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Electroweak corrections to double Higgs production at the LHC

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清華大學 Tsinghua University　中山大學 SUN YAT-SEN UNIVERSITY　東南大學 SOUTHEAST UNIVERSITY

Based on: arvix: 2311.16963

In cooperation with:
Li-Hong Huang, Rui-Jun Huang, Yan-Qing Ma, Huai-Min Yu

Introduction to Higgs Boson

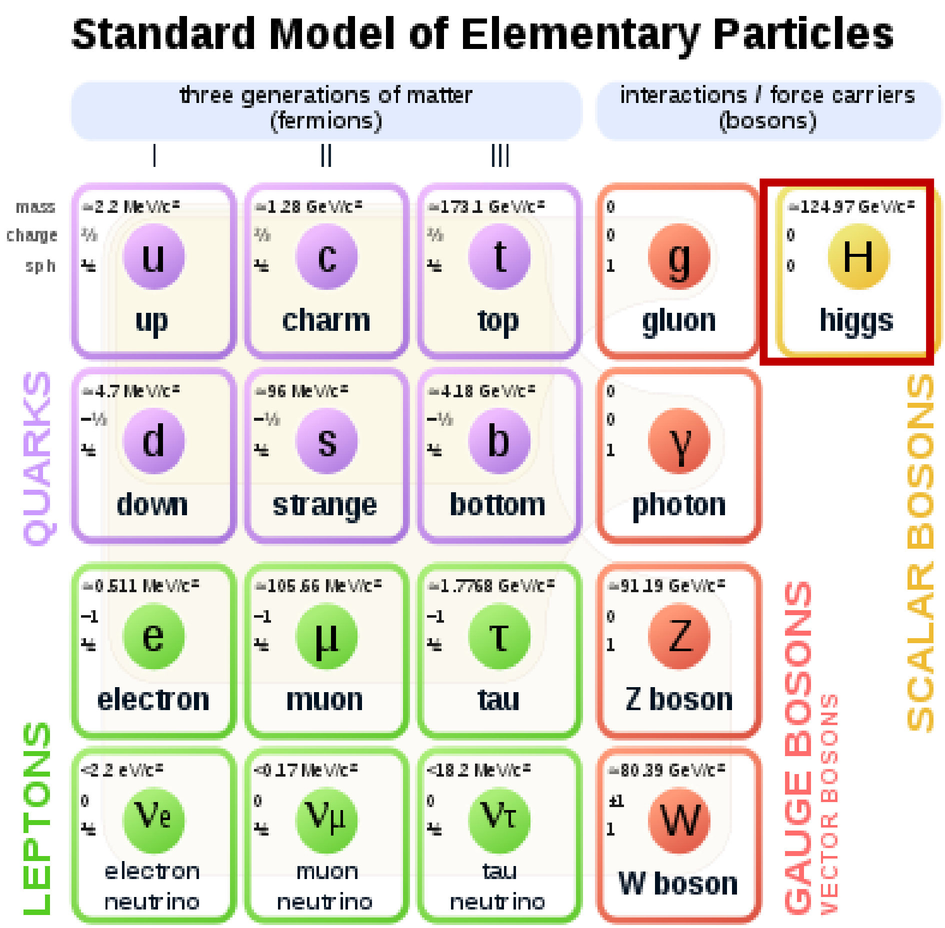
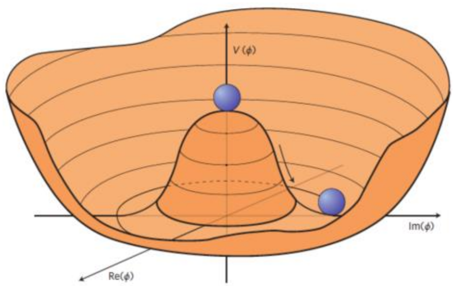


Figure taken from Wikipedia

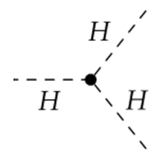
- Discovery of Higgs boson(2012,LHC): last fundamental particle in SM.
- Experiments at the ALTAS and CMS: agrees with result SM predicted.
- Problems not solved: electroweak symmetry breaking, Higgs coupling to SM particles/DM, hierarchy problem… Require new physics beyond SM.
- One promising way probing new physics: precision measurements of the properties of H (for e.g. Higgs self coupling).

Higgs self coupling

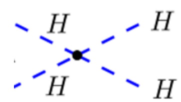


Plot taken from Ellis: 1312.5672

$$V(H) = \frac{m_H^2}{2} H^2 + \lambda_{HHH}^{SM} v H^3 + \frac{1}{4} \lambda_{HHHH}^{SM} H^4$$

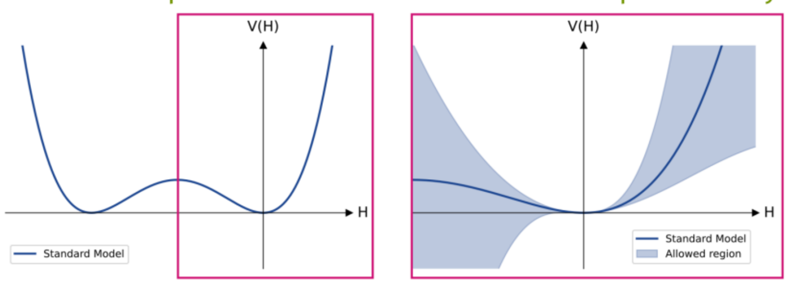


Feasible at HL-LHC



20??

