Dark matter candidates from U(1) hidden sectors

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This talk is based on

Amin Aboubrahim, WZF, Pran Nath, Zhu-Yao Wang, 2008.00529

Amin Aboubrahim, WZF, Pran Nath, Zhu-Yao Wang, 2103.15769

Amin Aboubrahim, WZF, Pran Nath, Zhu-Yao Wang, 2106.06494

Kai-Yu Zhang, WZF, 2204.08067

WZF, Zi-Hui Zhang, Kai-Yu Zhang 2312.03837

Overview

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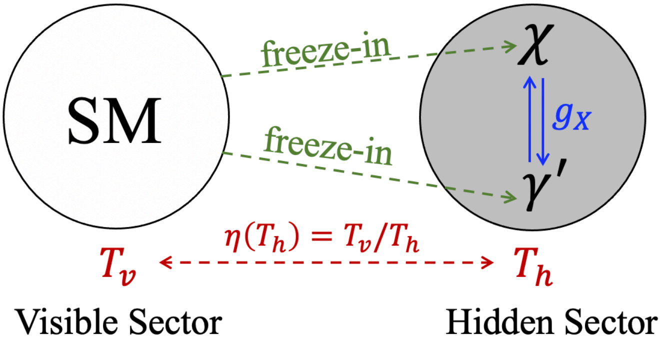
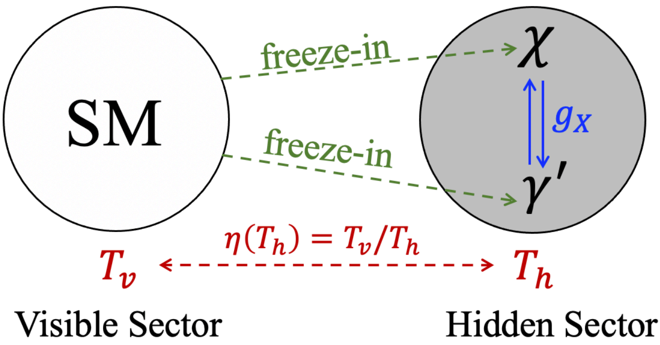
- Freeze-out scenarios – various problems: direct/indirect detection constraints, collider constraints...
- Turn to freeze-in – one major problem: why there exists such feeble coupling? A kinetic mixing just provides such smallness of the coupling constant.

A graphic illustration of the simplest $U(1)_X$ model



A graphic illustration of the simplest $U(1)_X$ model from freeze-in

A graphic illustration of the simplest $U(1)_X$ model



Because of the self-interaction inside the hidden sector, this simplest setup is difficult to calculate.

Why difficult?

- The thermal averaged cross-section of hidden sector interactions depend on the hidden sector temperature. We have no clue what the hidden sector temperature is.
- How to define the hidden sector temperature.
- How to setup a connection between the hidden sector temperature and the visible sector (SM) temperature.

Solution

In [Aboubrahim, WZF, Nath, Wang, 2008.005299], a general formalism was established to compute the *complete* evolution of the hidden sector (produced from freeze-in) particle number densities as well as the hidden sector temperature.

For a general hidden sector feebly coupled to the visible sector, its temperature T_h is linked to the visible sector temperature (the temperature of the observed Universe) T by a function $\eta(T_h) = T/T_h$.

The continuity equation derived from Friedmann equations is now modified to be

$$\frac{\mathrm{d}\rho_h}{\mathrm{d}t} + 3H\left(\rho_h + p_h\right) = j_h\,,$$
$$\frac{\mathrm{d}\rho_v}{\mathrm{d}t} + 3H\left(\rho_v + p_v\right) = -j_h\,,$$

where j_h is the source term arising from the freeze-in.

Temperature dependence

One can further deduce

$$\rho \frac{\mathrm{d}\rho_h}{\mathrm{d}T_h} = \left(\frac{\zeta_h}{\zeta} \rho_h - \frac{j_h}{4H\zeta} \right) \frac{\mathrm{d}\rho}{\mathrm{d}T_h} ,$$

$\zeta_h = \frac{3}{4}(1 + p_h/\rho_h)$ and $\zeta_h = 1$ for radiation dominated hidden sector.

