Emergence of nuclear clustering in *ab initio* nuclear theory

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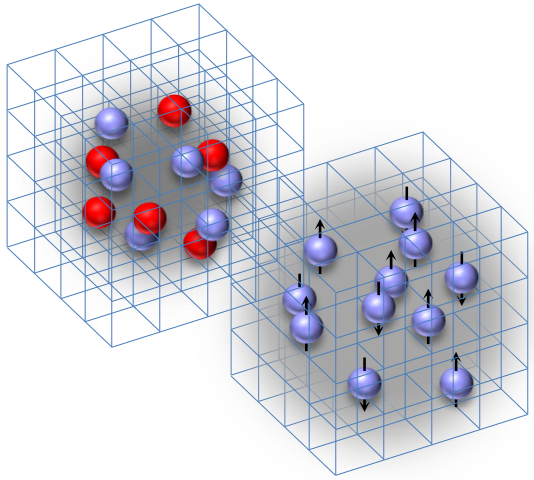
Exploring nuclear physics across energy scales 2024:
Interaction between nuclear structure and high energy nuclear collisions
Beijing - April 21, 2024



FRIB

MICHIGAN STATE UNIVERSITY

NUCLEI
Nuclear Computational Low-Energy Initiative
A SciDAC-4 Project

JÜLICH FORSCHUNGSZENTRUM

OAK RIDGE National Laboratory | LEADERSHIP COMPUTING FACILITY

Outline

Lattice effective field theory

Emergent geometry and duality of ^{12}C

Adiabatic projection and ^{4}He^{4}He scattering

A tale of two interactions

Wave function matching

^{16}O^{16}O collisions at RHIC and LHC energies

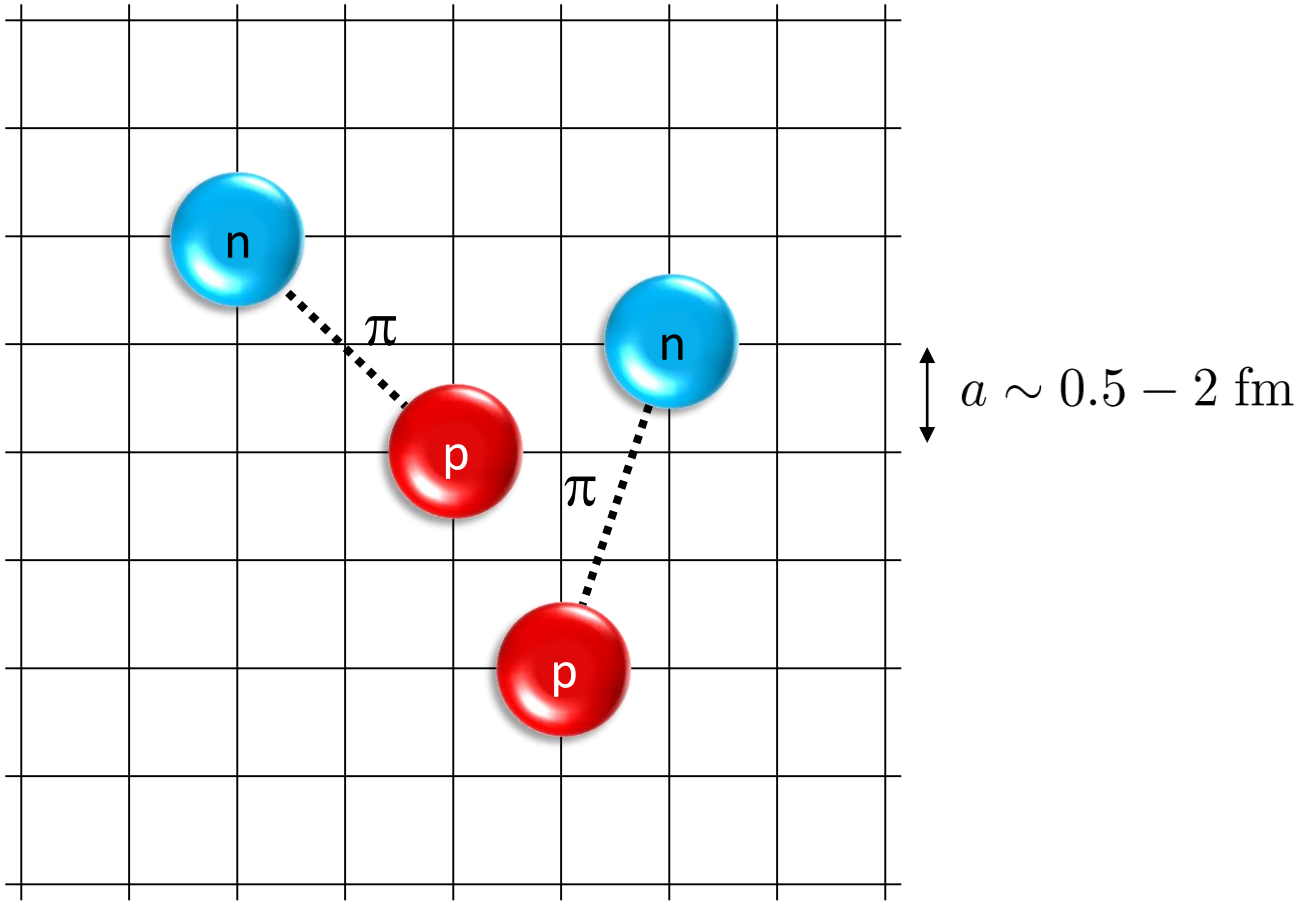
^{16}O^{16}O versus ^{20}Ne^{20}Ne collisions