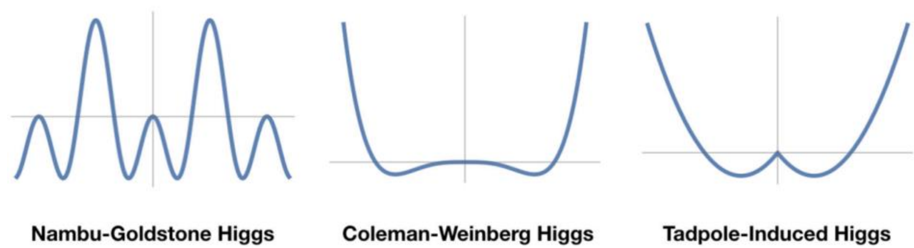
What the SM predicts vs what we know experimentally



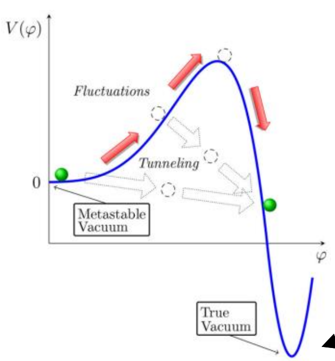
[using current ATLAS limits @ 95% CL]

Plot taken from Moser: Higgs 2023

New physics



Agrawal et al: 1907.02078



What would Universe be like in such vacuum?

big consequences for the Universe

Markkanen et al: 1809.06923

Measurements of Higgs boson coupling

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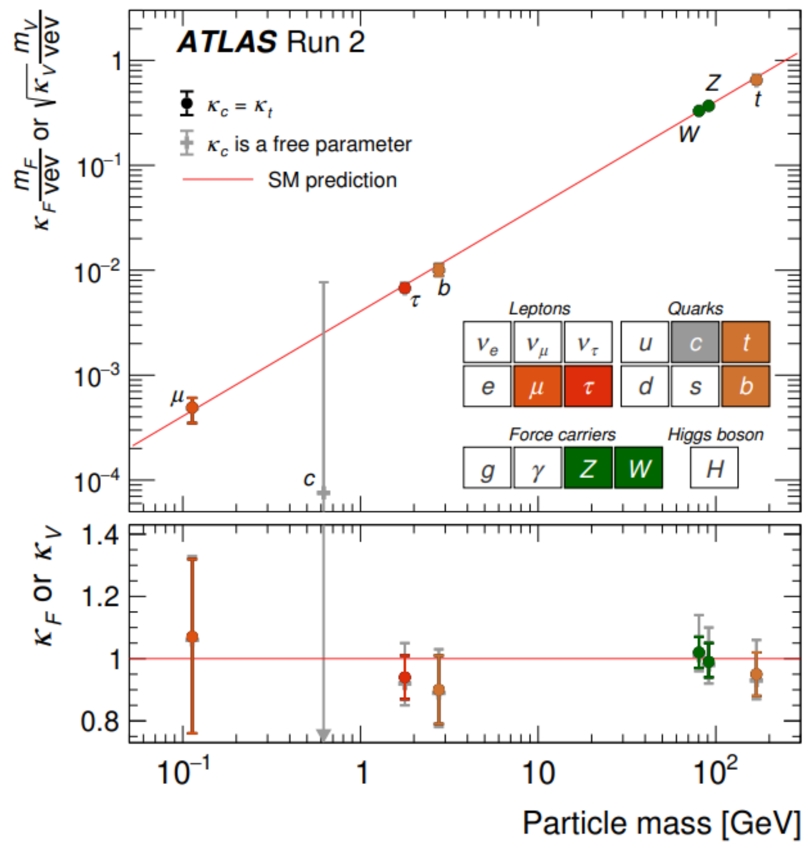
😊 $g_{Hf\bar{f}}$, g_{HVV}

- ➢ can be measured with high precision.

☹ λ_{HHH}, λ_{HHHH}

- ➢ require multi-Higgs production, small cross sections.
- ➢ Mixed with complicated background.



ALTAS:2207.00092

Run 2 δ_μ^{tot} [%]	HL-LHC δ_μ^{tot} (δ_μ^{th}) [%]
$-1.0 < \lambda/\lambda_{\text{SM}} < 6.6$	$0.5 < \lambda/\lambda_{\text{SM}} < 1.5$

