Phenomenological studies on netral *B*-meson decays into $J/\psi\ h_1$

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研究背景

在经典的夸克模型中，如果夸克及反夸克之间的轨道角动量 $L=1$，根据不同自旋及不同轨道-自旋耦合，就可以形成四类 p 波介子：标量介子 $J^{PC}=0^{++}$ (1^3P_0)、轴矢量介子 $J^{PC}=1^{++},1^{+-}$ $(1^3P_1,1^1P_1)$和张量介子 $J^{PC}=2^{++}$ (1^3P_2)。实验物理学家已经发现了相当多的标量、轴矢量介子和张量介子(图 2-2)，但是到目前为止，这些粒子的分类和内部结构仍然存在着诸多争议。 人们已经提出了很多方案用以描述这些粒子，她们还需要理论

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	-11.5	-24.6
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	38.7	36.0
$1\,^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^\dagger$	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^\dagger$	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0

HNNU

研究背景

轴矢介子（$J^{PC}=1^{++},1^{+-}$）

- LHCb通过$B\to J/\psi(2\pi^+2\pi^-)_{f_1(1285)}$衰变道得到了$B\to J/\psi f_1(1285)$衰变的分支比和混合角：

$$\mathcal{B}\left(B_s^0\to J/\psi\ f_1(1285)\right)_{\text{Exp}}=7.14_{-1.41}^{+1.36}\times10^{-5},\quad \varphi^{\text{Exp}}=\pm\left(24.0_{-2.7}^{+3.2}\right)^{\circ}.$$

[R. Aaij et al. [LHCb Collaboration], Phys. Rev. D, 2013, 112(9):091802.]

$$\mathcal{B}\left(B_s^0\to J/\psi\ f_1(1285)\right)_{Theo}=0.89_{-0.37}^{+0.48}\times10^{-4},$$ PQCD

[Liu X, Xiao Z J. Phys. Rev. D, 2014, 89(9):097503.]
[Li Y, Liu X et al. Eur. Phys. J. C, 2023, 83.]

- $b_1(1235), K_{1B}, a_1(1260), K_{1A}$

[Liu X. arXiv:2305,00713.]

- 轴矢量介子间的三个混合角可彼此约束，其中K_{1A}和K_{1B}的混合复杂，实验没有定论。

解析计算

$h_1(1170)$和$h_1(1415)$介子混合：$\begin{pmatrix} h_1(1170) \\ h_1(1415) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 8 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} N \\ S \end{pmatrix}$

QF basis: $N = (u\bar{u} + d\bar{d})/\sqrt{2}, S = s\bar{s}$　　SO basis: $1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}, 8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$

混合角关系: $\alpha = \theta_0 - \theta.$

1. 对于$B_d^0 \to J/\psi\, h_1$衰变道:

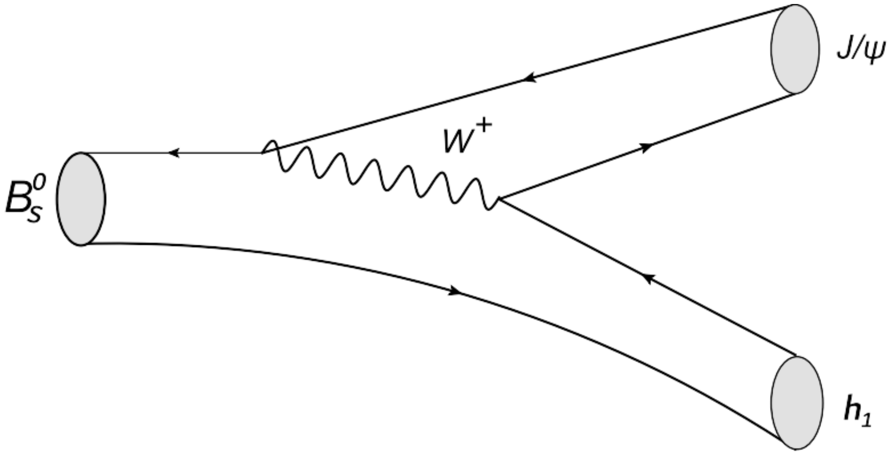
$$\mathcal{A}^h\left(B_d^0 \to J/\psi\, h_1(1170)\right) = A^h\left(B_d^0 \to J/\psi\, h_n\right)\cos\alpha$$

$$\mathcal{A}^h\left(B_d^0 \to J/\psi\, h_1(1415)\right) = -A^h\left(B_d^0 \to J/\psi\, h_n\right)\sin\alpha$$