Superfluidity

Outlook

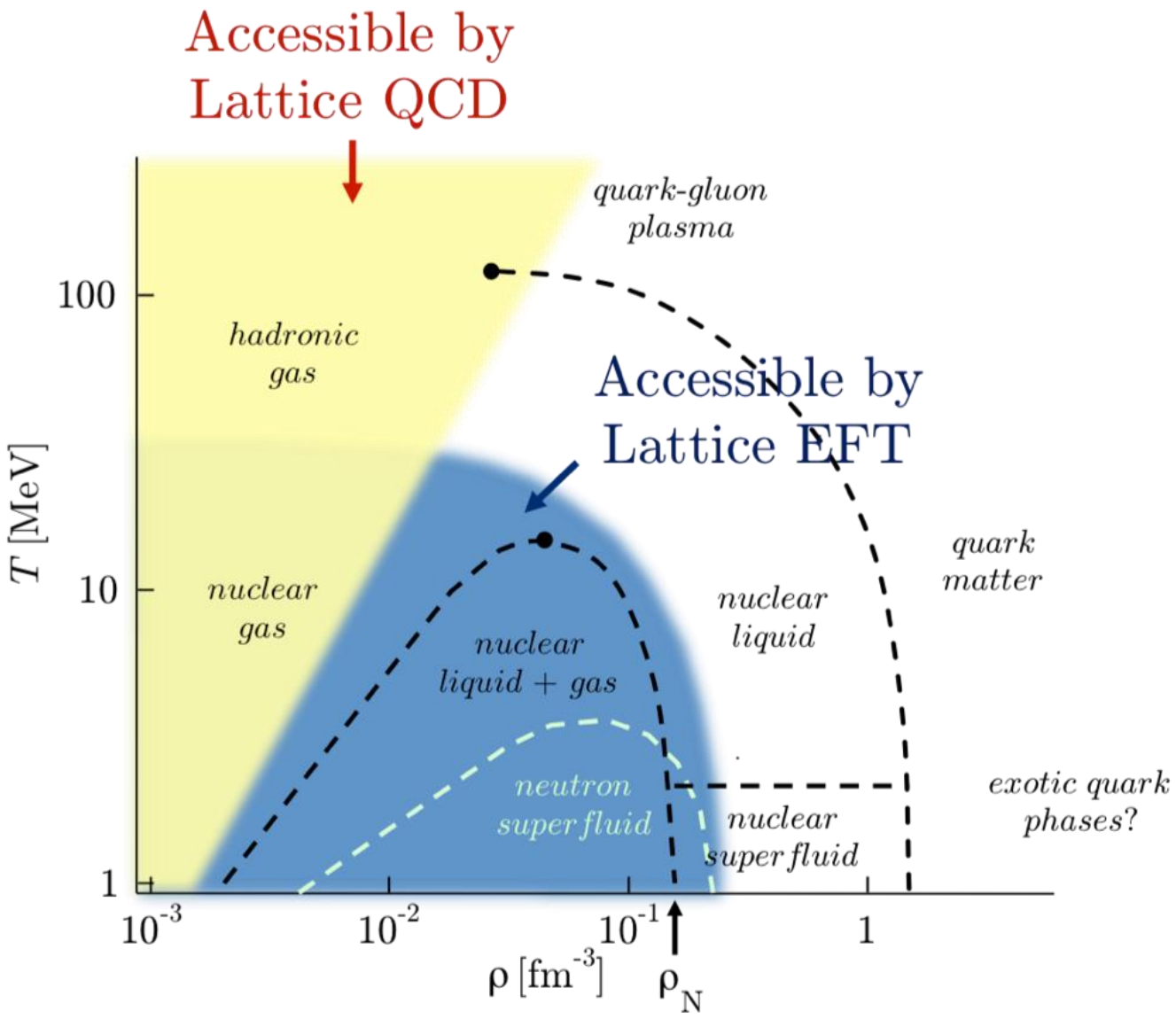
See Ulf's talk

Lattice effective field theory



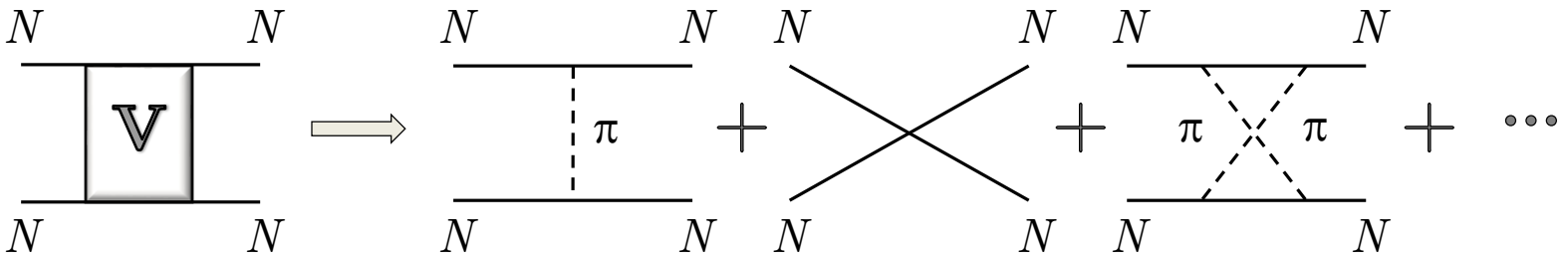
D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)

Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer


Accessible by
Lattice QCD
quark-gluon
plasma
hadronic