Jones: LHEP 2023 (2023) 442

Status of QCD corrections

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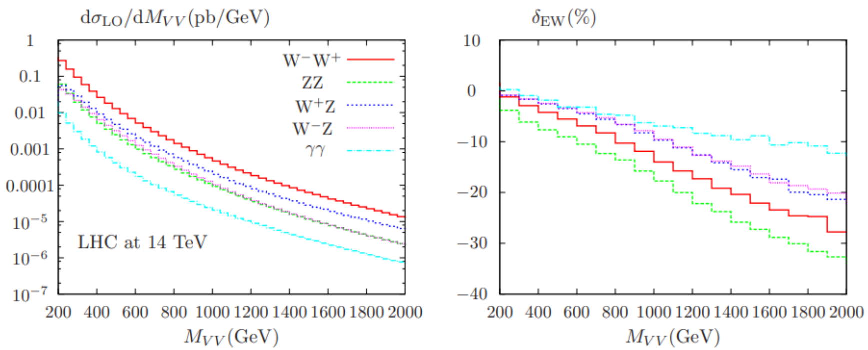
- NLO QCD
  - NLO QCD with full top-quark mass dependence, Borowka et al:1604.06447
  - NLO QCD matched to parton shower, Heinrich et al:1703.09252
  - NLO QCD with soft-gluon resummation, Ferrera et al: 1609.01691
- NNLO QCD
  - NNLO QCD in heavy-top limit (HTL) approximation, Florian et al:1305.5206
  - NNLO in HTL+ NLO with full top-quark mass dependence, Florian et al:2106.14050
  - NNLO QCD in HTL matched to parton shower, Alioli et al: 2212.10489
- NNNLO QCD
  - NNNLO QCD in HTL, Chen et al:1909.06808
  - NNNLO in HTL include the top-quark mass effects, Chen et al:1912.13001
  - NNNLO in HTL+ NLO with full top-quark mass dependence + soft-gluon resummation, Ajjath et al:2209.03914

Process	Theory	σ_{th} [pb]	δ_{th} [%]	δ_{PDF} [%]	δ_{α_s} [%]
$ggF\ HH$	N^3LO_{HTL} NLO_{QCD}	0.03105	$^{+2.2}_{-5.0}$	± 2.1	± 2.1

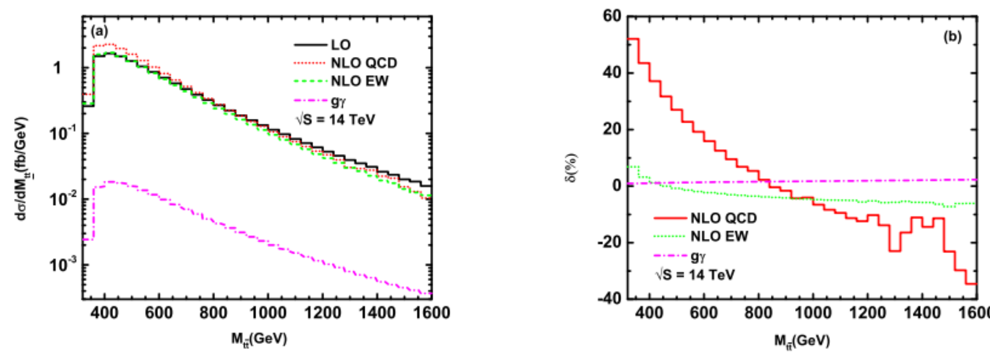
Jones: LHEP 2023 (2023) 442

Importance of EW corrections

- Unknown size of EW corrections
  - ➢ Biggest uncertainties from theoretical side
- NLO EW corrections are notably significant at high energy region
  - ➢ Sudakov enhancement: $\alpha \sim 0.7\% \rightarrow \frac{\alpha}{4\pi \sin\theta_W^2} \log^2(\frac{s}{m_Z^2})|_{s=2000^2} \sim 10\% \sim \alpha_s$

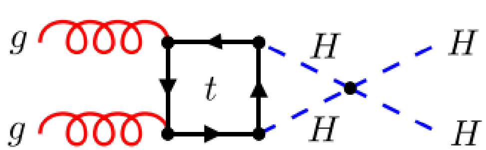


A Bierweiler et al:1305.5402



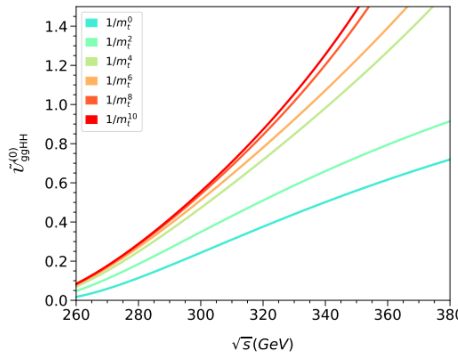
Zhang et al: 1407.1110

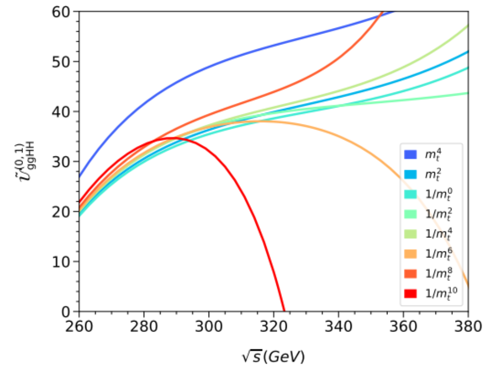
- Higgs quartic coupling only emerges at the NLO EW level
  - ➢ Constrained on λ_{HHHH}^{SM} indirectly from NLO EW correction