2. 对于$B_s^0 \to J/\psi\, h_1$衰变道:

$$\mathcal{A}^h\left(B_s^0 \to J/\psi\, h_1(1170)\right) = A^h\left(B_s^0 \to J/\psi\, h_s\right)\sin\alpha$$

$$\mathcal{A}^h\left(B_s^0 \to J/\psi\, h_1(1415)\right) = A^h\left(B_s^0 \to J/\psi\, h_s\right)\cos\alpha$$



Leading quark-level Feynman diagram

解析计算

大小 $$\cos^2\theta_{{}^1p_1} = \frac{4m_{K_{1B}}^2 - m_{b_1}^2 - 3m_{h_1(1170)}^2}{3(m_{h_1(1415)}^2 - m_{h_1(1170)}^2)}$$

其中， $$m_{K_{1B}}^2 = m_{K_1(1400)}^2 \sin^2\theta_{K_1} + m_{K_1(1270)}^2 \cos^2\theta_{K_1}$$

[Cheng H. Y, Phys. Lett. B, 2012, 707, 116-120.]

符号 $$\tan\theta_{{}^1p_1} = \frac{4m_{K_{1B}}^2 - m_{b_1}^2 - 3m_{h_1(1170)}^2}{2\sqrt{2}(m_{b_1}^2 - m_{K_{1B}}^2)}$$

$\lvert\theta_{K_1}\rvert$	33.0°	57.0°
$\alpha_{{}^3P_1}$	-13.3°	26.7°
$\theta_{{}^3P_1}$	22.0°	62.0°
$\alpha_{{}^1P_1}$	-4.3°	-23.3°
$\theta_{{}^1P_1}$	31.0°	12.0°

介子波函数

1. B介子波函数：

$$\Phi_B = \frac{i}{\sqrt{2N_C}}\left\{(\not{P} + m_B)\gamma_5 \phi_B(x, k_T)\right\}_{\alpha\beta}$$

B介子分布振幅：

$$\phi_B(x,b) = N_B x^2 (1-x)^2 \exp\left[-\frac{1}{2}\left(\frac{x m_B}{\omega_B}\right)^2 - \frac{\omega_B^2 b^2}{2}\right]$$

2. 矢量介子J/ψ的波函数及分布振幅：

$$\Phi^L_{J/\psi}(x) = \frac{1}{\sqrt{2N_C}}\left\{m_{J/\psi}\not{\epsilon}_L \phi^L_{J/\psi}(x) + \not{\epsilon}_L \not{P} \phi^t_{J/\psi}(x)\right\}_{\alpha\beta}$$

$$\Phi^T_{J/\psi}(x) = \frac{1}{\sqrt{2N_C}}\left\{m_{J/\psi}\not{\epsilon}_T \phi^v_{J/\psi}(x) + \not{\epsilon}_T \not{P} \phi^T_{J/\psi}(x)\right\}_{\alpha\beta}$$

$$\phi^L_{J/\psi}(x) = \phi^T_{J/\psi}(x) = 9.58 \frac{f_{J/\psi}}{2\sqrt{2N_C}} x(1-x)\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7}$$

$$\phi^t_{J/\psi}(x) = 10.94 \frac{f_{J/\psi}}{2\sqrt{2N_C}} (1-2x)^2 \left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7}$$

$$\phi^v_{J/\psi}(x) = 1.67 \frac{f_{J/\psi}}{2\sqrt{2N_C}} \left[1+(2x-1)^2\right]\left[\frac{x(1-x)}{1-2.8x(1-x)}\right]^{0.7}$$

介子波函数

3. 轴矢量介子($\boldsymbol{h_1}$)的波函数和分布振幅：

$$\Phi_A^L = \frac{1}{\sqrt{2N_c}}\gamma_5\left\{m_A \not{\epsilon}_L \phi_A(x) + \not{\epsilon}_L \not{P}\phi_A^t(x) + m_A\phi_A^s(x)\right\}_{\alpha\beta}$$

$$\Phi_A^T = \frac{1}{\sqrt{2N_C}}\gamma_5\left\{m_A \not{\epsilon}_T \phi_A^v(x) + \not{\epsilon}_T \not{P}\phi_A^T(x) + m_A i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu \not{\epsilon}_T^\nu n^\rho v^\sigma \phi_A^a(x)\right\}_{\alpha\beta}$$

