Imaging nuclei on "yoctosecond" time scale

""Exploring Nuclear Physics across Energy Scales 2024""

CCAST, Beijing

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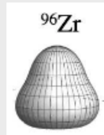
Jean-Paul Blaizot, IPhT University Paris- Saclay

IPhT SACLAY

Heavy ion collisions and nuclear structure

How does the low-energy structure of nuclei manifest itself in high-energy collisions?

Numerous evidences for the impact of "intrinsic" nuclear shapes
Spectacular example: Ru/Zr ratios



How do the observations made at colliders complement our knowledge of nuclear structure?

Large sensitivity to initial configurations of nucleons. Allows for precise determination of deformation parameters, neutron skin, etc
…and probably more

This talk will focus on fundamental issues, leaving aside many "details"….

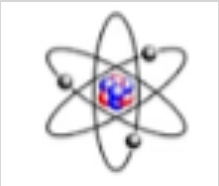
Why do fine details of nuclear structure survive the complexity of a nucleus-nucleus collision at high energy?

IMAGING NUCLEI on YOCTOSECOND time sale

Nobel prize 2023 (P. Agostini, F. Krausz, A l'Huillier)

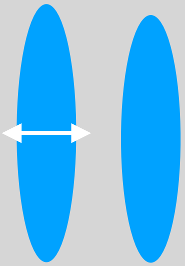
"for experimental methods that generate attosecond pulses of light for the study of electron dynamics in matter."

$$\mathrm{fs} = 10^{-15}\mathrm{s}$$ **[Molecule internal dynamics]**



$$\mathrm{as} = 10^{-18}\mathrm{s}$$ **[Electronic motion]**

[Heavy Ion collisions at LHC]



$$\mathrm{ys} = 10^{-24}\mathrm{s} \simeq 0.3\mathrm{fm/c}$$

$$\Delta x = \gamma(2R) \simeq 10^{-2}\mathrm{fm}$$

Very short time scale as compare to the time scale of internal nucleus dynamics

Typical time for a nucleon to cross a nucleus

$$\Delta t \simeq 10\,\mathrm{ys} \quad (v_F/c \approx 0.3)$$

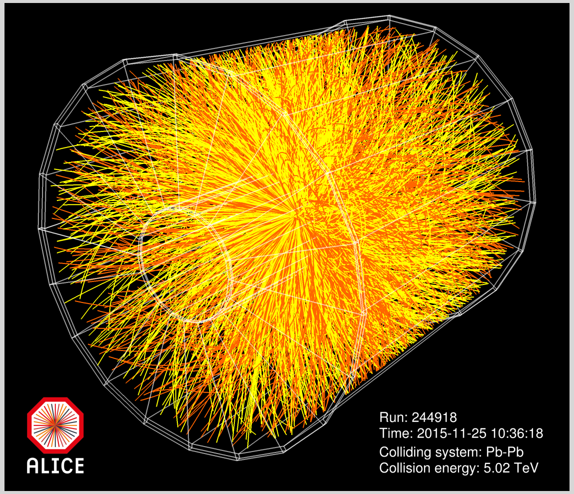
Time scale associated with excitation energy of 1 MeV

$$\Delta t \simeq 200\,\mathrm{fm/c} \approx 650\mathrm{ys}$$

(*)Title inspired by A. Ipp et al. "Yoctosecond pulses from the quark-gluon plasma", PRL 103.152301

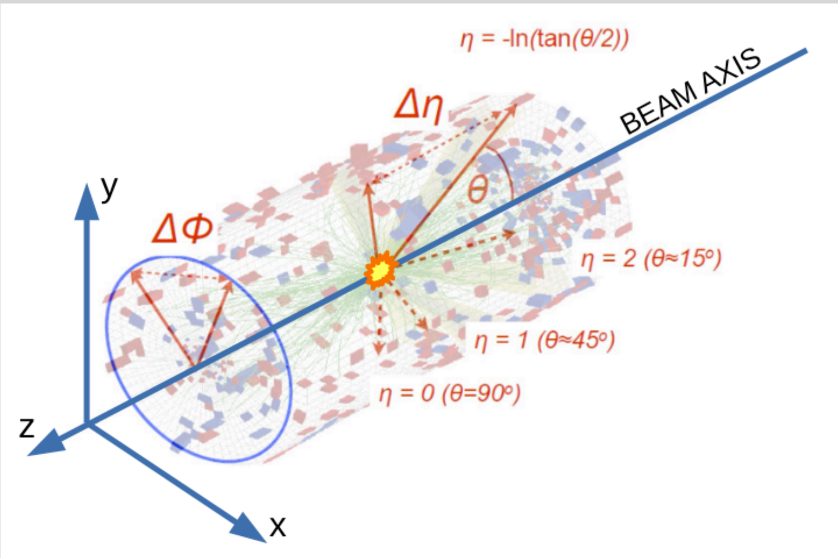
Azimuthal structure of particle production

What do we measure in HI collisions?



