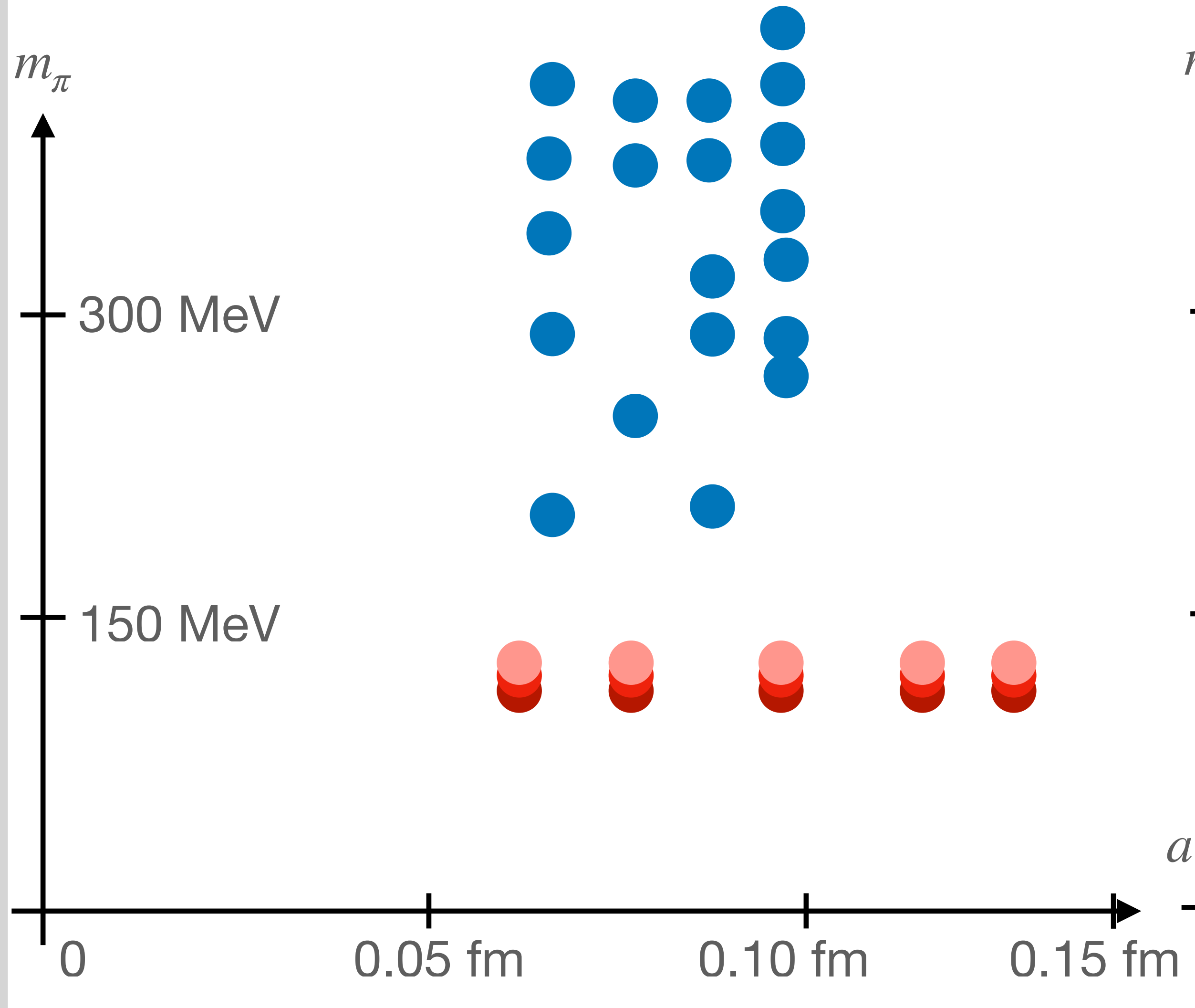




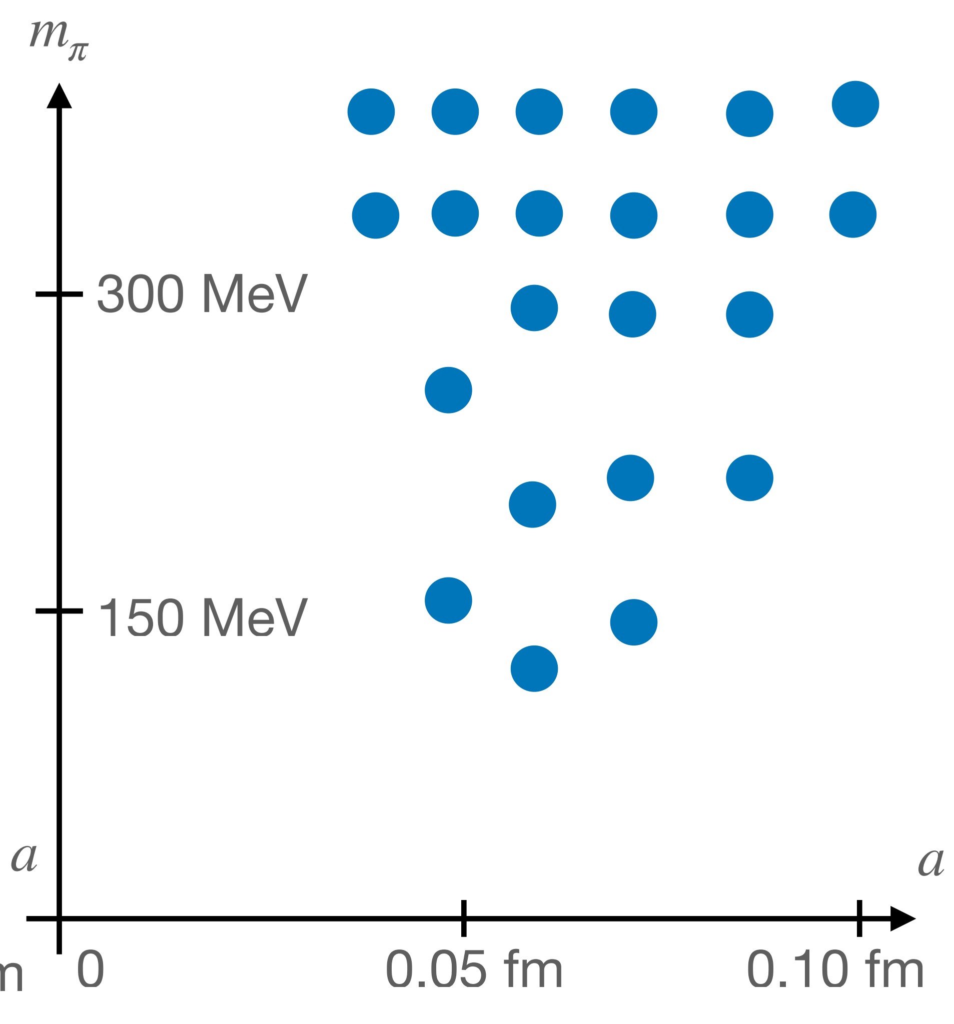




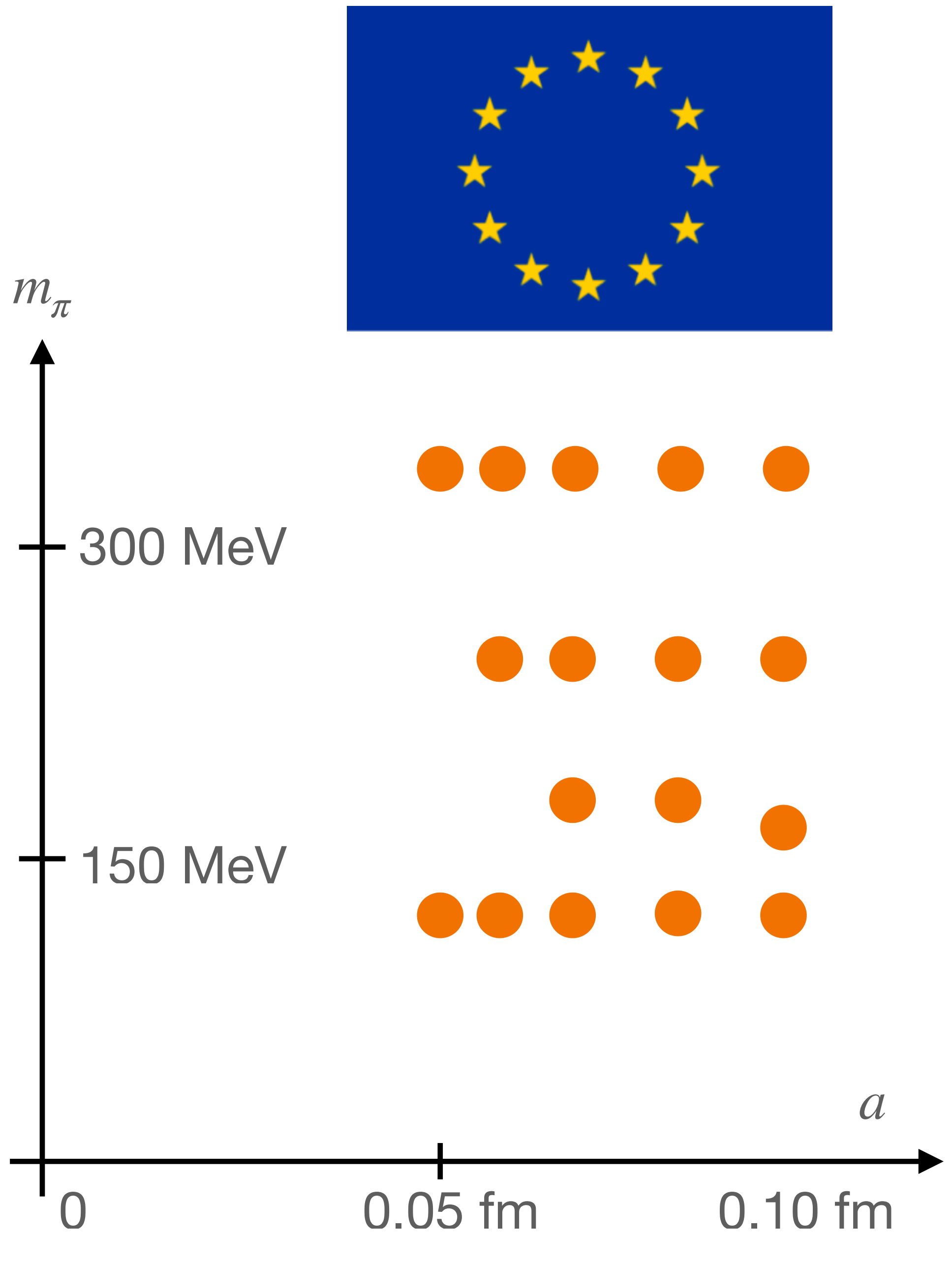
RBC/UKQCD collaboration



BMW collaboration

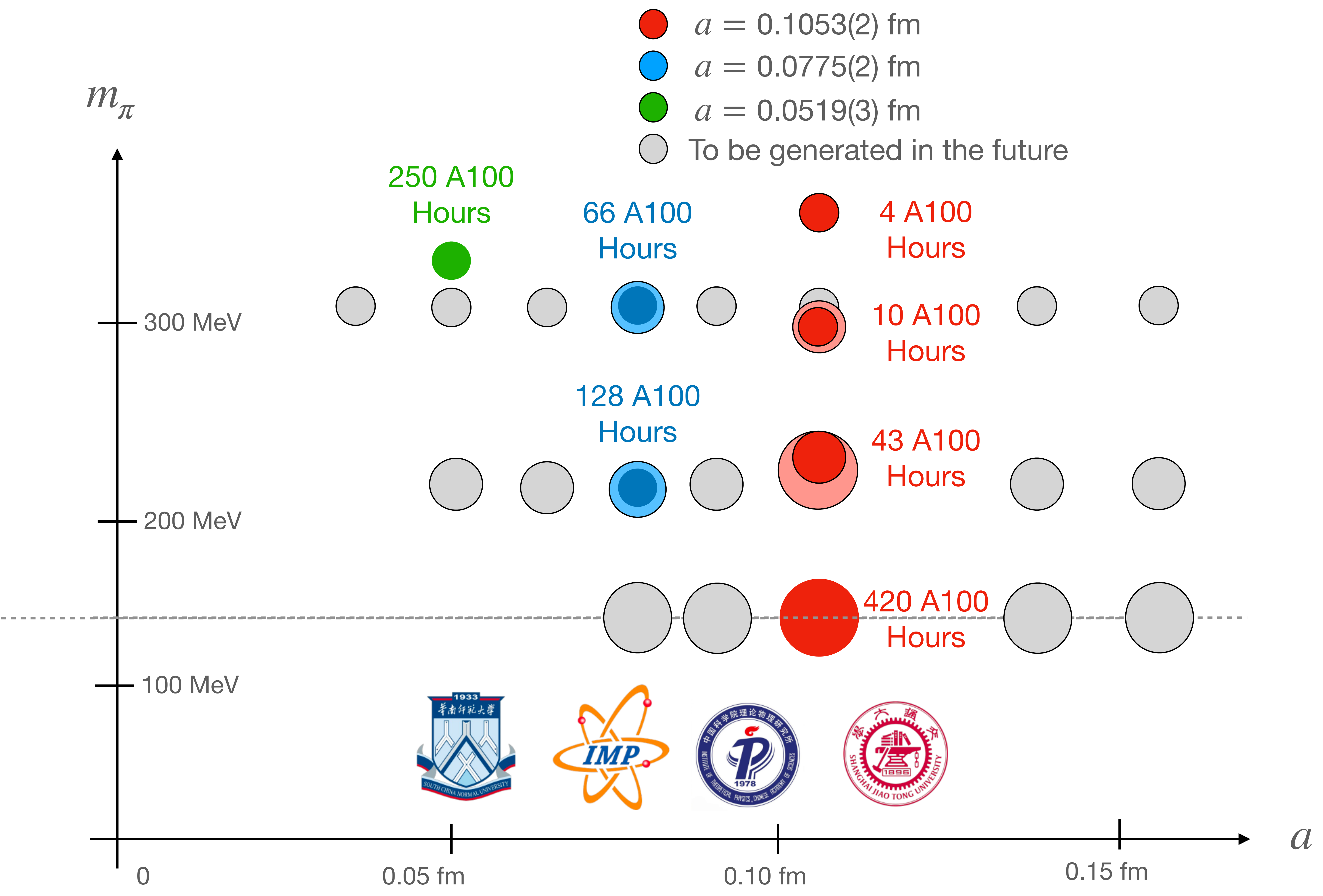


CLS collaboration



ETM collaboration

CLQCD ensembles

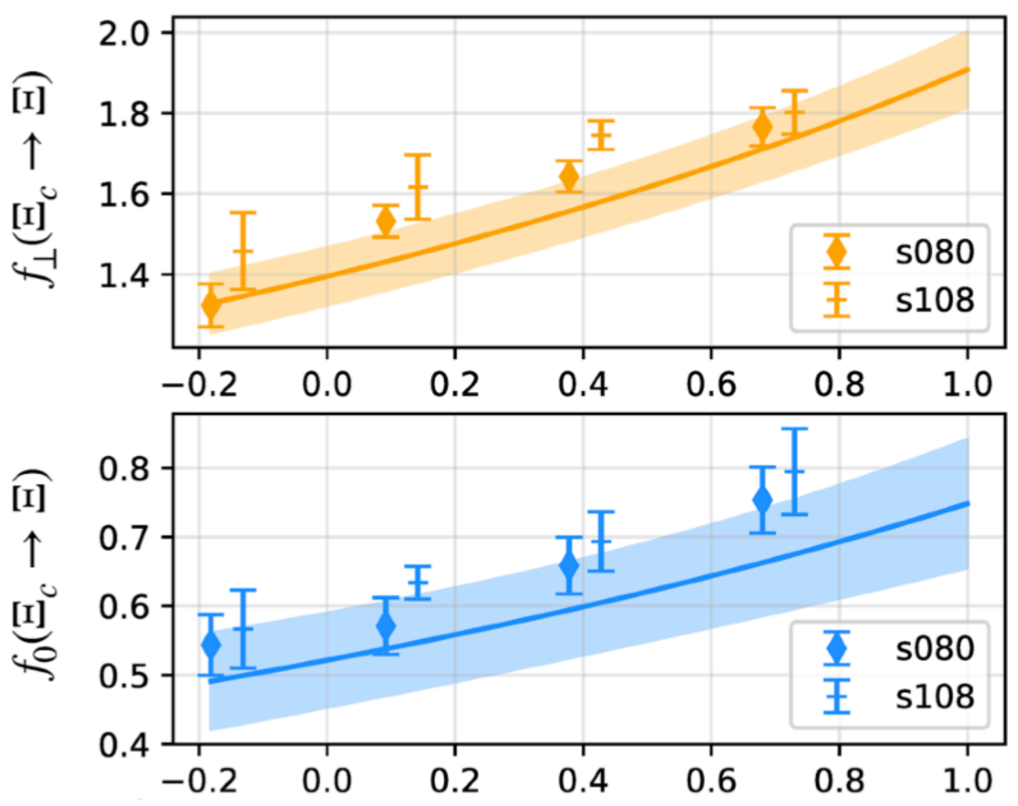


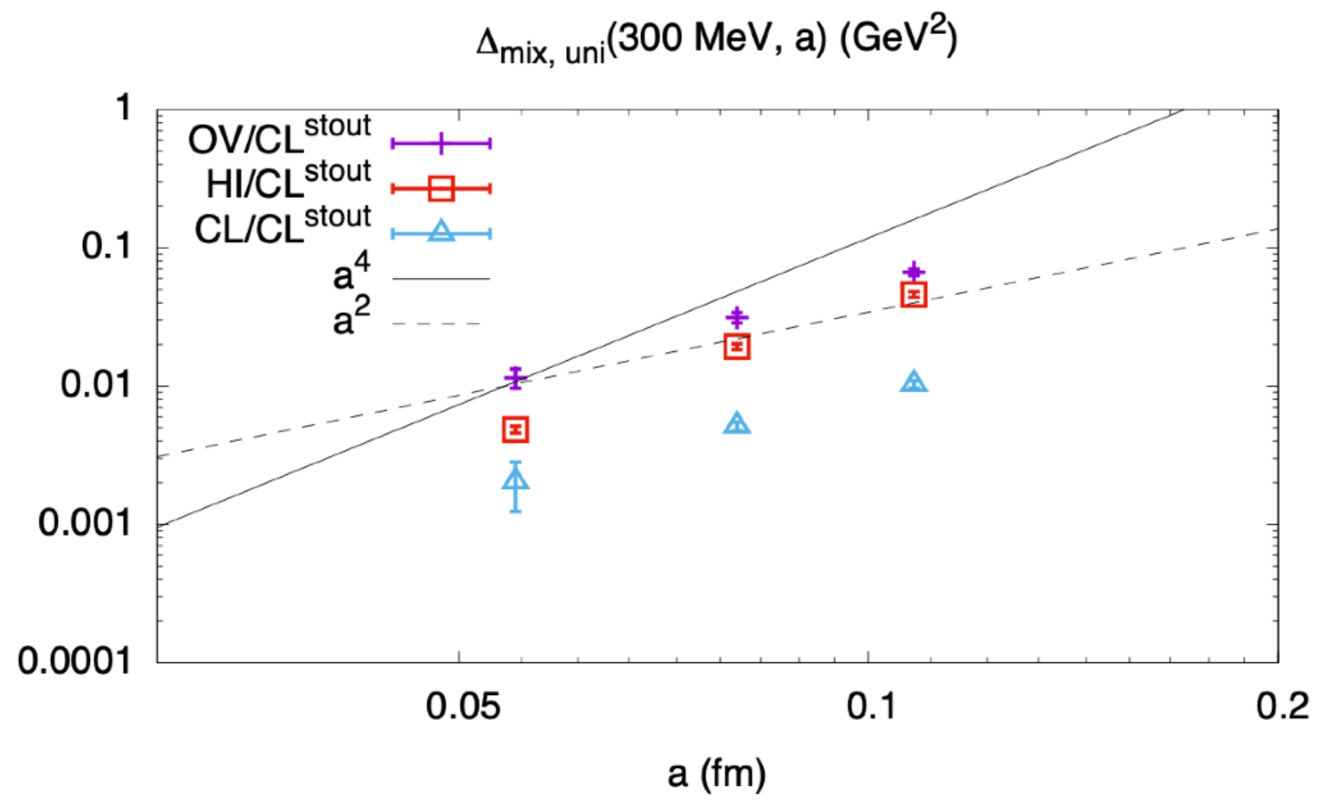
CLQCD choice and informations

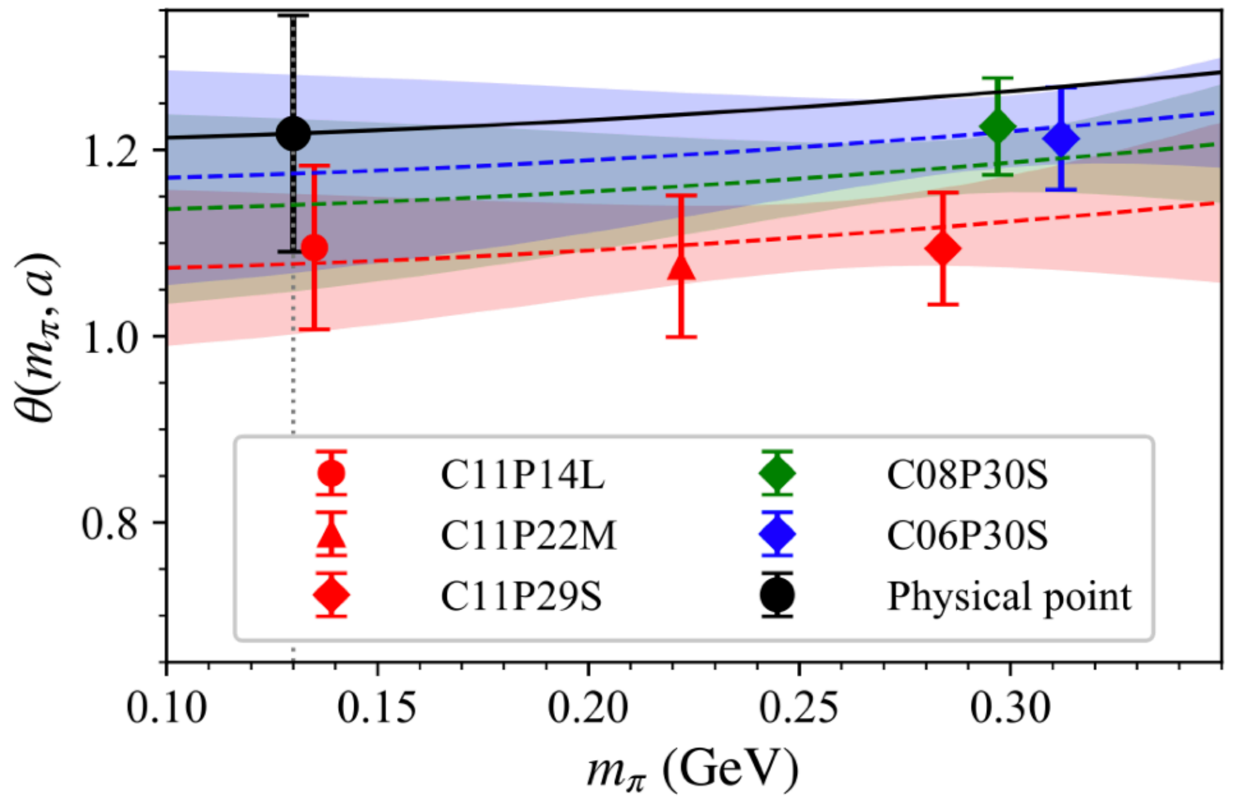
- Features:
- Maximum lattice size $48^3 \times 144$,
- Clover fermion action with stout smearing,
- Similar pion mass and volume at different lattice spacing:
- Cost:
- That of an independent configuration (per 10 traj.'s with $\tau = 1.0$, converted to A100 GPU hours) is shown on the figure;
- Needs ~1,000 configurations per ensemble;
- Currently used 658k A100 hours, equals to 3.3M Chinese Yuan with the market price.
- Working on the Sugon machines to avoid the embargo of A100 GPU.

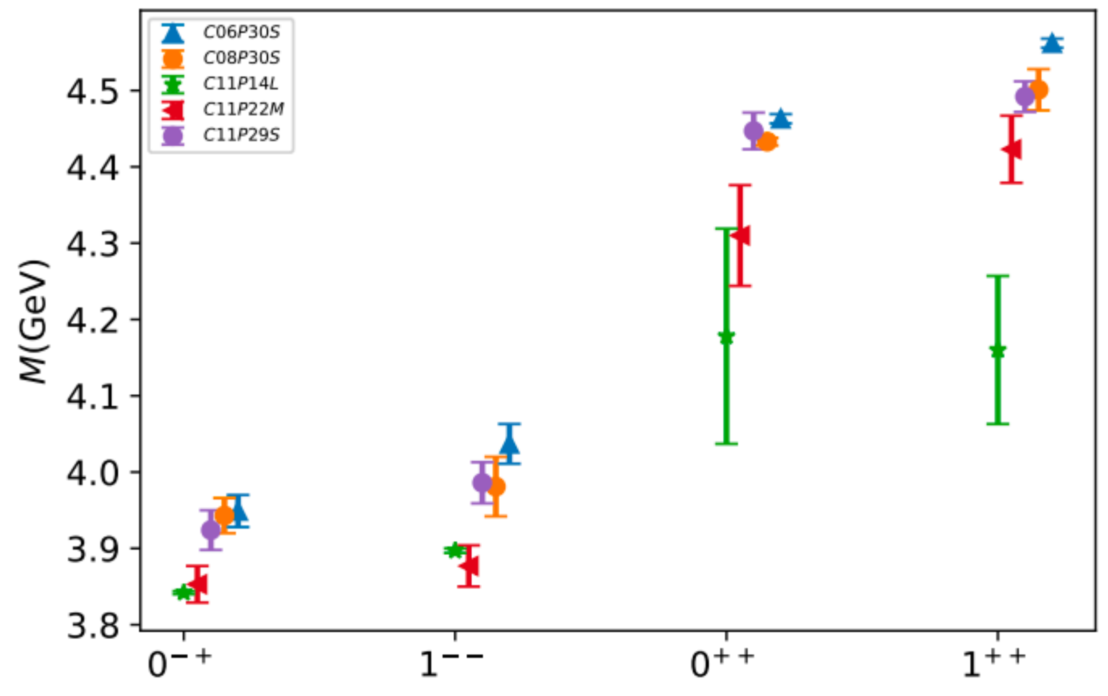
CLQCD ensembles

Published/accepted works with the CLQCD ensembles

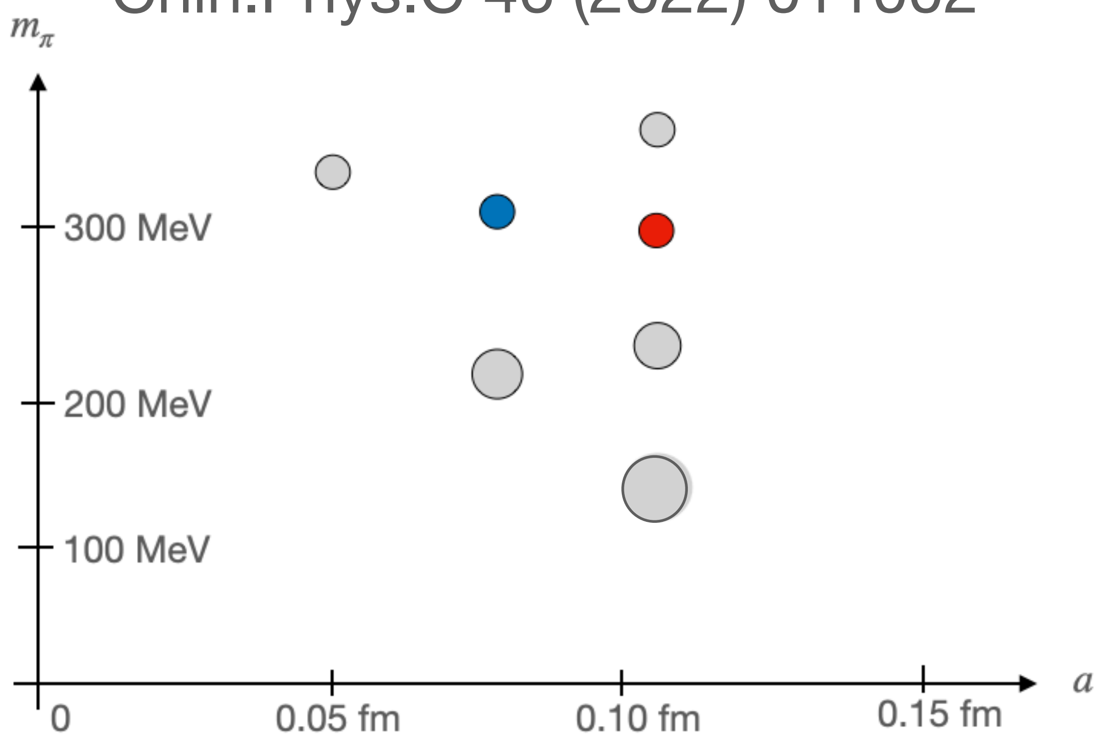




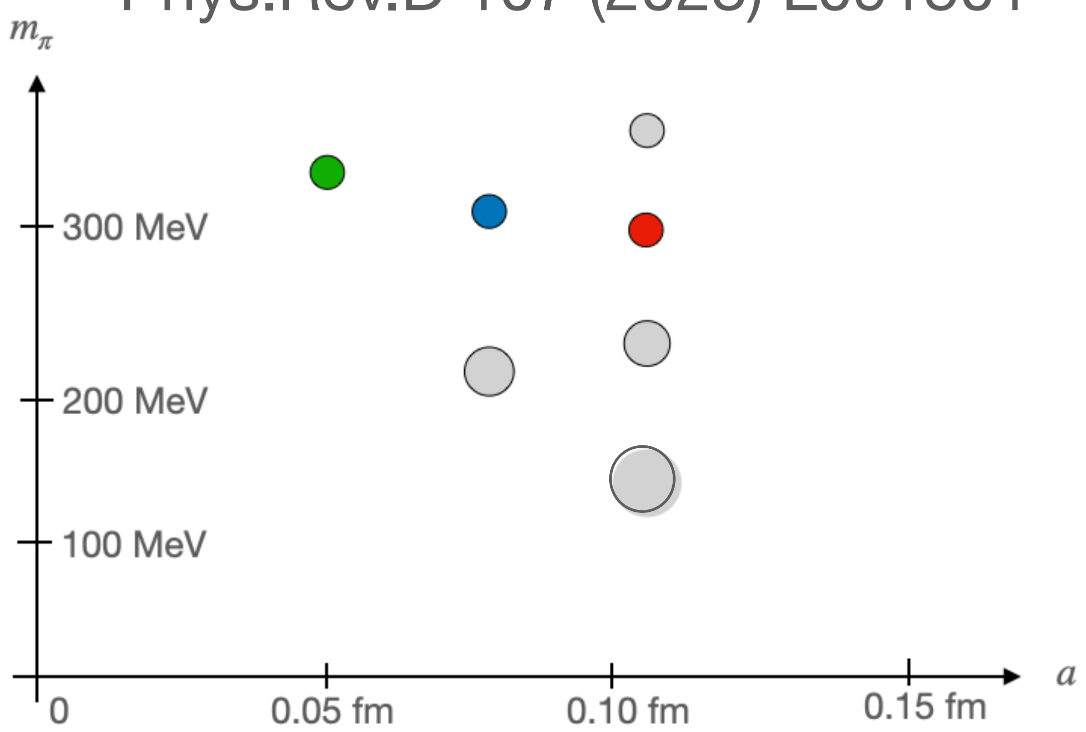




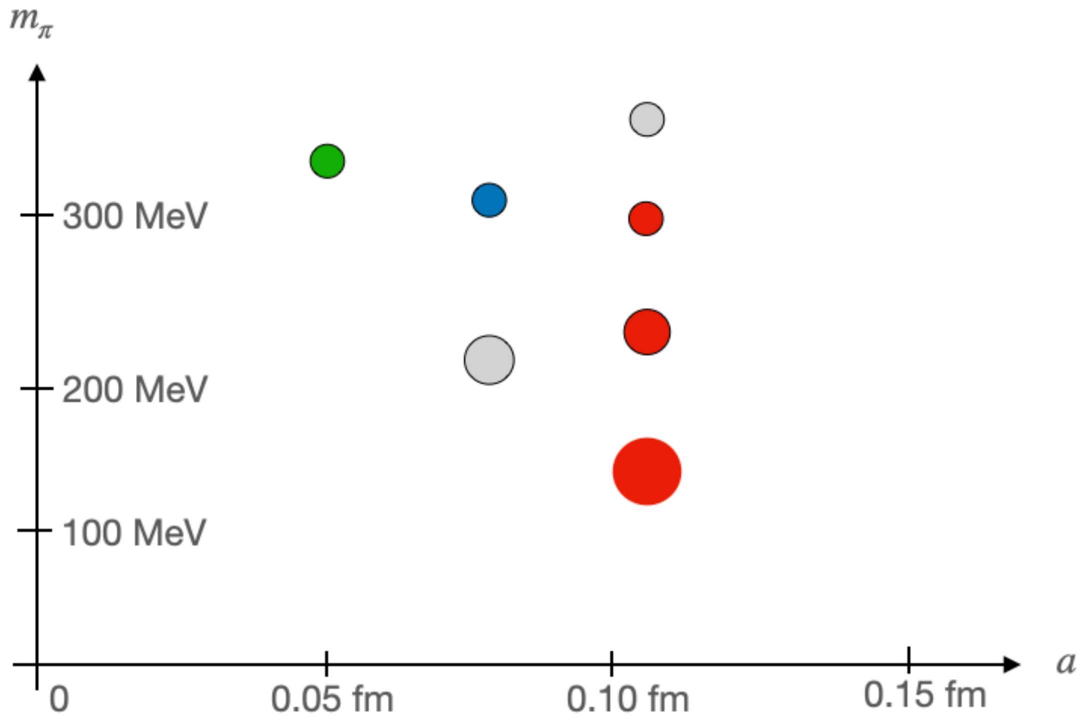
Q.A. Zhang, et.al.,
Chin.Phys.C 46 (2022) 011002



