The cosmological gravitational wave background from Pulsar timing array observations

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Ding Ran & Tian chi, arXiv: 2309.01643

紫金山暗物质研讨会

29–31 Dec 2023

Outline

- Introduction to CGWB
- CGWB anisotropies based on PTA observations
- Cross-correlations of CGWB with CMB and gravitational lensing
- Outlooks: Challenges and opportunities

SGWB

Diffuse gravitational-wave signal resulting from the superposition of numerous unresolved sources

Astrophysical (AGWB)

Unresolved astrophysical sources:

- astrophysical black hole
- compact binaries
- rotating neutron star
- core-collapse supernovae

Cosmological (CGWB)

Early Universe mechanisms:

- inflation, (pre) reheating,
- primordial black holes,
- cosmic strings, domain walls
- phase transitions

See Wang Sai's talk

Observations

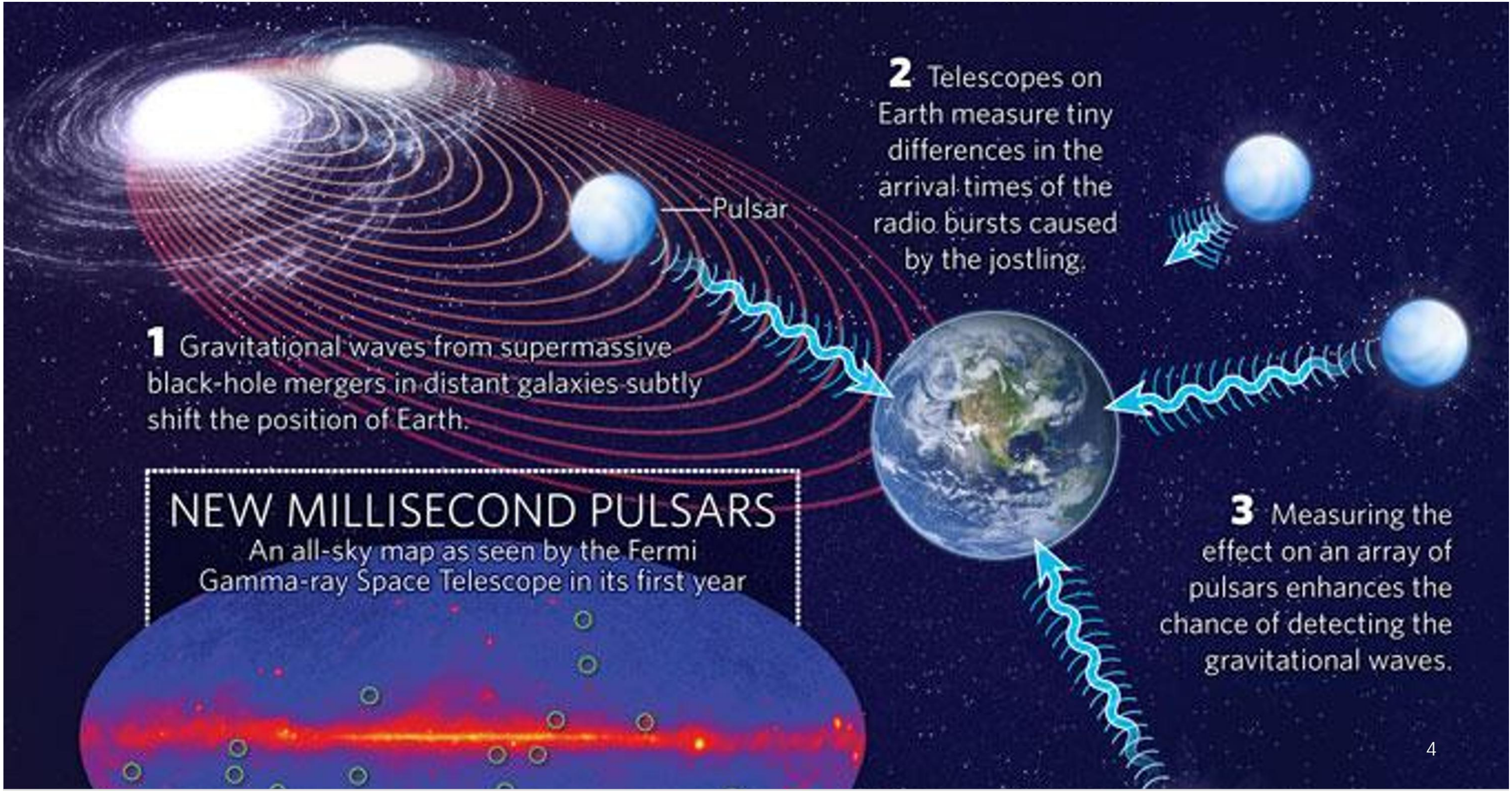
Interferometers (medium frequency)

- Ground-based : aLIGO/Virgo/KAGRA, ET, CE
- Space-based:LISA, Taiji, Tianqin, BBO, DECIGO

Pulsar timing array (low frequency)

- NANOGrav, CPTA , EPTA/InPTA, PPTA

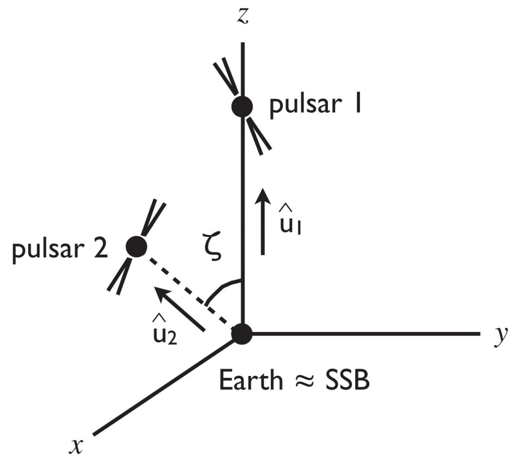
Pulsar timing array observations



Monopole measurement (isotropy) : Hellings-Downs Curve

Correlations of (Fourier) amplitudes of timing residual

$$\langle c_{ai} c_{bj} \rangle = \delta_{ij} \left(\delta_{ab} \varphi_{ai} + \Phi_{ab,i} \right)$$



See Xia Ziqing ’ s talk

intrinsic pulsar noise

$$\varphi_{ai} = \frac{A_a^2}{12\pi^2} \frac{1}{T} \left(\frac{f_i}{f_{\rm ref}} \right)^{-\gamma_a} f_{\rm ref}^{-3}$$

isotropic GWB with HD correlations

$$\Phi_{ab,i} = \Phi_{{\rm HD},i} \Gamma_{\rm HD} \left(\xi_{ab} \right)$$

Function of or $P(\Omega)$

antenna response

$$\Gamma(f, \xi_{ab}) \propto \Phi_{\rm HD} \int_{S^2} d^2\hat{\Omega} \left[\mathcal{F}^+ \left(\hat{p}_a, \hat{\Omega} \right) \mathcal{F}^+ \left(\hat{p}_b, \hat{\Omega} \right) + \mathcal{F}^\times \left(\hat{p}_a, \hat{\Omega} \right) \mathcal{F}^\times \left(\hat{p}_b, \hat{\Omega} \right) \right]$$

$$= \Phi_{\rm HD} \Gamma_{\rm HD} \left(\xi_{ab} \right)$$

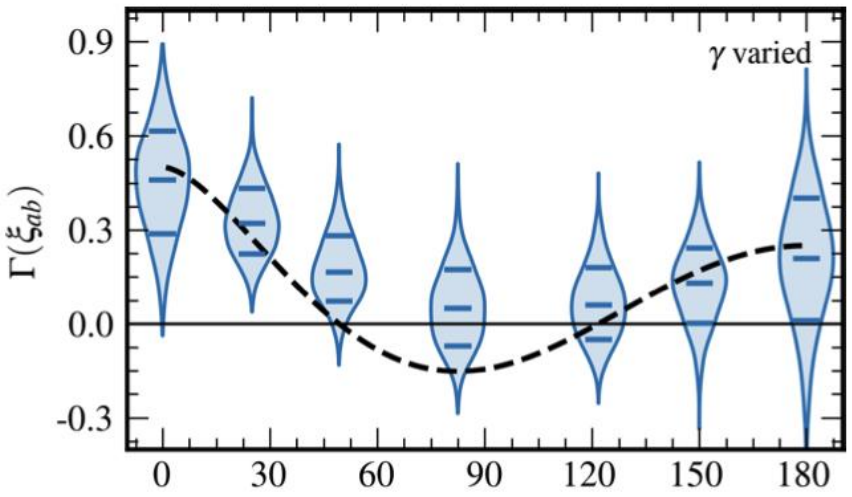
Hellings-Downs curve

$$\Phi_{{\rm HD},i} = \frac{A_{\rm HD}^2}{12\pi^2} \frac{1}{T} \left(\frac{f_i}{f_{\rm ref}} \right)^{-\gamma_{\rm HD}} f_{\rm ref}^{-3}$$