One-particle distribution

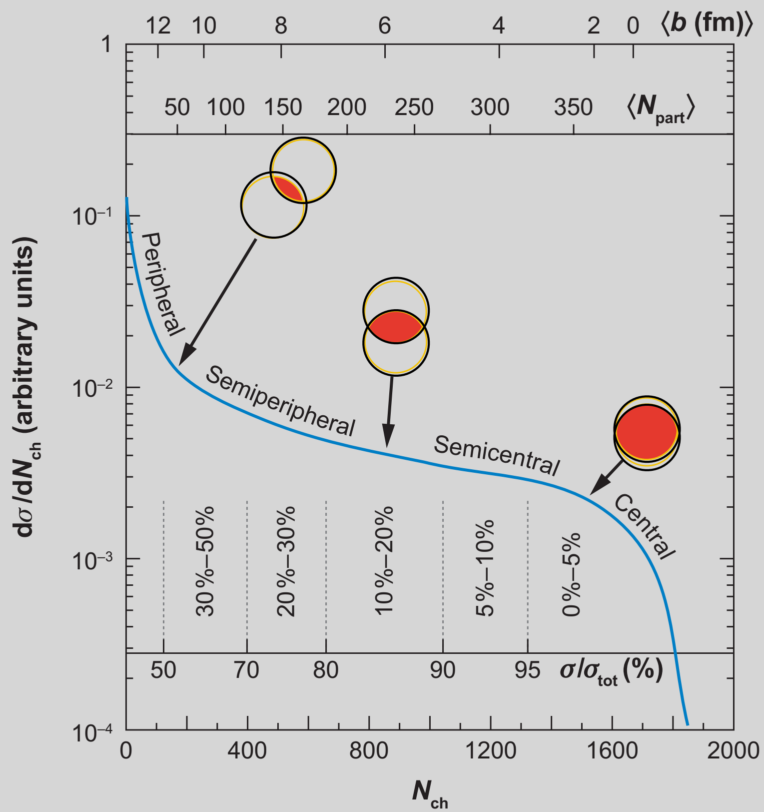
$$\frac{\mathrm{d}N}{p_T\,\mathrm{d}p_T\,\mathrm{d}\varphi\,\mathrm{d}\eta}$$



(from G. Giacalone, Talk ESNT, 2022)

Transverse energy distribution

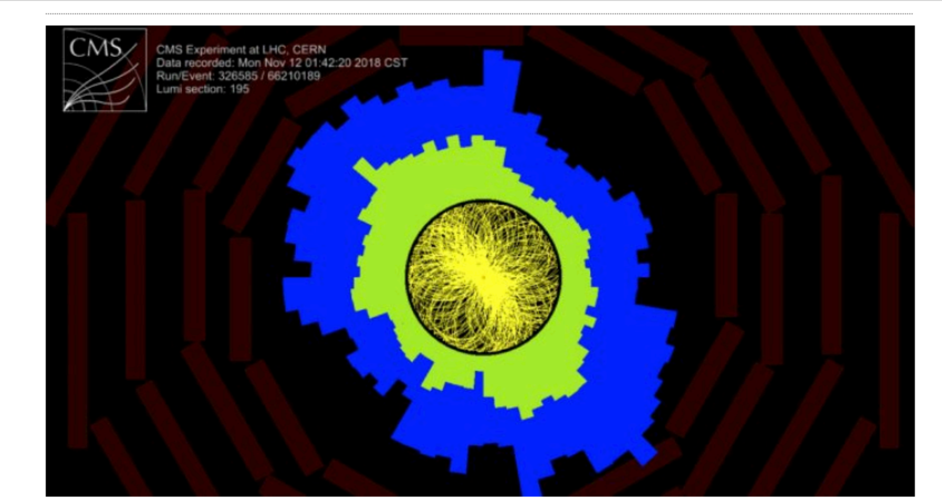
$$\frac{\mathrm{d}\sigma}{\mathrm{d}N_{\mathrm{ch}}} \sim \frac{\mathrm{d}\sigma}{\mathrm{d}E_T}$$



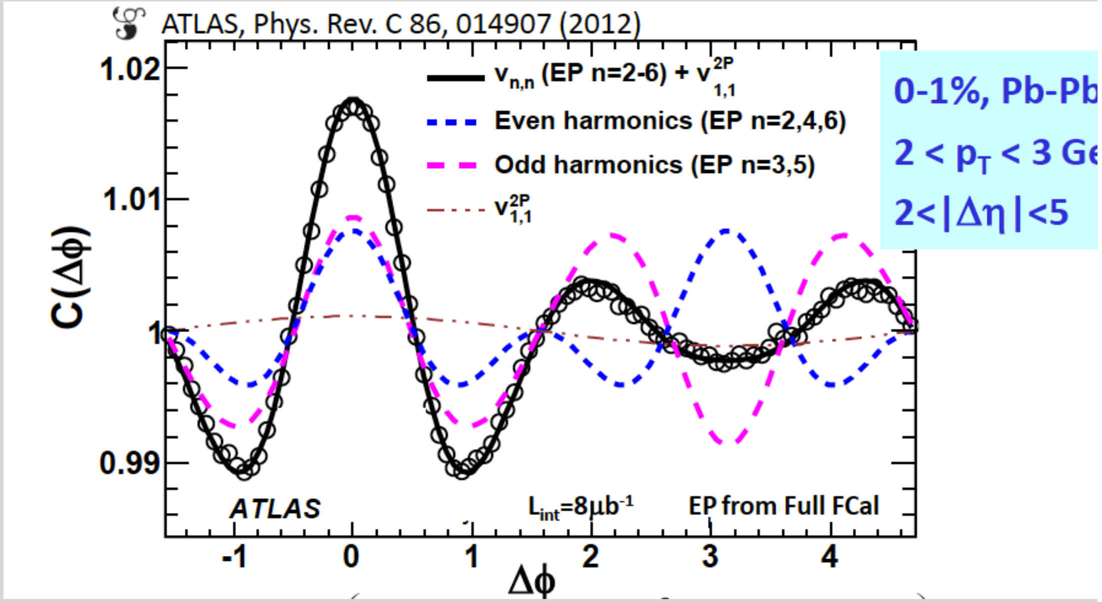
(Miller et al., Annu. Rev. Nucl. Part. Sci. 2007. 57:205–43)

Non trivial azimutal distribution (1)

Single event is not symmetric



Fourier analysis
"Multiple harmonics"



$$P_\Psi(\varphi) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} V_n \, e^{-in\varphi}$$

