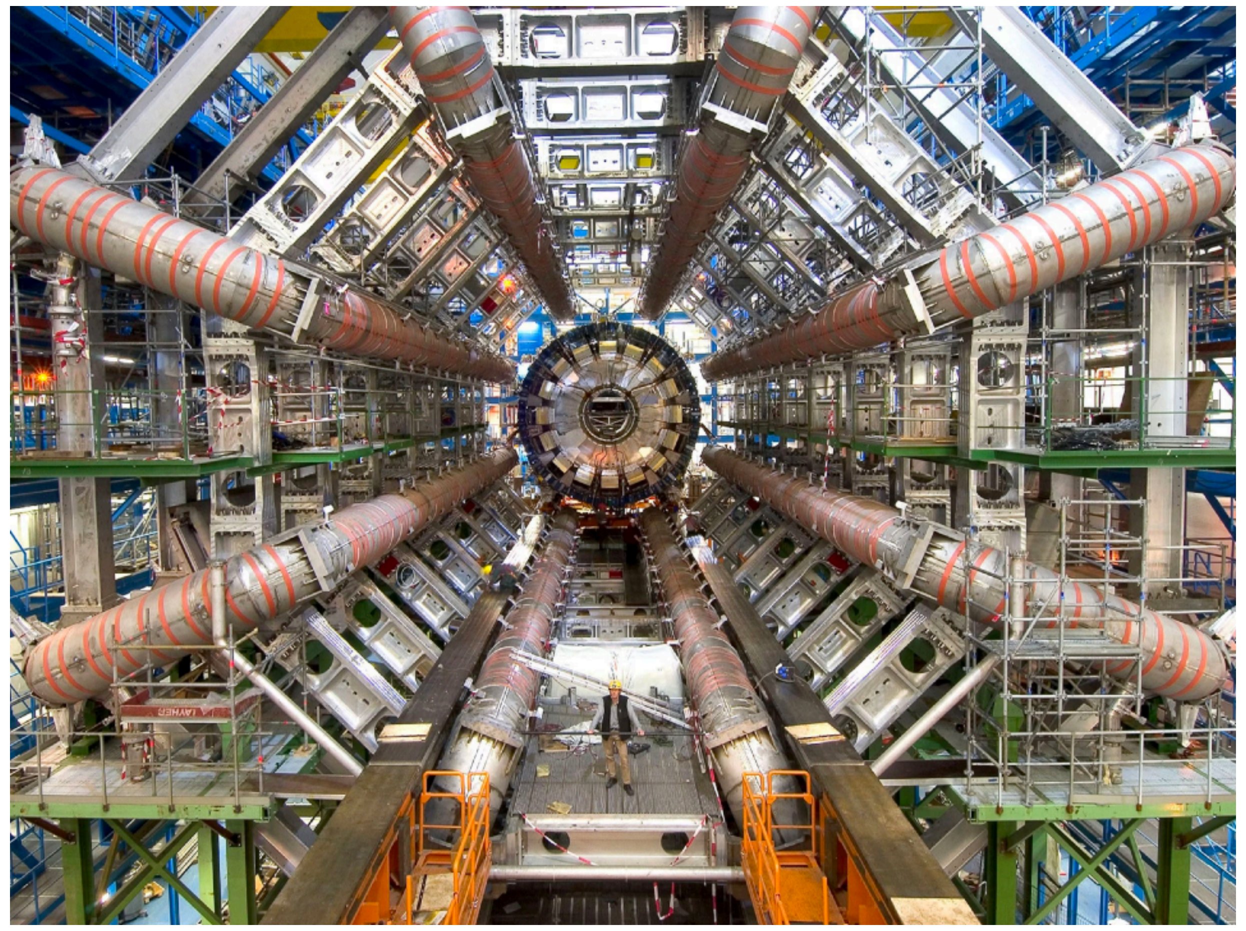
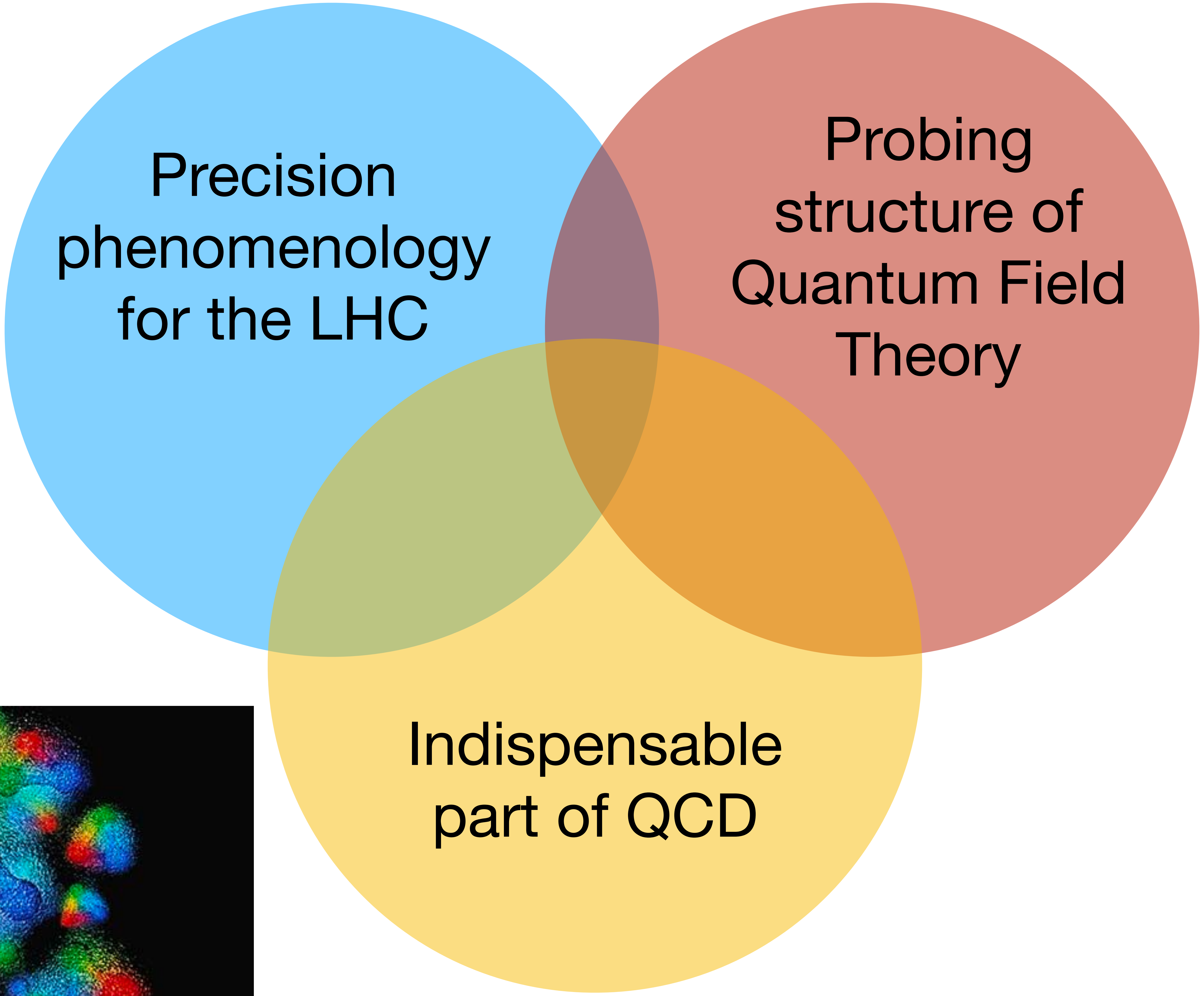
Progress for Perturbative QCD at the LHC

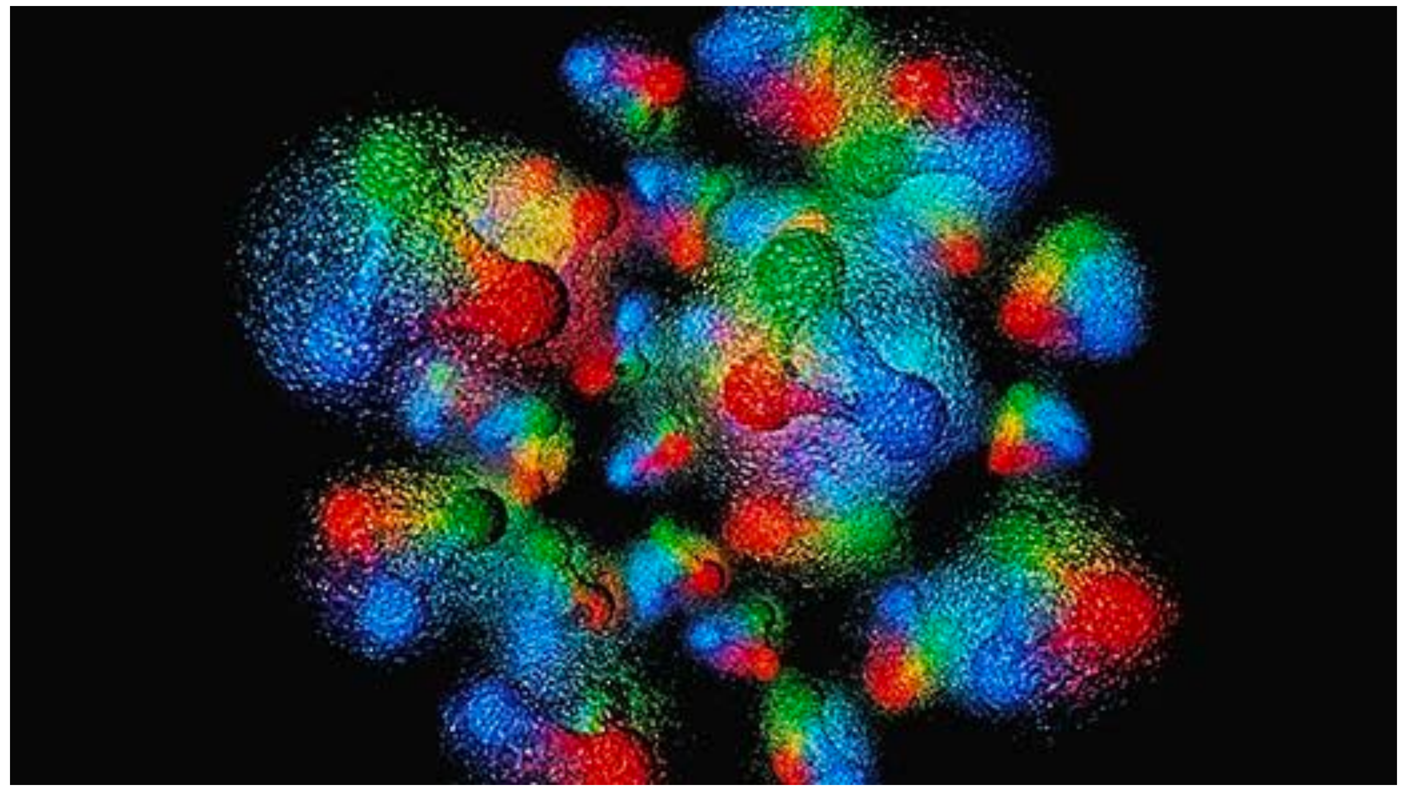
Hua Xing Zhu
Peking University

Southeast University, Nanjing
December 15-19, 2023

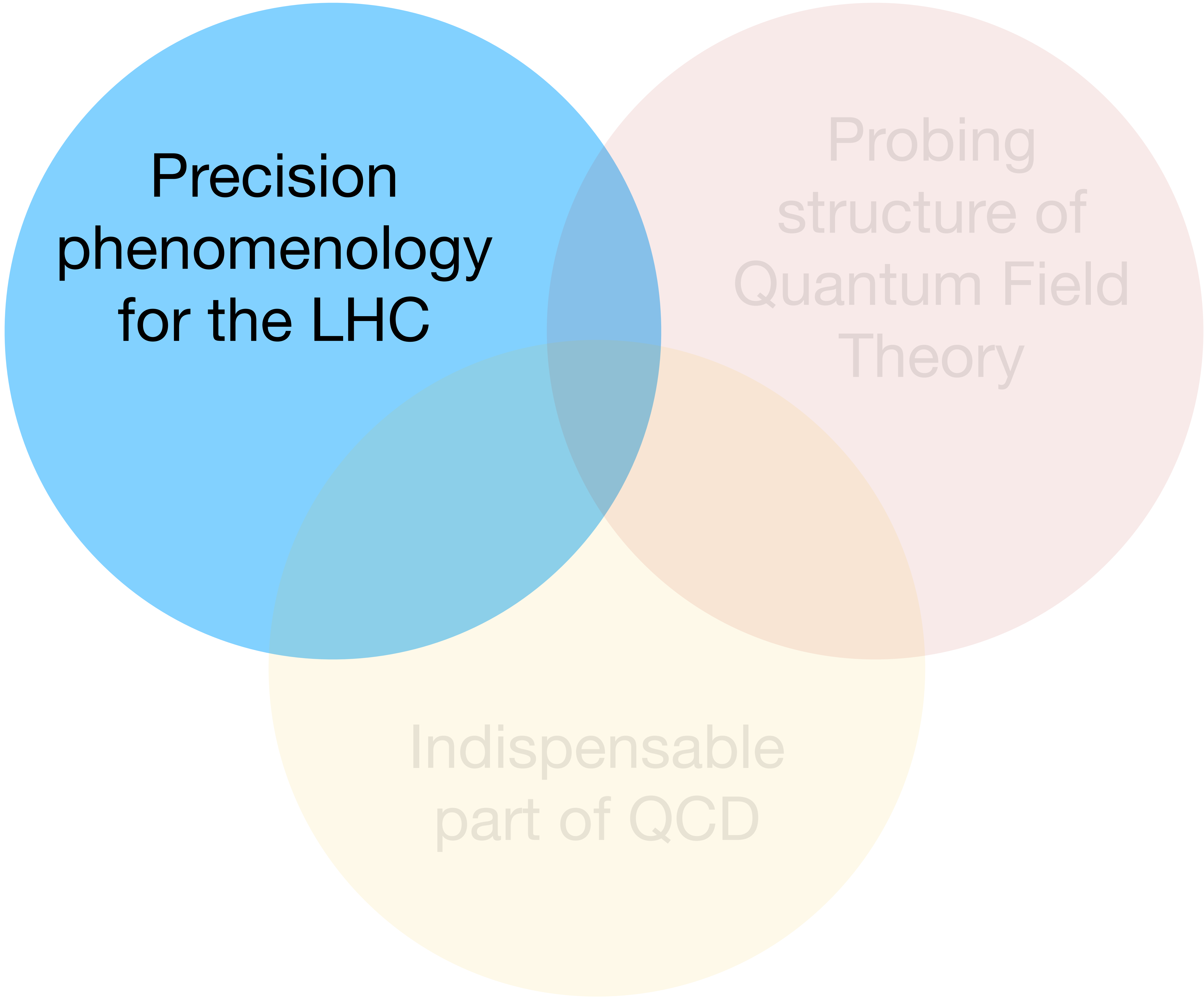
Plan of this talk



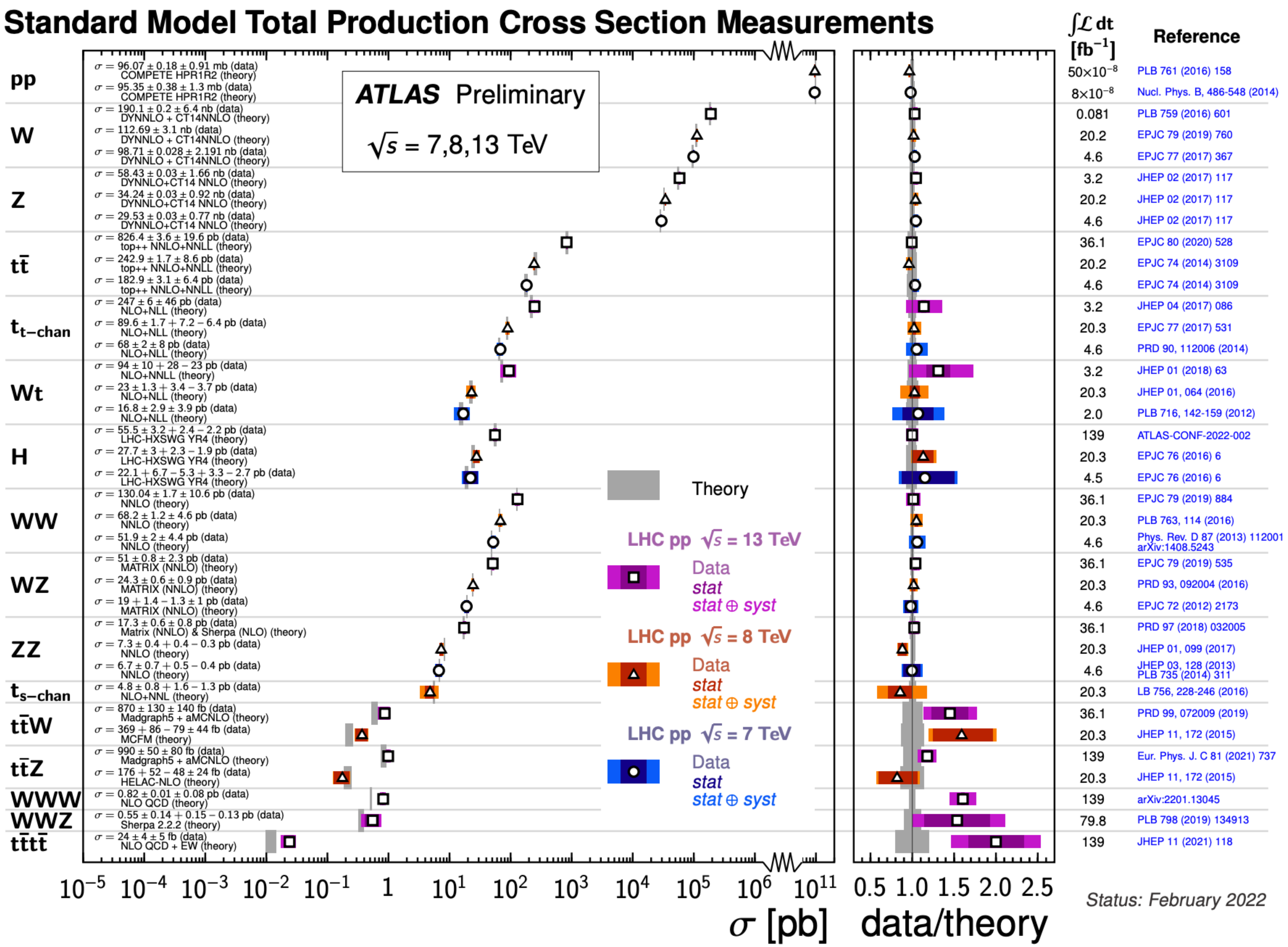




Plan of this talk



Success of the LHC precision program



A great triumph of The Standard Model

But we should let no stone left unturned

Stress-test The Standard Model to its extreme!

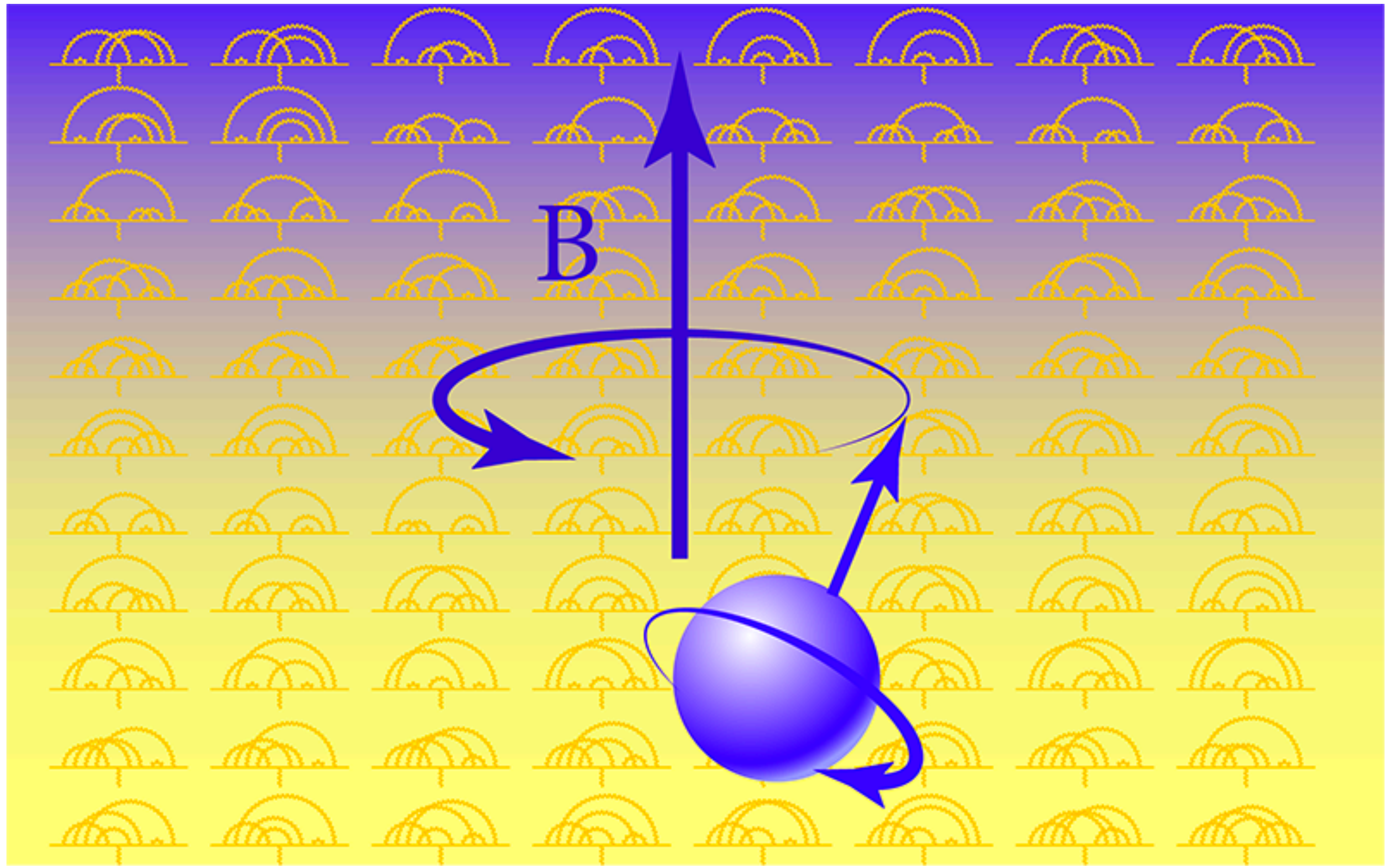
But let's recall some history

anomalous magnetic moment of electron

$$g = 2 + 2a_e$$

$$g = 2 + 2 \times \frac{\alpha}{2\pi} + \cdots$$
$$= 2 + 0.0023228 + \cdots$$

Lead to establishment of QED!