Status of NLO EW corrections

- Partial results
  - ➢ Two-loop box diagrams, Davies et al:2207.02587
  - ➢ Top-quark Yukawa corrections, Muhlleitner et al:2207.02524
  - ➢ Higgs self-coupling corrections, Borowka et al: 1811.12366
- Most recent work: Davies et al: 2308.01355
  - ➢ HTL and the convergent behaviour is not good.
  - ➢ Neglecting diagrams with massless fermion loops.
  - ➢ Only prediction at the matrix element squared level.
- Groups working on this topic:
  - ➢ See Hantian Zhang's talk at Higgs 2023: HTL + partial results
  - ➢ See Xiao Zhang's talk at Higgs 2023: partial results
  - ➢ See Thomas Stone's talk at Higgs 2023: partial results

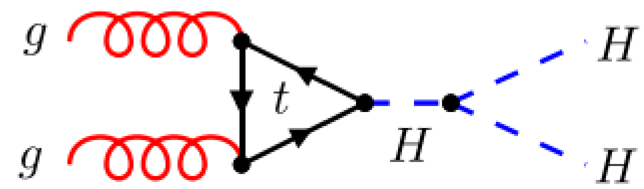




~65% corrections at $\sqrt{\hat{s}}$=260 GeV

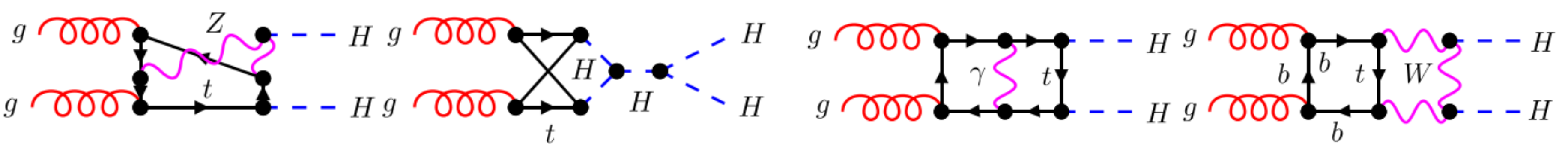
EW corrections to double H production at the LHC

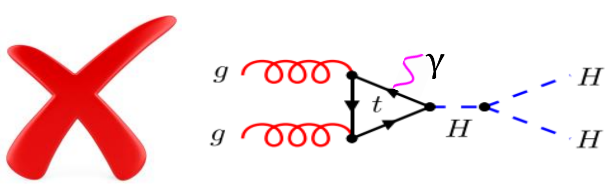
LO diagrams:



Typical Feynman diagrams at LO

NLO diagrams:





Forbidden due to Furry Theorem

Typical Feynman diagrams at NLO EW

Amplitudes of $g(p_1)g(p_2) \to H(p_3)H(p_4)$

Decomposition: $\mathcal{M}^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu} + \Delta_0^{\mu\nu} + \Delta_5^{\mu\nu}$

- $\Delta_0^{\mu\nu}$: depends on p_1^{μ} or p_2^{ν}. No contribution at the matrix element squred level.
- $\Delta_5^{\mu\nu}$: depends on Levi-Civita tensor. No contribution at the matrix element squred level at NLO EW.
- F_1, F_2: Form factors.

Calculation of form factors

Form factors can be decomposed into:

$$F_{1,2}(x) = \sum_i d_i(x)\, FI_i(x)$$

$x: m_H, m_t, m_W, m_Z,$
$\hat{s} = (p_1 + p_2)^2,$
$\hat{t} = (p_1 - p_3)^2.$

Reduce $FI_i(\hat{s})$ to master integrals (IBP):

$$\{FI_i(x)\} = \{\sum_k c_{i,k}(x) I_k(x)\}$$

- $d_i(x)$ and $c_{i,k}(x)$ are analytic.
- A huge number of I_k need to be calculated.

- The number of $\{I_k\} < \{FI_i\}$.
- The number of I_k is finite.
- We can construct the different equations for I_k and solve them.

A.V. Smirnov et al: 1004.4199

Different equations for I_k

Construct differential equations (DEs):$\vec{I}(x) = \{I_1(x), I_2(x) \ldots I_N(x)\}$