Using the fact $\rho = \rho_v + \rho_h$, and the total entropy of Universe is conserved and the Hubble parameter is given by

$$H^2 = \frac{8\pi G_N}{3} \left[\rho_v(T) + \rho_h(T_h) \right] ,$$

one can finally obtain

$$\frac{\mathrm{d}\eta}{\mathrm{d}T_h} = -\frac{A_v}{B_v} + \frac{\zeta \rho_v + \rho_h(\zeta - \zeta_h) + j_h/(4H)}{B_v[\zeta_h \rho_h - j_h/(4H)]} \frac{\mathrm{d}\rho_h}{\mathrm{d}T_h} ,$$

with A_v, B_v functions of T_h and g_{eff}.

The complete coupled Boltzmann equations for $U(1)_X$

$$
\begin{aligned}
\frac{\mathrm{d}Y_\chi}{\mathrm{d}T_h} =& -\frac{s}{H}\frac{\mathrm{d}\rho_h/\mathrm{d}T_h}{4\rho_h - j_h/H}\sum_{i\in\mathrm{SM}}\Big\{(Y_\chi^{\mathrm{eq}})^2\langle\sigma v\rangle^{T_h\eta}_{\chi\bar\chi\to i\bar i} + \frac{1}{s}Y_{\gamma^*}\langle\Gamma\rangle^{T_h\eta}_{\gamma^*\to\chi\bar\chi} \\
& + \theta(M_{\gamma'} - 2m_\chi)\big[-Y_\chi^2\langle\sigma v\rangle^{T_h}_{\bar\chi\chi\to\gamma'} + \frac{1}{s}Y_{\gamma'}\langle\Gamma\rangle^{T_h}_{\gamma'\to\chi\bar\chi}\big] \\
& - Y_\chi^2\langle\sigma v\rangle^{T_h}_{\chi\bar\chi\to\gamma'\gamma'} + Y_{\gamma'}^2\langle\sigma v\rangle^{T_h}_{\gamma'\gamma'\to\chi\bar\chi}\Big\}, \\
\frac{\mathrm{d}Y_{\gamma'}}{\mathrm{d}T_h} =& -\frac{s}{H}\frac{\mathrm{d}\rho_h/\mathrm{d}T_h}{4\rho_h - j_h/H}\sum_{i\in\mathrm{SM}}\Big\{Y_\chi^2\langle\sigma v\rangle^{T_h}_{\chi\bar\chi\to\gamma'\gamma'} - Y_{\gamma'}^2\langle\sigma v\rangle^{T_h}_{\gamma'\gamma'\to\chi\bar\chi} \\
& + \theta(M_{\gamma'} - 2m_\chi)\big[Y_\chi^2\langle\sigma v\rangle^{T_h}_{\bar\chi\chi\to\gamma'} - \frac{1}{s}Y_{\gamma'}\langle\Gamma\rangle^{T_h}_{\gamma'\to\chi\bar\chi}\big] \\
& + \theta(M_{\gamma'} - 2m_i)\big[Y_i^2\langle\sigma v\rangle^{T_h\eta}_{i\bar i\to\gamma'} - \frac{1}{s}Y_{\gamma'}\langle\Gamma\rangle^{T_h}_{\gamma'\to i\bar i}\big] \\
& + Y_i^2\langle\sigma v\rangle^{T_h\eta}_{i\bar i\to\gamma'\gamma} + 2Y_iY_{\gamma'}^{\mathrm{eq}}\langle\sigma v\rangle^{T_h\eta}_{i\gamma'\to i\gamma}\Big\}.
\end{aligned}
$$

Turn back to the $U(1)_X$ beyond the SM

- We focus on a $U(1)_X$ beyond the SM, where all SM particles are not charged under the $U(1)_X$.
- To setup the connection, kinetic mixing and/or mass mixing will be evoked.

History of kinetic mixing and millicharge dark matter

- Holdom 1986: The kinetic mixing between two massless $U(1)$'s can generate a millicharge.
- Goldberg and Hall 1986: millicharge dark matter.
- Feldman, Liu and Nath 2007: a kinetic mixing cannot generate a millicharge considering the extra $U(1)$ mixed with the full electroweak theory. Mass mixing can be the only source.