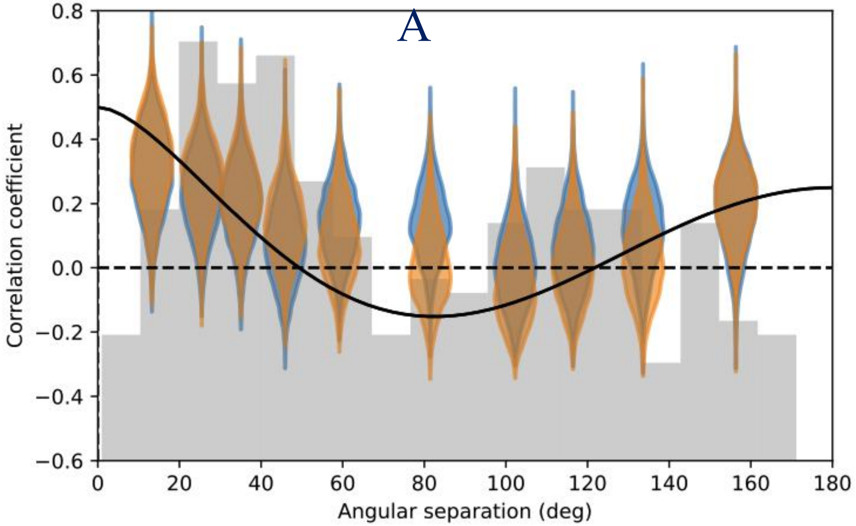
GWB auto-power spectrum

The Detections of the Hellings-Downs Curve

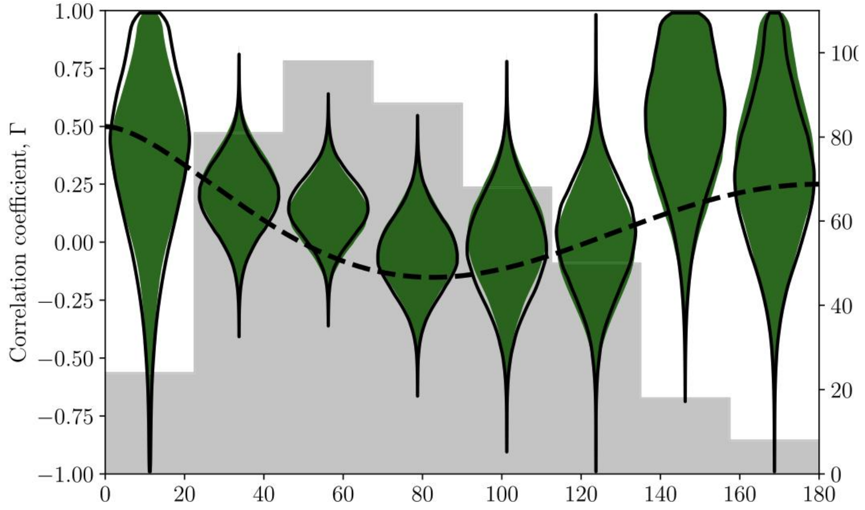
NANOGrav



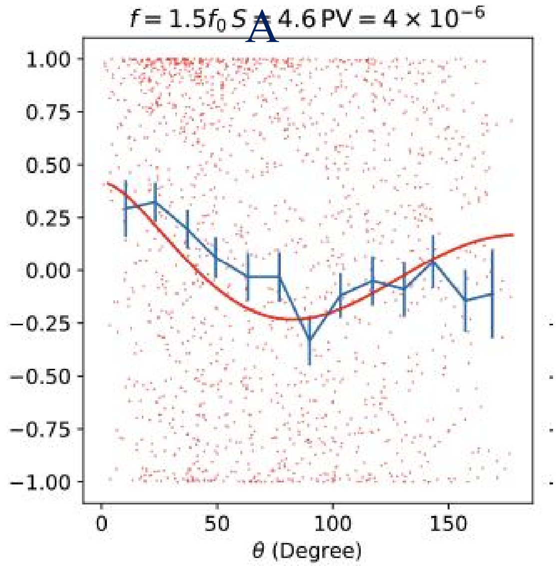
EPT
A



PPTA



CPT
A



Multipoles measurement (anisotropies)

power of the GWB has anisotropies

$$\Gamma(f,\xi_{ab}) \propto \int_{S^2} d^2\hat{\Omega}\Phi(f,\hat{\Omega}) \left[\mathcal{F}^+\left(\hat{p}_a,\hat{\Omega}\right)\mathcal{F}^+\left(\hat{p}_b,\hat{\Omega}\right)+\mathcal{F}^\times\left(\hat{p}_a,\hat{\Omega}\right)\mathcal{F}^\times\left(\hat{p}_b,\hat{\Omega}\right)\right]$$

$$=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\int_{S^2} d^2\hat{\Omega}a_{lm}(f)Y_{lm}(\hat{\Omega}) \left[\mathcal{F}^+\left(\hat{p}_a,\hat{\Omega}\right)\mathcal{F}^+\left(\hat{p}_b,\hat{\Omega}\right)+\mathcal{F}^\times\left(\hat{p}_a,\hat{\Omega}\right)\mathcal{F}^\times\left(\hat{p}_b,\hat{\Omega}\right)\right]$$

spherical harmonic basis

$$C_l = \frac{1}{2l+1}\sum_{m=-l}^{l}|a_{lm}|^2$$

pulsar number

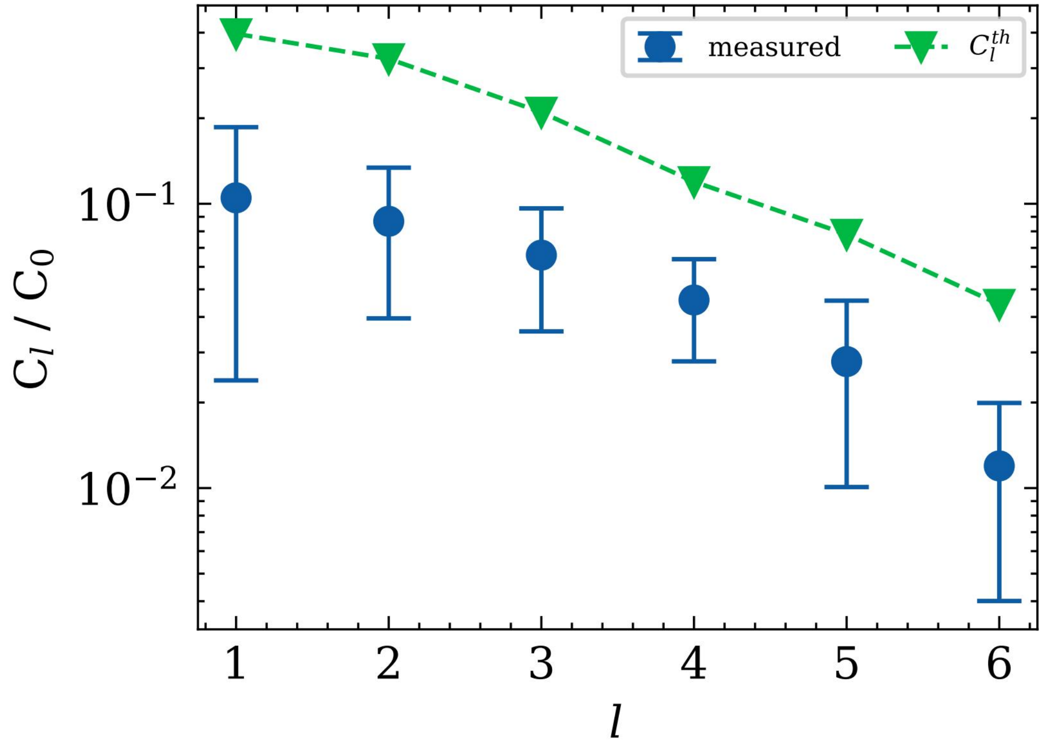
$$\sum_{l=0}^{l_{\max}}(2l+1) = (l_{\max}+1)^2 \sim N_{\mathrm{cc}} = N_{\mathrm{p}}\left(N_{\mathrm{p}}-1\right)/2$$