D.J. Zhao, et.al., χQCD,
Phys.Rev.D 107 (2023) L091501

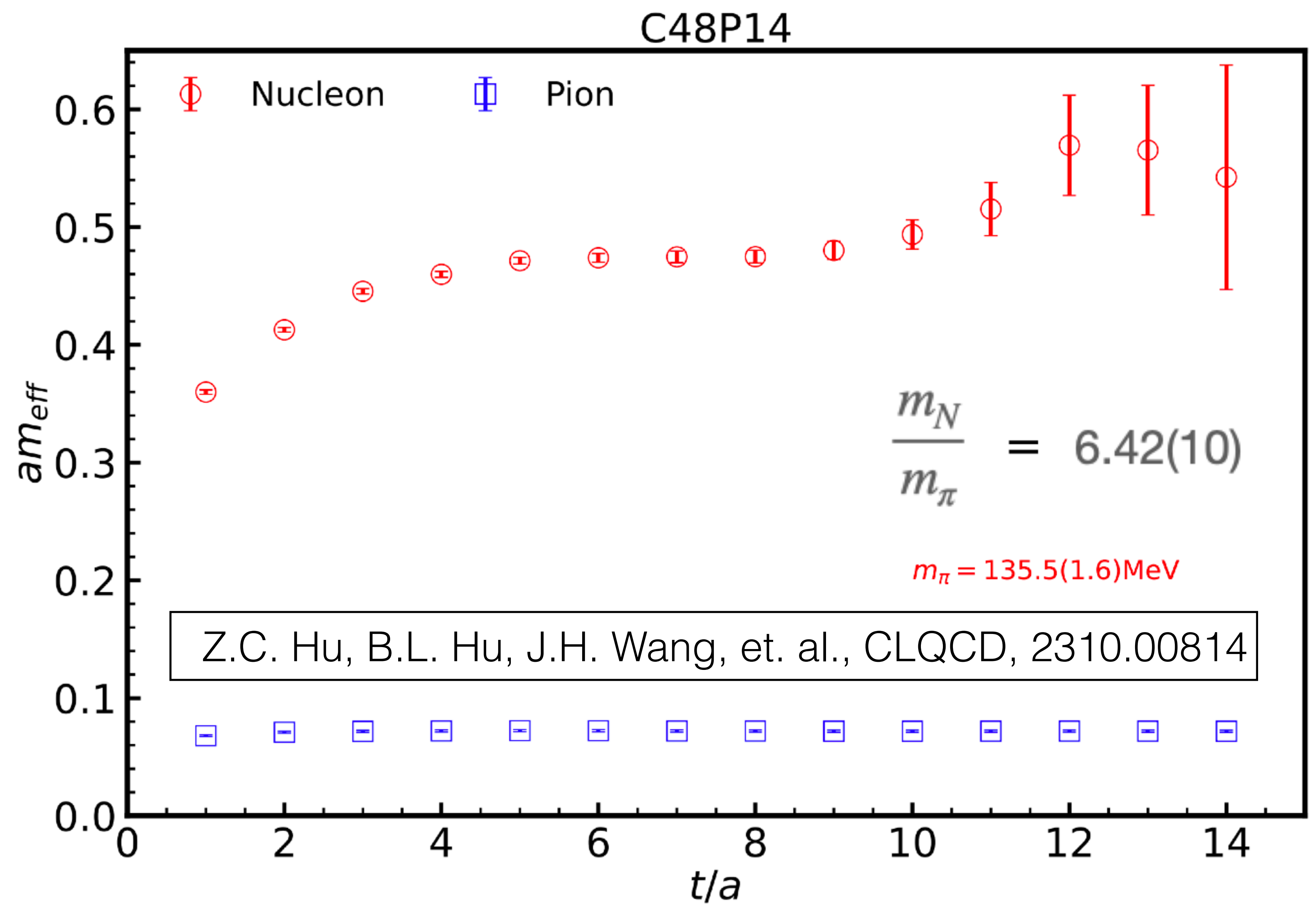


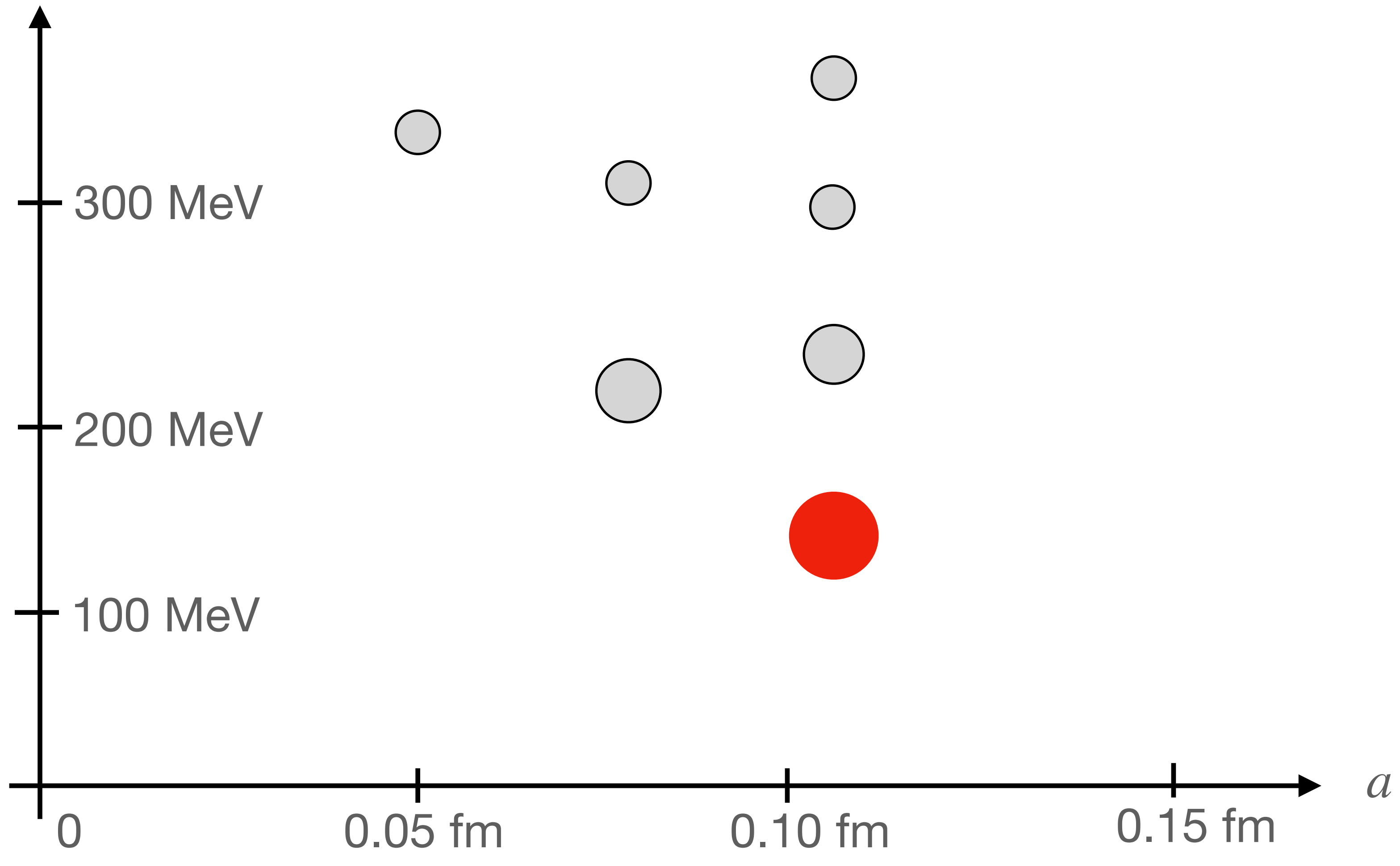
Q.A. Zhang, et.al.,
Phys.Lett.B 841 (2023) 137941



H. Liu, et.al.,
2207.00183, accepted by SCPMA

CLQCD ensembles





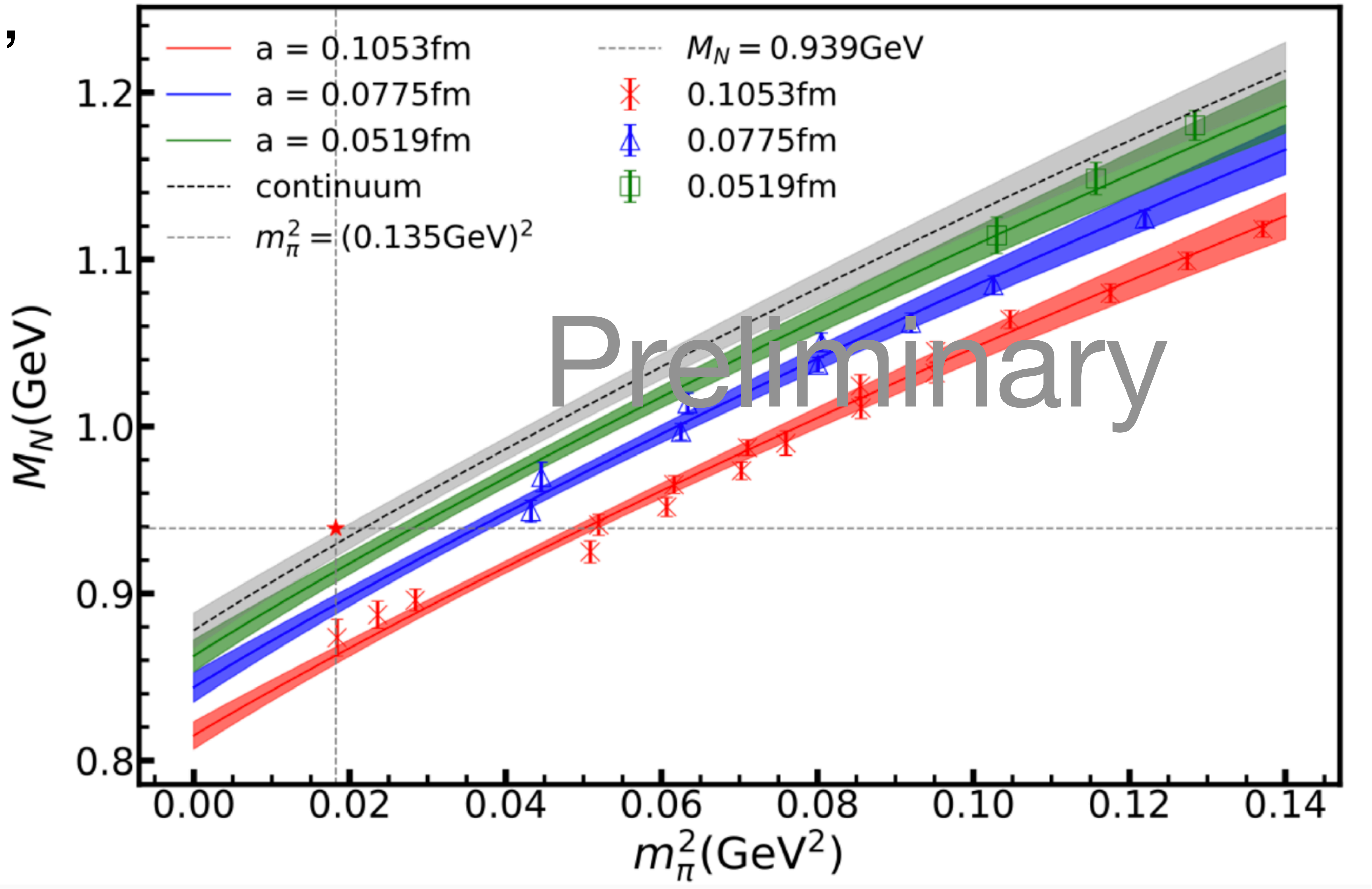
Nucleon mass v.s. pion mass

- At $a = 0.105(3)$ fm, we have

$$m_\pi = 135.5(1.6)\ \mathrm{MeV},$$

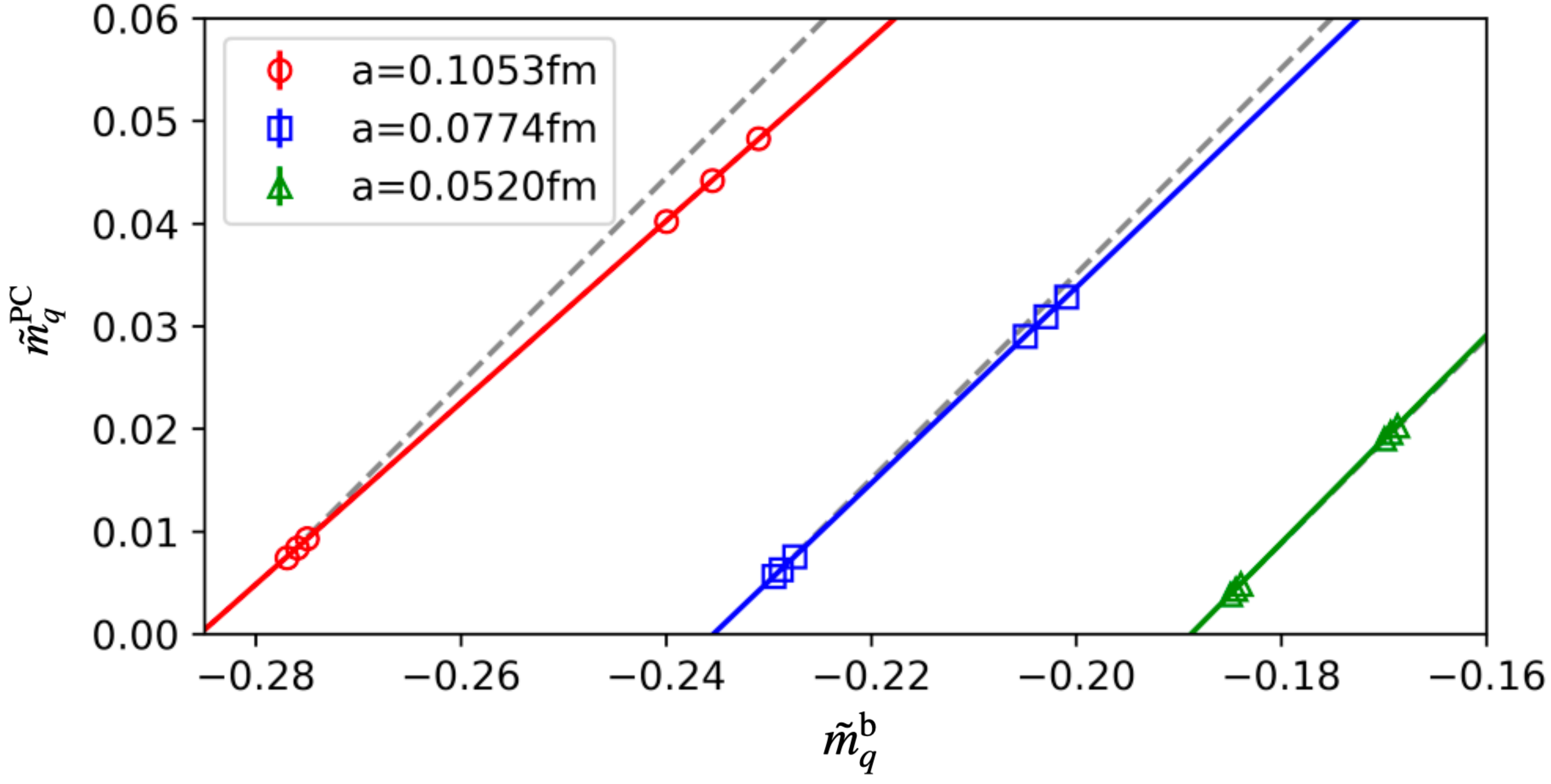
$$m_N = 870(12)\ \mathrm{MeV}.$$

- m_N are ~7% smaller than the physical value, and can reach the physical value after the continuum extrapolation.



