Nuclear Deformation in High Energy Nuclear Collisions

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Exploring nuclear physics across energy scales 2024: intersection between nuclear structure and high energy nuclear collisions



Figure 1. Global rare-earth-oxide production trends.

OUTLINE

1 – Role of nuclear deformation in heavy-ion collisions

2 – Implementations of nuclear deformation in high energy collisions

3 – Studying and exploiting nuclear deformation at high energy

1 – Role of nuclear deformation in heavy-ion collisions



[Gelis, IJMPE 24 (2015) 10, 1530008]

a QCD description of high energy collisions between hadrons may be feasible, provided we can provide "snapshots" of their partonic content at the time of the collision.

transition probability
from hadrons to X
$$\approx \sum_{\substack{\text{partons} \\ \{q,g\}}} \frac{\text{probability to find}}{\{q,g\} \text{ in } \{h_1,h_2\}} \otimes \left|\sum_{\substack{\text{Amplitudes} \\ \{q,g\} \to X}} Amplitudes\right|^2$$

called *initial state factorization*. Roughly speaking, the physical motivation for such a factorization is that the neglected terms are interferences between a hard process that occurs on the timescale of the collision and a process internal to one of the projectiles, happening on much longer timescales.

Snapshots of atomic nuclei

[Miller et al., Ann.Rev.Nucl.Part.Sci. 57 (2007) 205-243]



"snapshot" of the nucleon positions

Image of collapsed wave function of 10 Li atoms [from S. Brandstetter (PI Heidelberg)]



ANALYTICAL INSIGHTS

- THICKNESS FUNCTION:
$$t(\mathbf{x}) = \sum_{i=1}^{A} g(\mathbf{x}; \mathbf{x}_i, w)$$

Nucleon "form factor" at high energy.
w = nucleon size.
- ENERGY DEPOSITION:

$$\lim_{\tau \to 0^+} \tau e(\mathbf{x}) \propto \left(\frac{t_A(\mathbf{x}) + t_B(\mathbf{x})}{2}\right)^{q/p} \xrightarrow[p=0]{} [t_A(\mathbf{x})t_B(\mathbf{x})]^{q/2} \xrightarrow[q=2]{} t_A(\mathbf{x})t_B(\mathbf{x})$$
IP-Glasma (T=0)
"IP-Jazma"
"binary collisions" ...
nucleus A * nucleus B energy density
nucleus A * nucleus B energy density

DENSITY CORRELATIONS IN THE OVERLAP REGION

$$\langle \epsilon(\mathbf{x}) \rangle_{\text{ev}} = A^2 \left(\int_{\xi_i} P_{1\perp}(\xi_i) g(\mathbf{x} - \xi_i) \right)^2$$

$$\langle \epsilon(\mathbf{x})\epsilon(\mathbf{y})\rangle_{\rm ev} = \left(A \int_{\xi_i} P_{1\perp}(\xi_i)g(\mathbf{x}-\xi_i)g(\mathbf{y}-\xi_i)\right)$$

Energy density

$$+ (A^2 - A) \int_{\xi_i \neq \xi_j} P_{2\perp}(\xi_i, \xi_j) g(\mathbf{x} - \xi_i) g(\mathbf{y} - \xi_j) \bigg)^2$$

$$+ (A^2 - A) \int_{\xi_i \neq \xi_j} P_{2\perp}(\xi_i, \xi_j) g(\mathbf{x} - \xi_i) g(\mathbf{y} - \xi_j) \bigg)^2$$

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$$+ (A^2 - A) \int_{\xi_i \neq \xi_j} P_{2\perp}(\xi_i, \xi_j) g(\mathbf{x} - \xi_i) g(\mathbf{y} - \xi_j) \bigg)^2$$



[Giacalone, EPJA **59** (2023) 12, 297]

nuclear *n*-body density:
$$P_n(\mathbf{r}_1, \dots, \mathbf{r}_n) = \sum_{s,t} \int d\mathbf{r}_{n+1} \dots d\mathbf{r}_A |\Psi_A|^2$$

(\perp = integrated over "z") GROUND STATE

BONUS: CONNECTION TO EXPERIMENTAL OBSERVATIONS

- ECCENTRICITY:
$$\mathcal{E}_n = -rac{\int_{\mathbf{x}} \epsilon(\mathbf{x}) \ |\mathbf{x}|^n e^{in\phi_x}}{\int_{\mathbf{x}} \epsilon(\mathbf{x}) \ |\mathbf{x}|^n}$$