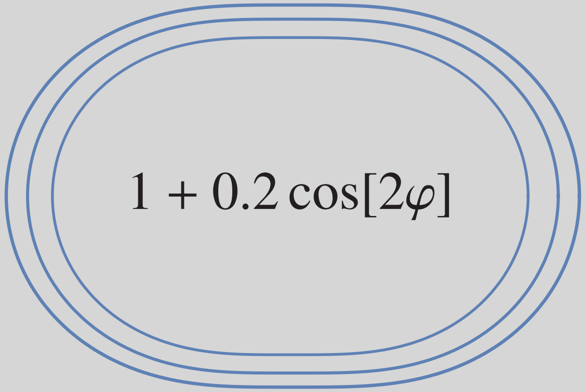
$$\frac{1}{N}\frac{dN}{d\varphi} = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} \upsilon_n \cos[n(\varphi - \Psi_n)]\right]$$

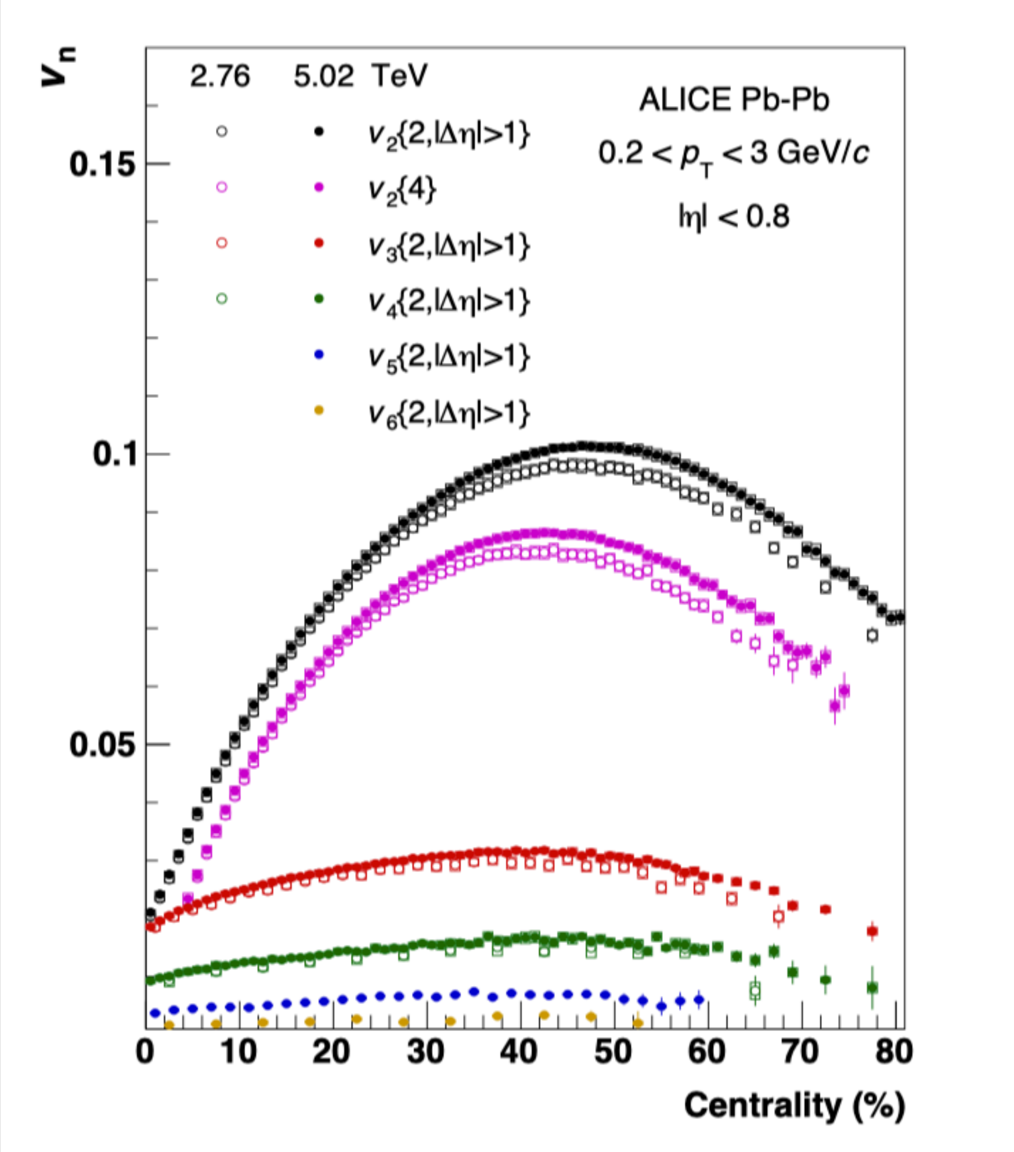
$$V_n = \int_0^{2\pi} d\varphi \, P_\Psi(\varphi) \, e^{in\varphi}$$

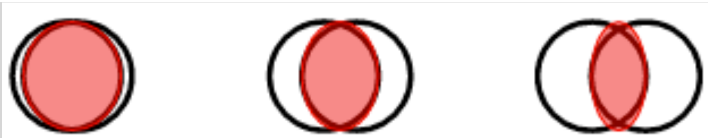
$$V_n = \upsilon_n \, e^{in\Psi_n}$$

Non trivial azimutal distribution (2)

$$\frac{1}{N}\frac{dN}{d\varphi} = \frac{1}{2\pi}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)]\right]$$







The magnitudes of the coefficients v_n are correlated with the impact parameter of the collision

THREE QUESTIONS

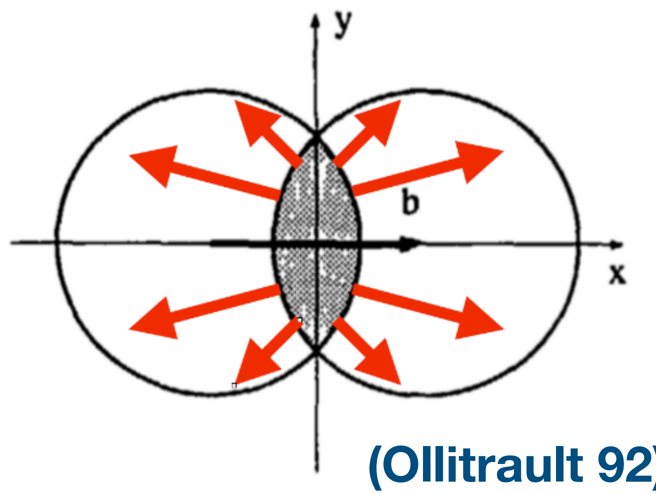
- WHAT IS THE PHYSICAL ORIGIN OF THE EFFET?
- HOW DOES ONE DETERMINE Vn ?
- WHAT HAPPENS AT SMALL IMPACT PARAMETER?