$$l_{\max} \sim N_{\mathrm{p}}$$

while taking into account angular resolution

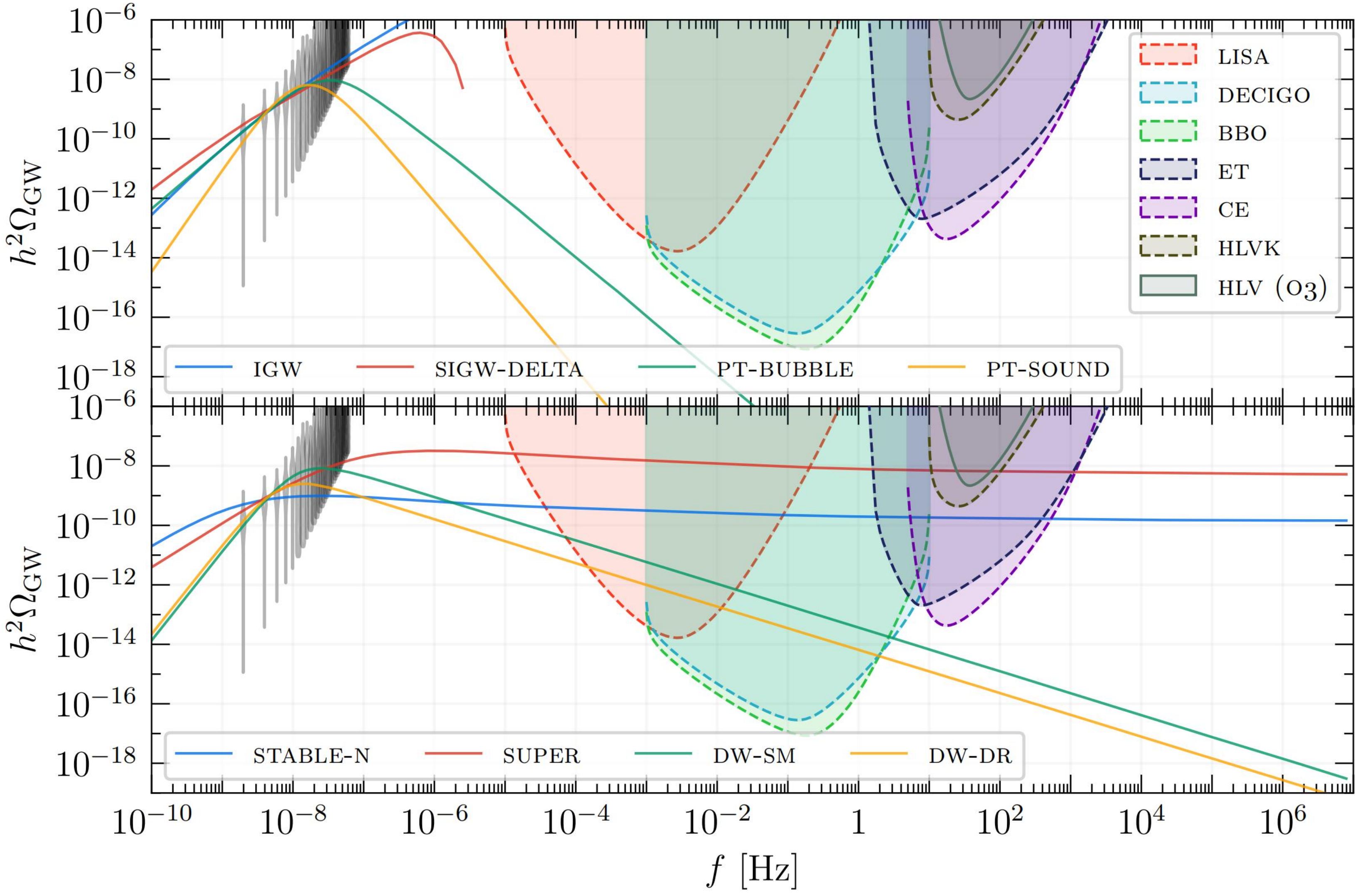
$$l_{\max} \sim \sqrt{N_{\mathrm{p}}}$$

The NANOGrav Collaboration, arXiv:2306.16221

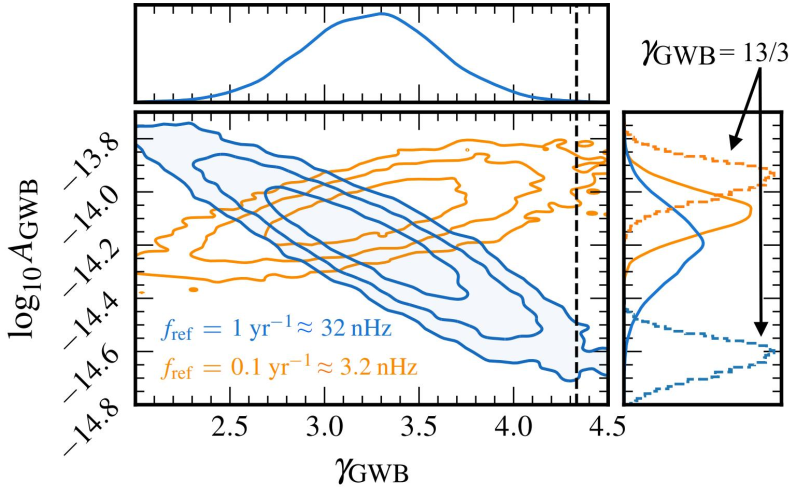


SGWB spectra of typicl new physics models

The NANOGrav Collaboration, arXiv:2306.16219

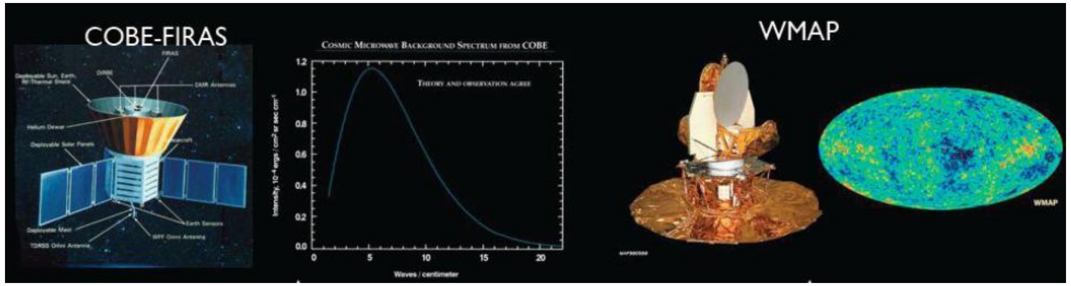


$$\Omega_{GW}(f) \equiv \frac{1}{\rho_c}\frac{d\rho_{GW}(f)}{d\ln f} = \frac{8\pi^4 f^5}{H_0^2}\frac{\Phi(f)}{\Delta f}$$



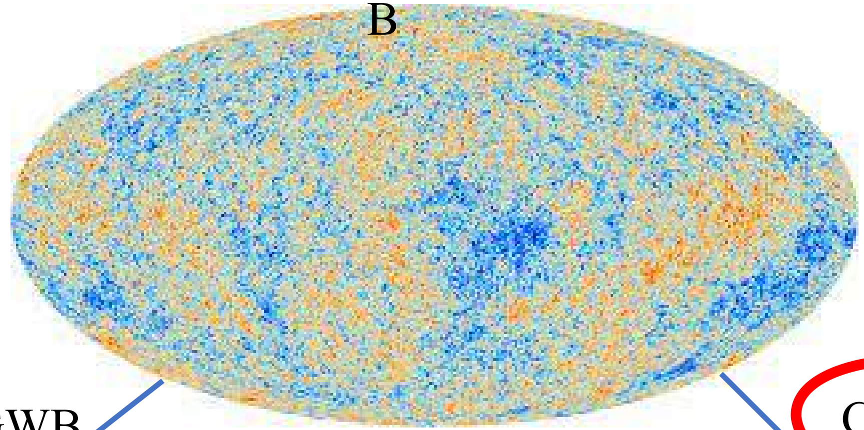
Is the GWB anisotropic?

