Ab initio description of deformed intermediate-mass nuclei

Benjamin Bally

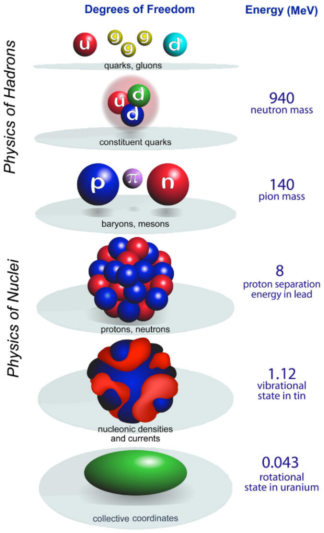
Exploring nuclear physics across energy scales

Beijing - 16/04/2024

cea irfu

Nuclear theory: hierarchy of scales

- Nuclear matter made of quarks and gluons



Courtesy of ORNL

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  - Impossible except $A \sim 1$
  - Even if possible, would be very inefficient
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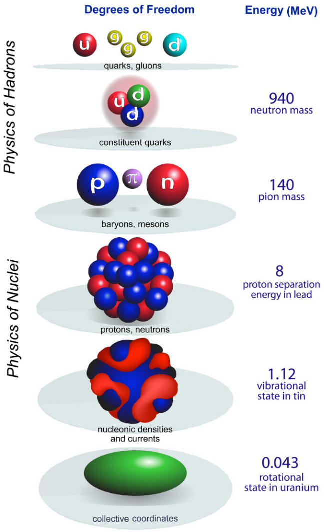

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- But description from QCD:
  - ◇ Impossible except $A \sim 1$
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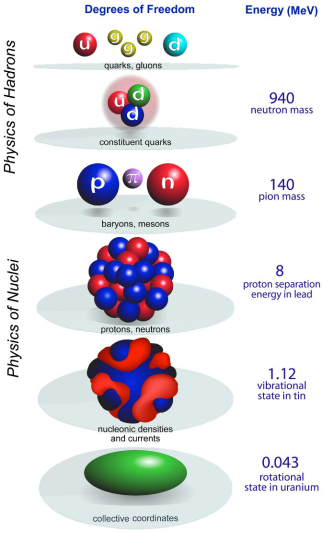
  - ◇ What would we learn?
- Define appropriate degrees of freedom for the scale
- Connect different scales

→ **Tower of Effective Field Theories**



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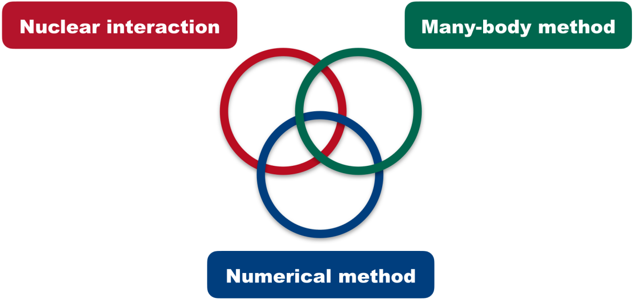

Courtesy of P. Arthuis

Effective field theory of the nuclear Hamiltonian

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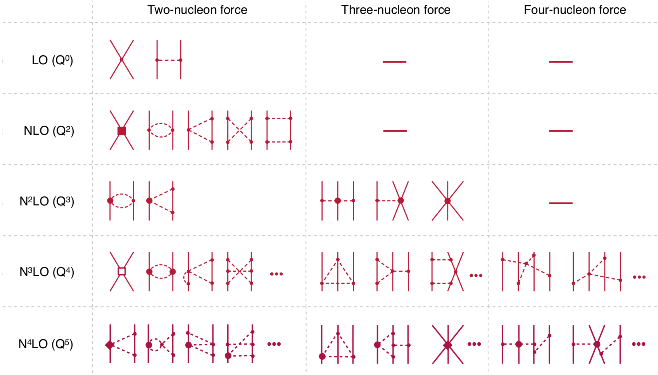

Epelbaum *et al.*, Front. Phys. 8, 98 (2020)

Effective field theory of the nuclear Hamiltonian

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- Not free of questions or problems!
  - Original "Weinberg" power counting not renormalizable!
  - Interactions not alway built consistently (different orders for 2N and 3N)
  - Need to go at N?LO
  - Multiplication of the number of interactions ("Skyrmization")