A deeper look at two $U(1)$ mixing

Kinetic mixing between two massless $U(1)$

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F_{X\,\mu\nu}F_X^{\mu\nu} - \frac{\boldsymbol{\delta}}{2}F_{\mu\nu}F_X^{\mu\nu}\,,$$
$$\mathcal{L}_{\rm int} = eA_\mu J^\mu_{\rm em} + g_X C_\mu J^\mu_{\rm d}\,.$$

The kinetic terms can be diagonalized by a non-unitary transformation (to keep the gauge kinetic term in the canonical form)

$$\begin{pmatrix} A \\ C \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\boldsymbol{\delta}^2}} & 0 \\ \frac{-\boldsymbol{\delta}}{\sqrt{1-\boldsymbol{\delta}^2}} & 1 \end{pmatrix} \begin{pmatrix} A_\gamma \\ A_X \end{pmatrix}.$$

In the physical eigenbasis, the interactions can be rewritten as

$$\mathcal{L}_{\rm int} = \frac{e}{\sqrt{1-\boldsymbol{\delta}^2}}A_\gamma J_{\rm em} + g_X\left(\frac{\boldsymbol{\delta}}{\sqrt{1-\boldsymbol{\delta}^2}}A_\gamma + A_X\right)J_{\rm d}\,.$$

A deeper look at two $U(1)$ mixing

For the case of kinetic mixing between a massless $U(1)$ and a massive $U(1)$, the Lagrangian reads

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F_{X\,\mu\nu}F_X^{\mu\nu} - \frac{\boldsymbol{\delta}}{2}F_{\mu\nu}F_X^{\mu\nu} - \frac{1}{2}M^2C^2\,,$$
$$\mathcal{L}_{\rm int} = eA_\mu J^\mu_{\rm em} + g_X C_\mu J^\mu_{\rm d}\,.$$

Now the only possible way of eliminating the kinetic mixing term is given by the following transformation

$$\begin{pmatrix} C \\ A \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\boldsymbol{\delta}^2}} & 0 \\ \frac{-\boldsymbol{\delta}}{\sqrt{1-\boldsymbol{\delta}^2}} & 1 \end{pmatrix} \begin{pmatrix} A_X \\ A_\gamma \end{pmatrix},$$

which gives rise to the interaction in the physical eigenbasis as

$$\mathcal{L}_{\rm int} = \frac{g_X}{\sqrt{1-\boldsymbol{\delta}^2}}A_X J_{\rm d} + e\left(\frac{\boldsymbol{\delta}}{\sqrt{1-\boldsymbol{\delta}^2}}A_X + A_\gamma\right)J_{\rm em}\,.$$

In this case, the dark particle carries exactly zero electric charge.

A deeper look at two $U(1)$ mixing

For the case of kinetic mixing between two massive $U(1)$'s (may produce a massless in the final mass eigenbasis), the Lagrangian reads

$$\begin{aligned}
\mathcal{L}_{\text{kin}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F_{X\,\mu\nu}F_X^{\mu\nu} - \frac{\boldsymbol{\delta}}{2}F_{\mu\nu}F_X^{\mu\nu}\,, \\
\mathcal{L}_{\text{mass}} &= -\frac{1}{2}M_2^2A^2 - \frac{1}{2}M_1^2C^2 - M_1M_2AC\,, \\
\mathcal{L}_{\text{int}} &= eA_\mu J^\mu_{\text{em}} + g_X C_\mu J^\mu_{\text{d}}\,.
\end{aligned}$$

To obtain the physical eigenbasis, one needs to diagonalize the kinetic mixing matrix and mass mixing matrix simultaneously for the gauge eigenbasis $V^T = (C, A)$,

$$\mathcal{K} = \begin{pmatrix} 1 & \boldsymbol{\delta} \\ \boldsymbol{\delta} & 1 \end{pmatrix}, \qquad M^2_{\text{St}} = \begin{pmatrix} M_1^2 & M_1M_2 \\ M_1M_2 & M_2^2 \end{pmatrix} = M_1^2 \begin{pmatrix} 1 & \epsilon \\ \epsilon & \epsilon^2 \end{pmatrix},$$

where we define the mass mixing parameter $\epsilon = M_2/M_1$.