CMB

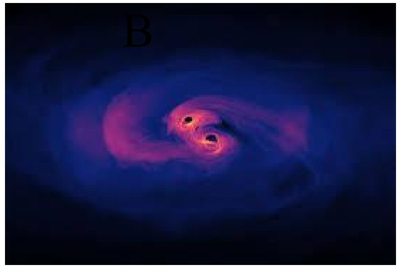


Global vs fluctuations

SGW
B

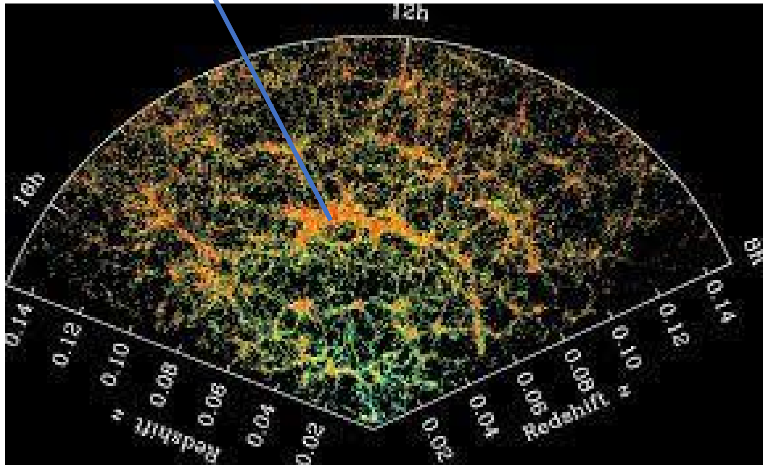


SMBH
B

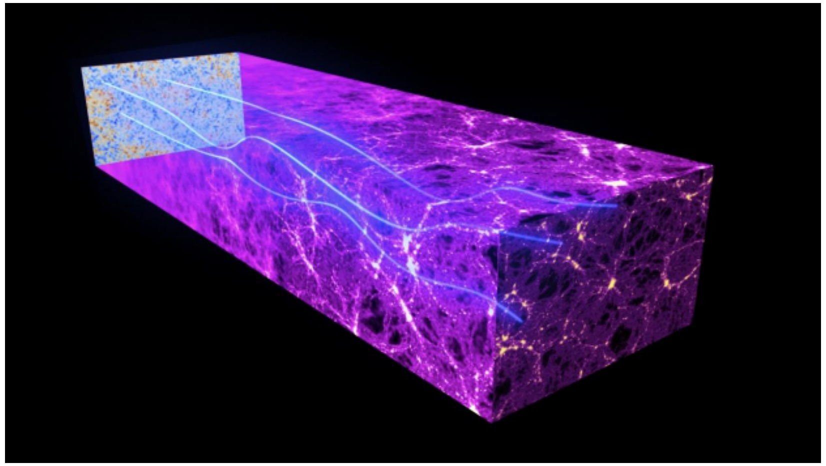


AGWB

CGWB



From sources



From propagation

Anisotropies in the CGWB

C. R. Contaldi, PLB, 771 (2017) 9–12

N. Bartolo, D. Bertacca, S. Matarrese, M. Peloso, A. Ricciardone, A. Riotto & G. Tasinato,PRD 100 no. 12, (2019) 121501

N. Bartolo, D. Bertacca, S. Matarrese, M. Peloso, A. Ricciardone, A. Riotto & G. Tasinato, PRD 102 no. 2, (2020) 023527

A. Ricciardone, L. V. Dall’Armi, N. Bartolo, D. Bertacca, M. Liguori, & S. Matarrese, PRL, 127 no. 27, (2021) 271301

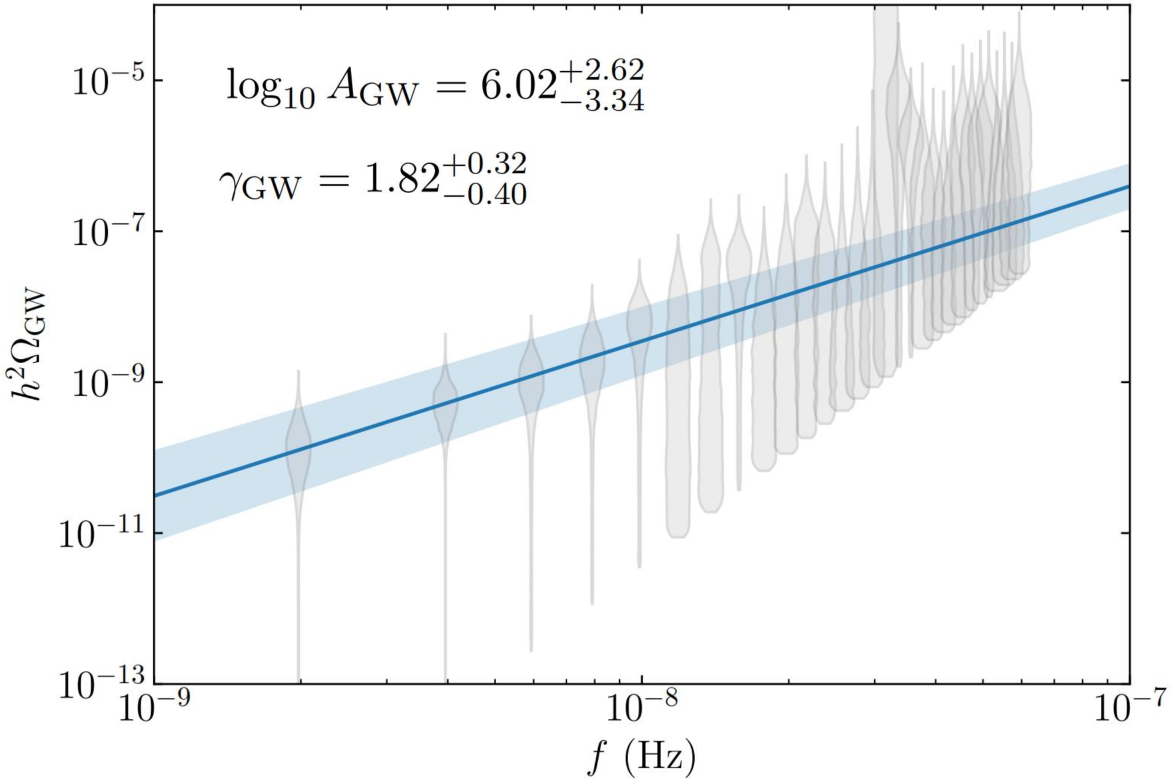
Yongping Li ,Fa Peng Huang ,Xiao Wang & Xinmin Zhang, PRD, 105, 083527 (2022)

F. Schulze, L. V. Dall’Armi, J. Lesgourgues, A. Ricciardone, N. Bartolo, D. Bertacca, C. Fidler & S. Matarrese, arXiv:2305.01602

Anisotropies in the CGWB

The NANOGrav 15-year Data

The NANOGrav Collaboration, arXiv:2306.16219



- assuming single power-law function

$$\Omega_{\rm GW} h^2 = A_{\rm GW} (f/{\rm Hz})^{\gamma_{\rm GW}}$$

- Weighted a non-gaussion PDF according to violin plot for each data point
- Construct likelihood to perform MCMC fitting

- GW energy density

$$\rho_{\rm GW}(\vec{x},\eta) = \int d\ln p \, d\Omega_{\hat{p}} \, p^4 f_{\rm GW}(\vec{x},p,\hat{p},\eta)$$

- GW spectrum

$$\Omega_{\rm GW}(\vec{x},p,\eta) \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln p} = \int d\Omega_{\hat{p}} \frac{p^4}{\rho_c} f_{\rm GW}(\vec{x},p,\hat{p},\eta)$$

- expand the distribution function

$$f_{\rm GW} = \bar{f}_{\rm GW}(p,\eta) - p\frac{\partial \bar{f}_{\rm GW}}{\partial p}\mathcal{G}(\vec{x},p,\hat{p},\eta)$$

- separate GW spectrum into isotropic + fluctuation

$$\Omega_{\rm GW}(\vec{x},p,\eta) \equiv \bar{\Omega}_{\rm GW}(p,\eta)\left[1+\delta_{\rm GW}(\vec{x},p,\eta)\right]$$

$$\delta_{\rm GW}(\vec{x},p,\hat{p},\eta) = \left[4 - \frac{\partial \ln \bar{\Omega}_{\rm GW}(p,\eta)}{\partial \ln p}\right]\mathcal{G}(\vec{x},p,\hat{p},\eta)$$

$$\Omega_{\rm GW} h^2 = A_{\rm GW} (f/{\rm Hz})^{\gamma_{\rm GW}}$$

$$\partial \ln \bar{\Omega}_{\rm GW}(p,\eta) / \partial \ln p = \gamma_{\rm GW}$$