Background

Chiral symmetry breaking in the clover fermion



Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

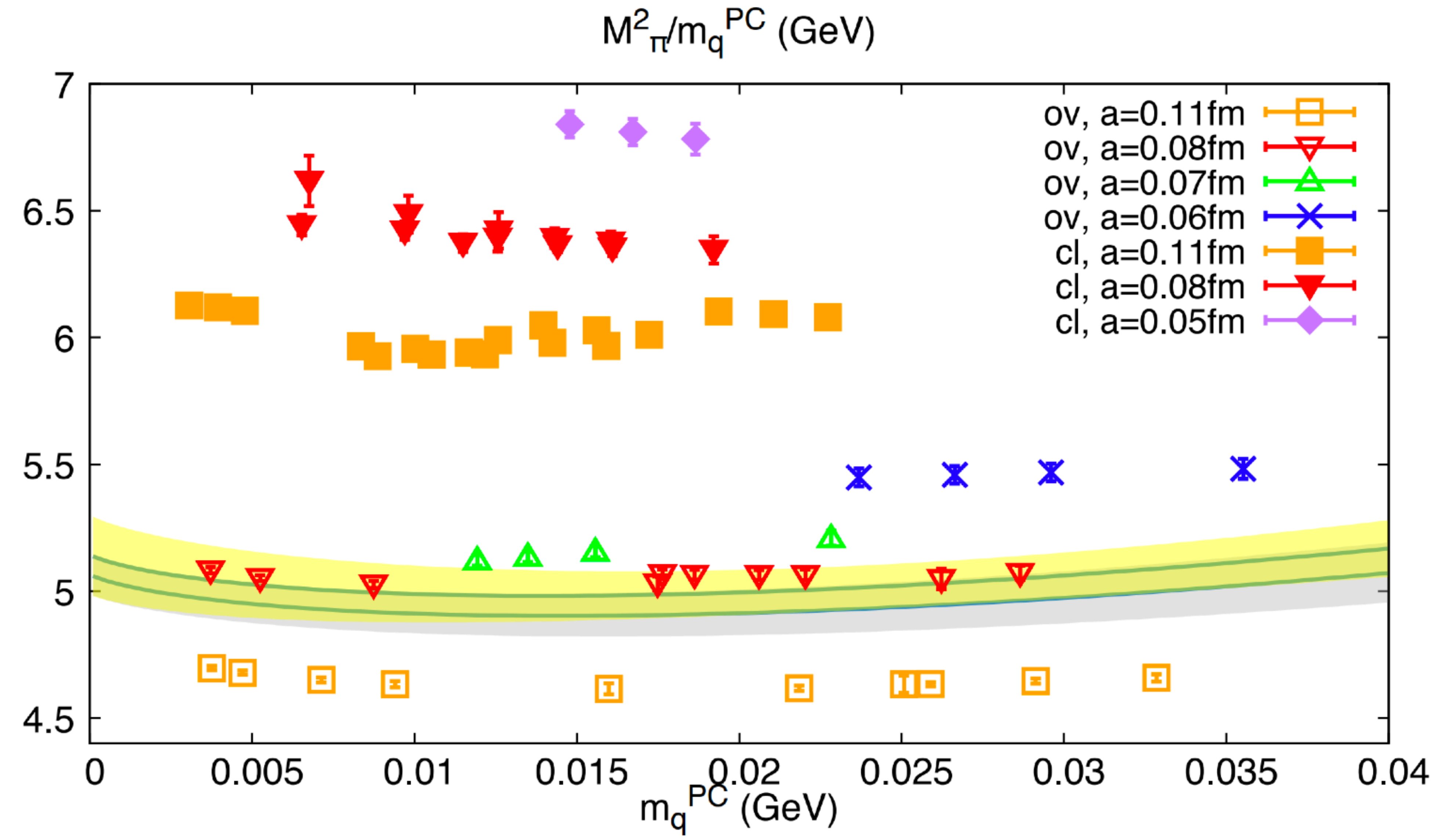
- Due to the additive α_s/a correction, the dimensionless bare quark mass $\tilde{m}_q^{\mathrm{b}} = m_q^{\mathrm{b}} a$ is negative.
- The renormalized quark mass should be defined as $m_q^R = Z_m(m_q^{\mathrm{b}} - m_{\mathrm{crti}})$, where m_{crti} is defined as the m_q^{b} which vanishes the pion mass.
- One can avoid this difficulty by defining the quark mass through PCAC relation:

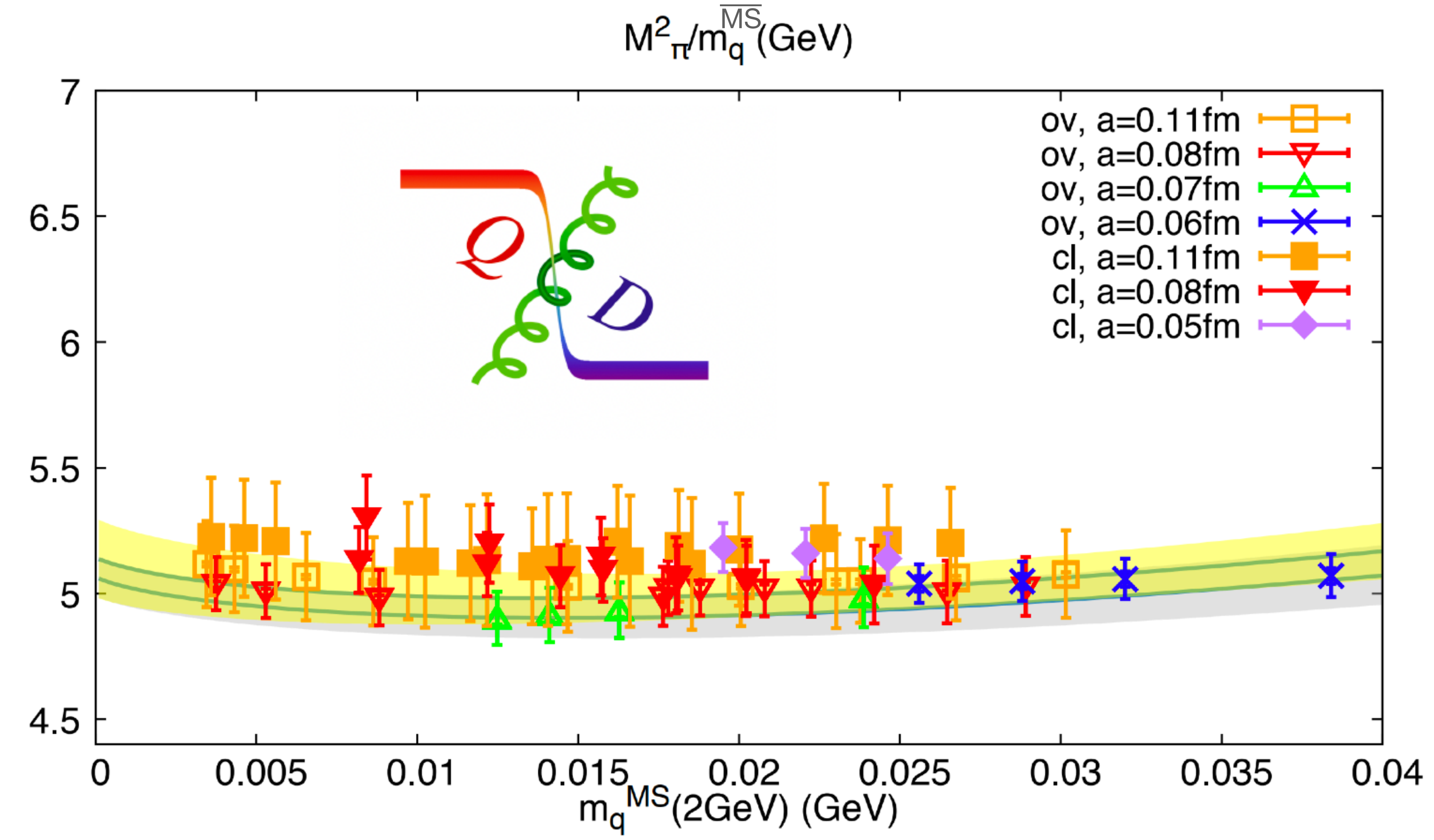
$$\langle 0 | \partial_4 A_4 | \mathrm{PS} \rangle = (m_q^{\mathrm{PC}} + m_{\bar{q}}^{\mathrm{PC}}) \langle 0 | P | \mathrm{PS} \rangle$$

T. Ishikawa, et.al., JLQCD, Phys.Rev.D78 (2008) 011502

- And then m_q^{PC} is always positive and can be renormalized as $m_q^R = Z_P/Z_A m_q^{\mathrm{PC}}$.

Renormalization and final results





Renormalized quark mass

- Non-perturbative renormalization to $\overline{\text{MS}}$ 2 GeV eliminates the regularization scale $1/a$ dependence of m_π^2/m_q.

- m_π^2/m_q using the clover fermion also turns out to be consistent with that using the overlap fermion.

- The large uncertainty of the renormalized m_π^2/m_q majorly comes from the missing higher order effect of the perturbative matching

$$\frac{Z_P^{\overline{\text{MS}}}}{Z_P^{\text{MOM}}} = 1 + 0.4244\alpha_s + 1.007\alpha_s^2 + 2.722\alpha_s^3 + 8.263\alpha_s^4 + \mathcal{O}(\alpha_s^5)$$

$$= \frac{1 - 2.611\alpha_s - 0.2813\alpha_s^2 - 0.3349\alpha_s^3}{1 - 3.036\alpha_s} + \mathcal{O}(\alpha_s^5),$$

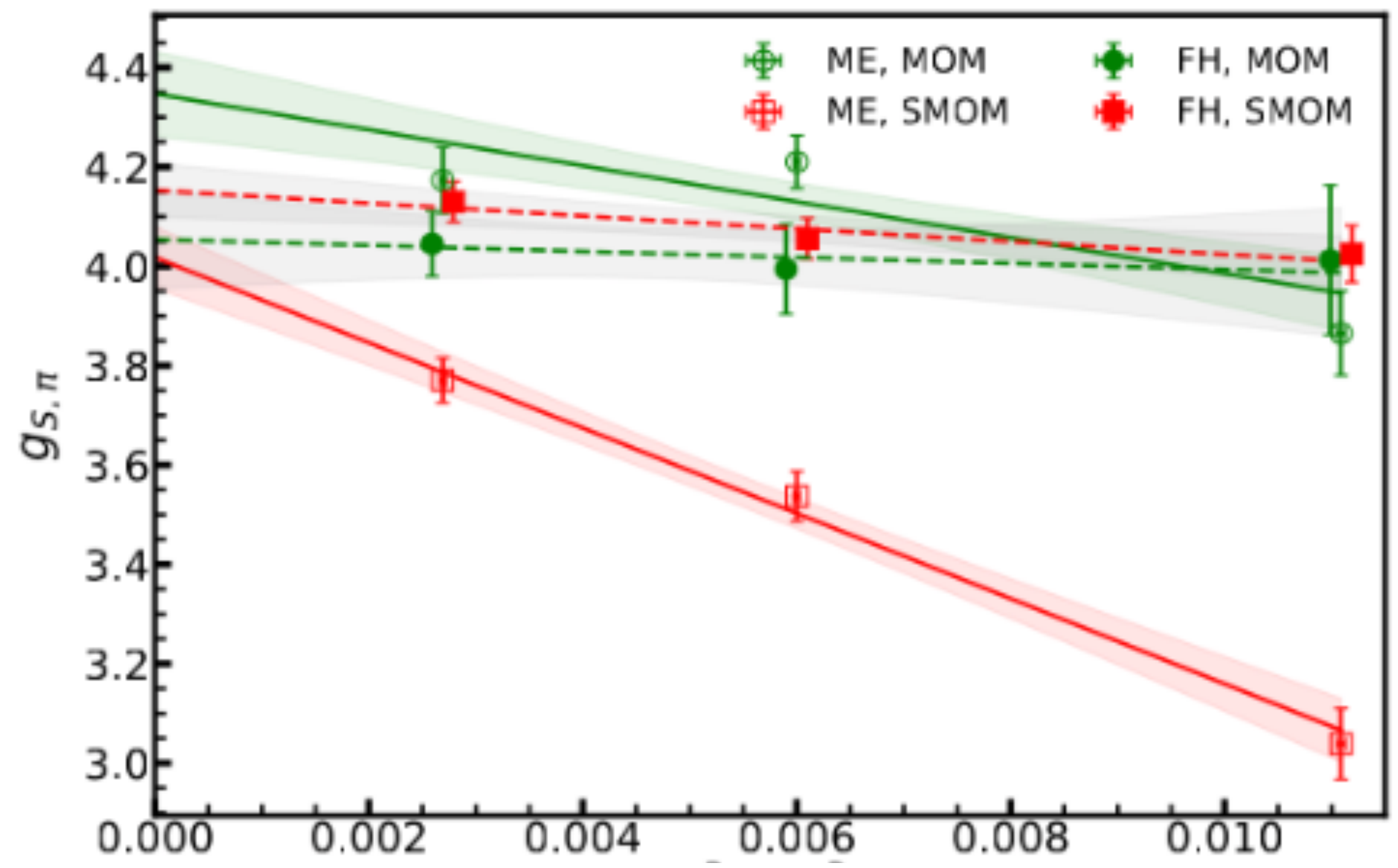
J.A. Gracey, Eur.Phys.J.C83 (2023) 181

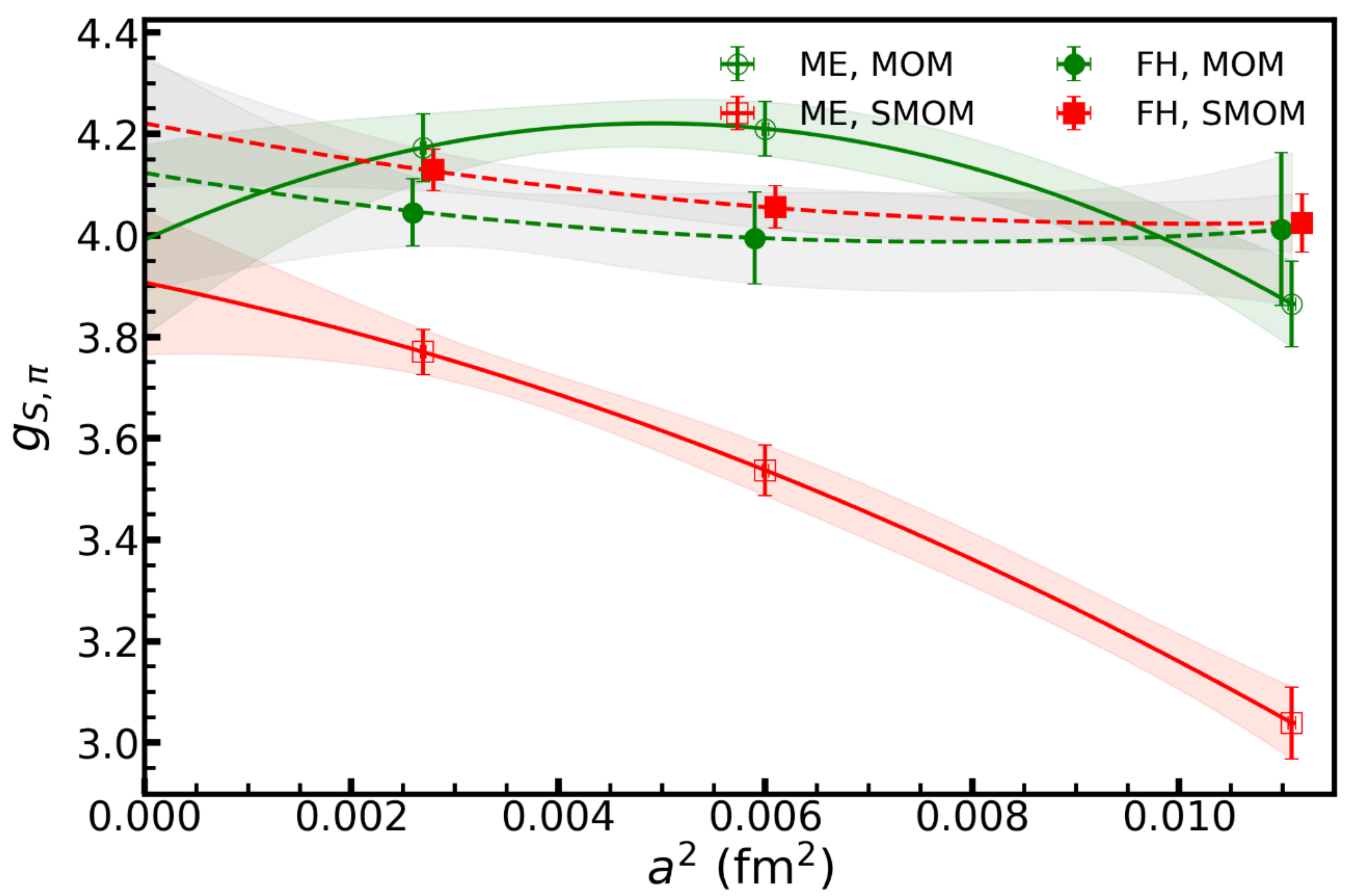
- and can be highly suppressed after the continuum extrapolation.