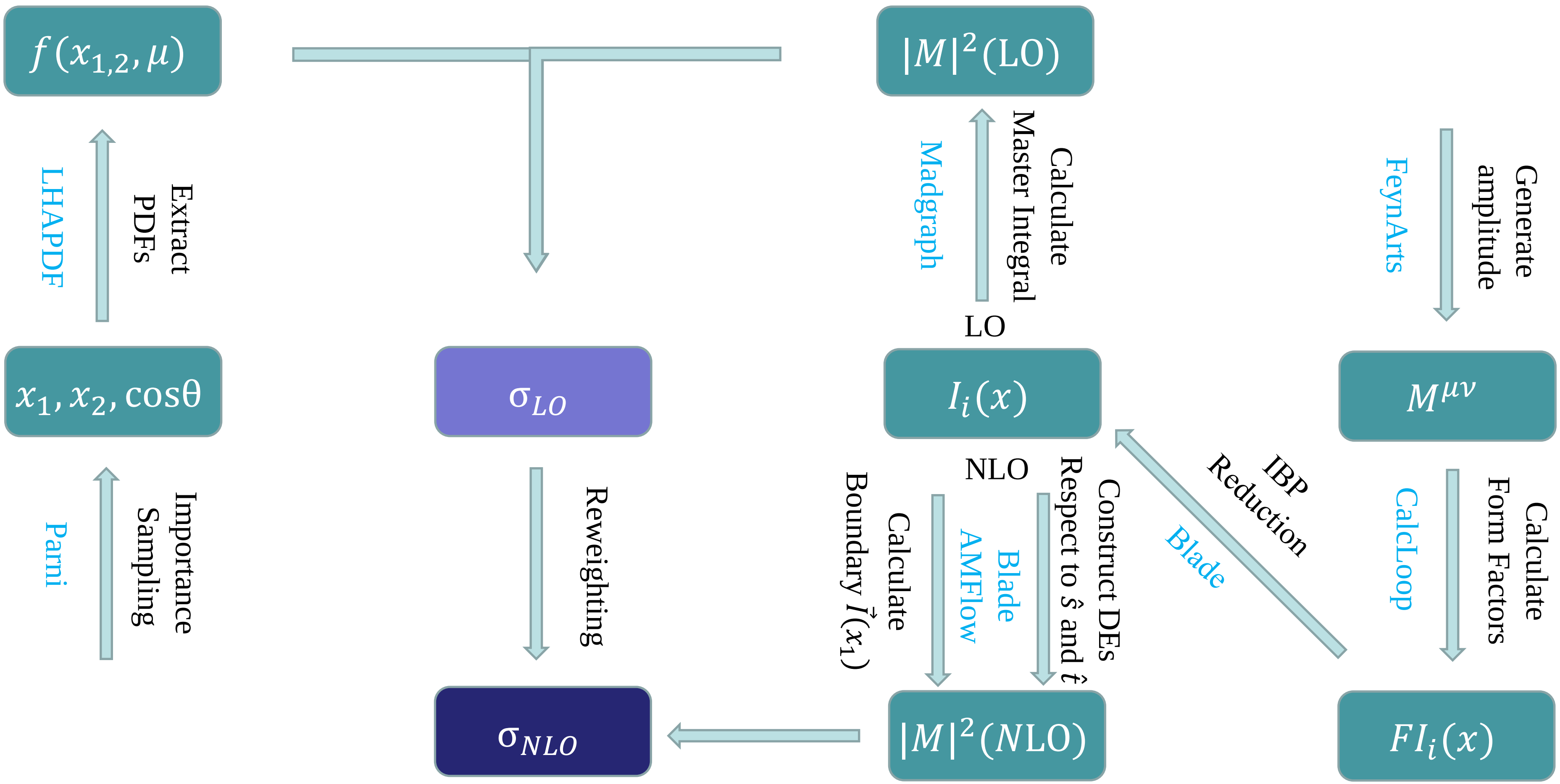
$$\frac{dI_m(x)}{dx} = \sum_n A_{m,n}(x)\, \mathrm{I}'_{\mathrm{n}}(x) \xrightarrow{\text{IBP}} \frac{d\vec{I}(x)}{dx} = A(x)\vec{I}(x)$$

- $\vec{I}(x)$ can be expanded as a power expansion near x_0,
  - regular:$S = \{0\}, k_0 = 0,$
  - singular:$S = \{-2\epsilon, 1+\epsilon \ldots\}, k_\mu \geq 0,$

$$I_i(x) = \sum_{\mu\in S}(x-x_0)^\mu \sum_{k=0}^{k_\mu} \log(x-x_0)^k \sum_{n=0}^{m} c_{i,\mu,k,n}(x-x_0)^n$$

- $c_{i,\mu,k,n}$ can be determined once any boundary $\vec{I}(x_1)$ are provided.
- $\vec{I}(x_1)$ can be determined by AMFlow
- Taking adequate expansion order m, we can eventually achieve predictions with high precision.
- $\vec{I}(x)$ can be evaluated at any points of x efficiently.

Calculation flowchart



Input Paramaters

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$$m_t = 172.69 \text{ GeV}$$

$$\frac{m_H^2}{m_t^2} = \frac{12}{23}, \frac{m_Z^2}{m_t^2} = \frac{23}{83}, \frac{m_W^2}{m_t^2} = \frac{14}{65},$$

$$G_\mu = 1.166378 \times 10^{-5} \text{ GeV}^{-2}$$

$$\alpha = \frac{\sqrt{2}}{\pi} G_\mu m_W^2 \left(1 - \frac{m_W^2}{m_Z^2}\right)$$

CKM=1

PDFs: NNPDF31_nlo_as_0118

on-shell renormalization: masses and fields; G_μ-scheme: Electromagnetic coupling

Denner et al:1912.06823

D=4-2ε， $\varepsilon = \pm 1/1000$

$$\sigma(\varepsilon) = a_0 + a_1\varepsilon + a_3\varepsilon^2 + \cdots$$

$$\sigma(0) \sim \frac{\sigma(+1/1000) + \sigma(-1/1000)}{2} = a_0 + a_3\varepsilon^2 + \ldots$$

Results: Total cross sections

μ	$M_{HH}/2$	$\sqrt{p_T^2 + m_H^2}$	m_H
LO	19.96(6)	21.11(7)	25.09(8)
NLO	19.12(6)	20.21(6)	23.94(8)
$\mathcal{K}$-factor	0.958(1)	0.957(1)	0.954(1)

LO and NLO EW corrected integrated cross sections (in fb) 14 TeV LHC.