Kinetic and mass mixing between two massive $U(1)$'s

The interaction terms are

$$\mathcal{L}_{\rm int} = \frac{1}{\sqrt{1-2\epsilon\boldsymbol{\delta}+\epsilon^2}}\frac{1}{\sqrt{1-\boldsymbol{\delta}^2}}A_X\left[(1-\epsilon\boldsymbol{\delta})g_X J_{\rm d} + eJ_{\rm em}(\epsilon-\boldsymbol{\delta})\right] + \frac{1}{\sqrt{1-2\epsilon\boldsymbol{\delta}+\epsilon^2}}A_\gamma(eJ_{\rm em}-\epsilon g_X J_{\rm d})\,.$$

String origin of the millicharge (small fractional charge and the fraction is proportional to the D-brane wrapping numbers on the cycles of the 6D internal manifold) [WZF, Shiu, Soler, Ye, 1401.5880, 1401.5890].

The extra $U(1)$ mixed with the full electroweak theory

Now $F_{\rm em}^{\mu\nu} \to F_Y^{\mu\nu}$:

The Lagrangian are written as: (1) Kinetic mixing

$$\mathcal{L}_{\rm mix}^{\rm kin} = -\frac{\delta}{2} F_{\mu\nu} F_X^{\mu\nu} \,,$$

(2) Mass mixing: Stueckelberg mass mixing [Cheng and Yuan 2007, Feldman, Liu and Nath 2007]

$$\mathcal{L}_{\rm mix}^{\rm st} = -\frac{1}{2}(M_1 C_\mu + M_2 B_\mu + \partial_\mu \sigma)^2 \,,$$

or extra Higgs mixing [Zhang, WZF, 2204.08067]

$$D_\mu H = \left(\partial_\mu - i g_2 T^a A_\mu^a - \frac{i}{2} g_Y Y B_\mu - \frac{i}{2} g_Y y C_\mu\right) H \,,$$

$$D_\mu \phi = (\partial_\mu - i g_X C_\mu)\, \phi \,,$$

Mass mixing: Extra Higgs case

In the gauge eigenbasis of the $U(1)_{y+Q}$, hypercharge and the neutral $SU(2)$ gauge field $V^T = (C, B, A_3)$, the mixing matrices can be written as

$$\mathcal{K} = \begin{pmatrix} 1 & \delta & 0 \\ \delta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; M^2 = \begin{pmatrix} g_X^2 u^2 + \frac{1}{4} y^2 g_Y^2 v^2 & \frac{1}{4} y g_Y^2 v^2 & -\frac{1}{4} y g_2 g_Y v^2 \\ \frac{1}{4} y g_Y^2 v^2 & \frac{1}{4} g_Y^2 v^2 & -\frac{1}{4} g_2 g_Y v^2 \\ -\frac{1}{4} y g_2 g_Y v^2 & -\frac{1}{4} g_2 g_Y v^2 & \frac{1}{4} g_2^2 v^2 \end{pmatrix}$$

$$\begin{pmatrix} C \\ B \\ A_3 \end{pmatrix} = \begin{pmatrix} \times & 0 & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} \begin{pmatrix} A' \\ A^\gamma \\ Z \end{pmatrix}$$

The photon may couple to a dark fermion originally carry a small amount of hypercharge y, although this coupling is not generated by the mixing effect.

Mass mixing: Stueckelberg case

In the gauge eigenbasis of the $U(1)_X$, hypercharge and the neutral $SU(2)$ gauge field $V^T = (C, B, A_3)$, the mixing matrices can be written as

$$\mathcal{K} = \begin{pmatrix} 1 & \delta & 0 \\ \delta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad M^2_{\rm St} = \begin{pmatrix} M_1^2 & M_1 M_2 & 0 \\ M_1 M_2 & M_2^2 + \frac{1}{4}v^2 g_Y^2 & -\frac{1}{4}v^2 g_2 g_Y \\ 0 & -\frac{1}{4}v^2 g_2 g_Y & \frac{1}{4}v^2 g_2^2 \end{pmatrix}.$$

$$\begin{pmatrix} C \\ B \\ A_3 \end{pmatrix} = c \begin{pmatrix} \times & -\frac{g_2 \epsilon}{\sqrt{g_2^2 + g_Y^2}} & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix} \begin{pmatrix} A' \\ A^\gamma \\ Z \end{pmatrix}$$

$$\mathcal{L} \sim \epsilon g_X Q_\chi \cos\theta_W \bar{\chi}\gamma^\mu \chi A^\gamma_\mu \equiv Q_\epsilon \bar{\chi}\gamma^\mu \chi A^\gamma_\mu\,,$$

where $\epsilon = M_2/M_1$ is the mass mixing parameter.

