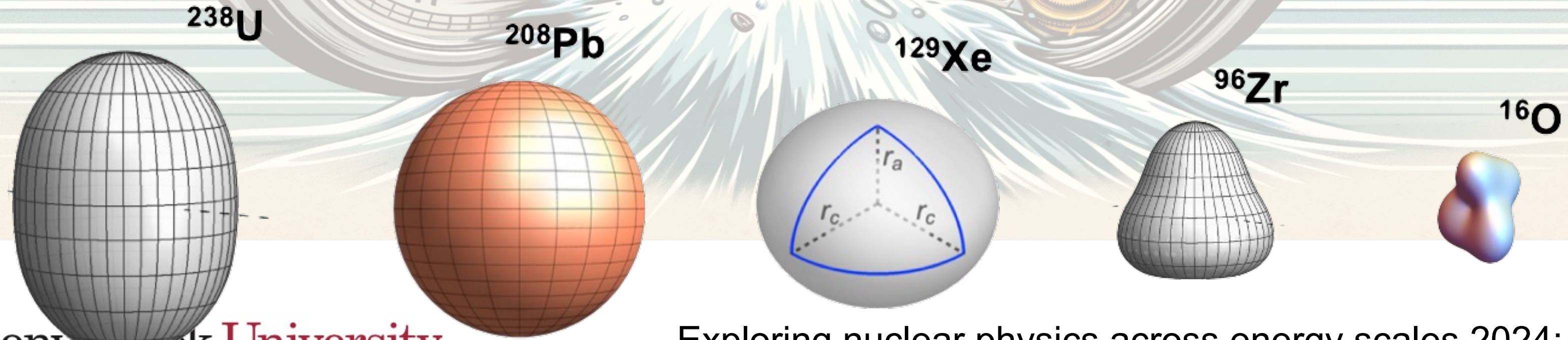
Probe structure of nuclei available at colliders

Jiangyong Jia

April 16, 2024

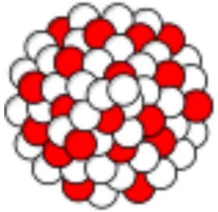


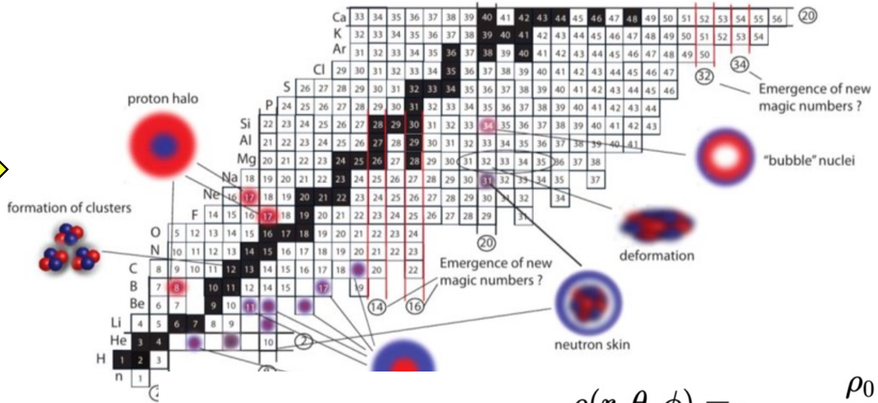
Stony Brook University

Exploring nuclear physics across energy scales 2024:
intersection between nuclear structure and high energy nuclear collisions

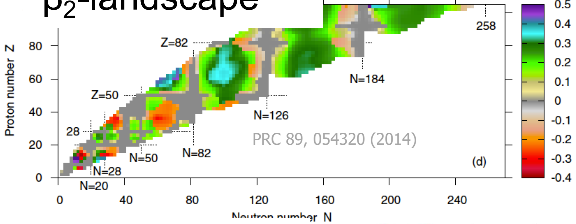
Atomic nuclei and their shapes

- Emergent phenomena of a many-body quantum system
  - clustering, halo, skin, bubble…
  - quadrupole/octupole/hexdecopole deformations
  - Non-monotonic evolution with N and Z

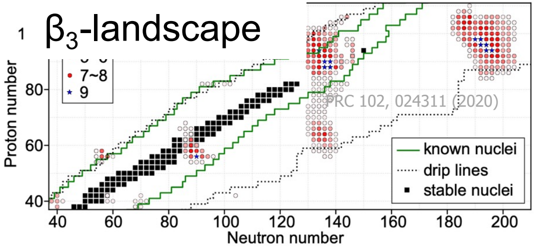




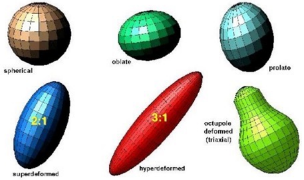
β_2-landscape



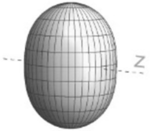
β_3-landscape



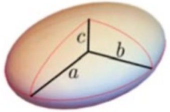
$$\rho(r,\theta,\phi)=\frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a_0}}$$

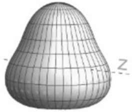


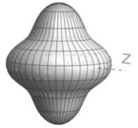
$$R(\theta,\phi)=R_0(1+\beta_2[\cos\gamma Y_{2,0}(\theta,\phi)+\sin\gamma Y_{2,2}(\theta,\phi)]+\beta_3 Y_{3,0}(\theta,\phi)+\beta_4 Y_{4,0}(\theta,\phi))$$



$0\leq\gamma\leq\pi/3$

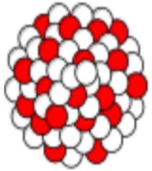




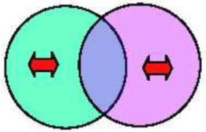


Degrees-of-freedom and timescales

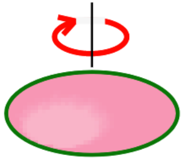
Heavy nuclei

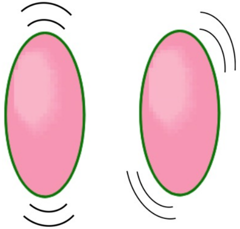


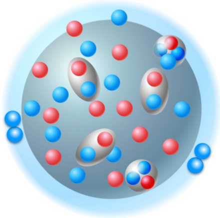
GR PR

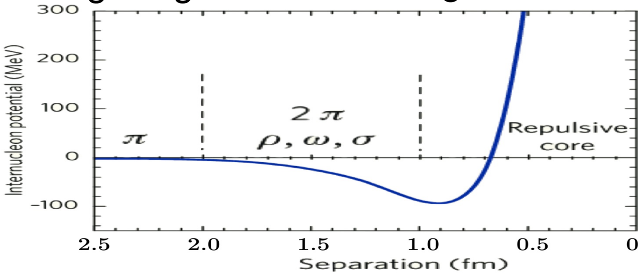


Long-range medium-range short-range

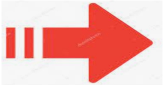








	rotation	vibration	clustering	nucleon correlations
$E = \frac{p^2}{2m}$ Energy scales: (MeV)	~0.04	~0.5-2	~a few	~1-50
Timescales: (fm/c)	10^4-10^3	~1000	~1000-100	~100 to a few



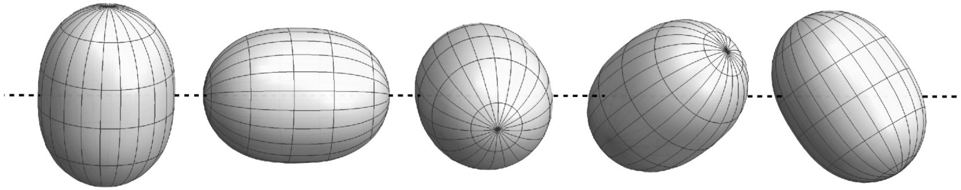
resonances quarks, gluons

$$\hbar = 197\text{MeV} \times \frac{\text{fm}}{c}$$

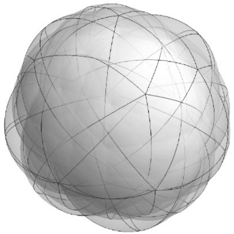
Nuclear shape in low-energy methods

Each DOF has zero-point fluctuations within an intrinsic timescale.

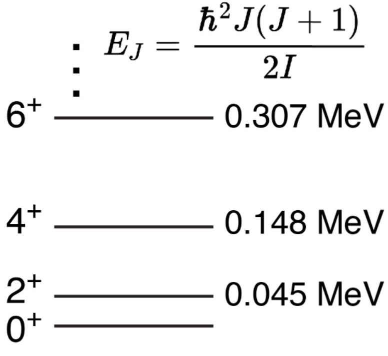
quantum fluctuations in orientations



time scale: $\tau \gtrsim I/\hbar \sim 10^3$ - 10^4 fm/c



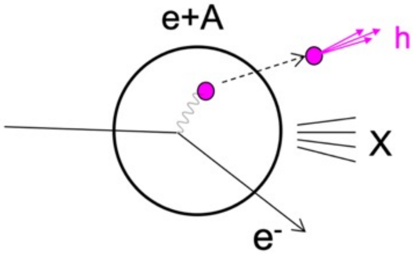
wavefunctions probed in spectroscopy method

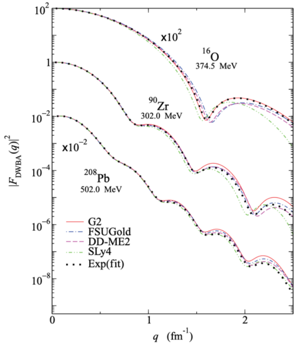


rotational band of ^{238}U

(non-invasive) spectroscopic methods probe a superposition of these fluctuations
Instantaneous nuclear shapes are not directly seen → intrinsic shape not observable

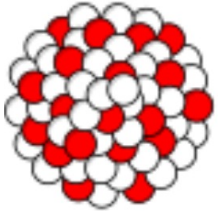
e+A scattering has very short timescales, but so far mostly imaged the one-body (charge) distribution. The impact of deformation appears as an increase in the radius





Taking a snapshot

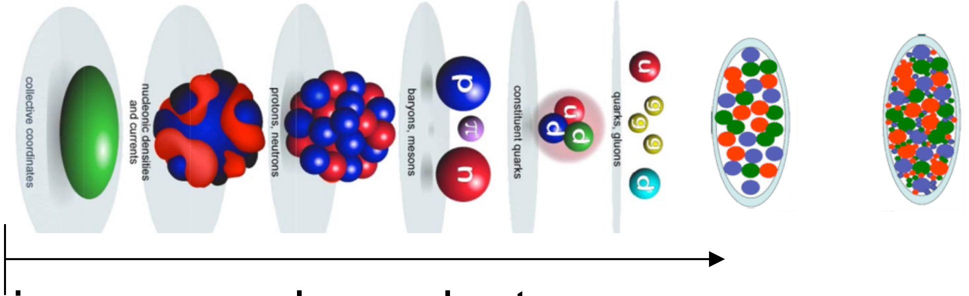
To see event-by-event shape directly, one must have access to instantaneous many-body distribution $\rho(\mathbf{r_1}, \mathbf{r_2} \ldots)$



$\tau_{\rm expo}$

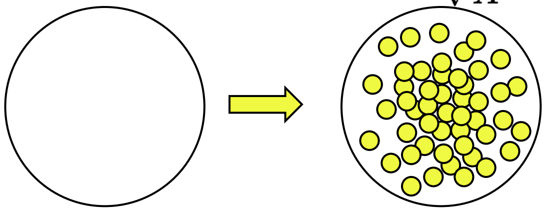
But we will see all DOFs longer than this timescale: $\tau > \tau_{\rm expo}$

Nucleons, hadrons, quark, gluons, gluon saturations



The concept of shape in principle collision energy dependent

$\epsilon_2 = 0$ $\qquad$ $\epsilon_2 \sim \frac{1}{\sqrt{A}}$

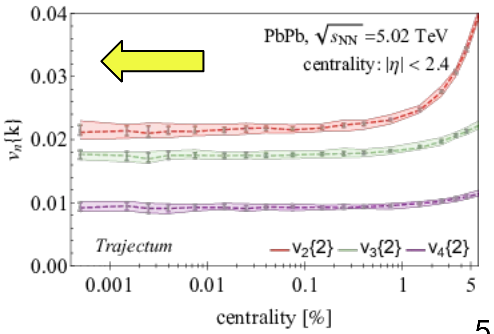


$\tau > 1000 \mathrm{fm/c}$ $\qquad$ $\tau < 1\ \mathrm{fm/c}$

Spherical Woods-Saxon $\qquad$ Sampled with A nucleons

$$\boldsymbol{\epsilon}_2 = \underbrace{\boldsymbol{\epsilon}_0}_{\sqrt{s}\text{-dependent fluctuations}} + \underbrace{\boldsymbol{p}(\Omega)\beta_2}_{\text{Global shape vibrational}} + \mathcal{O}(\beta_2^2)$$

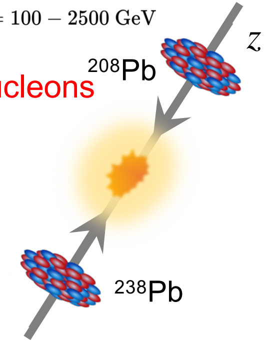
...



