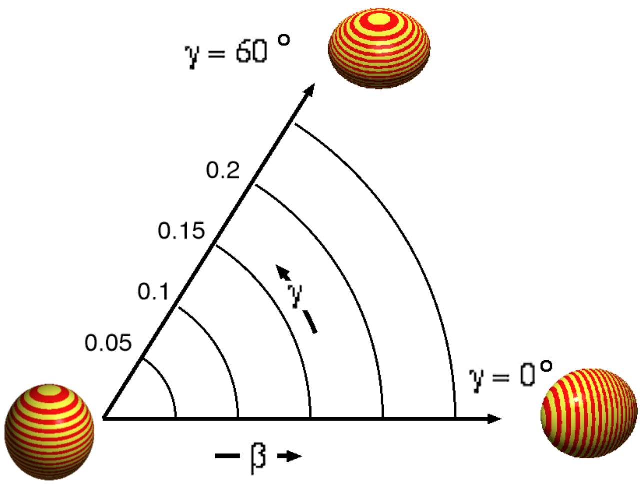
Overview of nuclear deformation and shape coexistence around ^{96}Zr and ^{96}Ru

- overall quadrupole deformation and shape coexistence
- triaxiality
- octupole collectivity

Nuclear shapes

- general description of a shape:
  $R(\theta, \phi) = R_0 \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} a_{\lambda,\mu} Y_{\lambda\mu}(\theta, \phi)\right]$
- important nuclear shapes:
  - $a_{2,\mu}$ quadrupole deformation (triaxial ellipsoid)
  - $a_{3,\mu}$ octupole deformation (pear shape)
- in the principal axes frame $a_{2,1} = a_{2,-1} = 0$ and only two parameters are enough to describe all possible quadrupole shapes:

$$a_{2,0} = \beta \cos \gamma$$
$$a_{2,2} = a_{2,-2} = \frac{\beta \sin \gamma}{\sqrt{2}}$$



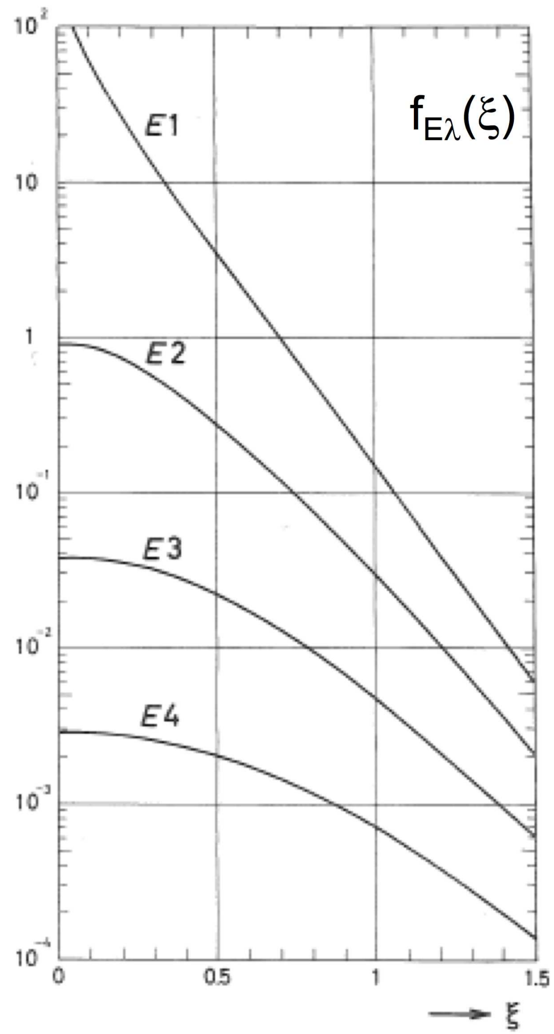
What observables are related to nuclear shapes?

- differences in root mean square charge radii (determined via laser spectroscopy for ground and isomeric states)
- level energies
  - energy of the first 2^+ state: the simplest measure of collectivity
- transition probabilities: $B(E2; 0^+ \rightarrow 2^+) = ((3/4\pi)eZR_0^2)^2\ {\beta_2}^2$
- quadrupole moments: measure of the charge distribution in a given state (always zero for spin 0 and 1/2, even if there is non-zero intrinsic deformation)
  - laser spectroscopy for long-lived states
  - reorientation effect in Coulomb excitation for short-lived states: influence of the quadrupole moment of an excited state on its excitation cross section
- deformation lengths from inelastic scattering: need for accurate potentials to describe the nuclear interaction between collision partners
- complete sets of E2 matrix elements:
  possibility to determine quadrupole invariants and level mixing
- monopole transition strengths: enhancements observed for shape coexistence with strong mixing

Coulomb excitation cross sections

Dependence on:

- strength of the electromagnetic field: atomic number of the collision partner
- beam energy
- difference in excitation energy between the initial and final levels
- scattering angle
- transition probabilities
- transition multipolarities
  - E2 excitation dominates, followed by E3; other multipolarities (including magnetic transitions) usually negligible in low-energy Coulomb-excitation process



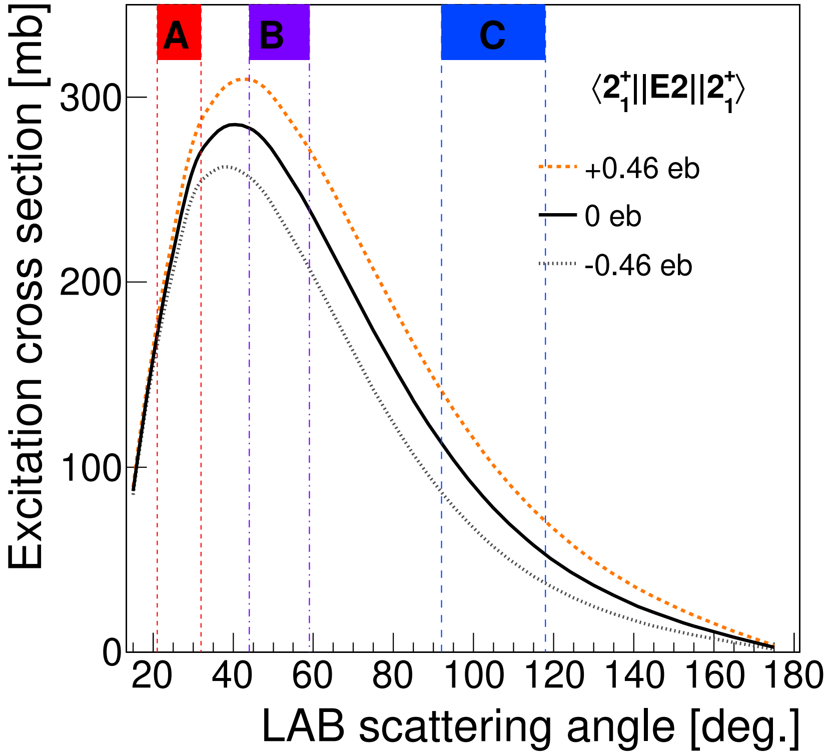
first perturbation: $\sigma = \left(\frac{Z_1 e}{\hbar v}\right)^2 a^{-2(\lambda+1)} B(E\lambda) \cdot f_{E\lambda}(\xi)$

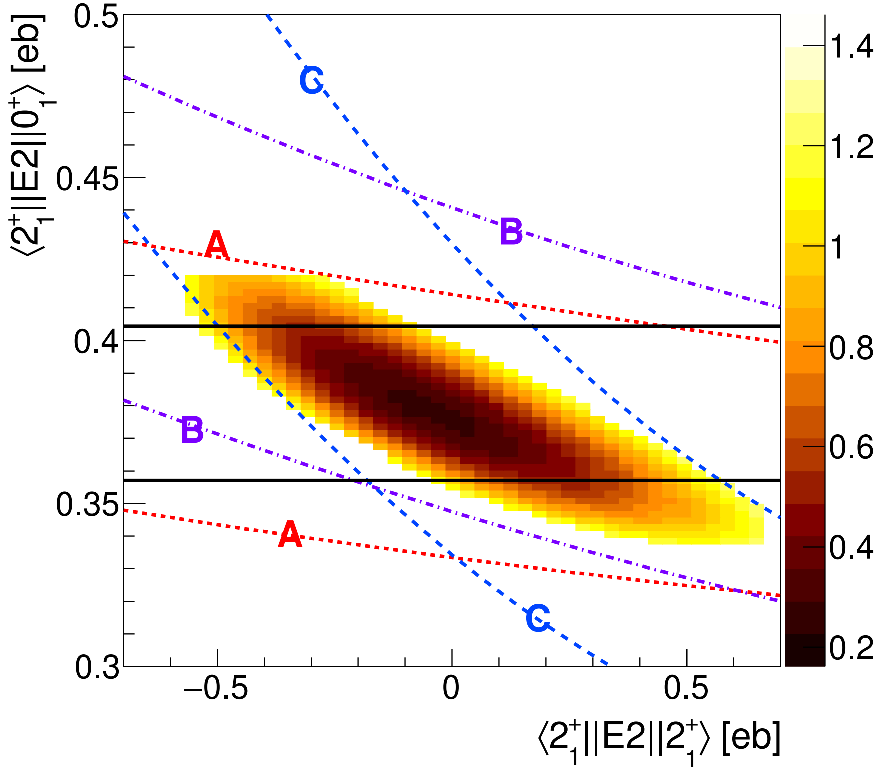
with adiabacity parameter $\xi = \frac{\Delta E a}{\hbar v}$

Measuring quadrupole moments of excited states

- reorientation effect: influence of the quadrupole moment on the excitation cross section

^{76}Zn, HIE-ISOLDE data from: A. Illana, MZ *et al.*, PRC 108, 044305 (2023)





- χ^2 comparison of measured cross sections with calculated ones
- independent lifetime measurements increase precision of extracted quadrupole moments

Quadrupole sum rules

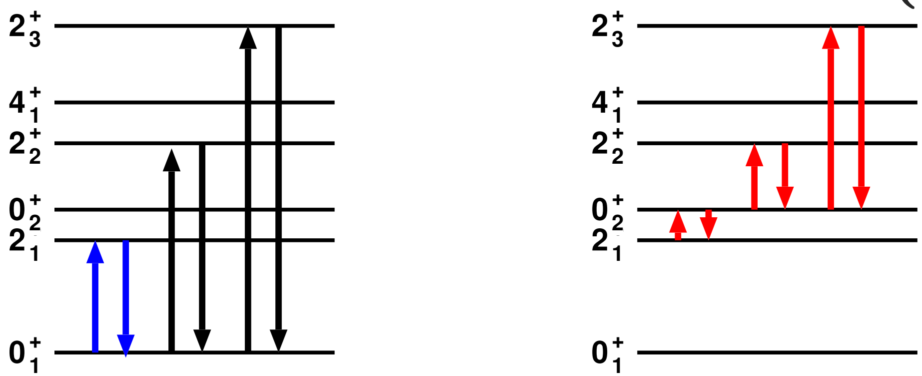
D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683
K. Kumar, PRL 28 (1972) 249

- electromagnetic multipole operators are spherical tensors – products of such operators coupled to angular momentum 0 are rotationally invariant

- in the intrinsic frame of the nucleus, the E2 operator may be expressed using two parameters Q and δ related to charge distribution:

$$
\begin{aligned}
E(2,0) &= Q\cos\delta \\
E(2,2) = E(2,-2) &= \frac{Q}{\sqrt{2}}\sin\delta \\
E(2,1) = E(2,-1) &= 0
\end{aligned}
$$

$$\frac{\langle Q^2\rangle}{\sqrt{5}} = \langle i|\,[E2\times E2]^0\,|i\rangle = \frac{1}{\sqrt{(2I_i+1)}}\sum_t \langle i||E2||t\rangle\langle t||E2||i\rangle \begin{Bmatrix} 2 & 2 & 0 \\ I_i & I_i & I_t \end{Bmatrix}$$



$\langle Q^2\rangle$: measure of the overall deformation;
for the ground state – extension of $B(E2; 0^+ \to 2^+) = ((3/4\pi)eZR_0^2)^2\ \beta_2{}^2$

Contributions to $\langle Q^2\rangle$ in ^{100}Mo: K. Wrzosek-Lipska *et al.*, PRC 86 (2012) 064305

$\langle Q^2 \rangle$ for ^{96}Zr and ^{96}Ru ground states

- Extensive lifetime measurements for low-spin states in ^{96}Zr and ^{96}Ru:
- ^{96}Zr: (n,n'γ) + (e,e') for 2_2^+; ^{96}Ru: (p,p'γ), (^{3}He,2nγ)

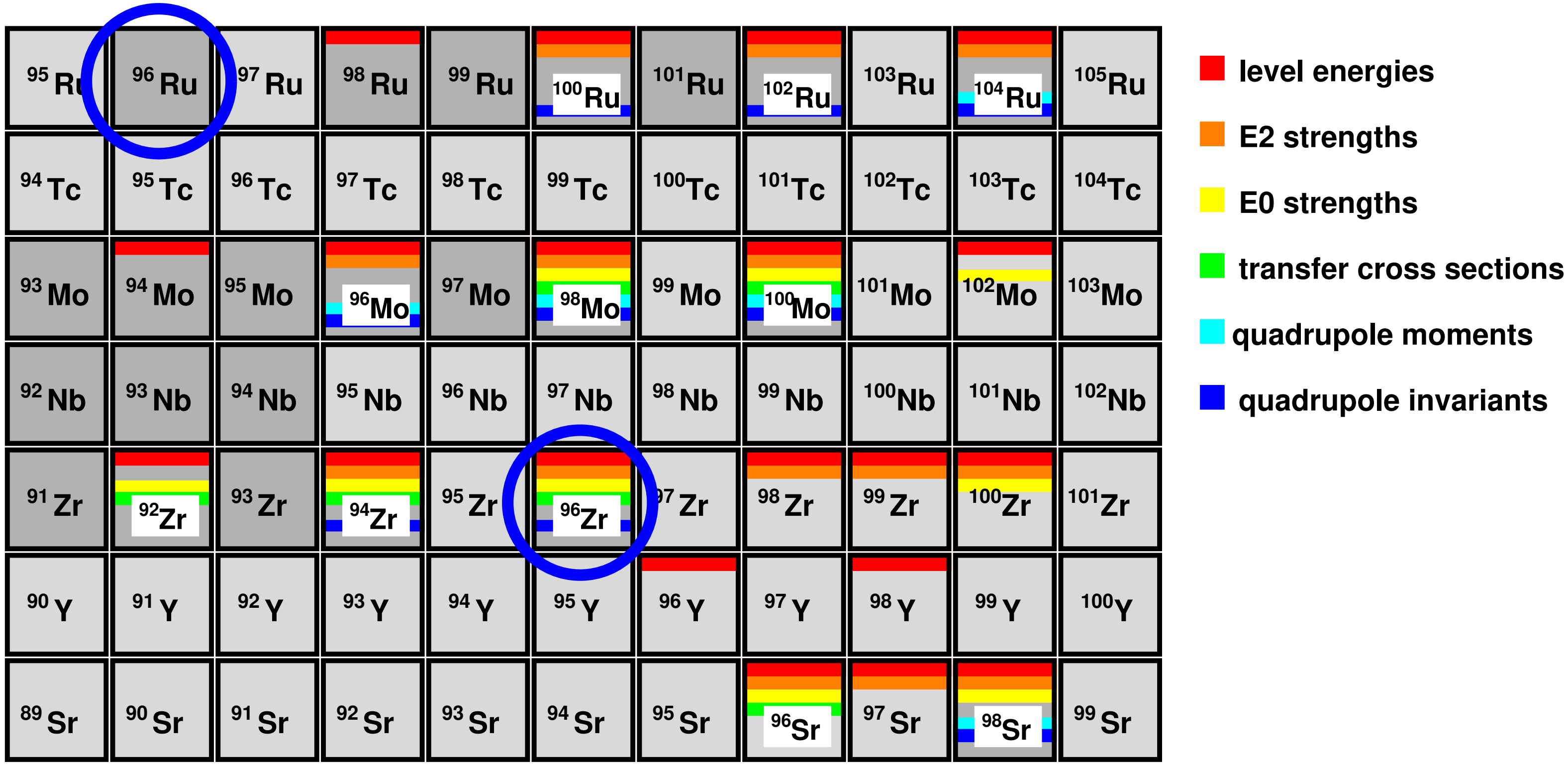
- ^{96}Zr:
  - B(E2; $2_1^+ \rightarrow 0_1^+$) = 2.3(3) W.u. $\rightarrow \langle 2_1^+ \| E2 \| 0_1^+ \rangle$ = 0.173(11) eb
  - B(E2; $2_2^+ \rightarrow 0_1^+$) = 0.26(8) W.u. $\rightarrow \langle 2_2^+ \| E2 \| 0_1^+ \rangle$ = 0.058(9) eb
  - $\langle Q^2 \rangle$ = 0.033(5)e^{2}b^2, β=0.06(1)