- Differences with varying scale choices are around 20%.
  - Huge scale uncertainties. Can be reduced by including QCD corrections.
- K-factor is insensitive to the scale choice.
  - EW corrections beyond NLO are on the order of a few thousandths.
- The statistical uncertainty for the K-factor is smaller than that of $\sigma_{LO,NLO}$.
  - K-factor can get a controllable error with far fewer events.

Results: Differential cross sections

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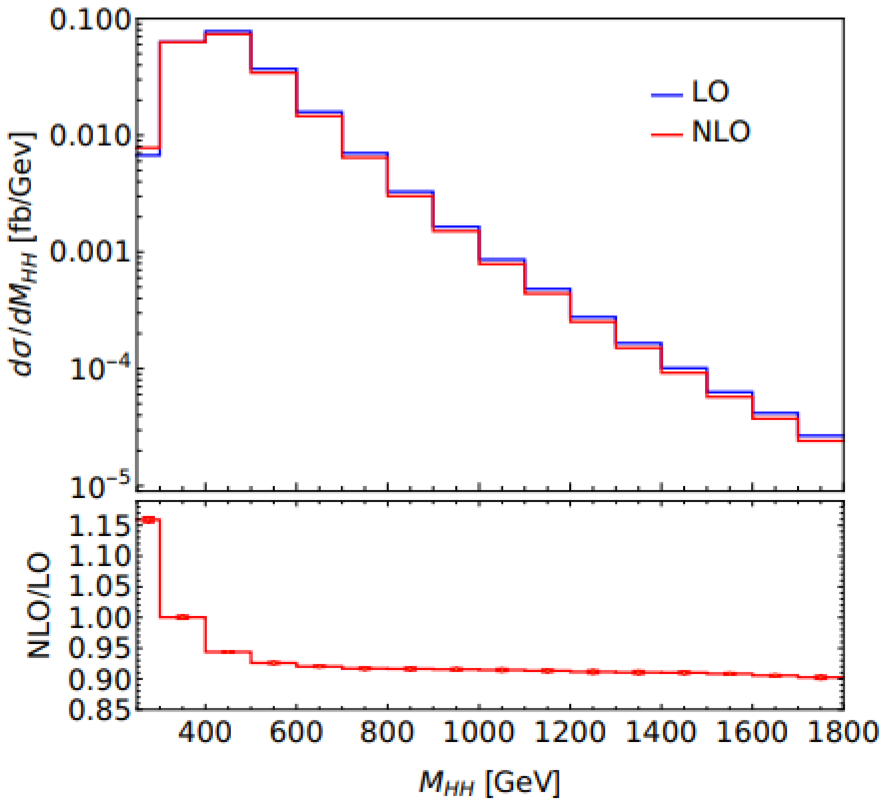
$$\Delta\sigma^{\rm NLO} = \Delta\mathcal{K}\Delta\sigma^{\rm LO}$$



$\Delta\sigma_{LO}$: based on 300k events
ΔK: based on 30k events

Results: Differential cross sections

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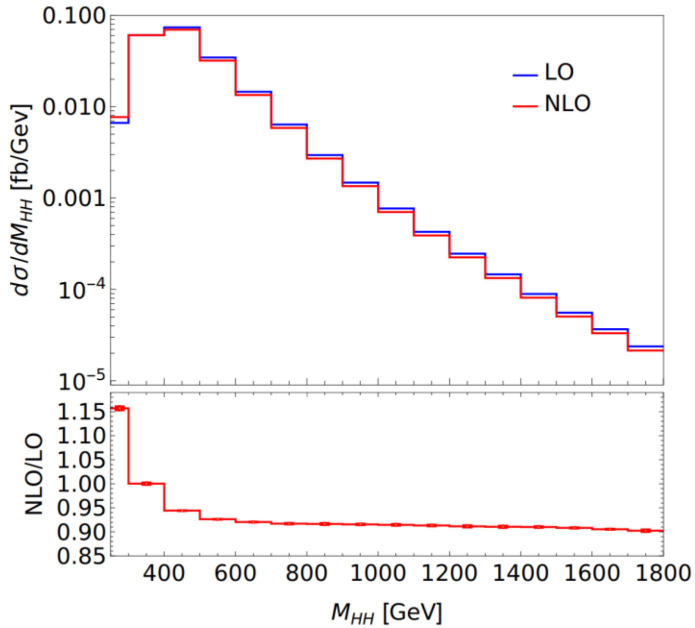
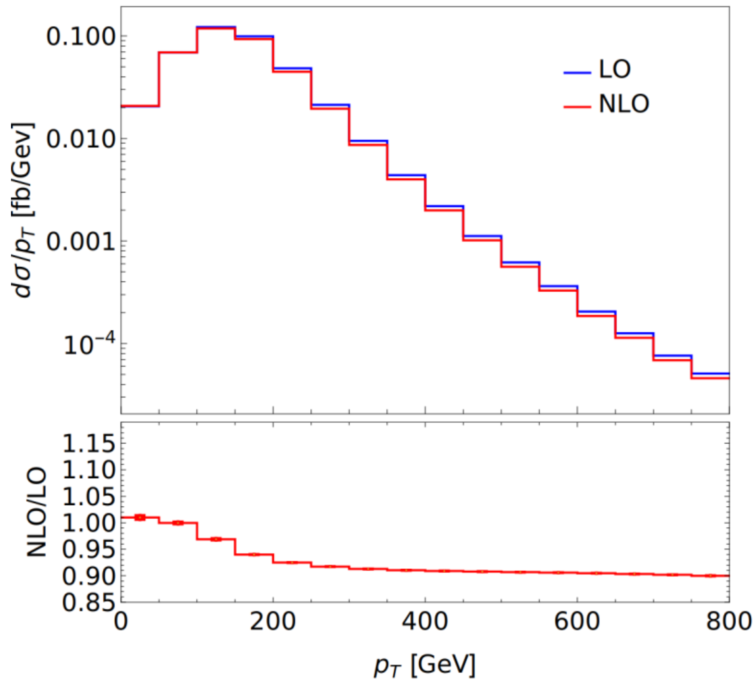