gas
Accessible by
Lattice EFT
quark
matter
nuclear
gas
nuclear
liquid
nuclear
liquid + gas
neutron
superfluid
nuclear
superfluid
exotic quark
phases?
100
10
1
T [MeV]
10^-3
10^-2
10^-1
1
ρ [fm^-3]
ρ_N

Chiral effective field theory

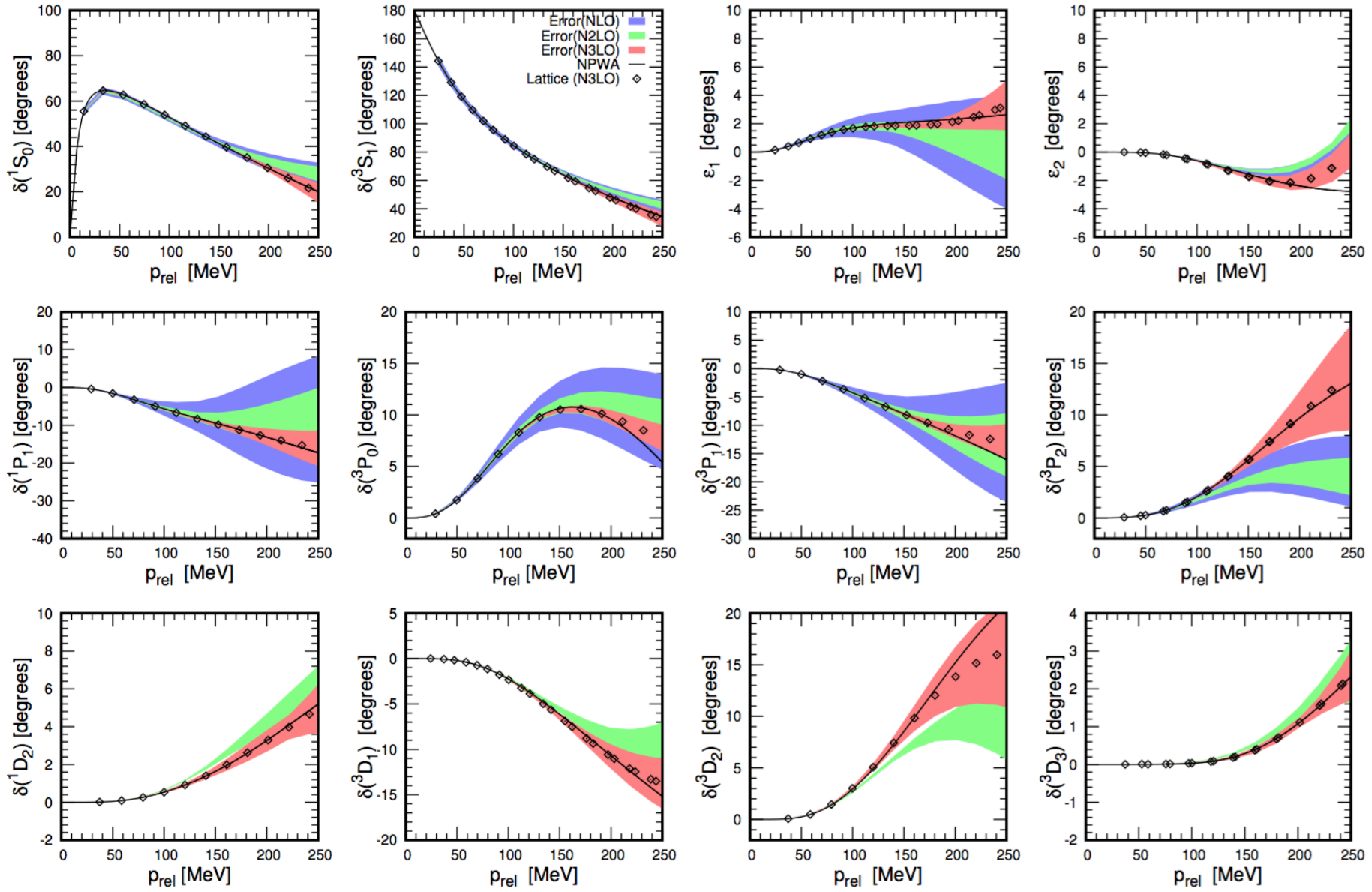
Construct the effective potential order by order