Systematic tool: EFT for decays

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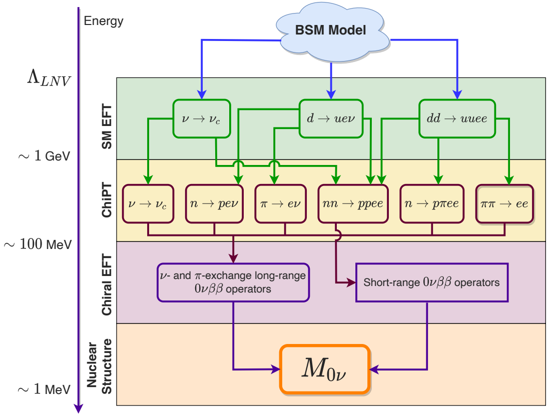
- Example: neutrinoless double-beta decay ⇒ **see talk by Jiangming**



Cirigliano *et al.*, J. Phys. G 49, 120502 (2022)

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  - Many-Body Perturbation Theory (MBPT)
  - Coupled Cluster (CC)
  - Self-Consistent Green's Function (SCGF)
  - In-Medium Similarity Renormalization Group (IMSRG)
  - Valence-Space IMSRG (VS-IMSRG)
  - Nuclear Lattice EFT (NLEFT) $\Rightarrow$ **see talks by Ulf and Dean**
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  - Projected Generator Coordinate Method + Perturbation Theory (PGCM-PT)
- Variants/generalization of these methods to describe
  - singly open-shell nuclei $\rightarrow$ pairing (Bogoliubov formalism)
  - doubly open-shell nuclei $\rightarrow$ deformation

Handling the three-body interaction

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  In general, we consider HO energy quanta $\rightarrow e_{3\mathrm{max}} = \max(e_i + e_j + e_k)$

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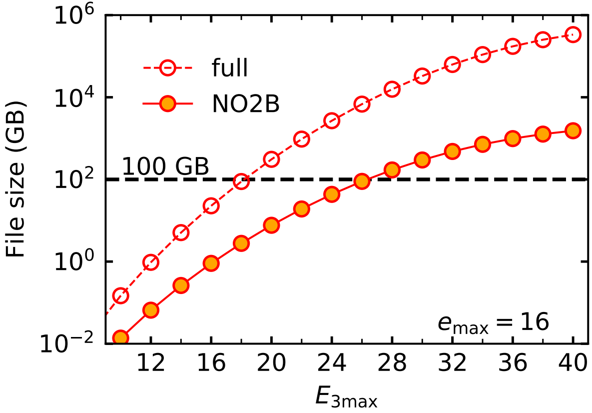
$$H \longrightarrow \tilde{H} = \tilde{H}^{0N} + \tilde{H}^{1N} + \tilde{H}^{2N}$$

$$\text{with, e.g., } \tilde{H}^{2N}_{ijlm} = V^{2N}_{ijlm} + \sum_{kn} V^{3N}_{ijklmn}\, \rho_{kn}$$

- Recent breakthrough in the computation of the effective two-body



Miyagi *et al.*, Phys. Rev. C 105, 014302 (2022)

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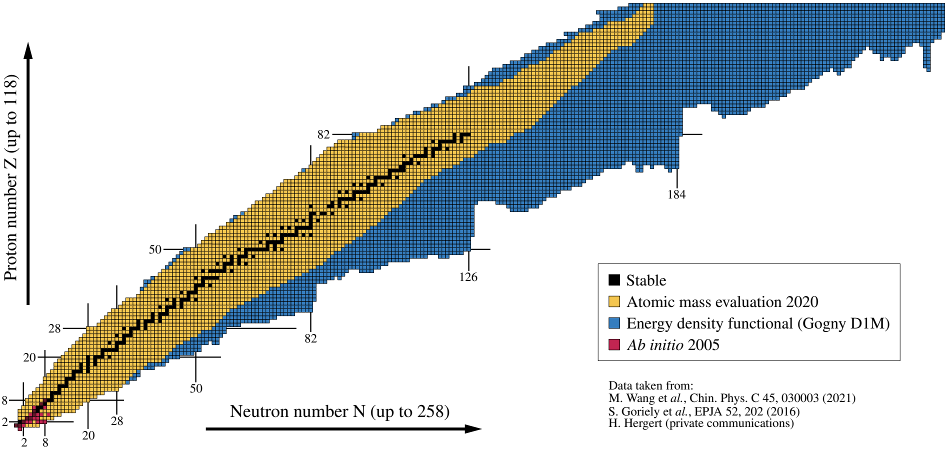
- Strategy 3: tensor factorization
  Tichai *et al.*, Phys. Rev. C 99, 034320 (2019)