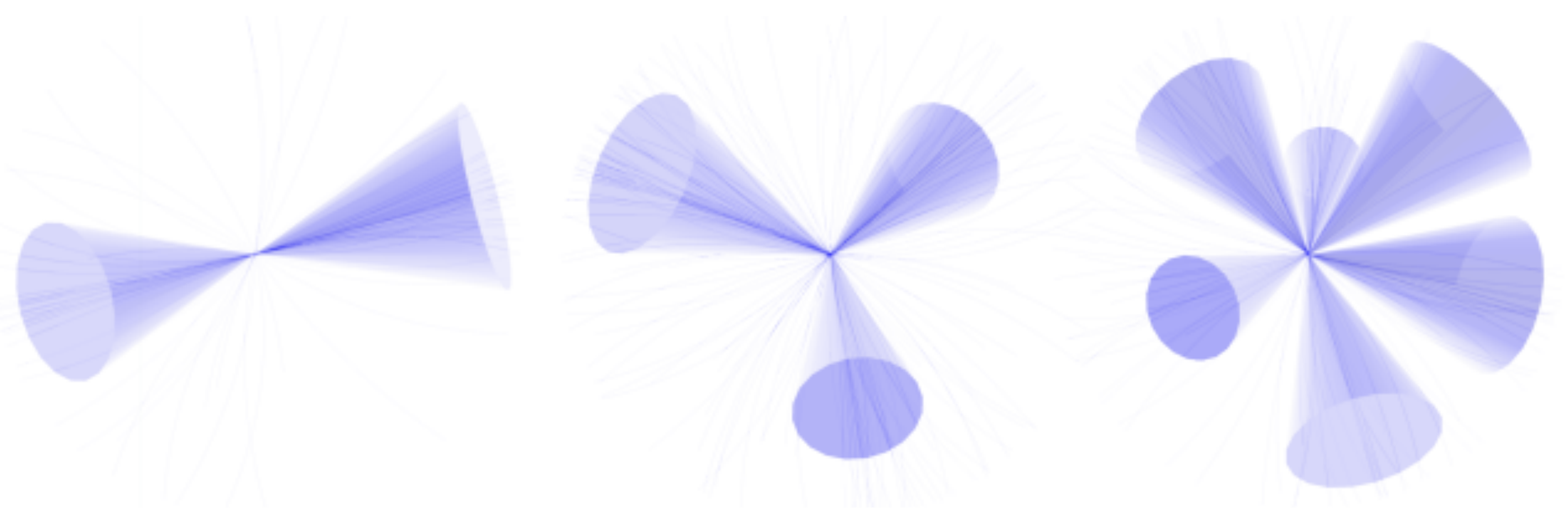


$$a_\mu = 0.001\,165\,920\,61(41)$$

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm hadron}$$
$$= 0.001\,165\,918\,04(51)$$

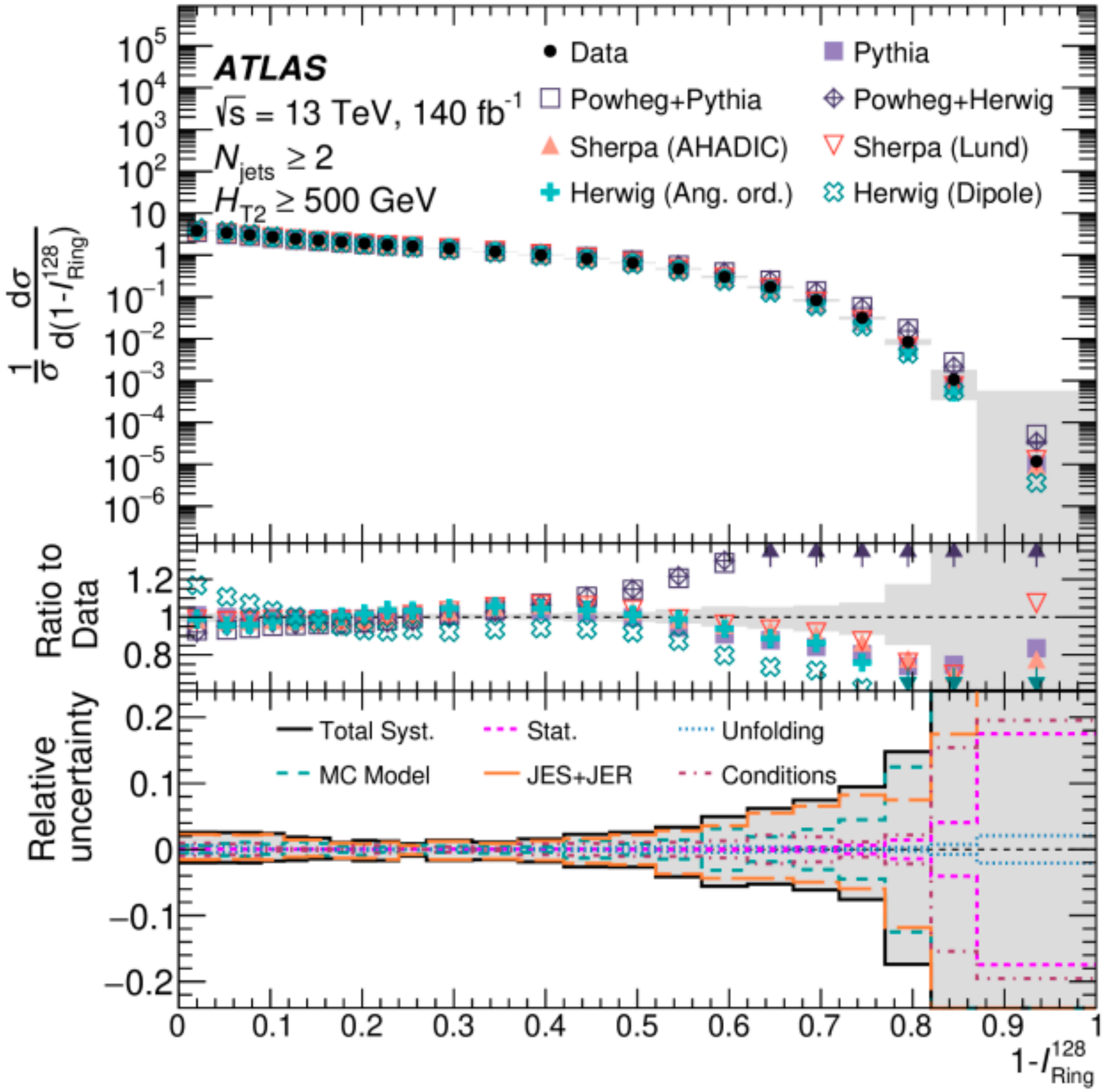
Looking for new physics at the 8th digit!

Deficiencies in theoretical predictions



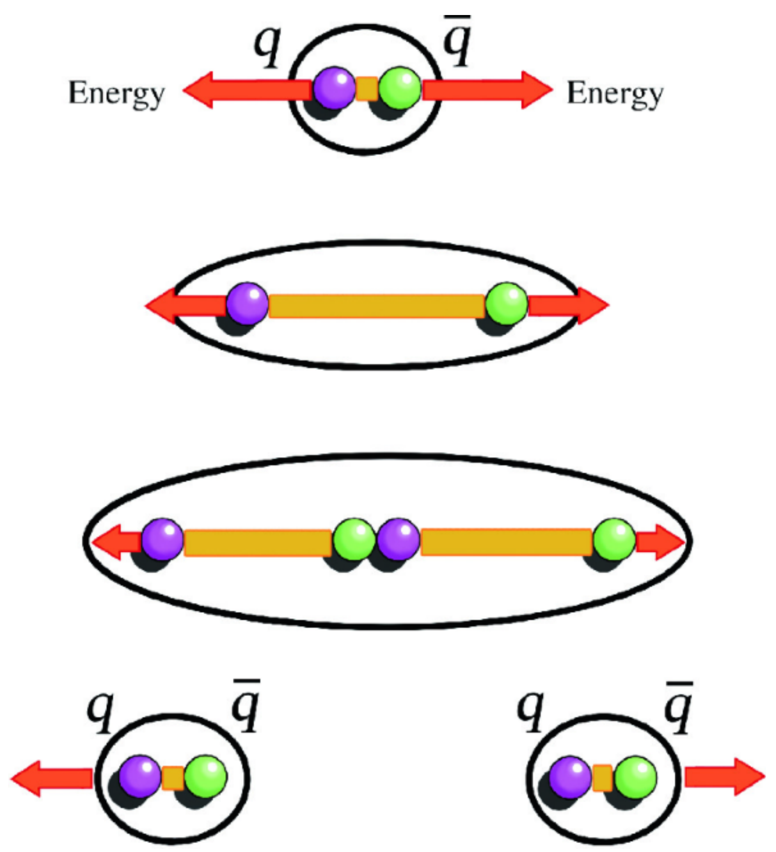
event isotropy: an event shape observable that measures the departure from perfect isotropy of collider event

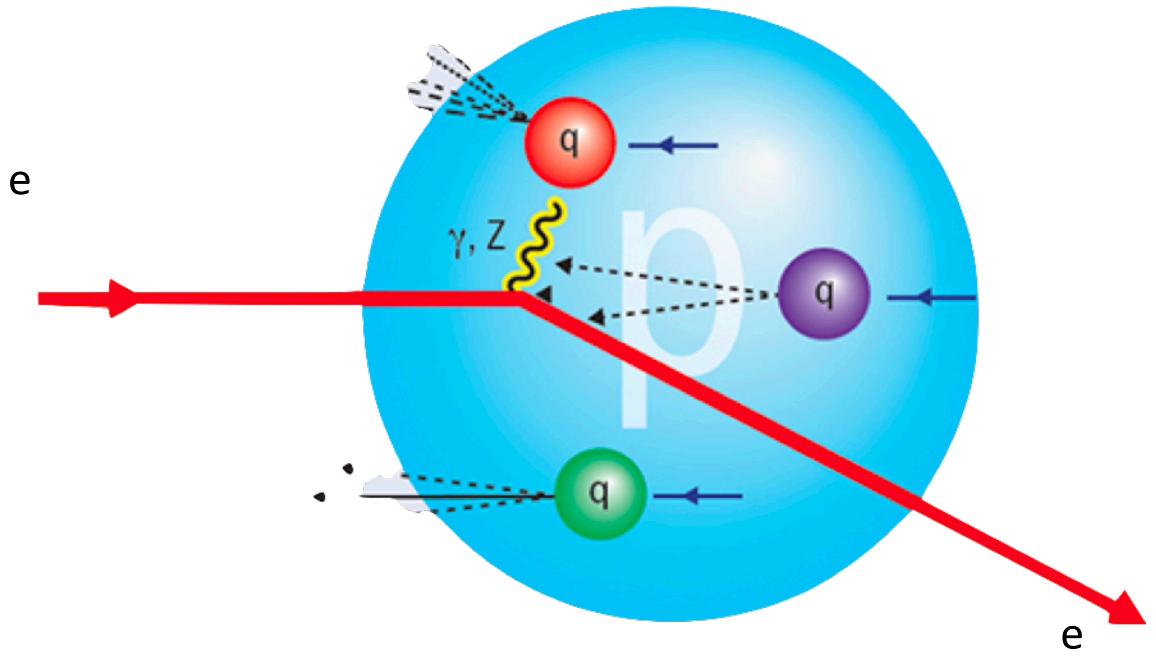
C. Cesarotti, J. Thaler

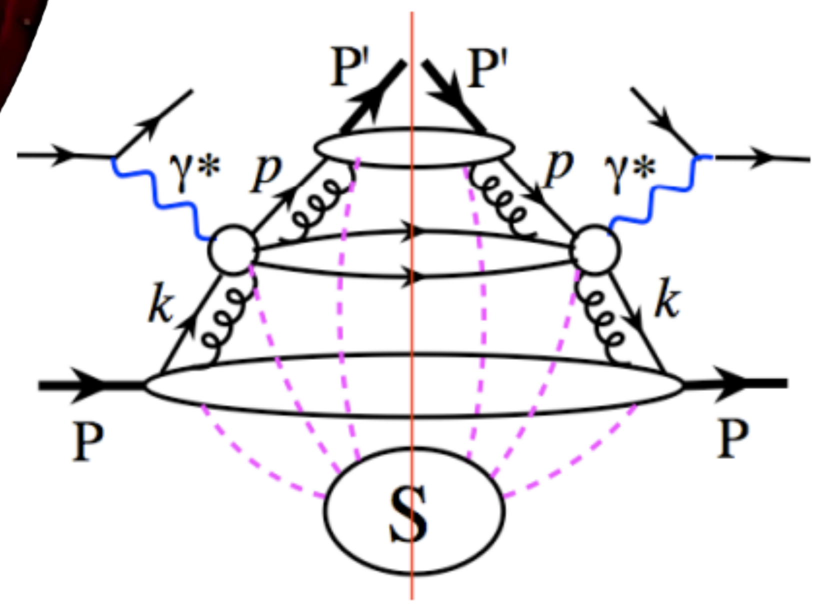


Theoretical predictions are far from perfect. Substantial space for progress!

Two faces of QCD





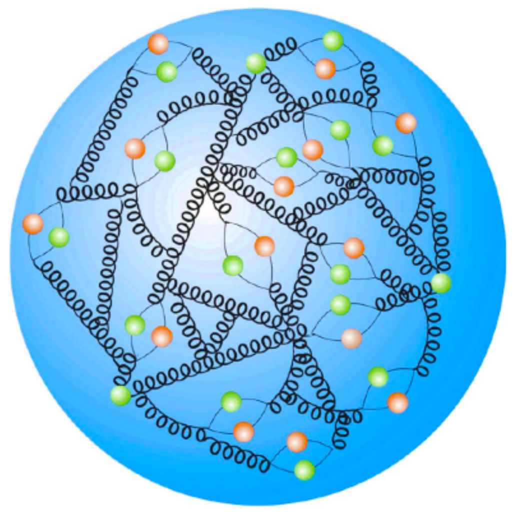


Feynman Diagrams
QCD Factorization
Effective Field Theories
Renormalization Group
Operator Product Expansion
……

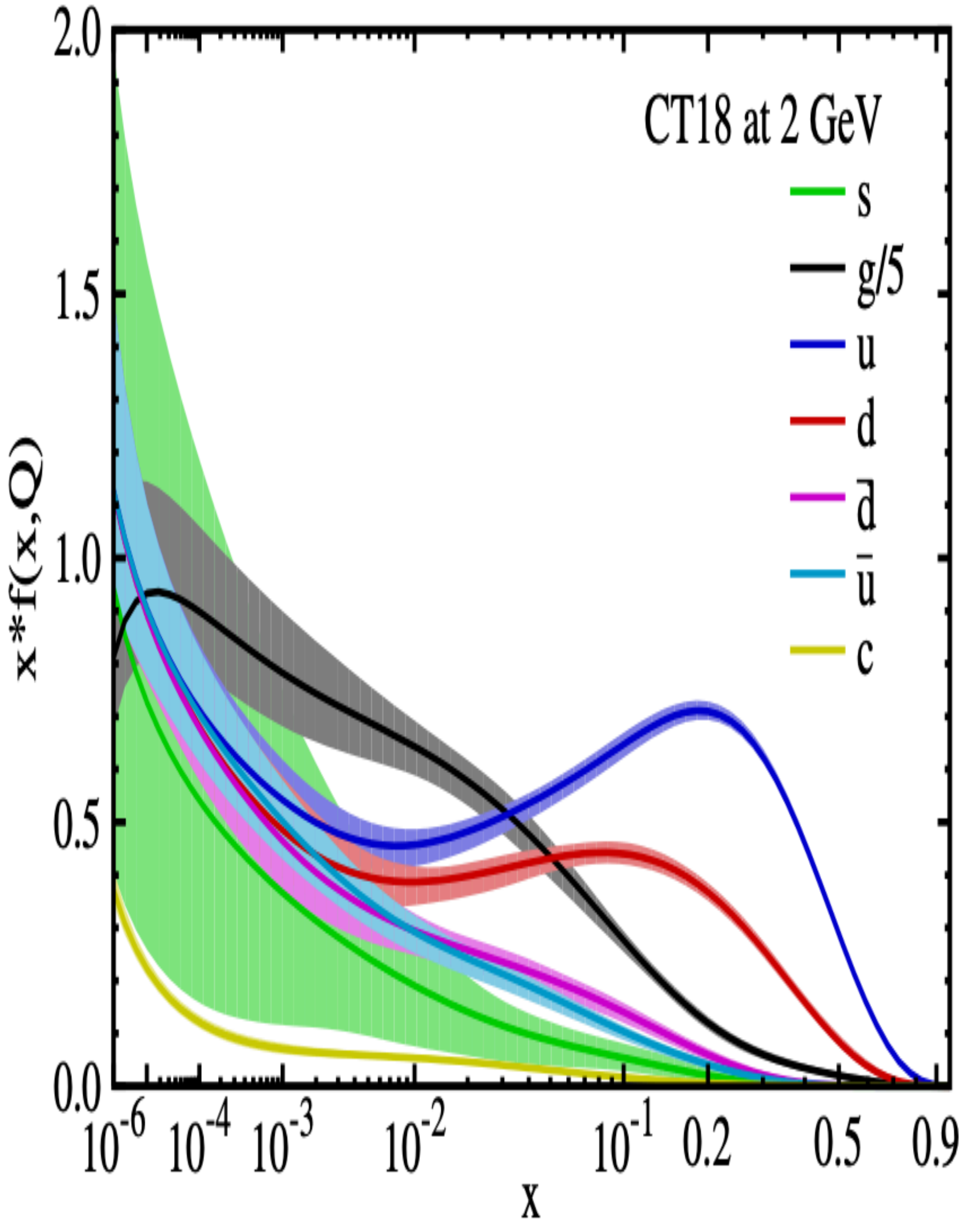
nonperturbative ⟷ factorization ⟷ perturbative

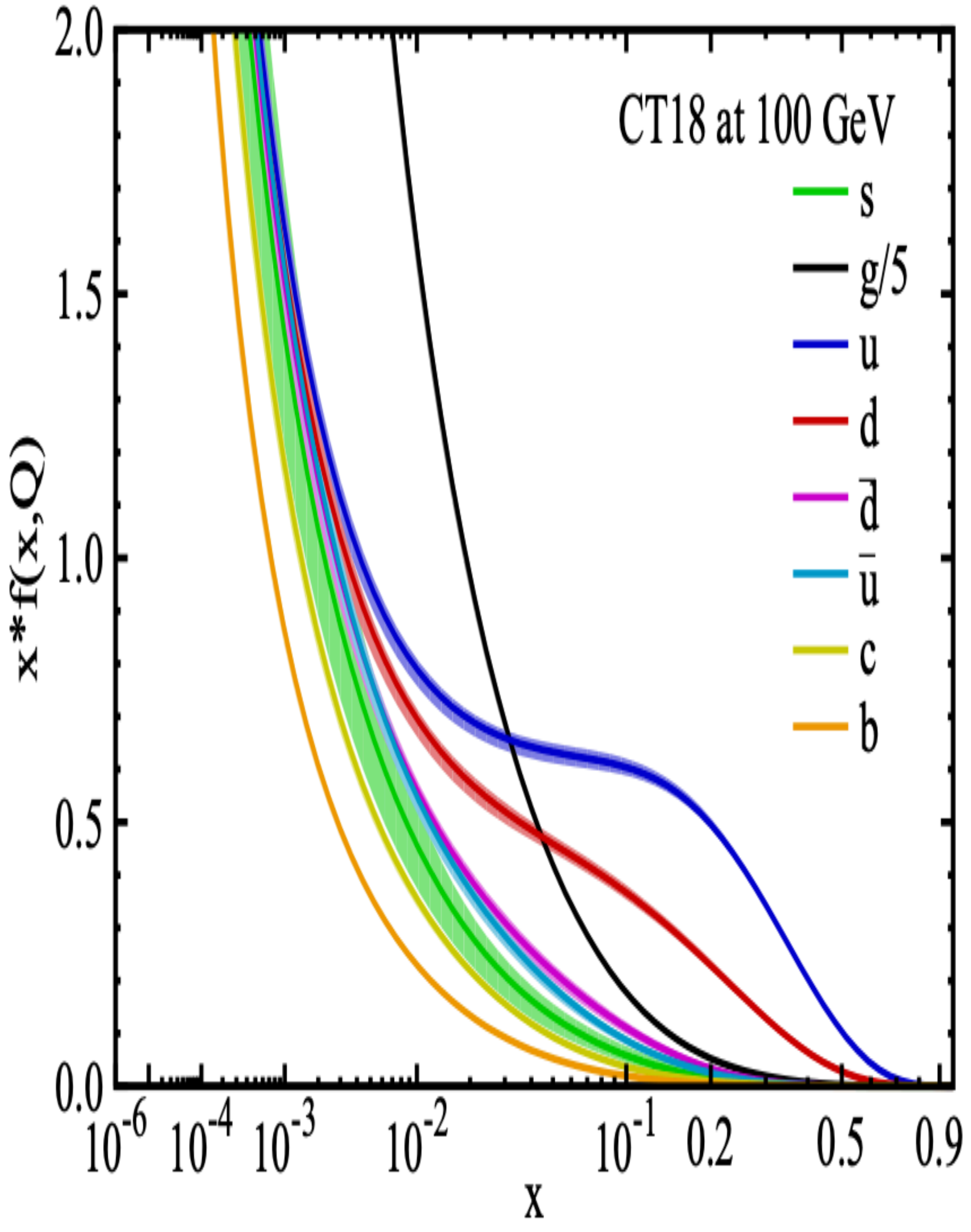
The two faces of QCD interwind together to form a full picture of QCD at the LHC

DGLAP evolution



$$\mu^2 \frac{\partial}{\partial \mu^2} f_i(x,\mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \sum_j \int_x^1 \frac{dy}{y} f_j(y,\mu^2) P_{ij}(x/y, \alpha_S(\mu^2))$$





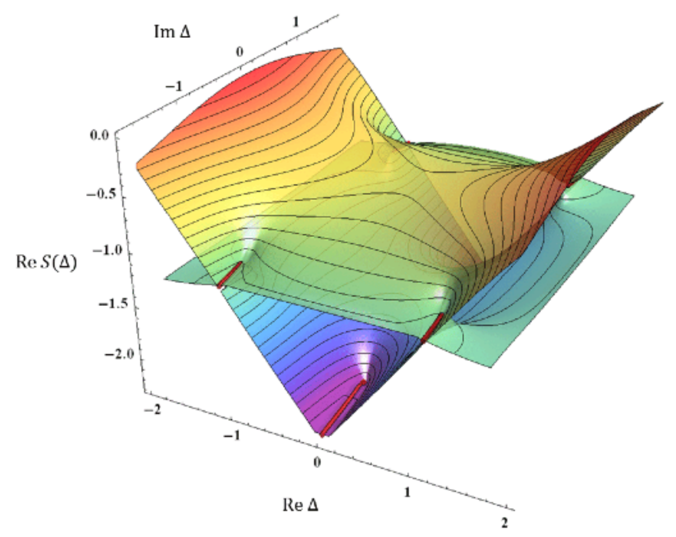
One loop: D. Gross, F. Wilczek, 1973 (twist operator)
Gribov, Lipatov; Altarelli, Parisi; Dokshitzer, 1976-1977

Two loops: G. Curci, W. Furmanski and R. Petronzio, 1980

Three loops: S. Moch, J. Vermaseren, A. Vogt, 2004

leading transcendentality: QCD => N=4 SYM

Integrability in N=4 SYM:



Towards 4 loops: S. Moch, B. Ruijl, T. Ueda, J. A. M. Vermaseren and A. Vogt: planar non-singlet

Towards DGLAP at 4 loops

Relation between twist-2 operator and DGLAP kernel

$$\gamma(n) = -\int_0^1 dx x^{n-1} P(x)$$

$$A_{ij} = \langle j(p)|O_i|j(p)\rangle \;\; \text{with } p^2 < 0$$

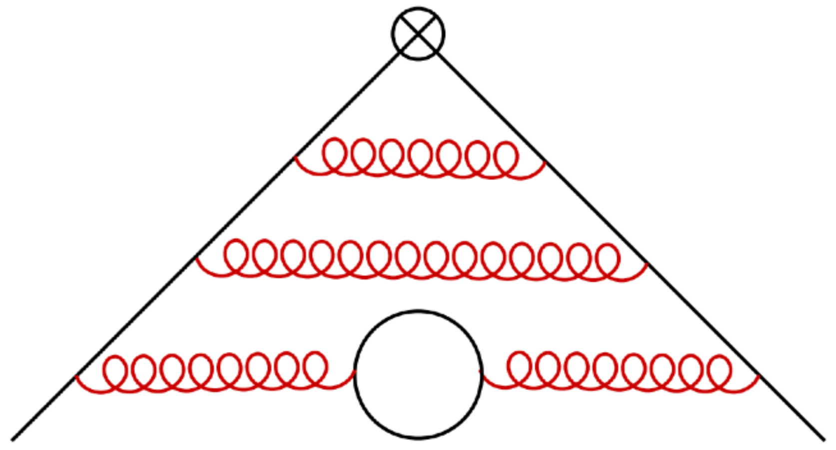
$$O_{\rm ns}(n) = \frac{i^{n-1}}{2}\left[\bar{\psi}_{i_1}\Delta\cdot\gamma(\Delta\cdot D)_{i_1 i_2}(\Delta\cdot D)_{i_2 i_3}\cdots(\Delta\cdot D)_{i_{n-1}i_n}\frac{\lambda_k}{2}\psi_{i_n}\right],\; k=3,\cdots N_f^2-1$$

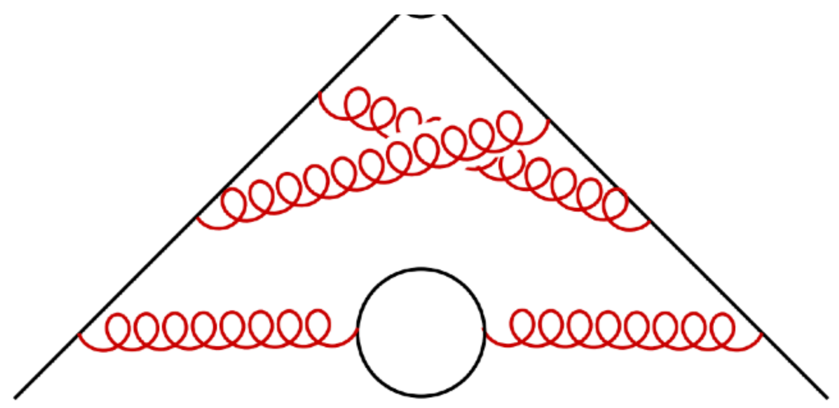
Computation with fixed moment in this way quickly explode in complexity

Turn fixed-order derivative into a generating function!