Flow-assisted nuclear shape imaging at high-energy

Nuclear structure

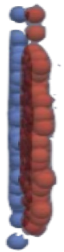
$\frac{\sqrt{s_{\mathrm{NN}}}}{2} = 100 - 2500\ \mathrm{GeV}$



400 nucleons

size & shape

Initial condition



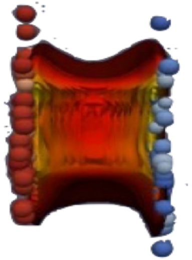
$\tau \sim 2R_0/\Gamma \sim 0.1\mathrm{fm}/c$

exposure

Fluid dynamics

Quark-Gluon Plasma

Final state

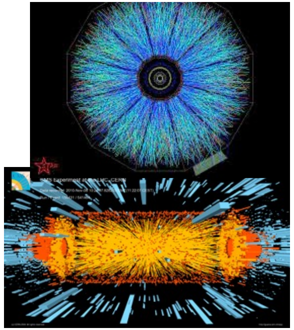


$\tau \sim 10\mathrm{fm}/c$

expansion

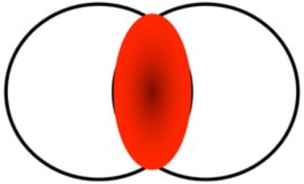
freestreaming

30,000 particles



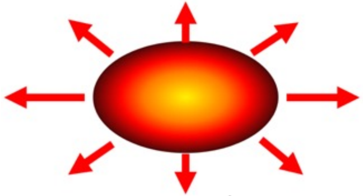
$\tau \sim 10^{15}\mathrm{fm}/c$

detection



$R_\perp^2 \propto \langle r_\perp^2 \rangle, \quad \mathcal{E}_n \propto \langle r_\perp^n e^{in\phi} \rangle$

size & shape



$$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n\, e^{-in\phi}\right)$$

observables

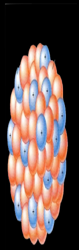
Event-by-event linear responses:

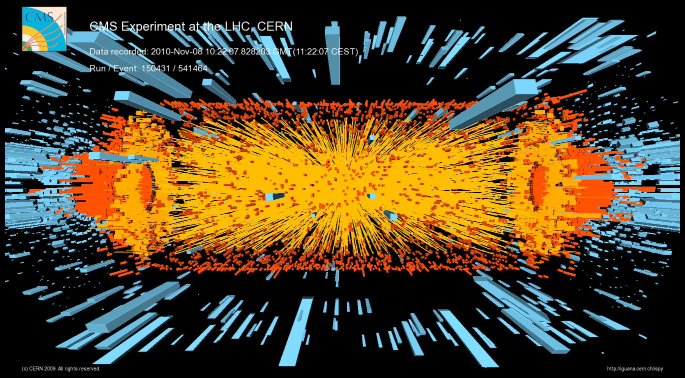
$$V_n \propto \mathcal{E}_n$$

$$\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp}$$

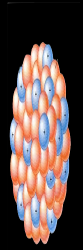
Key: 1) fast snapshot, 2) linear response, 3) large multiplicity for many-body correlations

Seen by ATLAS detector

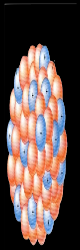


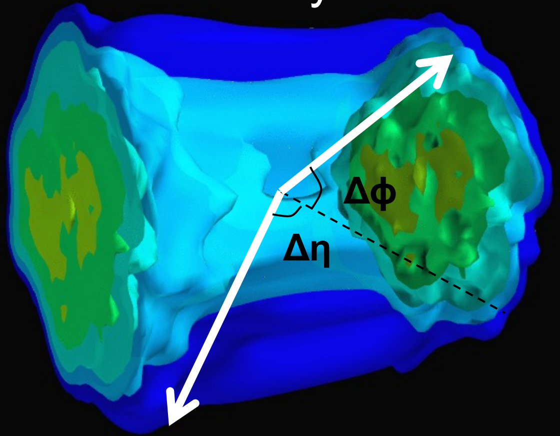


$t \sim 10^{-8}$ s

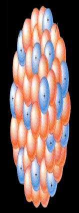


Collision dynamics





$t \sim 10 fm/c = 10^{-22}$ s



3D relativistic viscous hydrodynamics

Credit: Bjoern Schenke

Hydrodynamic EFT

Relativistic viscous Hydrodynamics (first-order):

Energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

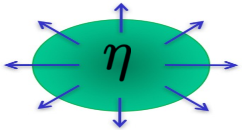
Equation-of-state P(∈) Bulk pressure Shear tensor

$\pi^{\mu\nu} = -\eta\sigma^{\mu\nu}$ η: shear viscosity

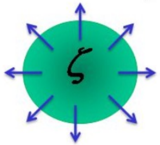
$\Pi = -\zeta\nabla^\perp_\lambda u^\lambda$ ζ : bulk viscosity

1) Collective flow driven by QCD eos: $F = -\nabla P(\epsilon)$

2) But resisted by viscosity

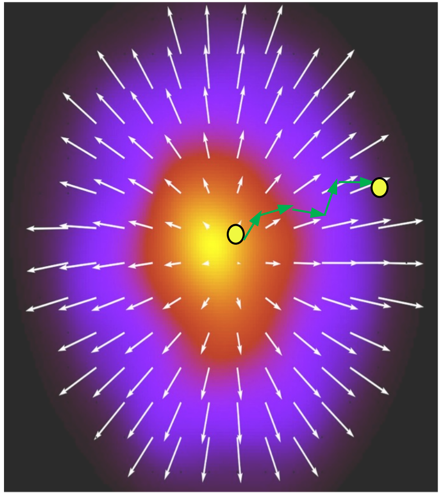


Reduce anisotropic flow



Reduce radial flow

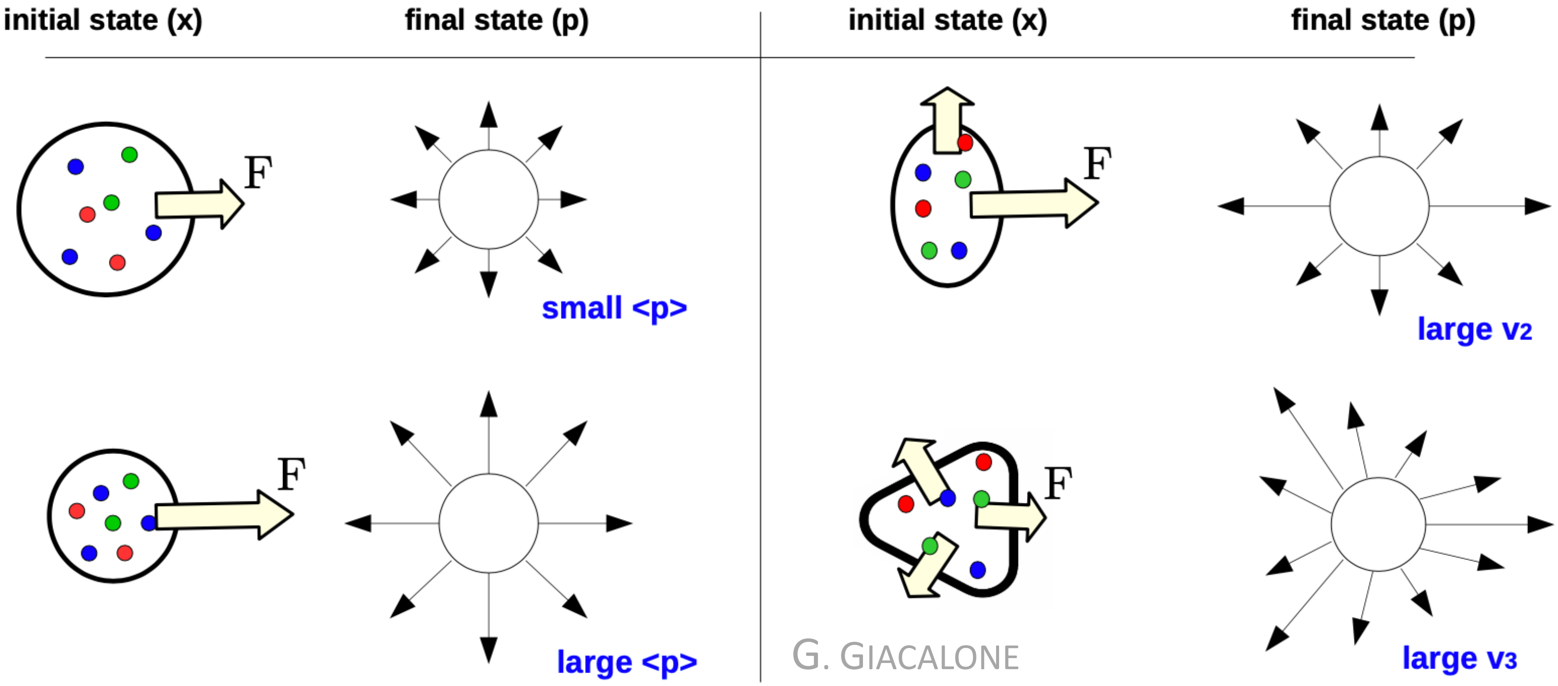
Valid when mean free path << system size



Initial geometry to Collective flow

Shape-flow transmutation via pressure-gradient force: $F = -\nabla P(\epsilon)$

$$\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp} \qquad \frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n\, e^{-in\phi}\right) \qquad V_n \propto \mathcal{E}_n$$



G. GIACALONE

Observables for flow fluctuations

- Single particle distribution

$$\frac{d^2N}{d\phi dp_\mathrm{T}} = N(p_T)\left[1 + 2\sum_n v_\mathrm{n}(p_T)\cos n(\phi - \Psi_n(p_T))\right]$$

$$= N(p_T)\left[\sum_{n=-\infty}^{\infty} V_\mathrm{n}(p_T)e^{in\phi}\right]$$

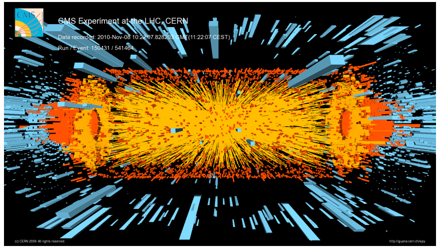
Flow vector: $\boldsymbol{V}_n = v_n e^{in\Psi_n}$

Radial flow

Anisotropic flow

- Two-particle correlation function

One real event



$$\left\langle \frac{d^2N_1}{d\phi dp_\mathrm{T}} \frac{d^2N_2}{d\phi dp_\mathrm{T}} \right\rangle \Rightarrow \langle \boldsymbol{V}_n(p_{T1})\boldsymbol{V}_n^*(p_{T2})\rangle \quad n - n = 0$$

- Multi-particle correlation function

$$\left\langle \frac{d^2N_1}{d\phi dp_\mathrm{T}} \cdots \frac{d^2N_m}{d\phi dp_\mathrm{T}} \right\rangle \Rightarrow \left\langle \boldsymbol{V}_{n_1}\boldsymbol{V}_{n_2}...\boldsymbol{V}_{n_m} \right\rangle \quad n_1 + n_2 + ... + n_m = 0$$

These moments completely describe the pdf: $p(\boldsymbol{V}_2, \boldsymbol{V}_3 \ldots) = \frac{1}{N_\mathrm{evts}} \frac{dN_\mathrm{evts}}{d\boldsymbol{V}_2 d\boldsymbol{V}_3 \ldots}$

Accessing information in intrinsic frame

$$\mathcal{E}_2 \equiv \varepsilon_2 e^{2i\Phi_2} \propto \int_{\mathbf{r}} \mathbf{r}^2 \rho(\mathbf{r}) \qquad d_\perp = 1/R_\perp \quad \delta d_\perp = d_\perp - \langle d_\perp \rangle$$

- We measure moments of p(1/R, ε_2, ε_3…) via p([p_T], v_2, v_3…)…
  - Mean $\langle d_\perp \rangle$ ⟺ $\langle p_T \rangle$
  - Variance: $\langle \varepsilon_n^2 \rangle, \left\langle (\delta d_\perp / d_\perp)^2 \right\rangle$ ⟺ $\langle v_n^2 \rangle, \left\langle (\delta p_T / p_T)^2 \right\rangle$
  - Skewness $\langle \varepsilon_n^2 \delta d_\perp / d_\perp \rangle, \left\langle (\delta d_\perp / d_\perp)^3 \right\rangle$ ⟺ $\langle v_n^2 \delta p_T / p_T \rangle, \left\langle (\delta p_T / p_T)^3 \right\rangle$
  - Kurtosis $\langle \varepsilon_n^4 \rangle - 2\langle \varepsilon_n^2 \rangle^2, \left\langle (\delta d_\perp / d_\perp)^4 \right\rangle - 3\left\langle (\delta d_\perp / d_\perp)^2 \right\rangle^2$ ⟺ $\langle v_n^4 \rangle - 2\langle v_n^2 \rangle^2, \left\langle (\delta p_T / p_T)^4 \right\rangle - 3\left\langle (\delta p_T / p_T)^2 \right\rangle^2$