Status of *ab initio* theory: 2005



Data taken from:
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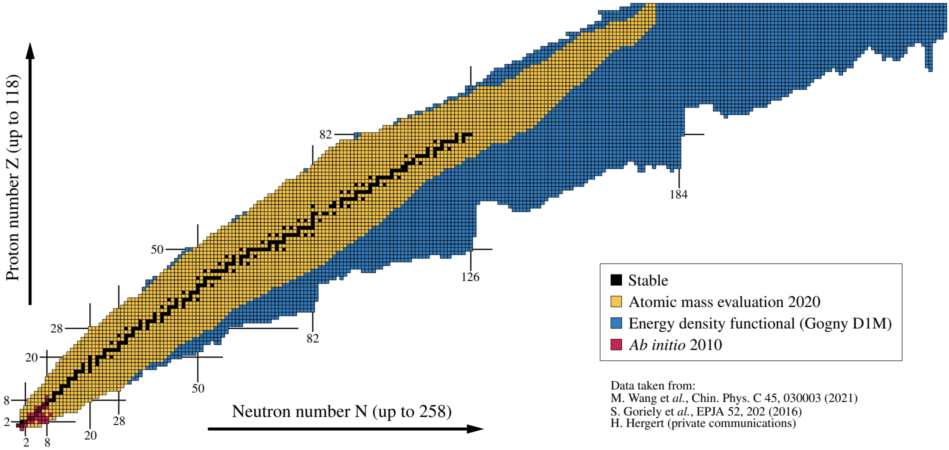

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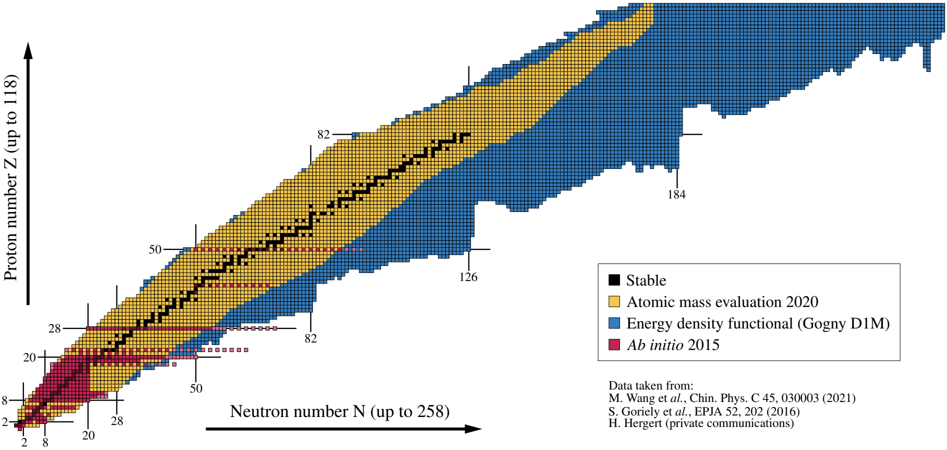

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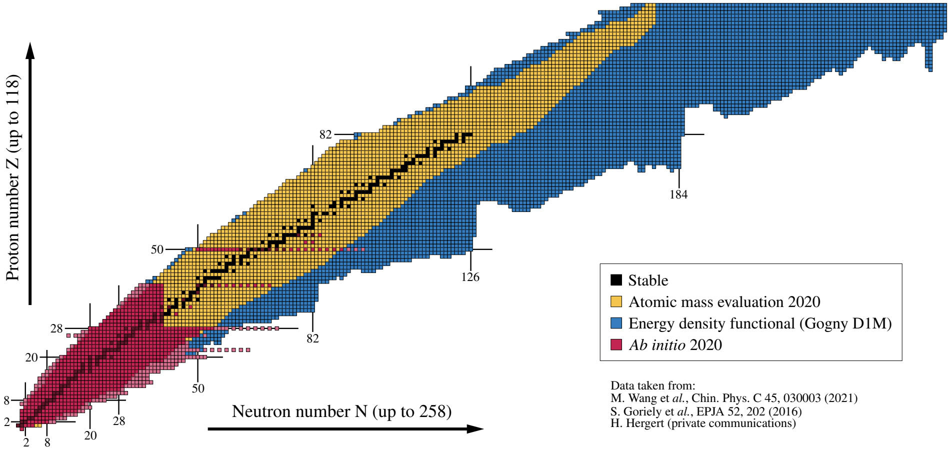