  ^{96}Ru:
  - B(E2; $2_1^+ \rightarrow 0_1^+$) = 18.4(4) W.u. $\rightarrow \langle 2_1^+ \| E2 \| 0_1^+ \rangle$ = 0.490(5) eb
  - B(E2; $2_2^+ \rightarrow 0_1^+$) = 0.16(4) W.u. $\rightarrow \langle 2_2^+ \| E2 \| 0_1^+ \rangle$ = 0.050(6) eb
  - $\langle Q^2 \rangle$ = 0.243(6)e^{2}b^2, β=0.155(4)

- $\langle Q^2 \rangle = q_0^2 \langle \beta_2^2 \rangle$; $q_0 = \frac{3}{4\pi} ZeR_0^2$ and $R_0 = 1.2A^{1/3}$ fm
- includes both dynamic and static deformation and assumes that mass and charge distributions are the same
- errors in ENSDF for ^{96}Ru: wrong B(E2; $2_2^+ \rightarrow 0_1^+$)=35 W.u, 2_4^+ lifetime 0.15 fs, 15 fs (it is 0.15 ps)

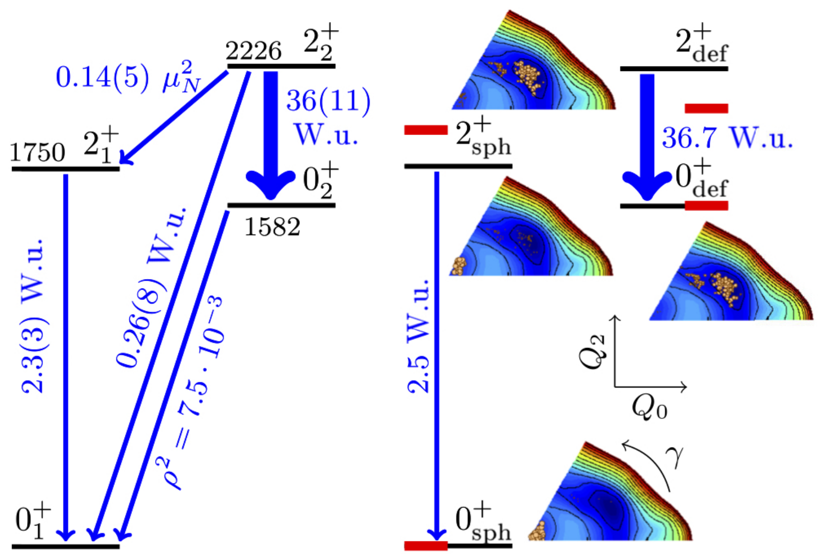
Shape coexistence: experimental information for A $\approx$ 100

- dramatic increase of ground-state deformation at N=60
- multitude of coexisting shapes predicted by theory

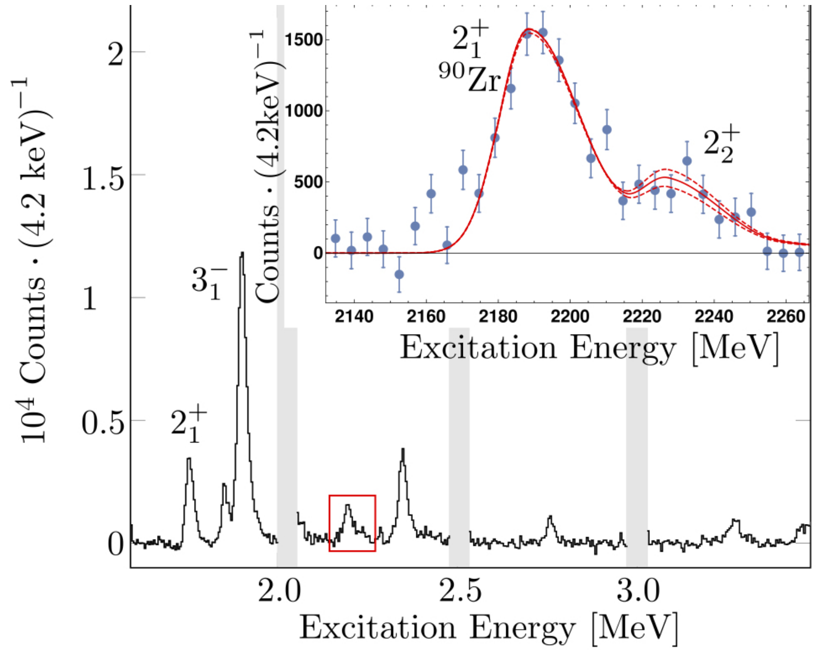


P. Garrett, MZ, E. Clément, Prog. Part, Nucl. Phys. 124, 123931 (2022)

Shape coexistence in ^{96}Zr – experimental information

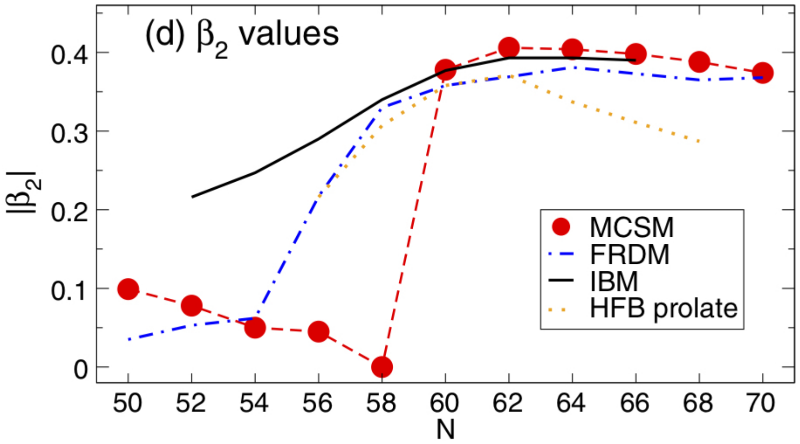


S. Kremer et al, Phys. Rev. Lett. 117 (2017) 172503



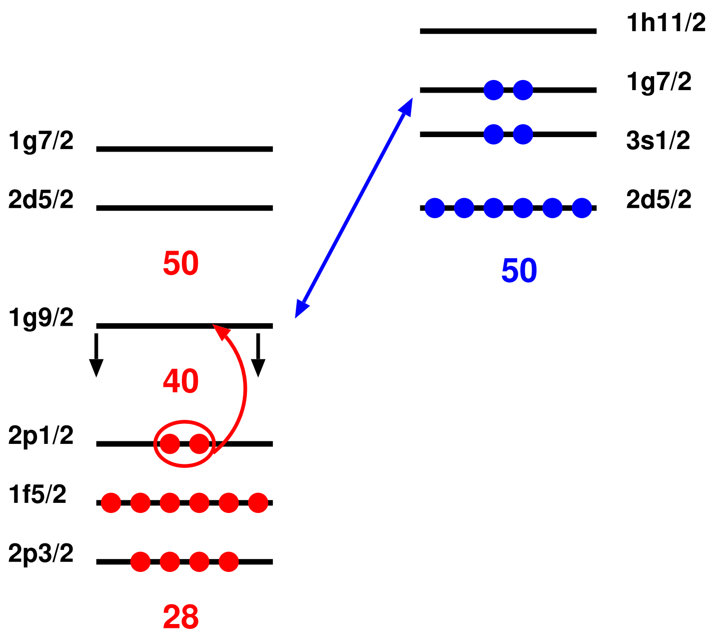
- B(E2; $2_2^+ \rightarrow 0_1^+$) measured using electron scattering, combined with known branching and mixing ratios:
  $\rightarrow$transition strengths from the 2_2^+ state
- B(E2; $2_1^+ \rightarrow 0_1^+$) = 2.3(3) Wu vs B(E2; $2_2^+ \rightarrow 0_2^+$) = 36(11) Wu: nearly spherical and a well-deformed structure ($\beta \approx 0.24$)
- very low mixing of coexisting structures: cos$^2\theta_0$=99.8%, cos$^2\theta_2$=97.5%,

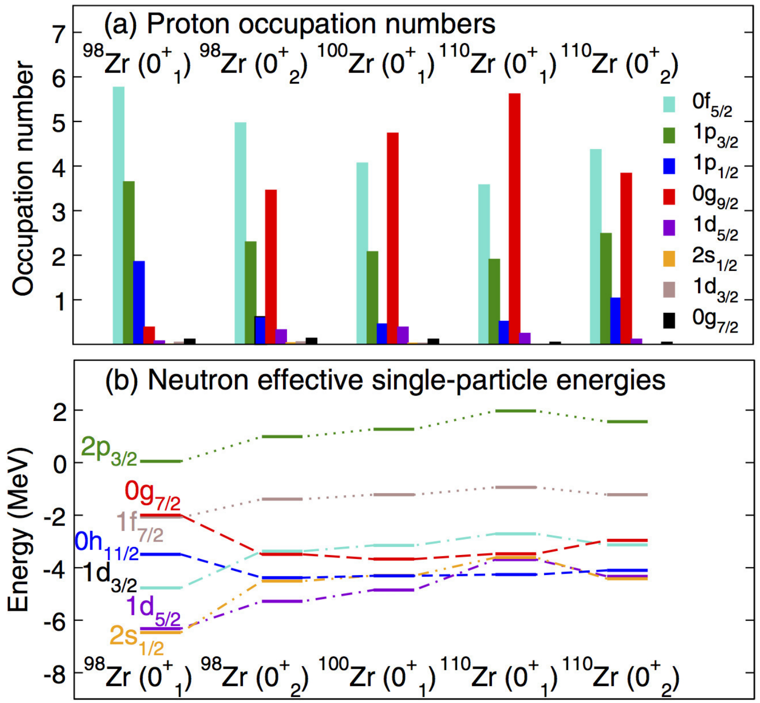
Shape coexistence and type-II shell evolution in Zr isotopes



T. Togashi et al, PRL 117, 172502 (2016)

- p-n tensor interaction reduces the Z=40 gap when $\nu g_{7/2}$ is being filled
- 0_2^+ states created by 2p-2h (+ 4p-4h...) excitation across Z=40
- very different configurations and small mixing of 0_1^+ and 0_2^+

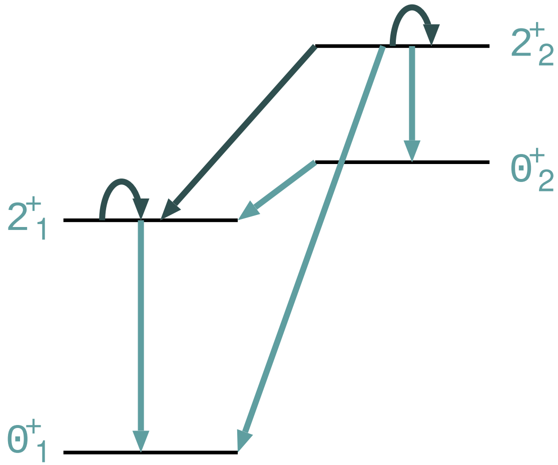




Two-state mixing model

- we assume that physical states are linear combinations of pure spherical and deformed configurations:

$| I_1^+ \rangle$ = +$\cos\theta_I \times | I_d^+ \rangle$ + $\sin\theta_I \times | I_s^+ \rangle$
$| I_2^+ \rangle$ = -$\sin\theta_I \times | I_d^+ \rangle$ + $\cos\theta_I \times | I_s^+ \rangle$



with transitions between the pure spherical and deformed states forbidden:

$\langle 2_d^+ \| E2 \| 0_s^+ \rangle$ = $\langle 2_d^+ \| E2 \| 2_s^+ \rangle$ = $\langle 2_s^+ \| E2 \| 0_d^+ \rangle$ = 0

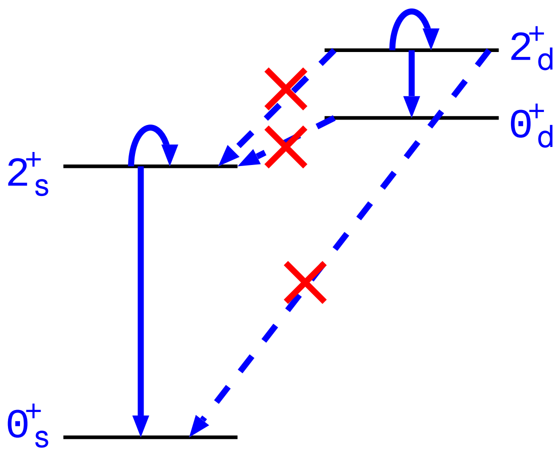
- the measured matrix elements can be expressed in terms of the "pure" matrix elements and the mixing angles:

$\langle 2_1^+ \| E2 \| 0_1^+ \rangle$ =
$\sin\theta_0 \sin\theta_2 \langle 2_s^+ \| E2 \| 0_s^+ \rangle + \cos\theta_0 \cos\theta_2 \langle 2_d^+ \| E2 \| 0_d^+ \rangle$

$\langle 2_1^+ \| E2 \| 0_2^+ \rangle$ =
$\cos\theta_0 \sin\theta_2 \langle 2_s^+ \| E2 \| 0_s^+ \rangle - \sin\theta_0 \cos\theta_2 \langle 2_d^+ \| E2 \| 0_d^+ \rangle$

$\langle 2_2^+ \| E2 \| 0_1^+ \rangle$ =
$\sin\theta_0 \cos\theta_2 \langle 2_s^+ \| E2 \| 0_s^+ \rangle - \cos\theta_0 \sin\theta_2 \langle 2_d^+ \| E2 \| 0_d^+ \rangle$

$\langle 2_2^+ \| E2 \| 0_2^+ \rangle$ =
$\cos\theta_0 \cos\theta_2 \langle 2_s^+ \| E2 \| 0_s^+ \rangle + \sin\theta_0 \sin\theta_2 \langle 2_d^+ \| E2 \| 0_d^+ \rangle$



E0 strengths, shape coexistence and mixing

- E0 transitions are sensitive to the changes in the nuclear charge-squared radii

- their strengths depends on the mixing of configurations that have different mean-square charge radii:

$$\rho^2(E0) = \frac{Z^2}{R^4}\cos^2\theta_0\ \sin^2\theta_0\ \left(\langle r^2\rangle_A - \langle r^2\rangle_B\right)^2$$

$$= \left(\frac{3Z}{4\pi}\right)^2 \cos^2(\theta_0)\sin^2(\theta_0)\cdot\left[\left(\beta_1^2-\beta_2^2\right)+\frac{5\sqrt{5}}{21\sqrt{\pi}}\left(\beta_1^3\cos\gamma_1-\beta_2^3\cos\gamma_2\right)\right]^2$$

J.L. Wood *et al.*, NPA 651, 323 (1999)

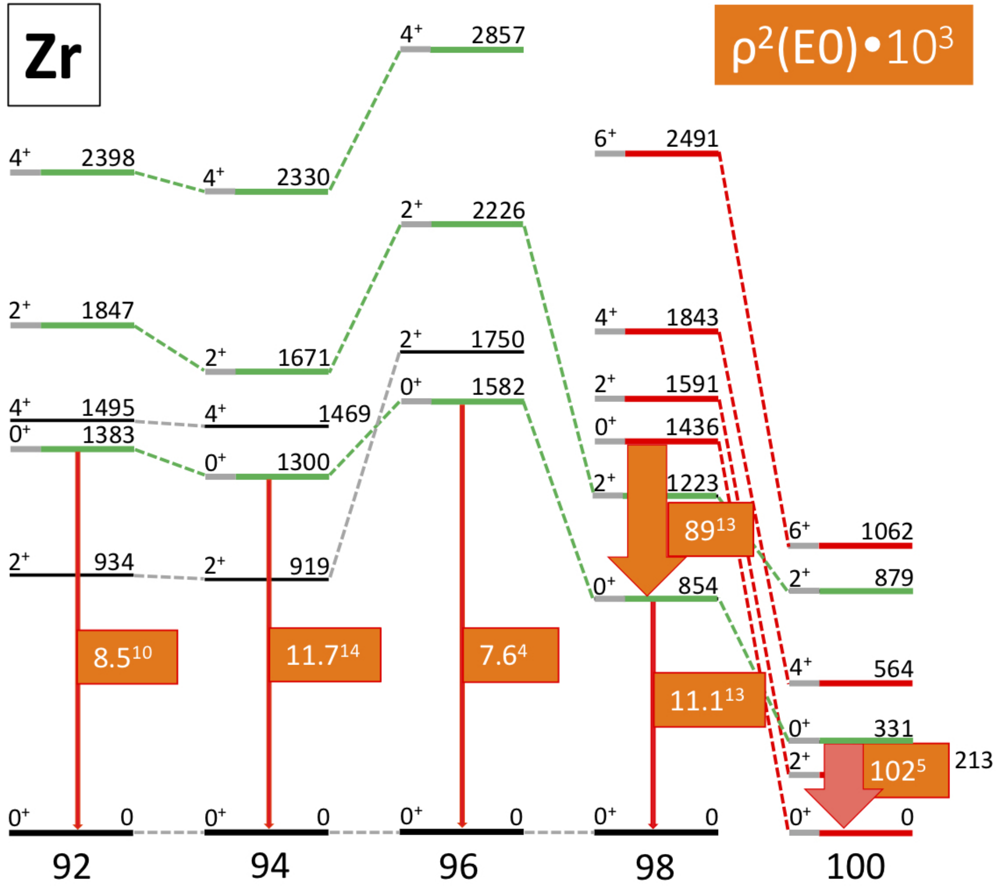
Example of ^{42}Ca: K. Hadyńska-Klęk *et al.*, PRC 97 (2018) 024326 (Coulomb excitation), J.L. Wood *et al.*, NPA 651, 323 (1999) (E0)

	from E2 matrix elements [KHK]	from $\rho^2(E0)$ [JLW] + sum rules results [KHK]
$\cos^2(\theta_0)$	0.88(4)	0.84(4)