Summary of the couplings from the mixing

A summary of the order of the couplings induced by the kinetic mixing and the mass mixing:

$$
\begin{aligned}
A^\gamma i\bar{i} &\sim eQ_i\,, & A^\gamma \chi\bar{\chi} &\sim \epsilon g_X\,,\\
Z i\bar{i} &\sim g_2\,, & Z\chi\bar{\chi} &\sim \delta g_X\,,\\
A' i\bar{i} &\sim |\delta-\epsilon| g_Y\,, & A'\chi\bar{\chi} &\sim g_X\,.
\end{aligned}
$$

How millicharge can be generated:

Millicharges can be generated in three ways:

1. The dark particle carries a tiny amount of hypercharge as a prior.
2. A kinetic mixing between a massless $U(1)$ and the hypercharge gauge field, and the generated millicharge is proportional to the kinetic mixing parameter.
3. The mass mixing between a massive $U(1)$ with the hypercharge gauge field (massive in the initial gauge eigenbasis) [Cheng and Yuan 2007, Feldman, Liu and Nath 2007]. In this case the kinetic mixing does not play any role in generating the millicharge, and the millicharge generated is proportional to the mass mixing parameter.

So a millicharge can be generated, what next?

Focus on sub-GeV mass region of dark matter, we consider six different cases:

Case		Model	$M_{\gamma'}$	m_χ	g_X	δ	ϵ	ε	$\Omega_\chi h^2$	$\Omega_{\gamma'} h^2$	$\tau_{\gamma'}$
1	$m_\chi > M_{\gamma'} > 2m_e$	a	20	100	0.0054	1×10^{-13}	1×10^{-10}	1.57×10^{-12}	0.120	0	0.616
		b	180	250	0.015	1×10^{-12}	1×10^{-9}	4.36×10^{-11}	0.120	0	6.82×10^{-4}
2	$2m_\chi > M_{\gamma'} > m_\chi > 2m_e$	c	100	60	1.59×10^{-5}	1×10^{-14}	5×10^{-11}	2.29×10^{-15}	0.120	0	0.491
3	$M_{\gamma'} > 2m_\chi > 2m_e$	d	100	10	0.01	1×10^{-14}	5.6×10^{-13}	1.62×10^{-14}	0.120	0	2.48×10^{-19}
4	$2m_e > m_\chi > M_{\gamma'}$	e	0.09	1	0.20	1×10^{-14}	1.27×10^{-12}	7.34×10^{-13}	7.42×10^{-12}	4.43×10^{-3}	2.67×10^{30}
5	$2m_e > 2m_\chi > M_{\gamma'} > m_\chi$	f	0.09	0.06	0.01	1×10^{-14}	1.27×10^{-12}	3.67×10^{-14}	5.94×10^{-3}	2.58×10^{-9}	2.67×10^{30}
		g	0.09	0.06	1.5×10^{-4}	1×10^{-14}	1.27×10^{-12}	5.50×10^{-16}	1.85×10^{-3}	3.03×10^{-3}	2.67×10^{30}
6	$2m_e > M_{\gamma'} > 2m_\chi$	h	1	0.05	0.001	4×10^{-13}	1×10^{-11}	2.9×10^{-14}	0.120	0	6.20×10^{-18}

The benchmark models we consider in this work for six different types of models. The lifetimes (in the unit of seconds) of the dark photon for each model are listed in the last column. $M_{\gamma'}$ and m_χ are in MeVs.