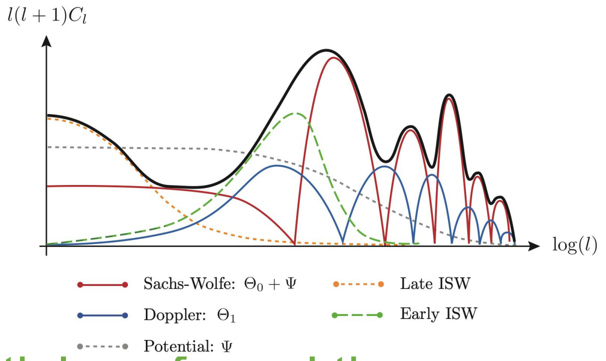
- Conformal Newtonian gauge

$$ds^2 = a^2(\eta)\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

- Boltzmann equation

$$\frac{df_{\rm GW}}{d\eta} = \frac{\partial f_{\rm GW}}{\partial \eta} + \hat{p}^i \partial_i f_{\rm GW} - p\frac{\partial f_{\rm GW}}{\partial p}\left[\Phi' - \hat{p}^i\partial_i \Psi\right] = 0$$

collisionless



- Boltzmann-Einstein equation

$$\mathcal{G}'(k,p,\mu,\eta) + ik\mu\mathcal{G}(k,p,\mu,\eta) = \Phi'(k,\eta) - ik\mu\Psi(k,\eta)$$

free-streaming | gravitational effect

- Line-of-sight integration

$$\mathcal{G}_l(k,p,\eta_0) \simeq \underbrace{\left[\mathcal{G}_0(k,p,\eta_{\rm in}) + \Psi(k,\eta_{\rm in})\right]}_{\rm SW} j_l\left[k(\eta_0 - \eta_{\rm in})\right] + \underbrace{\int_{\eta_{\rm in}}^{\eta_0} d\eta\left[\Psi'(k,\eta) + \Phi'(k,\eta)\right] j_l\left[k(\eta_0-\eta)\right]}_{\rm ISW}$$

initial conformal time where GW is produced

$$\eta_{\rm in} = 10^{-4}\,{\rm Mpc}$$

assuming adiabatic perturbations:

$$\mathcal{G}_0(k,\eta_{\rm in}) \simeq -\frac{2\Psi(k,\eta_{\rm in})}{4-\gamma_{\rm GW}}$$

$$\mathcal{G}_\ell(\vec{k},p,\eta_0) = T_\ell^{\rm GW}(\vec{k},p,\eta_{\rm in},\eta_0)\mathcal{R}(\vec{k})$$

- Transfer function

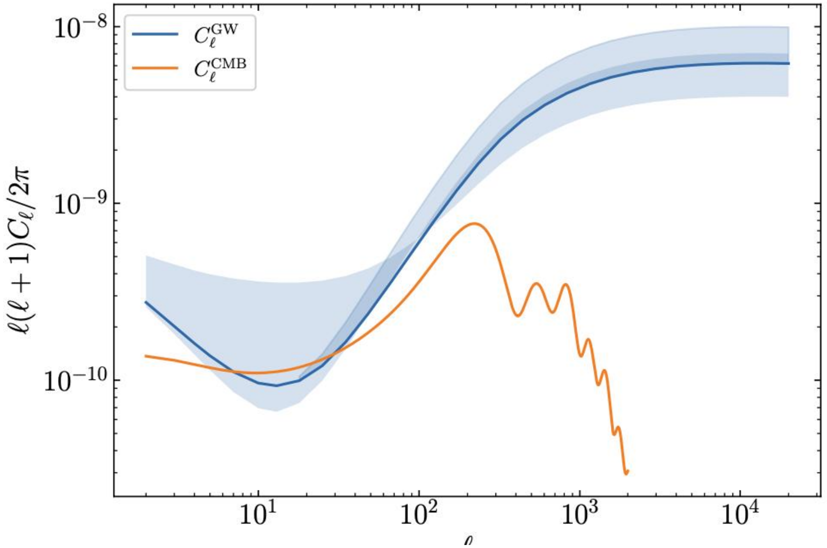
$$T_\ell^{\rm GW}(k,p,\eta_{\rm in},\eta_0) = T_\ell^{\rm SW}(k,p,\eta_{\rm in},\eta_0) + T_\ell^{\rm ISW}(k,\eta_{\rm in},\eta_0),$$

$$T_\ell^{\rm SW}(k,p,\eta_{\rm in},\eta_0) = \left[1 - \frac{2}{4-\gamma_{\rm GW}(p)}\right]\frac{\Psi(k,\eta_{\rm in})}{\mathcal{R}(\vec{k})} j_\ell\left[k(\eta_0-\eta_{\rm in})\right],$$

$$T_\ell^{\rm ISW}(k,\eta_{\rm in},\eta_0) = \int_{\eta_{\rm in}}^{\eta_0} d\eta \frac{\left[\Psi'(k,\eta)+\Phi'(k,\eta)\right]}{\mathcal{R}(\vec{k})} j_\ell\left[k(\eta_0-\eta)\right].$$

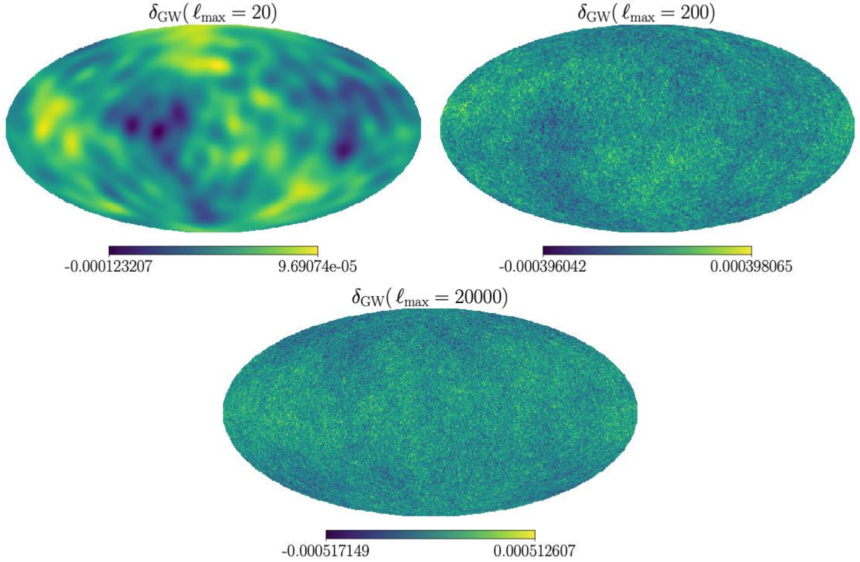
- Anisotropic power spectrum $$C_\ell^{\mathrm{GW}} = 4\pi \int \frac{dk}{k} \mathcal{P}_{\mathcal{R}}(k) \left| T_\ell^{\mathrm{GW}}(k, p, \eta_{\mathrm{in}}, \eta_0) \right|^2$$

We integrate metric perturbations generated by CLASS convoluted with spherical Bessel functions.



The anisotropies of the of CGWB has the following feature (and assumptions):

1. Homogeneous sources (?)
2. Start propagating at much earlier time ($\eta_{in} \ll \eta_{rec}$)
3. No collision terms (no diffusion damping)
4. Adiabatic initial perturbations(?)
5. SW term is modulated by the auto-spectrum



Cross-correlation with CMB

$$\langle a_{\ell m}^{\mathrm{GW}} a_{\ell' m'}^{\mathrm{CMB}*} \rangle \equiv \delta_{\ell\ell'} \delta_{mm'} C_\ell^{\mathrm{GW}\times\mathrm{CMB}}$$

- CMB multipole moments

$$\Theta_\ell(k,\eta_*,\eta_0) \simeq \underbrace{\left[\Theta_0(k,\eta_*) + \Phi(k,\eta_*)\right]}_{\mathrm{SW}} j_\ell\left[k\left(\eta_0-\eta_*\right)\right] + \underbrace{\frac{-iv_b(k,\eta_*)}{k}}_{\mathrm{Doppler}} j_\ell'\left[k\left(\eta_0-\eta_*\right)\right]$$

$$+ \underbrace{\int_{\eta_*}^{\eta_0} d\eta \left[\Psi'(k,\eta) + \Psi'(k,\eta)\right]}_{\mathrm{ISW}} j_\ell\left[k\left(\eta_0-\eta\right)\right]$$

- CMB transfer function

$$\Delta_\ell^{\mathrm{CMB}}(k,\eta_*,\eta_0) = \Delta_\ell^{\mathrm{SW}}(k,\eta_*,\eta_0) + \Delta_\ell^{\mathrm{DOP}}(k,\eta_*,\eta_0) + \Delta_\ell^{\mathrm{ISW}}(k,\eta_*,\eta_0),$$

$$\Delta_\ell^{\mathrm{SW}}(k,\eta_*,\eta_0) = \frac{\left[\Theta_0(k,\eta_*) + \Phi(k,\eta_*)\right]}{\mathcal{R}(\vec{k})} j_\ell\left[k\left(\eta_0-\eta_*\right)\right],$$

$$\Delta_\ell^{\mathrm{DOP}}(k,\eta_*,\eta_0) = -\frac{iv_b(k,\eta_*)}{k\mathcal{R}(\vec{k})} j_\ell'\left[k\left(\eta_0-\eta_*\right)\right],$$