D. J. Zhao, et. al., χQCD & CLQCD, in preparation

Chiral symmetry breaking and renormalization

Restore of chiral symmetry in the continuum





- Renormalized quark mass $m_q^R = Z_A/Z_P m_q^{\rm PC}$ with 317 MeV pion mass at three lattice spacings:
  - The intermediate renormalization scheme dependence is 3.1(1.5)%.
  - RI/MOM scheme has smaller discretization error.
  - Feynman-Hellman theorem can extract $g_{S,\pi}$ as

$$g_{S,\pi}^{\rm FH} = \frac{1}{2}\frac{\partial m_\pi(m_q)}{\partial m_q} \simeq \frac{Z_P}{Z_A}\frac{m_\pi}{4m_q^{\rm PC}} + \mathcal{O}(m_q, a^2)$$

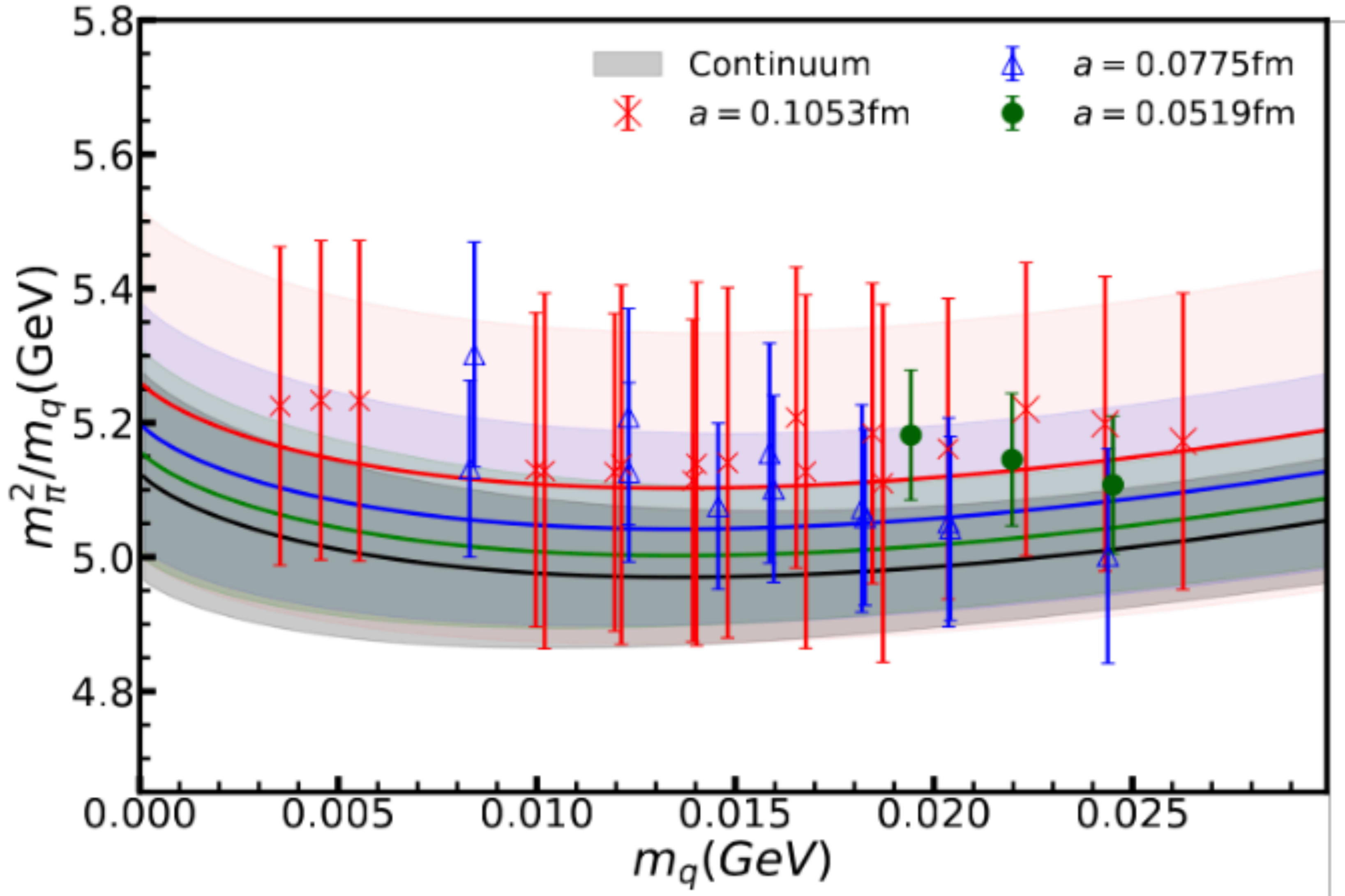
  which is 4.04(6)(12) for $m_\pi = 317\ \rm MeV$ in the continuum.

- Renormalized $g_{S,\pi}^{\rm R,ME} = Z_S \frac{\langle \pi | S | \pi \rangle_{\rm conn}}{\langle \pi | \pi \rangle}$ based on the direct calculation:
  - The intermediate renormalization scheme dependence is 7.6(2.3)% (linear a^2 correction) or 2.0(5.8)% ($a^2 + a^4$ corrections).
  - $g_{S,\pi}^{\rm ME}$ using RI/MOM scheme has smaller discretization error, and agree with $g_{S,\pi}^{\rm R,FH}$ within 2σ at all the lattice spacings.

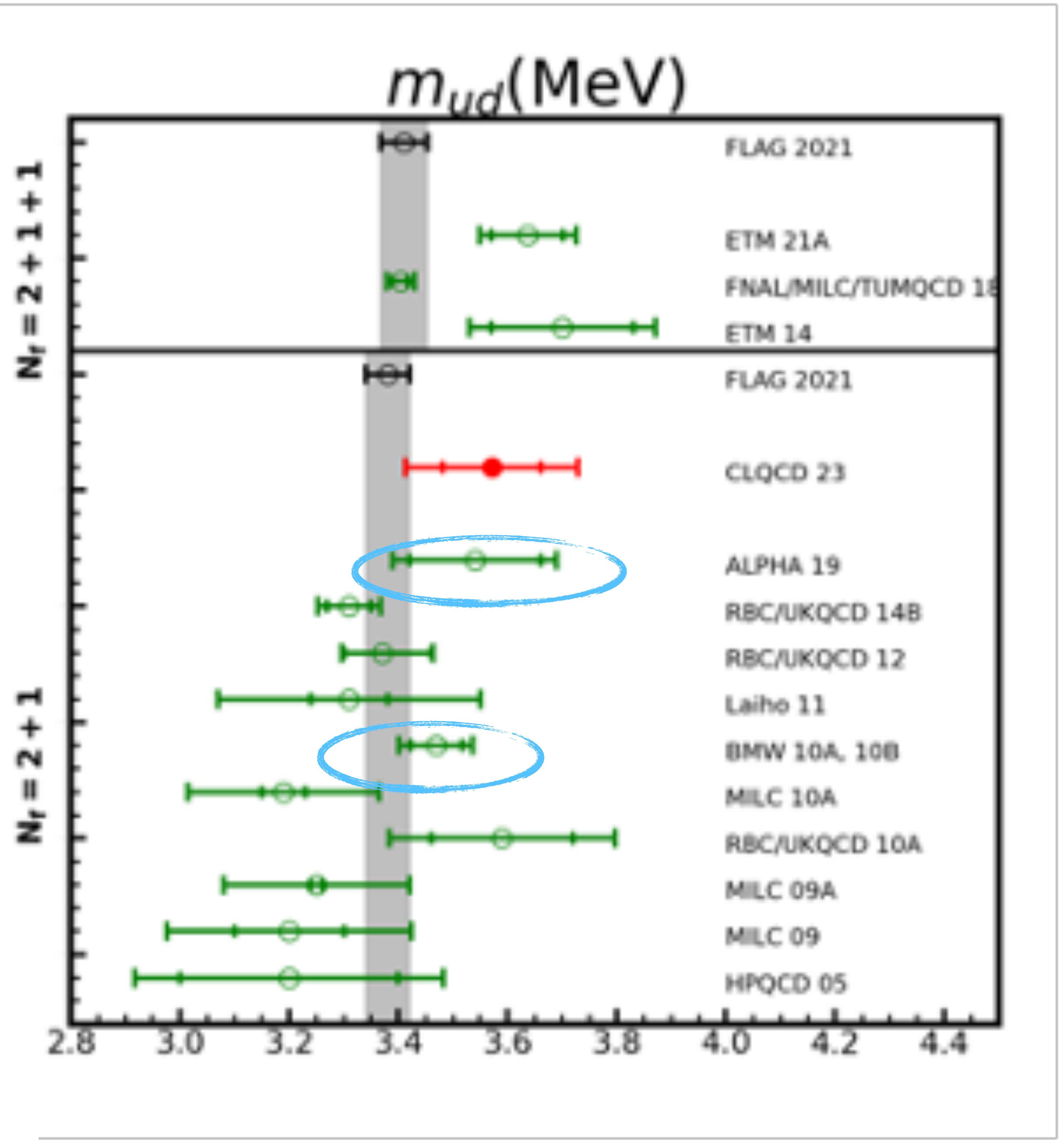
Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

Chiral symmetry breaking and renormalization

Global fit of the pion mass



$$m^2_{\pi,\mathrm{vv}} = \Lambda_\chi^2 2y_\mathrm{v}\{1 + \frac{2}{N_f}[(2y_\mathrm{v} - y_\mathrm{s})\ln(2y_\mathrm{v}) + (y_\mathrm{v} - y_\mathrm{s})]$$
$$+ 2y_\mathrm{v}(2\alpha_8 - \alpha_5) + 2y_\mathrm{s}N_f(2\alpha_6 - \alpha_4)\}$$
$$\times [1 + c_L^\pi e^{-m_\pi L} + c_s^\pi(m^2_{\eta_s} - m^2_{\eta_s,\mathrm{phys}})]$$
$$\times (1 + d_a^\pi a^2),$$
$$\Lambda_\chi = 4\pi F,\ y = \frac{\Sigma m}{F^2\Lambda_\chi^2}$$

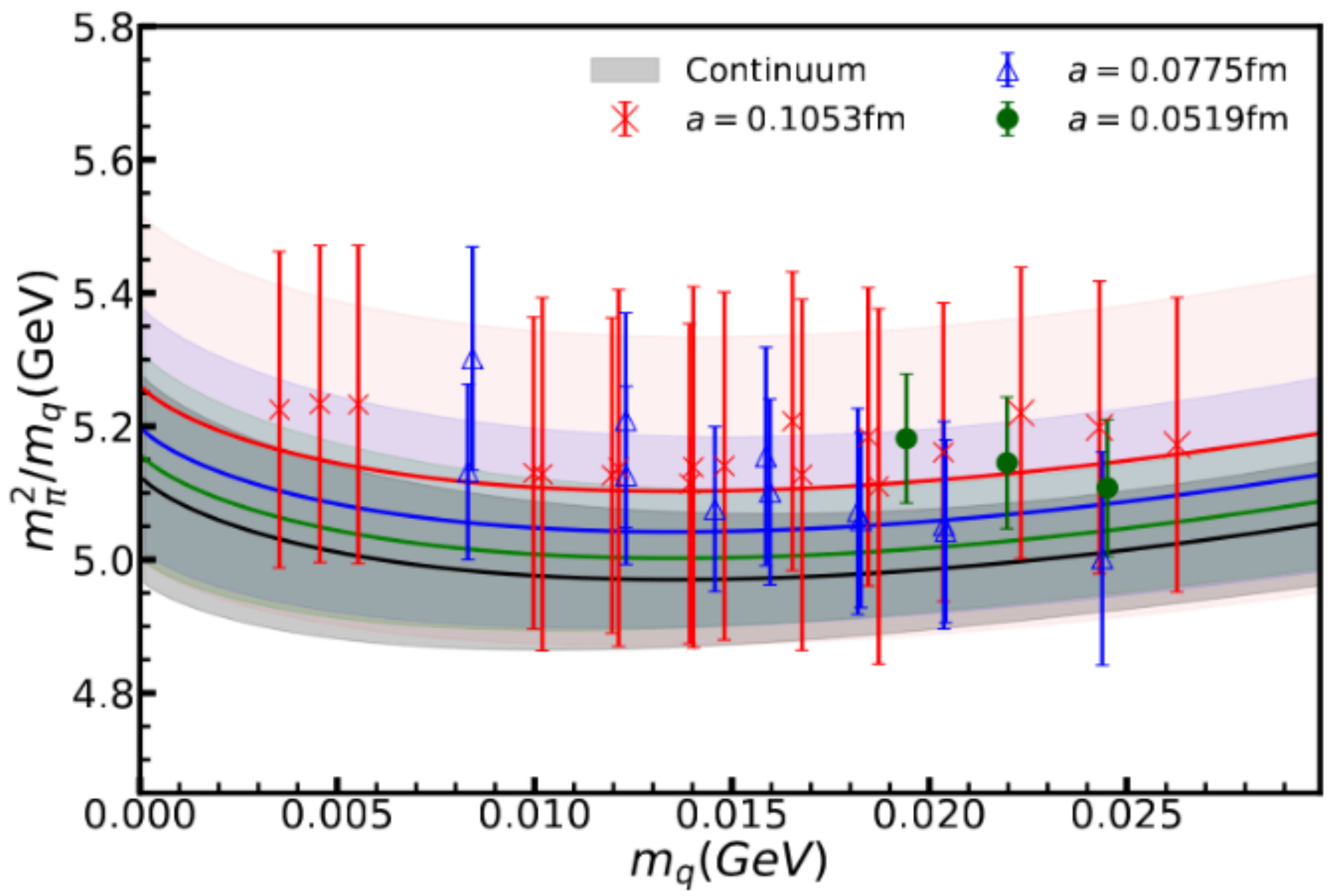


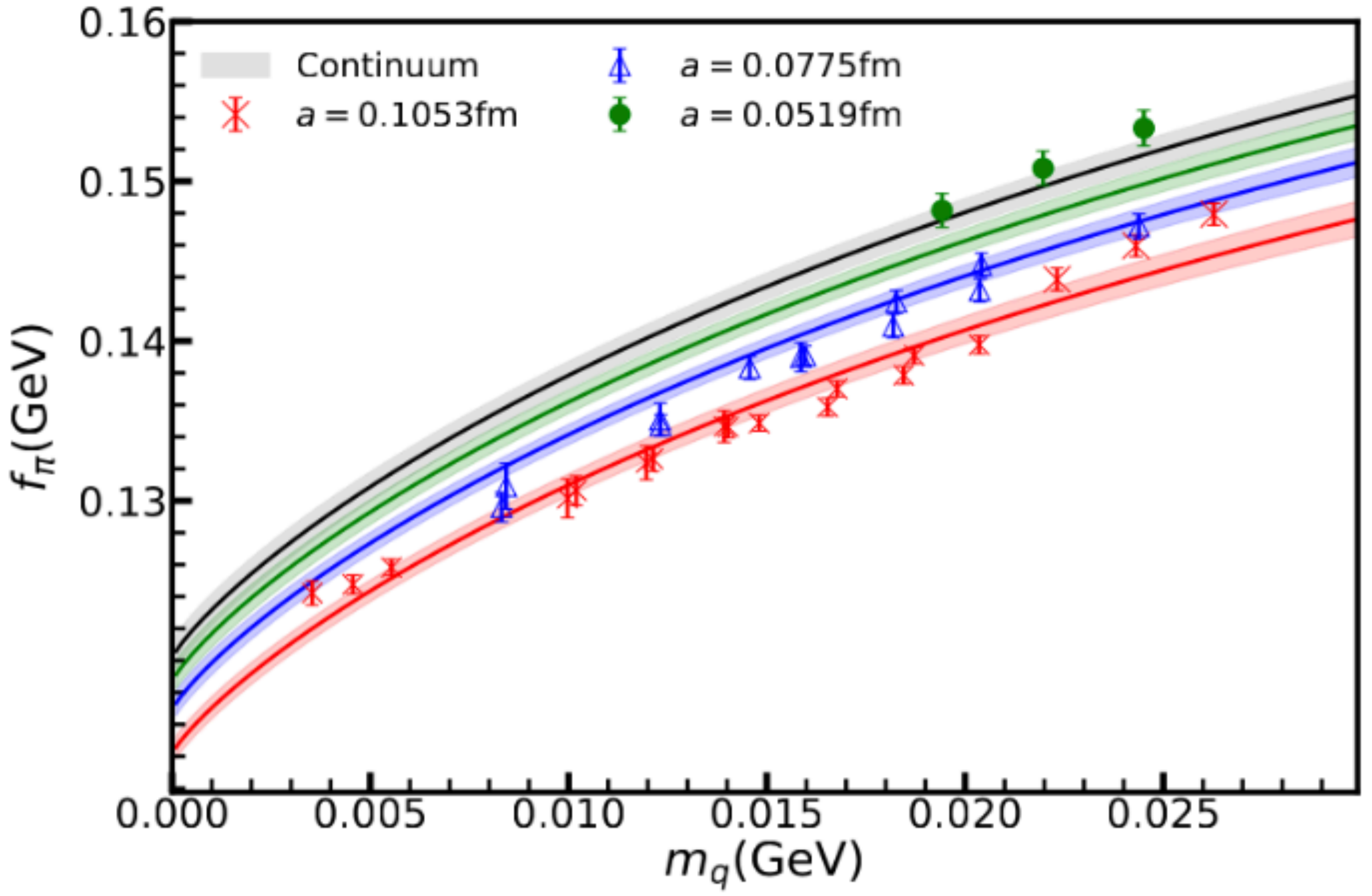
- Present CLQCD prediction of the u-d averaged light quark masses is consistent with the lattice averages within 5% uncertainty.
- Most of the uncertainties come from the non-perturbative renormalization and further improvements are in progress.
- All the finite volume, discretization and sea quark mass effects have been taken into account.