T. Gehrmann, A. von Manteuffel, V. Sotnikov, Tong-Zhi Yang, 2023

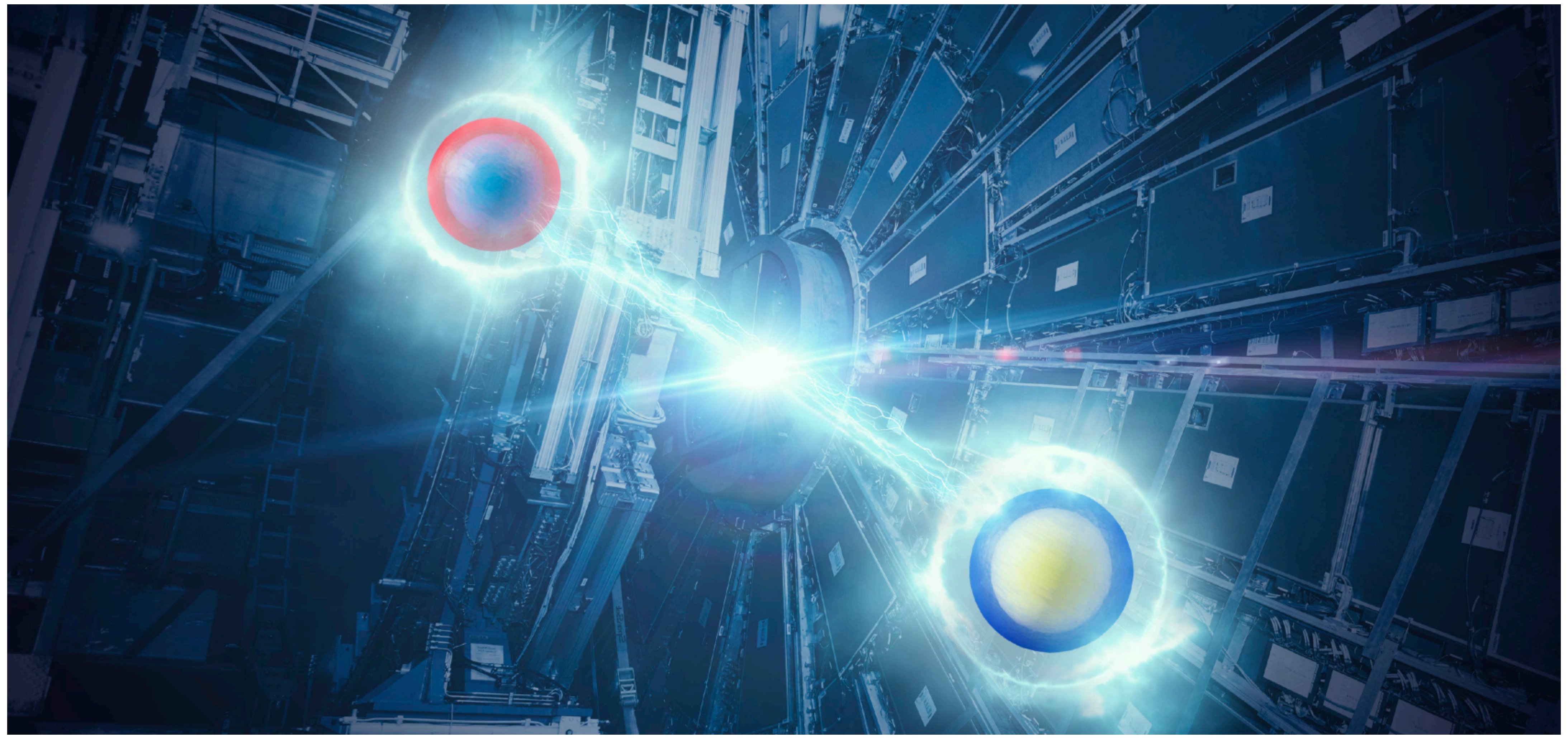
$$(\Delta\cdot p)^{n-1} \to \sum_{n=1}^{\infty} t^n (\Delta\cdot p)^{n-1} = \frac{t}{1-t\Delta\cdot p}$$





Non-planar Nf coefficient are now calculable!

Precision top quark physics

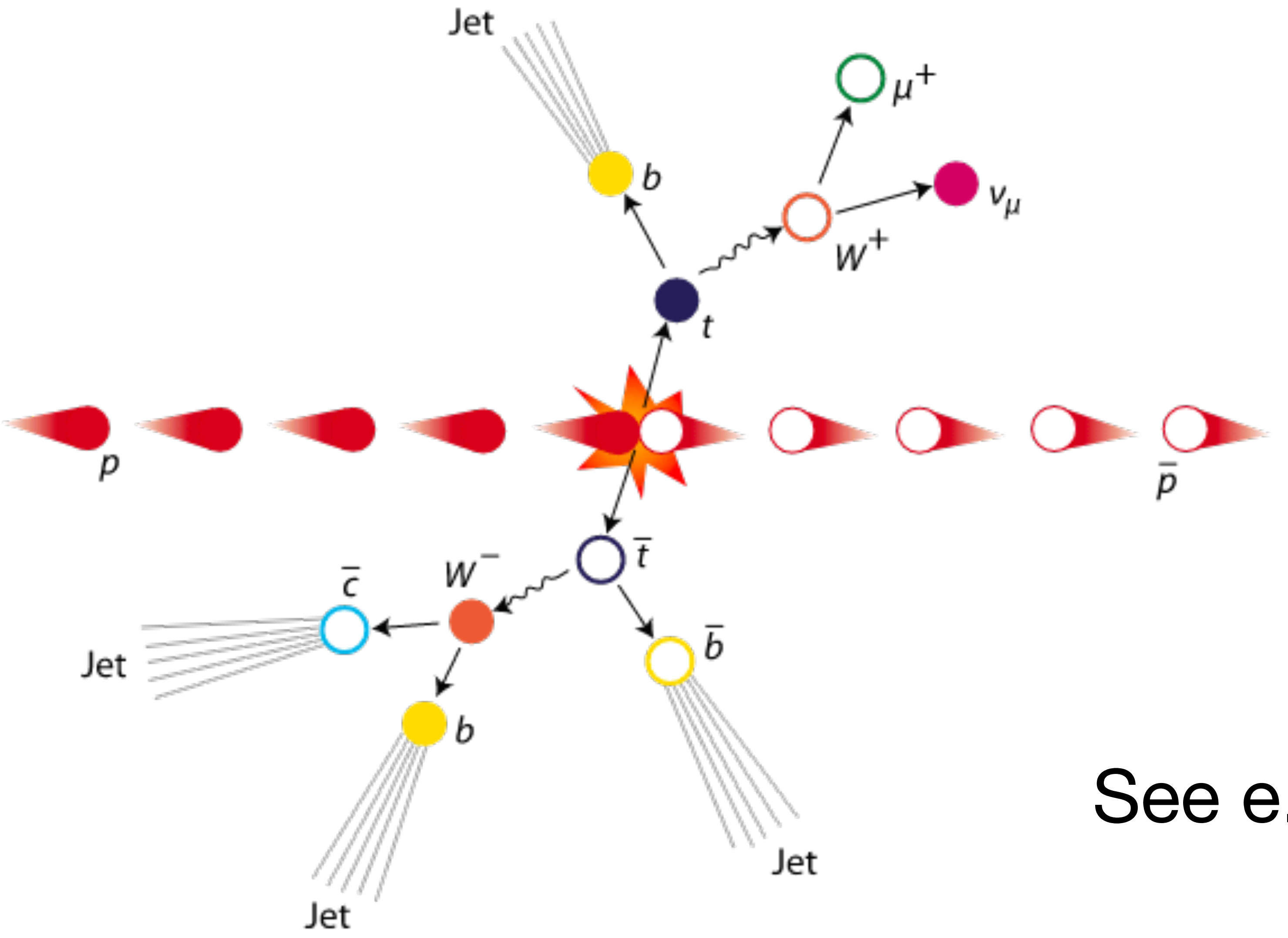


ATLAS achieves highest-energy detection of quantum entanglement

28 September 2023 I By ATLAS Collaboration

Call for production and decay of top pair to high precision!

Bernreuther, Brandenburg, Zong-Guo Si, P. Uwer, 2004



See e.g. Kun Cheng's talk

Top quark decay:

NLO: M. Jezabek and J. H. Kuhn (1989, approx.)
A. Czarnecki, (1990, approx.)
Chong Sheng Li, J. Oakes, T. C. Yuan, (1991, exact)

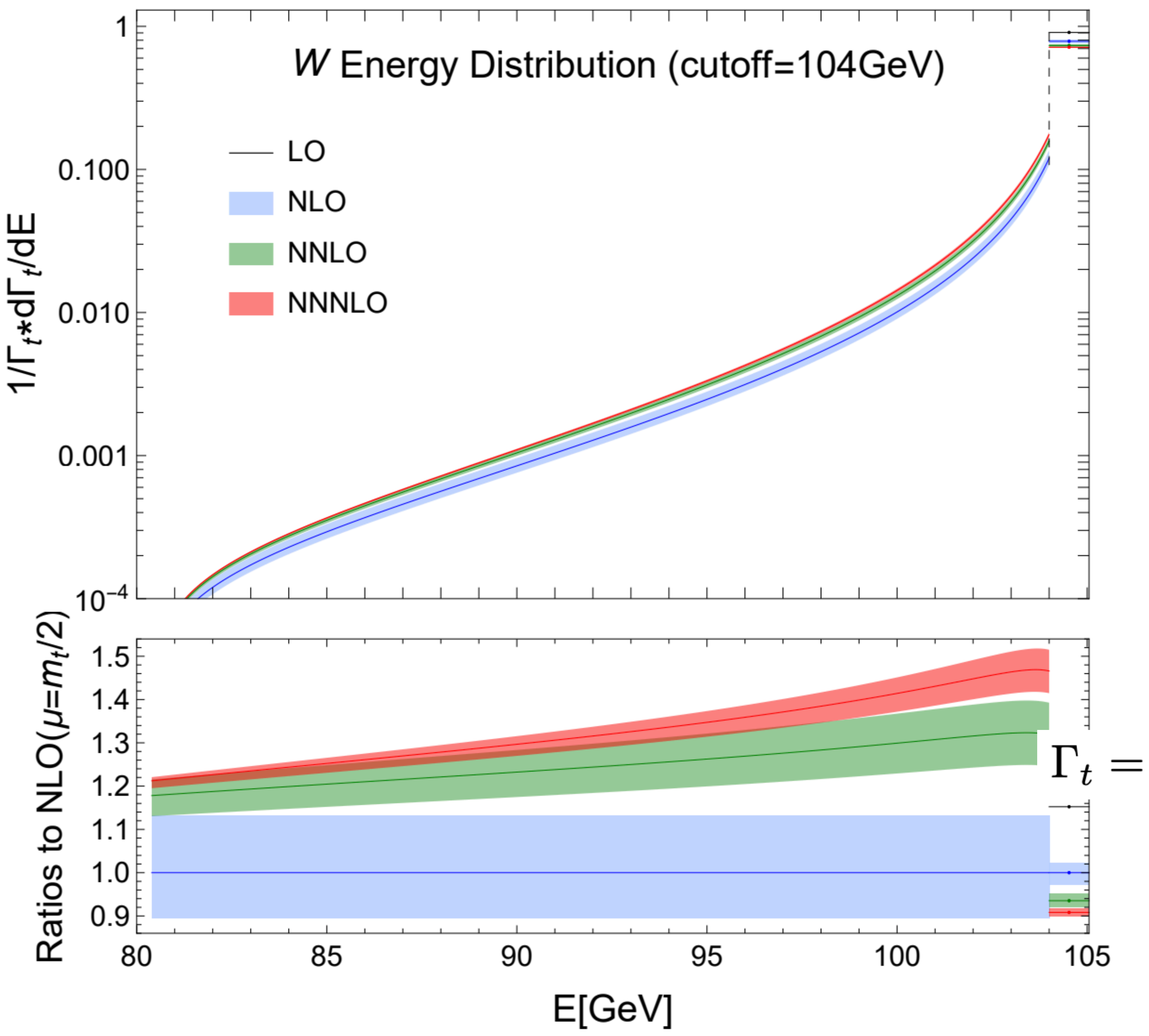
NNLO: Jun Gao, Chong Sheng Li, HXZ, 2012
M. Brucherseifer, F. Caola, and K. Melnikov 2013

Long-Bing Chen, Hai Tao Li, Jian Wang, Yefan Wang, 2022 (analytic)

Top decay at N3LO

Top-Quark Decay at Next-to-Next-to-Next-to-Leading Order in QCD

Long Chen,[1,*] Xiang Chen,[2,†] Xin Guan,[2,‡] and Yan-Qing Ma[2,3,§]



$$\Gamma_t = 1.48642 - 0.140877 - 0.023305 - 0.007240 \text{ GeV}$$
$$= 1.31500 \text{ GeV}, \qquad (5)$$

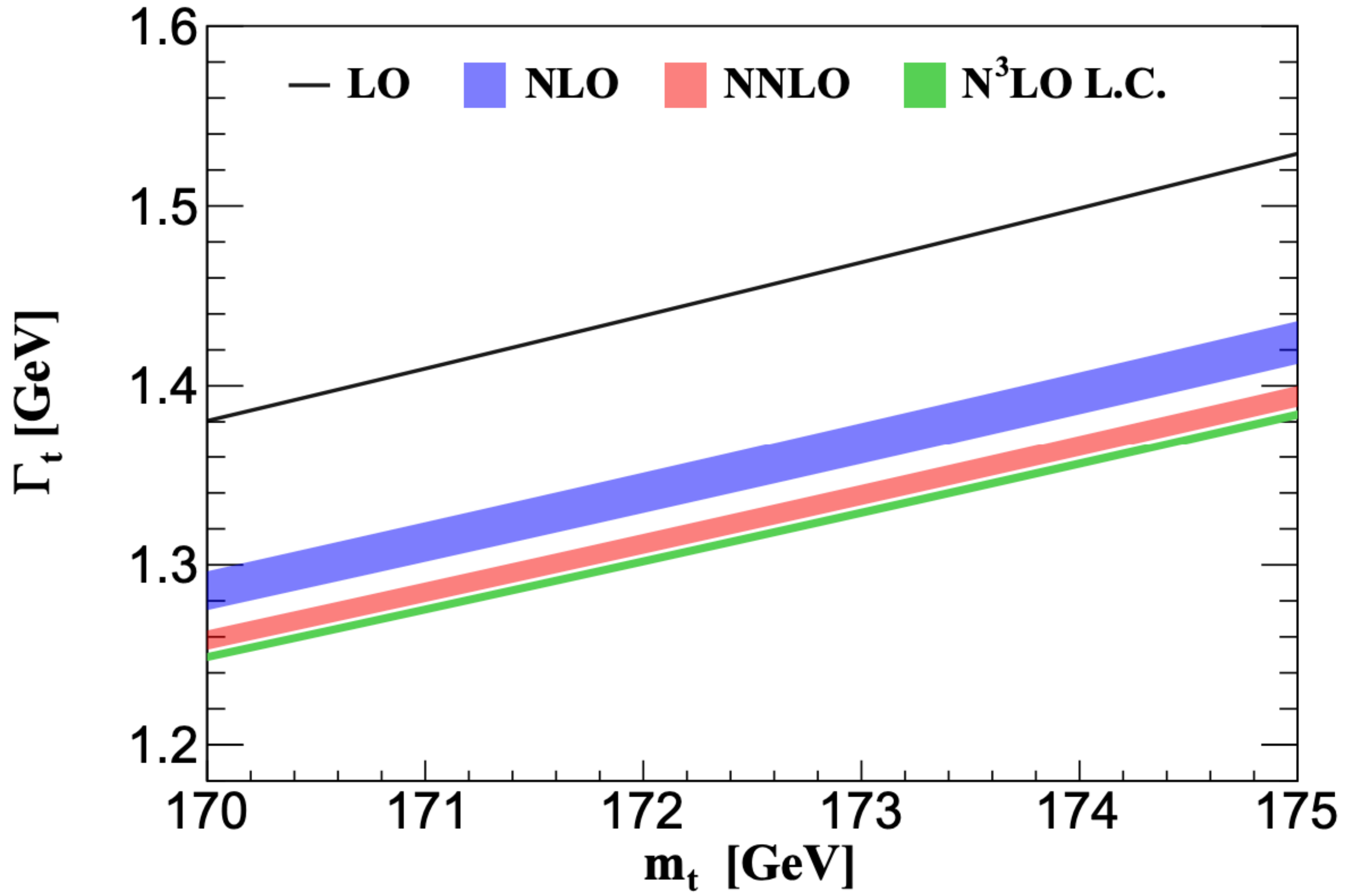
$$\Gamma_t = 1.36 \pm 0.02 \; (\text{stat.})^{+0.14}_{-0.11} \; (\text{syst.}) \text{ GeV}$$

current experimental uncertainty

See also Jian Wang's talk

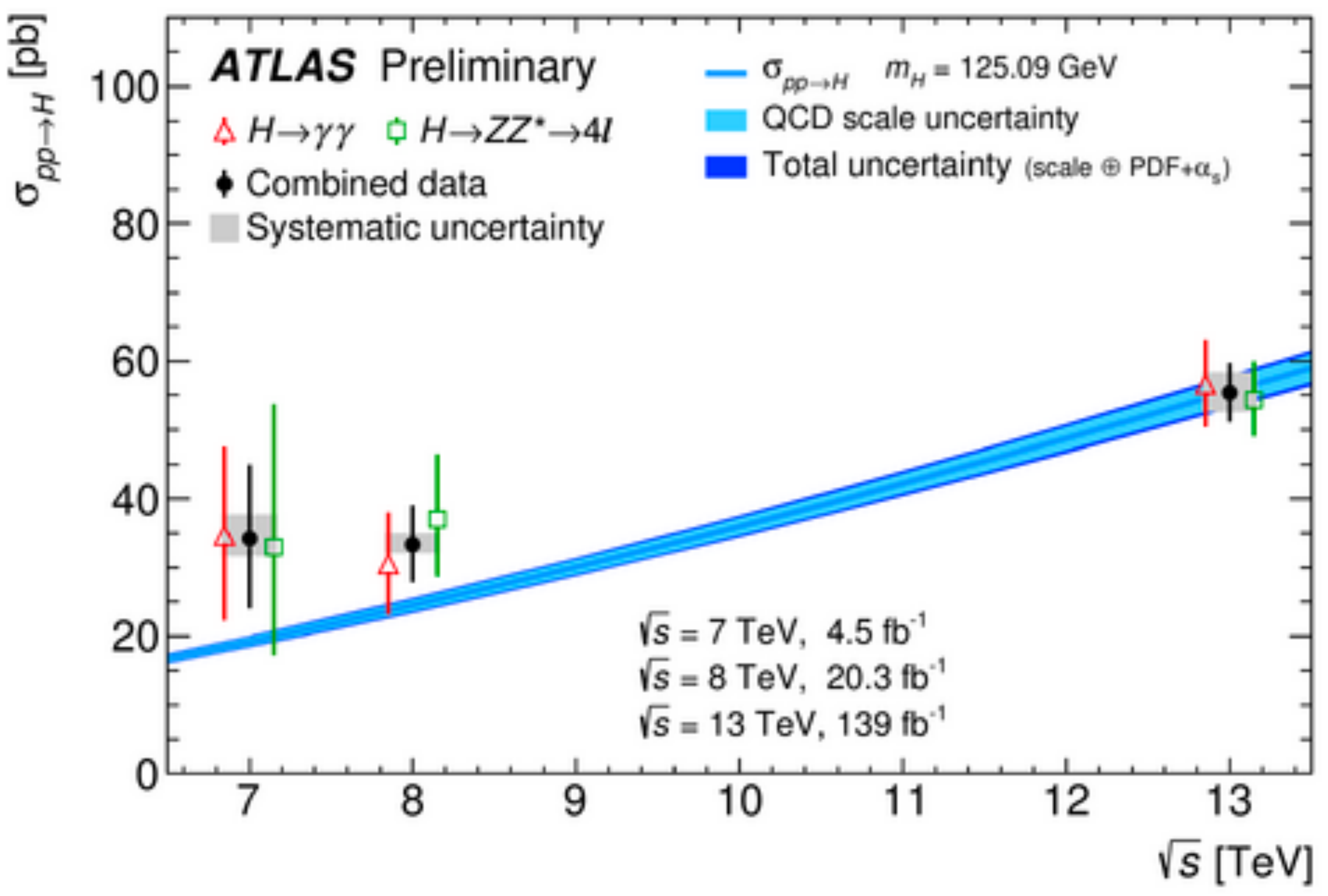
Analytic three-loop QCD corrections to top-quark and semileptonic $b \to u$ decays

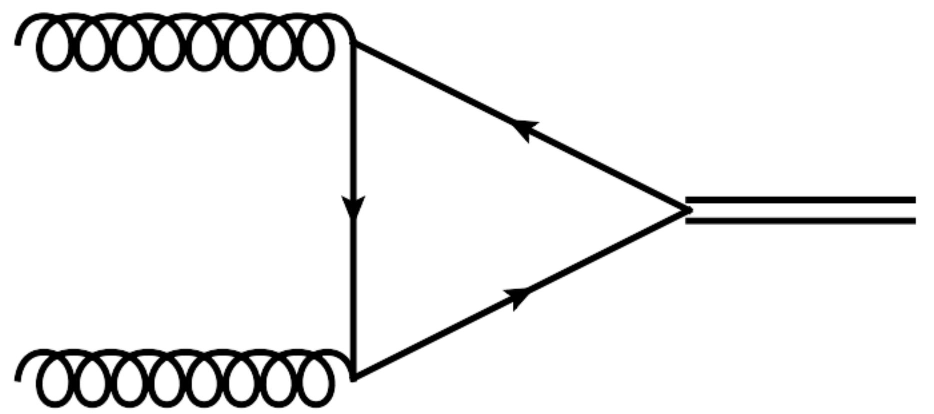
Long-Bin Chen,[1] Hai Tao Li,[2,*] Zhao Li,[3,4,5,†] Jian Wang,[2,‡] Yefan Wang,[2,§] and Quan-feng Wu[3,4,¶]



	$\delta_b^{(i)}$	$\delta_W^{(i)}$	$\delta_{\text{EW}}^{(i)}$	$\delta_{\text{QCD}}^{(i)}$	Γ_t [GeV]
LO	-0.273	-1.544	–	–	1.459
NLO	0.126	0.132	1.683	-8.575	$1.361^{+0.0091}_{-0.0130}$
NNLO	*	0.030	*	-2.070	$1.331^{+0.0055}_{-0.0051}$
N^{3}LO	*	0.009	*	-0.667	$1.321^{+0.0025}_{-0.0021}$