  …

- Higher moments probe the frame-independent many-body distributions 1902.07168

$$\langle \varepsilon_2^2 \rangle = \langle \mathcal{E}_2 \mathcal{E}_2^* \rangle \approx \frac{\int_{\mathbf{r}_1, \mathbf{r}_2} (\mathbf{r}_1)^2 (\mathbf{r}_2^*)^2 \rho(\mathbf{r}_1, \mathbf{r}_2)}{\left(\int_{\mathbf{r}} |\mathbf{r}|^2 \langle \rho(\mathbf{r}) \rangle\right)^2} \qquad \langle \varepsilon_2^2 \delta d_\perp / d_\perp \rangle \approx -\frac{\int_{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3} (\mathbf{r}_1)^2 (\mathbf{r}_2^*)^2 |\mathbf{r}_3^2| \rho(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)}{\left(\int_{\mathbf{r}} |\mathbf{r}|^2 \langle \rho(\mathbf{r}) \rangle\right)^2 \int_{\mathbf{r}} |\mathbf{r}|^2 \langle \rho(\mathbf{r}) \rangle}$$

$$\rho(\mathbf{r}_1, \mathbf{r}_2) = \langle \delta\rho(\mathbf{r}_1) \delta\rho(\mathbf{r}_2) \rangle = \langle \rho(\mathbf{r}_1) \rho(\mathbf{r}_2) \rangle - \langle \rho(\mathbf{r}_1) \rangle \langle \rho(\mathbf{r}_2) \rangle \qquad \rho(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \langle \delta\rho(\mathbf{r}_1) \delta\rho(\mathbf{r}_2) \delta\rho(\mathbf{r}_3) \rangle$$

Coulomb Explosion Imaging in Chemistry

Instantaneous stripping of electrons (thin foil or x-ray laser), and then let atoms explode under mutual coulomb repulsion

Science 244, 4903 (1989)

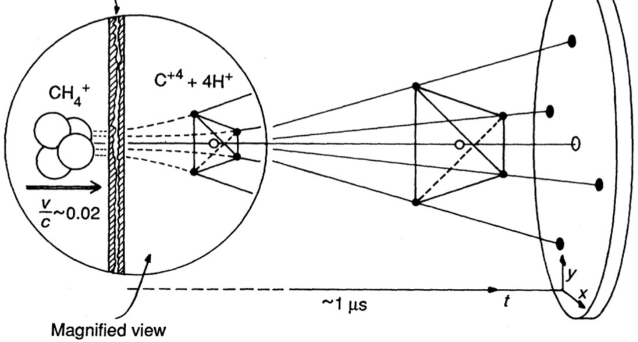
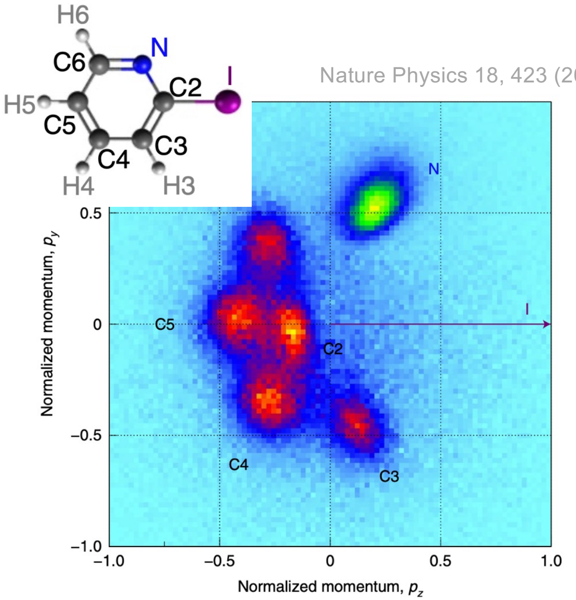
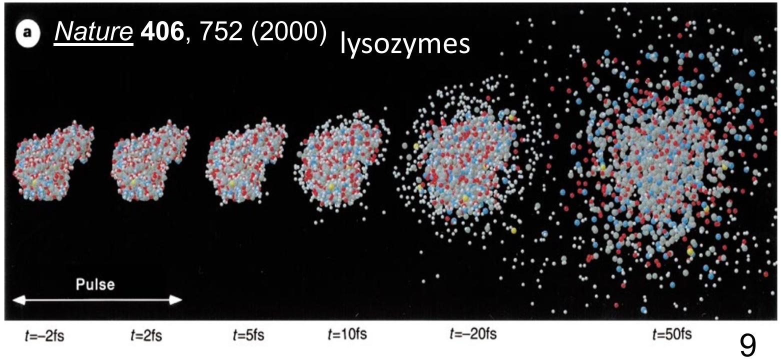


Fig. 1. A schematic view of a Coulomb explosion experiment. When a swift molecule passes through a thin solid film, it loses all of its binding electrons. The remaining positive ions repel each other, thus transforming the microstructure (as seen in the magnified view) into a macrostructure that can be measured precisely with an appropriate detector. The measured traces (x, y, t) of each fragment nucleus for individual molecules are then transformed into the original molecular structure.

Nature Physics 18, 423 (2022)





"Nuclear explosion imaging" is 10^6-10^9 times faster.

Applications of nuclear structure imaging

- Demonstration of methodology → deformation of Uranium238
- Layout the general strategy of structure imaging
- Establishing the precision via isobar collisions: ^{96}Ru+^{96}Ru vs ^{96}Zr+^{96}Zr
- Future opportunities.

Impact of quadrupole deformation in U+U

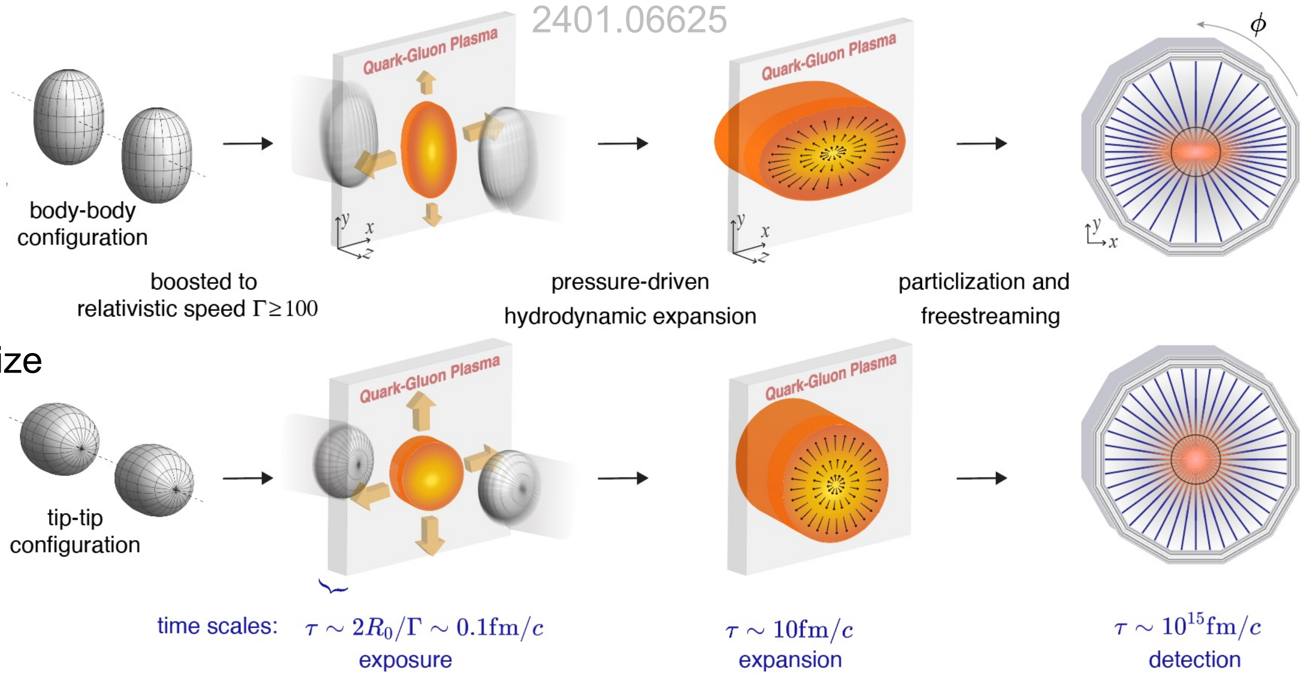
Collision geometry depends on the orientations: Head-on collisions has two extremes body-body or tip-tip collisions

Body-body: large-eccentricity large-size

$v_2 \nearrow \quad p_T \searrow$

Tip-tip : small-eccentricity small-size

$v_2 \searrow \quad p_T \nearrow$

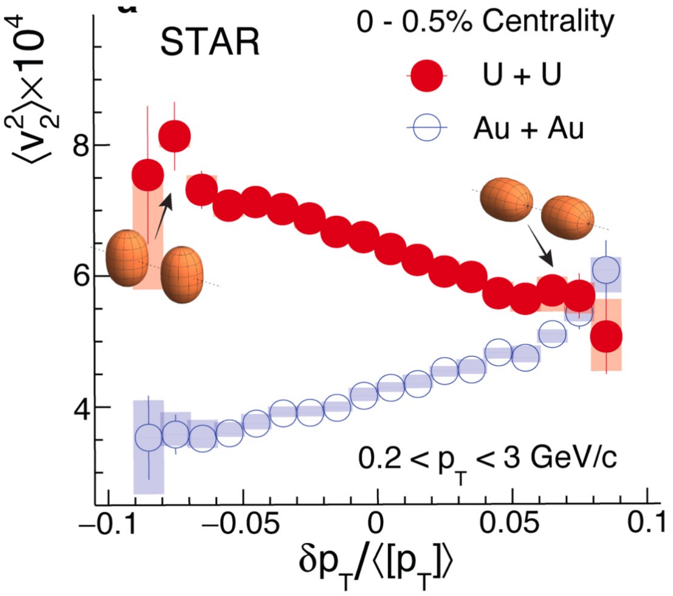


$$\langle v_2^2 \rangle = a_1 + b_1 \beta_2^2 \,,$$
$$\langle (\delta p_T)^2 \rangle = a_2 + b_2 \beta_2^2 \,,$$
$$\langle v_2^2 \delta p_T \rangle = a_3 - b_3 \beta_2^3 \cos(3\gamma)$$

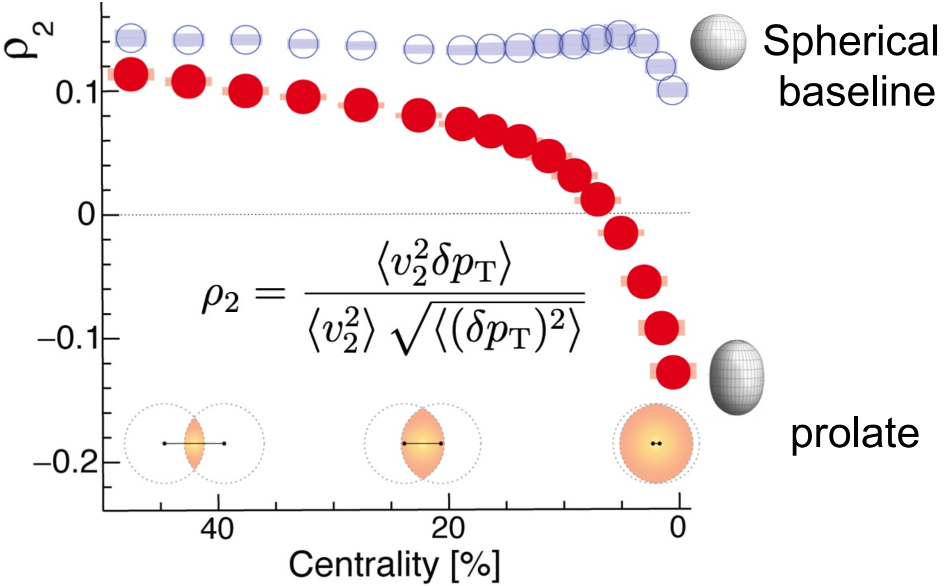
Deformation enhances the fluctuations of v_2 and $[p_T]$.
Also leads to anticorrelation between v_2 and $[p_T]$.