– perturbative expansion: $\epsilon(\mathbf{x}) = \langle \epsilon(\mathbf{x})
angle + \delta \epsilon(\mathbf{x})$

[Blaizot, Broniowski, Ollitrault, PLB 738 (2014) 166-171]

- MEAN SQUARED ANISOTROPY:
INITIAL STATE

$$\langle \mathcal{E}_n \mathcal{E}_n^* \rangle = \frac{\int_{\mathbf{x}, \mathbf{y}} |\mathbf{x}|^n |\mathbf{y}|^n e^{in(\phi_x - \phi_y)} \langle \epsilon(\mathbf{x}) \epsilon(\mathbf{y}) \rangle}{\left(\int_{\mathbf{x}} \langle \epsilon(\mathbf{x}) \rangle |\mathbf{x}|^n \right)^2} \propto \langle V_n V_n^* \rangle$$
FINAL STATE



Independent fermions:

ions:
$$h_i = \frac{p_i^2}{2m} + V(r_i) \longrightarrow P_A(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = P_1(\mathbf{r}_1) P_1(\mathbf{r}_2) \dots P_1(\mathbf{r}_A)$$

All information in the one-body density: $P_1(r) \propto rac{1}{1+e^{(r-R)/a}}$ e.g. Woods-Saxon



Information from nuclear structure = radial profile + number of nucleons

However, nuclei display strong spatial correlations = shapes



Correlations from random rotation of an intrinsic deformed object

$$P_{1}(\mathbf{r}_{1}) = \int_{\Omega} \rho_{\Omega}(\mathbf{r}_{1})$$

$$P_{2}(\mathbf{r}_{1}, \mathbf{r}_{2}) = \int_{\Omega} \rho_{\Omega}(\mathbf{r}_{1})\rho_{\Omega}(\mathbf{r}_{2}) \neq P_{1}(\mathbf{r}_{1})P_{1}(\mathbf{r}_{2})$$







2 – Implementations of nuclear deformation in high energy collisions





Intrinsic one-body density from mean-field states

$$\rho(\mathbf{r}) = \sum_{s,t} \left. \left\langle \Phi(\vec{\beta}) \right| \left. a_{s,t}^{\dagger}(\mathbf{r}) a_{s,t}(\mathbf{r}') \right. \left| \Phi(\vec{\beta}) \right\rangle \right|_{\mathbf{r}=\mathbf{r}'}$$

[Bally *et al.*, PRL **128** (2022) 8, 082301] [Bally, Giacalone, Bender, EPJA **58** (2022) 9, 187] [Bally, Giacalone, Bender EPJA **59** (2023) 3, 58] [Ryssens *et al.*, PRL **130** (2023) 21, 212302]

Fit some appropriate function

$$p(r,\theta,\phi) \propto \frac{1}{1+\exp\left(\left[r-R(\theta,\phi)\right]/a\right)}, \ R(\theta,\phi) = R_0 \left[1+\frac{\beta_2}{2}\left(\cos\gamma Y_{20}(\theta)+\sin\gamma Y_{22}(\theta,\phi)\right)+\frac{\beta_3}{2}Y_{30}(\theta)+\frac{\beta_4}{2}Y_{40}(\theta)\right]$$
e.g. Woods-Saxon

 γ

20°

- 10°

 0°

Deformation of 129Xe at the LHC



Evidence of triaxiality in ¹²⁹Xe



Probing three-body correlations



NUCLEAR THREE-BODY DENSITY

$$P_{3}(\mathbf{r}_{1},\mathbf{r}_{2},\mathbf{r}_{3}) = \sum_{s,t} \int d^{3}\mathbf{r}_{4} \dots d^{3}\mathbf{r}_{A} |\Psi_{A}(\mathbf{r}_{1}\dots\mathbf{r}_{A})|^{2} \quad (\mathbf{r}_{1} \dots \mathbf{r}_{A})|^{2}$$

[Jia, PRC **105** (2022) 4, 044905] [Giacalone, EPJA **59** (2023) 12, 297]



MOMENTUM-ANISOTROPY CORRELATION

$$\rho_n \propto \operatorname{cov}([p_t], v_n^2) = \left\langle \frac{\sum_{i \neq j \neq k} (p_i - \langle [p_t]_{ev} \rangle) e^{in(\phi_j - \phi_k)}}{N_{ch, ev} (N_{ch, ev} - 1) (N_{ch, ev} - 2)} \right\rangle_{ev}$$

$$\rho_n \propto -\cos(3\gamma)\beta_2^3$$

Breakthrough of 2021: data from "isobar collisions" is released.