Precision Higgs production



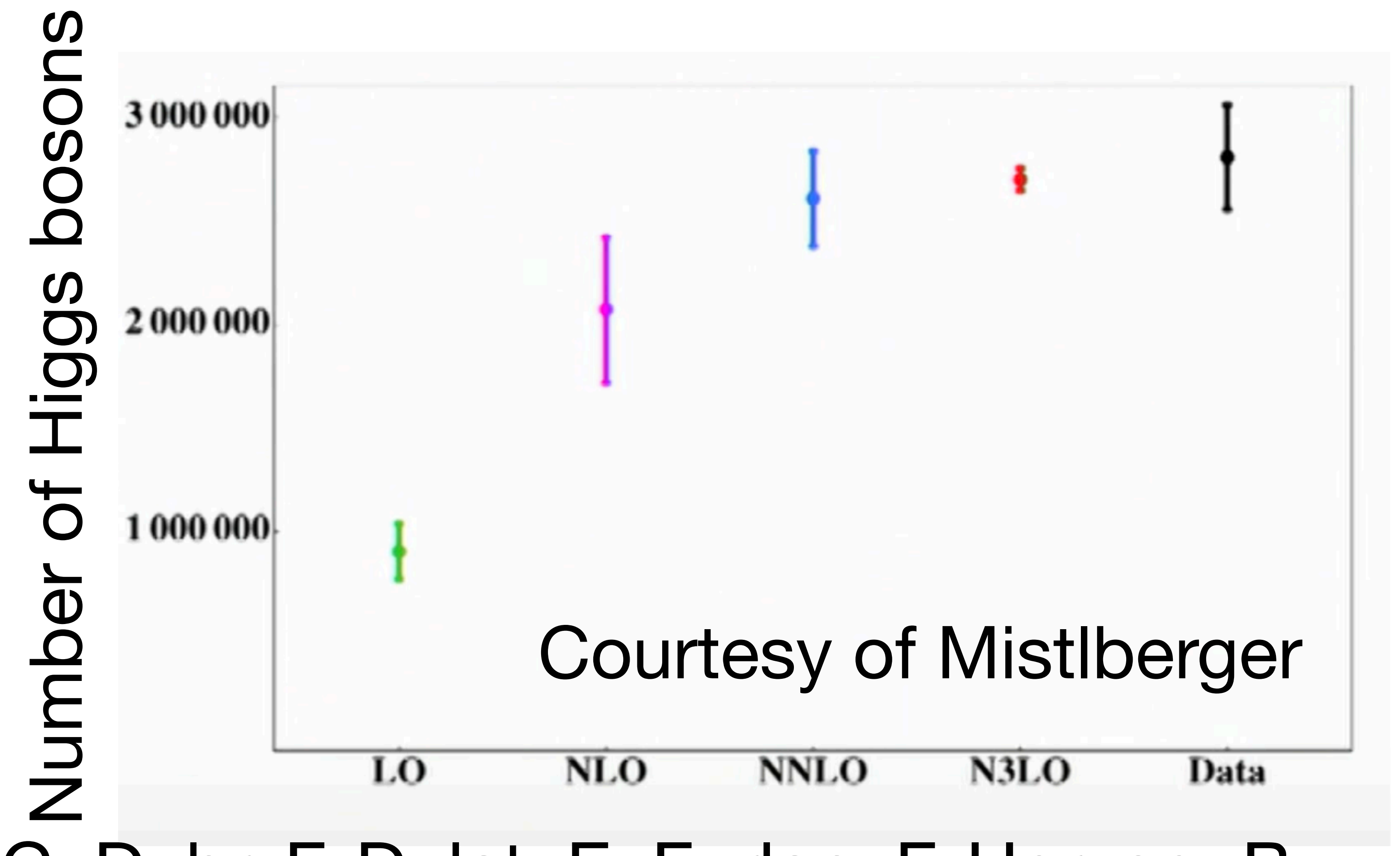


LO **NLO** **NNLO** **N3LO**

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(0)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} \ldots$$

Expect ten times of more data in the full LHC run. Call for even more precise theory prediction!

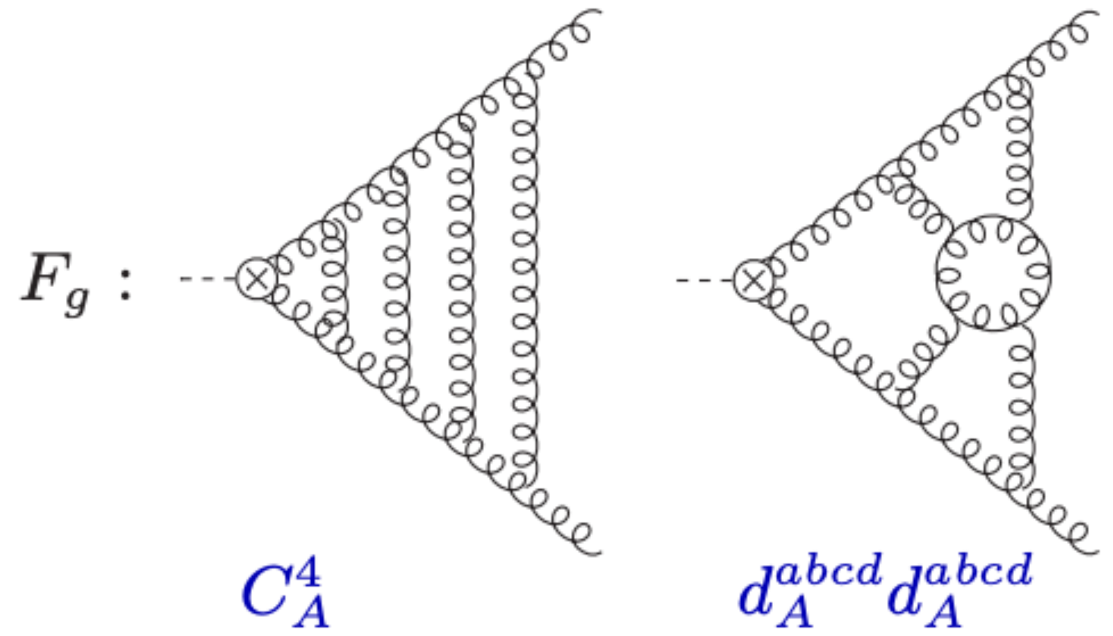
Order	Integrals
0	1
1	100
2	50000
3	517531178



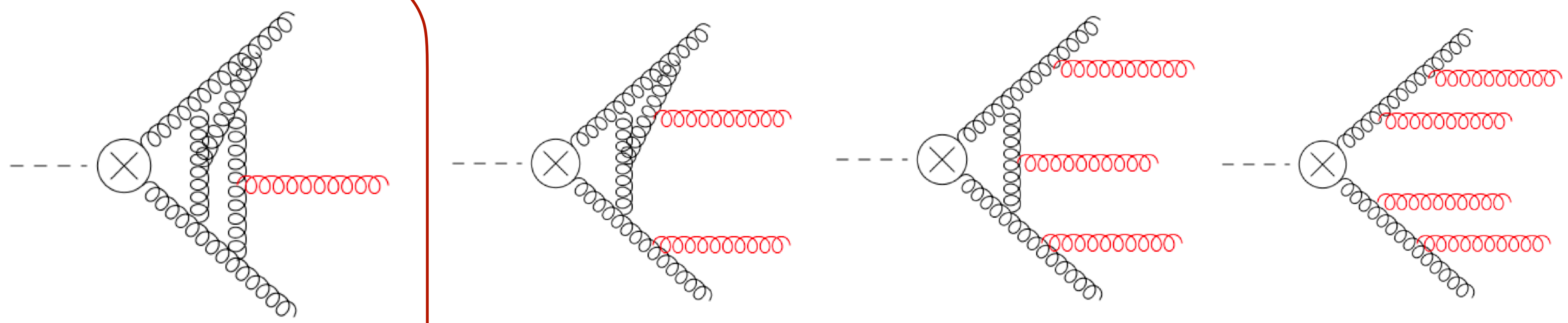
B. Anastasiou, C. Duhr, F. Dulat, E. Furlan, F. Herzog, B. Mistlberger, T. Gehrmann, A. Lazopolous, 2013-2015
B. Mistlberger, 2018

Towards Higgs production at N4LO

Anatomy of N4LO Higgs cross section near threshold



quadruple virtual



real-triple virtual

double real -double virtual

triple real-virtual

quadruple real

integral reduciton

canonical DE

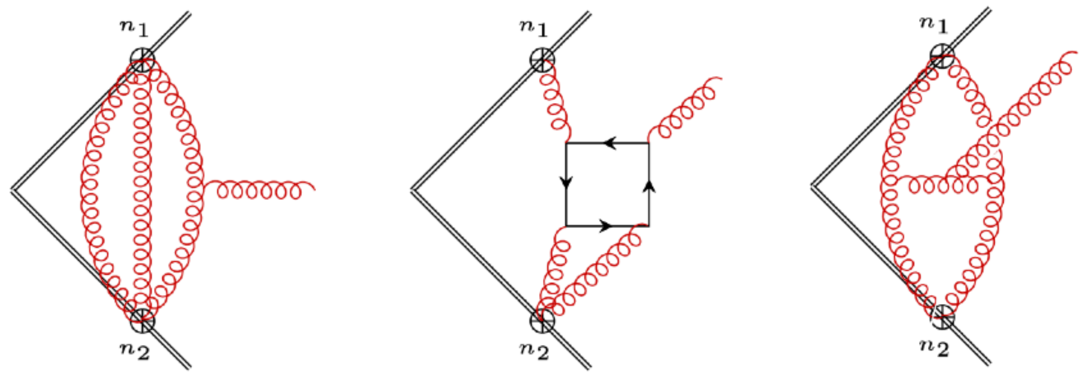
finite basis

<=> N=4 SYM

$$F_g^{(4)}\Big|_{\epsilon^0} = C_A^4\Big(-\frac{2591}{90}\zeta_{5,3}+\frac{1018949}{90}\zeta_5\zeta_3-\frac{35689}{27}\zeta_3^2\zeta_2+\frac{18282694}{7875}\zeta_2^4-\frac{27705161}{504}\zeta_7+\frac{1160731}{270}\zeta_5\zeta_2-\frac{1928564}{405}\zeta_3\zeta_2^2$$
$$-\frac{1296845}{1458}\zeta_3^2-\frac{727183}{1134}\zeta_2^3+\frac{6161623}{243}\zeta_5-\frac{3233651}{729}\zeta_3\zeta_2+\frac{54443689}{14580}\zeta_2^2+\frac{839716507}{104976}\zeta_2-\frac{84995881}{52488}\zeta_3+\frac{96887974603}{3779136}\Big)$$
$$+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(260\,\zeta_{5,3}-5092\,\zeta_5\zeta_3-16\,\zeta_3^2\zeta_2-\frac{496766}{525}\zeta_2^4-\frac{6776}{3}\zeta_7-5016\,\zeta_5\zeta_2+\frac{2992}{3}\zeta_3\zeta_2^2+\frac{31588}{3}\zeta_3^2$$
$$+\frac{1073972}{945}\zeta_2^3-6460\,\zeta_5+\frac{6752}{9}\zeta_3\zeta_2+\frac{24616}{45}\zeta_2^2-\frac{4682}{27}\zeta_2-\frac{1310}{9}+\frac{68410}{9}\zeta_3\Big)$$
+ contributions with closed fermion loop from Ref. [35].

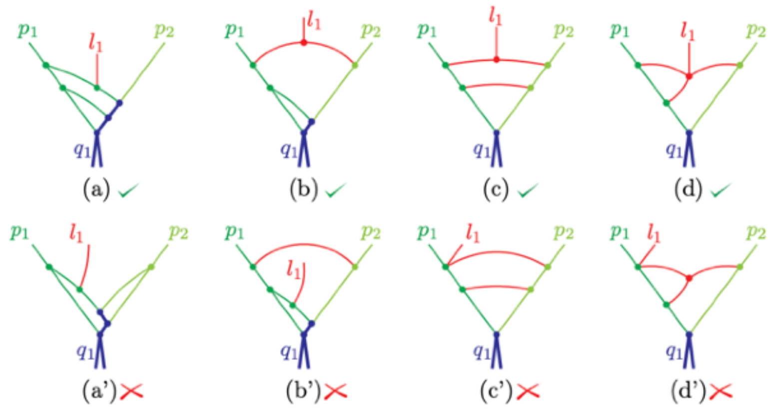
R. Lee, A. von Manteuffel, R. Schabinger, A. Smirnov, V. Smirnov, M. Steinhauser, 2022

SCET



Wen Chen, Ming-xing Luo, Tong-Zhi Yang, HXZ, 2023

Expansion by region



F. Herzog, Yao Ma, B. Mistlberger, A. Suresh, 2023

Soft photon theorem

Amplitude with two charged line emitting a soft photon

$$M_3(\{p_i\}, k, \epsilon(k)) \;=\; \sum_i \delta_i\, e_i\, \frac{p_i^\mu}{p_i \cdot k}\; \left[\epsilon_\mu(k) \;-\; (k_\mu \epsilon^\nu(k) - \epsilon_\mu(k) k^\nu)\; O_\nu\,(p_i, k)\right] M_2(\{p_i\})$$

Weinberg's soft theorem, 1965

Low, 1958; Burnett, Kroll, 1968

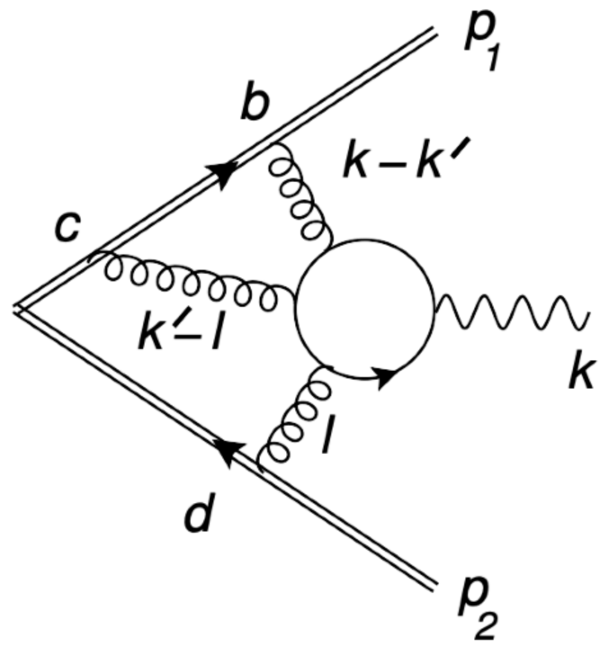
It is widely believed that Weinberg's soft photon theorem is exact to all orders.

The soft photon theorem [1–5] relates the leading infrared behavior of scattering amplitudes with and without single soft photon emission

$$\langle p_{m+1}, \ldots | a_\alpha(q) \mathcal{S} | p_1, \ldots \rangle = S_0 \langle p_{m+1}, \ldots | \mathcal{S} | p_1, \ldots \rangle + \mathcal{O}(q^0) \qquad (1.1)$$

where p_k is the momentum of the kth particle and a_α annihilates the momentum $q \to 0$ photon. The soft factor S_0 (equation (2.1) below) has a pole in q . The formula (1.1) is exact as long as there are no magnetic monopoles among the asymptotic particles. In this paper we argue that the general form of the relation (1.1) remains valid in the presence of monopoles, but the formula for S_0 is corrected. Electromagnetic duality transformations

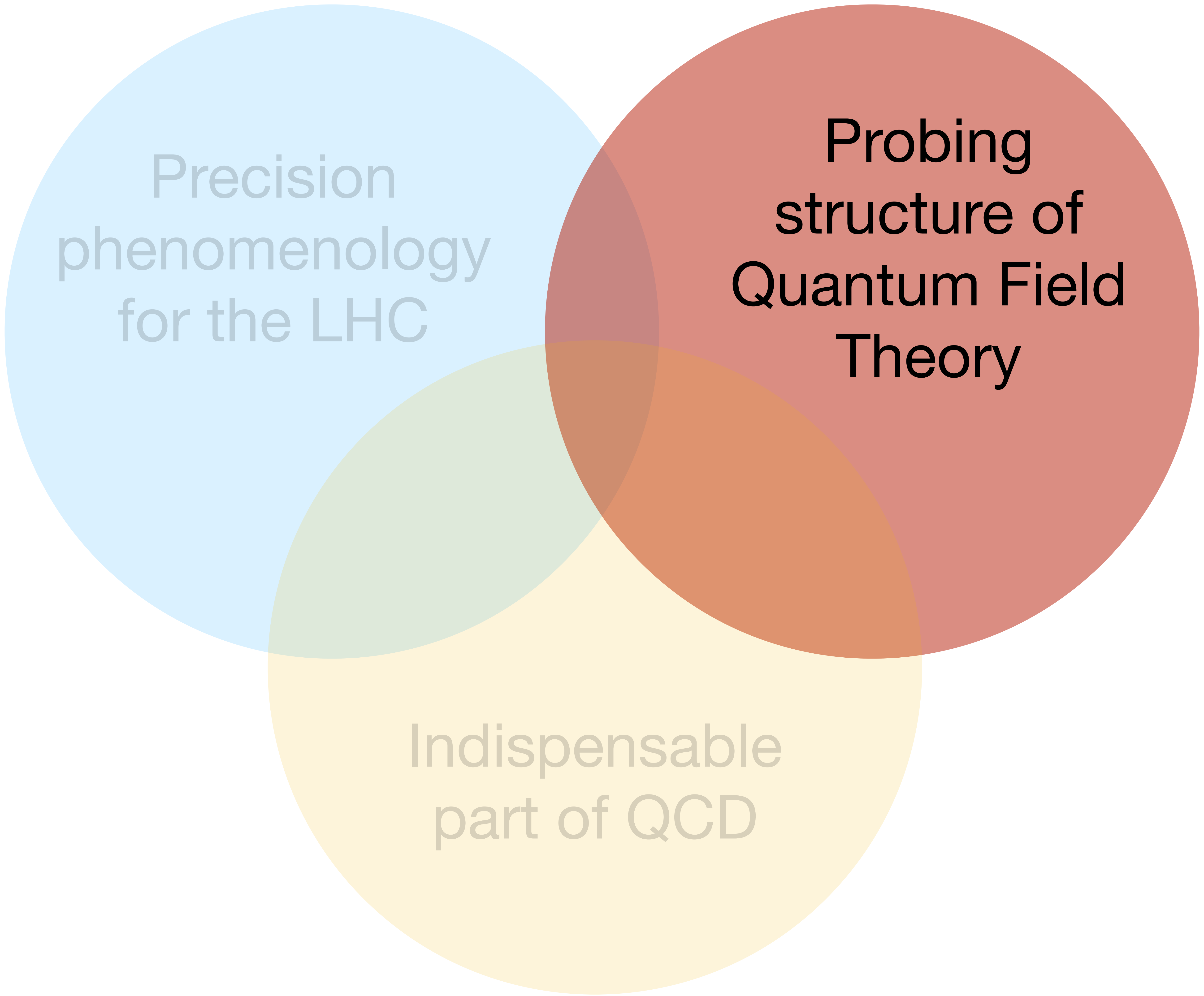
A. Strominger, 2015



But actual calculation reveals that it's non-vanishing!

Yao Ma, G. Sterman, A. Venkata, 2023

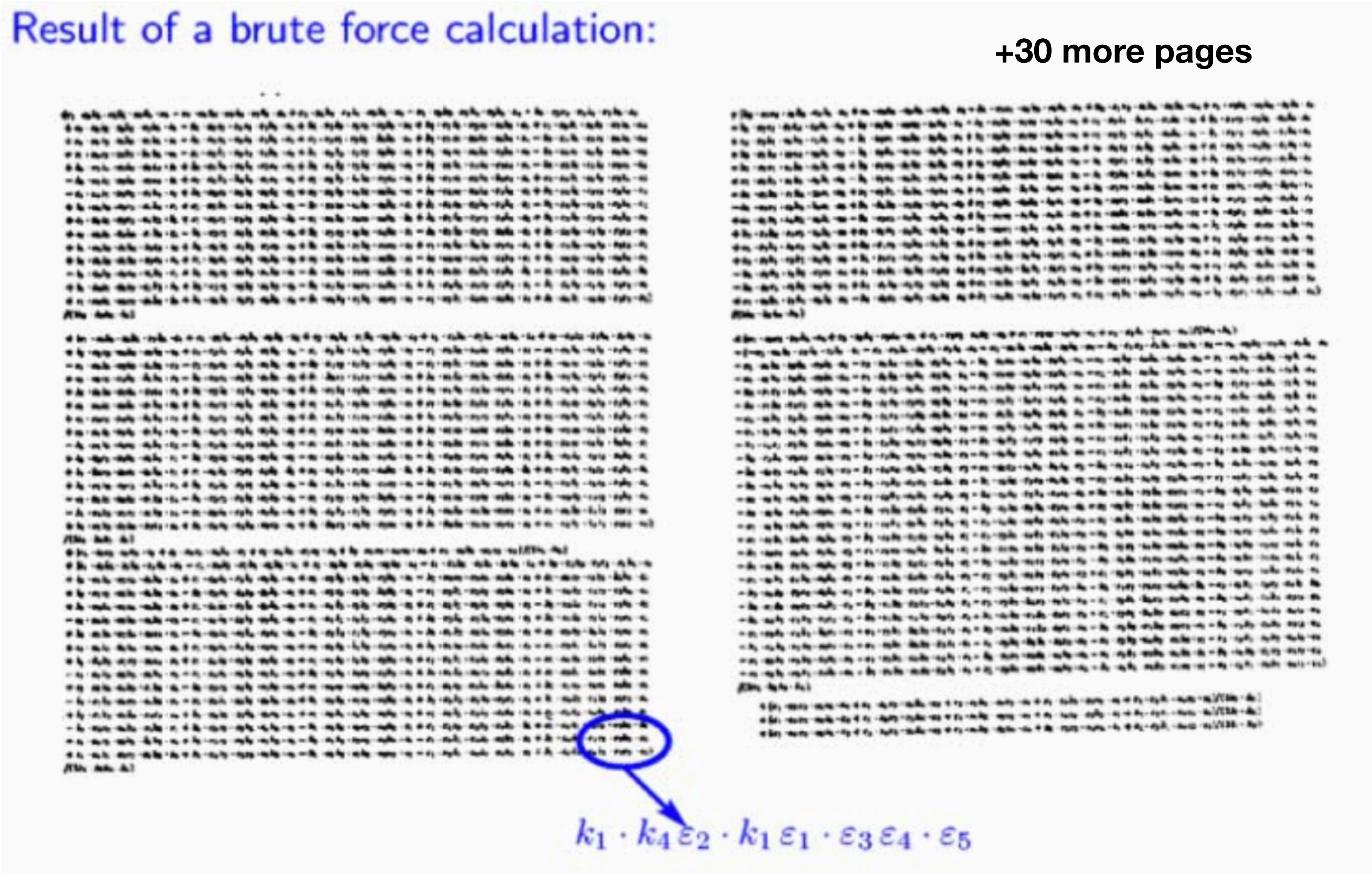
Plan of this talk



Unreasonable simplicity of Quantum Field Theory

The Chinese Magic

Zhan Xu, Da-Hua Zhang, Li Chang, 1985



$$A(1^+, 2^+, \cdots, i^-, \cdots, j^-, \cdots, n^+) = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n-1, n \rangle}$$

Parke, Taylor, 1988

The symbol magic

Goncharov

$$R_6^{(2)}(u_1, u_2, u_3) = \sum_{i=1}^{3} \left(L_4(x_i^+, x_i^-) - \frac{1}{2} \mathrm{Li}_4(1 - 1/u_i) \right)$$
$$- \frac{1}{8} \left(\sum_{i=1}^{3} \mathrm{Li}_2(1 - 1/u_i) \right)^2 + \frac{1}{24} J^4 + \frac{\pi^2}{12} J^2 + \frac{\pi^4}{72}. \quad (3)$$

$$u_1 = \frac{s_{12} s_{45}}{s_{123} s_{345}}, \quad u_2 = \frac{s_{23} s_{56}}{s_{234} s_{123}}, \quad u_3 = \frac{s_{34} s_{61}}{s_{345} s_{234}}, \qquad x_i^\pm = u_i x^\pm, \quad x^\pm = \frac{u_1 + u_2 + u_3 - 1 \pm \sqrt{\Delta}}{2 u_1 u_2 u_3},$$