Impact of quadruple deformation

Seen directly by comparing $^{238}U+^{238}U$ with near-spherical $^{197}Au+^{197}Au$



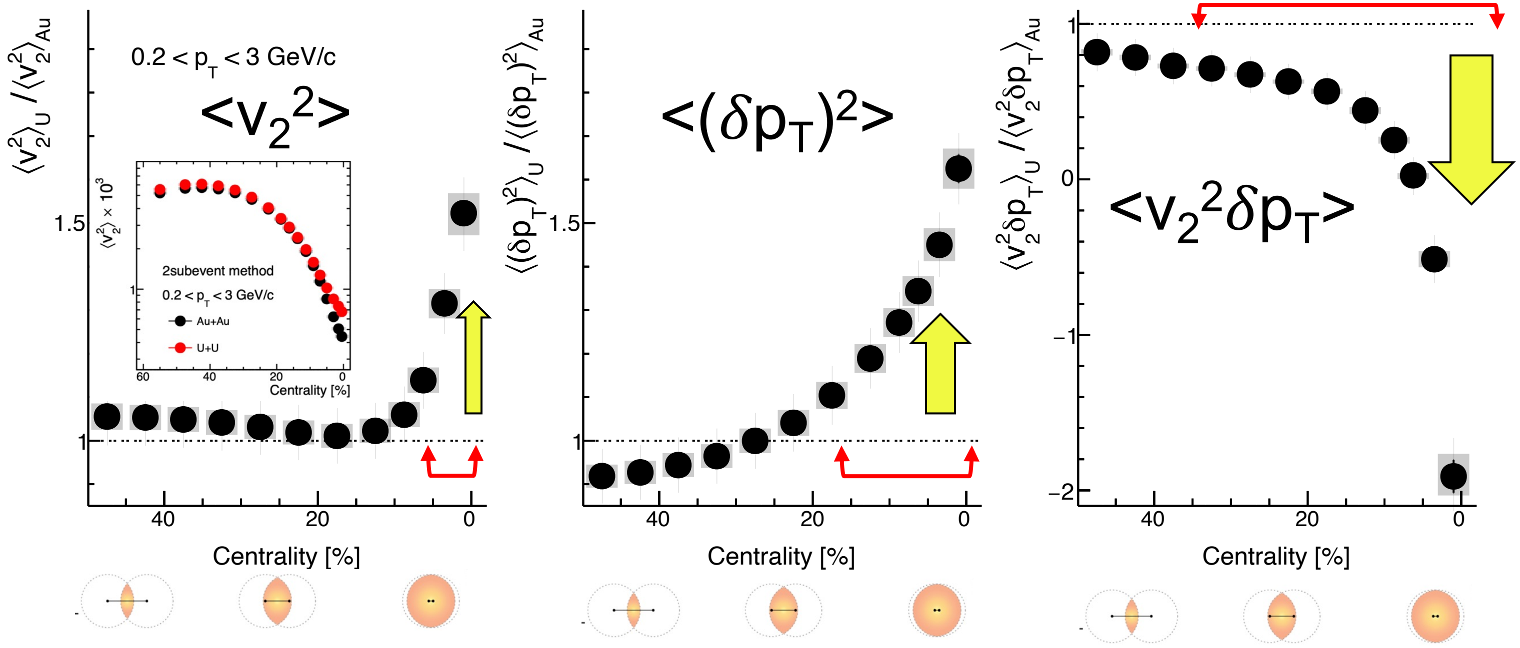
2401.06625



Near-spherical → flat ρ_2 vs centrality

Strongly prolate → decreasing ρ_2 vs centrality

Ratio of observables $R_{\mathcal{O}} = \langle \mathcal{O} \rangle_{\mathrm{U+U}} / \langle \mathcal{O} \rangle_{\mathrm{Au+Au}}$



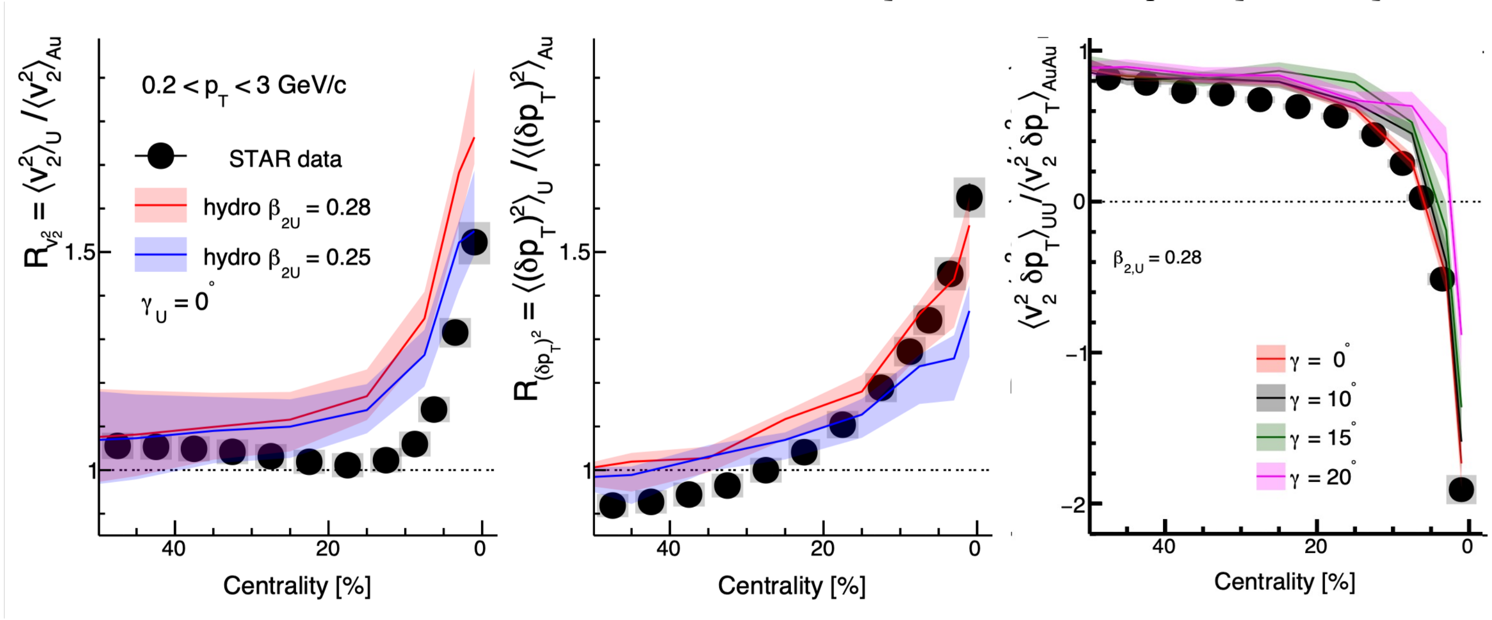
Ratios isolate impacts of the initial state, including nuclear structures!

U deformation dominates the UCC (ultra-central collisions)
→50%-70% modification on $<v_2^2>$ and $<(\delta p_T)^2>$, 300% for $<v_2^2\delta p_T>$

More smooth centrality dependence for $<(\delta p_T)^2>$ than $<v_2^2>$

v_2 is dominated by v_2^{RP} (unaffected by deformation), which has residual impact in UCC

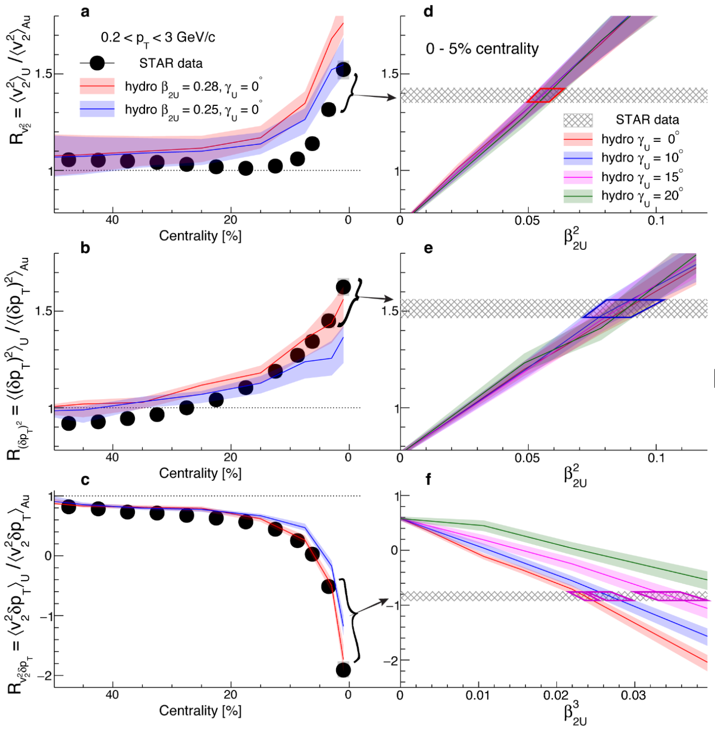
Ratio of observables $R_{\mathcal{O}} = \langle \mathcal{O} \rangle_{\mathrm{U+U}} / \langle \mathcal{O} \rangle_{\mathrm{Au+Au}}$

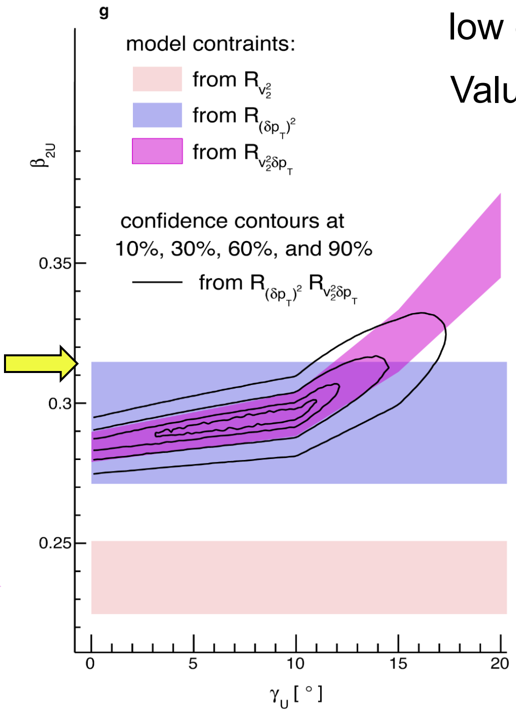


Compare with state-of-the-art IPGlasma+Music+UrQMD hydro model 2005.14682

- Increase of $<v_2^2>$ in model is less sharp → overestimate ratio of v_2^{RP} → lower bound β_{2U}.
- The $<(\delta p_T)^2>$ and $<v_2^2 \delta p_T>$ data seem to prefer values close to β_{2U} =0.28.
- $<v_2^2 \delta p_T>$ has additional sensitivity to γ_U → simultaneously constrain the β_{2U} and γ_U.

Constraining Uranium shape parameters 2401.06625





low energy estimate $\beta_{2\mathrm{U}} = 0.287 \pm 0.009$

Value could be smaller due to possible β_4.

$\beta_{2\mathrm{U}} \sim 0.25 - 0.26$ 2302.13617

Structure models suggest triaxiality, which seems to be preferred by the HI data. PRC 54, 2356 (1996)

arXiv:2303.11299

But we cannot distinguish between rigid triaxiality and triaxial fluctuations This can be done in the future using six-particle correlations: $v_2\{6\}$, $\langle v_2^4 \delta p_T^2 \rangle_c$.

2301.03556
2403.07441

Relations confirmed!

$$\langle v_2^2 \rangle = a_1 + b_1 \beta_2^2 \,,$$
$$\langle (\delta p_\mathrm{T})^2 \rangle = a_2 + b_2 \beta_2^2 \,,$$
$$\langle v_2^2 \delta p_\mathrm{T} \rangle = a_3 - b_3 \beta_2^3 \cos(3\gamma)$$

Derive a confidence contour

$$\beta_{2\mathrm{U}} = 0.297 \pm 0.013$$
$$\gamma_\mathrm{U} = 8.6^\circ \pm 4.8^\circ$$

Ratios cancel final state effects

- Vary the shear/bulk viscosity in Music hydro model
  - Flow signal change by more than factor of 2, yet the ratio unchanged.