TALK BY J. JIA

X and Y are isobars.

X+X collisions produce QGP with same properties as Y+Y collisions.

Ratios of observables (O) should be unity...

 $\frac{\mathcal{O}_{X+X}}{\mathcal{O}_{Y+Y}} \stackrel{?}{=} 1$

[STAR collaboration, PRC **105** (2022) 1, 014901] [Giacalone, Jia, Somà, PRC **104** (2021) 4, L041903]

Departure from unity is mainly due to nuclear structure.

Extremely precise measurements.

Quadrupole and octupole deformations



[Giacalone, Jia, Zhang, PRL **127** (2021) 24, 242301] [Jia, PRC **105** (2022) 1, 014905] [Jia, Zhang, PRL **128** (2022) 2, 022301]







Octupole requires correlations "beyond mean field". A feature of the 2-body density.



But how do we sample it?

Evaluate mean field state at the maximum of g² / minimum of projected energy. Sample nucleons.

[Bally *et al.*, PRL **128** (2022) 8, 082301] [Bally, Giacalone, Bender EPJA **59** (2023) 3, 58]



Implementation from zero-point fluctuations

$$|\Psi(t)\rangle = |0\rangle + \sum_{n} c_{n} |n\rangle e^{-i\omega_{n}t}$$
$$\rho(\mathbf{r}, t) = \langle \Psi(t) | \sum_{i=1}^{A} \delta(\mathbf{r} - \mathbf{r}_{i}) | \Psi(t) \rangle$$

TALK BY P. RING

To leading order in the perturbation:

$$\rho(\mathbf{r},t) = \rho_0(\mathbf{r}) + \sum_n \left\{ c_n \delta \rho^n(\mathbf{r}) e^{-i\omega_n t} + \text{c.c.} \right\}$$

[Ring, Schuck, Nuclear Many Body Problem]

Transition densities computed within relativistic nuclear effective theory

Equations-of-Motion Method and the Extended Shell Model D. J. ROWE Rev. Mod. Phys. **40**, 153 – Published 1 January 1968 **Citing Articles (948)**

[Litvinova, Schuck, PRC 107 (2023) 2, 029903]

The dominant softness of 96 Ru is of the quadrupole character, with the lowest quadrupole state at ≈ 833 (880) keV in the experiment [9] (theory), which is well separated from the higher excited states. In 96 Zr, the lowest 2_1^+ and 3_1^- states which define the dominant fluctuation shape are nearly degenerate, appearing at the energies ≈ 1751 (1790) keV and ≈ 1897 (1990) keV





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Opportunities from O+O

1 – Variational Monte Carlo – Auxiliary Field Diffusion Monte Carlo (VMC-AFDMC)

MC solution of Schrödinger eq. from time evolution of trial wave function.

[Lonardoni *et al.*, PRC **97** (2018) 4, 044318] [Lim *et al.*, PRC **99** (2019) 4, 044904]

2 – Nuclear Lattice Effective Field Theory (NLEFT)

MC solution of Schrödinger eq. on a lattice.

[Lu *et al.*, PLB **797** (2019) 134863] [Summerfield *et al.*, PRC **104** (2021) 4, L041901]

sampled nucleons include up to A-body correlations



3 - ab initio Projected Generator Coordinate Method (ab initio PGCM)

Wave function from variational calculation (as in density functional theory).

Provides a deformed density.

$$\delta \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0$$

TALK BY U. MEISSNER

[Frosini *et al.*, EPJA **58** (2022) 4, 62 EPJA **58** (2022) 4, 63 EPJA **58** (2022) 4, 64] TALK BY B. BALLY



Deformations?





VMC has strongest clustering/short-range correlations

[Zhang et al., 2404.08385]

[Broniowski, Rybczyński, PRC 100 (2019) 6, 064912]



Oxygen-oxygen collisions will discern models of clustering and short-range correlations.

3 – Studying and exploiting nuclear deformation at high energy



EIC (or UPC) program – Same paradigm?