Recall our Boltzmann equations:

The complete coupled Boltzmann equations for $U(1)_X$

$$
\begin{aligned}
\frac{\mathrm{d}Y_\chi}{\mathrm{d}T_h} = &-\frac{s}{H}\frac{\mathrm{d}\rho_h/\mathrm{d}T_h}{4\rho_h - j_h/H}\sum_{i\in\mathrm{SM}}\Big\{(Y_\chi^{\mathrm{eq}})^2\langle\sigma v\rangle^{T_h\eta}_{\chi\bar\chi\to i\bar i} + \frac{1}{s}Y_{\gamma^*}\langle\Gamma\rangle^{T_h\eta}_{\gamma^*\to\chi\bar\chi} \\
&+\theta(M_{\gamma'}-2m_\chi)\big[-Y_\chi^2\langle\sigma v\rangle^{T_h}_{\bar\chi\chi\to\gamma'} + \frac{1}{s}Y_{\gamma'}\langle\Gamma\rangle^{T_h}_{\gamma'\to\chi\bar\chi}\big] \\
&-Y_\chi^2\langle\sigma v\rangle^{T_h}_{\chi\bar\chi\to\gamma'\gamma'} + Y_{\gamma'}^2\langle\sigma v\rangle^{T_h}_{\gamma'\gamma'\to\chi\bar\chi}\Big\}, \\
\frac{\mathrm{d}Y_{\gamma'}}{\mathrm{d}T_h} = &-\frac{s}{H}\frac{\mathrm{d}\rho_h/\mathrm{d}T_h}{4\rho_h - j_h/H}\sum_{i\in\mathrm{SM}}\Big\{Y_\chi^2\langle\sigma v\rangle^{T_h}_{\chi\bar\chi\to\gamma'\gamma'} - Y_{\gamma'}^2\langle\sigma v\rangle^{T_h}_{\gamma'\gamma'\to\chi\bar\chi} \\
&+\theta(M_{\gamma'}-2m_\chi)\big[Y_\chi^2\langle\sigma v\rangle^{T_h}_{\bar\chi\chi\to\gamma'} - \frac{1}{s}Y_{\gamma'}\langle\Gamma\rangle^{T_h}_{\gamma'\to\chi\bar\chi}\big] \\
&+\theta(M_{\gamma'}-2m_i)\big[Y_i^2\langle\sigma v\rangle^{T_h\eta}_{i\bar i\to\gamma'} - \frac{1}{s}Y_{\gamma'}\langle\Gamma\rangle^{T_h}_{\gamma'\to i\bar i}\big] \\
&+Y_i^2\langle\sigma v\rangle^{T_h\eta}_{i\bar i\to\gamma'\gamma} + 2Y_iY_{\gamma'}^{\mathrm{eq}}\langle\sigma v\rangle^{T_h\eta}_{i\gamma'\to i\gamma}\Big\}, \\
\frac{\mathrm{d}\eta}{\mathrm{d}T_h} = &-\frac{A_v}{B_v} + \frac{\zeta\rho_v + \rho_h(\zeta-\zeta_h) + j_h/(4H)}{B_v[\zeta_h\rho_h - j_h/(4H)]}\frac{\mathrm{d}\rho_h}{\mathrm{d}T_h}.
\end{aligned}
$$

The destination of the dark photon

Decay channels of the dark photon for $M_{\gamma'} < 2m_e$: to neutrinos, to three photons

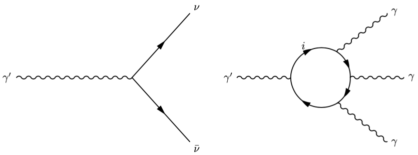


Figure: A display of dark photon decay channels for $M_{\gamma'} < 2m_e$, including the decay to pair of neutrino and anti-neutrino, and to three photons. Considering various constraints, the decay of the dark photon to neutrinos due to the mixing effect is always suppressed compared to the three-photon decay channel.

Although the dark photon's lifetime is extended beyond the age of the Universe, it can still undergo decay, even in minuscule amounts. This decay contributes to the isotropic diffuse photon background (IDPB), and thus the model may suffer stringent constraints.

IDPB

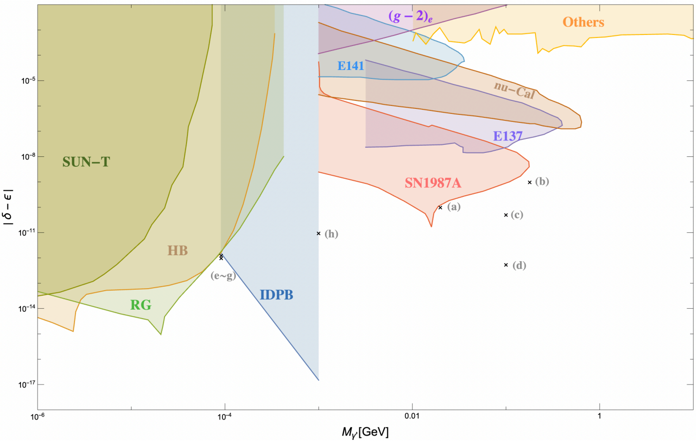


Figure: A display of current constraints (colored regions) on the absolute value of the kinetic mixing parameter minus the mass mixing parameter.