Correct choice of variables

$$L_4(x^+, x^-) = \frac{1}{8!!} \log(x^+ x^-)^4$$
$$+ \sum_{m=0}^{3} \frac{(-1)^m}{(2m)!!} \log(x^+ x^-)^m (\ell_{4-m}(x^+) + \ell_{4-m}(x^-))$$

$$\ell_n(x) = \frac{1}{2} \left(\mathrm{Li}_n(x) - (-1)^n \, \mathrm{Li}_n(1/x) \right)$$

Remarkable relation between special functions

A. Goncharov, M Spradlin, C. Vergu, A. Volovich, 2010

Special functions in Feynman integrals

Who ordered those functions?

$$\log(x) \longrightarrow \mathrm{Li}_2(x) \longrightarrow \mathrm{Li}_n(x)\,, \mathrm{HPL}[\{n_1, n_2, \cdots, n_m\}, x] \longrightarrow$$

$$K(w^2) = \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-w^2x^2)}}\,, \qquad 0 < w^2 < 1\,,$$

$$E(w^2) = \int_0^1 dx \sqrt{\frac{1-w^2x^2}{1-x^2}}\,, \qquad 0 < w^2 < 1\,,$$

differential equations

Tkachov, 1981

$$F(x,\epsilon) = \sum_i R_i(x,\epsilon) I_i(x,\epsilon) \longrightarrow$$

Remiddi, Gehrmann, 1999

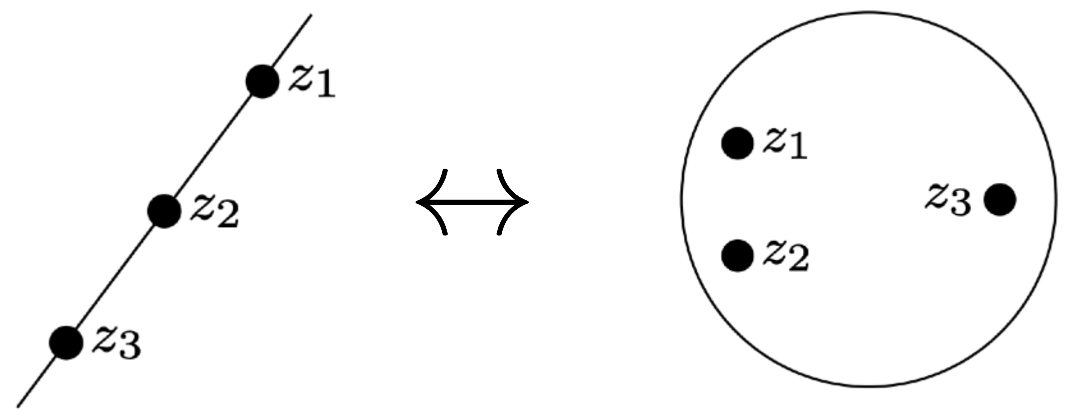
$$\frac{d}{dx} I_i(x,\epsilon) = \sum_j A_{ij}(x,\epsilon) I_j(x,\epsilon) \longrightarrow$$

Henn, 2013

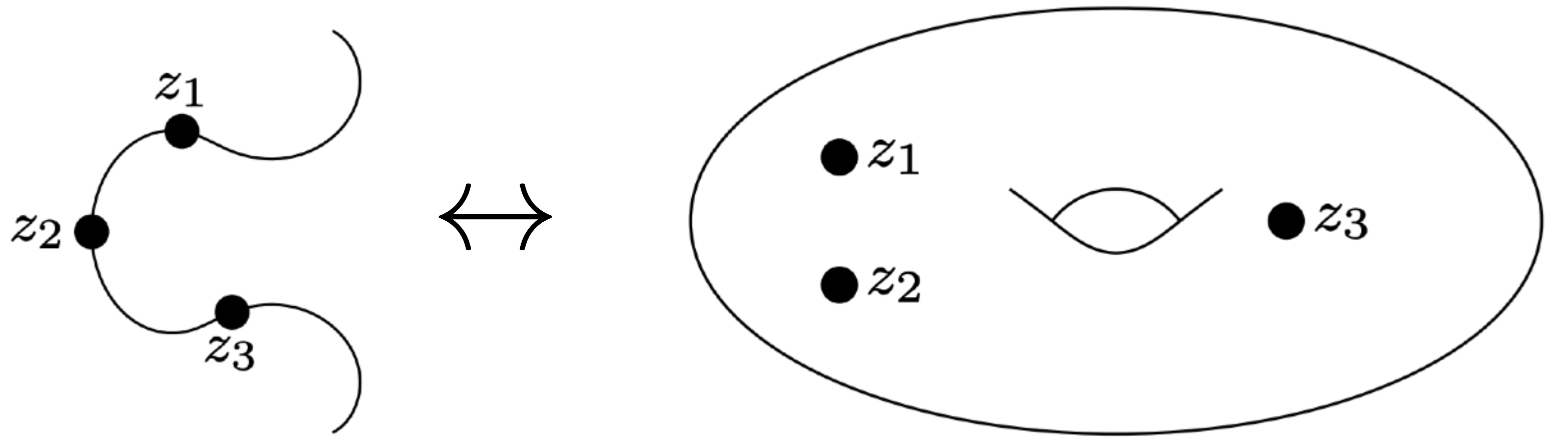
$$\frac{d}{dx} \tilde{I}_i(x,\epsilon) = \epsilon \sum_j A_{ij}(x) \tilde{I}_j(x,\epsilon)$$

$$G(z_1,\ldots,z_k;\lambda) = \int_0^{\lambda} \frac{d\lambda_1}{\lambda_1 - z_1} \int_0^{\lambda_1} \frac{d\lambda_2}{\lambda_2 - z_2} \cdots \int_0^{\lambda_{k-1}} \frac{d\lambda_k}{\lambda_k - z_k}\,, \qquad z_k \neq 0$$

Geometry



genus 0 complex curve



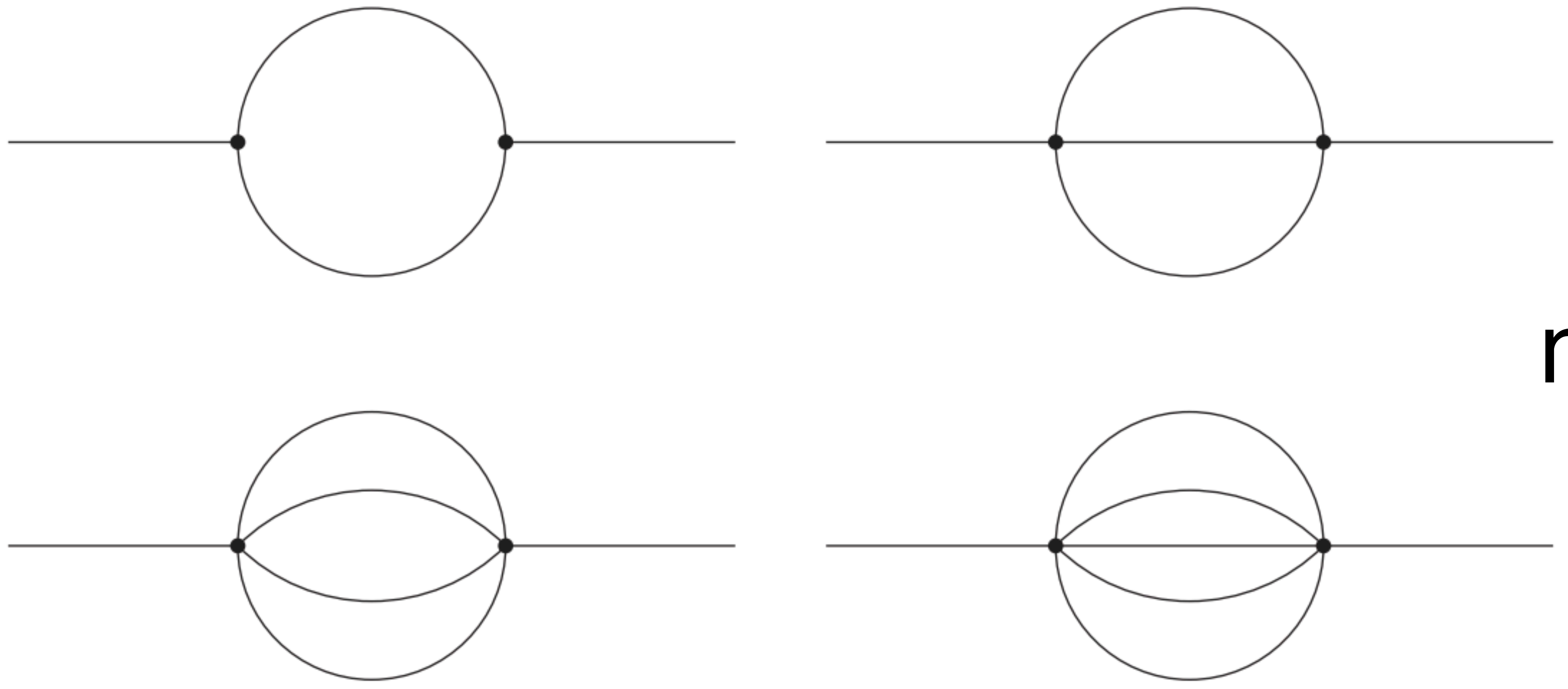
genus 1 complex curve

→ higher genus curve

→ higher dimensional varieties

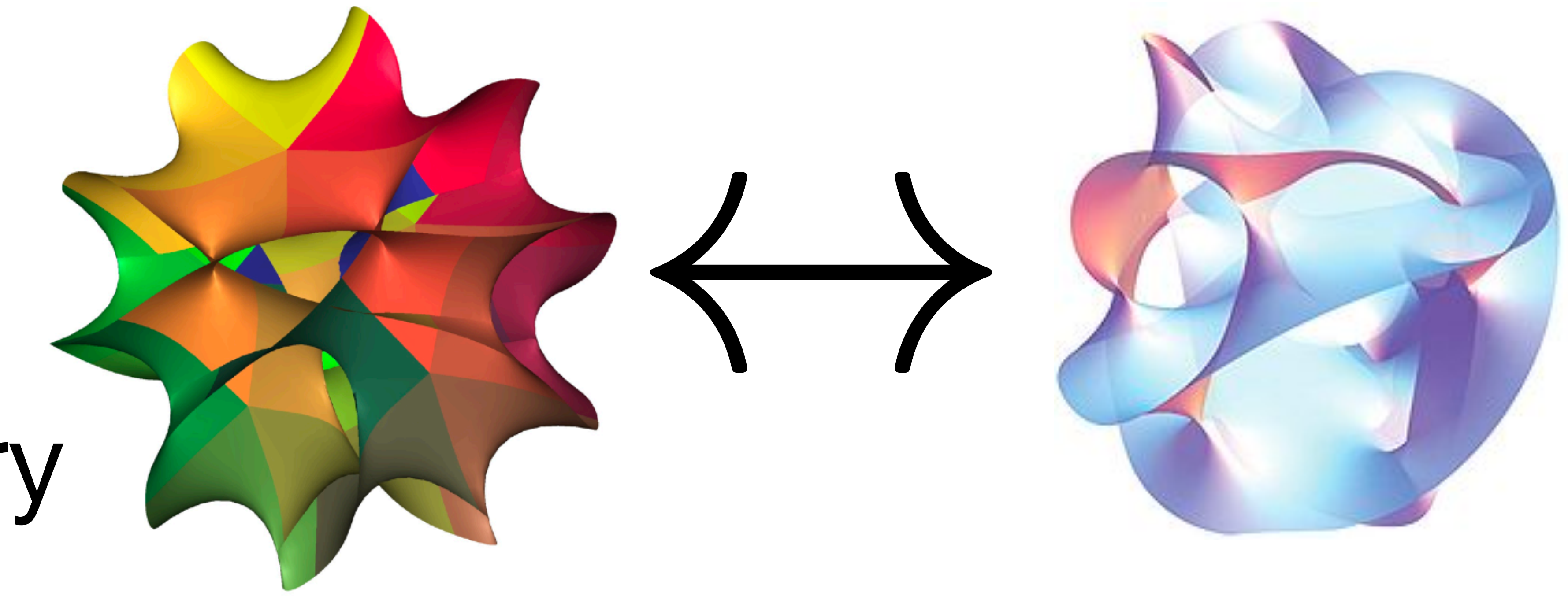
Calabi-Yau manifolds

banana integrals up to four loops



S. Pogel, Xing Wang, S. Weinzierl, 2022, 2023

mirror symmetry



$$\theta = q\frac{d}{dq}$$

$l = 0:\quad 1$

$l = 1:\quad \theta$

$l = 2:\quad \theta\cdot\theta$

$l = 3:\quad \theta\cdot\theta\cdot\theta$

$l = 4:\quad \theta\cdot\theta\cdot\frac{1}{Y_2}\cdot\theta\cdot\theta$

$l = 5:\quad \theta\cdot\theta\cdot\frac{1}{Y_2}\cdot\theta\cdot\frac{1}{Y_2}\cdot\theta\cdot\theta$

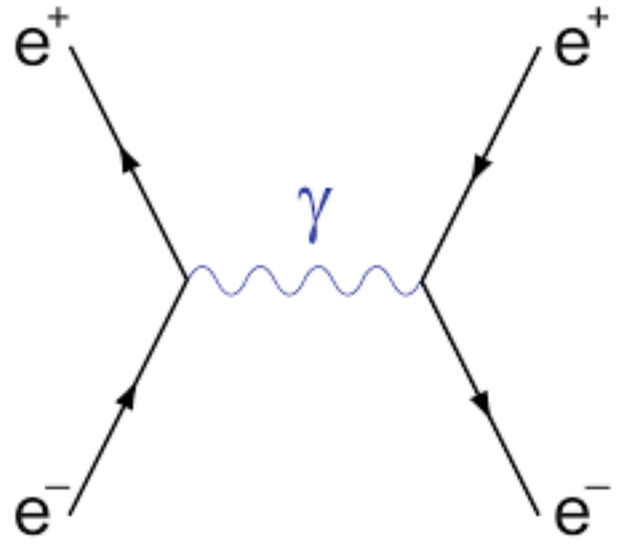
$l = 6:\quad \theta\cdot\theta\cdot\frac{1}{Y_2}\cdot\theta\cdot\frac{1}{Y_3}\cdot\theta\cdot\frac{1}{Y_2}\cdot\theta\cdot\theta$

$l = 7:\quad \theta\cdot\theta\cdot\frac{1}{Y_2}\cdot\theta\cdot\frac{1}{Y_3}\cdot\theta\cdot\frac{1}{Y_3}\cdot\theta\cdot\frac{1}{Y_2}\cdot\theta\cdot\theta$

$$J\frac{d}{dy}M = \varepsilon \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & F_{11} & 1 & 0 & 0 & & 0 & 0 \\ 0 & F_{21} & F_{22} & Y_2 & 0 & & 0 & 0 \\ 0 & F_{31} & F_{32} & F_{33} & Y_3 & & 0 & 0 \\ \vdots & & & & & \ddots & & \vdots \\ 0 & F_{(l-2)1} & F_{(l-2)2} & F_{(l-2)3} & F_{(l-2)4} & \cdots & Y_{l-2} & 0 \\ 0 & F_{(l-1)1} & F_{(l-1)2} & F_{(l-1)3} & F_{(l-1)4} & \cdots & F_{(l-1)(l-1)} & 1 \\ * & * & * & * & * & \cdots & * & * \end{pmatrix} M.$$

All loops iterated solution of banana integrals!

Bhabha and Moller scattering

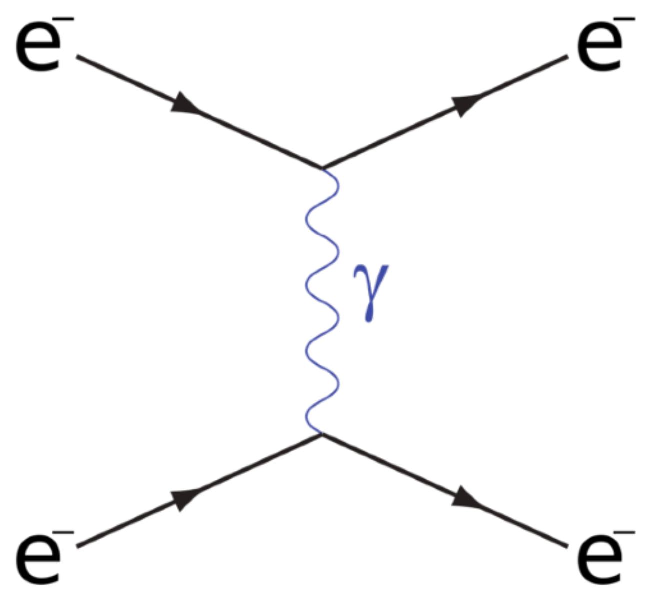


Bhabha scattering

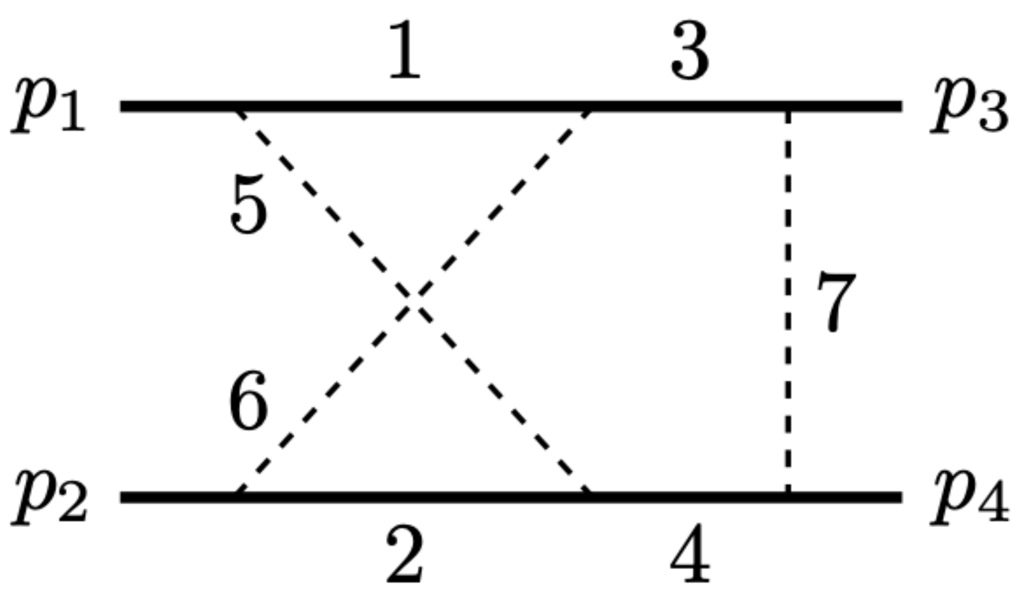
Two most familiar process in QFT 101

relevant for LEP
luminosity calibration

relevant for PRad II
exp. theory uncertainty

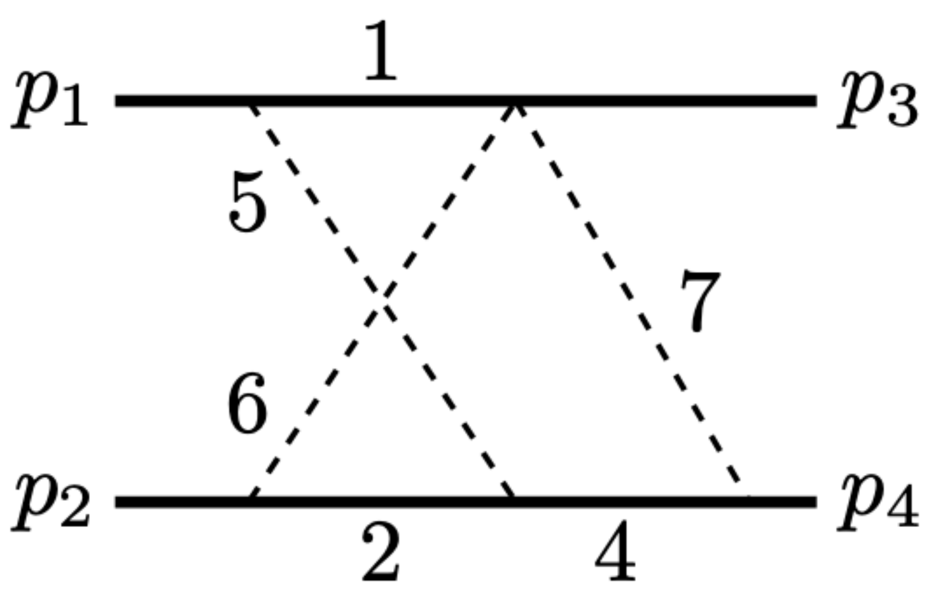


Moller scattering



alphabet in
differential equation

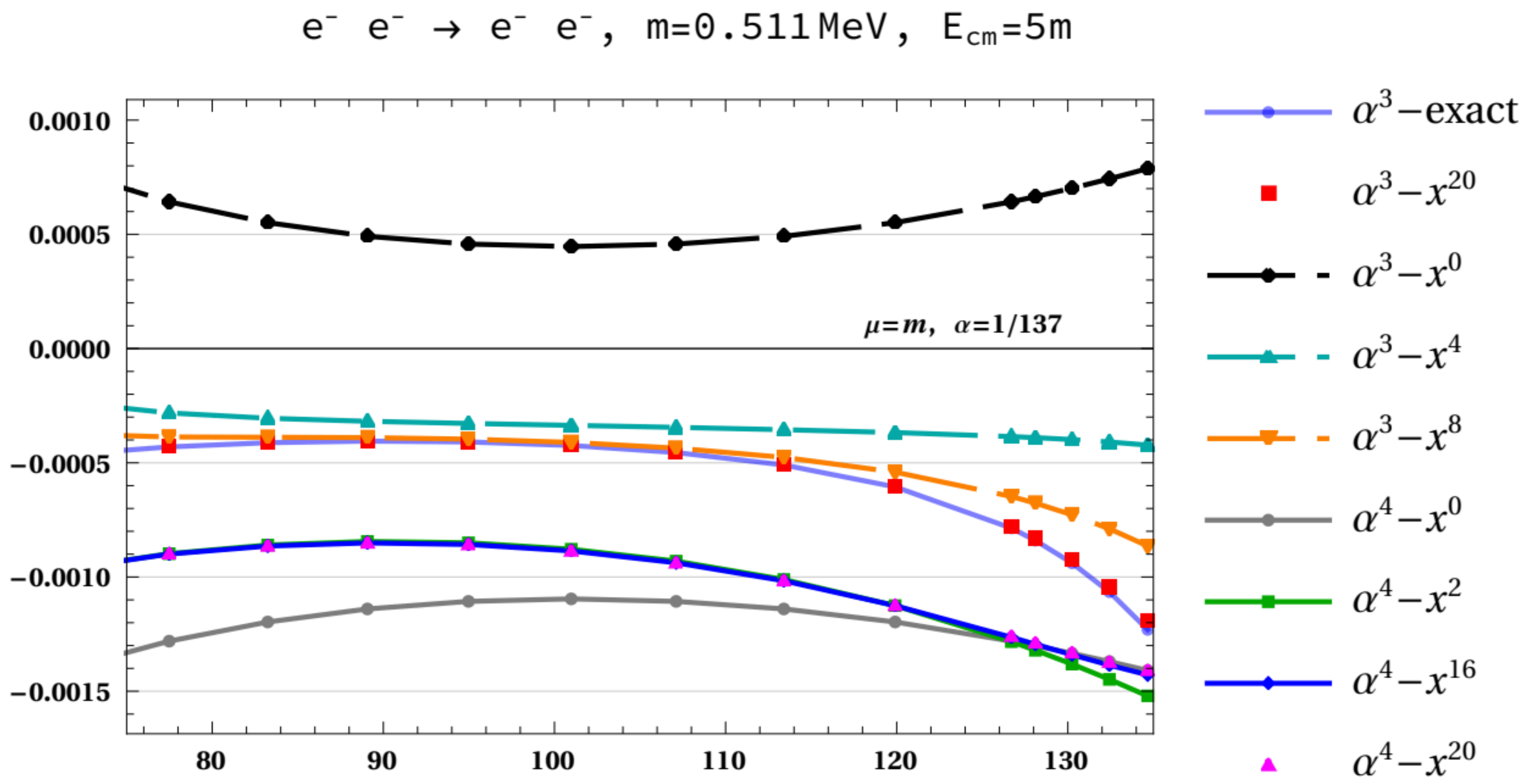
$$\{\sqrt{x^2-1},\ \sqrt{x^2-t_4},\ \sqrt{1+t_4},\ \sqrt{t_4},\ \sqrt{1-t_4}\}$$



$$K(k') \equiv \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-k'^2 t^2)}}$$