[Mäntysaari et al., PRL 131 (2023) 6, 062301]



[Caldwell, Kowalski, PRC **81** (2010) 025203] [Giacalone, EPJA **59** (2023) 12, 297]





Consistency of nuclear phenomena across scales

BAYESIAN ANALYSIS
$$\begin{cases} \Pr(p \& D) = \Pr(p) \times \Pr(D|p) = \Pr(D) \times \Pr(p|D) \\ \text{prior} \times \text{likelihood} = \text{evidence} \times \text{post} \end{cases}$$

[e.g. Paquet, arXiv:2310.17618]

posterior

Beautiful example and opportunity: The neutron distribution of ²⁰⁸Pb is poorly known.

$$\rho(r) \propto \frac{1}{1 + e^{(r-R)/\underline{a}}}$$

Protons: density from low-energy scattering. [Zenihiro *et al.*, PRC 82 (2010) 044611]

Neutrons: same R as protons, infer *a* from LHC data.



Correlation between nuclear properties and initial-state parameters

TALK BY G. NIJS





[PREX Collaboration, PRL **126** (2021) 17, 172502] [Hu *et al.*, Nature Phys. 18 (2022) 10, 1196-1200]

Generalize to deformation parameters - Refining initial-state model

CERN-TH-2024-021 **The unexpected uses of a bowling pin: exploiting** ²⁰Ne isotopes for precision characterizations of collectivity in small systems <u>Giuliano Giacalone</u>,^{1,*} <u>Benjamin Bally</u>,² <u>Govert Nijs</u>,³ <u>Shihang Shen</u>,⁴ <u>Thomas Duguet</u>,^{5,6} <u>Jean-Paul Ebran</u>,^{7,8} <u>Serdar Elhatisari</u>,^{9,10} <u>Mikael Frosini</u>,¹¹ <u>Timo A. Lähde</u>,^{12,13} <u>Dean Lee,¹⁴ <u>Bing-Nan Lu</u>,¹⁵ <u>Yuan-Zhuo Ma</u>,¹⁴ <u>Ulf-G. Meißner</u>,^{10,16,17} <u>Jacquelyn Noronha-Hostler</u>,¹⁸ <u>Christopher Plumberg</u>,¹⁹ <u>Tomás R. Rodríguez</u>,²⁰ <u>Robert Roth</u>,^{21,22} <u>Wilke van der Schee</u>,^{3,23,24} and <u>Vittorio Somà⁵</sub></u></u>





Ancillary files (details):

- NLEFT_dmin_0.5fm_negativeweights_Ne.h5
- NLEFT_dmin_0.5fm_negativeweights_O.h5
- NLEFT_dmin_0.5fm_positiveweights_Ne.h5
- NLEFT_dmin_0.5fm_positiveweights_0.h5
- PGCM_clustered_dmin0_Ne.h5
- PGCM_clustered_dmin0_O.h5
- PGCM_uniform_dmin0_Ne.h5
- PGCM_uniform_dmin0_O.h5

Error cancellations – Quantitative predictions for a small system





Theory with reliable systematic error:



TALK BY G. NIJS

Systematics are from hydrodynamics. Ideal to test physics beyond "standard model".

The unexpected uses of a bowling pin – SMOG2

TALK BY G. GRAZIANI



For deformed nuclei, flat eccentricity up to ~20% centrality





High-energy collisions and the quest for nuclear interactions

TALKS BY K. GODBEY, X. ZHANG



[Giacalone, Nijs, van der Schee, PRL 131 (2023) 20, 20]



Reconstructing the strong nuclear force

$$H_{\mathrm{SU}(4)} = H_{\mathrm{free}} + \frac{1}{2!}C_2 \sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^2 + \frac{1}{3!}C_3 \sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^3$$

$$\tilde{\rho}(\mathbf{n}) = \sum_{i} \tilde{a}_{i}^{\dagger}(\mathbf{n}) \tilde{a}_{i}(\mathbf{n}) + \sum_{|\mathbf{n}'-\mathbf{n}|=1} \sum_{i} \tilde{a}_{i}^{\dagger}(\mathbf{n}') \tilde{a}_{i}(\mathbf{n}')$$

$$\tilde{a}_i(\mathbf{n}) = a_i(\mathbf{n}) + \sum_{|\mathbf{n}'-\mathbf{n}|=1} a_i(\mathbf{n}')$$

Parameters can be extracted from LHC and RHIC data on 160 collisions

Is it important? Different nuclei offering different sensitivities?