- good agreement of the $\cos^2(\theta_0)$ values obtained with the two methods

E0 strengths in Zr and Ru isotopes

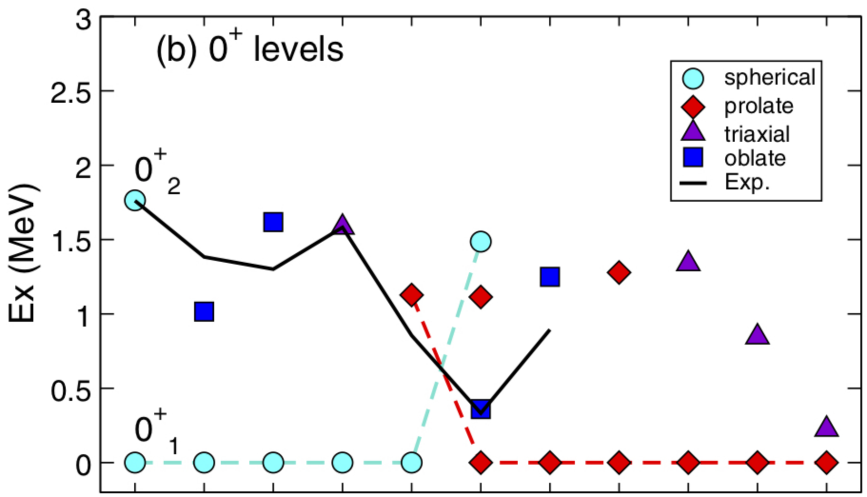
T. Kibedi *et al.*, Prog. Part. Nucl. Phys. 120 (2021)



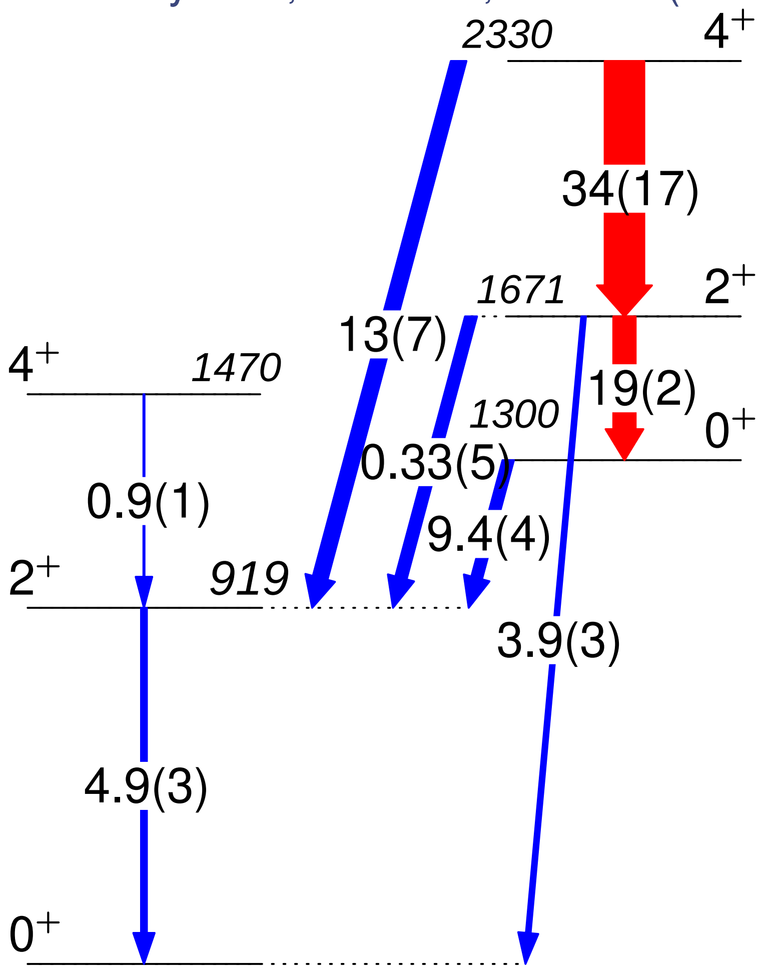
- ^{100}Ru: 11(2) 10^{-3} between 0_2^+ and 0_2^+, no data for lighter Ru isotopes

Shape coexistence in ^{94}Zr

A. Chakraborty et al, PRL 110, 022504 (2013)



T. Togashi et al, PRL 117, 172502 (2016)

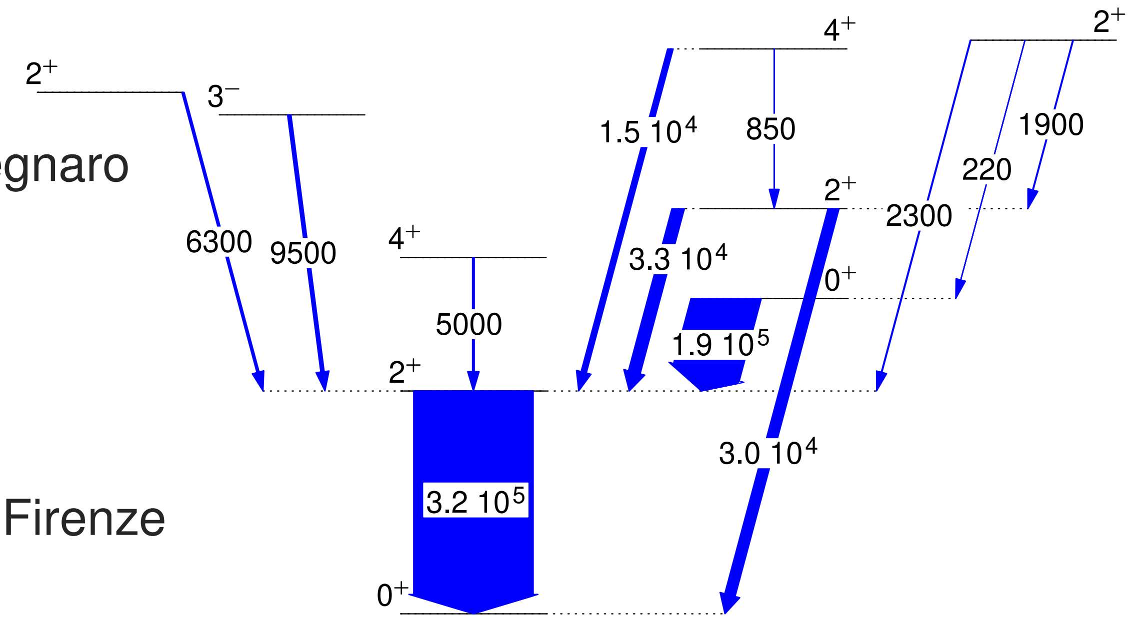


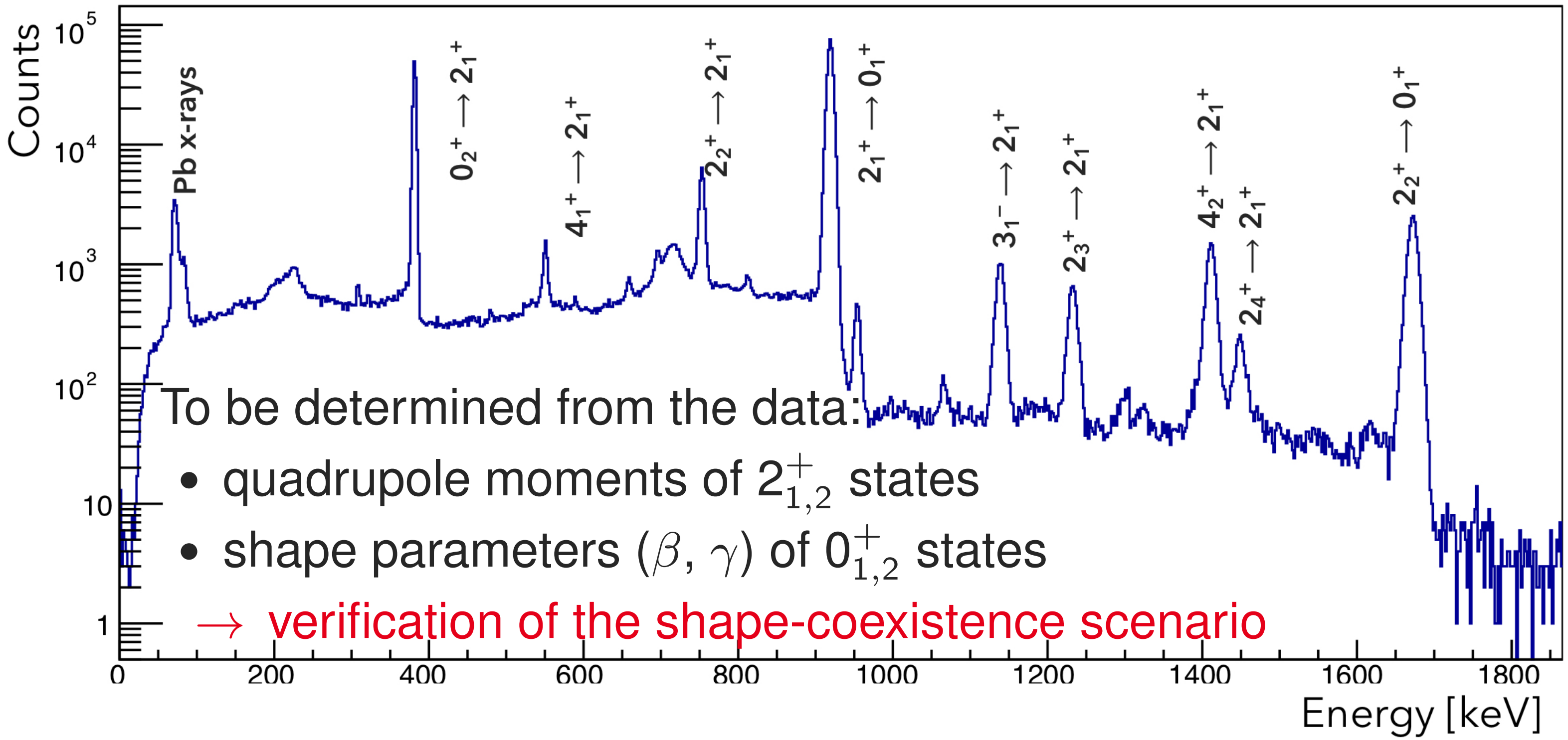
- MCSM calculations suggest a variety of shapes appearing at low excitation energy in Zr nuclei
- ^{94}Zr selected as the first candidate for a detailed experimental investigation
- oblate deformed structure predicted to be built on the 0_2^+ state

- high-statistics β-decay study at TRIUMF: observation of a strong $2_2^+ \rightarrow 0_2^+$ transition (19 W.u.) – a deformed band built on 0_2^+

Coulomb excitation of ^{94}Zr

- experiment performed at LNL Legnaro (March 2018)
- GALILEO + SPIDER
- ^{94}Zr beam on ^{208}Pb target
- analysis: Naomi Marchini, INFN Firenze





To be determined from the data:

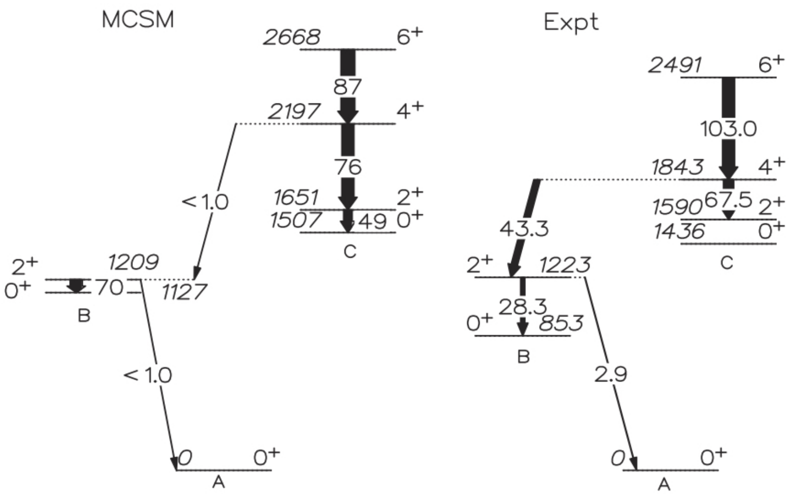
- quadrupole moments of $2^+_{1,2}$ states
- shape parameters (β, γ) of $0^+_{1,2}$ states

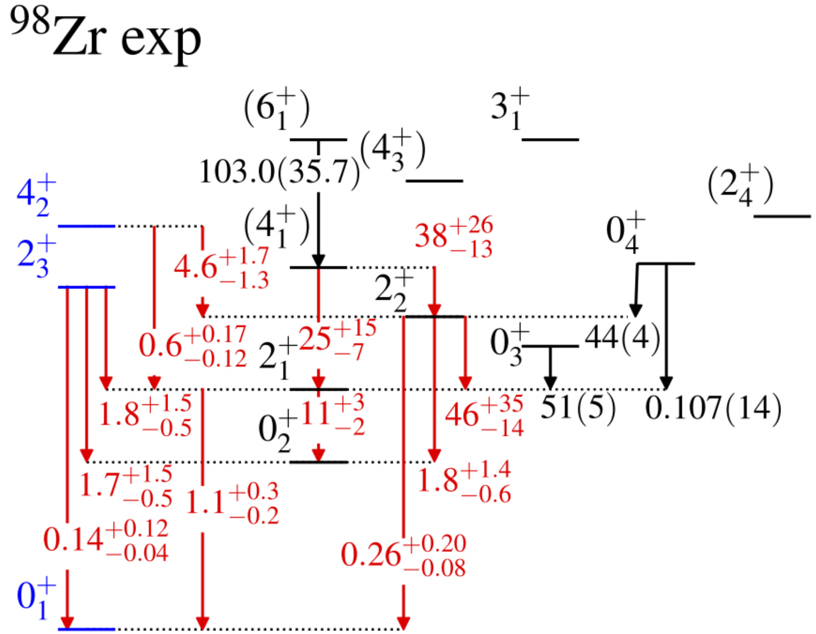
$\rightarrow$ verification of the shape-coexistence scenario

Lifetime measurements in ^{98}Zr

- Lifetimes measured in ^{9}Be induced fission of ^{238}U, and ^{96}Zr+^{18}O 2p transfer