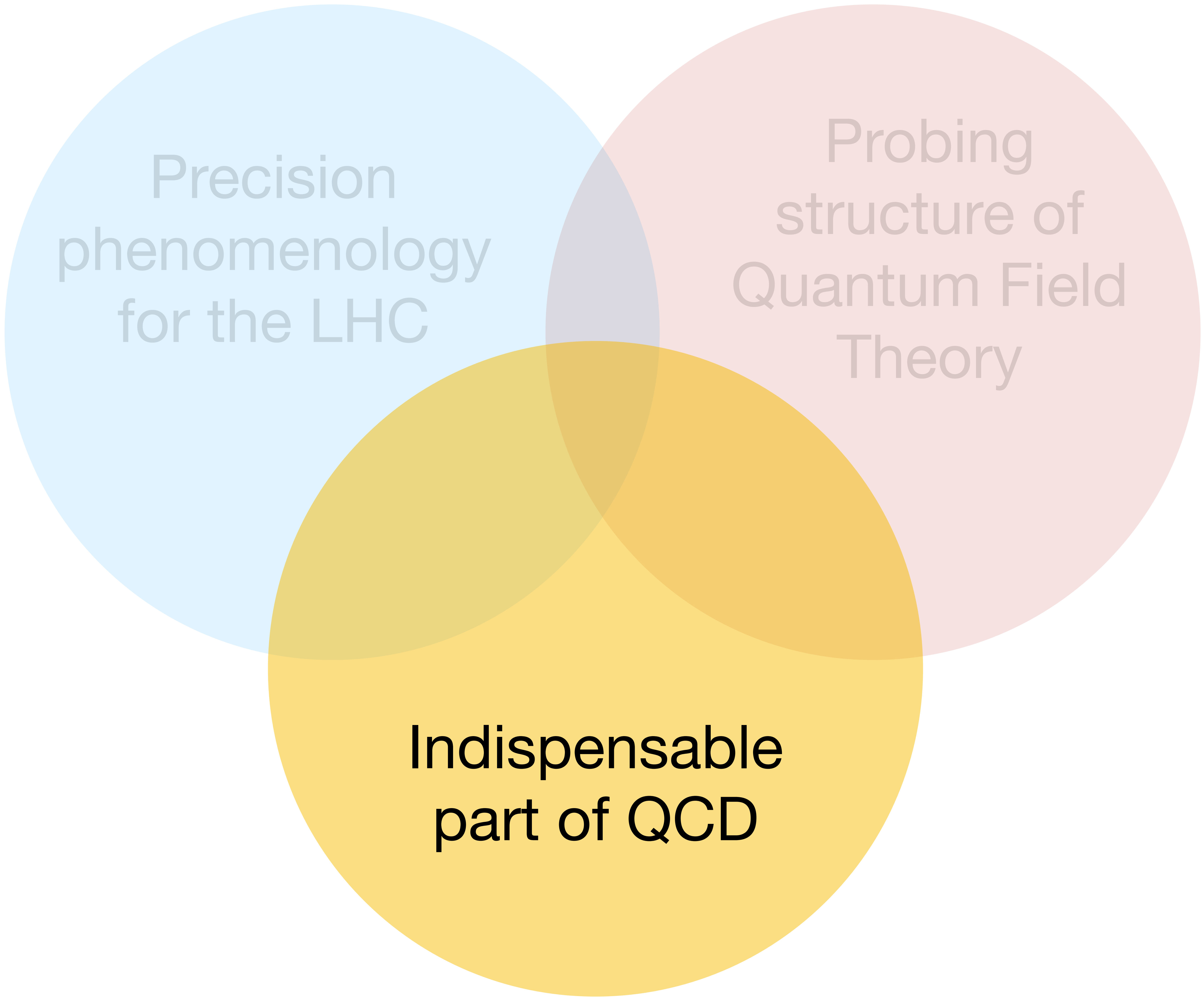
$$\partial_{t_4} f = 2\frac{1-t_4}{\sqrt{t_4}(1+t_4)^{3/2}}\,\mathrm{K}(t_4)$$

$$\mathcal{F}(x,t_4) = \mathrm{K}(t_4)\partial_{t_4}\left[\frac{1}{\mathrm{K}(t_4)}\int_{-1}^{x}\frac{\mathrm{d}X}{\sqrt{(X^2-1)(X^2-t_4)}}\right]$$

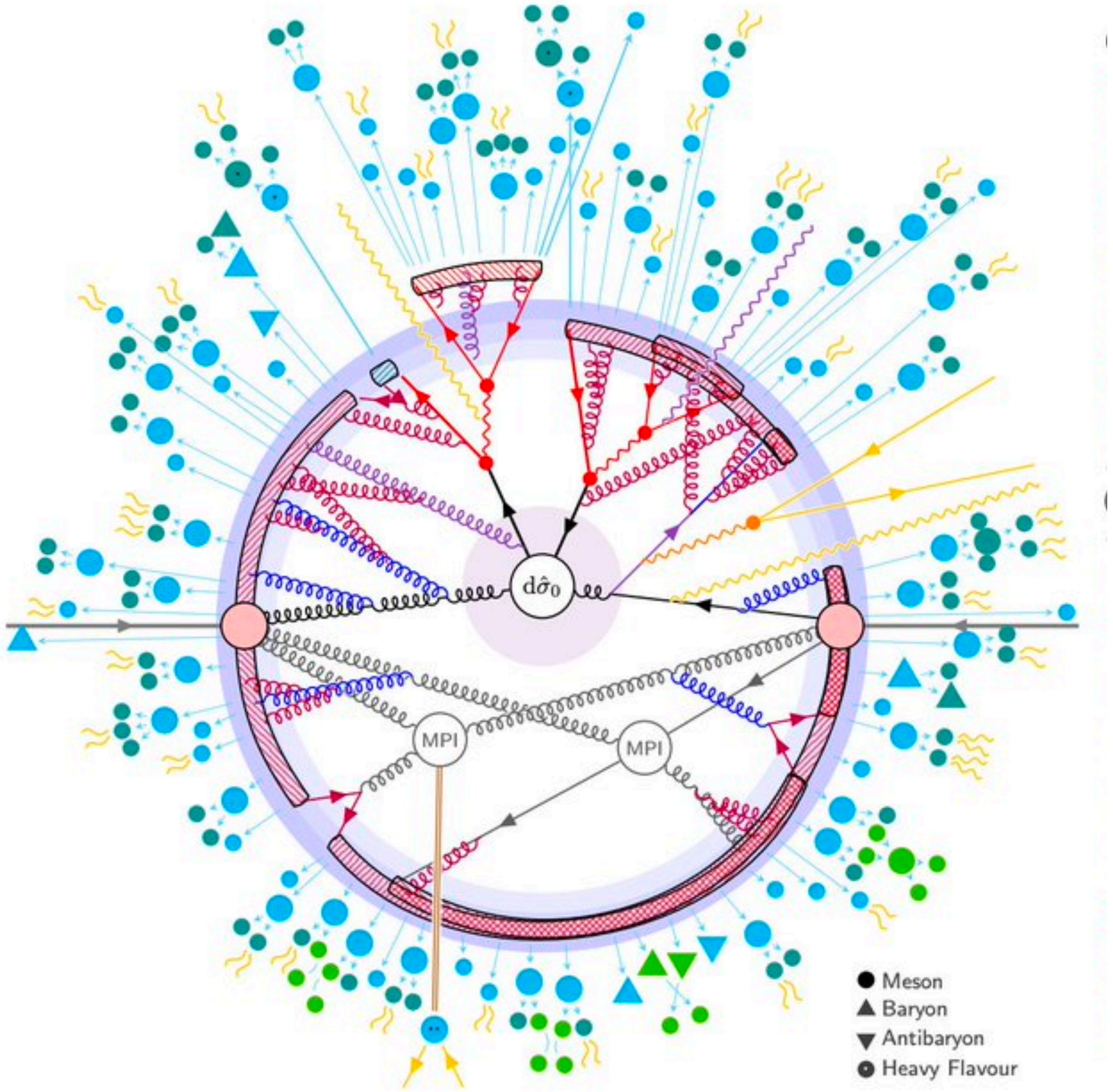


M. Delto, C. Duhr, L. Tancredi, Yu Jiao Zhu, 2023

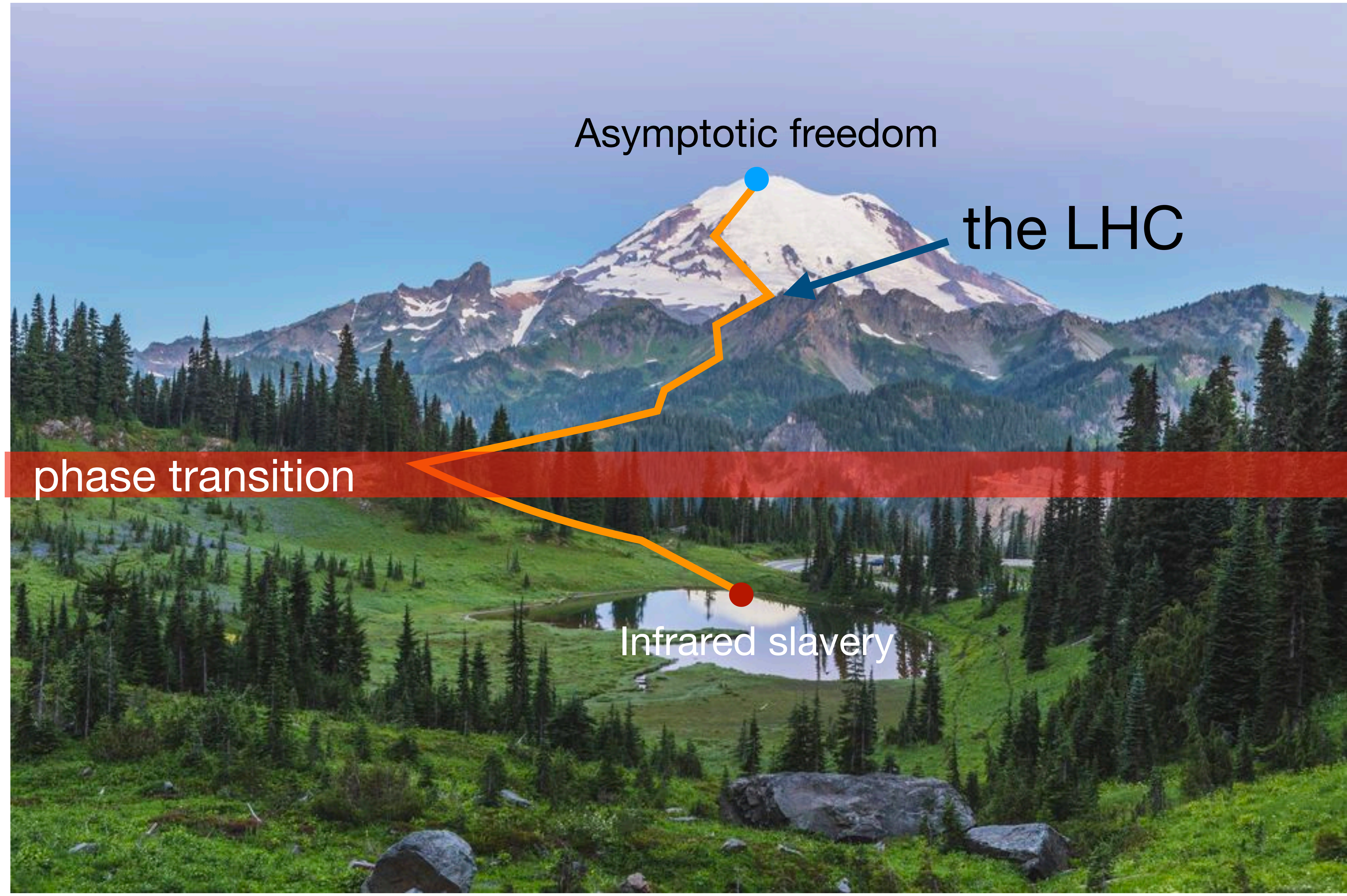
Plan of this talk



RG flow of QCD probed by the LHC



from Pythia event generator

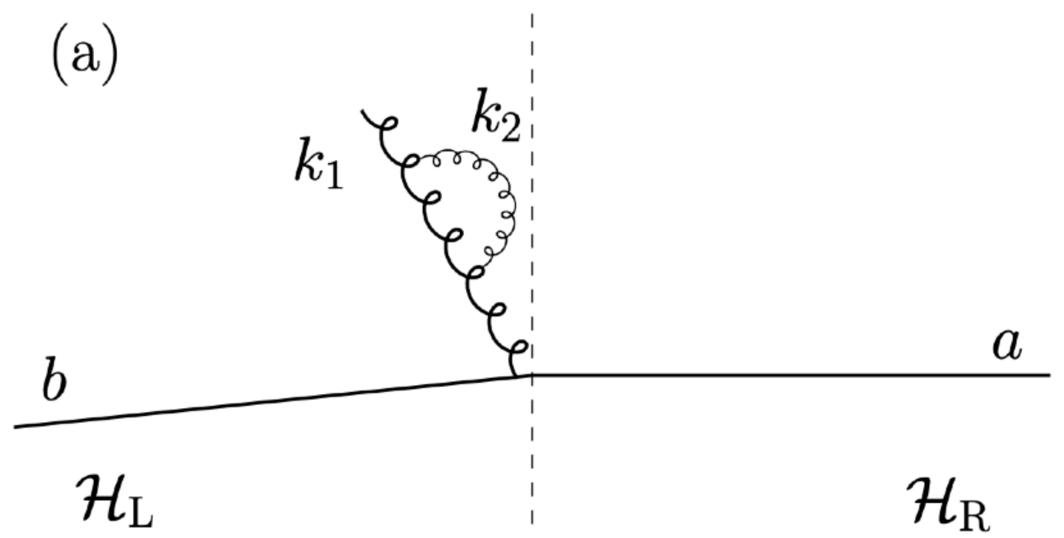


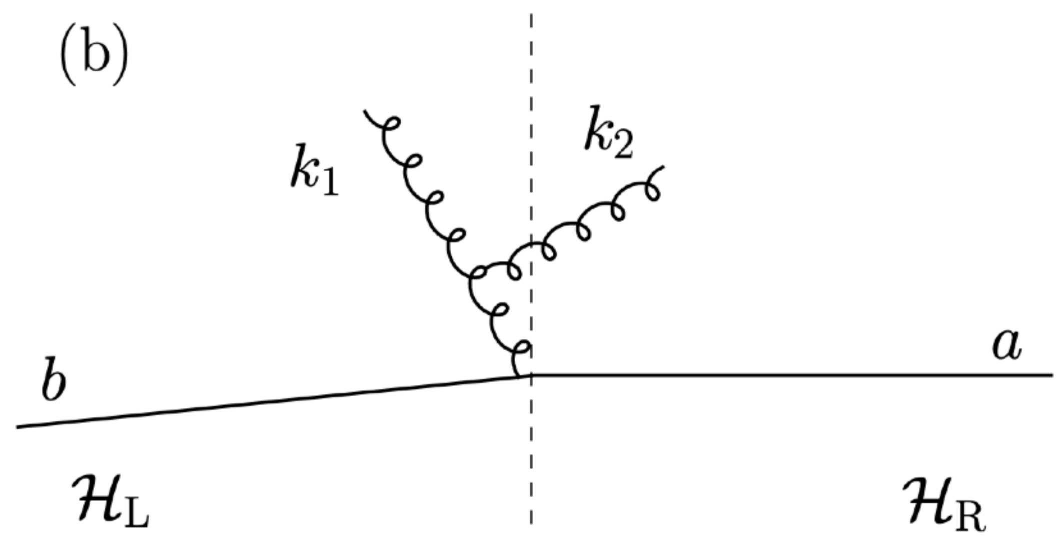
Spectacular view along the journey from UV to IR!

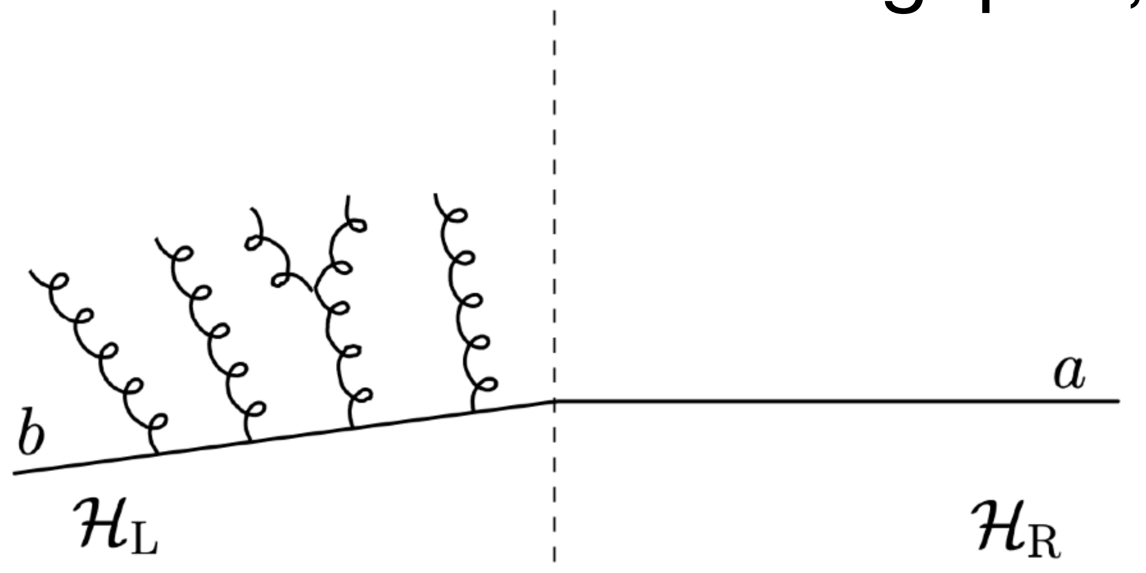
The non-global world

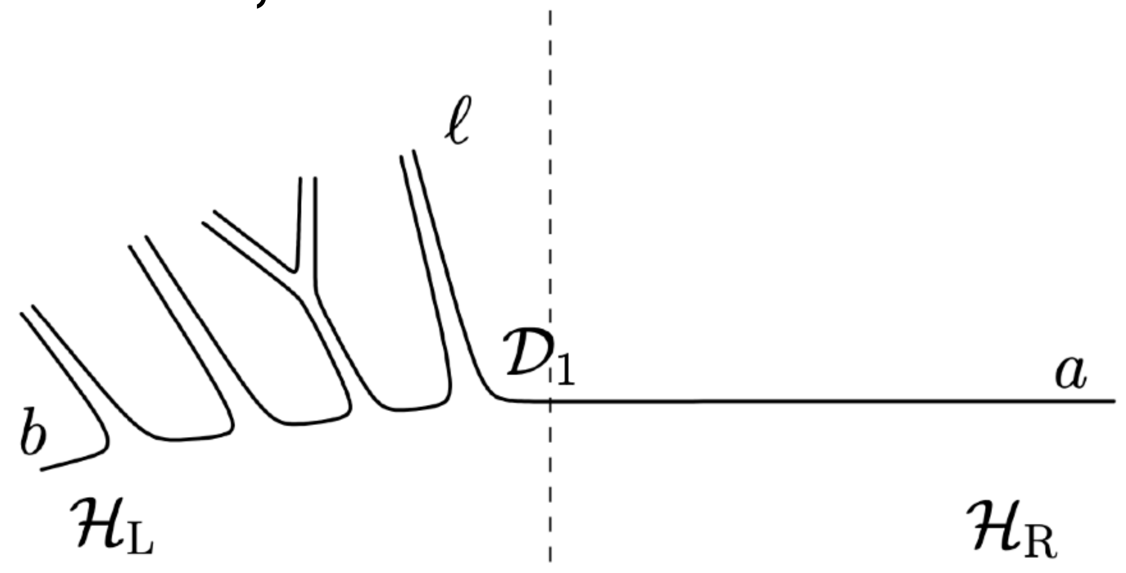
light jet mass distribution

M. Dasgupta , G. Salam, 2001





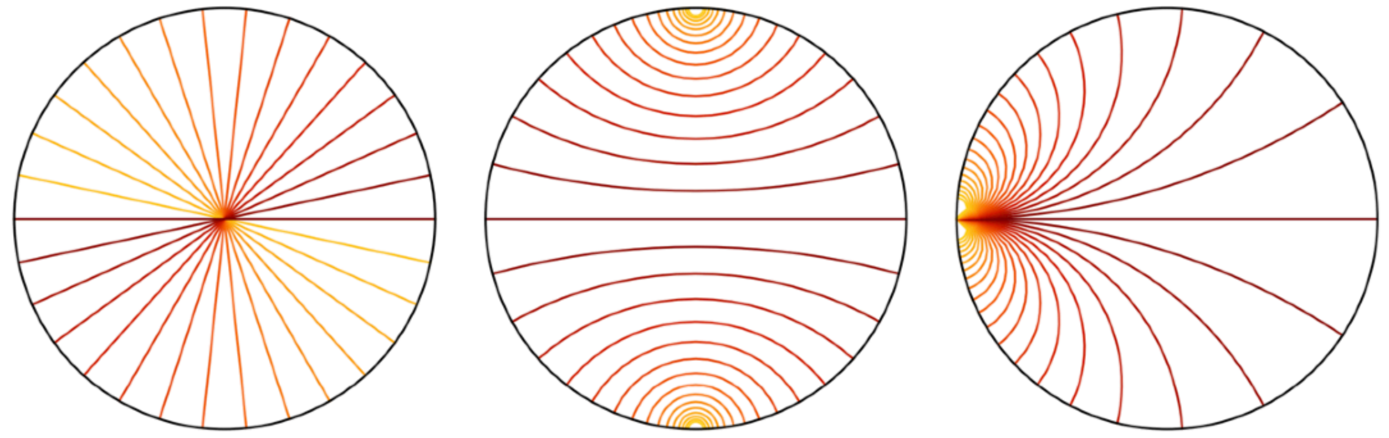




Banfi-Marchesini-Syme equation

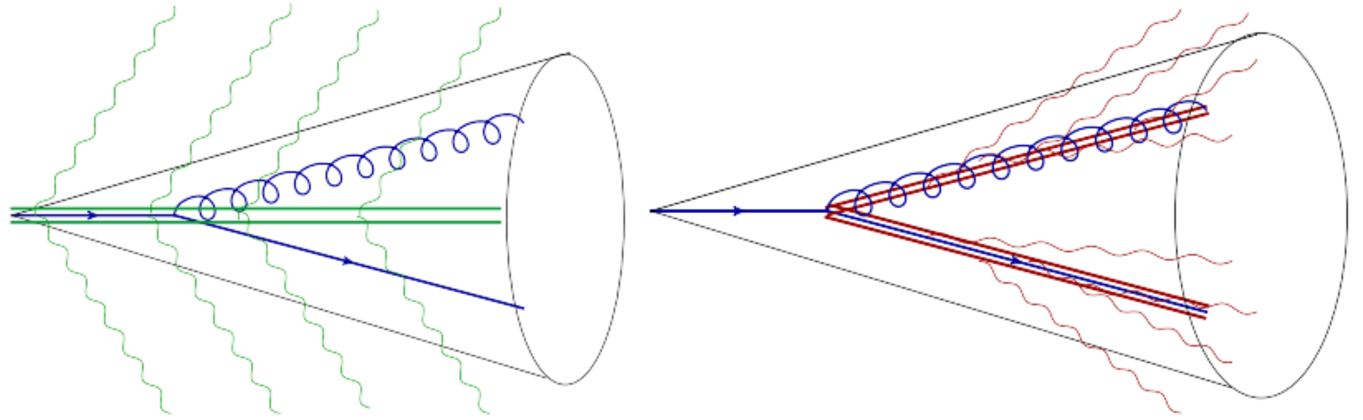
$$\partial_L G_{ab}(L) = \int \frac{d\Omega_j}{4\pi} \frac{1-\cos\theta_{ab}}{(1-\cos\theta_{aj})(1-\cos\theta_{jb})} \left[\theta_L(j) G_{aj}(L) G_{jb}(L) - G_{ab}(L)\right]$$