$V^{\rm OPEP}$ Contact interactions $V^{\rm TPEP}$

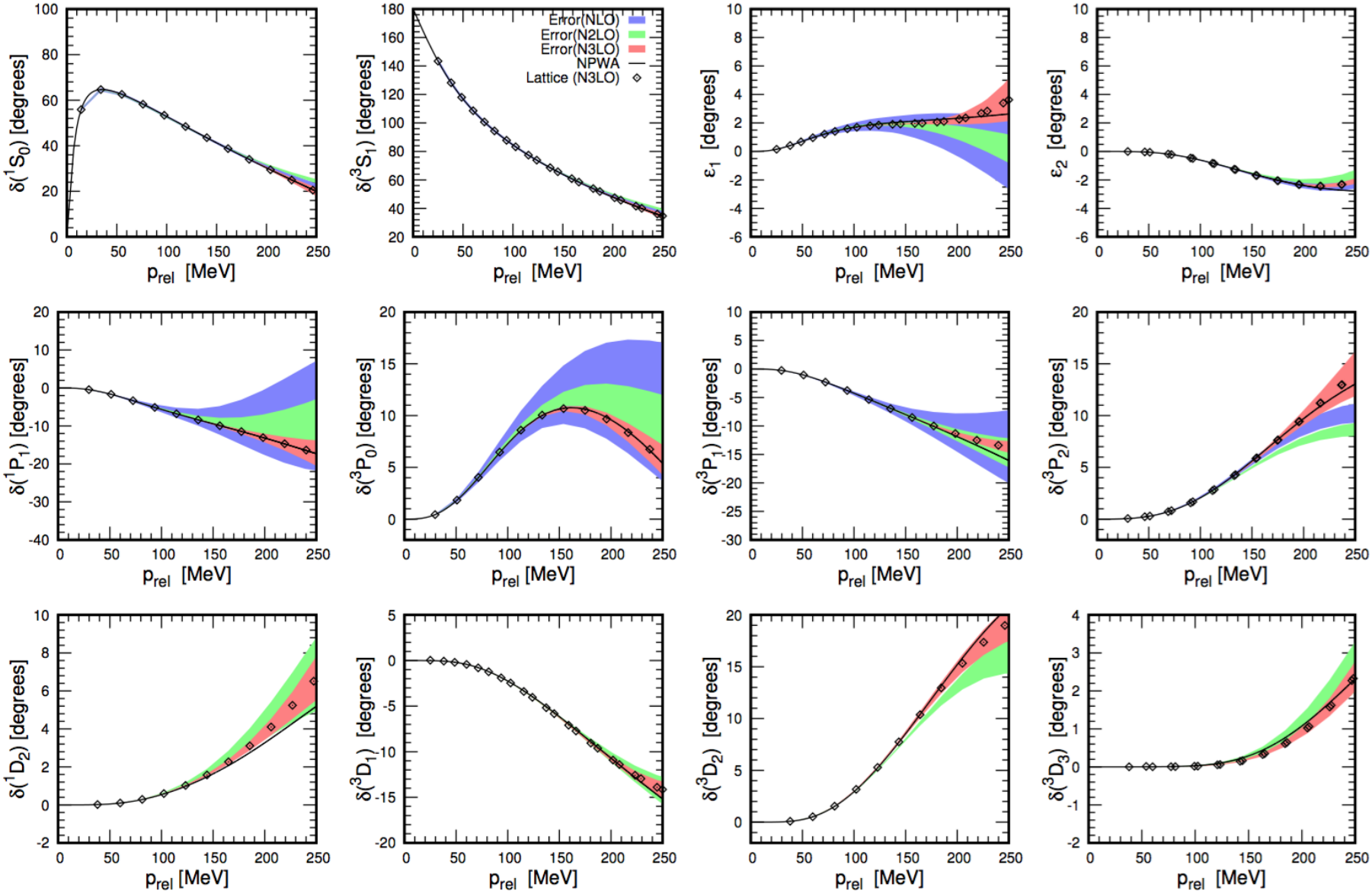
Leading order (LO) Next-to-leading order (NLO)