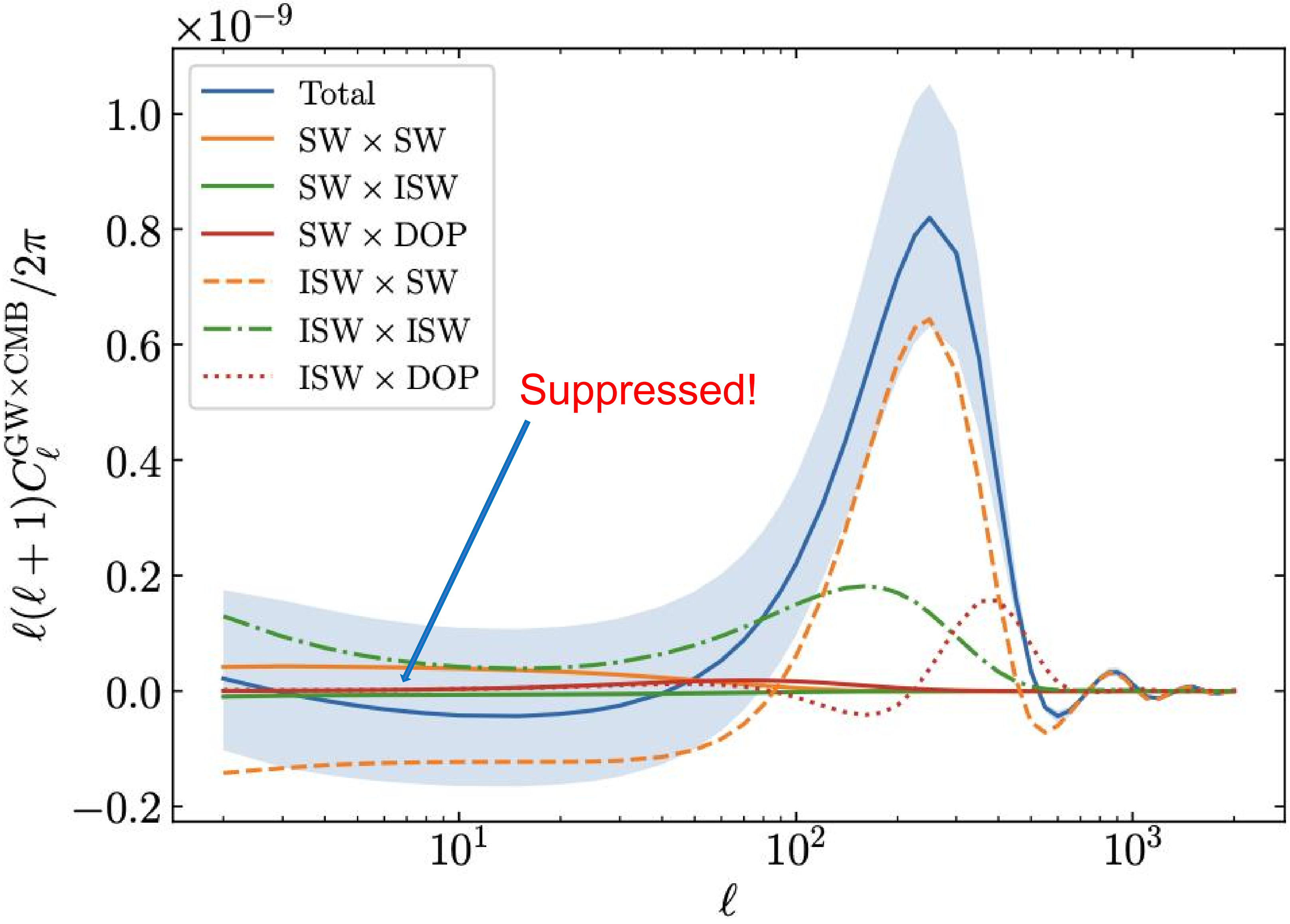
$$\Delta_\ell^{\mathrm{ISW}}(k,\eta_*,\eta_0) = \int_{\eta_*}^{\eta_0} d\eta \frac{\left[\Psi'(k,\eta) + \Phi'(k,\eta)\right]}{\mathcal{R}(\vec{k})} j_\ell\left[k\left(\eta_0-\eta\right)\right].$$

- GW x CMB Cross-correlation

$$C_\ell^{\mathrm{GW}\times\mathrm{CMB}} = 4\pi \int \frac{dk}{k} \mathcal{P}_{\mathcal{R}}(k) \left[T_\ell^{\mathrm{GW}}(k,\eta_{\mathrm{in}},\eta_0)\, \Delta_\ell^{\mathrm{CMB}}(k,\eta_*,\eta_0)\right]$$

$$= C_\ell^{\mathrm{SW}\times\mathrm{SW}} + C_\ell^{\mathrm{SW}\times\mathrm{DOP}} + C_\ell^{\mathrm{SW}\times\mathrm{ISW}} + C_\ell^{\mathrm{ISW}\times\mathrm{SW}} + + C_\ell^{\mathrm{ISW}\times\mathrm{DOP}} + C_\ell^{\mathrm{ISW}\times\mathrm{ISW}}$$

GW x CMB Cross-correlation

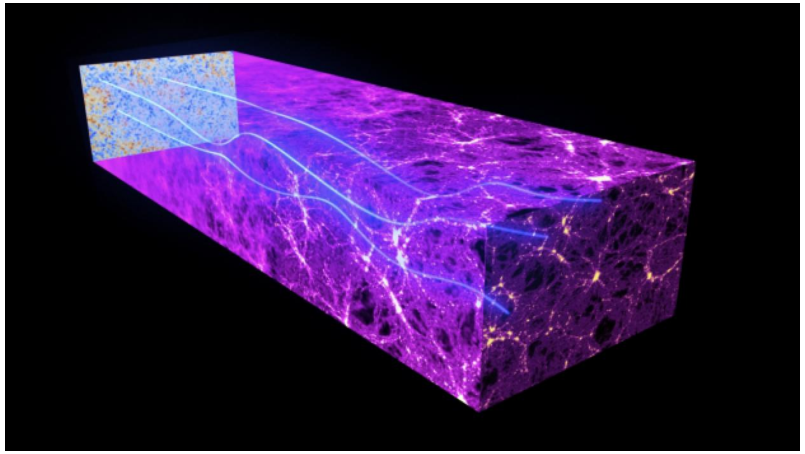


CMB lensing

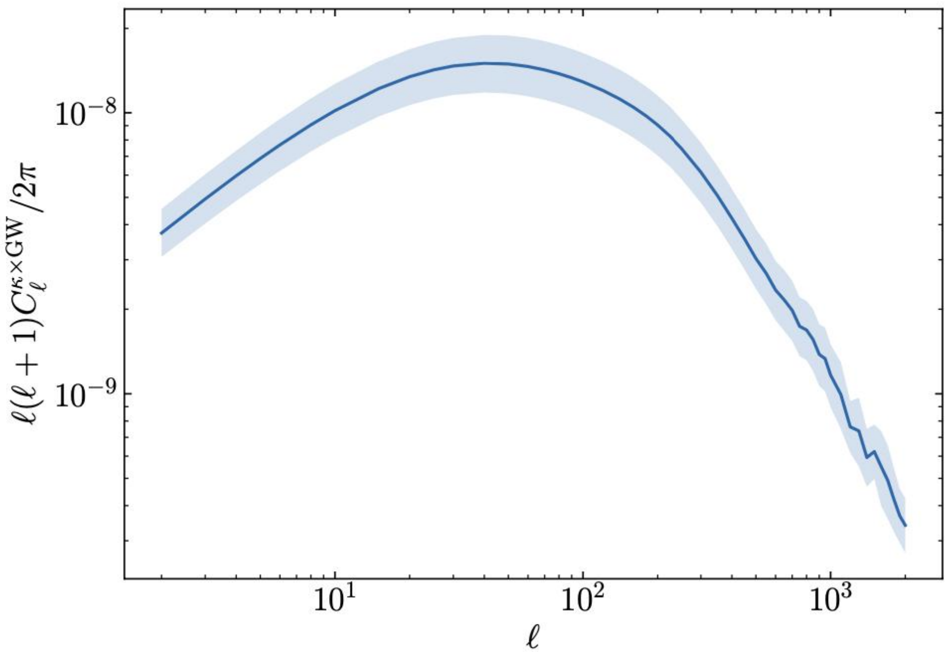
- Lensing potential $$\psi = -2\int_{\eta_0-\eta_*}^{\eta_0} d\eta \frac{(\eta-\eta_*)}{(\eta_0-\eta_*)(\eta_0-\eta)}\Psi(k,\eta)$$
- Lensing convergence $$\kappa = \int_{\eta_0-\eta_*}^{\eta_0} d\eta \frac{(\eta_0-\eta)(\eta-\eta_*)}{\eta_0-\eta_*}k^2\Psi(k,\eta)$$
- Lensing transfer function