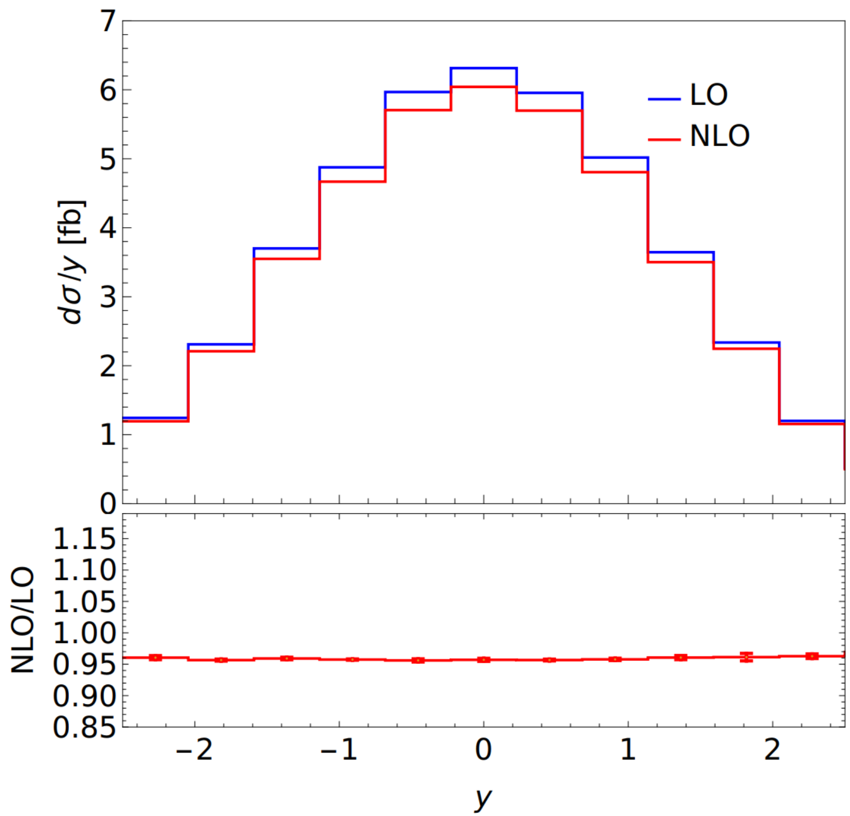
- Big positive corrections at the HH threshold.
  - $\sigma_{LO}(\sqrt{\hat{s}} = 2m_H) \sim 0$. Enhancement from Log(v) at NLO EW.
- -10% correction at high energy region.
  - EW Sudakov effects.
- Tiny cross section at high energy region
  - Gluon PDFs are highly suppressed at high energy region.

Results: Differential cross sections



- Positive corrections at the beginning of the spectrum.
  - ➢ The events in this region are mixed with high $\sqrt{\hat{s}}$ and low $\sqrt{\hat{s}}$.
- -10% correction at high energy region.
  - ➢ EW Sudakov effects.

Results: Differential cross sections



- Flat corrections at around -4%.
    - ➢ Similar to the total cross section

Summary

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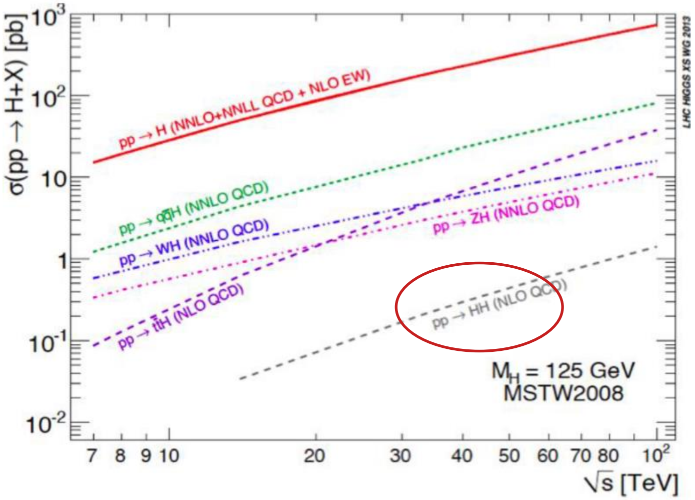
- Higgs self coupling is important to identify the Higgs potential and to probe new physics.
- The study of $\sigma(HH)$ is the best way to extract the Higgs self coupling.
- Our full calculation includes all the diagrams and all the mass effects.
- -4% EW corrections at total cross section level.
- For dimensionful observables, EW corrections reach up to +15% at the beginning of the spectrum and −10% in the tail.
- Our results suggest that the remained uncertainties from theoretical side is overall about few percent and it's precise enough for the measurements at the HL-HLC.