$a = 1.315\,\mathrm{fm}$

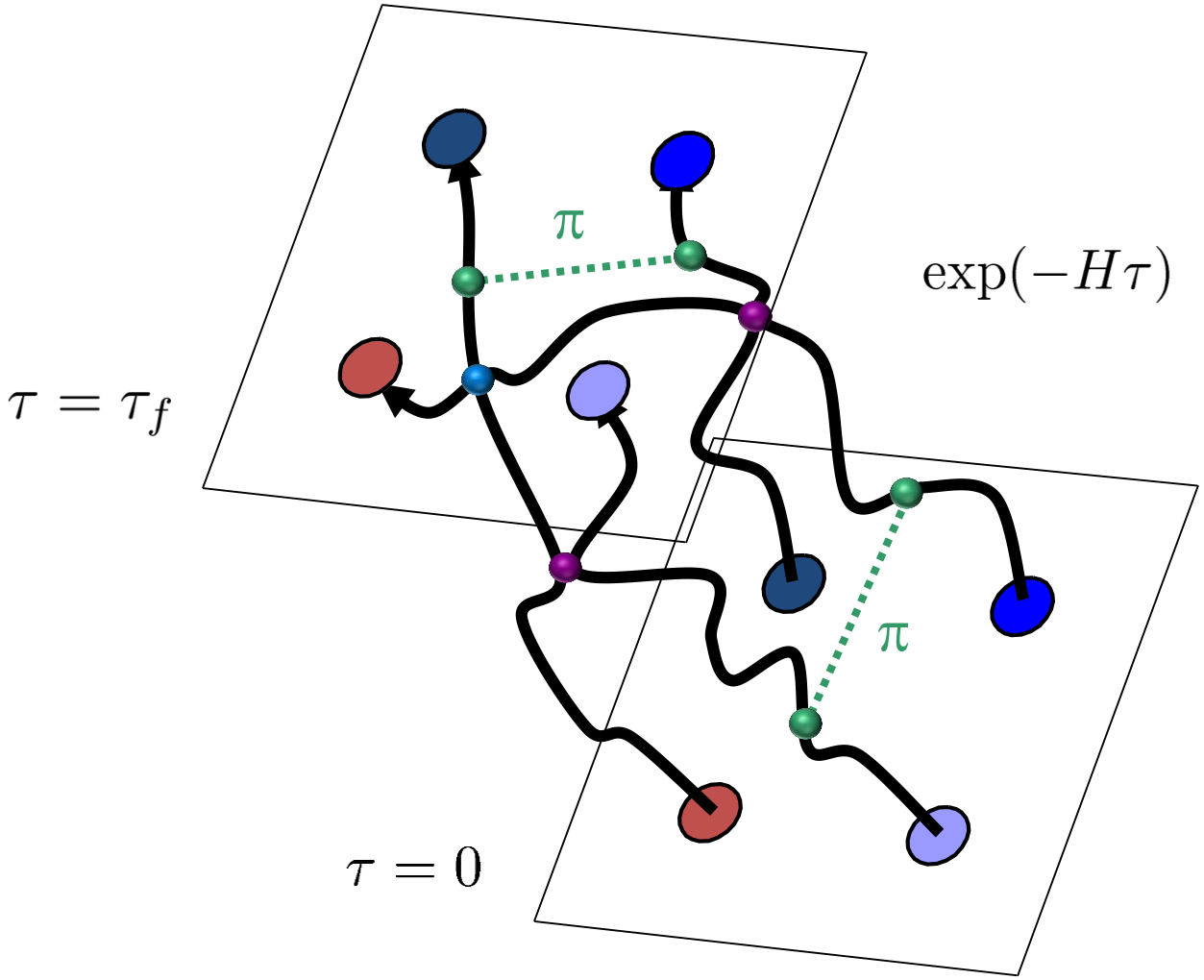


Li, Elhatisari, Epelbaum, D.L., Lu, Meißner, PRC 98, 044002 (2018)