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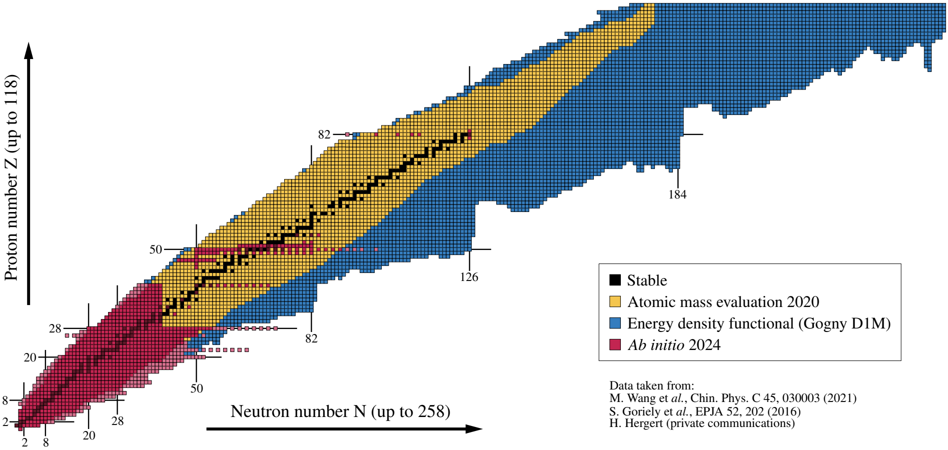
Status of *ab initio* theory: 2024



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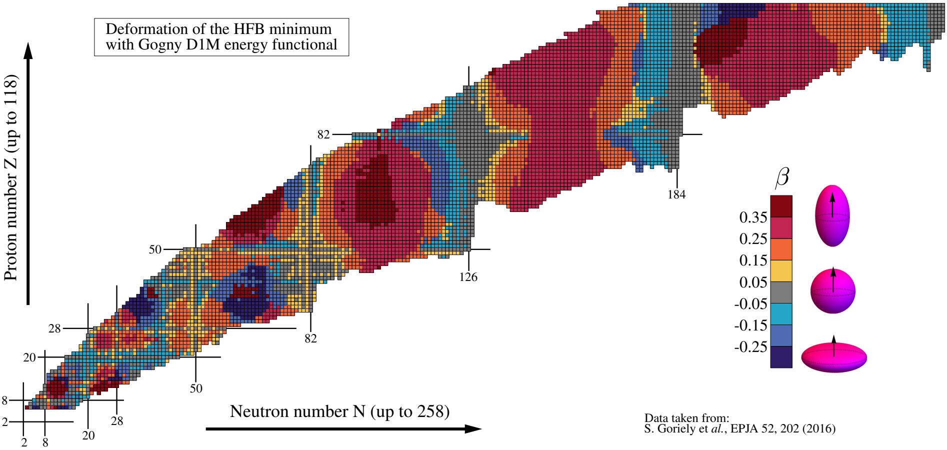
Deformation is widespread



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- Exact eigenstate $|\Psi\rangle$ expanded from a (simpler) reference state $|\Phi\rangle$

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- Such deformed states break the symmetries of H
  $\Rightarrow$ symmetries have to be restored

  Duguet, J. Phys. G 42, 025107 (2015)

  Duguet *et al.*, J. Phys. G 44, 015103 (2017)

  Qiu *et al.*, Phys. Rev. C 99, 044301 (2018)

Deformed MBPT (dMBPT)

- Perturbation theory applied to a many-body reference state
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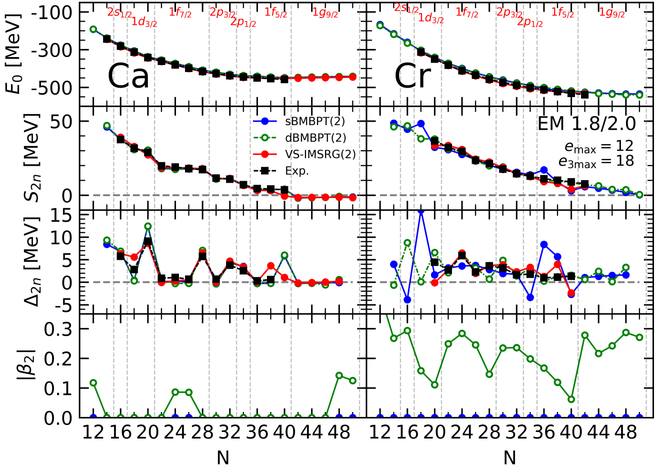
- Method is computationally cheap $\rightarrow$ large-scale calculations possible