P. Singh et al., PRL 121, 192501 (2018) V. Karayonchev et al., PRC 102, 064314 (2020)

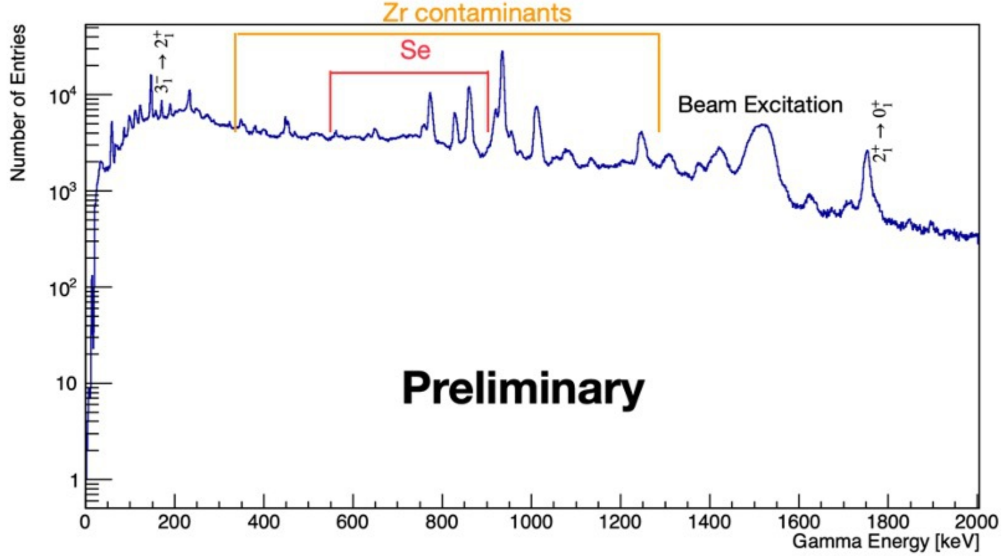




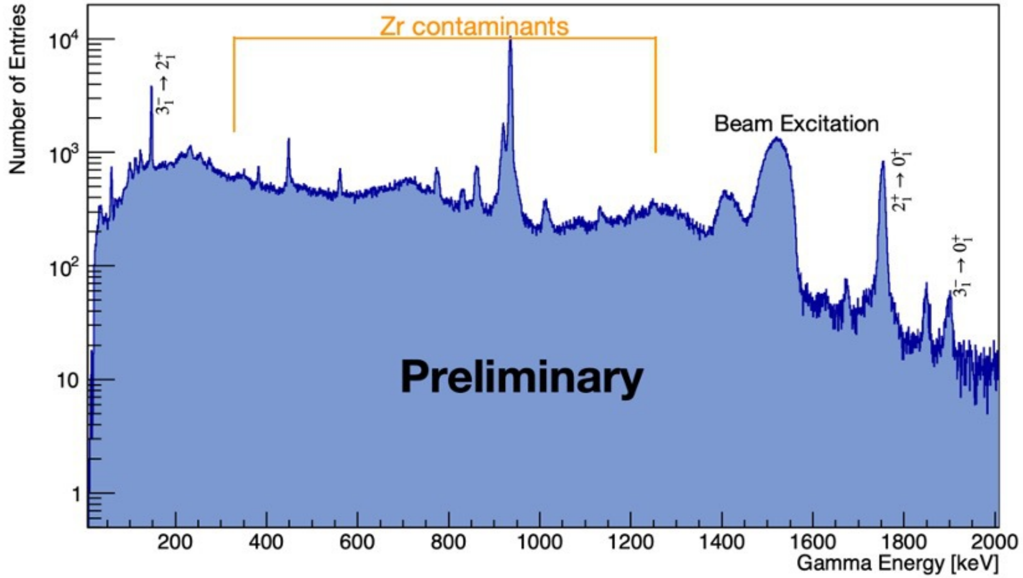
- substantial differences in measured lifetimes and interpretations
- $2_2^+ \rightarrow 0_3^+$ is expected to be either enhanced in-band transition, or a forbidden three- to two-phonon transition
- combination of 2_2^+ lifetime and branching ratio points to an unphysical value of 500 W.u.
- β-decay data from TRIUMF (under analysis) expected to resolve this issue

Coulomb excitation of ^{96}Zr with AGATA at LNL

- problems to get the ^{96}Zr material for the targets due to the Russia-Ukraine war; obtained targets with lower isotopic enrichement than reported



- data analysis: cut on excitation energy to remove the fusion-evaporation background



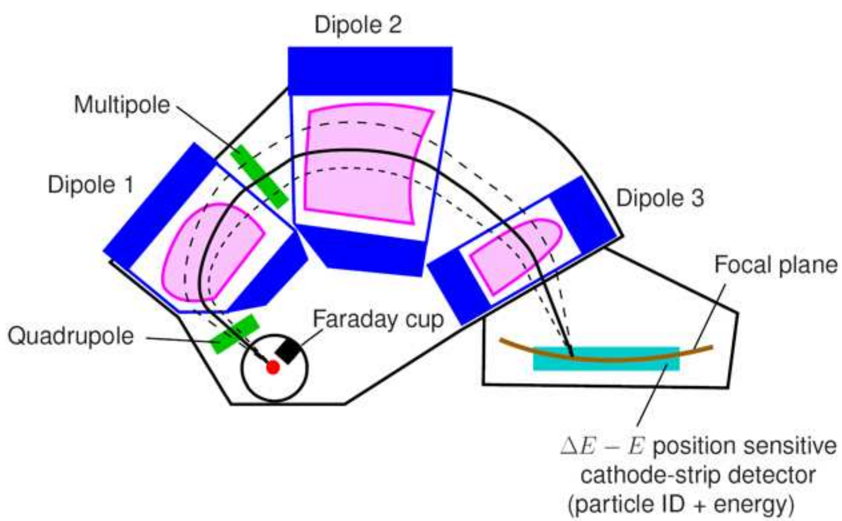
- analysis in progress (N. Marchini, F. Ercolano)

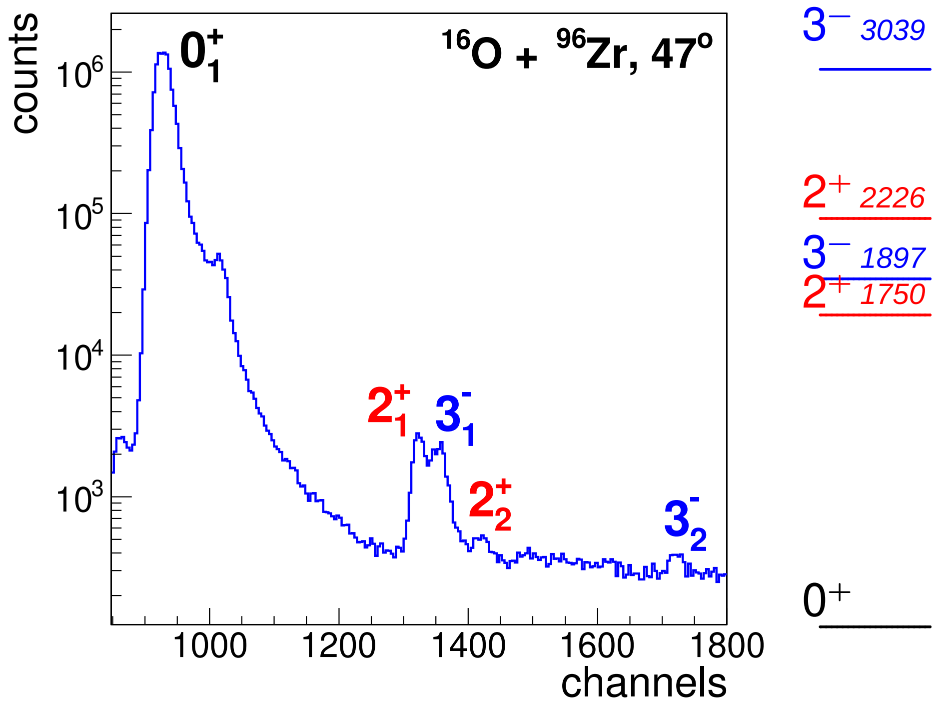
- aim: extraction of quadrupole moments in ^{96}Zr

Coulomb excitation with the Q3D spectrometer

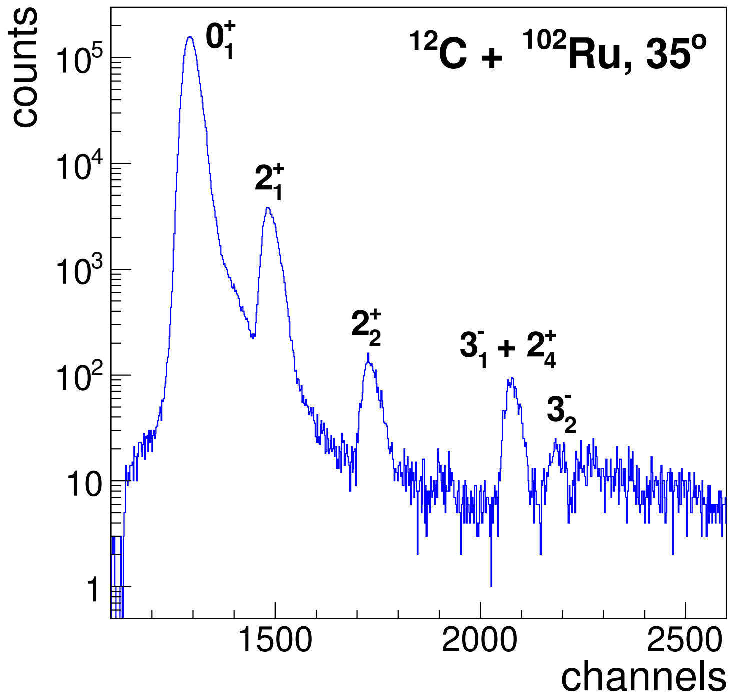
- Coulomb-excitation measurements with magnetic spectrometers common in 1970s, but completely abandoned in favour of γ-ray spectroscopy
- still a very attractive option, especially to populate higher-lying low-spin states: very high beam intensities (100 pnA) can compensate for low cross sections
- campaigns with ^{12}C, ^{16}O beams: direct measurement of 2^+ and 3^- population $\rightarrow$ precise B(E2; $2^+_i \rightarrow 0^+_1$) and B(E3; $3^-_i \rightarrow 0^+_1$) values

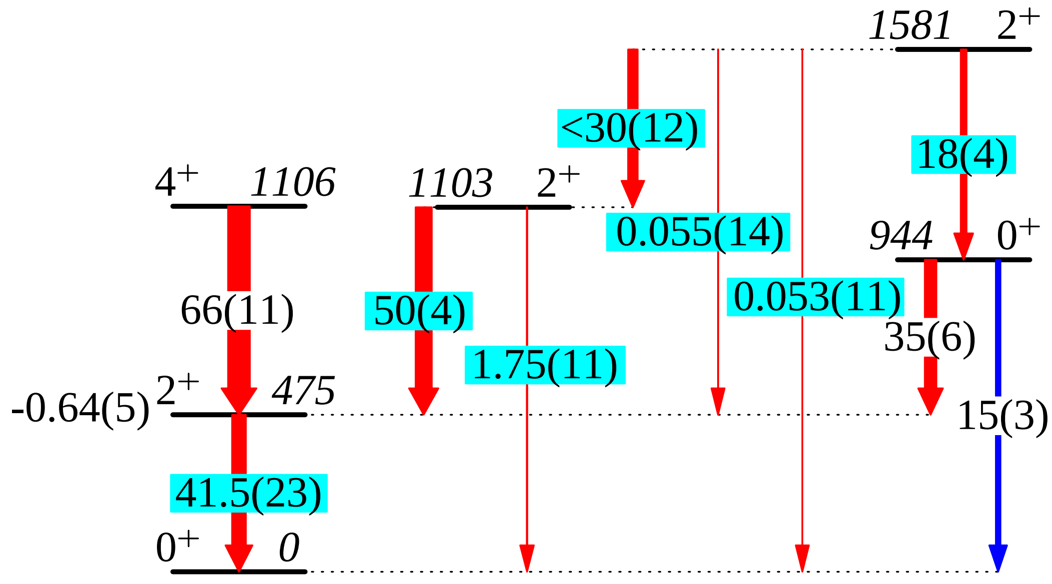
Q3D magnetic spectrometer, MLL





Results: shape coexistence in ^{102}Ru

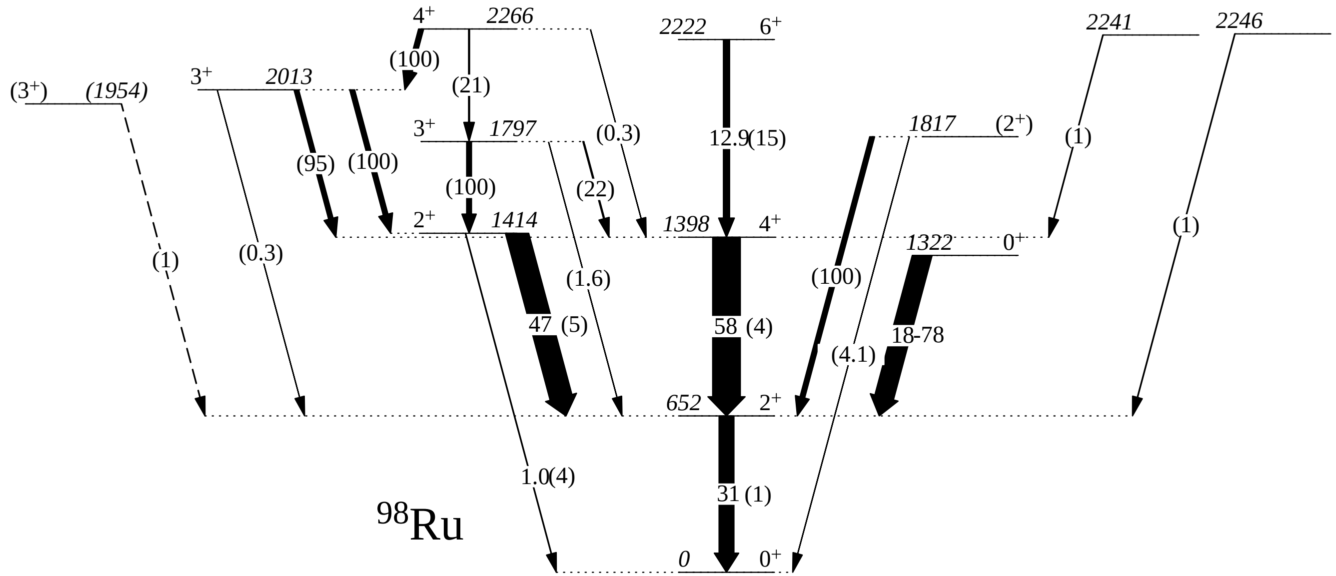




P. Garrett, MZ et al, PRC 106, 064307 (2022)

- first measurement of the B(E2; $2_3^+ \rightarrow 0_1^+$) value
- combined with known branching ratios yields B(E2) values in the two bands differing by a factor of 2
- coexistence of two structures with different overall deformation ($\beta \approx$ 0.24 and $\beta \approx$ 0.18)

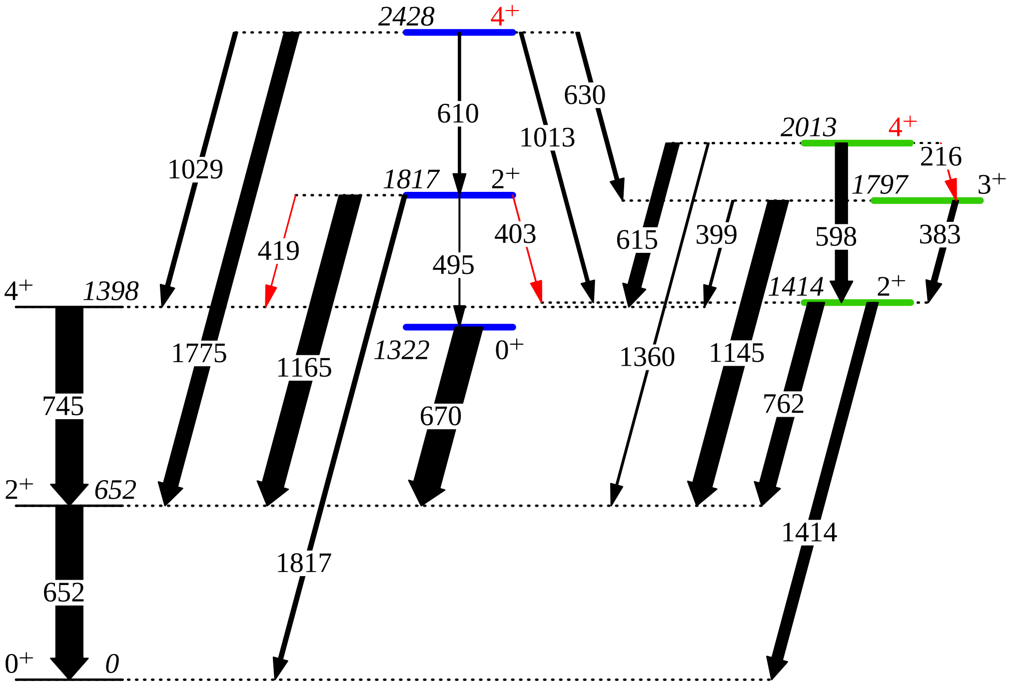
^{98}Ru level scheme a few years ago

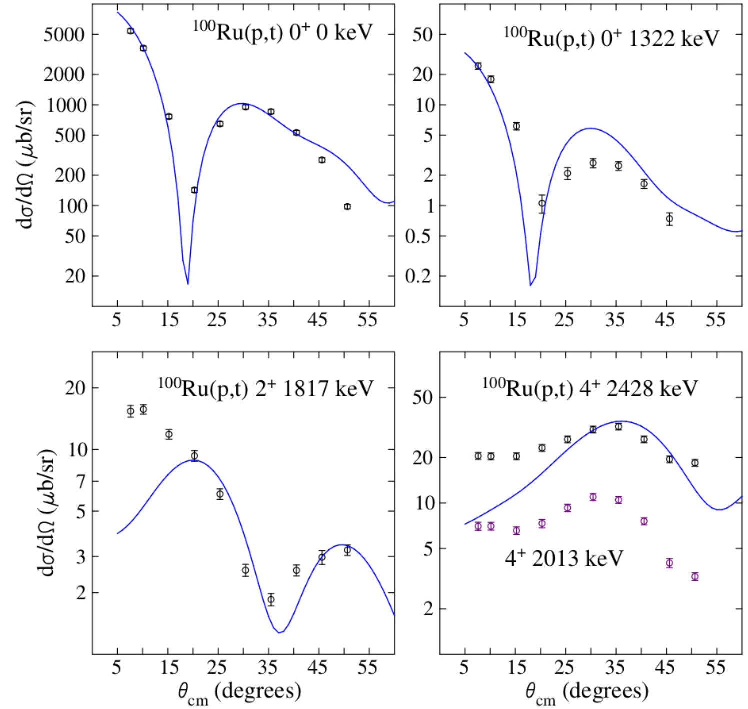


- highly unlikely that there are three closely-lying 3^+ states
- level scheme incomplete with missing decays and spin assignments