$a = 0.987\,\mathrm{fm}$



Euclidean time projection



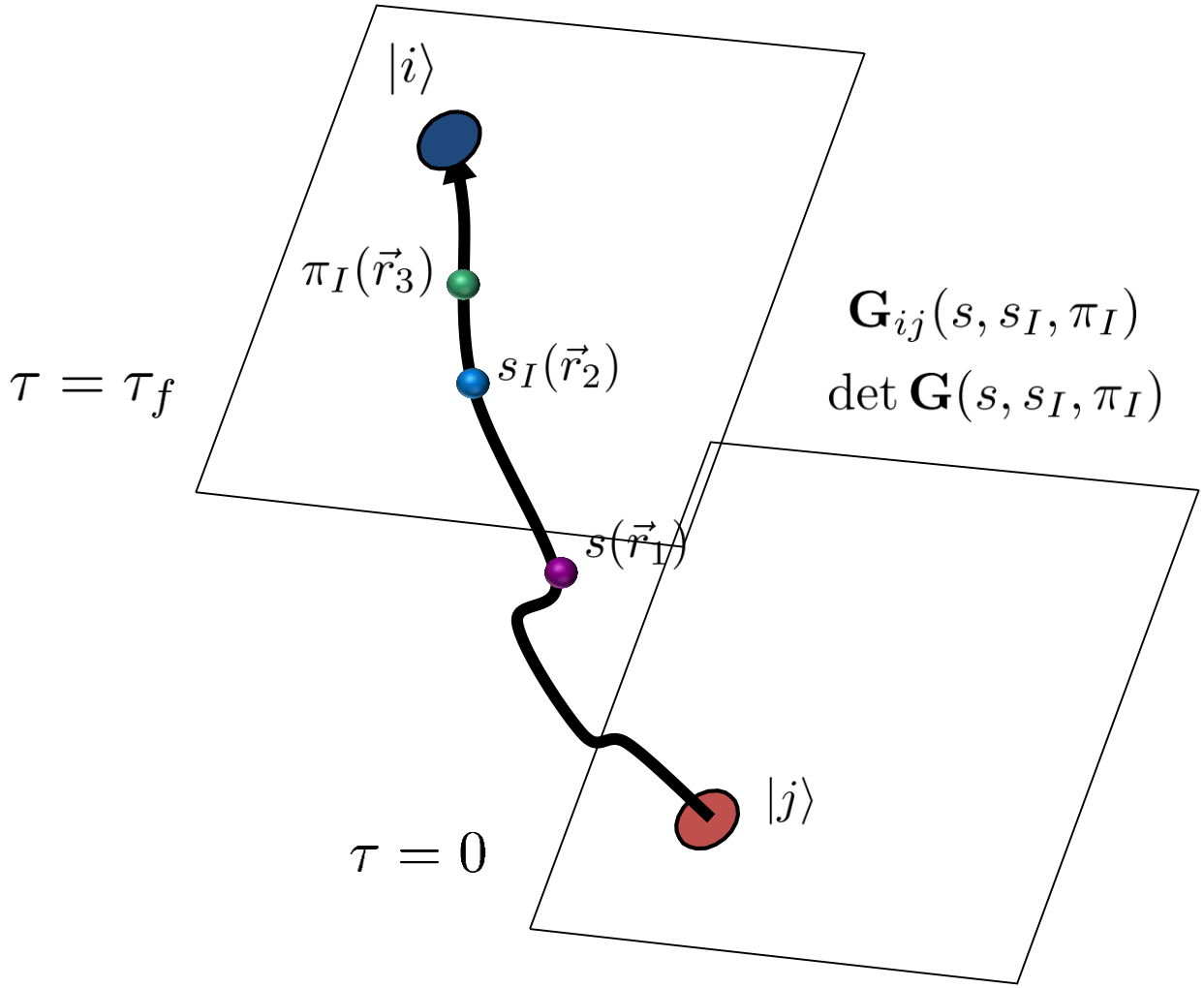
Auxiliary field method

We can write exponentials of the interaction using a Gaussian integral identity

$$\exp\left[-\frac{C}{2}(N^\dagger N)^2\right] \qquad \times \quad (N^\dagger N)^2$$

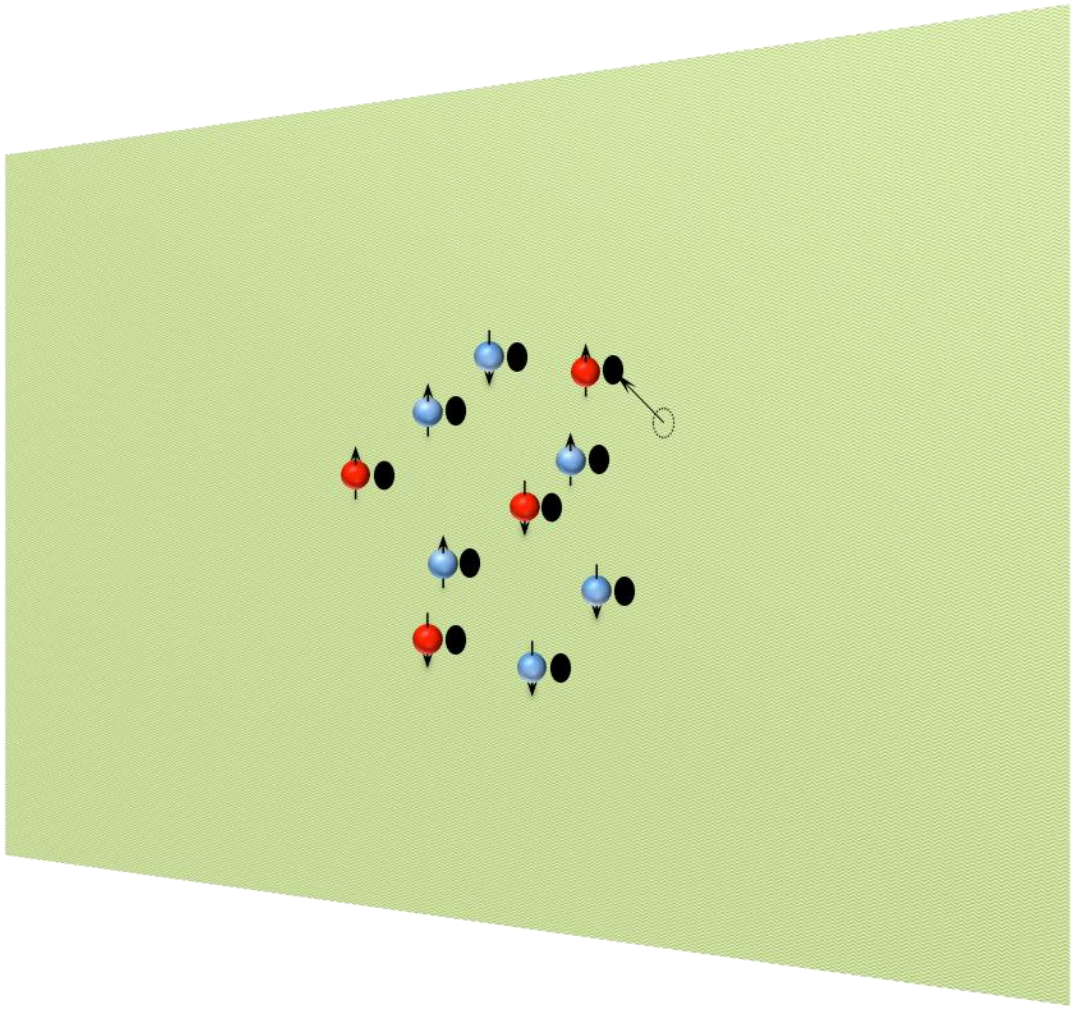
$$= \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} ds \exp\left[-\frac{1}{2}s^2 + \sqrt{-C}\, s(N^\dagger N)\right] \qquad \rangle\!\bullet \quad sN^\dagger N$$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.