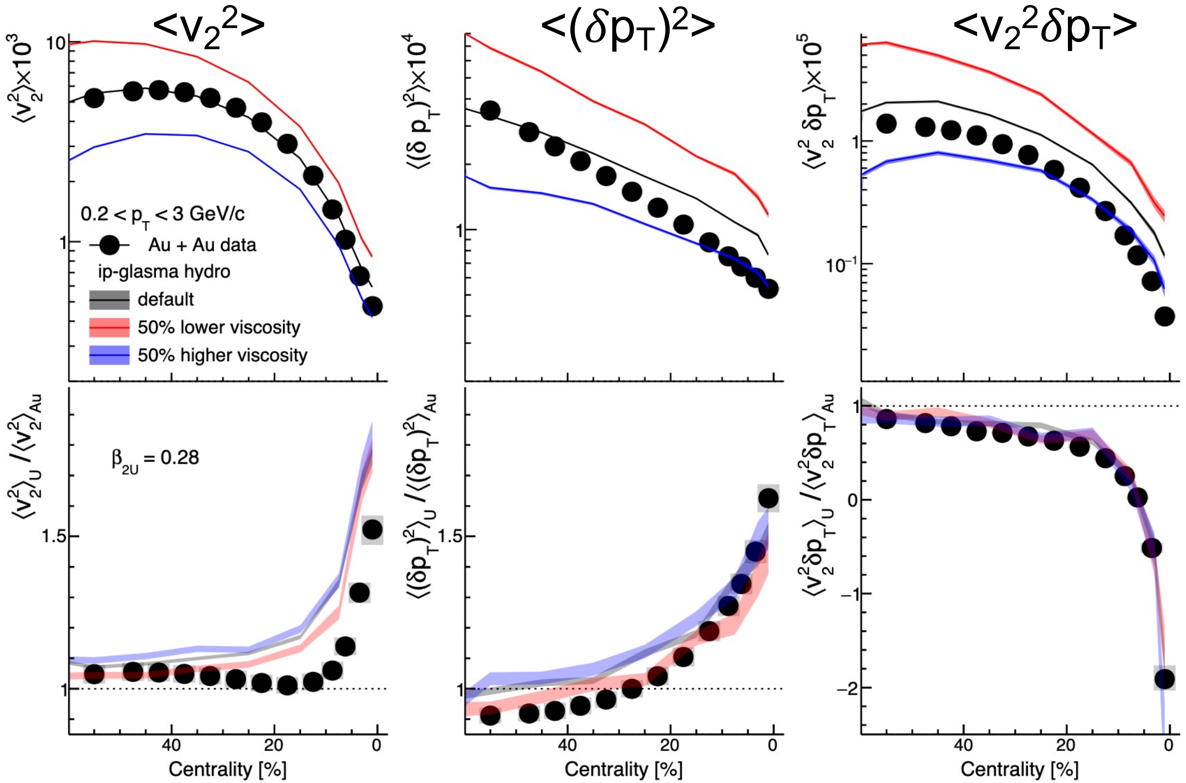
$$v_2 \propto \varepsilon_2$$
$$\delta p_T \propto -\delta R$$

$$\frac{v_{2,\mathrm{U}}}{v_{2,\mathrm{Au}}} \approx \frac{\varepsilon_{2,\mathrm{U}}}{\varepsilon_{2,\mathrm{Au}}}$$

$$\frac{\langle(\delta p_T)^2\rangle_{\mathrm{U}}}{\langle(\delta p_T)^2\rangle_{\mathrm{Au}}} \approx \frac{\langle(\delta R)^2\rangle_{\mathrm{U}}}{\langle(\delta R)^2\rangle_{\mathrm{Au}}}$$

$$\frac{\langle v_2^2\delta p_T\rangle_{\mathrm{U}}}{\langle v_2^2\delta p_T\rangle_{\mathrm{Au}}} \approx \frac{\langle \varepsilon_2^2\delta R\rangle_{\mathrm{U}}}{\langle \varepsilon_2^2\delta R\rangle_{\mathrm{Au}}}$$

Robust probe of initial state!



A general strategy for nuclear shape imaging

Flow observable = k $\otimes$ initial condition (structure)

QGP response,
a smooth function of N+Z

Structure of colliding nuclei,
non-monotonic function of N and Z

Compare two systems of similar mass but different structure

Two-particle observable: $R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1 \Delta\beta_2^2 + c_2 \Delta\beta_3^2 + c_3 \Delta R_0 + c_4 \Delta a$

Deviation from unity depends only on their structure differences

arXiv: 2111.15559

Available collision systems

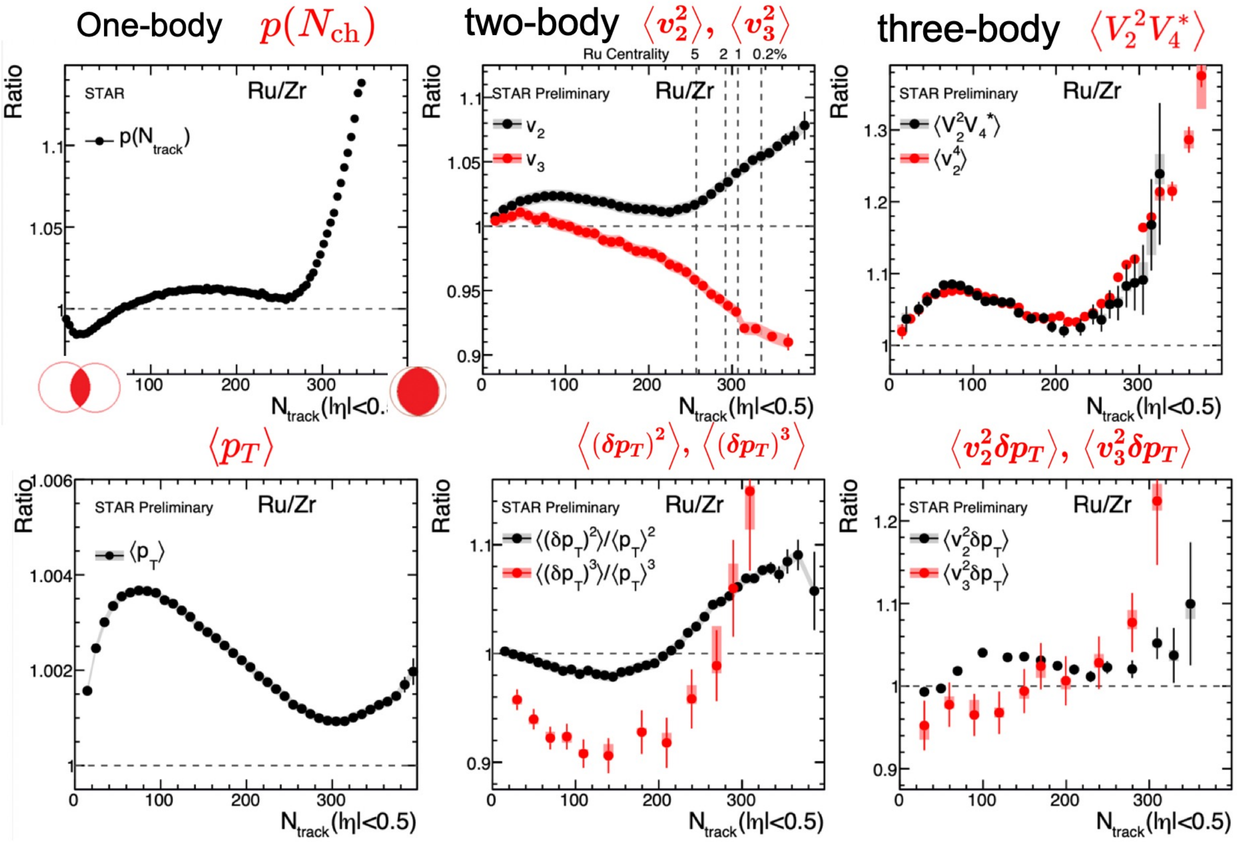
Nuclear Structure←→Initial Condition←→QGP dynamics/properties

RHIC $\sqrt{s}$=200GeV		LHC $\sqrt{s}$=5000 GeV
$^{197}Au+^{197}Au$ vs $^{238}U+^{238}U$ β_{2U} γ_U β_{3U} β_{4U}	Establish methodology • Large sensitivity	$^{129}Xe+^{129}Xe$ vs $^{208}Pb+^{208}Pb$ β_{2Xe} γ_{Xe} Neutron skin See talk of G. Nils, C. Zhang, Y. Zhou, H. Xu
$^{96}Ru+^{96}Ru$ vs $^{96}Zr+^{96}Zr$ β_{2Ru} β_{3Zr} large skin	Establish precision • 0.2% measurement error vs 5-15% signal • High-order observables	See talk of C. Zhang, H. Xu
$d+^{197}Au$ vs $^{16}O+^{16}O$	Structure of light nuclei • Cluster, subnucleon structure. • Benchmark ab-initio models	$^{16}O+^{16}O$ vs $^{20}Ne+^{20}Ne$? See talk of G. Giacolone
p+p, $p+^{27}Al$, $p+^{197}Au$, $^{3}He+^{197}Au$, $^{63}Cu+^{63}Cu$, $^{63}Cu+^{197}Au$	What can we learn from these?	p+p, $p+^{16}O$, $p+^{208}Pb$

What interesting species to consider & what questions do they answer?

Isobar ^{96}Ru+^{96}Ru and ^{96}Zr+^{96}Zr collisions at RHIC 200 GeV

QM2022 poster, Chunjian Zhang

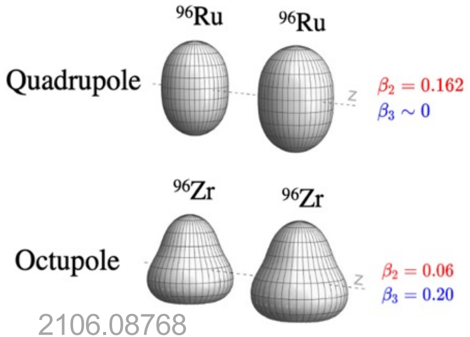


$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}}$$

Structure influences everywhere

Nuclear structure is inherently part of Heavy ion problem

Nuclear structure via v_2-ratio and v_3-ratio



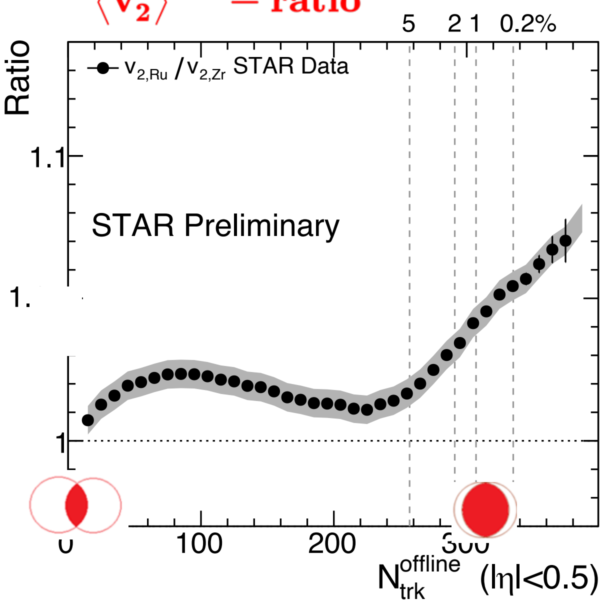
2106.08768

Species	β_2	β_3	a_0	R_0
Ru	0.162	0	0.46 fm	5.09 fm
Zr	0.06	0.20	0.52 fm	5.02 fm
difference	$\Delta\beta_2^2$	$\Delta\beta_3^2$	Δa_0	ΔR_0
	0.0226	-0.04	-0.06 fm	0.07 fm

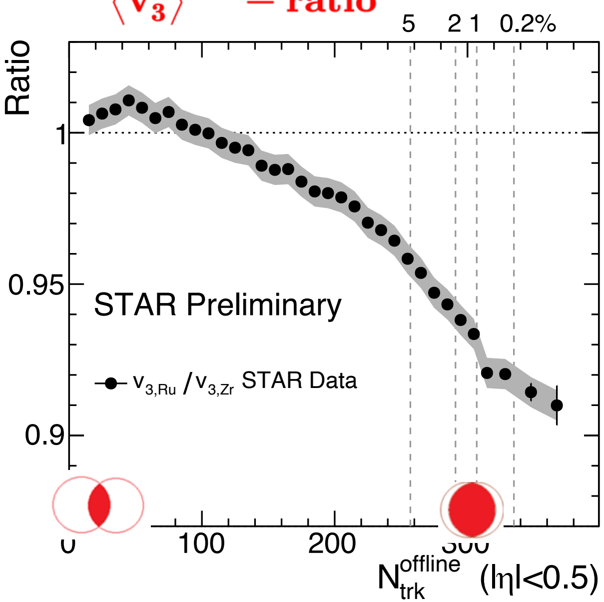
$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

Simultaneously constrain four structure parameters

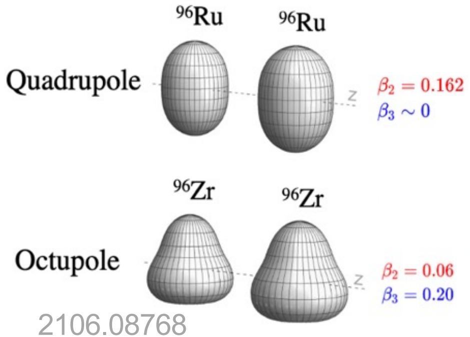
$\langle \mathbf{v_2^2} \rangle^{1/2}$ **— ratio**