Reevaluation of ^{98}Ru level scheme

P. Garrett et al., PLB 809, 135762 (2020)



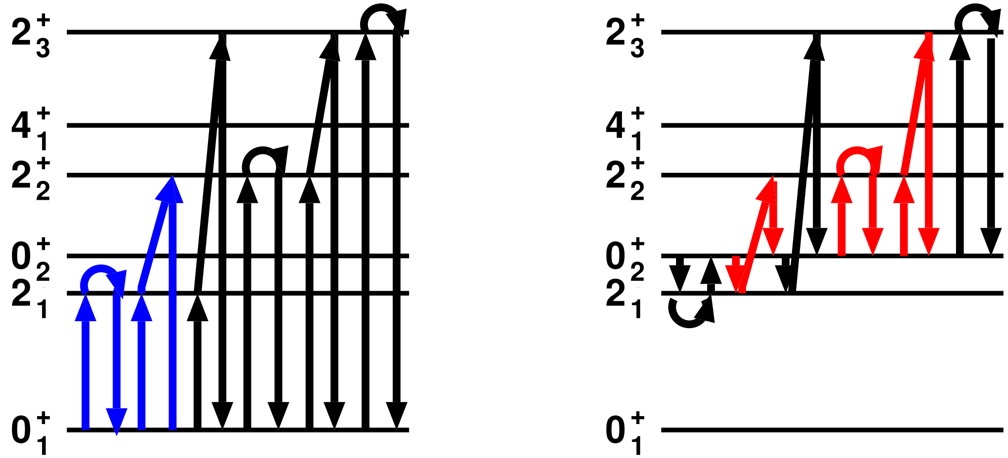


- combined β-decay study (iTHEMBA Labs) and (p,t) transfer (MLL)
- resulting level scheme suggestive of shape coexistence and triaxiality

Quadrupole sum rules: triaxiality

D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683
K. Kumar, PRL 28 (1972) 249

$$\sqrt{\frac{2}{35}}\langle Q^3 \cos 3\delta\rangle = \langle i|\{[E2\times E2]^2 \times E2\}^0|i\rangle$$
$$= \frac{1}{(2I_i+1)} \sum_{t,u} \langle i||E2||u\rangle\langle u||E2||t\rangle\langle t||E2||i\rangle \begin{Bmatrix} 2 & 2 & 2 \\ I_i & I_t & I_u \end{Bmatrix}$$



$\langle \cos 3\delta \rangle$: measure of triaxiality

- relative signs of E2 matrix elements are needed: can we get them experimentally?

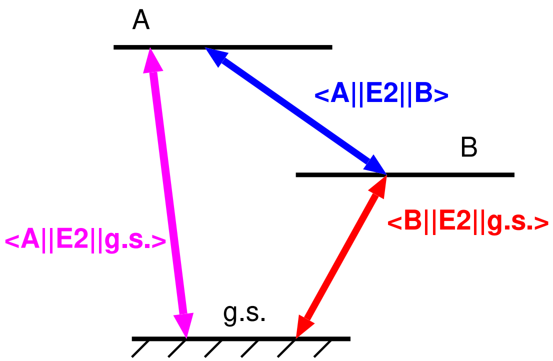
Contributions to $\langle Q^3 \cos 3\delta\rangle$ in ^{100}Mo: K. Wrzosek-Lipska *et al.*, PRC 86 (2012) 064305

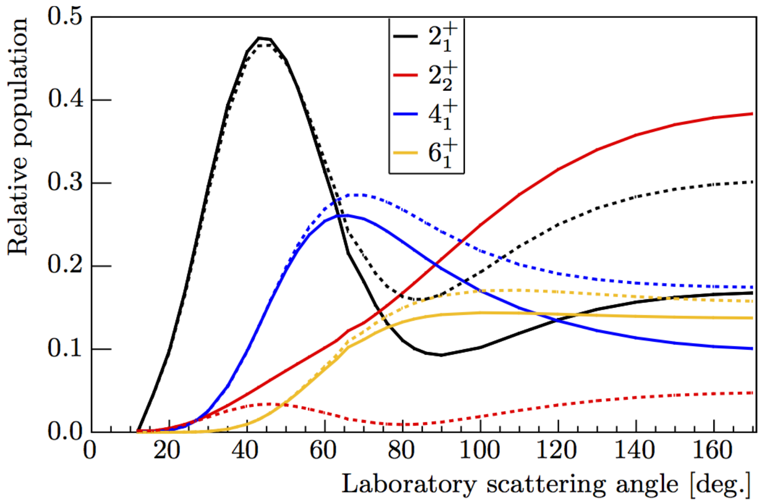
Relative signs of E2 matrix elements

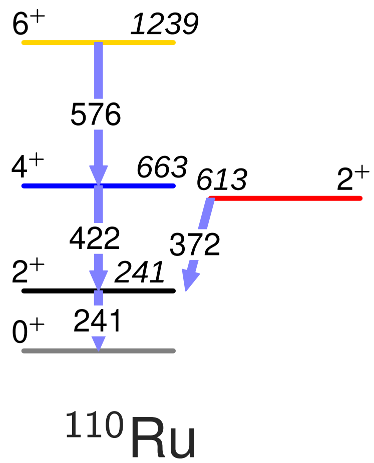
- Coulomb-excitation cross section are sensitive to relative signs of MEs: result of interference between single-step and multi-step amplitudes

- excitation amplitude of state A: $a_A \sim \langle A||E2||g.s.\rangle + \langle B||E2||g.s.\rangle\langle A||E2||B\rangle$

- excitation probability ($\sim a_A^2$) contains interference terms $\langle A||E2||g.s.\rangle\langle B||E2||g.s.\rangle\langle A||E2||B\rangle$







- negative $\langle 2_1^+||E2||2_2^+\rangle$ (solid lines): much higher population of 2_2^+ at high CM angles

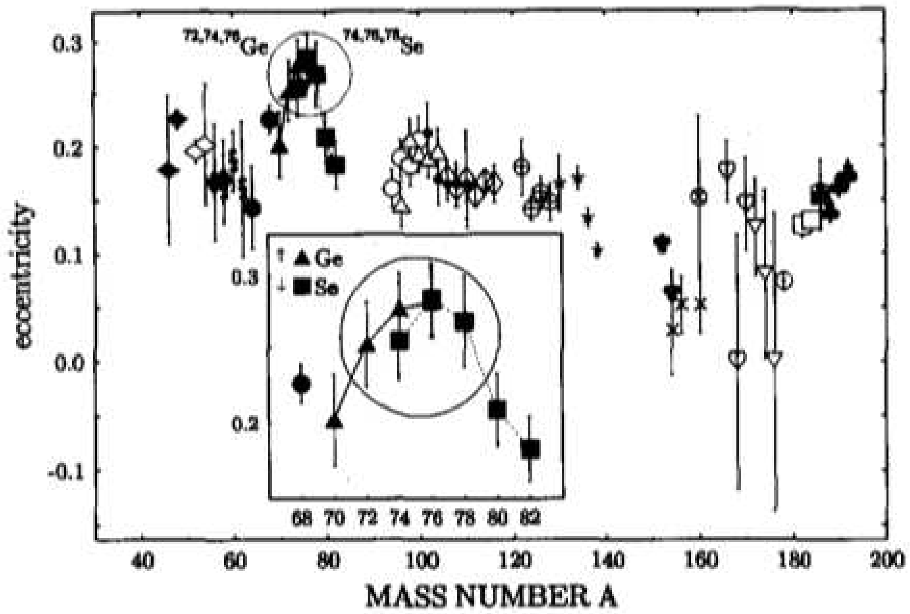
- sign of a product of matrix elements is an observable

Quadrupole sum rules: triaxiality

A. Andrejtscheff *et al*, Phys. Lett. B 329 (1994) 1

For the ground state, two terms dominate the sum:

$$\langle\cos 3\delta\rangle \approx -\sqrt{\frac{7}{10}}\langle Q^2_{0_1^+}\rangle^{-3/2}\Big(\left|\langle 0_1^+\|E2\|2_1^+\rangle\right|^2\langle 2_1^+\|E2\|2_1^+\rangle$$
$$+2\langle 0_1^+\|E2\|2_1^+\rangle\langle 2_1^+\|E2\|2_2^+\rangle\langle 2_2^+\|E2\|0_1^+\rangle\Big)$$

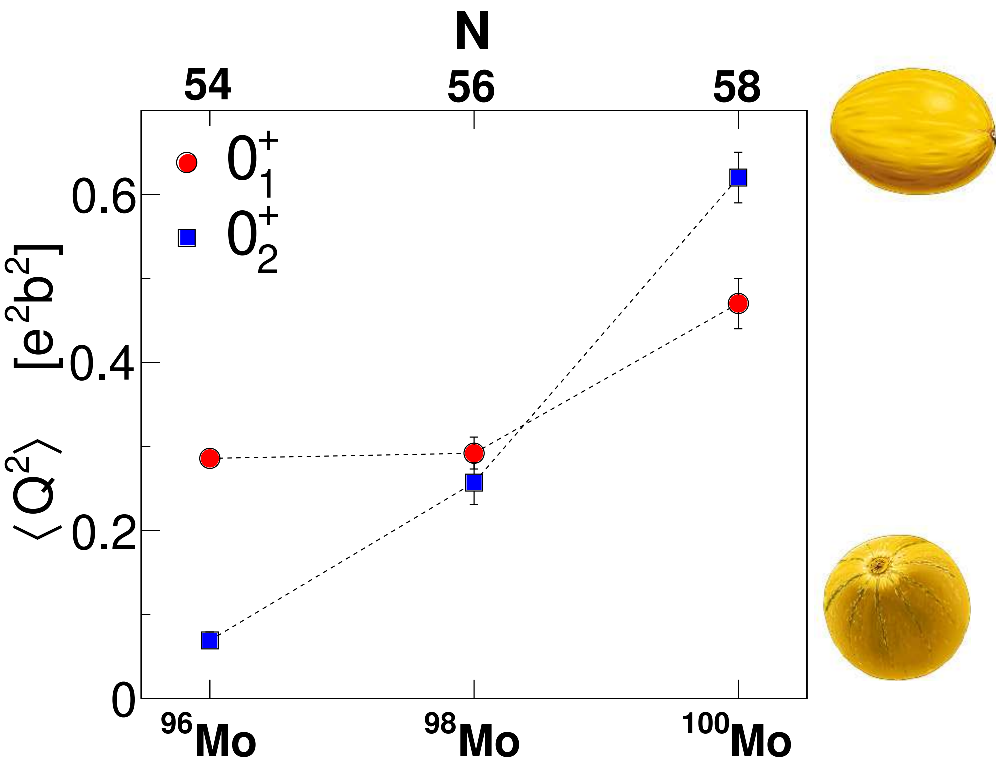


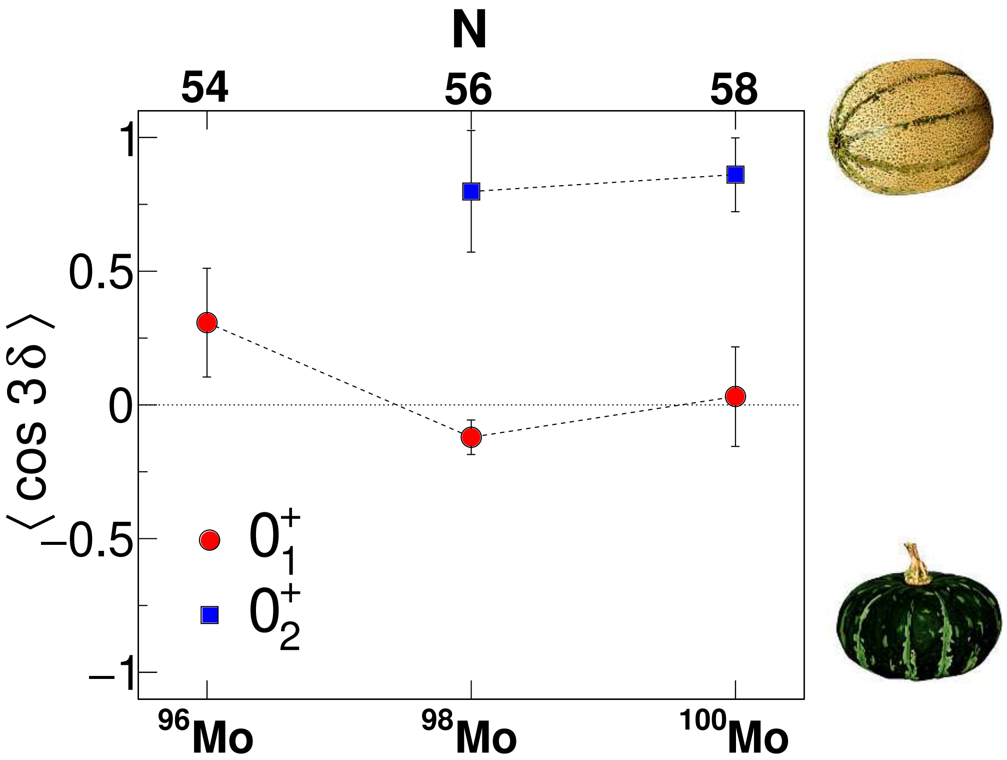
still, sign of the $\langle 0_1^+\|E2\|2_1^+\rangle\langle 2_1^+\|E2\|2_2^+\rangle\langle 2_2^+\|E2\|0_1^+\rangle$ product is necessary