$|i\rangle$
$\pi_I(\vec{r}_3)$
$s_I(\vec{r}_2)$
$s(\vec{r}_1)$
$|j\rangle$
$\tau = \tau_f$
$\tau = 0$
$\mathbf{G}_{ij}(s, s_I, \pi_I)$
$\det \mathbf{G}(s, s_I, \pi_I)$

See Bingnan's talk

Pinhole algorithm



Seeing Structure with Pinholes

Consider the density operator for nucleon with spin i and isospin j

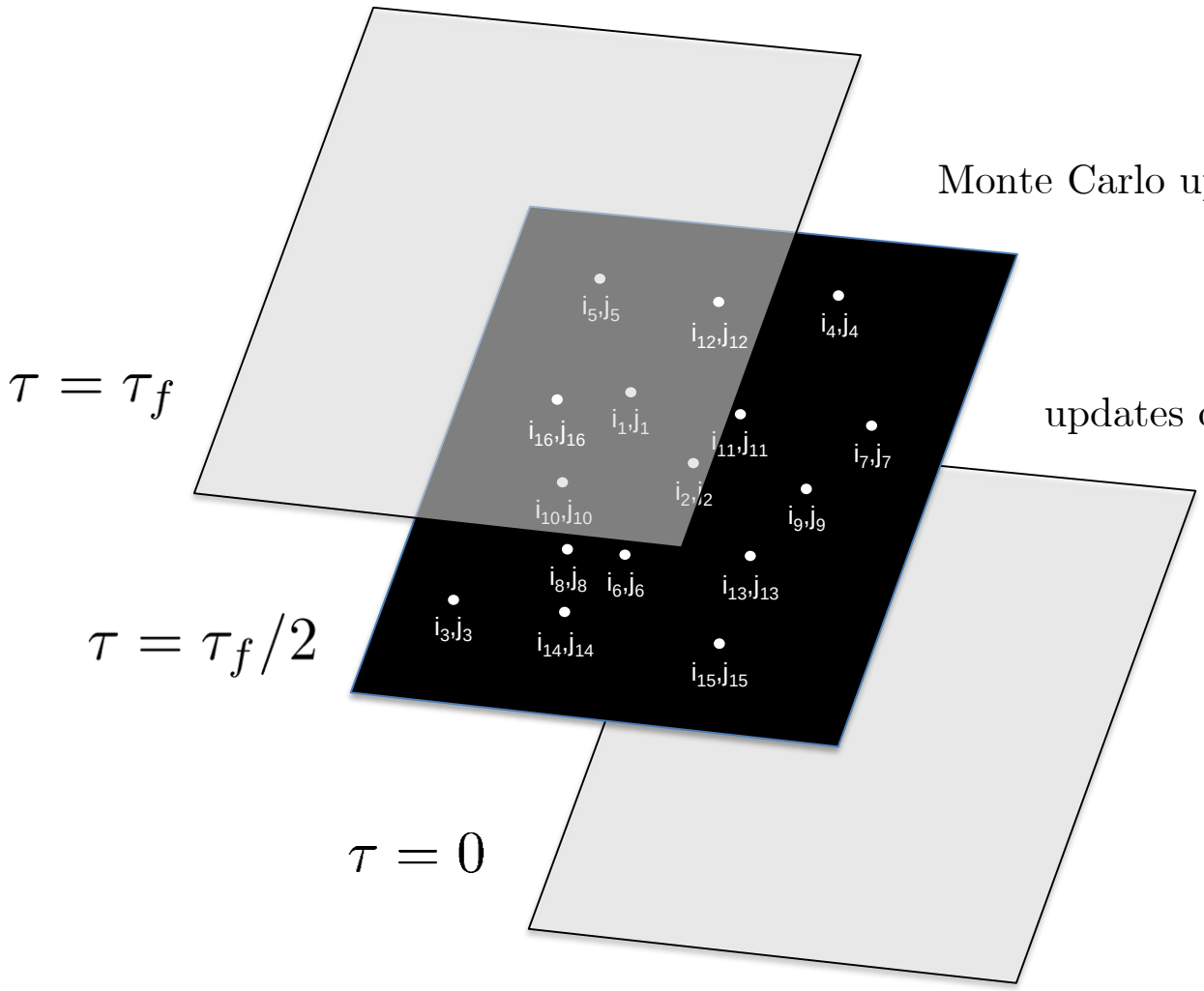
$$\rho_{i,j}(\mathbf{n}) = a^{\dagger}_{i,j}(\mathbf{n}) a_{i,j}(\mathbf{n})$$