$\langle \mathbf{v_3^2} \rangle^{1/2}$ **— ratio**



Nuclear structure via v_2-ratio and v_3-ratio



2106.08768

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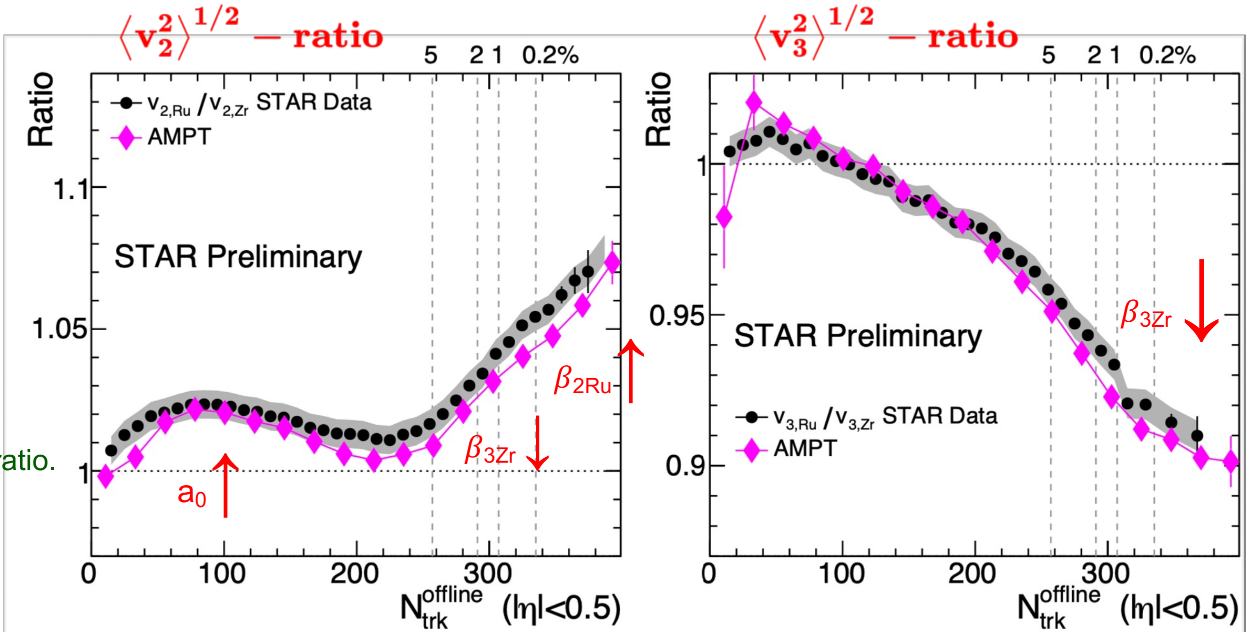
$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1 \Delta\beta_2^2 + c_2 \Delta\beta_3^2 + c_3 \Delta R_0 + c_4 \Delta a$$

Simultaneously constrain four structure parameters

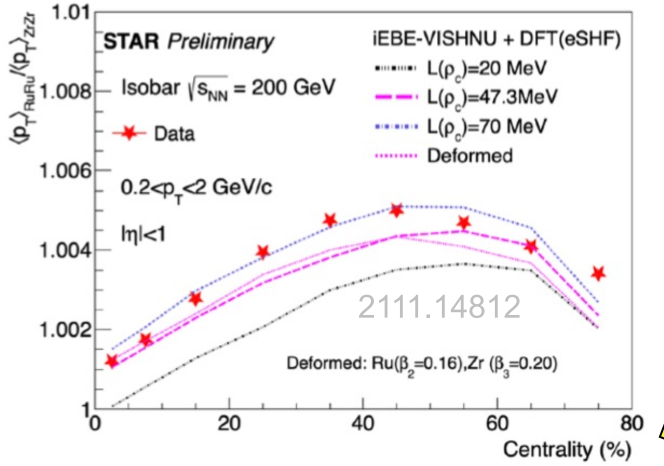
- $\beta_{2Ru} \sim 0.16$ increase v_2, no influence on v_3 ratio
- $\beta_{3Zr} \sim 0.2$ decrease v_2 and v_3 ratio
- $\Delta a_0 = -0.06$ fm increase v_2 mid-central,
- Radius $\Delta R_0 = 0.07$ fm slightly affects v_2 and v_3 ratio.

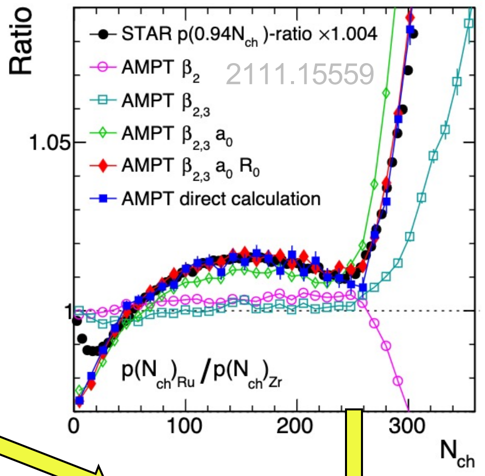
Is ^{96}Zr octupole deformed?

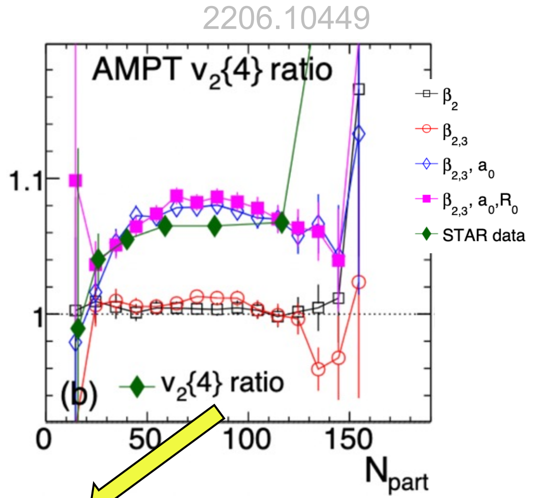


Imaging the radial structures

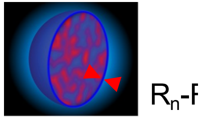
- Radial parameters R_0, a_0 are properties of one-body distribution → $\langle p_T \rangle$, $\langle N_{ch} \rangle$, $v_2^{RP} \sim v_2\{4\}$, σ_{tot},

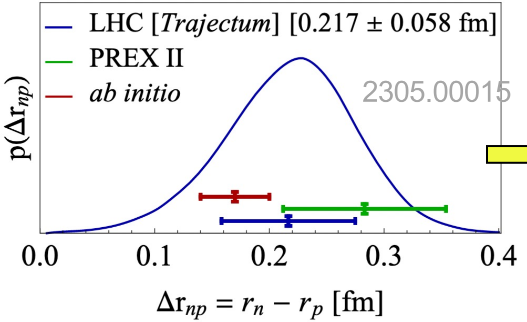


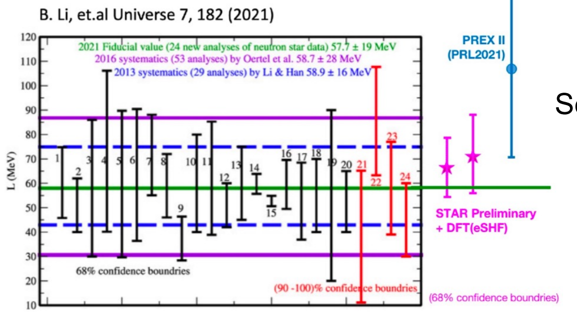




Constrain neutron skin and symmetry energy





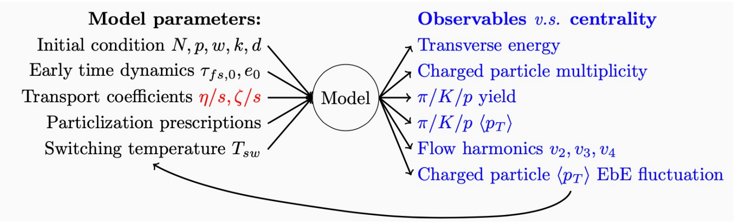


See talks of H. Xu, Z. Xu

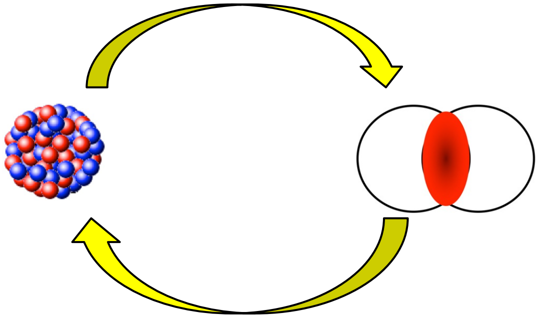
Opportunities

Low-energy: complexity & interpretation depend on location in nuclide chart
High-energy: fast snapshot of nucleon distribution for any collision species.

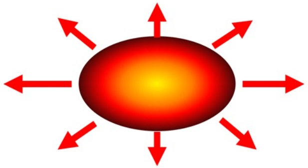
Current extraction of QGP properties are limited by the initial condition



constrain initial condition via nuclei with known structure



Better constraints on properties of QGP

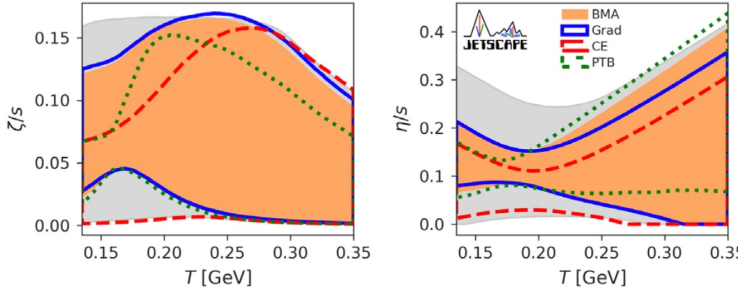


Constrain the structure for nuclei of interest with known initial condition

Systematic approaches for this purpose exist

Many potential applications
1) Odd mass nuclei
2) Higher-order deformations
3) Shape fluctuations/coexistence
4) Neutrinoless double-beta decay

…



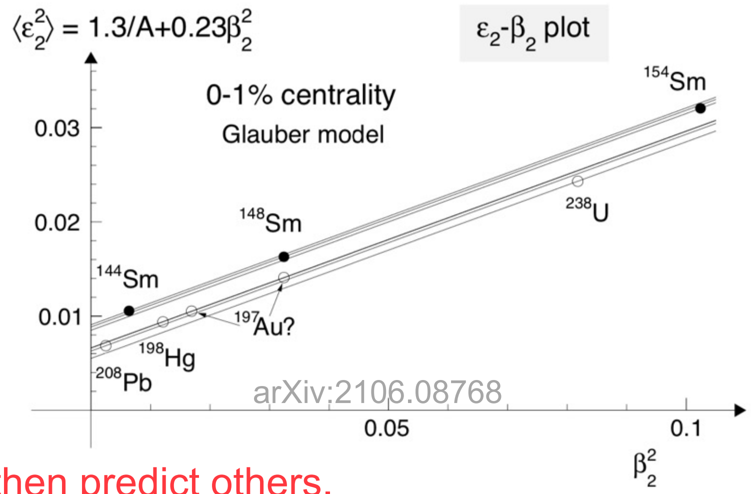
How to obtain the shape of nuclei of interest

In central collisions

$$\left\langle \epsilon_2^2 \right\rangle = a' + b'\beta_2^2 \qquad a' = \left\langle \varepsilon_2^2 \right\rangle_{|\beta_2=0} \propto 1/A$$

$$\left\langle v_2^2 \right\rangle = a + b\beta_2^2 \qquad a = \left\langle v_2^2 \right\rangle_{|\beta_2=0} \propto 1/A$$

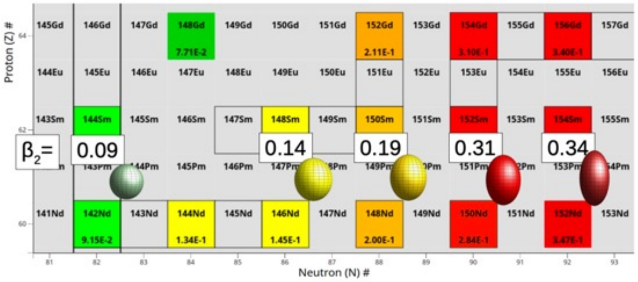
b', b are ~ independent of system



Systems with similar A fall on the same curve.

Fix a and b with two isobar systems with known β_2, then predict others.