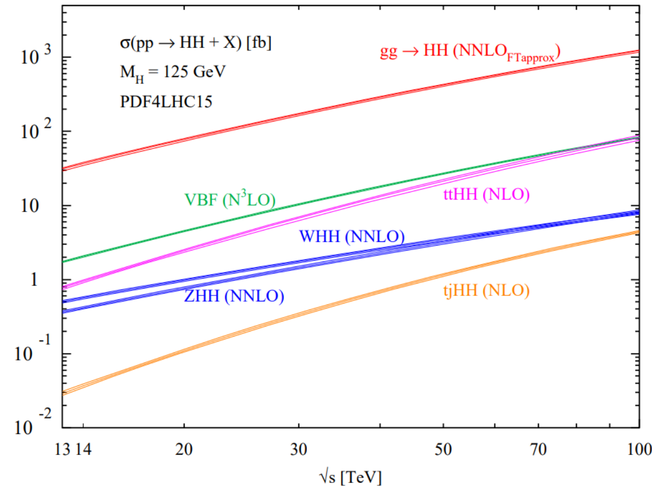
Thanks for your attention!

Higgs production at the LHC

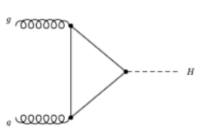
Some Higgs hadroproduction channels:



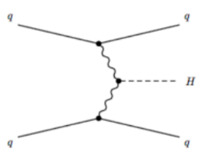
Contino et al: 1606.09408



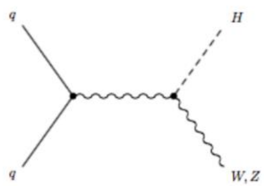
Micco et al: 1910.00012



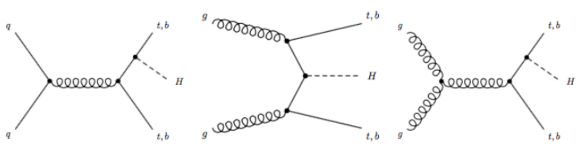
$gg \to H$



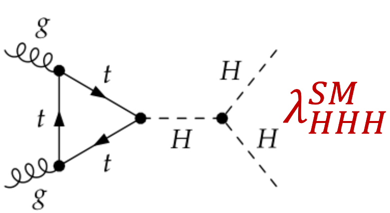
Vector boson fusion



Associated $V + H$ production



$\bar{t}t + H$ production



Double Higgs production

Cross section of $pp \to HH$

- Due to the dominant gluon luminosity $f_{g/p}(x,\mu)$, other channels are at least one order magnitude lower.

$$\sigma^{\rm LO(NLO)} = \frac{1}{512\pi}\int_0^1 {\rm d}x_1 \int_0^1 {\rm d}x_2 \int_{\hat{t}_-}^{\hat{t}_+} {\rm d}\hat{t} \times \frac{1}{\hat{s}^2} f_{g/p}(x_1,\mu) f_{g/p}(x_2,\mu)\hat{\sigma}^{\rm LO(NLO)},$$

$$\hat{\sigma}^{\rm LO} = \sum_{i=1}^{2} |F_i^{(0)}|^2 \,,$$

$$\hat{\sigma}^{\rm NLO} = \sum_{i=1}^{2} |F_i^{(0)}|^2 + F_i^{(0)} F_i^{(1)*} + F_i^{(0)*} F_i^{(1)} \,,$$

Amplitudes of $g(p_1)g(p_2) \to H(p_3)H(p_4)$

$$\mathcal{M}^{\mu\nu} = F_1 T_1^{\mu\nu} + F_2 T_2^{\mu\nu}$$

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu\, p_2^\mu}{p_1 \cdot p_2} \,,$$
$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_T^2\,(p_1 \cdot p_2)} \big[2\,(p_1 \cdot p_2)\, p_3^\nu\, p_3^\mu - 2\,(p_1 \cdot p_3)\, p_3^\nu\, p_2^\mu - 2\,(p_2 \cdot p_3) p_3^\mu\, p_1^\nu + m_H^2\, p_1^\nu\, p_2^\mu \big] \,,$$

$$P_1^{\mu\nu} = +\frac{1}{4}\frac{D-2}{D-3} T_1^{\mu\nu} - \frac{1}{4}\frac{D-4}{D-3} T_2^{\mu\nu} \,,$$

$$P_2^{\mu\nu} = -\frac{1}{4}\frac{D-4}{D-3} T_1^{\mu\nu} + \frac{1}{4}\frac{D-2}{D-3} T_2^{\mu\nu} \,,$$

$$F_1 = P_1^{\mu\nu}\mathcal{M}_{\mu\nu} \,,\; F_2 = P_2^{\mu\nu}\mathcal{M}_{\mu\nu} \,.$$