[Li R-H, Lv C-D, Wang W. Phys. Rev. D, 2009, 79(3):034014.]

$$\phi_A(x) = \frac{3f_A}{\sqrt{2N_c}}x(1-x)\left[3a_1^{\parallel}(2x-1)\right]$$

$$\phi_A^T(x) = \frac{3f_A}{\sqrt{2N_c}}x(1-x)\left[1 + a_2^{\perp}\frac{3}{2}\left(5(2x-1)^2 - 1\right)\right]$$

$$\phi_A^s(x) = \frac{3f_A}{2\sqrt{2N_c}}\frac{d}{dx}\left[x(1-x)\right]$$

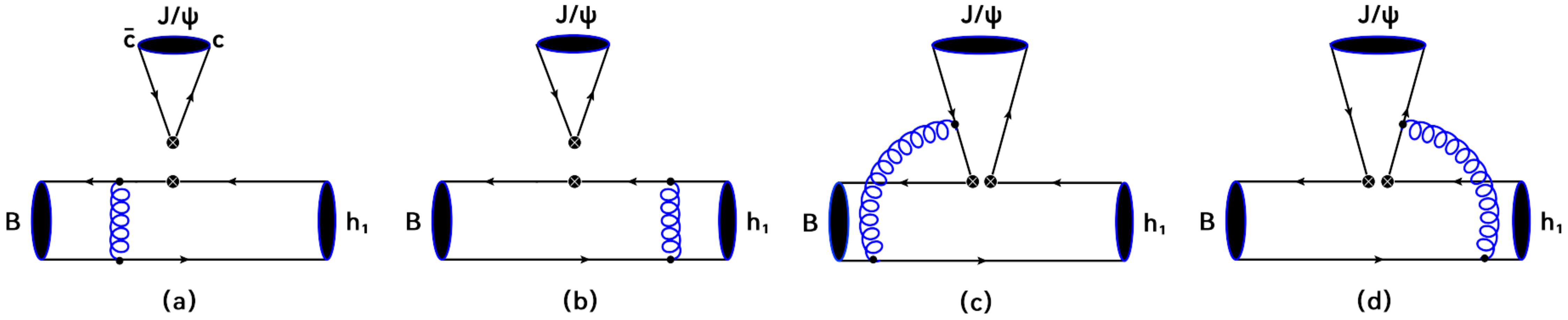
$$\phi_A^t(x) = \frac{3f_A}{2\sqrt{2N_c}}\left[(2x-1)^2\right]$$

$$\phi_A^v(x) = \frac{3f_A}{4\sqrt{2N_c}}\left[a_1^{\parallel}(2x-1)^3\right]$$

$$\phi_A^a(x) = \frac{3f_A}{4\sqrt{2N_c}}\frac{d}{dx}\left[x(1-x)\left(a_1^{\parallel}(2x-1)\right)\right]$$

HNNU

费曼图



可因子化发射图　　　　　不可因子化发射图

基于费曼振幅，结合CKM矩阵元和Wilson系数，最终获得到不同夸克-味态的衰变振幅：

[Cheng H Y, Yang K-C. Phys. Rev. D, 2002, 65(9):094023.]

$$\xi A^h\left(B^0_{d(s)} \to J/\psi\, h_{n(s)}\right) = F^h_{J/\psi}\left\{V^*_{cb}V_{cd(s)}\tilde{a}^h_2 - V^*_{tb}V_{td(s)}\left(\tilde{a}^h_3 + \tilde{a}^h_5 + \tilde{a}^h_7 + \tilde{a}^h_9\right)\right\}$$

$$+M^h_{J/\psi}\left\{V^*_{cb}V_{cd(s)}C_2 - V^*_{tb}V_{td(s)}\left(C_4 - C_6 - C_8 + C_{10}\right)\right\}$$

数值结果

The *CP*-averaged branching ratios for neutral *B*-meson decays into $J/\psi h_1$.