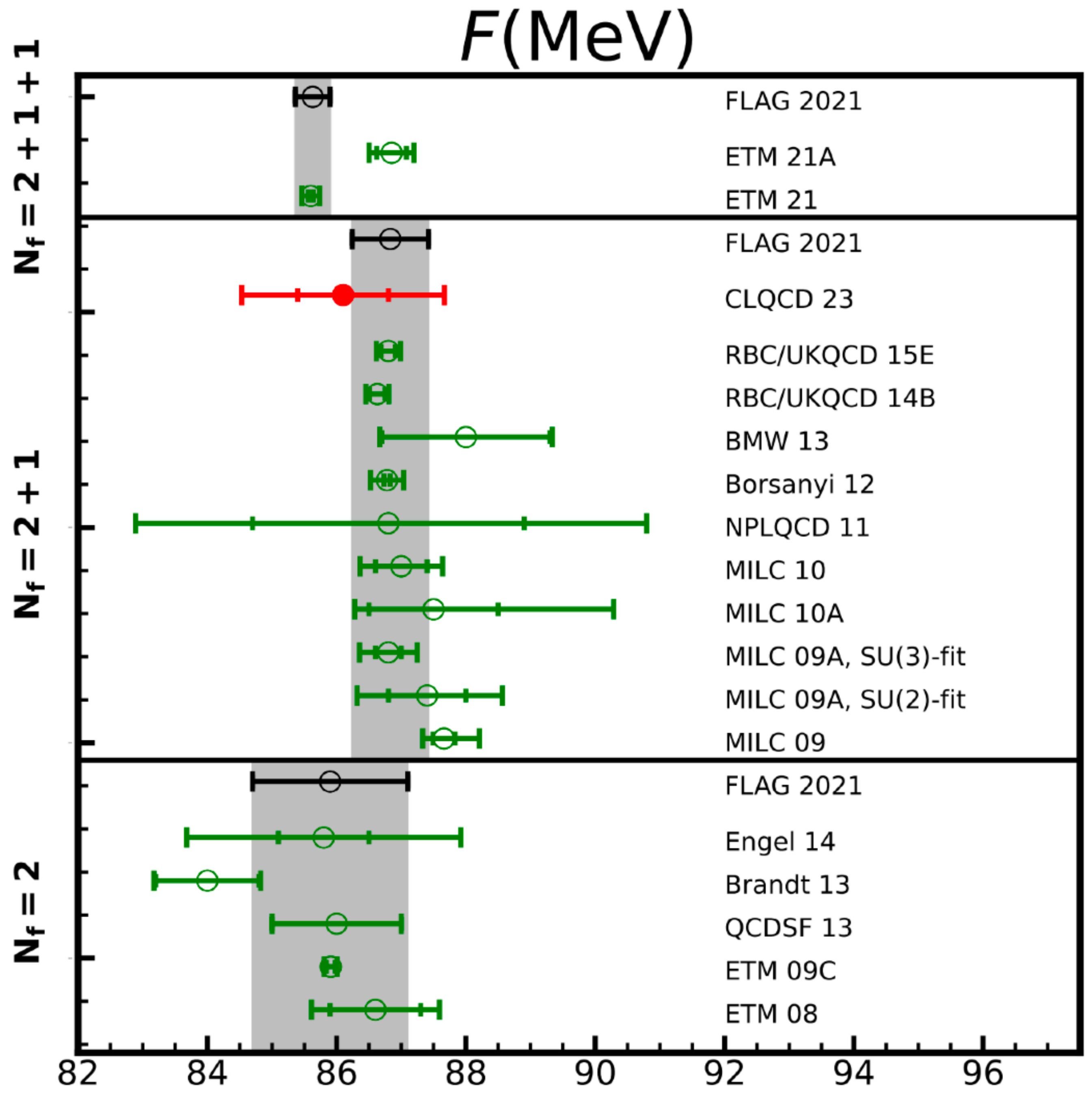
Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

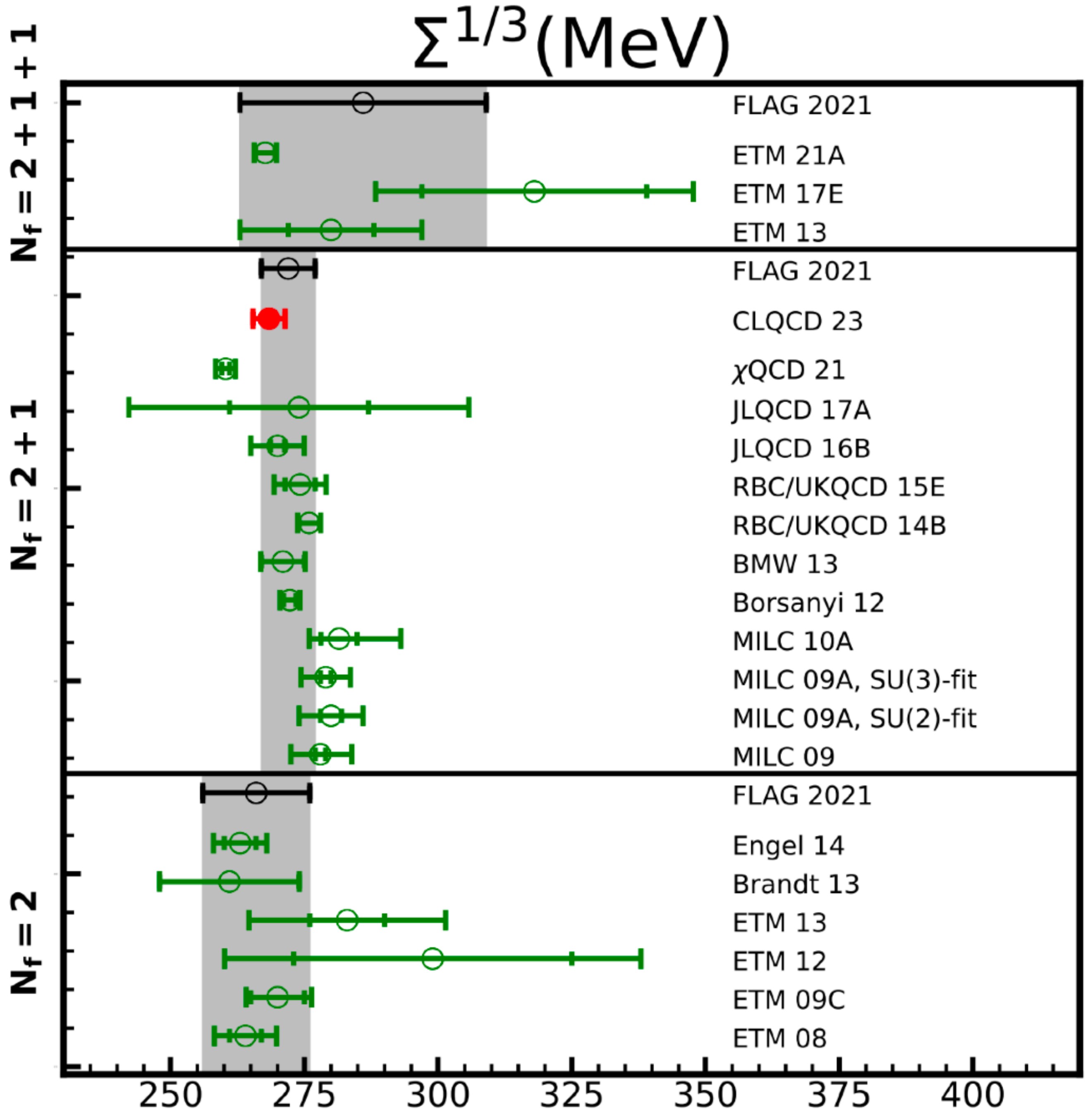
Renormalization and final results

Global fit of the low energy constants









$$m^2_{\pi,\mathrm{vv}} = \Lambda_\chi^2 2y_\mathrm{v}\{1+\frac{2}{N_f}[(2y_\mathrm{v}-y_\mathrm{s})\ln(2y_\mathrm{v})+(y_\mathrm{v}-y_\mathrm{s})] + 2y_\mathrm{v}(2\alpha_8-\alpha_5)+2y_\mathrm{s}N_f(2\alpha_6-\alpha_4)\} \times\left[1+c_L^\pi e^{-m_\pi L}+c_s^\pi(m^2_{\eta_s}-m^2_{\eta_s,\mathrm{phys}})\right] \times(1+d_a^\pi a^2),$$

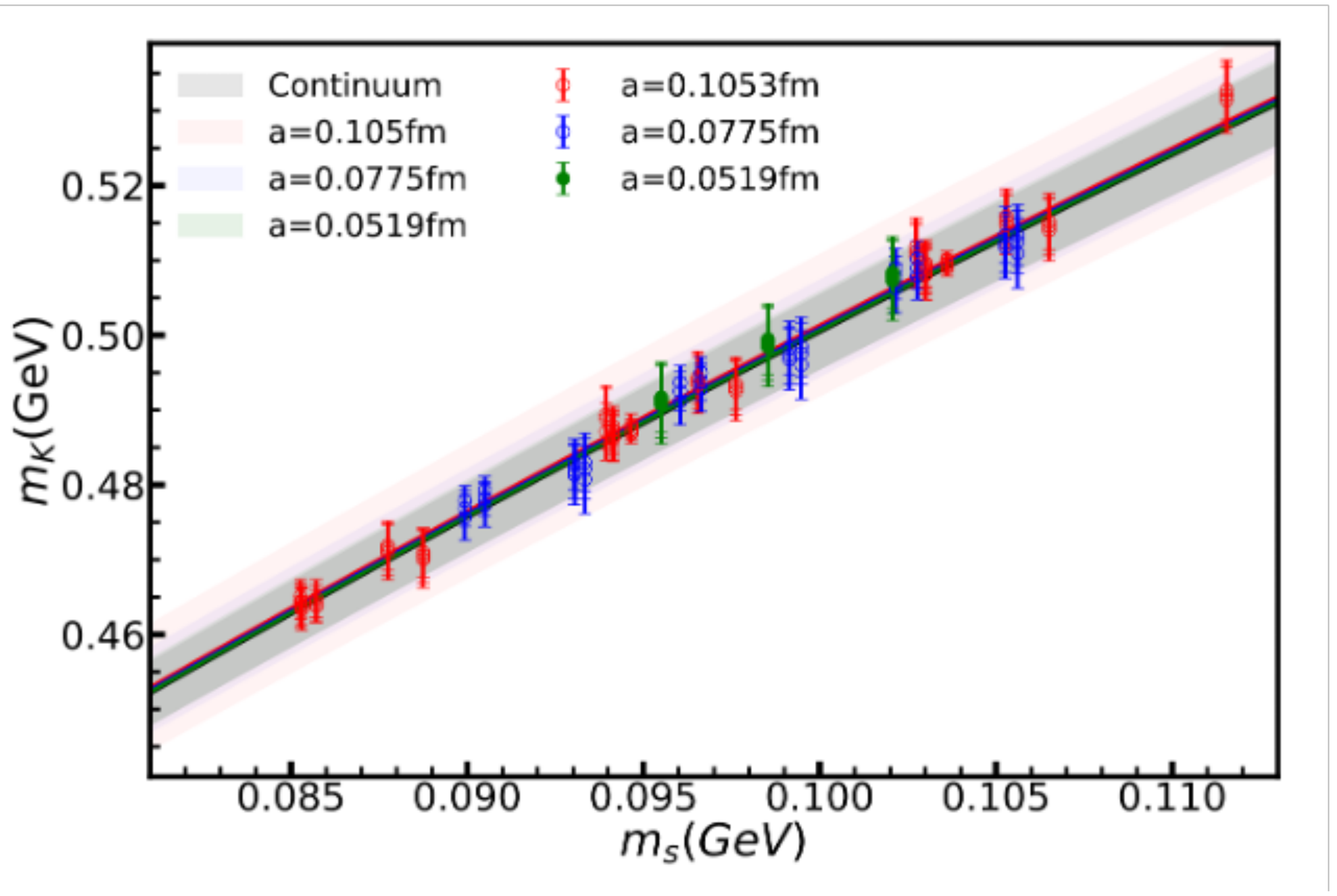
$$\Lambda_\chi = 4\pi F,\ y = \frac{\Sigma m}{F^2\Lambda_\chi^2}$$

$$F_{\pi,\mathrm{vv}} = F(1-\frac{N_f}{2}(y_\mathrm{v}+y_\mathrm{s})\ln(y_\mathrm{v}+y_\mathrm{s})+y_\mathrm{v}\alpha_5+y_\mathrm{s}N_f\alpha_4) \times\left[1+d_L^\pi e^{-m_\pi L}+d_s^\pi(m^2_{\eta_s}-m^2_{\eta_s,\mathrm{phys}})\right] \times(1+d_a^\pi a^2)$$

- Global fit of all the ensembles to obtain the quark mass dependence of m_π and f_π in the continuum and infinite volume limit, which allows us to extract the χPT low energy constants.

Chiral symmetry breaking and renormalization

Global fit of the kaon mass



$$m_K^2(m_l^{\rm v}, m_l^{\rm s}, m_s^{\rm v}, m_s^{\rm s}, a) = (b_s^{\rm v} m_s^{\rm v} + b_s^{\rm s} m_s^{\rm s} + b_l^{\rm v} m_l^{\rm v} + b_l^{\rm s} m_l^{\rm s})$$
$$\times \left[1 + c_l^K m_l^{\rm v} + c_m^K a^2 + c_L^K \exp\left(-m_K L\right)\right],$$

$$m_u^{\rm phys} + m_d^{\rm phys} = 2 m_l^{\rm phys}.$$

$$m_K(m_d^{\rm phys}, m_l^{\rm phys}, m_s^{\rm phys}, m_s^{\rm phys}, 0) = m_{K^0,\rm QCD},$$
$$m_K(m_u^{\rm phys}, m_l^{\rm phys}, m_s^{\rm phys}, m_s^{\rm phys}, 0) = m_{K^\pm,\rm QCD},$$

val sea val sea

P.Zyla et,al, PTEP(2020)083C01 (PDG2020):

- $m_p = 938.27 \text{ MeV} = m_{p,\rm QCD} + 1.00(16) \text{ MeV} + \ldots;$
- $m_n = 939.57 \text{ MeV};$
- $m_\pi^0 = 134.98 \text{ MeV};$
- $m_\pi^+ = 139.57 \text{ MeV} = m_\pi^0 + 4.53(6) \text{ MeV} + \ldots;$
- $m_K^0 = 497.61(1) \text{ MeV} = m_{K,\rm QCD}^0 + 0.17(02) \text{ MeV} + \ldots;$
- $m_K^+ = 493.68(2) \text{ MeV} = m_{K,\rm QCD}^+ + 2.24(15) \text{ MeV} + \ldots$