Conformal symmetry of BMS equation
Y. Hatta, T. Ueda, 2013



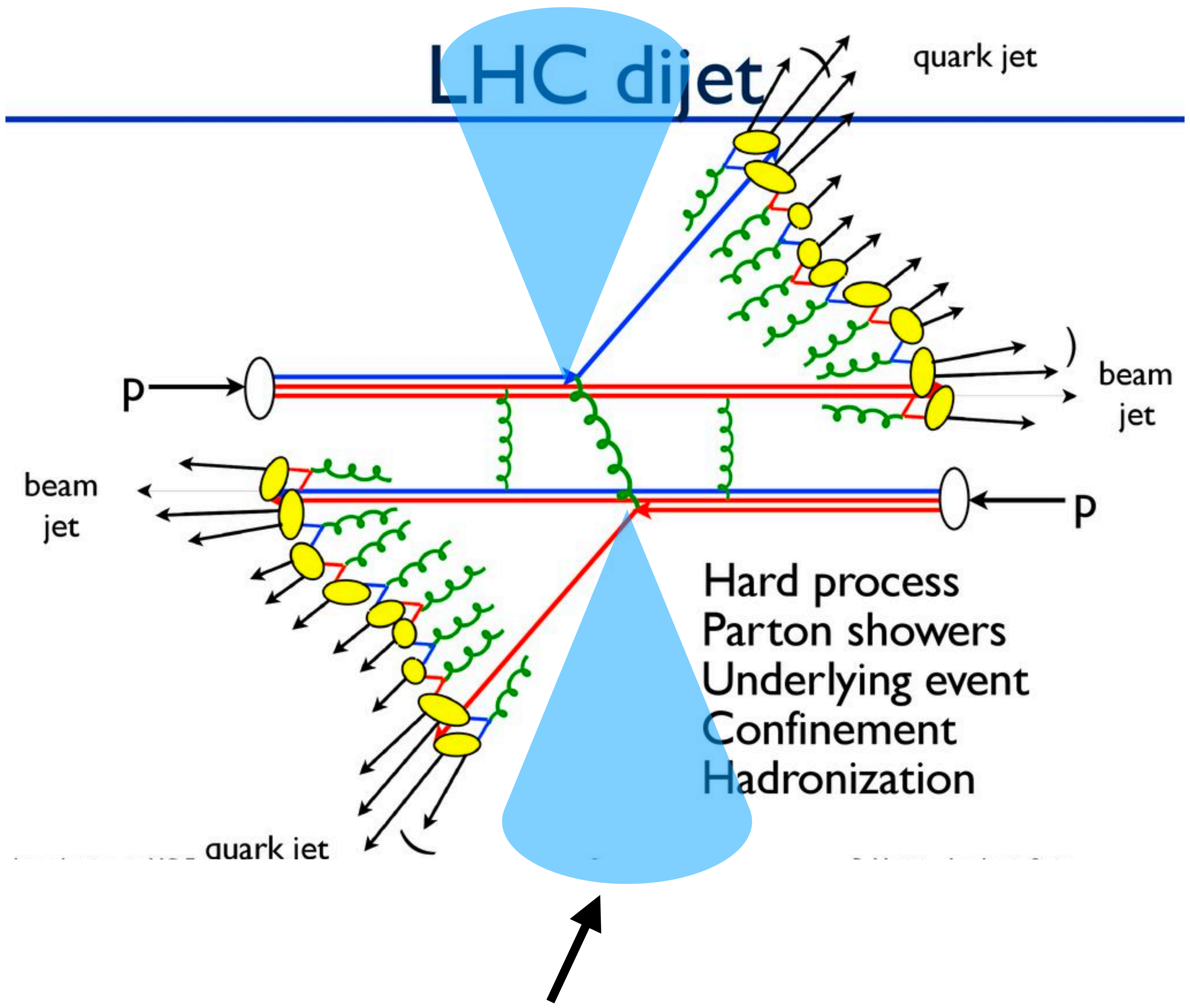
Five-loop perturbative solution:
M. Schwartz, HXZ, 2014

Effective field theory for non-global logarithms:
T. Becher, M. Neubert, Ding Yu Shao, 2015

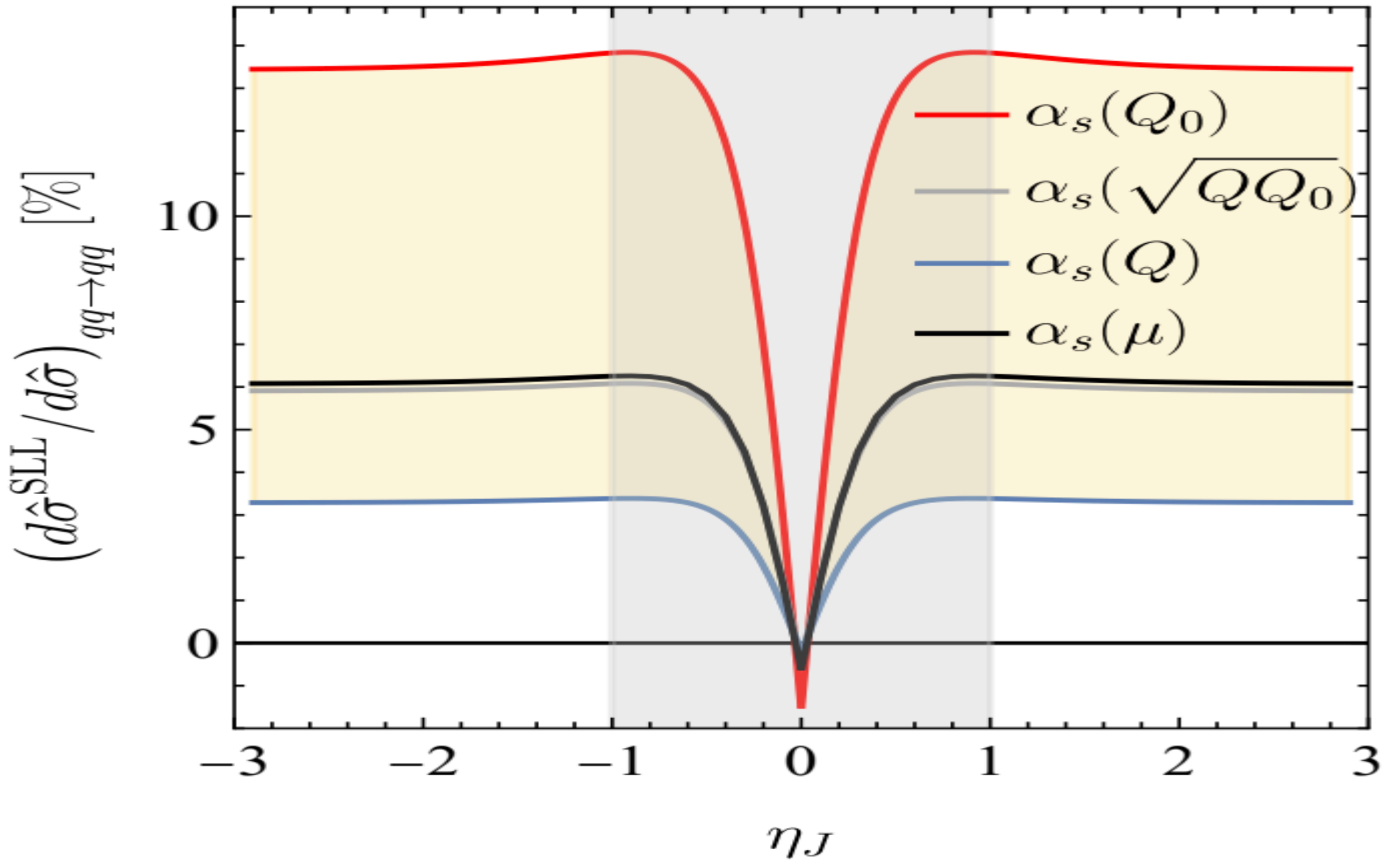


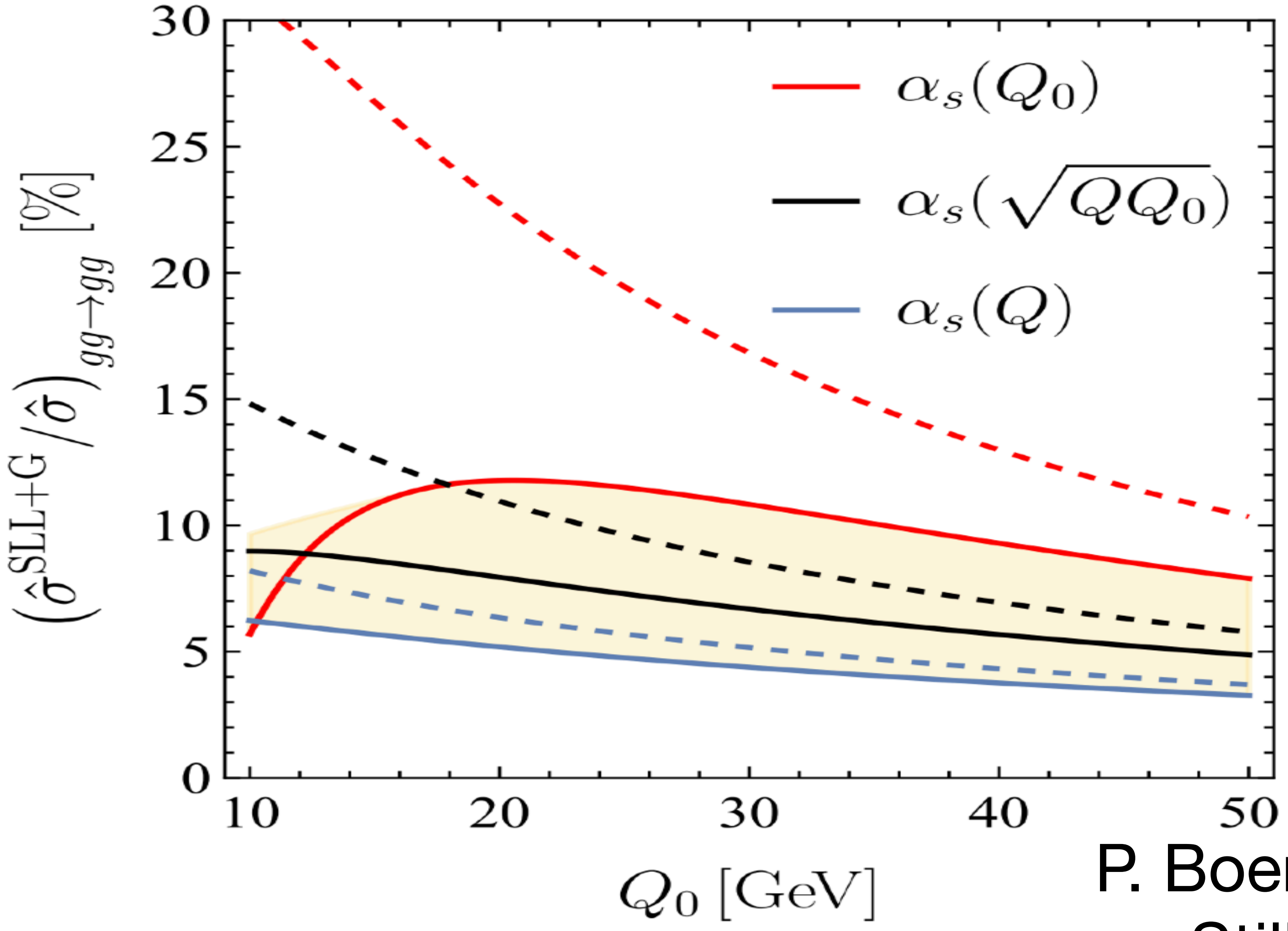
Non-global resummation for the LHC

T. Becher, M. Neubert, Ding Yu Shao, M. Stillger, 2023



gap between jet





Two contracted indices	
$\boldsymbol{A}^{(j)}_{2,F}$	$(\boldsymbol{F}_1-\boldsymbol{F}_2)\cdot\boldsymbol{T}_j$
$\boldsymbol{A}^{(j)}_{2,D}$	$(\boldsymbol{D}_1-\boldsymbol{D}_2)\cdot\boldsymbol{T}_j$

Three contracted indices	
$\boldsymbol{A}^{(j)}_{3f,F,F}$	$if^{abc}\,\boldsymbol{F}_1^a\,\boldsymbol{F}_2^b\,\boldsymbol{T}_j^c$
$\boldsymbol{A}^{(j)}_{3f,D,D}$	$if^{abc}\,\boldsymbol{D}_1^a\,\boldsymbol{D}_2^b\,\boldsymbol{T}_j^c$
$\boldsymbol{A}^{(j)}_{3f,F,D}$	$if^{abc}\left(\boldsymbol{F}_1^a\boldsymbol{D}_2^b-\boldsymbol{F}_2^a\boldsymbol{D}_1^b\right)\boldsymbol{T}_j^c$
$\boldsymbol{A}^{(j)}_{3d,F,D}$	$d^{abc}\left(\boldsymbol{F}_1^a\boldsymbol{D}_2^b-\boldsymbol{F}_2^a\boldsymbol{D}_1^b\right)\boldsymbol{T}_j^c$

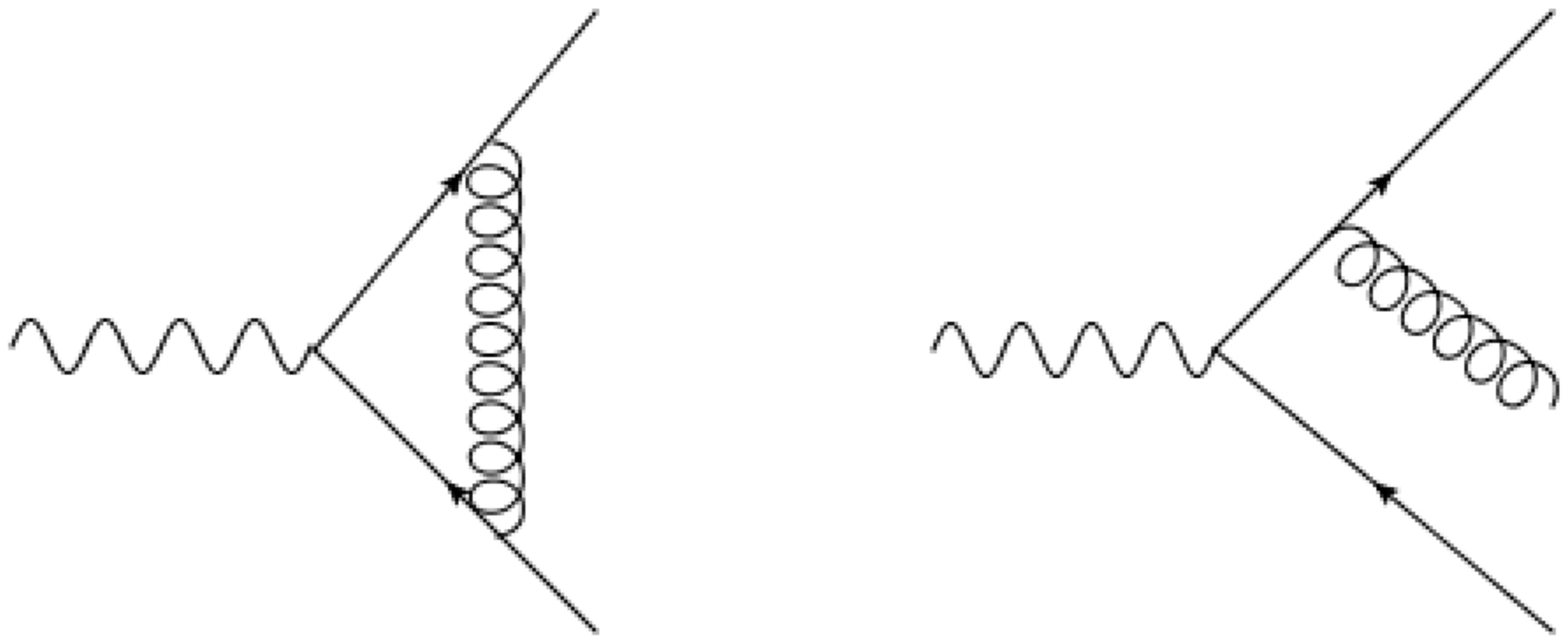
Four contracted indices	
$\boldsymbol{A}^{(j)}_{4,F,\Delta}$	$\left(\boldsymbol{F}_1^a\,\boldsymbol{\Delta}_2^{ab}-1\leftrightarrow2\right)\boldsymbol{T}_j^b$
$\boldsymbol{A}^{(j)}_{4,F,\nabla}$	$\left(\boldsymbol{F}_1^a\,\boldsymbol{\nabla}_2^{ab}-1\leftrightarrow2\right)\boldsymbol{T}_j^b$
$\boldsymbol{A}^{(j)}_{4,F,FF}$	$\left(\boldsymbol{F}_1^a\,\{\boldsymbol{F}_2^a,\boldsymbol{F}_2^b\}-1\leftrightarrow2\right)\boldsymbol{T}_j^b$
$\boldsymbol{A}^{(j)}_{4,F,FD}$	$\left(\boldsymbol{F}_1^a\,\{\boldsymbol{F}_2^a,\boldsymbol{D}_2^b\}-1\leftrightarrow2\right)\boldsymbol{T}_j^b$
$\boldsymbol{A}^{(j)}_{4,D,\Delta}$	$\left(\boldsymbol{D}_1^a\,\boldsymbol{\Delta}_2^{ab}-1\leftrightarrow2\right)\boldsymbol{T}_j^b$
$\boldsymbol{A}^{(j)}_{4,D,\nabla}$	$\left(\boldsymbol{D}_1^a\,\boldsymbol{\nabla}_2^{ab}-1\leftrightarrow2\right)\boldsymbol{T}_j^b$
$\boldsymbol{A}^{(j)}_{4,D,FF}$	$\left(\boldsymbol{D}_1^a\,\{\boldsymbol{F}_2^a,\boldsymbol{F}_2^b\}-1\leftrightarrow2\right)\boldsymbol{T}_j^b$
$\boldsymbol{A}^{(j)}_{4,D,FD}$	$\left(\boldsymbol{D}_1^a\,\{\boldsymbol{F}_2^a,\boldsymbol{D}_2^b\}-1\leftrightarrow2\right)\boldsymbol{T}_j^b$

Five contracted indices			
$\boldsymbol{A}^{(j)}_{5f,\Delta,\Delta}$	$if^{abc}\,\boldsymbol{\Delta}_1^{ad}\,\boldsymbol{\Delta}_2^{bd}\,\boldsymbol{T}_j^c$		
$\boldsymbol{A}^{(j)}_{5f,\nabla,\nabla}$	$if^{abc}\,\boldsymbol{\nabla}_1^{ad}\,\boldsymbol{\nabla}_2^{bd}\,\boldsymbol{T}_j^c$		
$\boldsymbol{A}^{(j)}_{5f,\Delta,\nabla}$	$if^{abc}\left(\boldsymbol{\Delta}_1^{ad}\,\boldsymbol{\nabla}_2^{bd}-1\leftrightarrow2\right)\boldsymbol{T}_j^c$	$\boldsymbol{A}^{(j)}_{5d,\Delta,\nabla}$	$d^{abc}\left(\boldsymbol{\Delta}_1^{ad}\,\boldsymbol{\nabla}_2^{bd}-1\leftrightarrow2\right)\boldsymbol{T}_j^c$
$\boldsymbol{A}^{(j)}_{5f,\Delta,FF}$	$if^{abc}\left(\boldsymbol{\Delta}_1^{ad}\,\{\boldsymbol{F}_2^b,\boldsymbol{F}_2^d\}-1\leftrightarrow2\right)\boldsymbol{T}_j^c$	$\boldsymbol{A}^{(j)}_{5d,\Delta,FF}$	$d^{abc}\left(\boldsymbol{\Delta}_1^{ad}\,\{\boldsymbol{F}_2^b,\boldsymbol{F}_2^d\}-1\leftrightarrow2\right)\boldsymbol{T}_j^c$
$\boldsymbol{A}^{(j)}_{5f,\Delta,FD}$	$if^{abc}\left(\boldsymbol{\Delta}_1^{ad}\,\{\boldsymbol{F}_2^b,\boldsymbol{D}_2^d\}-1\leftrightarrow2\right)\boldsymbol{T}_j^c$	$\boldsymbol{A}^{(j)}_{5d,\Delta,FD}$	$d^{abc}\left(\boldsymbol{\Delta}_1^{ad}\,\{\boldsymbol{F}_2^b,\boldsymbol{D}_2^d\}-1\leftrightarrow2\right)\boldsymbol{T}_j^c$
$\boldsymbol{A}^{(j)}_{5f,\nabla,FF}$	$if^{abc}\left(\boldsymbol{\nabla}_1^{ad}\,\{\boldsymbol{F}_2^b,\boldsymbol{F}_2^d\}-1\leftrightarrow2\right)\boldsymbol{T}_j^c$	$\boldsymbol{A}^{(j)}_{5d,\nabla,FF}$	$d^{abc}\left(\boldsymbol{\nabla}_1^{ad}\,\{\boldsymbol{F}_2^b,\boldsymbol{F}_2^d\}-1\leftrightarrow2\right)\boldsymbol{T}_j^c$
$\boldsymbol{A}^{(j)}_{5f,\nabla,FD}$	$if^{abc}\left(\boldsymbol{\nabla}_1^{ad}\,\{\boldsymbol{F}_2^b,\boldsymbol{D}_2^d\}-1\leftrightarrow2\right)\boldsymbol{T}_j^c$	$\boldsymbol{A}^{(j)}_{5d,\nabla,FD}$	$d^{abc}\left(\boldsymbol{\nabla}_1^{ad}\,\{\boldsymbol{F}_2^b,\boldsymbol{D}_2^d\}-1\leftrightarrow2\right)\boldsymbol{T}_j^c$

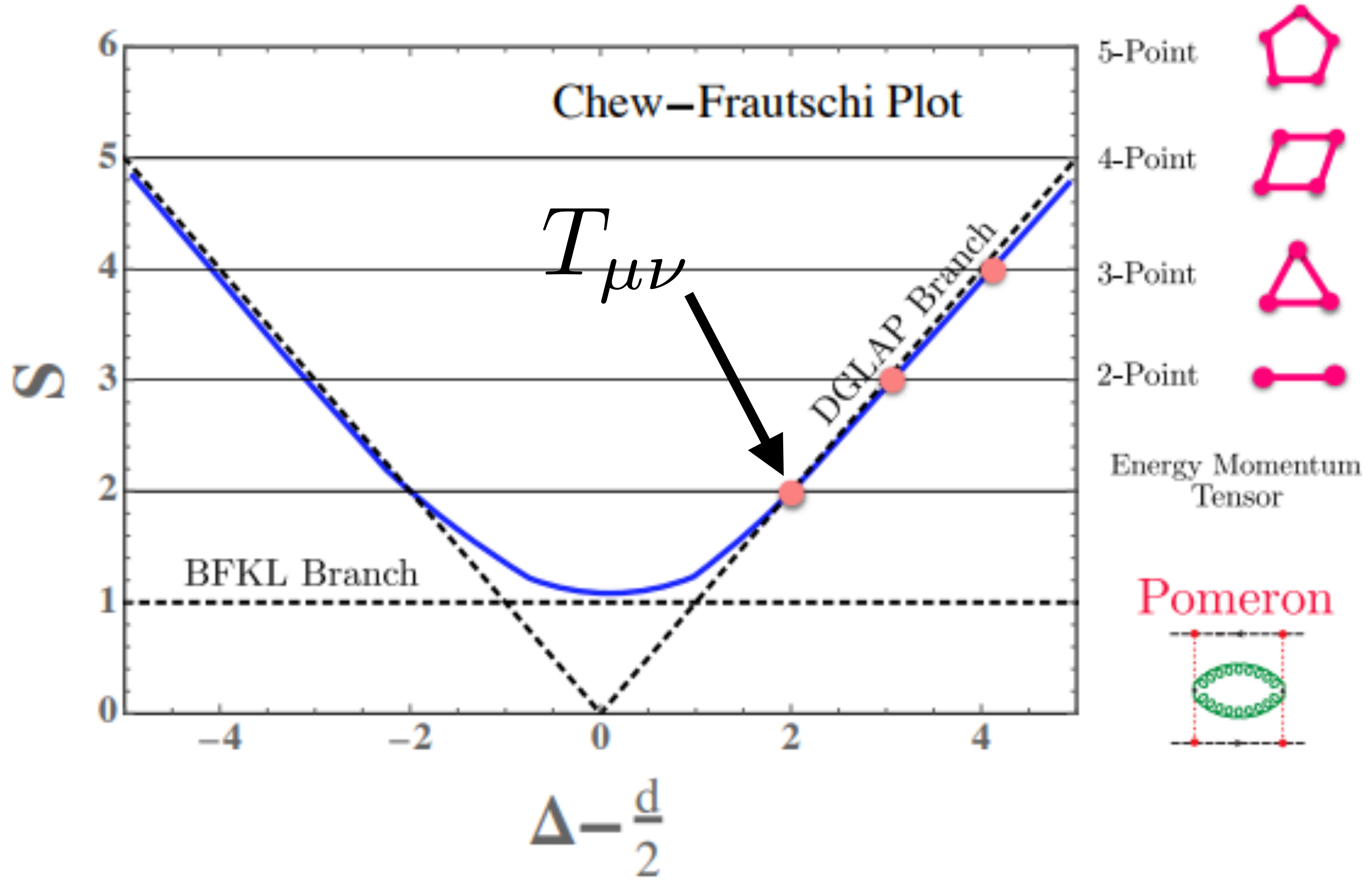
P. Boer, P. Hager, M. Neubert, M. Stillger, Xiaofeng Xu, 2023