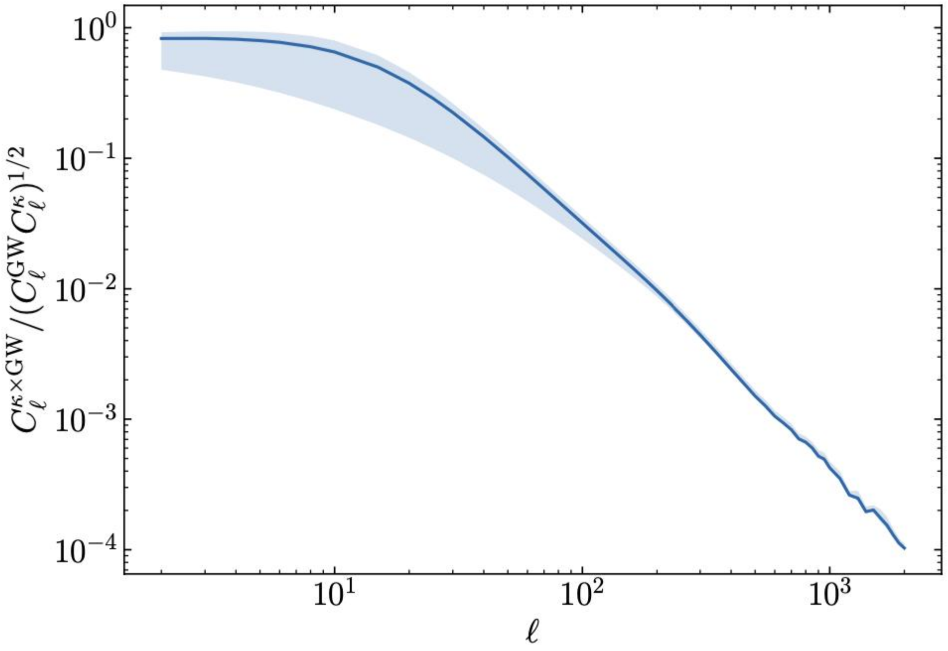
$$\Delta_\ell^\psi(k,\eta_*,\eta_0) = \int_{\eta_0-\eta_*}^{\eta_0} d\eta \frac{\Psi(k,\eta)}{\mathcal{R}(\vec{k})}\frac{\eta-\eta_*}{(\eta_0-\eta_*)(\eta_0-\eta)} j_\ell\left[k\left(\eta_0-\eta\right)\right],$$
$$\Delta_\ell^\kappa(k,\eta_*,\eta_0) = \int_{\eta_0-\eta_*}^{\eta_0} d\eta \frac{\Psi(k,\eta)}{\mathcal{R}(\vec{k})}\frac{(\eta_0-\eta)(\eta-\eta_*)}{\eta_0-\eta_*} j_\ell\left[k\left(\eta_0-\eta\right)\right].$$

CMB lensing



See Luo Wentao ' s talk





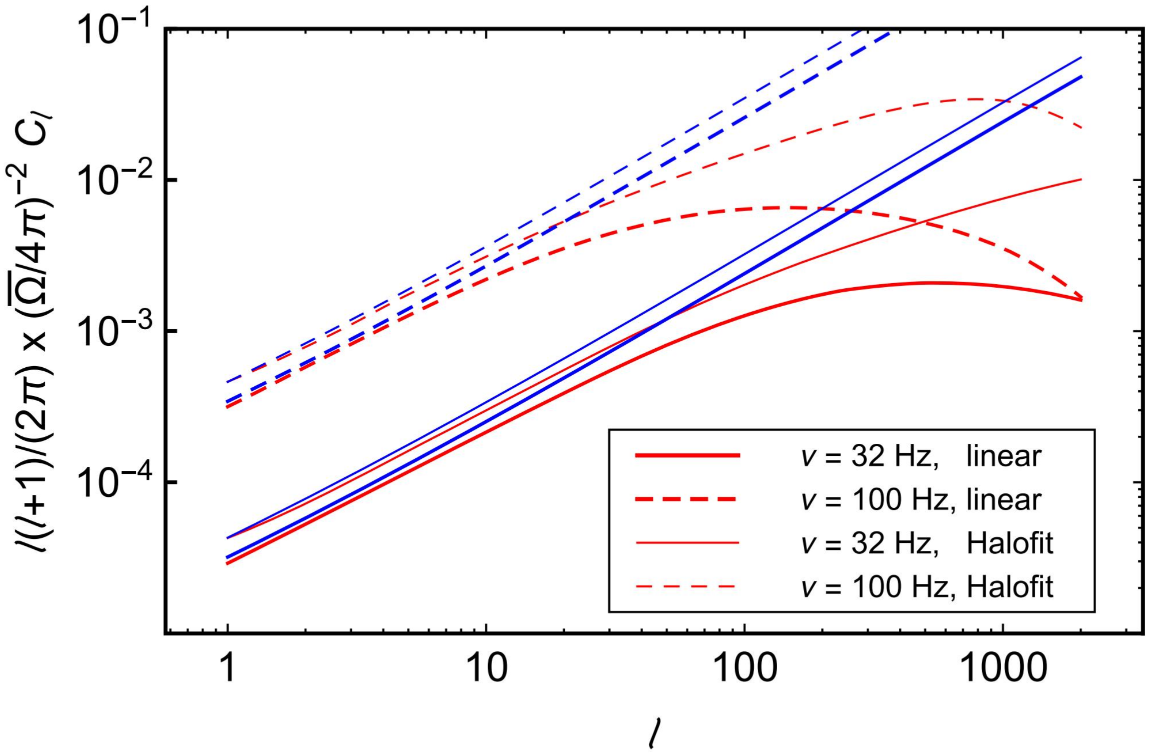
Compared with AGWB

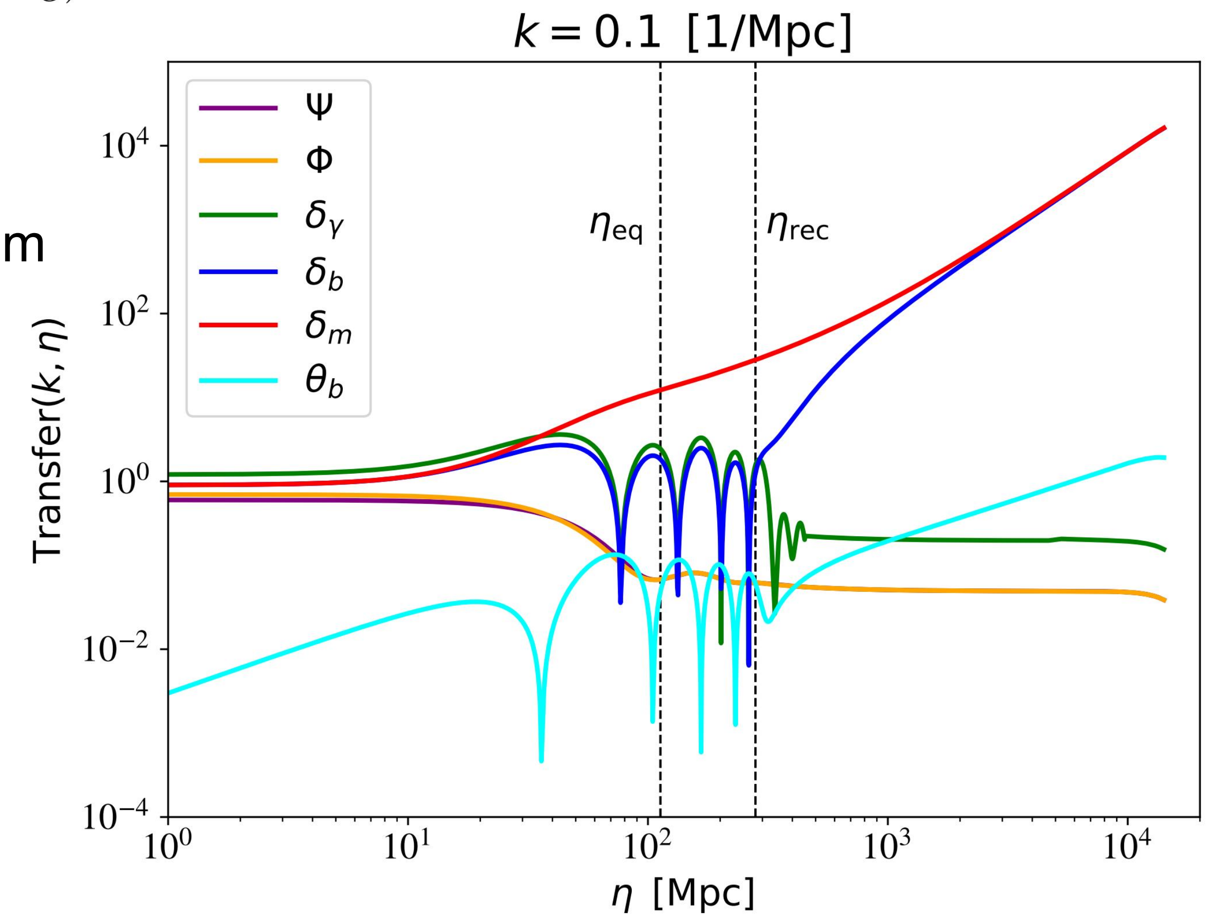
BH luminosity function $\mathcal{A}(\eta,\nu_O) \equiv a^4 \bar{n}_G(\eta) \int d\theta_G \mathcal{L}_G(\eta,\nu_G,\theta_G)$