Ground-state triaxiality in $^{96-100}$Mo

MZ *et al.*, Nucl. Phys. A 712 (2002) 3

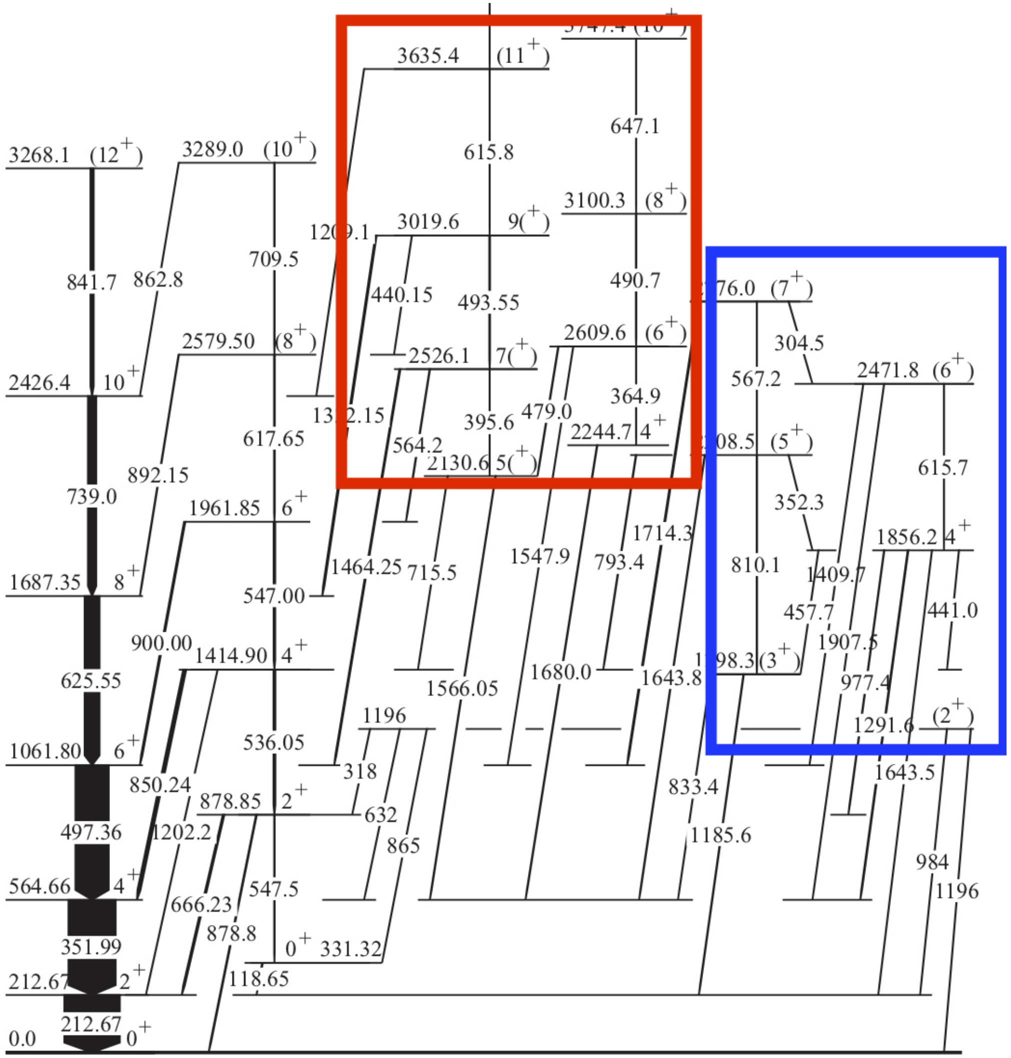
K. Wrzosek-Lipska *et al.*, PRC 86 (2012) 064305





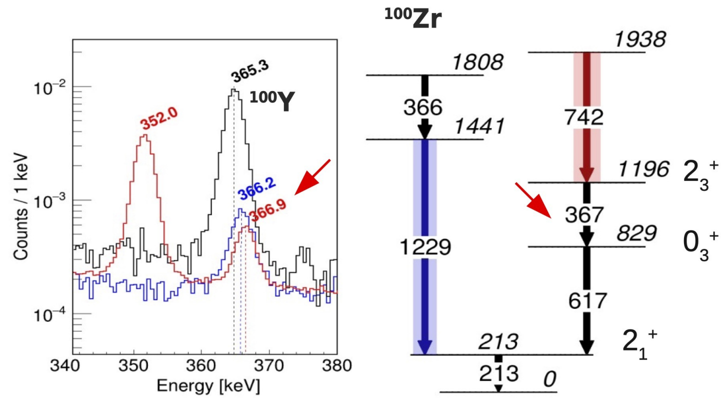
- ground states of the Mo isotopes triaxial (average shape, may result from dynamic effects)
- shape coexistence of the deformed and triaxial ground state with a more axial 0_2^+ increasing in deformation with N

Gamma and 'triaxial' structures in ^{100}Zr



W. Urban et al, PRC 100, 014319 (2019)

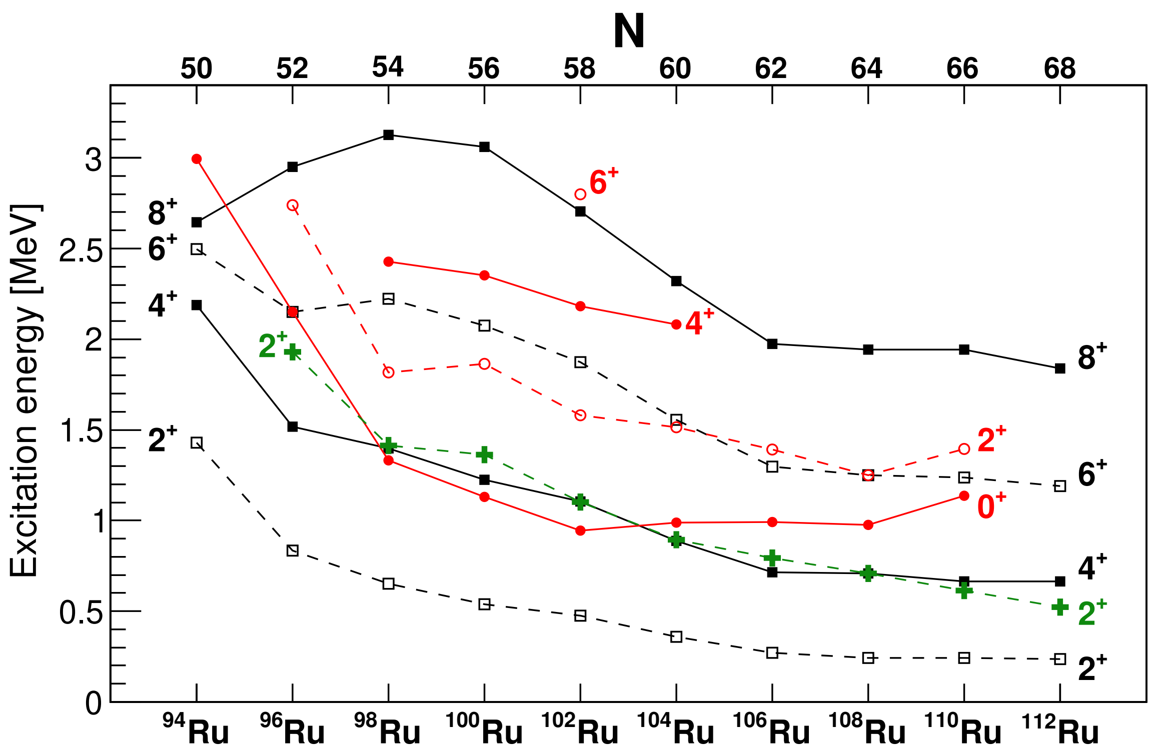
- "gamma" band proposed (related to the softness in the γ degree of freedom) and "triaxial" band (related to a rotation of an non-axial shape)
- transitions to low-spin states missing, or even candidates missing



- our new results from β decay at TRIUMF (D. Kalaydjieva, PhD thesis) suggest that the presumed head of the "gamma" band is instead a member of a deformed structure built on the 0_3^+ state

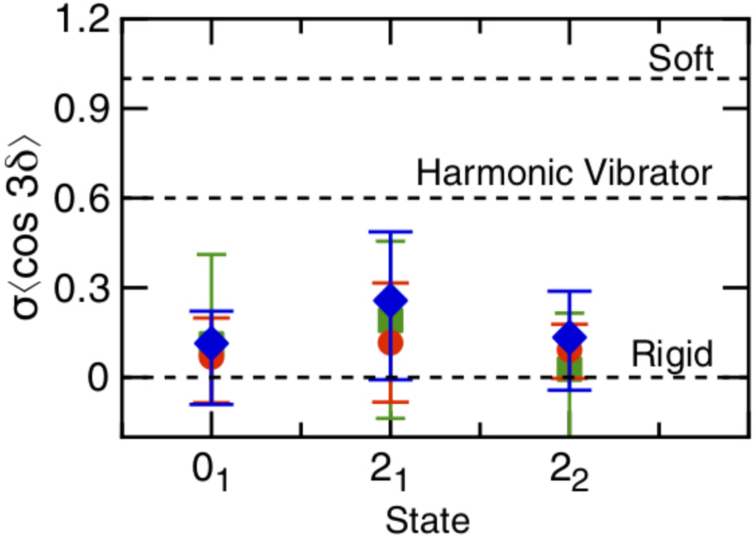
Energy systematics in Ru isotopes

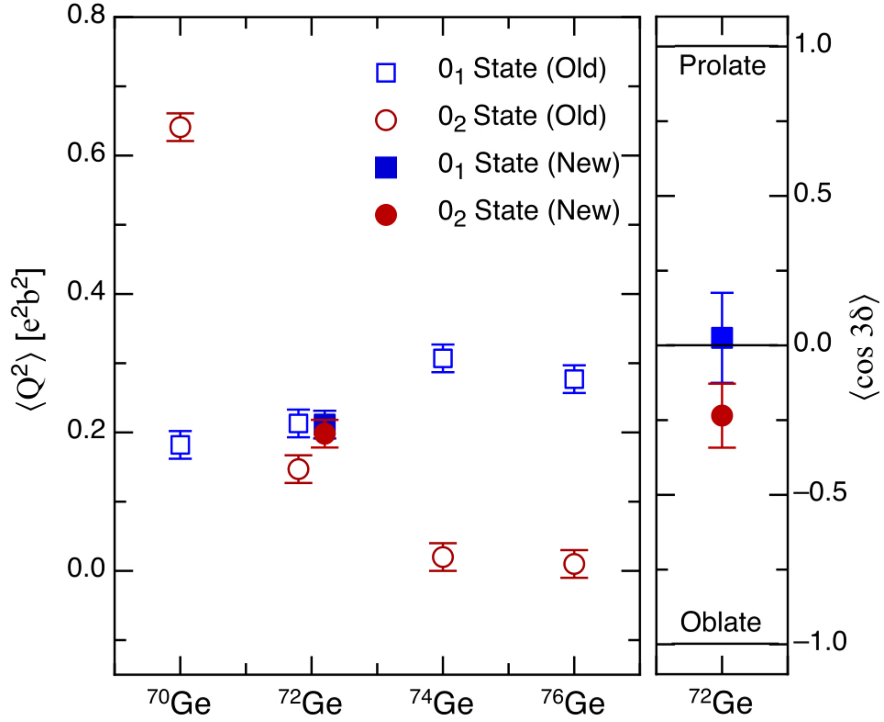
- transition from potentially γ-rigid ^{110,112}Ru (D. Doherty et al, PLB 776, 334 (2017)) to γ-soft nuclei
- parabolic intrusion of potentially shape-coexisting shapes
- experimental data on shape coexistence less detailed than in the Zr, Mo isotopic chains



Higher-order quadrupole invariants – example of ^{72,76}Ge

A.D. Ayangeakaa *et al.*,
PRL 123, 102501 (2019)
PLB 754, 254 (2016)





- ^{76}Ge: unique example of determination of softness in γ from experimental data

- ^{72}Ge: much higher number of transitions observed in a new measurement
$\rightarrow$ slight change of the deduced invariants due to extra states entering the sum

Experimental information on octupole collectivity in even-even nuclei

- energy of the first 3^- state (first hint)

- $B(E3; 3^-_1 \rightarrow 0^+_1)$ value; $B(E3; I_i \rightarrow I_f) = \frac{7}{16\pi}(I_f030|I_i0)^2 Q_3^2$
  $Q_3 = \frac{3}{\sqrt{7\pi}} Z\, e\, R_0^3 \beta_3$

- negative-parity states decay predominantly by fast E1 transitions; large B(E1) values usually correlate with octupole collectivity, but the inverse is not true

- lifetime of a negative-parity state is a very poor indicator of octupole collectivity

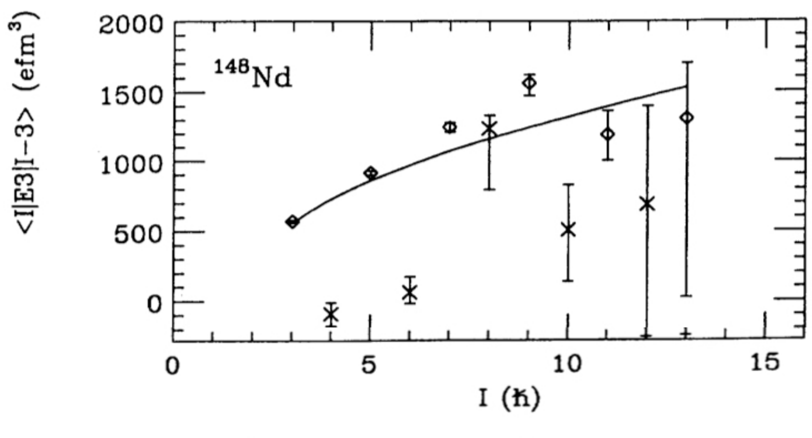
- direct E3 decay is rarely observed

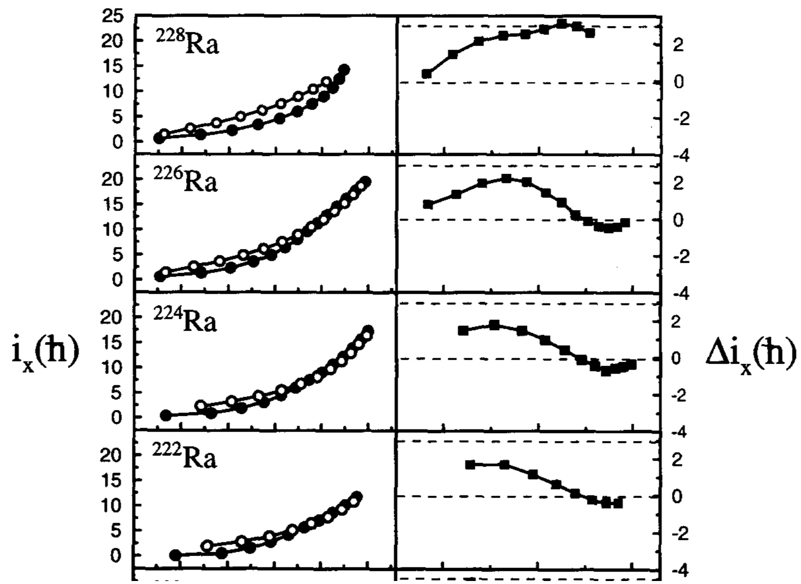
- Coulomb excitation and inelastic scattering are the methods of choice to determine E3 strength

Rigid octupole deformation versus octupole vibration

- apart from actinides, E3 collectivity is usually attributed to surface vibrations
- rigid octupole deformation can be claimed on the basis of B(E3) values between the ground-state band and the negative-parity band, or identical rotational alignments in these bands ($\rightarrow$ interleaving of positive and negative-parity states)

J.F.C. Cocks et al. / Nuclear Physics A 645 (1999) 61–91



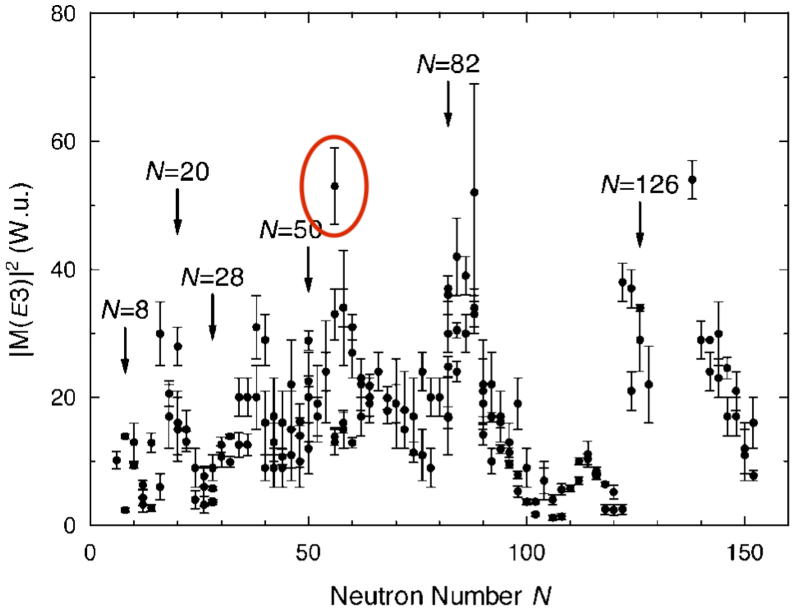


R. Ibbotson et al, PRL 71, 27 (1993)

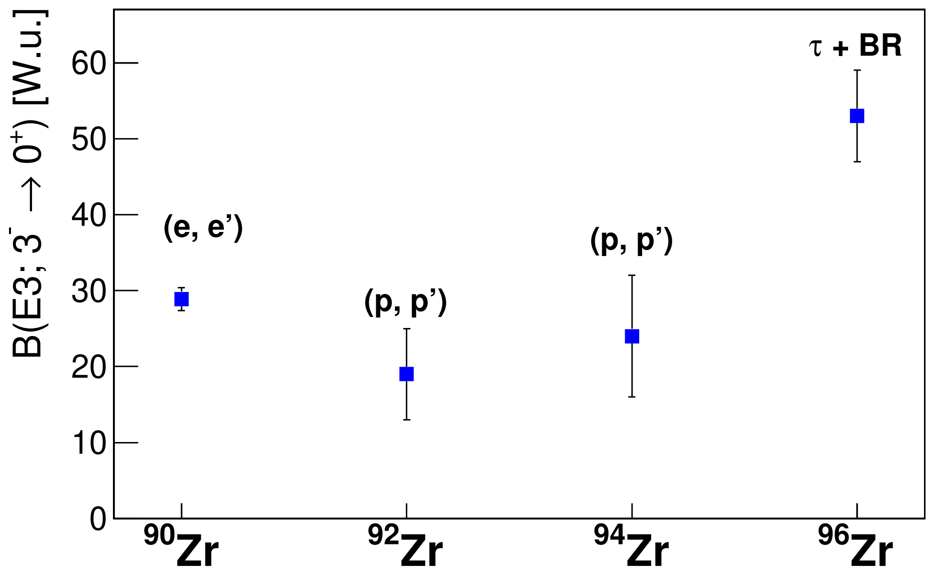
More info: P. A. Butler and W. Nazarewicz Rev. Mod. Phys. 68, 349 (1996); P. Butler, Proc. R. Soc. A 476, 202 (2020)