Decay Modes	$\|\alpha\| = 4.3^\circ$	$\|\alpha\| = 23.3^\circ$
$\mathcal{B}(B_d^0 \to J/\psi h_1(1170))$	1.43×10^{-4}	1.21×10^{-4}
$\mathcal{B}(B_d^0 \to J/\psi h_1(1415))$	0.97×10^{-6}	1.98×10^{-5}
$\mathcal{B}(B_s^0 \to J/\psi h_1(1170))$	1.21×10^{-6}	2.48×10^{-5}
$\mathcal{B}(B_s^0 \to J/\psi h_1(1415))$	1.52×10^{-4}	1.05×10^{-4}

数值结果

$$B\left(\bar{B}\to J/\psi K^{*}\bar{K}\right)=0.4\times10^{-5},$$

[He D Z, Sun H et al. Phys. Rev. D, 2021, 103(9):094007.]

$$B\left(h_1(1415)\to K^{*}\bar{K}\right)\sim\frac{1}{3}$$

[Du M C, Zhao Q. Phys. Rev. D, 2021, 104(3):036008.]

$$R=\frac{B\left(\bar{B}\to J/\psi K^{*}\bar{K}\right)}{B\left(B\to J/\psi h_1(1415)\right)}\sim\frac{1}{5}$$

后续工作

- 检查解析计算和数值计算过程，尝试可行性衰变道；
- 考虑 $h_1(1170), h_1(1415)$ 介子的次级衰变，进行数值对比；
- 计算出极化分数和CP破坏等其它物理量；

Thank you

附录

$$\begin{pmatrix} h_1(1170) \\ h_1(1415) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 \\ 8 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \end{pmatrix} \begin{pmatrix} N \\ S \end{pmatrix}$$

$$= \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} N \\ S \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} 1 \\ 8 \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}} \end{pmatrix} \begin{pmatrix} N \\ S \end{pmatrix}$$

$K_1(1270)$ 和 $K_1(1400)$ 混合: $$\begin{pmatrix} K_1(1270) \\ K_1(1400) \end{pmatrix} = \begin{pmatrix} \sin\theta_{K_1} & \cos\theta_{K_1} \\ \cos\theta_{K_1} & -\sin\theta_{K_1} \end{pmatrix} \begin{pmatrix} K_{1A} \\ K_{1B} \end{pmatrix}$$

附录

Input Quantities:

$$f_{J/\psi}=0.405, h_1=0.18\pm0.012, h_8=0.19\pm0.01, m_{h_1}=1.23\pm0.07, m_{h8}=1.37\pm0.07$$

$$m_{J/\psi}=3.097, m_{h_1(1170)}=1.17, m_{h_1(1415)}=1.42,$$

Analogy $\eta-\eta'$ mixing:

$$\begin{pmatrix}1\\8\end{pmatrix}=\begin{pmatrix}\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}}\\ \sqrt{\frac{1}{3}} & -\sqrt{\frac{2}{3}}\end{pmatrix}\begin{pmatrix}N\\S\end{pmatrix} \Longrightarrow \begin{aligned} f_{h_q}&=0.256^{+0.005}_{-0.007}\\ f_{h_s}&=0.051^{+0.007}_{-0.008}\end{aligned}$$

[Du M C, Zhao Q. Phys. Rev. D, 2021, 104(3):036008.]

附录

SO basis:

1. 对于$B_d^0 \to J/\psi\, h_1$ 衰变道：

$$\mathcal{A}^h\left(B_d^0 \to J/\psi\, h_1(1170)\right) = A^h\left(B_d^0 \to J/\psi\, h_n\right)\left[\frac{1}{\sqrt{3}}\left(\sqrt{2}\cos\theta + \sin\theta\right)\right]$$

$$\mathcal{A}^h\left(B_d^0 \to J/\psi\, h_1(1415)\right) = A^h\left(B_d^0 \to J/\psi\, h_n\right)\left[\frac{1}{\sqrt{3}}\left(\cos\theta - \sqrt{2}\sin\theta\right)\right]$$

2. 对于$B_s^0 \to J/\psi\, h_1$ 衰变道：

$$\mathcal{A}^h\left(B_s^0 \to J/\psi\, h_1(1170)\right) = A^h\left(B_s^0 \to J/\psi\, h_s\right)\left[\frac{1}{\sqrt{3}}\left(\cos\theta - \sqrt{2}\sin\theta\right)\right]$$

$$\mathcal{A}^h\left(B_s^0 \to J/\psi\, h_1(1415)\right) = -A^h\left(B_s^0 \to J/\psi\, h_s\right)\left[\frac{1}{\sqrt{3}}\left(\sqrt{2}\cos\theta + \sin\theta\right)\right]$$

其中，

$$h_n = \left(u\bar{u} + d\bar{d}\right)/\sqrt{2}\,, h_s = s\bar{s}$$

HNNU