PHYSICAL ORIGIN OF THE EFFECT

- HYDRODYNAMIC FLOW
- GLAUBER PICTURE OF "INITIAL CONDITIONS"
- SHAPE OF NUCLEI MATTERS

HYDRODYNAMIC FLOW (1)

The shape of the collision zone determines pressure gradients which in turn produce acceleration of particles



(Ollitrault 92)

$$u_x = u\cos\varphi \qquad u_y = u\sin\varphi$$

(u =flow velocity)

$$\nabla_x P \gg \nabla_y P \longrightarrow |u_x| \gg |u_y|$$

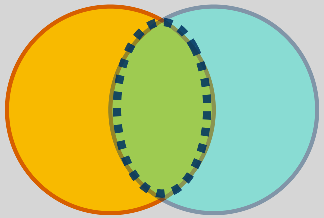
$$\langle \cos 2\varphi \rangle = \langle \cos^2\varphi - \sin^2\varphi \rangle > 0$$

Hence a non vanishing value of the "elliptic flow" v2

$$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}\varphi} = \frac{1}{2\pi}\left[1 + 2v_2\cos[2(\varphi - \Psi_2)]\right] \qquad v_2 = \langle \cos 2\varphi \rangle = \int \frac{\mathrm{d}\varphi}{2\pi}\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}\varphi}\cos(2\varphi)$$

HYDRODYNAMIC FLOW (2)

- The magnitude of v2 is sensitive to the "shape" of the interaction region (in the transverse plane)



$$\varepsilon_2 = \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$

Average done over the energy density in the transverse plane

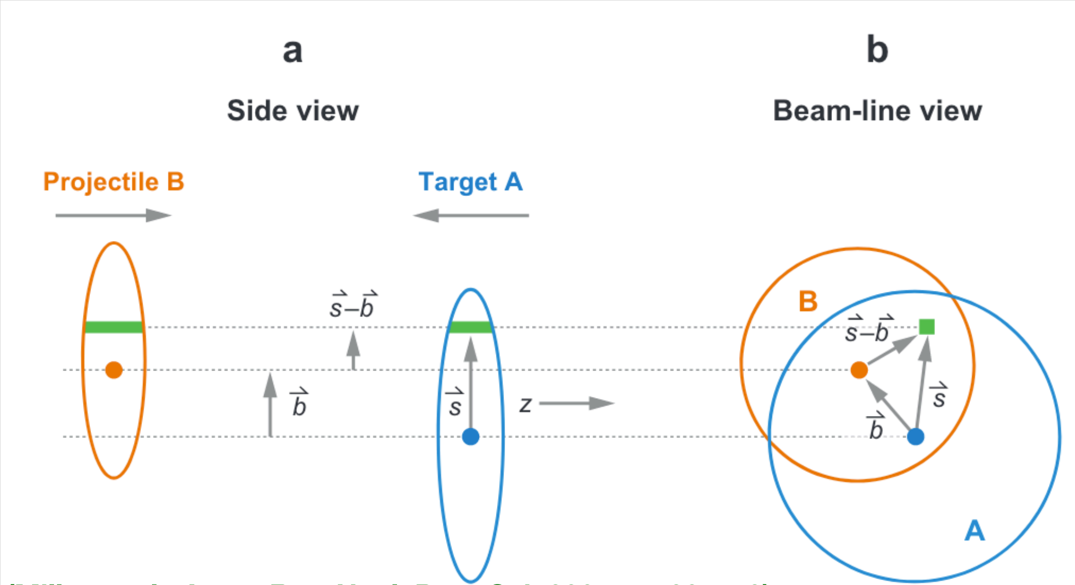
- Hydrodynamics response is such that $v_2 \propto \varepsilon_2$

- Hence the dependence of v2 on the impact parameter

- Hydrodynamics then provides a link between the initial energy density and the final observed particles.

- Similar considerations apply (with subtleties) to higher moments, e.g. $v_3 \propto \varepsilon_3$

Glauber picture of "initial conditions"



(Miller et al., Annu. Rev. Nucl. Part. Sci. 2007. 57:205–43)

Straight line trajectories

- forward scattering
- inelastic nn cross section

T_A depends on the positions of nucleons (projected in the transverse plane) at the "instant" of the collision

$$T_A(s) = \int \mathrm{d}z \, \rho_A(s, z) \qquad T_{AB}(b) = \int \mathrm{d}^2 s \, T_A(s) \, T_B(s - b)$$