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

Renormalization and final results

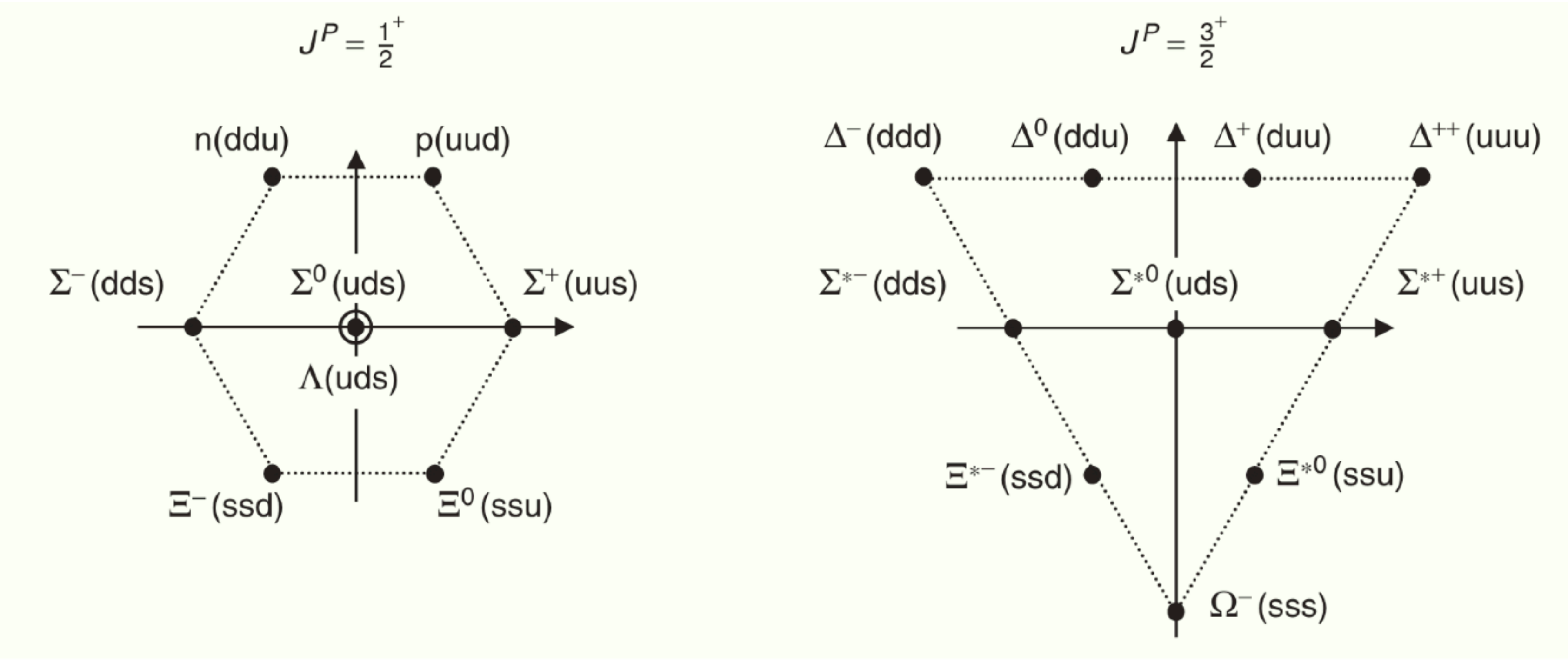
Quark mass of three light flavors

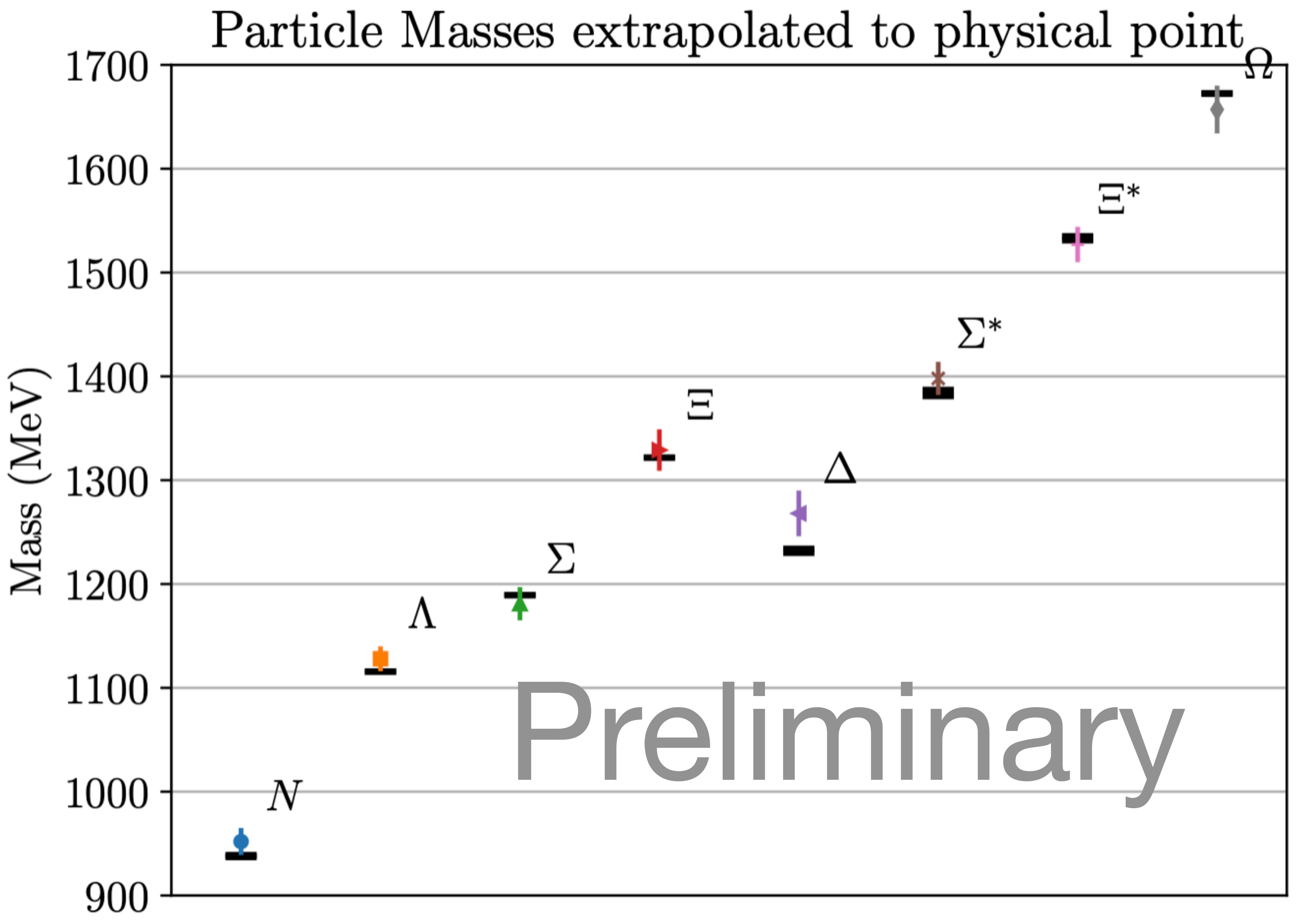
Summary of the lattice methodology.—The lattice setup used for this project is very similar to Ref. [13] and is based on our set of lattice QCD simulations presented in Ref. [6]. It is composed of 47 $N_f = 2+1$ QCD ensembles with pion masses down to 120 MeV, 5 lattice spacings down to 0.054 fm, and 16 different volumes up to $(6\ \text{fm})^3$.

BMWc, PRL 117(2016)0820001

Z.C. Hu, B.L. Hu, J.H. Wang, et. al., CLQCD, 2310.00814

CLQCD ensembles





Octet and decuplet baryons

- **Octet:**
  - $m_0 = 0.90(6)$ GeV;
  - $a_l = 3.0(9), b_l = 1.6(9)$;
  - $a_s = 1.9(2), b_s = 1.4(2)$;
  - $c_l = 1.4(2), c_s = 0.1(4)$;

$$m_N = m_0 + (a_l + b_l + 2c_l)m_l + c_s m_s$$

$$m_\Lambda = m_0 + (\frac{a_l + 4b_l}{3} + 2c_l)m_l + (\frac{2a_l - b_l}{3} + c_s)m_s$$

$$m_\Sigma = m_0 + (a_l + 2c_l)m_l + (b_l + c_s)m_s$$

$$m_\Xi = m_0 + (b_l + 2c_l)m_l + (a_l + c_s)m_s$$

- **Decuplet:**
  - $\bar{m}_0 = 1.19(3)$ GeV;
  - $\bar{a} = 1.4(1)$;
  - $\bar{c}_l = 2.2(2), \bar{c}_s = 0.5(4)$.

$$m_\Delta = \bar{m}_0 + (3\bar{a} + 2\bar{c}_l)m_l + \bar{c}_s m_s$$

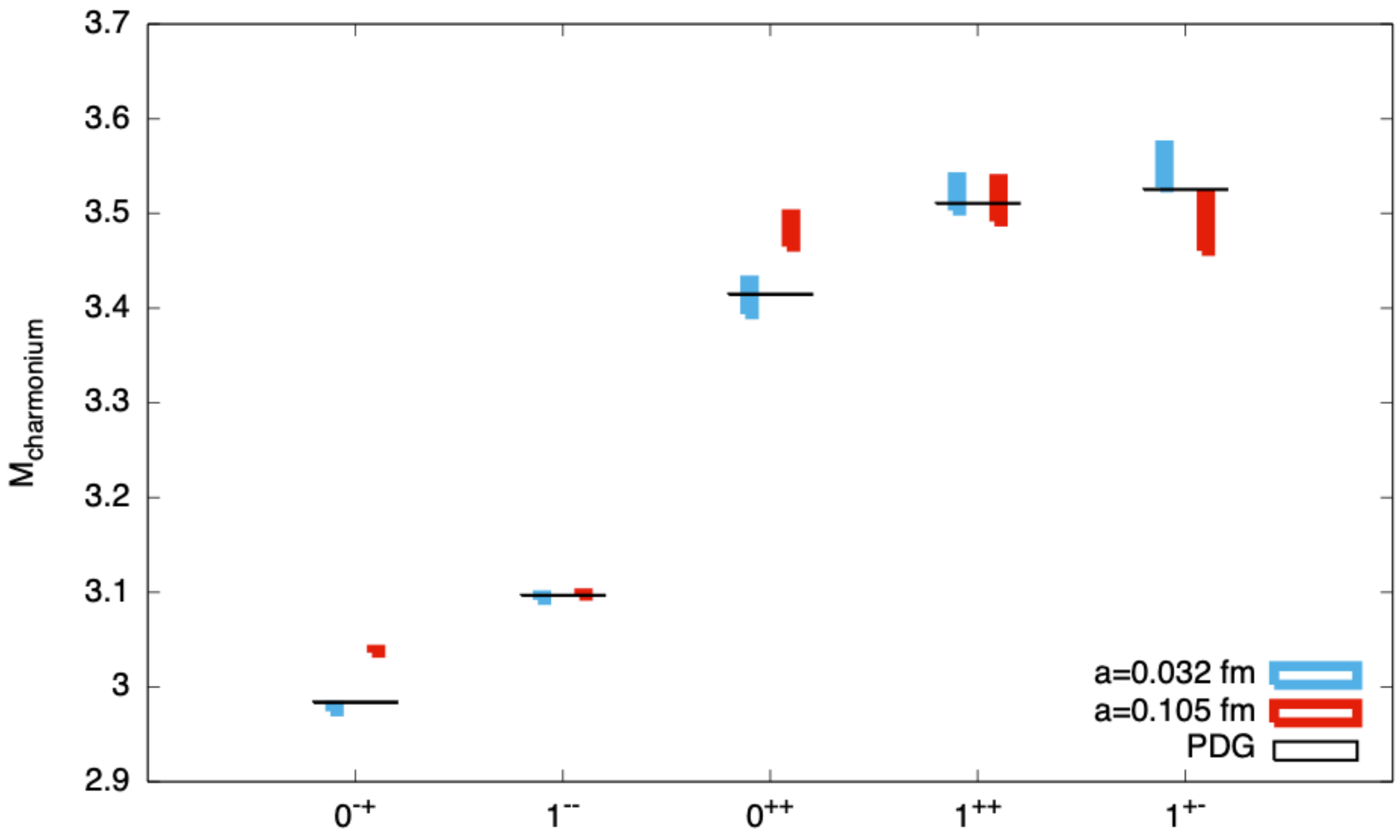
$$m_{\Sigma*} = \bar{m}_0 + (2\bar{a} + 2\bar{c}_l)m_l + (\bar{a} + \bar{c}_s)m_s$$

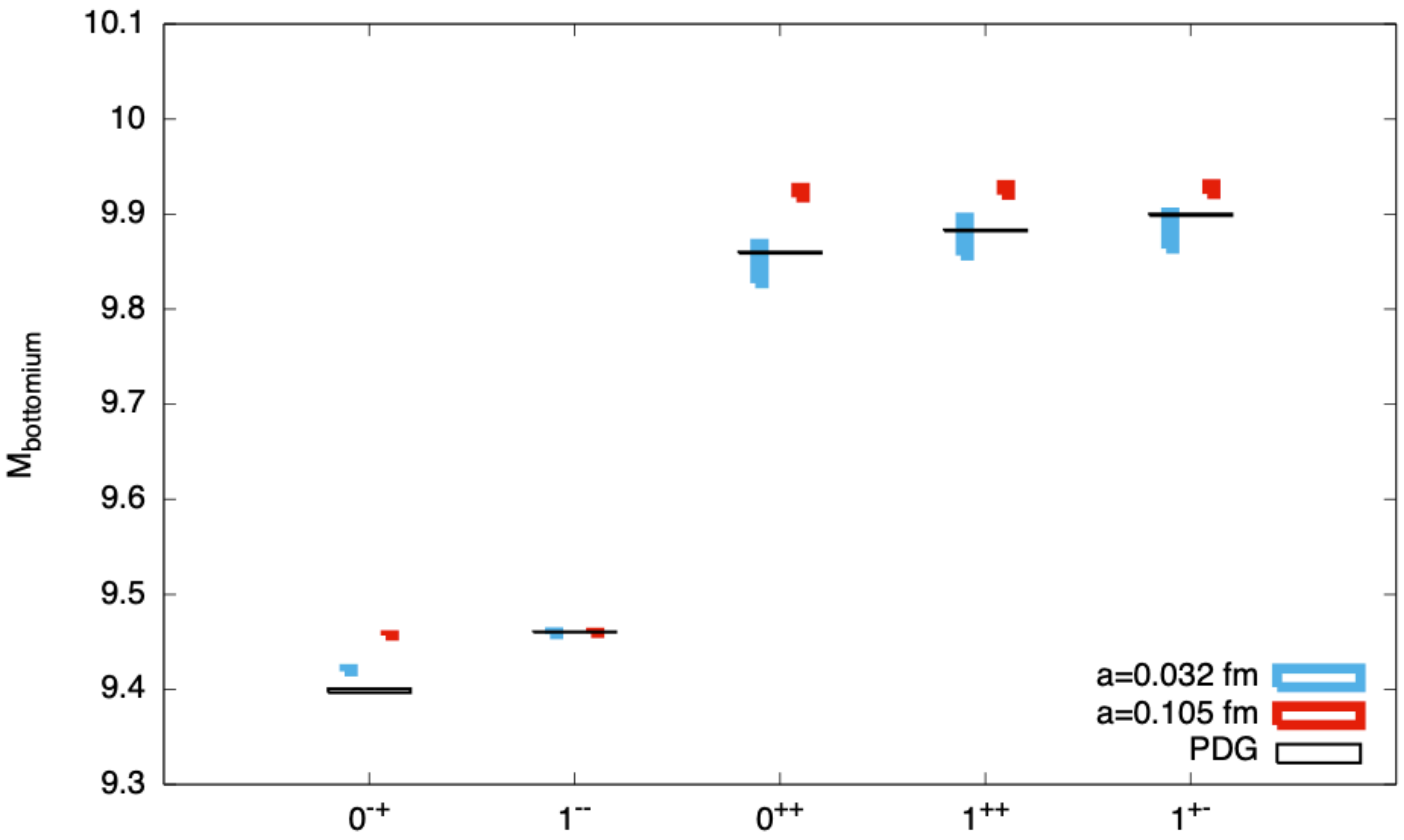
$$m_{\Xi*} = \bar{m}_0 + (\bar{a} + 2\bar{c}_l)m_l + (2\bar{a} + \bar{c}_s)m_s$$