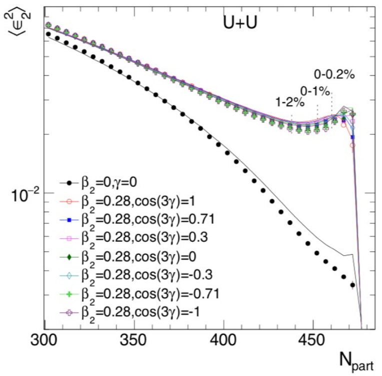
Transition from nearly-spherical to well-deformed nuclei when size increase by less than 7%. Using HI to access the multi-nucleon correlations leading to such shape evolution,



How to constrain triaxiality

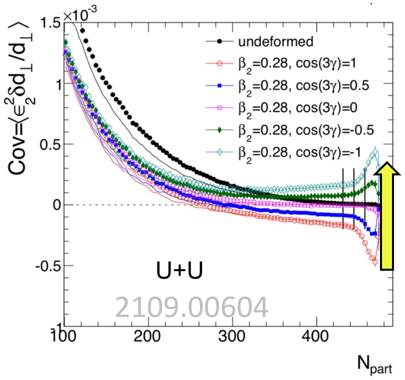
variances insensitive to γ

$$\langle v_2^2 \rangle \propto a + b\beta_2^2$$



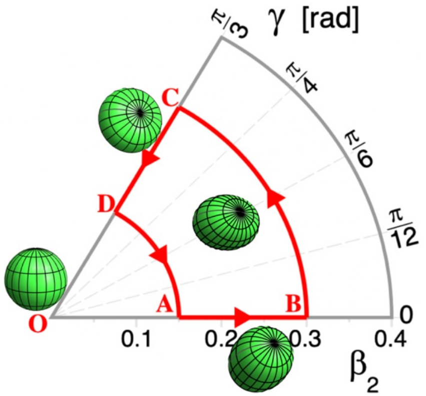
Skewness sensitive to γ

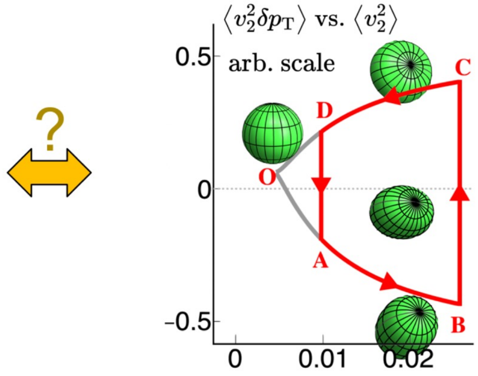
$$\langle v_2^2 \delta p_{\mathrm{T}} \rangle \propto a - b\cos(3\gamma)\beta_2^3$$



Use variance to constrain β_2, then use skewness to constrain γ

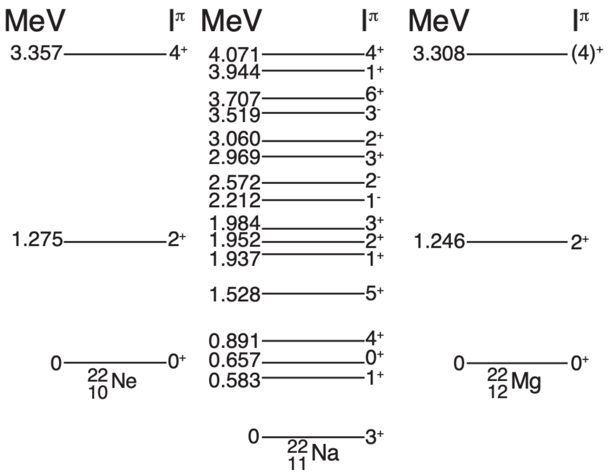
Map from (β_2,γ) plane to HI observables:





System scan to map out this trajectory: calibrate coefficients with species with known β,γ, then predict for species of interest.

Odd N or Z nuclei



nuclear shape is often presumed to be similar to adjacent even-even nuclei.

their spectroscopic data are more complex e.g. by the coupling of the single unpaired nucleon with the nuclear core.

by comparing the flow observables of odd-mass nuclei to selected even-even neighbors with established shapes, the high-energy approach avoids this complication.

Higher-order deformations β_3 and β_4

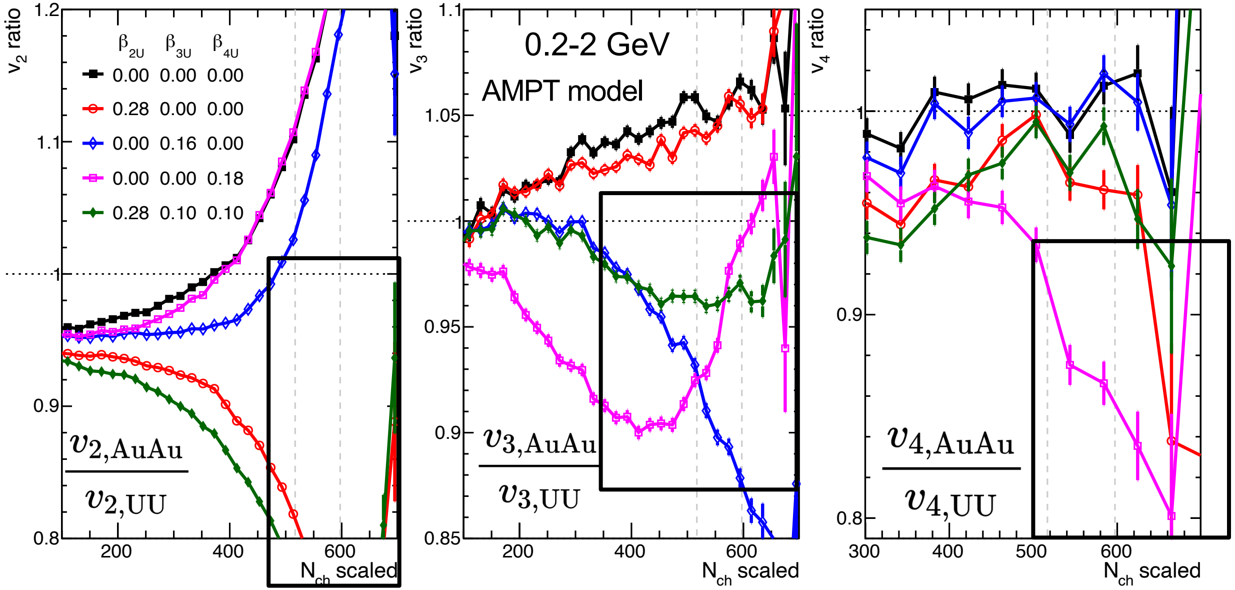
Ratio of v_n in UCC region are mainly sensitive to β_n

Uranium 238

β_2	β_3	β_4
0.286[12]	0.078[13]	0.07 – 0.09[14, 15]

Expectation:

$$\langle v_2^2 \rangle \approx a_2 + b_{2,2}\beta_2^2 + b_{2,3}\beta_3^2$$
$$\langle v_3^2 \rangle \approx a_3 + b_{3,3}\beta_3^2 + b_{3,4}\beta_4^2$$
$$\langle v_4^2 \rangle \approx a_4 + b_{4,4}\beta_4^2$$



β_{4U} constrained using v_4 ratio in central region

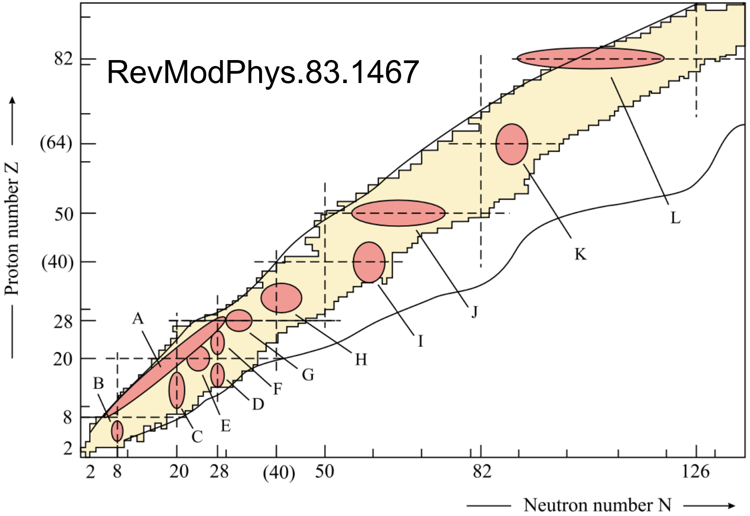
order of v_3 reversed by considering non-zero β_{3U} β_{4U}

v_2 ratio is mostly affected by β_{2U}, but also β_{3U}

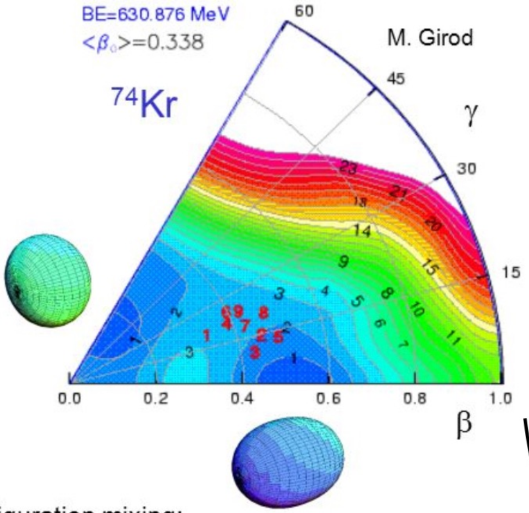
Shape fluctuation and coexistence

nuclei can have several low-lying states with different intrinsic shapes

probe the shape entanglement?



Each collision picks out one shape component of the ground state WF



Configuration mixing:

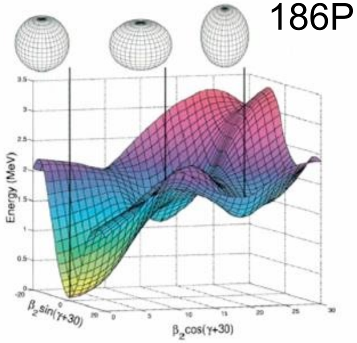
$$|0_1^+\rangle = +\cos\eta\,|0_{\text{pro}}^+\rangle + \sin\eta\,|0_{\text{obl}}^+\rangle$$

$$|0_2^+\rangle = -\sin\eta\,|0_{\text{pro}}^+\rangle + \cos\eta\,|0_{\text{obl}}^+\rangle$$

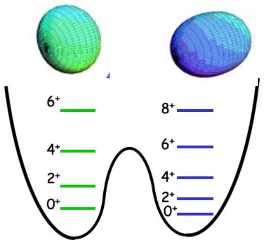
electric monopole (E0) transition

$$\langle 0_2^+\|\mathrm{M}(E0)\|0_1^+\rangle \propto \sin\eta\cos\eta\,(\beta_{\text{pro}}^2 - \beta_{\text{obl}}^2)$$

186Pb

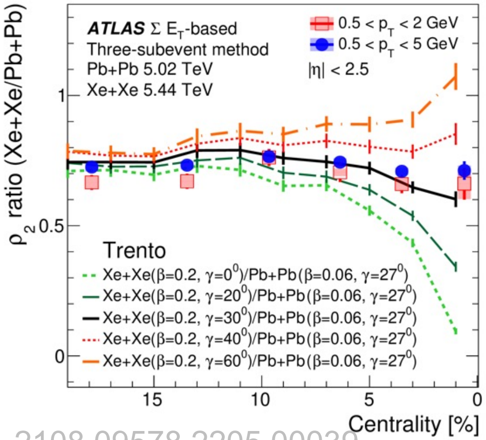


Nature **405**,430 (2000)



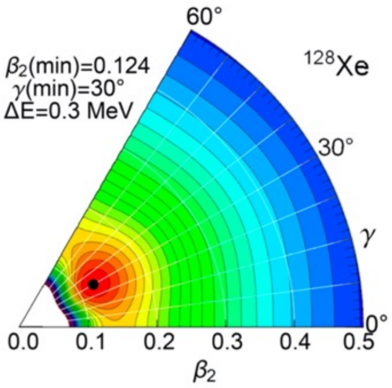
Shape fluctuations via high-order correlations.

$$R = \frac{\langle v_2^2\delta[p_T]\rangle_{^{129}Xe}}{\langle v_2^2\delta[p_T]\rangle_{^{208}Pb}}$$



2108.09578,2205.00039

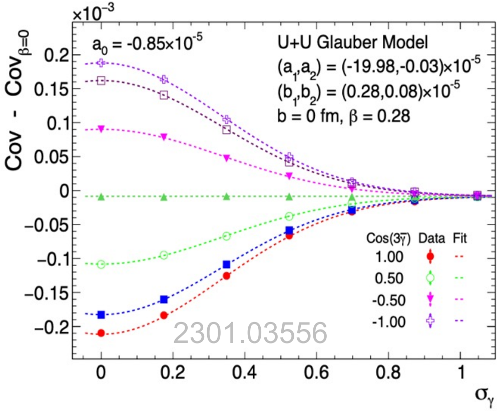
Large shape fluct., esp along γ



PhysRevC.101.064318

Fluctuation in γ washes out difference between prolate and oblate, such that all results approach triaxial case

$$\langle\cos(3\gamma)\rangle \approx e^{-\frac{9\sigma_\gamma^2}{2}}\cos(3\bar{\gamma})$$



2301.03556

Kumar sum rule

$$\langle\beta^2\rangle = \frac{16\pi^2}{9A^2R_0^4}\langle\hat{Q}^2\rangle$$

2-p correlation

$$\sigma\langle\beta^2\rangle/\langle\beta^2\rangle = \sigma\langle\hat{Q}^2\rangle/\langle\hat{Q}^2\rangle$$