Two ingredients in a Glauber calculation

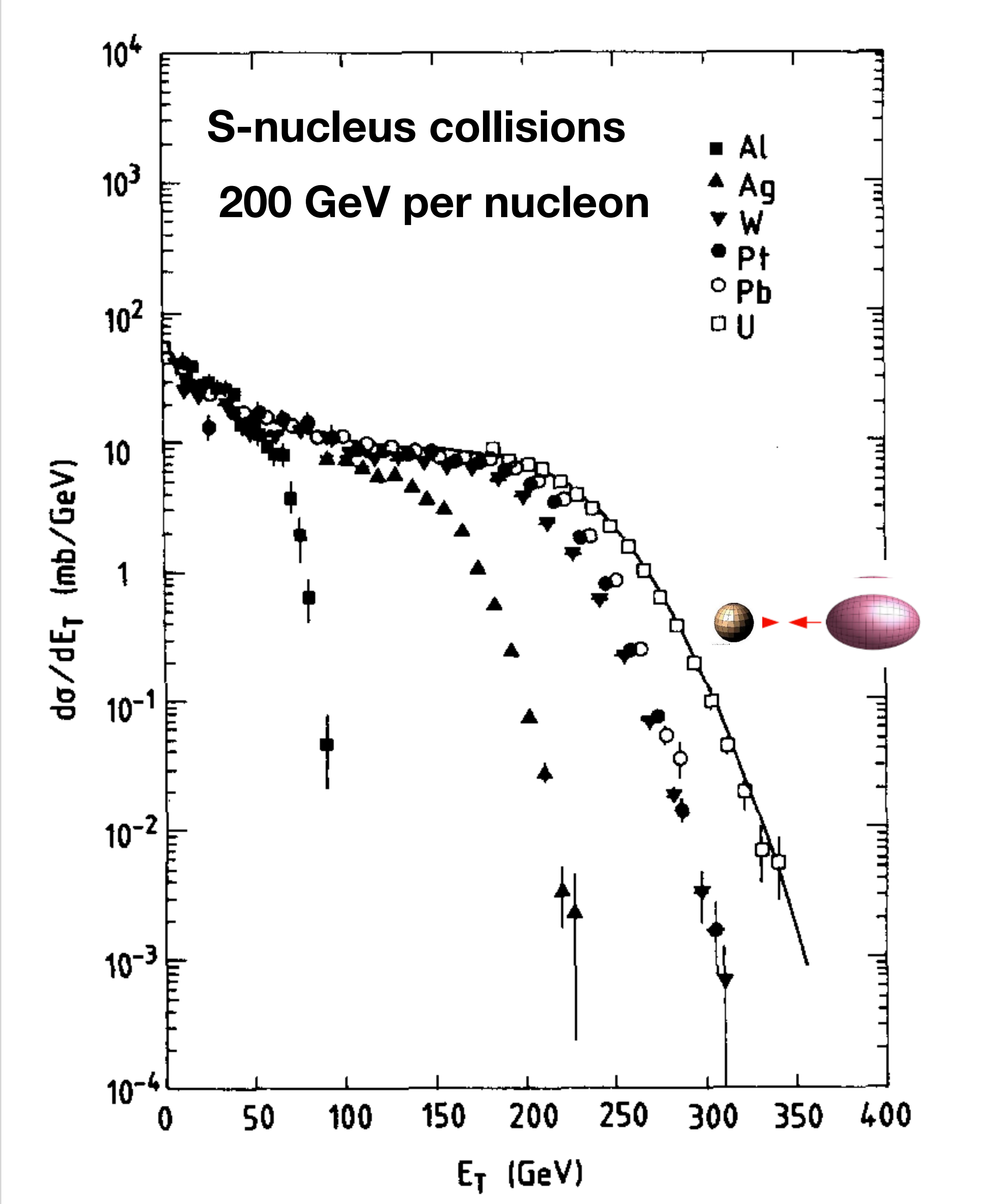
- the inelastic nn cross section
- the one-body density

NB. Some modelling is involved in relating energy density to these functions

Shape of nuclei matters

Shape of nucleus matters

The tail of the transverse energy distribution depends on the orientation of the Uranium nucleus



HELIOS collaboration, (CERN SPS)
Phys. Lett. B 214 (1988) 295

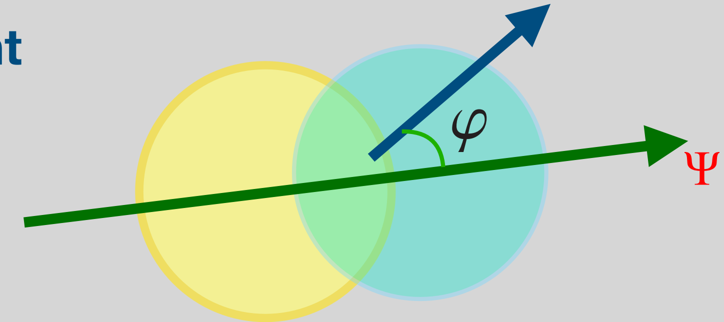
NB. Analogous finding in electron scattering

HOW DO WE DETERMINE Vn ?

- TWO KINDS OF AVERAGE
- CORRELATIONS GENERATED BY AVERAGE OVER THE EVENTS

TWO KINDS OF AVERAGE

Distribution of particles produced in a single event



$$P_{\Psi}(\varphi_1, \varphi_2, \cdots, \varphi_N)$$

In a given event emissions of particles are uncorrelated

$$P_{\Psi}(\varphi_1, \varphi_2, \cdots, \varphi_N) = p_{\Psi}(\varphi_1) \cdots p_{\Psi}(\varphi_N)$$

One-point function

$$p_{\Psi}^{(1)}(\varphi) = \int d\varphi_2 \cdots d\varphi_N \, P_{\Psi}(\varphi_1, \varphi_2, \cdots, \varphi_N) = p_{\Psi}(\varphi)$$

For elliptic flow alone

$$p_{\Psi}(\varphi) = \frac{1}{2\pi} \left[1 + 2v_2 \cos 2(\varphi - \Psi)\right]$$

If there were enough particles in the event, one could reconstruct the one-point distribution, that is v2 and Psi.