- Does it converge?
  At least deformed $|\Phi\rangle$ is a better starting point for deformed nuclei

Deformed MBPT (dMBPT)

- Application to deformed case ongoing



Alberto Scalesi *et al.*, to be published (2024)

Deformed CC (dCC)

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  Hagen *et al.*, Rep. Prog. Phys. 77, 096302 (2014)

  - $T = T_1 + T_2 + T_3 + \ldots + T_A$ with, e.g., $T_2 = \frac{1}{4}\sum_{ijkl} t_{ijkl}\, a_i^\dagger a_j^\dagger a_l a_k$

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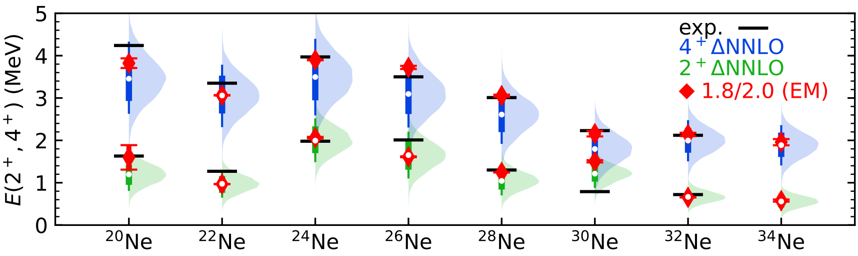
- Extension to deformed reference states with symmetry projection!



Ekström *et al.*, arXiv:2305.06955 (2023)

Deformed IMSRG (dIMSRG)

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  Hergert *et al.*, Phys. Rep. 621, 165 (2016)
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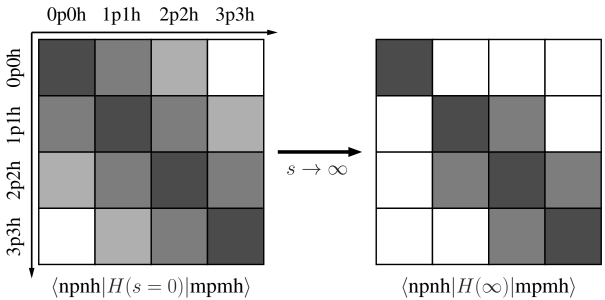

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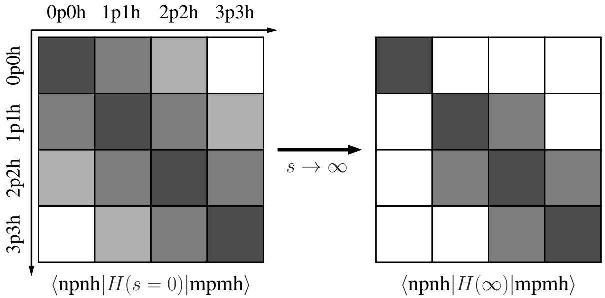

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  - ◇ Flow generates higher-body terms → truncation IMSRG(k)

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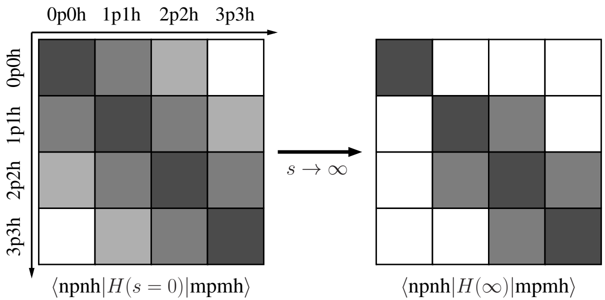

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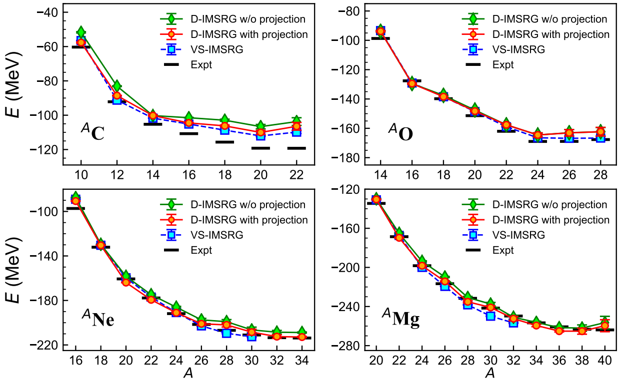
- Extension to deformed nuclei