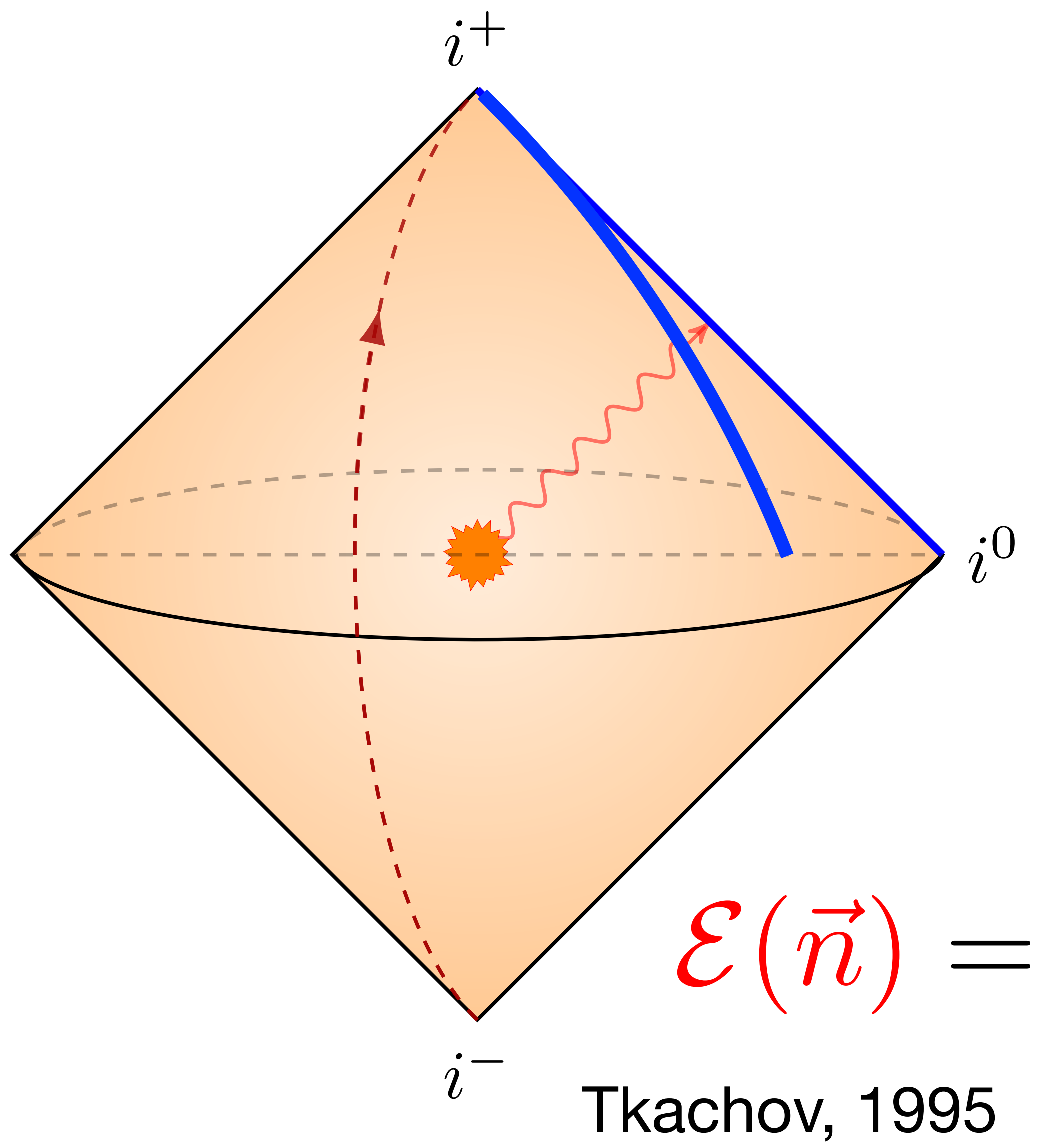
Extending the space flat-space observable

QFT 101: gauge theory amplitude not observable;
cross section are



Hao Chen, I. Moult, Xiaoyuan Zhang, HXZ, 2020





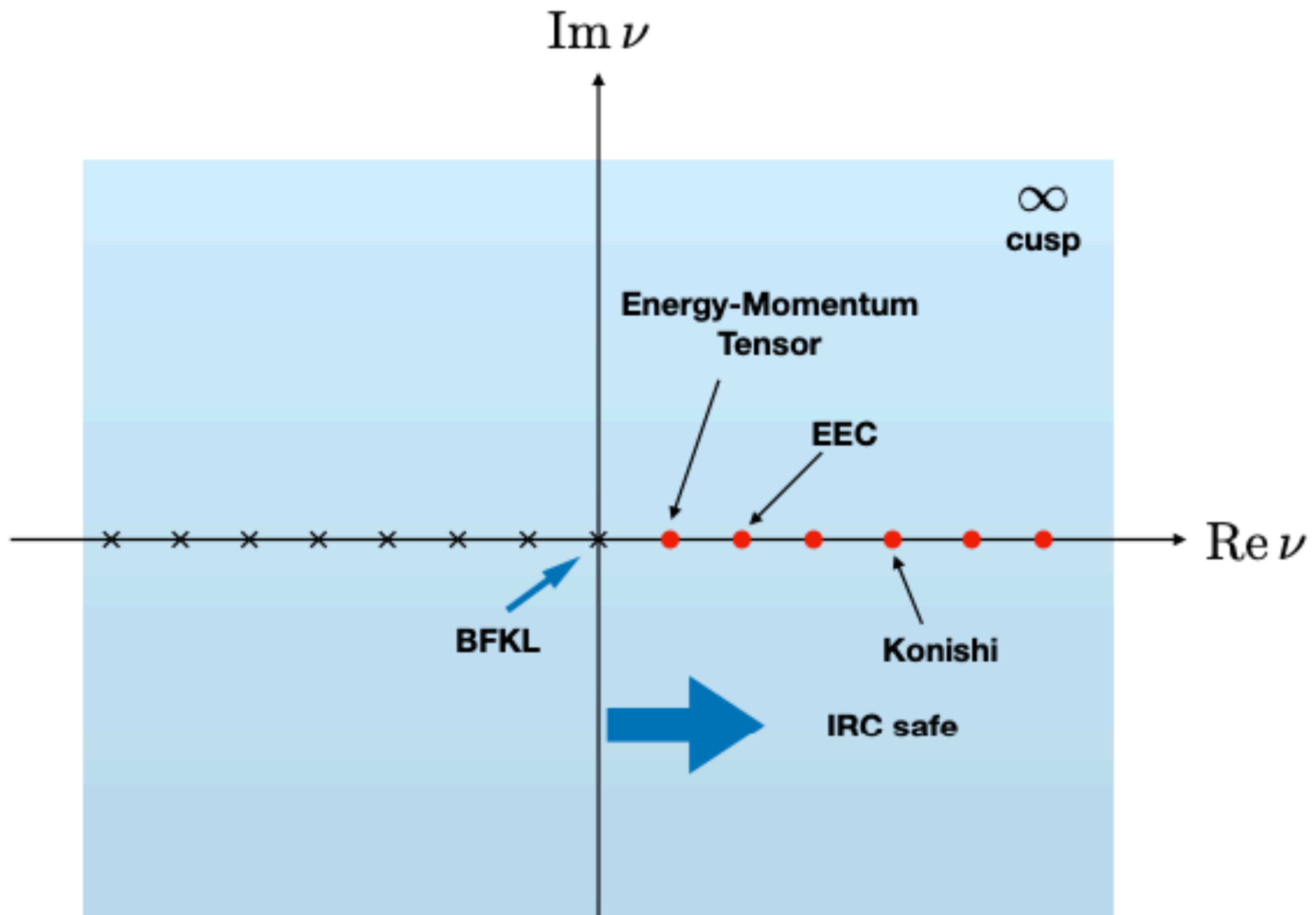
But cross section are not the only observable;

correlation function of asymptotic lightray operators are also finite

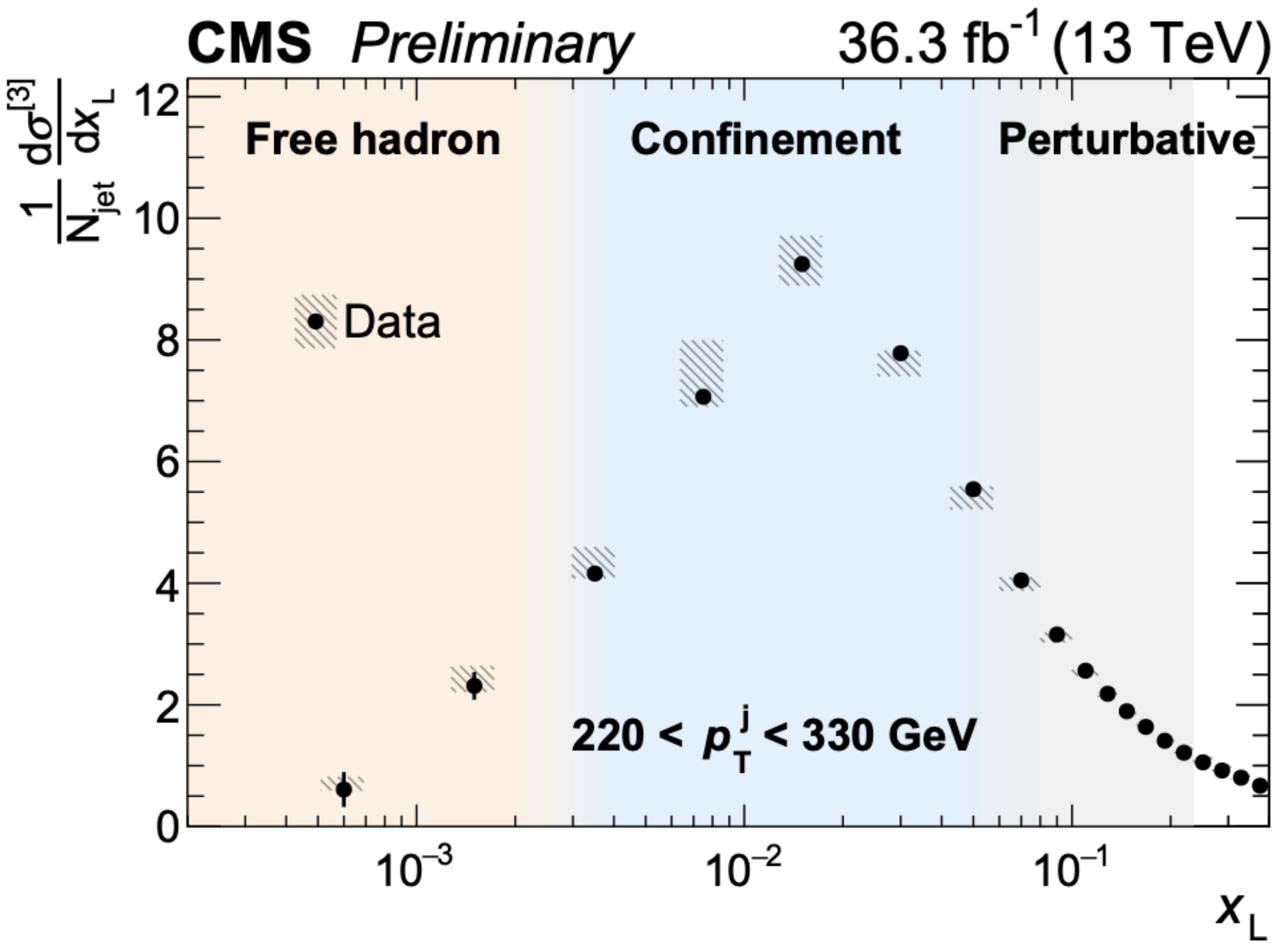
The simplest class are energy correlators

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \vec{n}_i T^{0i}(t, r\vec{n})$$

Tkachov, 1995
Hofman, Maldacena, 2008



Energy correlators at works



time

Decrease in transverse momentum transfer between probed hadron

https://cds.cern.ch/record/2866560/files/SMP-22-015-pas.pdf

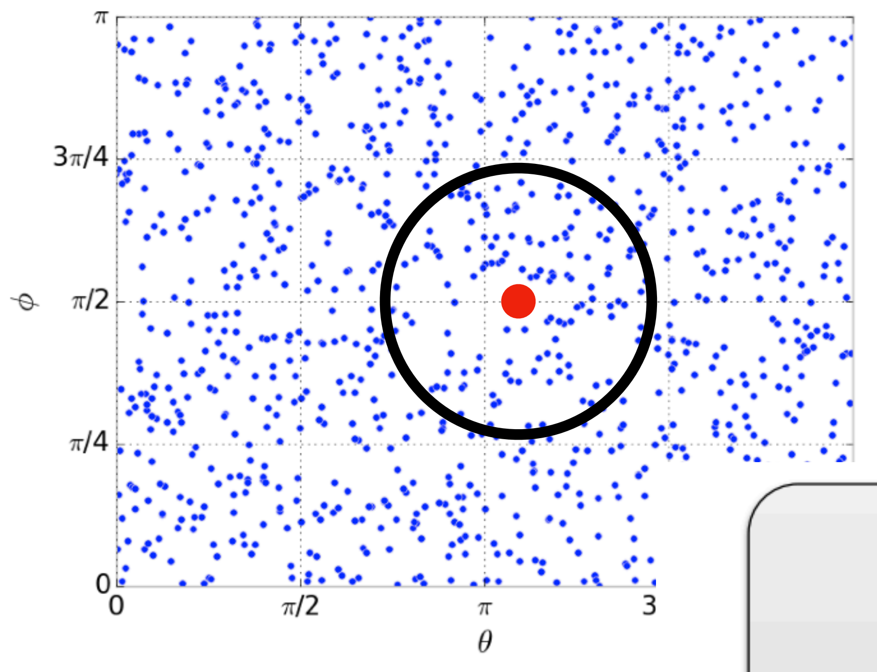
$$\alpha_S(m_Z) = 0.1229^{+0.0014}_{-0.0012}(\text{stat.})^{+0.0030}_{-0.0033}(\text{theo.})^{+0.0023}_{-0.0036}(\text{exp.})$$

Large R_L (perturbative region)

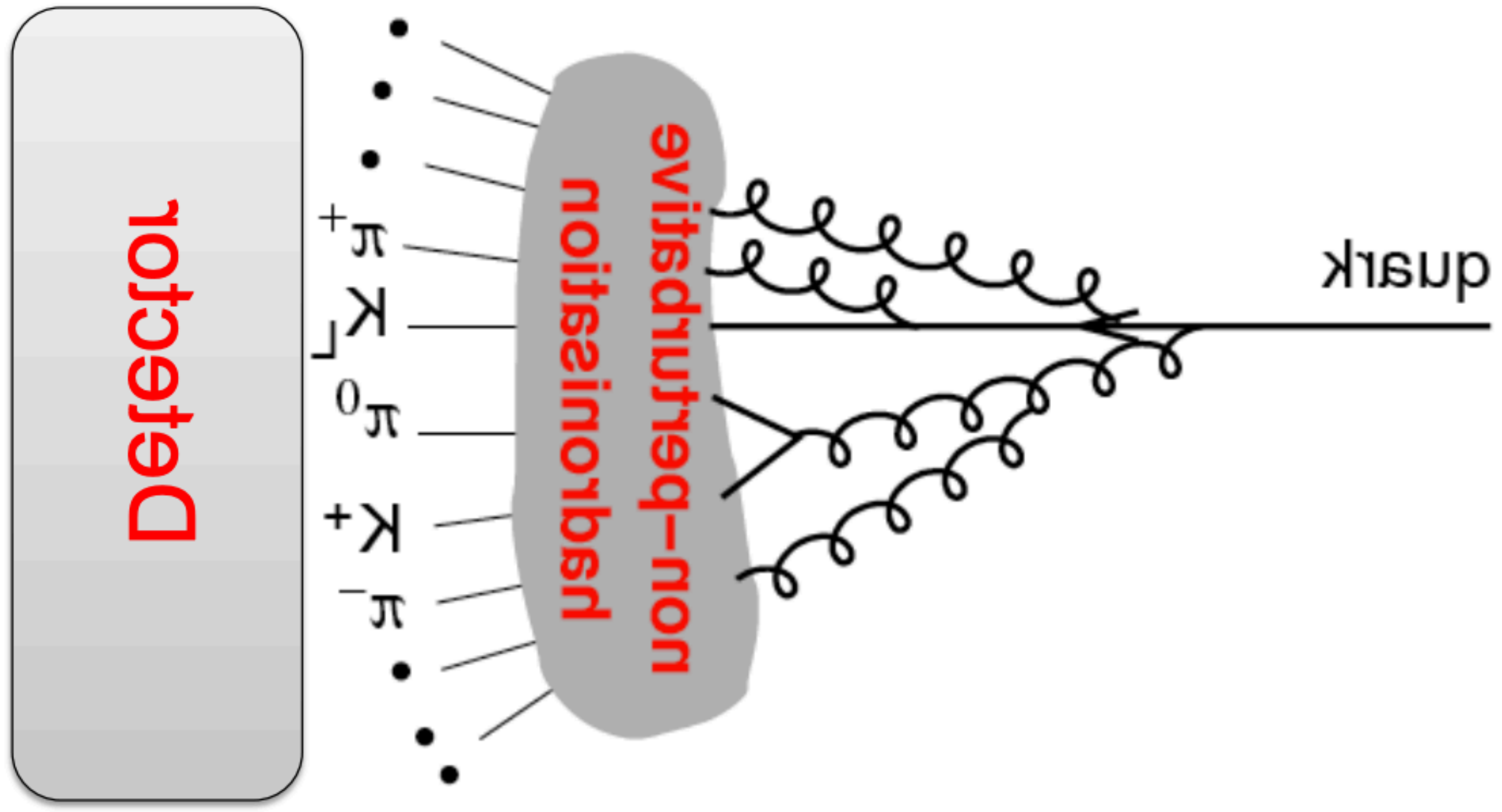
scaling law

$$\lim_{\hat{n}_2 \to \hat{n}_1} \mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = \sum c_i \theta^{\tau_i - 4} \mathbb{O}_i(\hat{n}_1) \text{ + running coupling}$$

very small R_L (free hadrons)

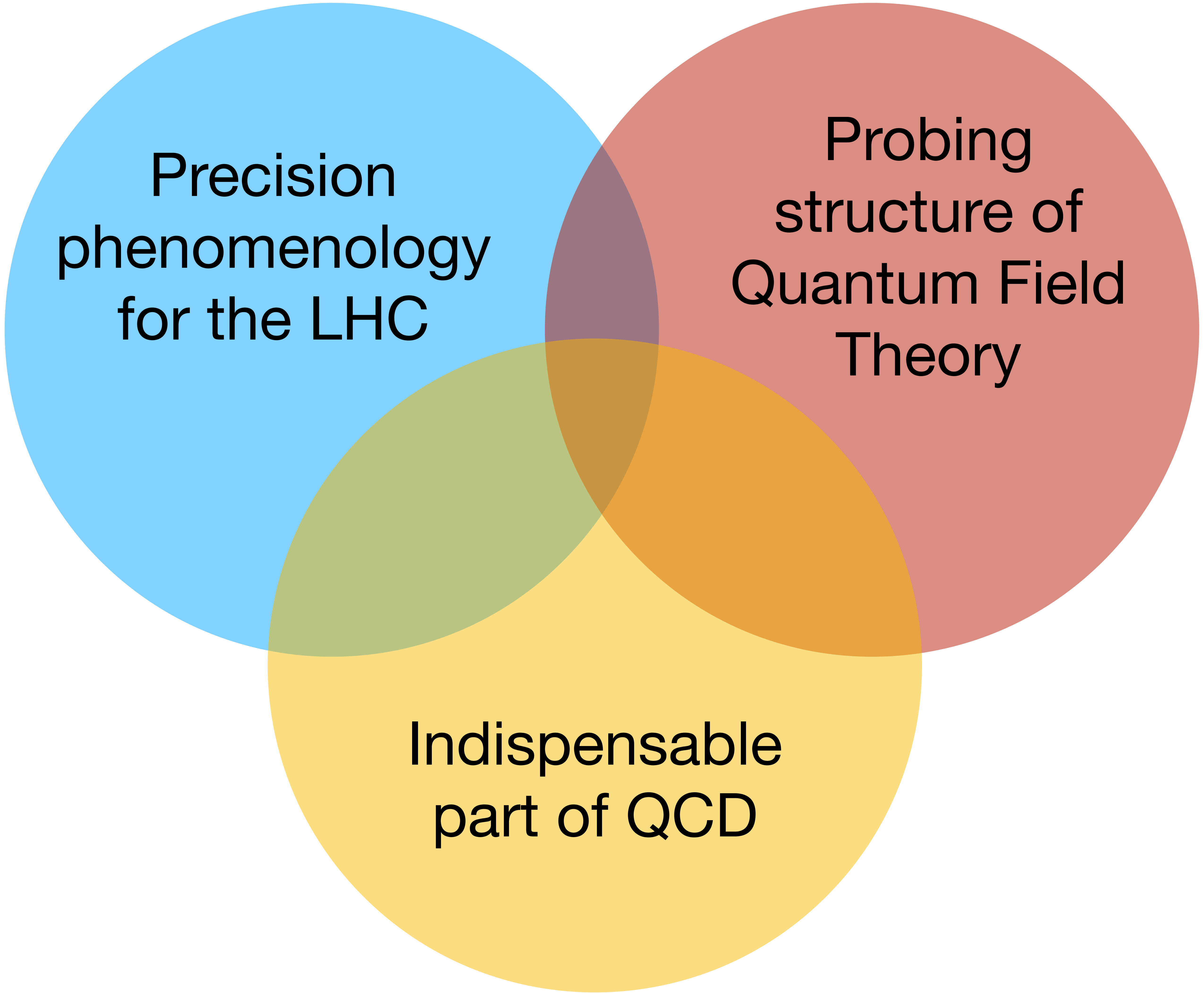


Starting from any given point, the number of points correlated with it grow linearly with radius R



Most precise measurement from jet substructure. Uncertainties dominated by theory!

Summary



This year marks the discovery of QCD for 50 years.

QCD gave rise to the pursuit of understanding the strong force via perturbation theory.

We have witnessed remarkable **continuous** progress in the past 50 years.

Stay tuned for more exciting results from the future!