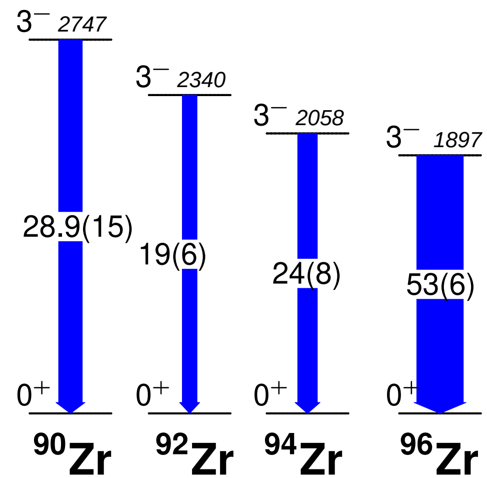
Octupole collectivity in Zr isotopes: anomalous value for ^{96}Zr

- evaluated $B(E3; 3_1^- \rightarrow 0_1^+)$ strength for ^{96}Zr strikingly high (53(6) W.u.), comparable with those known for nuclei with rigid pear shapes
- observed trend of $B(E3; 3_1^- \rightarrow 0_1^+)$ values in Zr isotopes inconsistent with 3_1^- energies and hard to explain



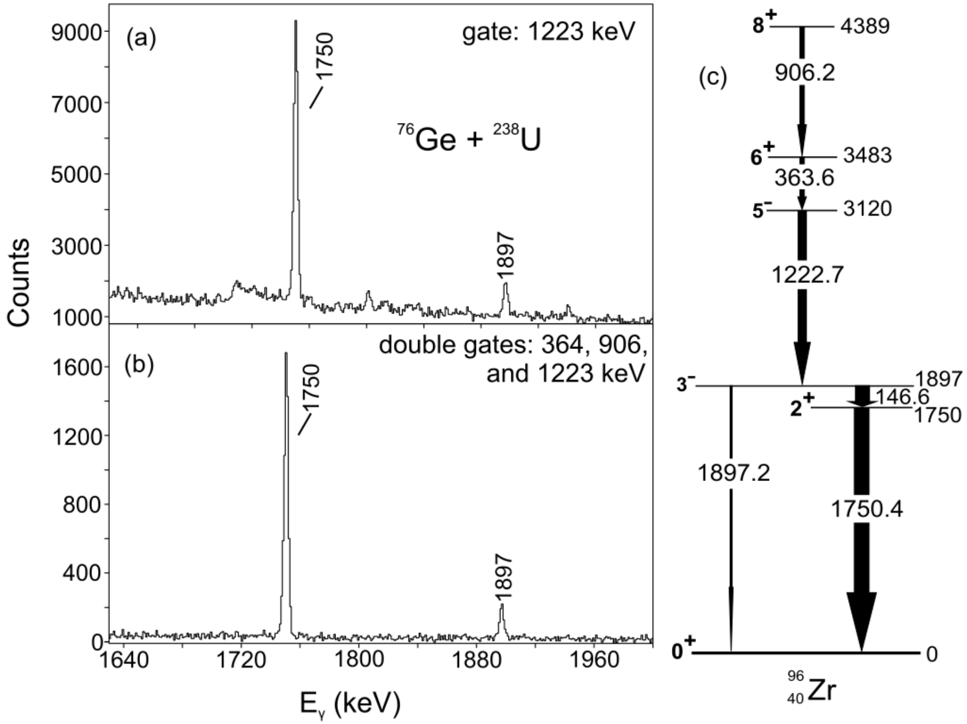
T. Kibédi and R.H. Spear, At. Data Nucl. Data Tables 80, 35 (2002)





Revision of the E3 strength in ^{96}Zr

- determination of E3 strength in ^{96}Zr using gamma-ray spectroscopy requires two measurements:
  - lifetime ($\approx$ 70ps – plunger measurements)
  - branching ratio E3/E1
- if the 147 keV / 1897 keV intensity ratio is directly measured, the efficiency must be known precisely
  - walk effect, conversion at 147 keV

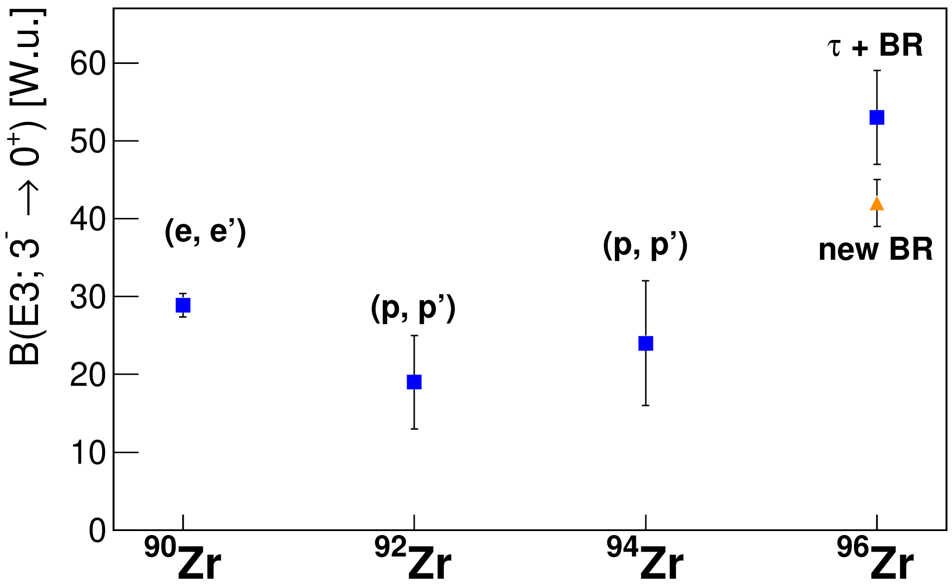


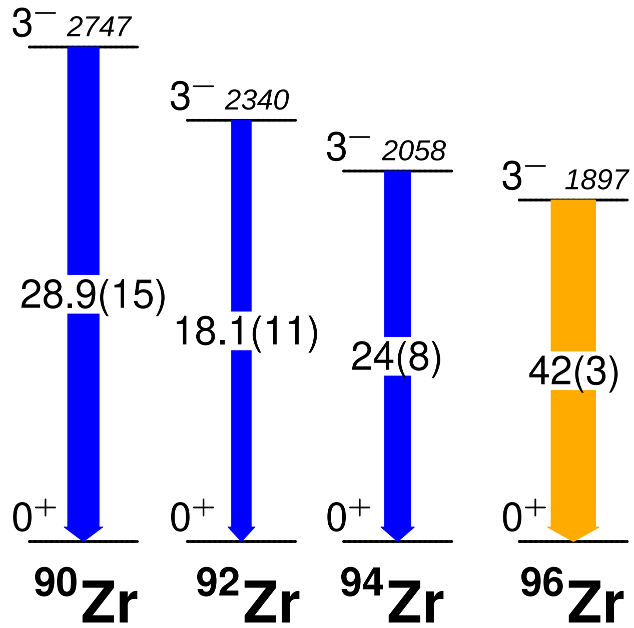
- new measurement – gating from above and comparison of 1750 keV and 1897 keV intensities

Ł. Iskra et al, Phys. Lett. B 788 (2019) 396

Octupole collectivity in Zr isotopes: new BR measurement for ^{96}Zr

- new measurement of E1/E3 branching ratio in ^{96}Zr (Ł. Iskra et al, Phys. Lett. B 788 (2019) 396) points to lower octupole collectivity, but the overall trend remains puzzling

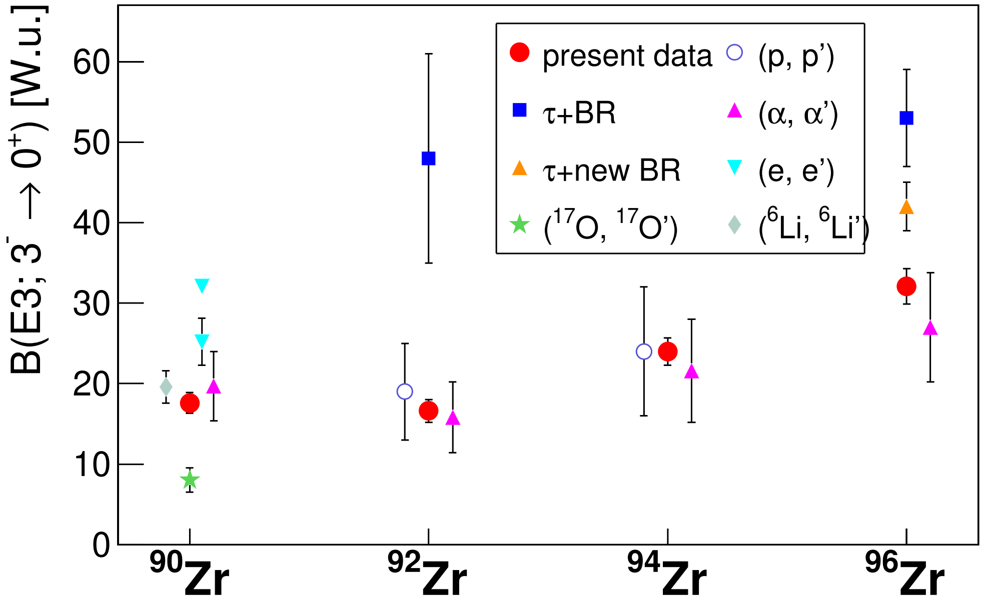


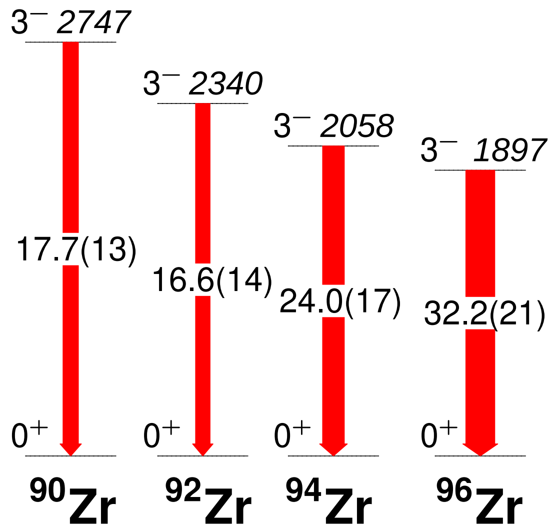


→ new systematic study of quadrupole and octupole collectivity in stable Zr isotopes at MLL

Results: octupole collectivity in Zr isotopes

- overall trend of B(E3; $3_1^- \rightarrow 0_1^+$) values in Zr more consistent with evolution of 3_1^- energies than that of evaluated values



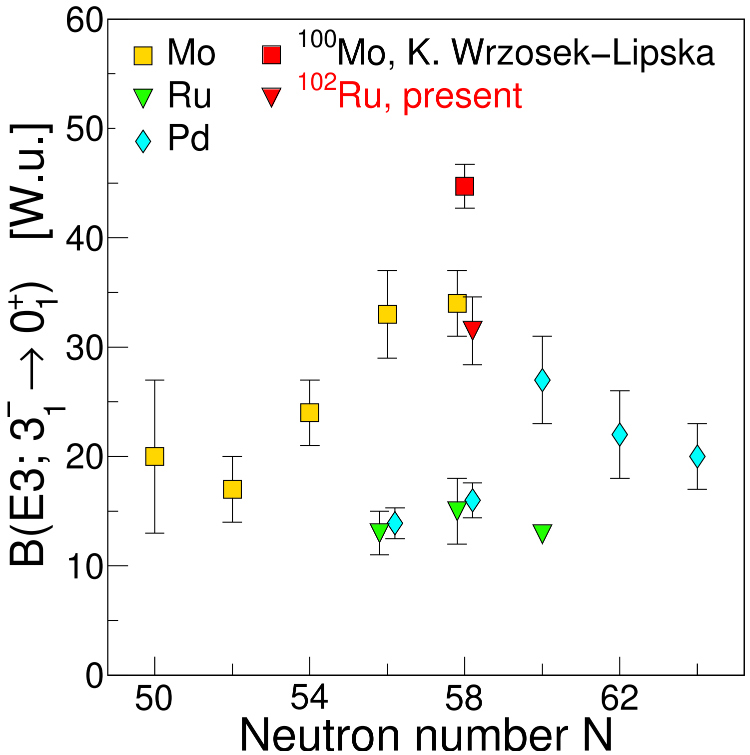


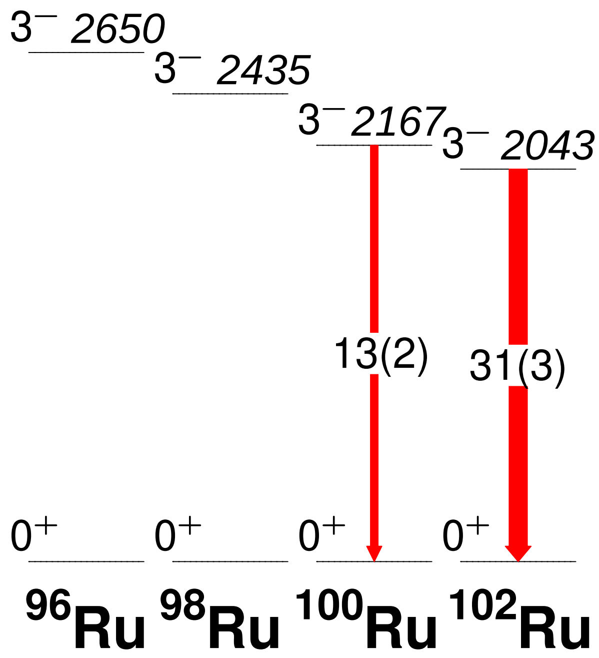
- similarities with results of (α, α') (D. Rychel et al, Z. Phys. A 326, 455 (1987) – the only other systematic study of β_3 in Zr)

- caution – preliminary result of B(E3; $3_1^- \rightarrow 0_1^+$) measurement in ^{96}Zr from AGATA much closer to the result of L. Iskra than to our new value; under investigation

Octupole collectivity in Ru isotopes

- no B(E3) values for Ru isotopes lighter than ^{100}Ru
- smooth evolution of 3^- energies
- conflicting B(E3) results in Ru and Mo nuclei



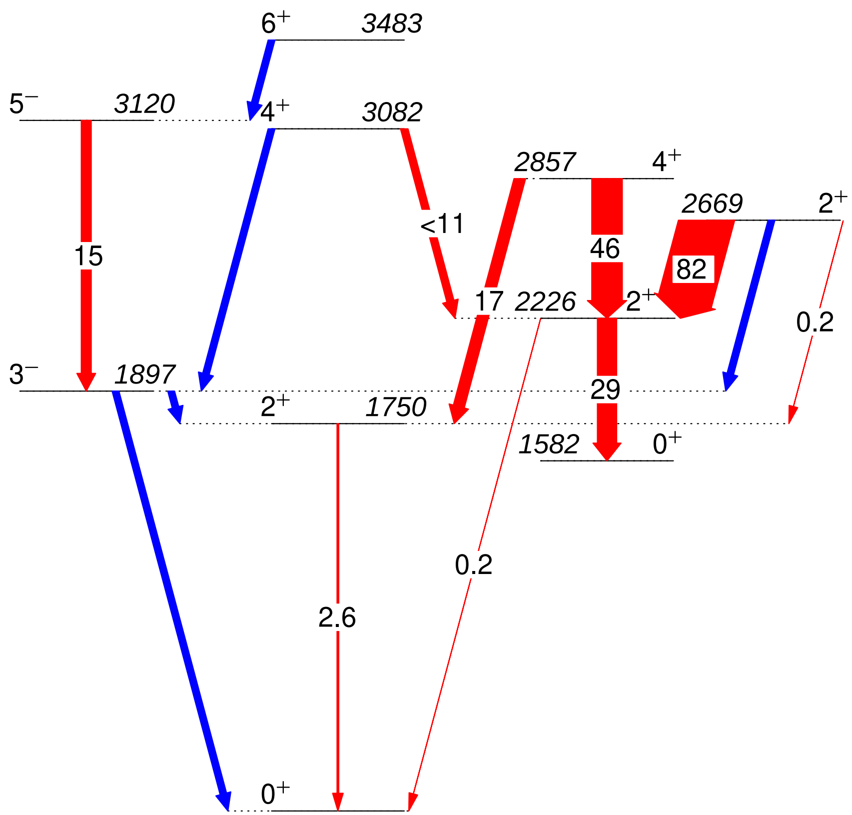


P. Garrett, MZ et al, PRC 106, 064307 (2022)

Remaining questions regarding ^{96}Zr

Revised branching and mixing ratios in ^{96}Zr: J. Wiśniewski et al, Phys. Rev. C 108, 024302 (2023)

- which 4^+ belongs to which band? if 4_1^+ is part of the deformed structure, why is its decay to the 2_1^+ so strong (mixing between bands should be weak)?
- the $2_3^+ \rightarrow 2_2^+$ decay seems surprisingly enhanced
- E1 transitions from presumably collective states compete with E2 ones; in particular, the 6^+ state decays predominantly via E1; is it related to a two-phonon octupole vibration?