EMERGENCE OF CORRELATIONS

- **Start from uncorrelated 2-point function**

$$p_\Psi^{(2)}(\varphi_1, \varphi_2) = p_\Psi^{(1)}(\varphi_1) p_\Psi^{(1)}(\varphi_2)$$

- **Integration over Ψ generates correlations**

$$p^{(2)}(\varphi_1, \varphi_2) = \int \mathrm{d}\psi \, p_\Psi^{(2)}(\varphi_1, \varphi_2) = 1 + 2v_2^2 \cos 2\Delta\varphi = p^{(2)}(\Delta\varphi)$$

$$\Delta\varphi \equiv \varphi_1 - \varphi_2$$

NB. $p^{(1)}(\varphi_1) = \int \frac{\mathrm{d}\varphi_2}{2\pi} p^{(2)}(\varphi_1, \varphi_2) = \int \frac{\mathrm{d}\Psi}{2\pi} p_\Psi^{(1)} = \frac{1}{2\pi}$

- **One can determine v2 from a correlation function**
  (count pairs instead of single particles)

$$\langle \sum_{i \neq j} \cos 2(\Delta\varphi_{ij}) \rangle = N(N-1) \int \frac{\mathrm{d}\Delta\varphi}{2\pi} p^{(2)}(\Delta\varphi) \cos(2\Delta\varphi) = N(N-1) v_2^2$$

WHAT HAPPENS AT SMALL IMPACT PARAMETER?

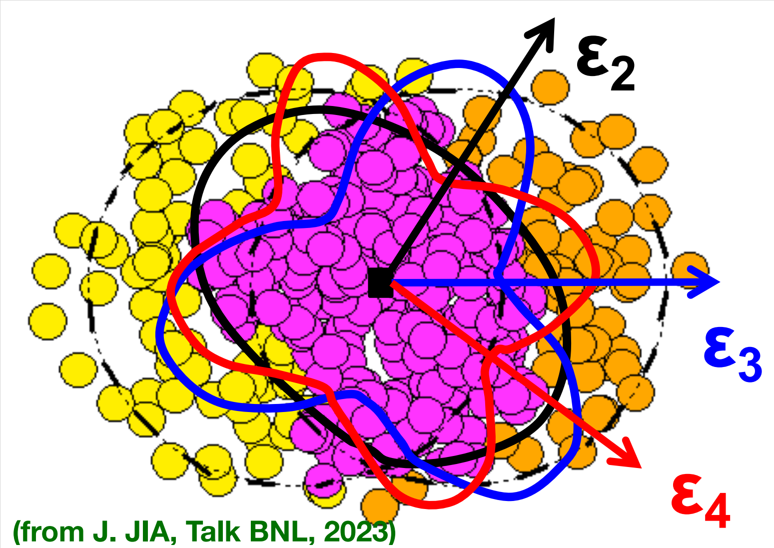
- FLUCTUATIONS
- SHAPES STILL MATTER

GLAUBER INITIAL FLUCTUATIONS

The initial energy density (in the transverse plane) is not smooth

Typical distribution of "participants" in a Glauber calculation at some finite impact parameter

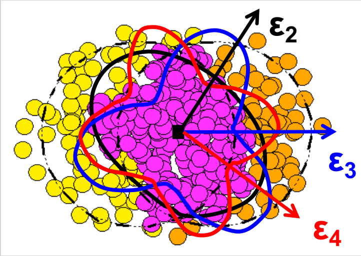
NB. Obtained from sampling the one-body density



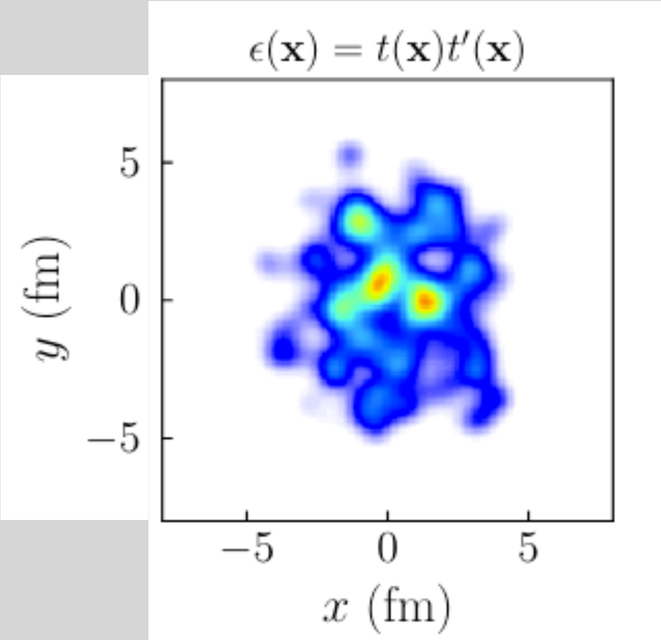
Thus, even at vanishing impact parameter (symmetric system) there can be a sizeable v2

NB. There are potentially many sources of fluctuations. The dominant ones seem to be those associated to the nucleon positions at the instant of the collisions. Hence the connection to nuclear structure.

MORE ON FLUCTUATIONS



- Energy deposition is a random process, with local fluctuations in energy density



- Average over events include correlations due to these fluctuations (in addition to those related to orientation of collision plane).

- It appears that the pattern of fluctuations is strongly correlated with that of initial positions, as given by the Glauber sampling

- Short wavelength fluctuations average out. What remains after some evolution are the long wavelength fluctuations (low multipoles, "collective variables") that characterize the "shape" of the collision zone.

- Hydrodynamical evolution preserves that information, which is carried to the momentum distribution (by pressure gradients).

$v_2 \propto \varepsilon_2$ $\quad$ $v_3 \propto \varepsilon_3$ $\quad$ (higher multipoles are non linearly coupled)

Intermediate summary

☆ In a given event particles are emitted independently of each other. Their momenta are a combination of a thermal component (isotropic in the local rest frame) and a collective flow velocity.

☆ The average over events introduce correlations between the particles, which can be determined experimentally. They give access to the flow harmonics of the momentum distribution (v1,v2,… v6).

☆ These flow harmonics turn out to be directly sensitive to the "shape" of the collision zone. This shape determines the pattern of pressure gradients that controls the hydrodynamic evolution.