$$m_{\Omega*} = \bar{m}_0 + 2\bar{c}_l m_l + (3\bar{a} + \bar{c}_s)m_s$$

Explore heavy flavors

Charmonium and Bottomium





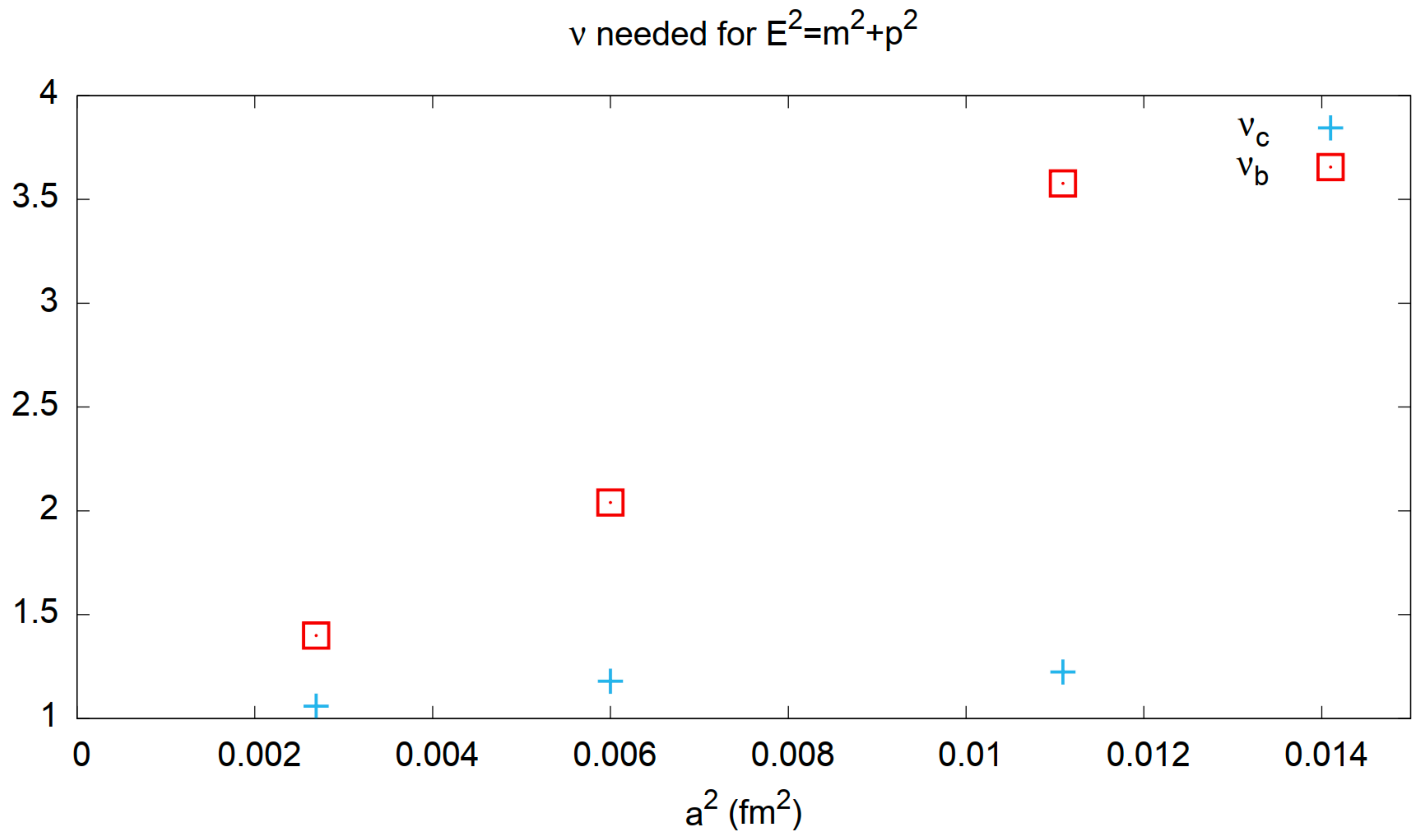
H.Y. Du, et. al., CLQCD, in preparation

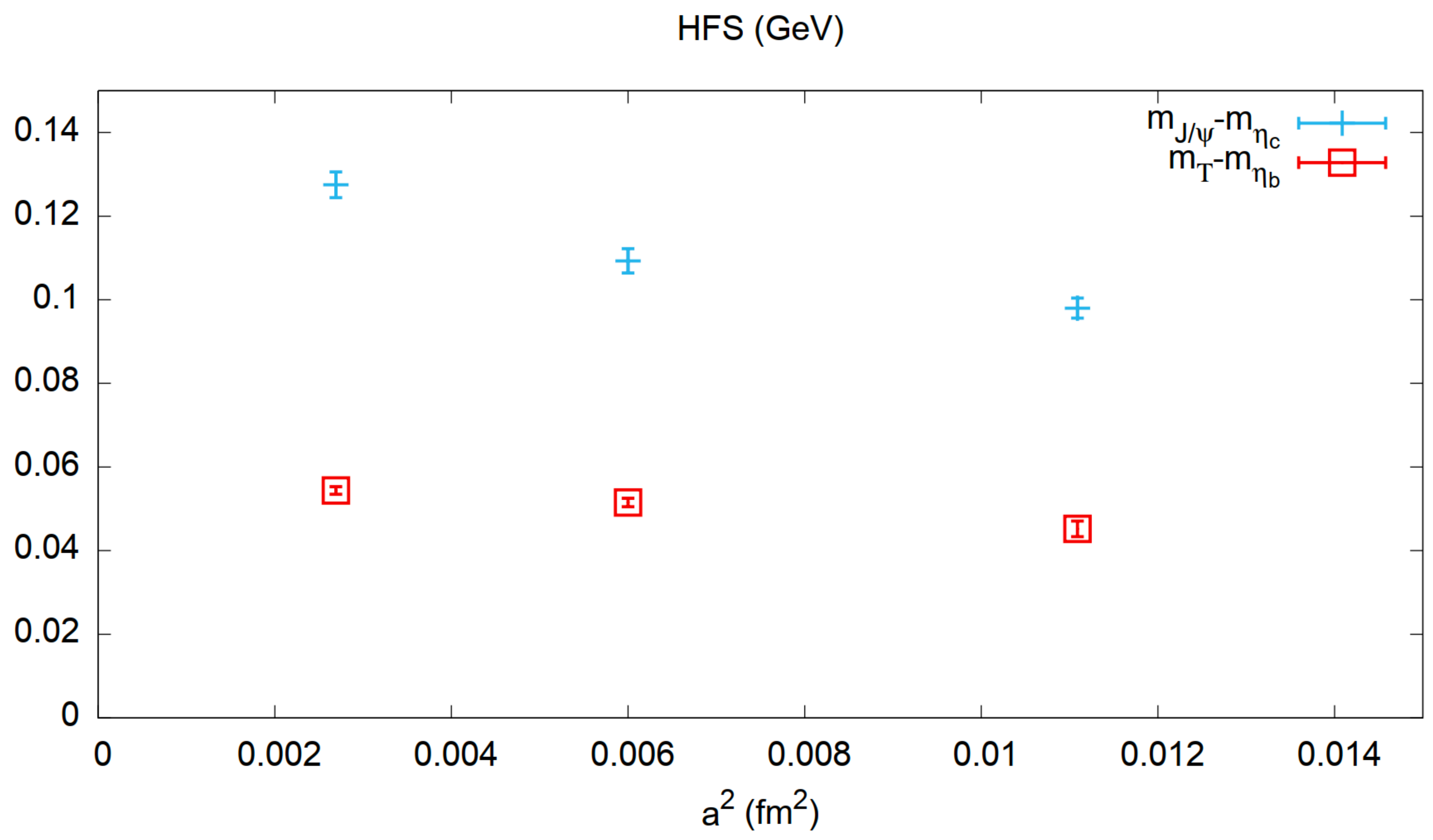
- At $a = 0.105$ fm, the discretization error can make the 1P-1S fine splitting to be ~10% larger, and the hyperfine splitting to be 50% (charm) and 95% (bottom) smaller.
- The situation can also be significantly improved when the lattice spacing becomes smaller.

Hyperfine splitting from Clover fermion

Anisotropic action

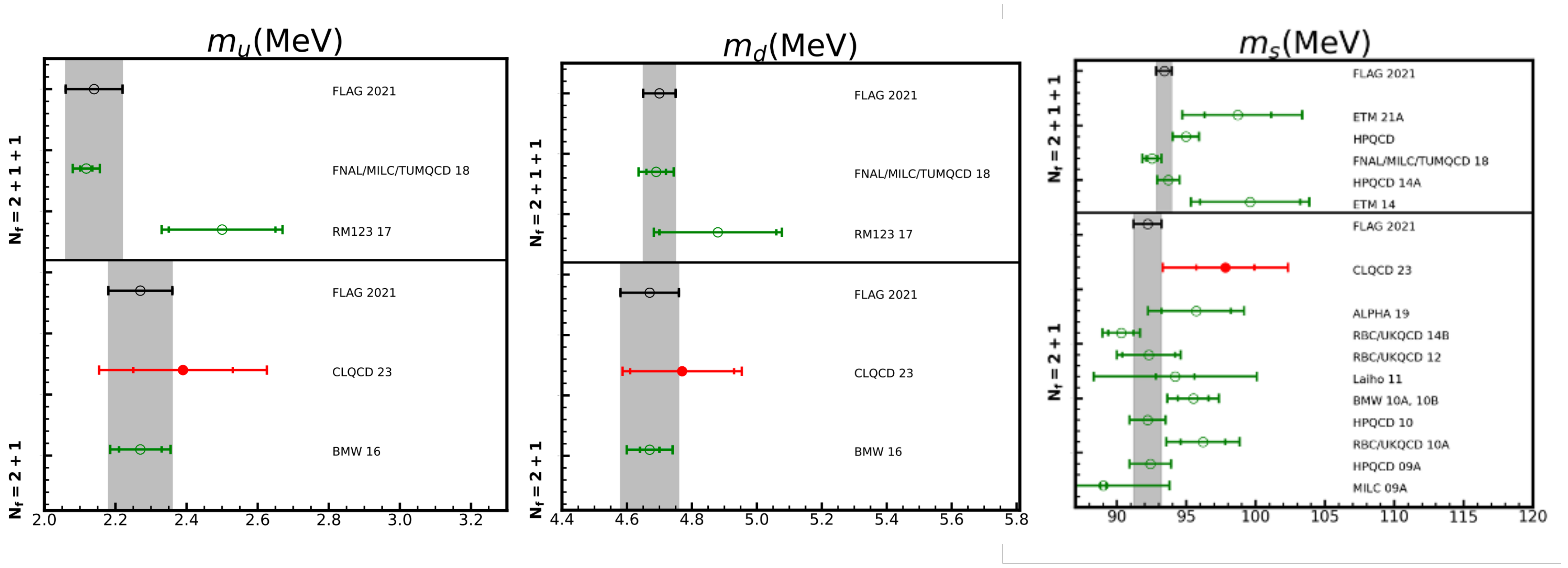
$$S_Q = a^4 \sum_x \bar{Q} \left[m_Q + \gamma_0 \nabla_0 - \frac{a}{2} \nabla_0^{(2)} + \nu \sum_{i=1}^{3} \left(\gamma_i \nabla_i - \frac{a}{2} \nabla_i^{(2)} \right) - c_E \frac{a}{2} \sum_{i=1}^{3} \sigma_{0i} F_{0i} - c_B \frac{a}{4} \sum_{i,j=1}^{3} \sigma_{ij} F_{ij} \right] Q \, .$$



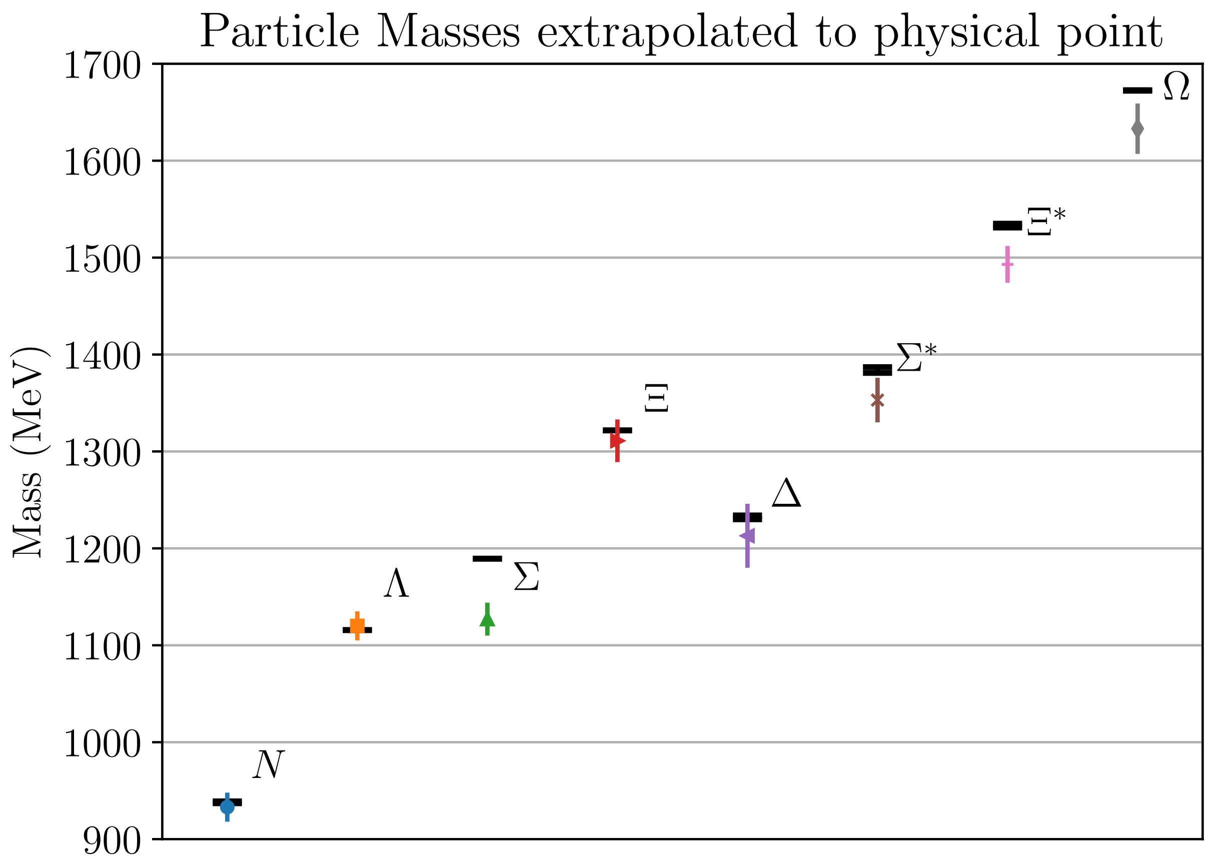


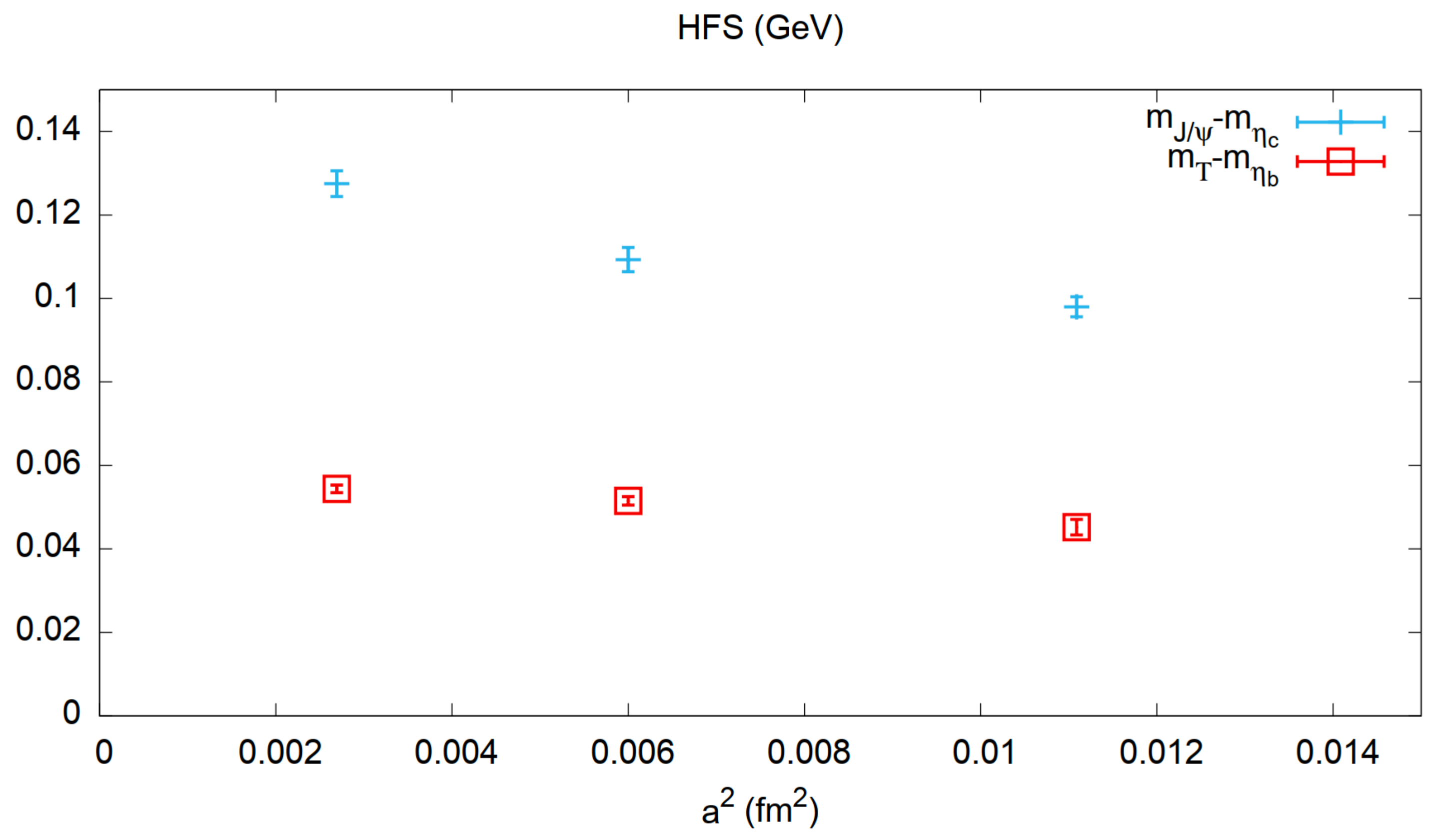
Summary

- We chose the clover fermion and Symanzik gauge actions to generate the Lattice QCD ensembles at multiple lattice spacing, pion mass and volume, and figured out the proper renormalization to restore the chiral symmetry at 5% level.



- Current prediction of quark masses and low energy quantities agree with the lattice averages within 5%, and more accurate studies are on-going.





- The heavy quark can have huge discretization error at coarse lattice spacings, while further improvement on the fermion action is also on-going.