Remaining questions regarding ^{96}Zr

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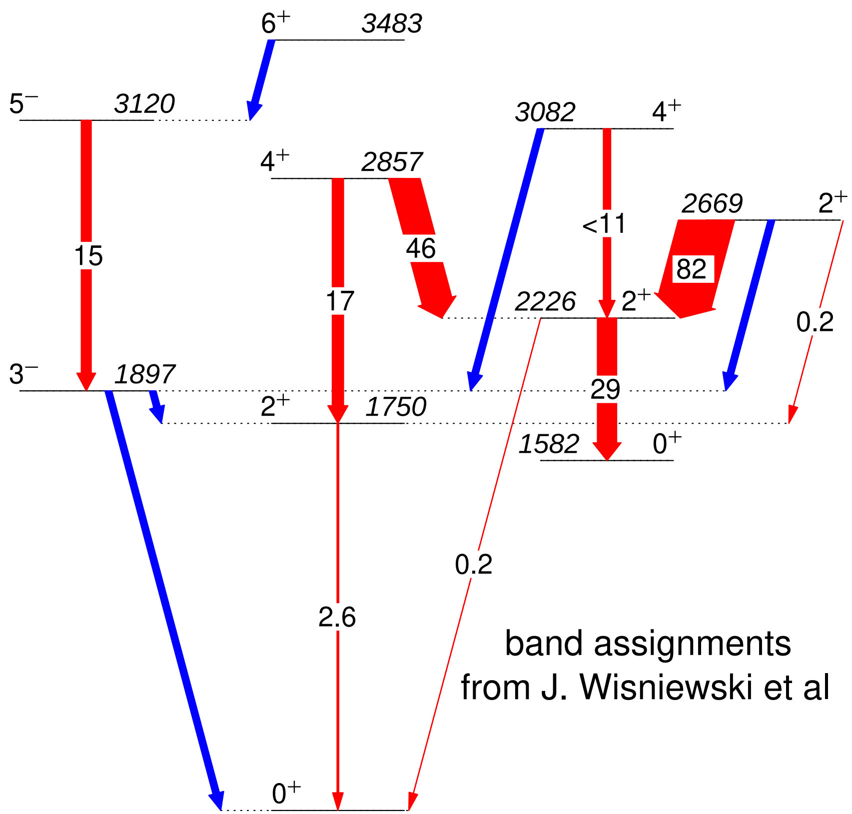


band assignments from J. Wisniewski et al

Outlook: complementary measurements on ^{96}Zr

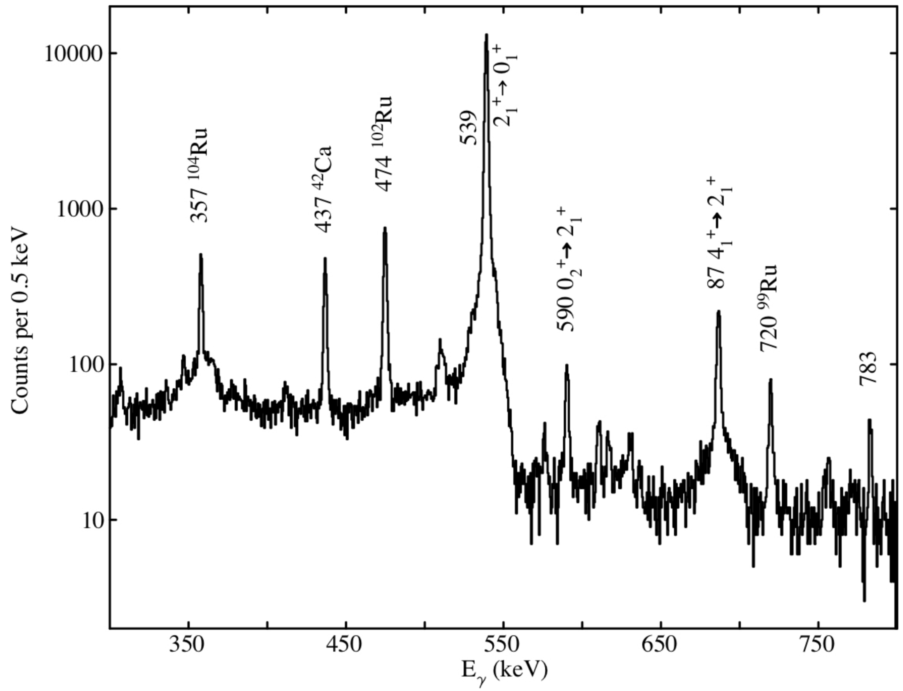
- combination of a lifetime study with safe and unsafe Coulomb-excitation cross-section measurement with a ^{96}Zr beam (AGATA@LNL, MZ, N. Marchini et al) – to be performed with AGATA at LNL in May 2024

- (p,p') on ^{96}Zr (AGATA@LNL, November 2023, D. Stramaccioni et al) – search for the direct $6^+ \to 3^-$ decay in order to verify the hypothesis of the 6^+ state being a double octupole phonon state

- β decay into ^{96}Zr (TRIUMF, December 2023, M. Rocchini, MZ et al) – precise measurement of branching and mixing ratios in the decay of spin-0,1,2,3 states

Coulomb excitation of ^{100}Ru

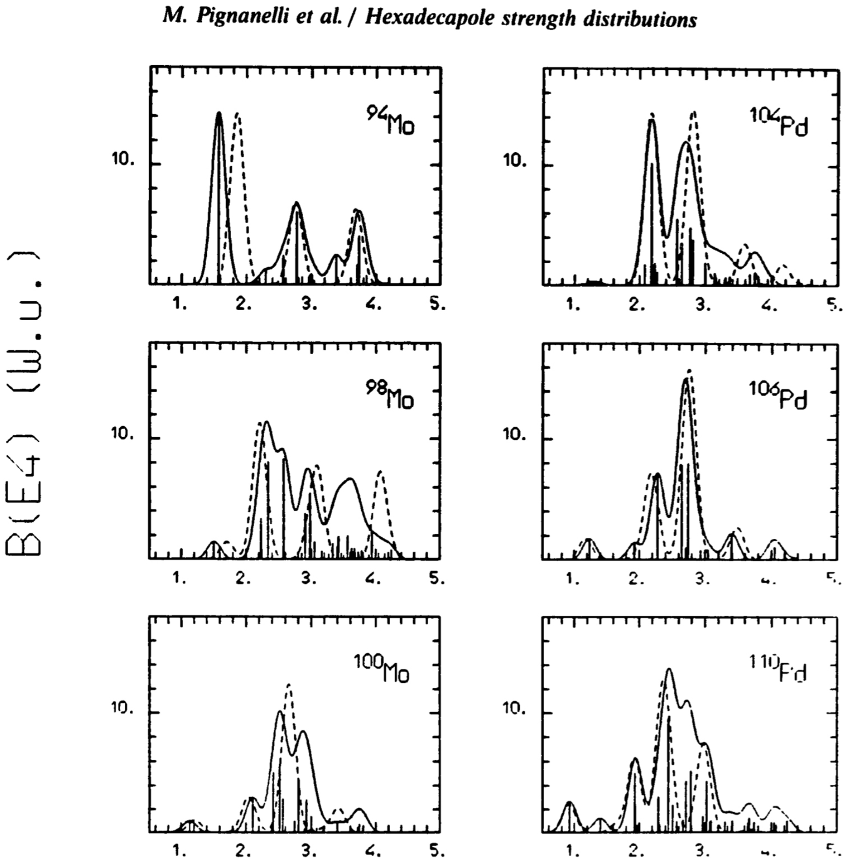
- low-energy Coulomb excitation of ^{100}Ru with a ^{32}S beam performed at HIL Warsaw in April 2022 (PI P. Garrett, K. Wrzosek-Lipska, MZ)
- in order to better constrain the properties of the 2_2^+ state, data will be completed by a second measurement with a ^{14}N beam
- additional lines in the spectrum due to target oxidation
- decay of the 3_1^- state at the observation limit



Outlook: challenges for future Coulomb-excitation studies

- abundance: 5.54% ^{96}Ru, 2.80% ^{96}Zr
- difficult to get material with high enrichment (even more since the war has started); to my knowledge, no suppliers offer ^{96,98}Ru
- difficult to produce Ru and Zr targets (material often available in oxide form, Ru targets produced by electrodeposition proven very fragile)
- high excitation energies in ^{96}Zr and ^{96}Ru with respect to other isotopes make it more difficult to populate levels of interest

Hexadecapole strength in A≈ 100 nuclei



M. Pignanelli et al, NPA 540, 27 (1992)

Do we know all states that should enter the sum?

- especially for the (E2 x E2 x E2), where terms can cancel out – can we say that terms involving higher lying levels (the 2^+_4 state etc) do not significantly influence the rotational invariant?
  - if such state were coupled to the state in question via a large E2 matrix element, it would be populated in the experiment
  - comparison with GBH calculations for ^{100}Mo: $\langle Q^2 \rangle$, $\langle Q^3 \cos(3\delta) \rangle$ calculated by acting with an operator on calculated wave functions and from theoretical values of matrix elements, limited to the same three intermediate states

$\Rightarrow$ difference below 3% for both 0^+ states

	GBH		exp
$0^+_1 : \overline{\beta}$	0.20	0.20	0.22 ±0.01
$0^+_1 : \overline{\gamma}$	27°	27°	29°±3°
$0^+_2 : \overline{\beta}$	0.24	0.24	0.25 ±0.01
$0^+_2 : \overline{\gamma}$	18°	17°	10°±3°

K. Wrzosek-Lipska, PRC 86 (2012) 064305

PHYSICAL REVIEW C **86**, 064305 (2012)

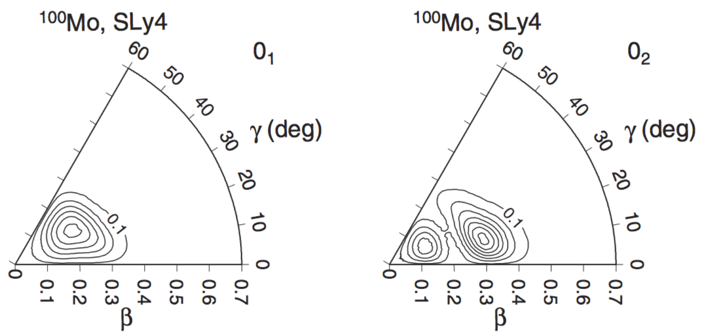
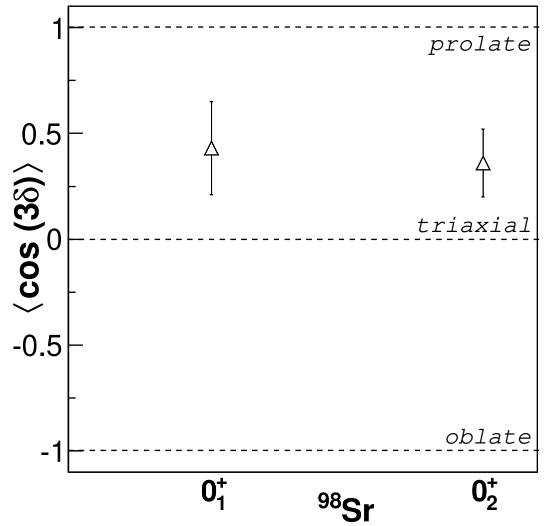
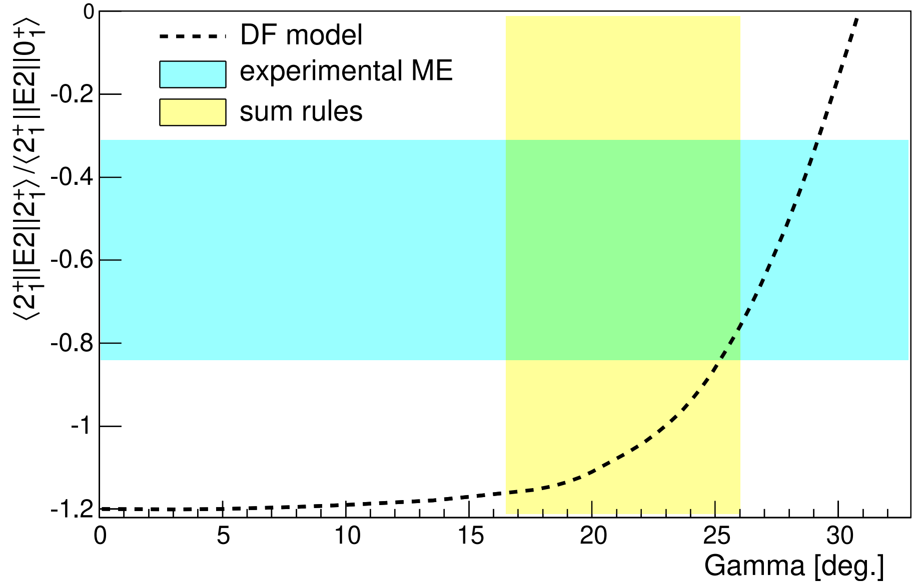


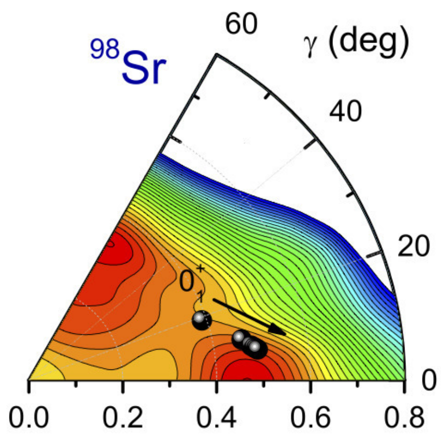
FIG. 15. Probability density [Eq. (26)] for the 0^+_1 and 0^+_2 states for the Skyrme SLy4 interaction. The contour interval is 0.3.

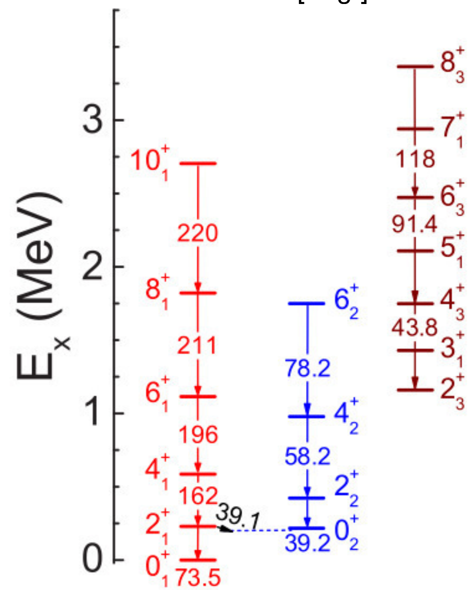
Triaxiality in ^{98}Sr





- gamma ≈ 25° would explain the reduction of $Q_s(2_1^+)$ in ^{98}Sr
- but where is the gamma band?





J. Xiang *et al.*, PRC 93, 054324 (2016), 5DCH with PC-PK1 interaction