$$\left(\ell+\frac{1}{2}\right)C_\ell(\nu_O) \simeq \left(\frac{\nu_O \mathcal{A}(\eta_O,\nu_O) b(\eta_O)}{4\pi\rho_c}\right)^2 \int_{k_{\min}} P_\delta(k)dk$$

matter power spectrum

G. Cusin, I. Dvorkin, C. Pitrou & J-P. Uzan, PRL, 120, 231101 (2018)





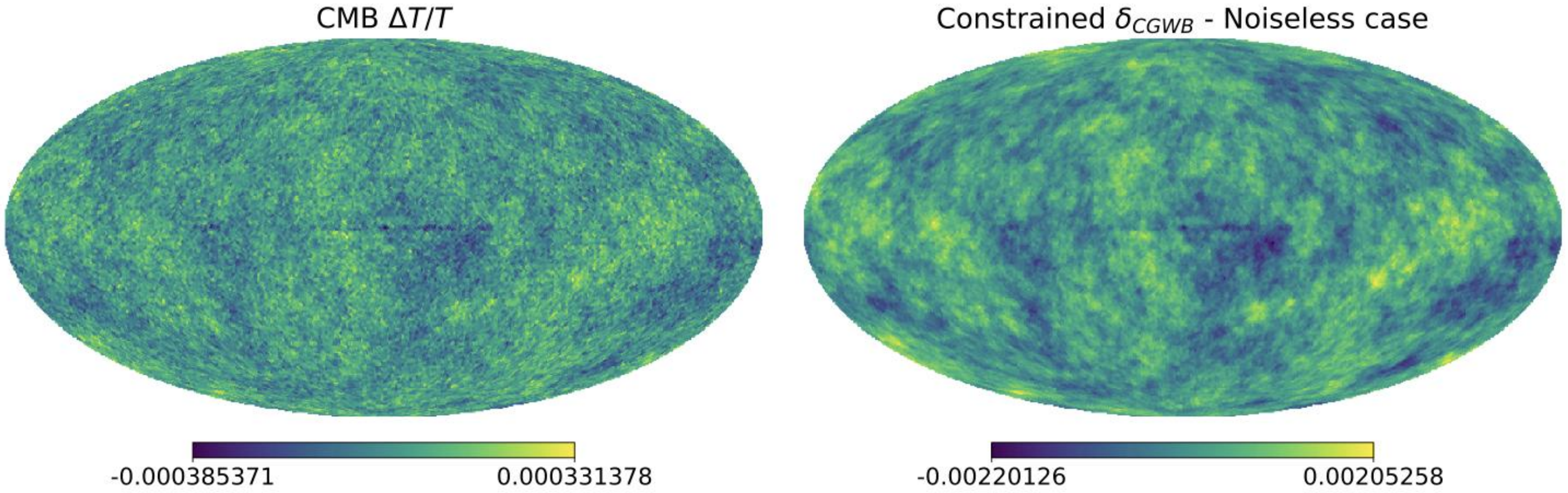
Summary

- Compute the anisotropic power spectrum of the CGWB based on the NANOGrav-15 years data
  - Anisotropies are at the same level with the CMB
- Cross-correlating with CMB and CMB-lensing
  - Suppressed cross-spectrum at lower ℓs
  - Unsuppressed cross-spectrum between the CGWB and the CMB-lensing

Outlooks

1: reconstruction from the CMB?

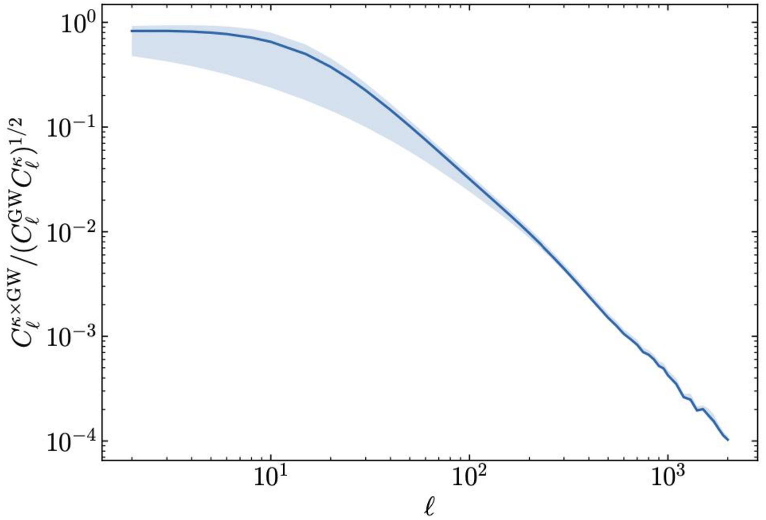
Reconstruction of δ_{GW} from the CMB



Ricciardone, et. al. PRL 2021 arXiv: 2106.02591

May not be feasible due to the suppressed CC.

But CC with lensing is NOT suppressed!



2: Disentangle AGWB and CGWB

Does cross-correlation help disentangle AGWB and CGWB signal?

Traditional cosmological tracers:

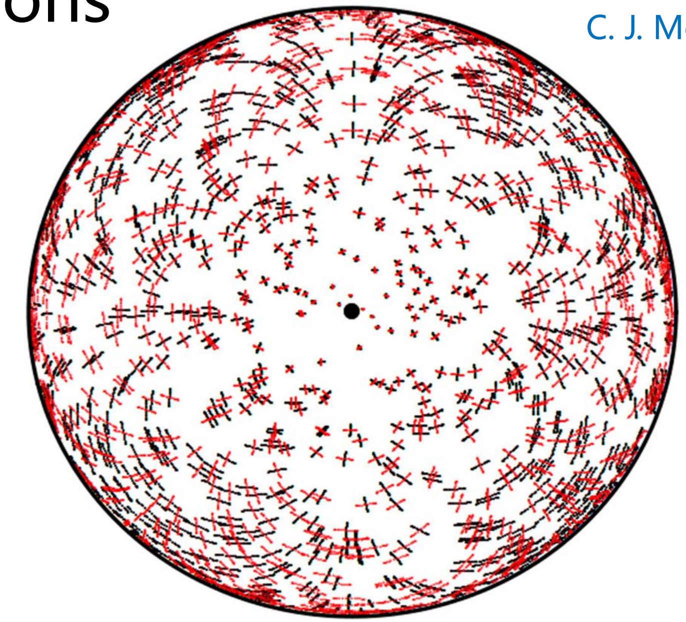
- CMB
- CMB-lensing
- Galaxy clustering
- …

Anisotropic GWB

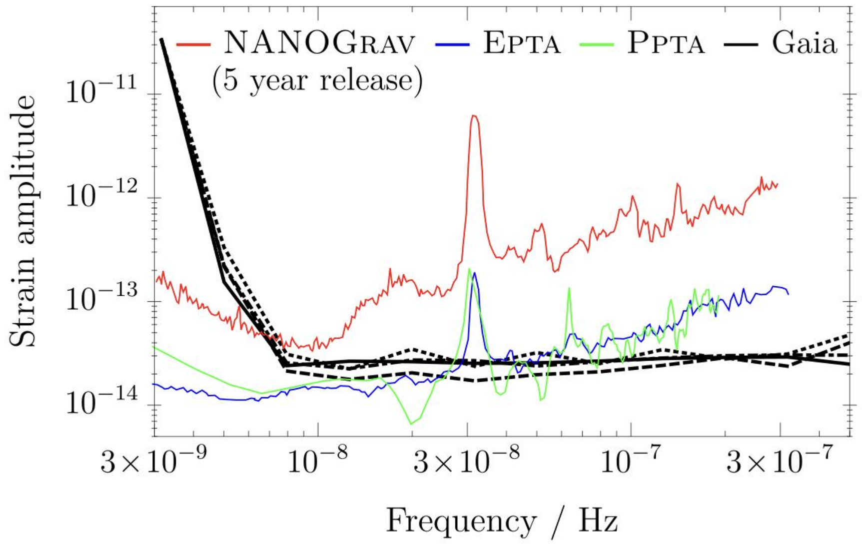
- AGWB
- CGWB

3: Astrometric detections

$N_{\rm star} \sim 10^9$



C. J. Moore, D. P. Mihaylov A. Lasen & G.Gilmore, PRL 119, 261102 (2017)



Thanks for your attention