More can be measured:

- select particles
- transverse momentum
- longitudinal fluctuations
- covariance (pT,v2)
- etc

Connection to nuclear structure

Why are nuclei "deformed"

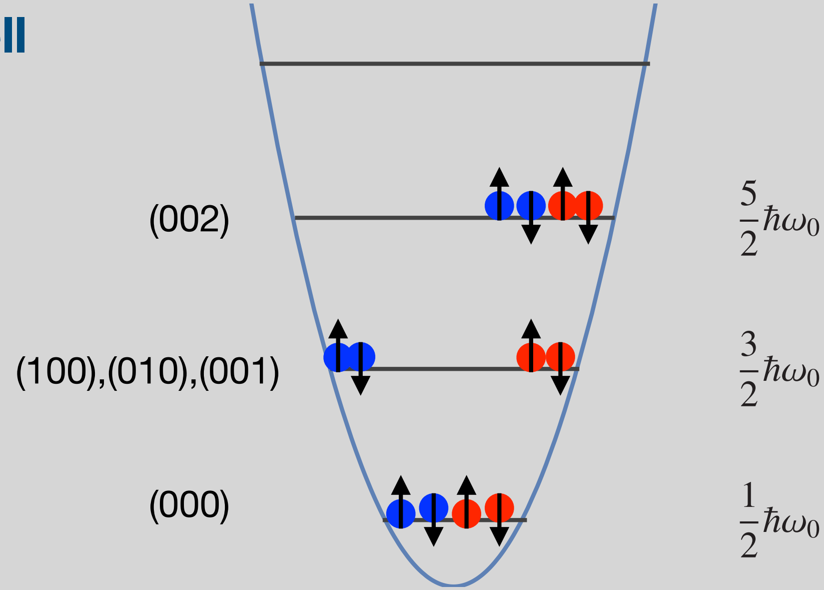
- **If nuclei were "liquid drops", their equilibrium shapes would be spherical**
  (the qualification "deformed" refers to deviation from spherical shape)

- **Deformation is intimately connected with single particle motion in a self-consistent mean field**

- **Independent nucleons in a harmonic potential well**

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}\left(\omega_x x^2 + \omega_y y^2 + \omega_z z^2\right)$$

$$\varepsilon_{n_x n_y n_z} = (n_x + 1/2)\hbar\omega_x + (n_y + 1/2)\hbar\omega_y + (n_z + 1/2)\hbar\omega_z$$



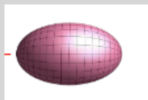
- **The frequencies of the oscillator adjust so as to make the velocity distribution isotropic**

- **Isotropic filling yields a spherical potential**

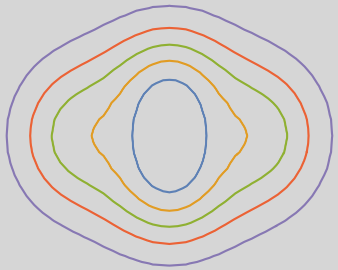
$$^{16}\mathrm{O} \ : \quad \omega_x = \omega_y = \omega_z = \omega_0$$

- **Anisotropic filling yields a deformed potential**

$$^{20}\mathrm{Ne} \ : \quad \omega_x = \omega_y, \ \omega_z = \frac{7}{11}\omega_x$$



The deformed shape is energetically favoured



Is deformation "real" ?
Can it be observed ?

- A deformed nucleus is characterised by a non vanishing quadrupole moment of the one-body density

$$Q = r^2 P_2(\cos\theta)$$

$$\langle Q \rangle = \int \mathrm{d}^3 r \, \rho(\vec{r}) \, Q(\vec{r}) \neq 0 \qquad \rho(\vec{r}) = \int \mathrm{d}^3 r_2 \cdots \mathrm{d}^3 r_N \, |\Phi(\vec{r}, \vec{r}_2, \cdots, \vec{r}_N)|^2$$

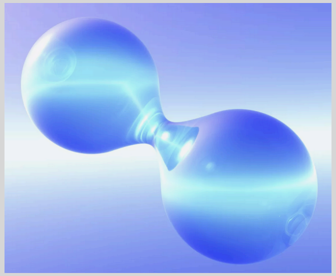
where $\Phi(\vec{r}_1, \vec{r}_2, \cdots, \vec{r}_N)$ is the deformed independent particle wave function.

- Note that Φ cannot be the ground state of the nuclear Hamiltonian

$$\langle \Psi_{J=0} | Q | \Psi_{J=0} \rangle = 0$$

- Way out: Φ is to be considered as an "intrinsic" state, function of intrinsic coordinates. The full wave function contains a factor that describe the collective rotation of the system.

[Note analogy with a diatomic molecule]



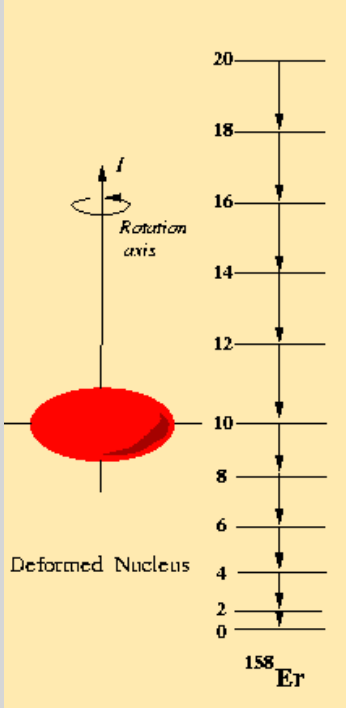
- **Yields the so-call collective (or "unified") model**