Case 1, model a: $M_{\gamma'} = 20\text{MeV}, M_\chi = 100\text{MeV}$

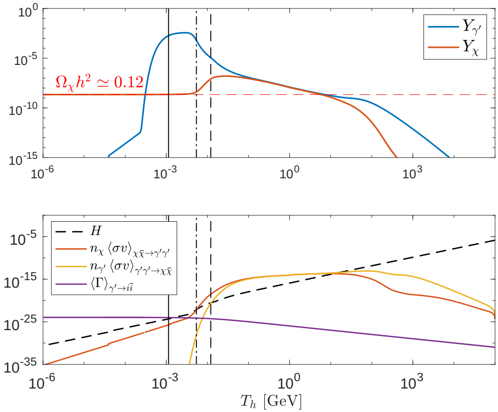


Figure: One can see apparent dark freeze-out and the hidden sector interactions reach equilibrium (inside the hidden sector).

Case 2, model c: $M_{\gamma'} = 100\text{MeV}, M_\chi = 60\text{MeV}$

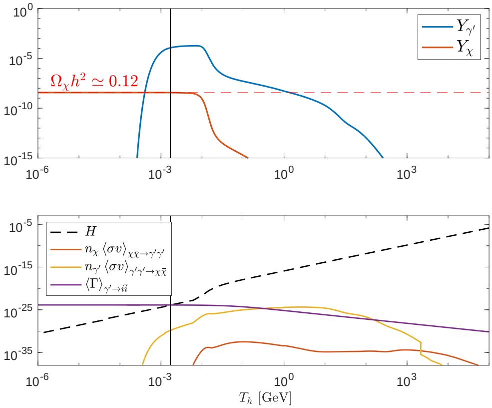


Figure: In this case we choose a rather small $g_X \sim 10^{-5}$, thus hidden sector interactions never reach equilibrium inside the hidden sector. However, these ultraweak interactions still play significant role.

Case 2, model c: $M_{\gamma'} = 100\text{MeV}, M_\chi = 60\text{MeV}$

Comparison of different calculations	$\Omega_\chi h^2$
All freeze-in processes included	0.1195
Plasmon decay process excluded	0.1195
Four-point γ' freeze-in processes excluded	0.0643
Pure freeze-in for χ	10^{-9}

Table: A comparison of different calculations of the dark matter relic density for the benchmark model c.

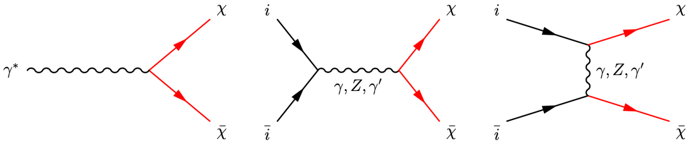


Figure: Freeze-in processes for the dark fermion χ.

Dark photon freeze-in

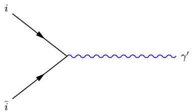


Figure: Three-point freeze-in processes for the dark photon γ'.

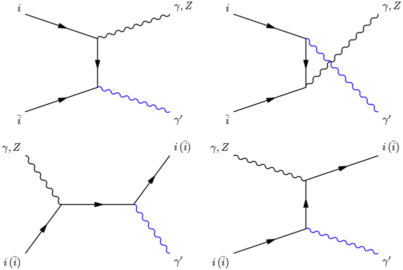


Figure: Four-point freeze-in processes for the dark photon γ'.

Case 2, model c, Conclusion

1. Four-point freeze-in processes are always important.
2. As long as the self-interactions inside hidden sector are present, one cannot calculate the freeze-in of the hidden sector particles independently. Namely, there is no such limit called “pure freeze-in”.
3. Plamon effect is not significant.