Yuan *et al.*, Phys. Rev. C 105, L061303 (2022)



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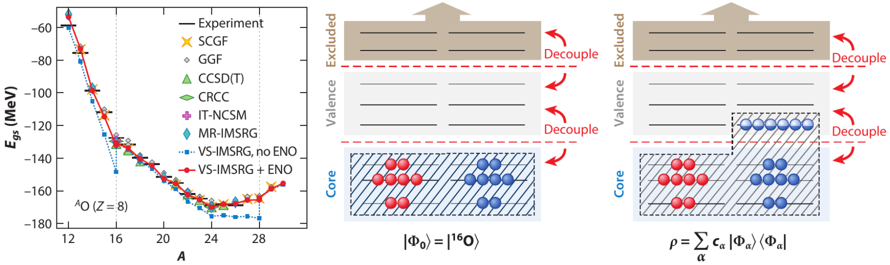

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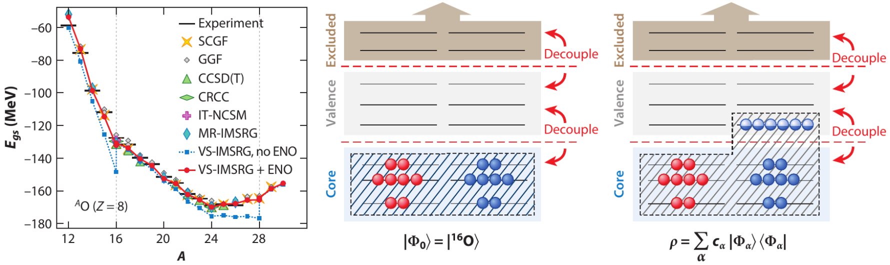
  - Exact diagonalization within the valence space
- Can reuse SM know-how and numerical codes!
  ⇒ Large contribution to recent progress in *ab initio* reach



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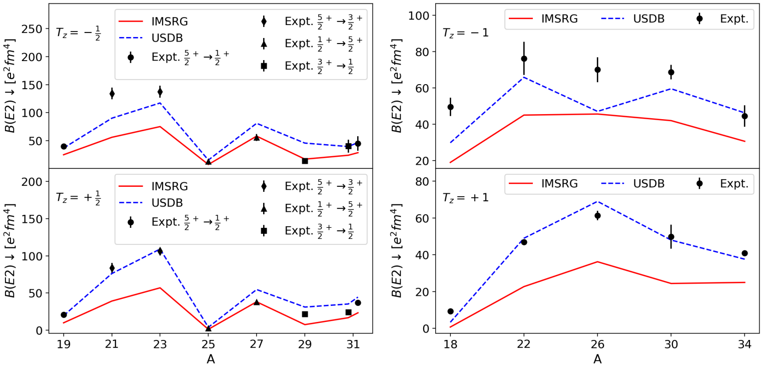
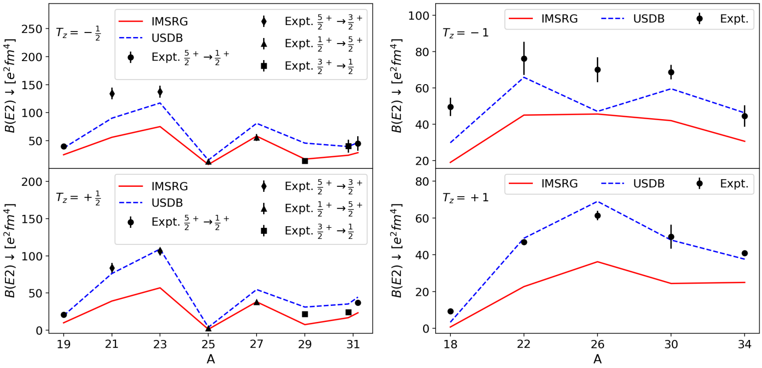


Stroberg *et al.*, Phys. Rev. c 105, 034333 (2022)

- Problems to take into account collectivity
  - ◇ Includes all correlations inside the valence space but ...
  - ◇ ... still depends on IMRSG(k) for the decoupling



Stroberg *et al.*, Phys. Rev. c 105, 034333 (2022)