- **For an axially symmetric nucleus**

$$\Psi_{JM}(\Omega, q) \propto \mathcal{D}^{J}_{M0}(\Omega)\Phi(q)$$

Ω **denotes the Euler angles that specify the orientation of the intrinsic state**

q **denotes the intrinsic coordinates**

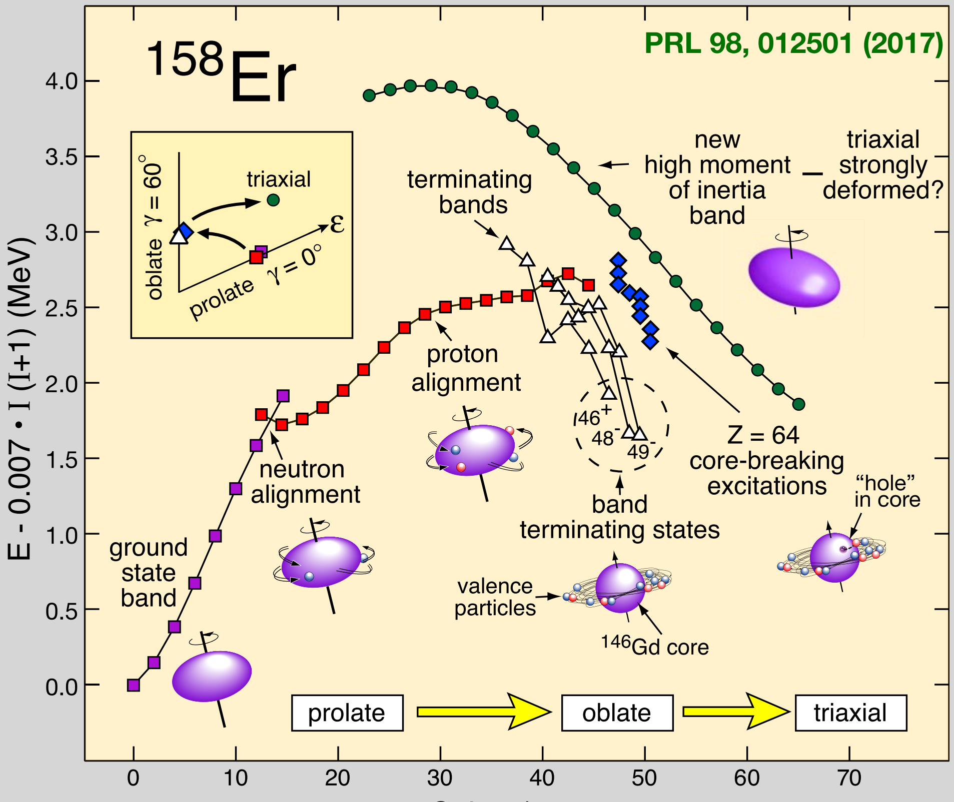


- **Rotational bands are observed**

$$E_J \propto J(J+1)$$

- **Further indirect evidences of deformation:**

- **Electromagnetic transitions (e.g. E2 transitions which involve quadrupole moment)**
- **Nuclear spectroscopy (host of phenomena very suggestive of nuclear deformation and collective rotation**
- **Coulomb excitation**



Conceptual issues

- The collective model is physically well motivated, but it remains a model. No unique description. Collective coordinates are (most often) redundant.

- Descriptions based on self-consistent mean fields (or density functional theories) involve spontaneous symmetry breaking.

- Symmetry breaking in finite systems is an approximate concept. Symmetry has to be restored, one way or another (collective model, projection techniques, etc.

- One could even describe nuclear properties without any reference to an intrinsic state. This is the case for instance of shell model wave functions.

(see e.g. A. Poves et al. "Limits on assigning a shape to a nucleus", arXiv: 1906.07542)

Deformation can be inferred from invariant moments (Kumar 1972)

$$\langle Q \rangle = 0 \quad \langle Q^2 \rangle \neq 0 \qquad \left(\langle Q^4 \rangle - \langle Q^2 \rangle^2, \langle Q^6 \rangle - \langle Q^3 \rangle^2 \right) \longrightarrow (\Delta\beta, \Delta\gamma)$$

One touches here a general issue, that of the choice of basis in quantum mechanics. In some basis the "physics" is more "manifest" than in others…

Conceptual issues

- **Deformation is a one-body concept. It is a property of the intrinsic state.**

- **In the intrinsic state the nucleons are uncorrelated**

$$P_\Omega(r_1, r_2, \cdots, r_N) = \left|\Phi_\Omega^{\rm int}(r_1, r_2, \cdots, r_N)\right|^2$$

- **Averaging over the collective wave function generates correlations (of all orders).**

$$P(r_1, r_2, \cdots, r_N) = \int \frac{\mathrm{d}\Omega}{4\pi} \left|\Phi_\Omega^{\rm int}(r_1, r_2, \cdots, r_N)\right|^2$$

NB. This average projects onto a spherical state

Note the analogy with the determination of the Vn.

- **The Glauber calculation samples the one-body density associated to the intrinsic state**

$$p_\Omega^{(1)}(r) = \int \mathrm{d}^3 r_2 \cdots \mathrm{d}^3 r_N \, P_\Omega(r, r_2, \cdots, r_N)$$

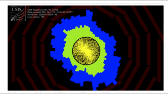
BUT WHAT DOES THE YOCTOSECOND SNAPSHOT CORRESPOND TO?

Summary

Correlation between nuclear deformation and collective flow is well establish

We may take this as an "empirical fact"

$$\frac{1}{N}\frac{dN}{d\varphi}$$



Shape of a given event

$$V_n = v_n\, e^{in\Psi_n}$$

Averaging over events

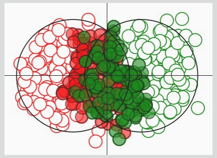
$$v_n$$

Various **correlation functions** can be measured

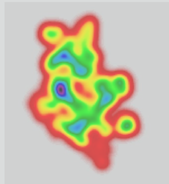
Nuclear structure input
- concept of "deformation"
- what is the physical meaning of this picture?

Uncertainties:
- initial conditions
- thermalisation
- viscous corrections
- small systems

Glauber calculation



Hydrodynamic simulations



$$v_2 \propto \varepsilon_2$$

$$v_n$$

Many issues remain to be investigated to fully understand this connection