Case 4 and 5, model e: $M_{\gamma'} = 0.09\text{MeV}, M_\chi = 0.06\text{MeV}$

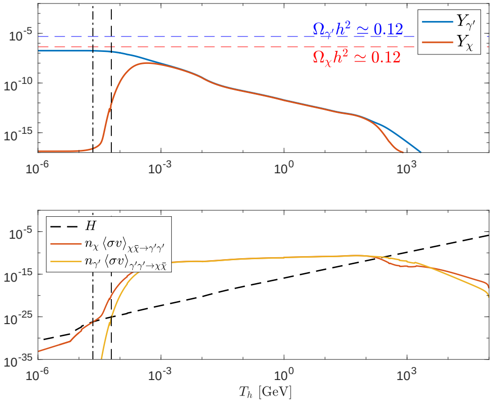


Figure: In this case the parameters are chosen to obtain a maximum value for the dark photon dark matter occupation ($\sim 5\%$).

Case 4 and 5, Plasmon effect

Benchmark models	Plasmon decay included		Plasmon contribution excluded	
	$\Omega_\chi h^2$	$\Omega_{\gamma'} h^2$	$\Omega_\chi h^2$	$\Omega_{\gamma'} h^2$
Model e	7.4193×10^{-12}	4.4344×10^{-3}	7.4191×10^{-12}	4.4327×10^{-3}
Model f	5.9415×10^{-3}	2.5842×10^{-9}	5.9397×10^{-3}	2.5807×10^{-9}

The results of dark matter relic densities including or excluding the plasmon decay contribution for benchmark models e and f. In these two benchmark models, both the dark fermion χ and the dark photon γ' are dark matter candidates.

Full occupation of the dark photon dark matter

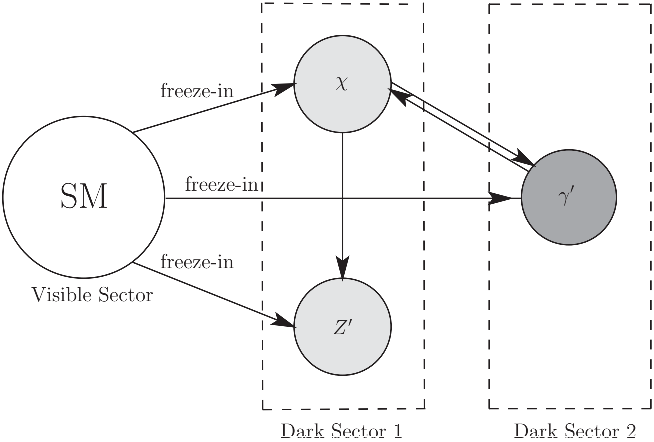


Figure: Model involving two $U(1)$ hidden sectors, discussed in [Aboubrahim, WZF, Nath, Wang, 2103.15769].

Conclusion page 1

1. The effective term $\mathcal{L} \sim -\frac{\delta}{2} F^{\text{em}}_{\mu\nu} F^{\mu\nu}_X$ is difficult to generated from a renormalized model in UV. Thus considering $\mathcal{L} \sim -\frac{\delta}{2} F^{\text{em}}_{\mu\nu} F^{\mu\nu}_X$ is not appropriate, especially from the theoretical perspective.
2. Even one consider such mixing term, the millicharge cannot be generated if the extra $U(1)$ is massive.
3. The millicharge can be only generated in three ways:
    1. The dark particle carries a tiny amount of hypercharge as a prior.
    2. A kinetic mixing between a massless $U(1)$ and the hypercharge gauge field, and the generated millicharge is proportional to the kinetic mixing parameter.
    3. The mass mixing between a massive $U(1)$ with the hypercharge gauge field, and the generated millicharge is proportional to the mass mixing parameter.

Conclusion page 2

Key findings which may apply to general freeze-in scenarios:

1. Four-point freeze-in processes must be kept at all times, even the three-point freeze-in production channels are present for the same freeze-in particle.
2. The hidden sector interactions never reach equilibrium, does not indicate such interactions don't occur. On the contrary, these interactions inside the hidden sector play significant role in determining the dark particle number densities.
3. Thus, the hidden sector interactions (even ultraweak) must be taken into account at all times.

Thank You!