4-p correlation

$$\cos 3\gamma = -\sqrt{\frac{7}{2}}\frac{\langle\hat{Q}^3\rangle}{\langle\hat{Q}^2\rangle^{3/2}}$$

3-p correlation

$$\frac{\sigma^2(\cos 3\gamma)}{(\cos 3\gamma)^2} = \frac{\sigma^2\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle^2} + \frac{9}{4}\frac{\sigma^2\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle^2} - 3\frac{\langle\hat{Q}^5\rangle - \langle\hat{Q}^3\rangle\langle\hat{Q}^2\rangle}{\langle\hat{Q}^3\rangle\langle\hat{Q}^2\rangle}$$

6-p correlation

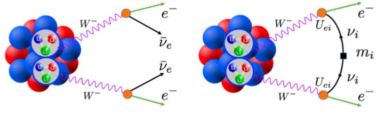
Heavy ion observables:

arXiv:2301.03556

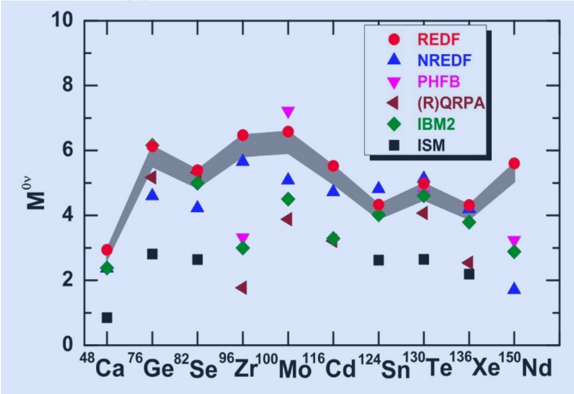
$$\frac{\langle\varepsilon_2^2\rangle}{\frac{3}{4\pi}\langle\beta_2^2\rangle} \Longleftrightarrow \frac{\langle\varepsilon_2^4\rangle - 2\langle\varepsilon_2^2\rangle^2}{-\frac{9}{112\pi^2}\left(7\langle\beta_2^2\rangle^2 - 5\langle\beta_2^4\rangle\right)}$$

$$\frac{\langle\varepsilon_2^2(\delta d_\perp/d_\perp)\rangle}{-\frac{3\sqrt{5}}{112\pi^{3/2}}\langle\cos(3\gamma)\beta_2^3\rangle} \Longleftrightarrow \frac{\left(\langle\varepsilon_2^6\rangle - 9\langle\varepsilon_2^4\rangle\langle\varepsilon_2^2\rangle + 12\langle\varepsilon_2^2\rangle^3\right)/4}{\frac{81}{256\pi^3}\left[\langle\beta_2^2\rangle^3 - \frac{45}{14}\langle\beta_2^4\rangle\langle\beta_2^2\rangle - \frac{1175}{6006}\langle\beta_2^6\rangle + \underline{\frac{25}{3003}\langle\cos(6\gamma)\beta_2^6\rangle}\right]}$$

Neutrinoless double-beta decay



2νββ: observed **0νββ: not yet**



Phase space Mass parameter

$$[T^{0\nu}_{1/2}]^{-1} = G_{0\nu}(Q,Z)|M_{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$

Nuclear matrix element

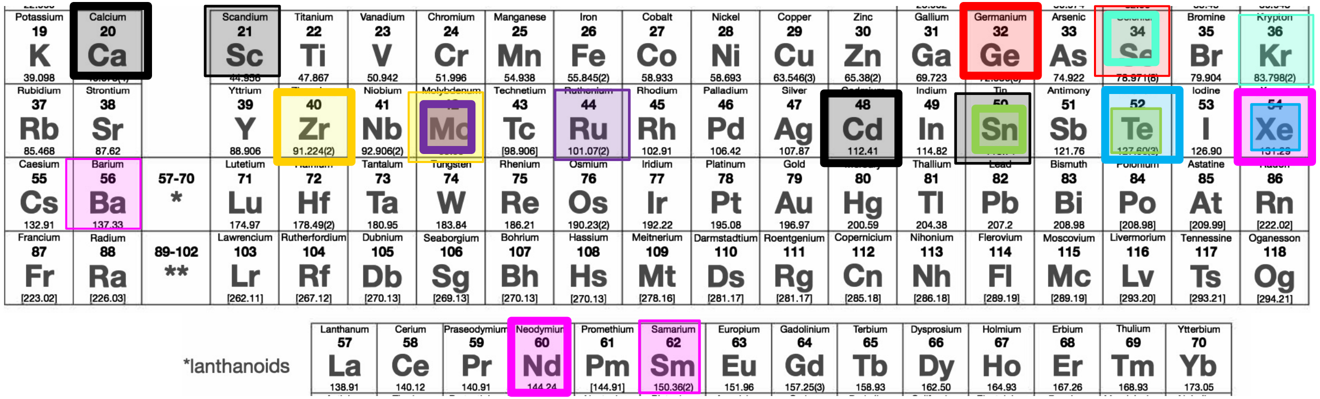
Need to know the overlap of nuclear wavefunction between initial nuclei and its final isobar nuclei.

x2-3 difference in matrix element leads to x10 change in lifetime

Challenge: modeling nucleon correlations in nuclear structure including quadruple and pairing correlations

HI collision could measure structure differences precisely

Talks by CF Jiao, J.M Yao



Summary

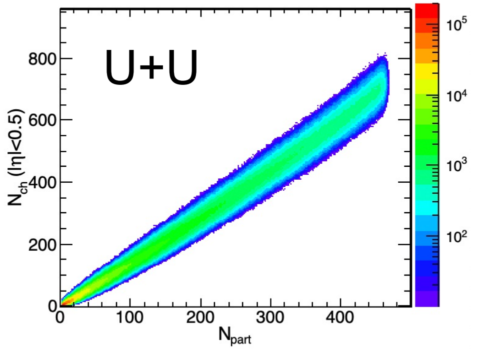
- Nuclear structure imaging could be a discovery tool for nuclear structure and high-energy nuclear physics
- High- and low-energy techniques together enable study of evolution of nuclear structure across energy and time scales.
- Future research could leverage collider facilities to conduct experiments with selected isobaric or isobar-like pairs

2102.08158

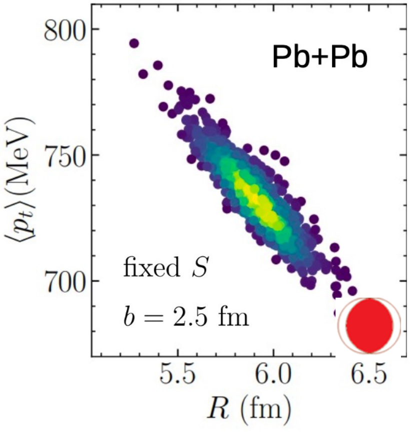
A	isobars	A	isobars	A	isobars	A	isobars	A	isobars	A	isobars
36	Ar, S	80	Se, Kr	106	Pd, Cd	124	Sn, Te, Xe	148	Nd, Sm	174	Yb, Hf
40	Ca, Ar	84	Kr, Sr, Mo	108	Pd, Cd	126	Te, Xe	150	Nd, Sm	176	Yb, Lu, Hf
46	Ca, Ti	86	Kr, Sr	110	Pd, Cd	128	Te, Xe	152	Sm, Gd	180	Hf, W
48	Ca, Ti	87	Rb, Sr	112	Cd, Sn	130	Te, Xe, Ba	154	Sm, Gd	184	W, Os
50	Ti, V, Cr	92	Zr, Nb, Mo	113	Cd, In	132	Xe, Ba	156	Gd,Dy	186	W, Os
54	Cr, Fe	94	Zr, Mo	114	Cd, Sn	134	Xe, Ba	158	Gd,Dy	187	Re, Os
64	Ni, Zn	96	Zr, Mo, Ru	115	In, Sn	136	Xe, Ba, Ce	160	Gd,Dy	190	Os, Pt
70	Zn, Ge	98	Mo, Ru	116	Cd, Sn	138	Ba, La, Ce	162	Dy,Er	192	Os, Pt
74	Ge, Se	100	Mo, Ru	120	Sn, Te	142	Ce, Nd	164	Dy,Er	196	Pt, Hg
76	Ge, Se	102	Ru, Pd	122	Sn, Te	144	Nd, Sm	168	Er,Yb	198	Pt, Hg
78	Se, Kr	104	Ru, Pd	123	Sb, Te	146	Nd, Sm	170	Er,Yb	204	Hg, Pb

Correlation between initial and final state

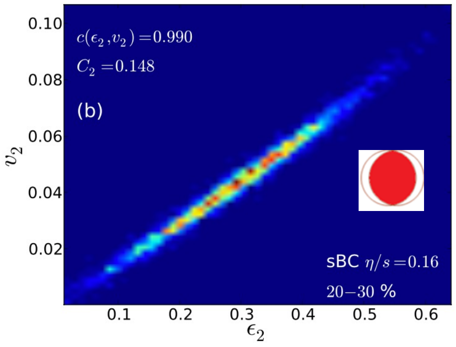
$$N_{ch} \propto N_{part}$$



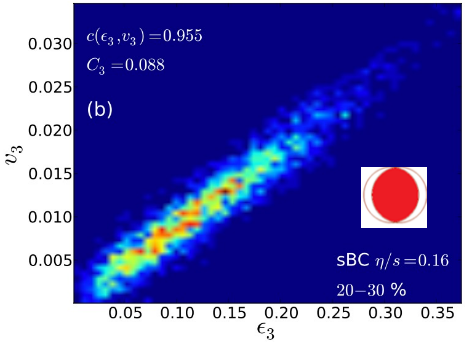
$$\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp}$$



$$V_2 \propto \mathcal{E}_2$$



$$V_3 \propto \mathcal{E}_3$$



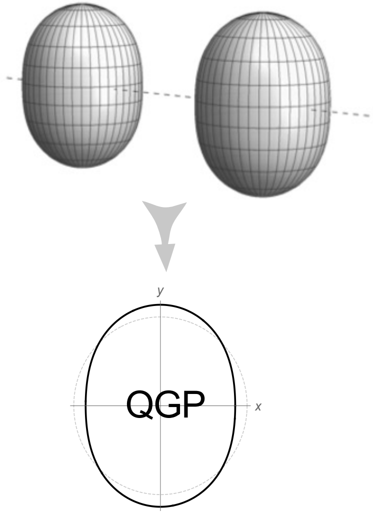
Nice correlation at very high energy

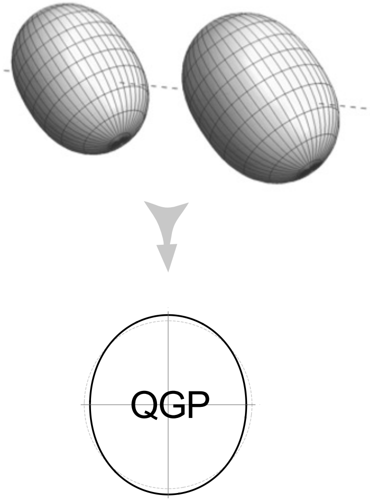
Connecting initial condition to nuclear shape

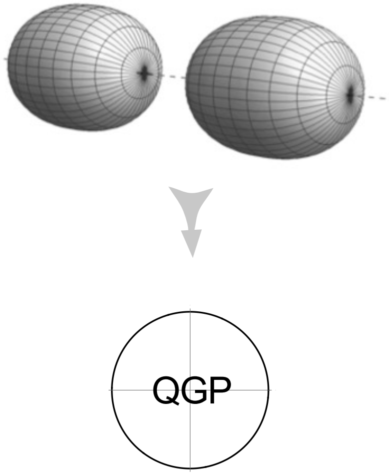
Body-Body

$$\mathcal{E}_2 = \epsilon_2 e^{i2\Phi} \propto \left\langle \boldsymbol{r}_\perp^2 e^{i2\phi} \right\rangle$$

Tip-Tip







$\varepsilon_2 \sim 0.95\beta_2$

$\varepsilon_2 \sim 0.48\beta_2$

$\varepsilon_2 \sim 0$

$$\boldsymbol{\epsilon}_2 = \underbrace{\boldsymbol{\epsilon}_0}_{\text{undeformed}} + \underbrace{\boldsymbol{p}(\Omega_1, \Omega_2)}_{\text{phase factor}} \beta_2 + \mathcal{O}(\beta_2^2)$$

$$\left\langle \epsilon_2^2 \right\rangle \approx \left\langle \epsilon_0^2 \right\rangle + 0.2\beta_2^2$$

Shape depends on Euler angle $\Omega = \varphi\theta\psi$

Sensitivity to other structure parameters

In ultra-central collisions, ratios are controlled by $\beta_{2\mathrm{U}}$ and $\boldsymbol{\gamma}_{\mathrm{U}}$.

In non-central collisions, v_2 ratio is sensitive to nuclear skin