Projected Generator Coordinate Method (PGCM)

- Approximate eigenstates of H with variational ansatz

$$|\Theta_{\epsilon}^{\Lambda M}\rangle = \sum_{qK} f_{\epsilon K}^{\Lambda M}(q) P_{MK}^{\Lambda} |\Phi(q)\rangle \quad \text{where} \quad \Lambda \equiv Z, N, J, \pi$$

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  - ◇ Series of constrained HFB minimization, e.g., $\langle \Phi(q) | Q | \Phi(q) \rangle = q$

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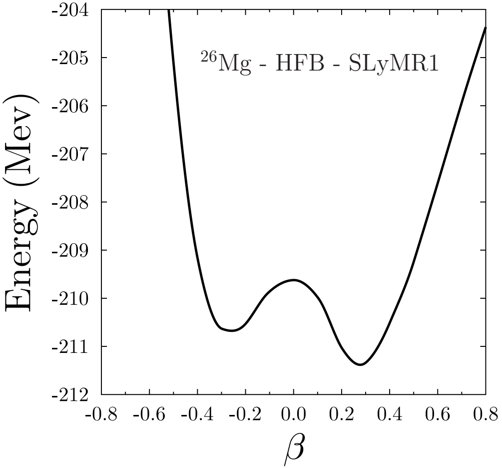
- Advantages:
  - Multi-reference approach
  - Symmetry conserving → good quantum numbers
  - Scales with A as mean field ... but with a LARGE prefactor

PGCM: schematic example



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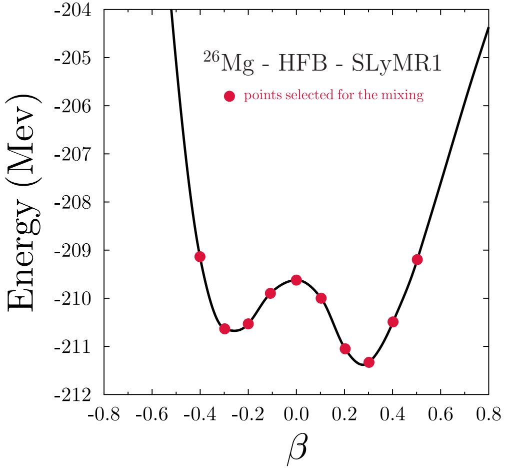

PGCM: coupling to expansion schemes

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  - Expansion on top of multi-reference state is more complicated/costly
- Two methods recently developed
  - IMSRG + PGCM → In-Medium GCM

    Yao et al., Phys. Rev. Lett. 124, 232501 (2020)
  - PGCM + Perturbation Theory (PGCM-PT)

    Frosini *et al.*, Eur. Phys. J. A 58, 62 (2022), Frosini *et al.*, Eur. Phys. J. A 58, 63 (2022), Frosini *et al.*, Eur. Phys. J. A 58, 64 (2022)
  - (the two can be combined!)

    Frosini *et al.*, Eur. Phys. J. A 58, 64 (2022)

In-Medium GCM: application to ^{76}Ge

- Workflow:
  - Perform PGCM calculation $\rightarrow |\Phi^{ZNJ\pi}\rangle$
  - Evolve H through IMSRG using $|\Phi^{ZNJ\pi}\rangle$ as reference state $\rightarrow H(\infty)$
  - Perform new PGCM calculation with $H(\infty)$

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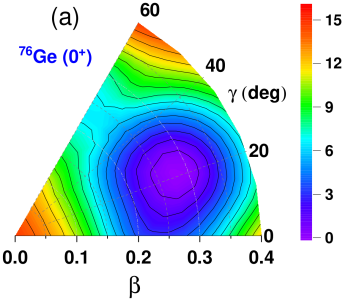
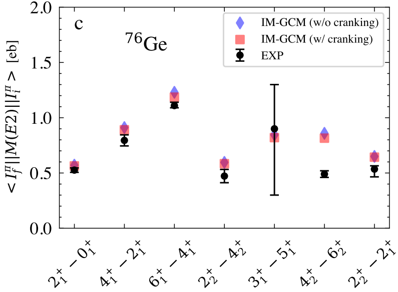
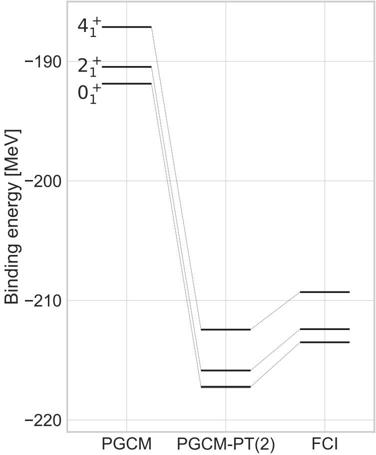




Belley et al., arXiv:2308.15634 (2023) + Phys. Rev. Lett., in production (2024)

- Perturbation Theory on top of PGCM state
  - Multi-reference & symmetry conserving
  - State dependent PT
  - Scaling much worse than PGCM (need better partitioning)



Frosini *et al.*, Eur. Phys. J. A 58, 64 (2022)

PGCM-PT: application to ^{20}Ne

- Perturbation Theory on top of PGCM state
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  - State dependent PT
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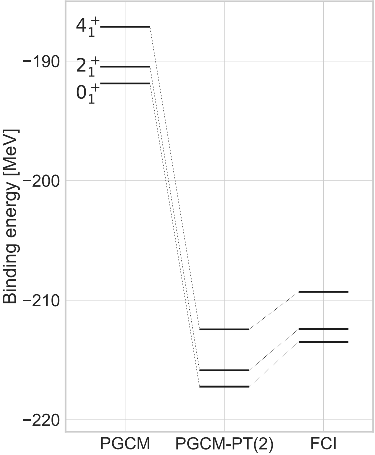
- Plain PGCM enough for relative quantities

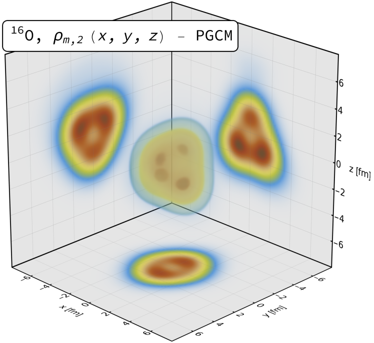


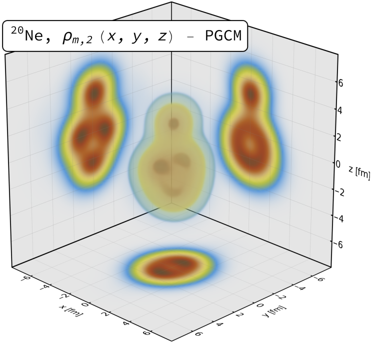
Frosini *et al.*, Eur. Phys. J. A 58, 64 (2022)

Calculations for ion-ion collisions

- State-of-the-art PGCM and NLEFT calculations for ^{16}O and ^{20}Ne
  Giacalone *et al.*, arXiv:2402.05995 (2024)

- The two methods agree quite well!





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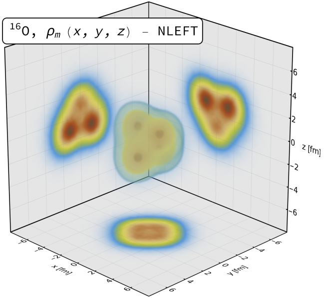

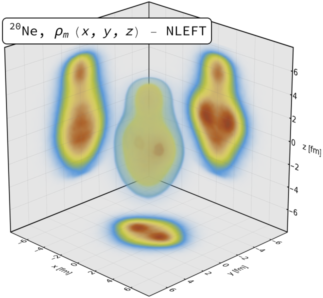

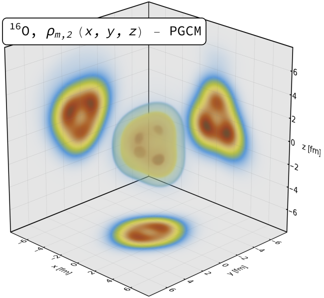

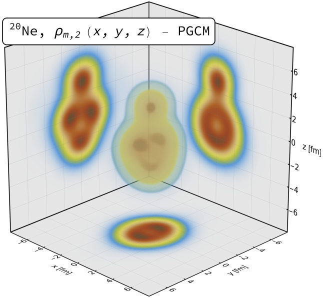

Conclusion

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  - include static deformation correlations in the reference states
  - include dynamic correlations through the expansion
  - need to restore the symmetries!

- Collaboration with heavy-ion community started but could be reinforced

  Summerfield *et al.*, PRC 104, L041901 (2021)

  Mäntysaari *et al.*, PRL 131, 062301 (2023)

  Giacalone *et al.*, arXiv:2402.05995 (2024)