We construct the normal-ordered A-body density operator

$$\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots \mathbf{n}_A) = :\rho_{i_1,j_1}(\mathbf{n}_1)\cdots\rho_{i_A,j_A}(\mathbf{n}_A):$$

In the simulations we do Monte Carlo sampling of the amplitude

$$A_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots \mathbf{n}_A,t) = \langle\Psi_I|e^{-Ht/2}\rho_{i_1,j_1,\cdots i_A,j_A}(\mathbf{n}_1,\cdots \mathbf{n}_A)e^{-Ht/2}|\Psi_I\rangle$$

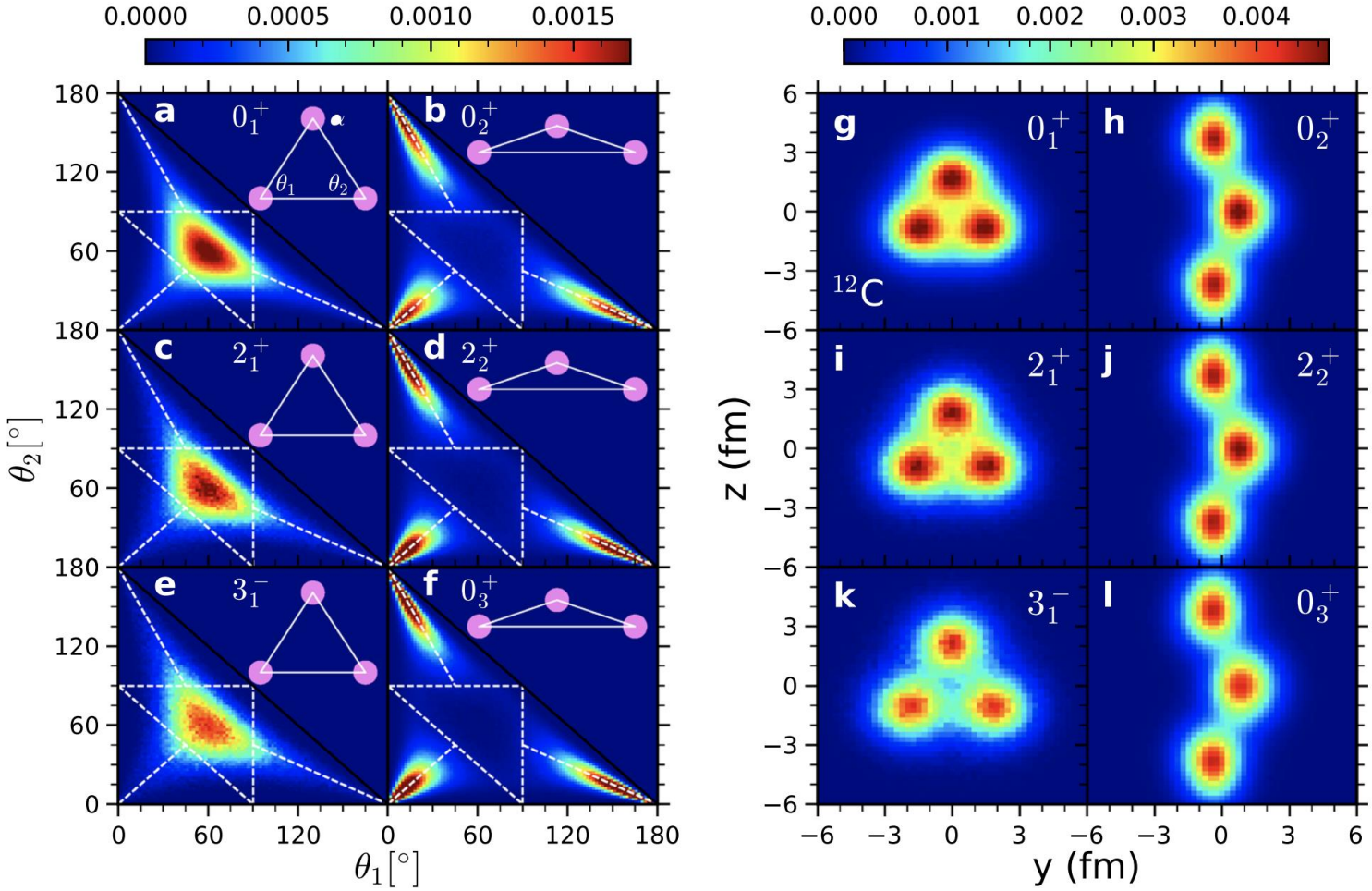


Monte Carlo updates of pinholes

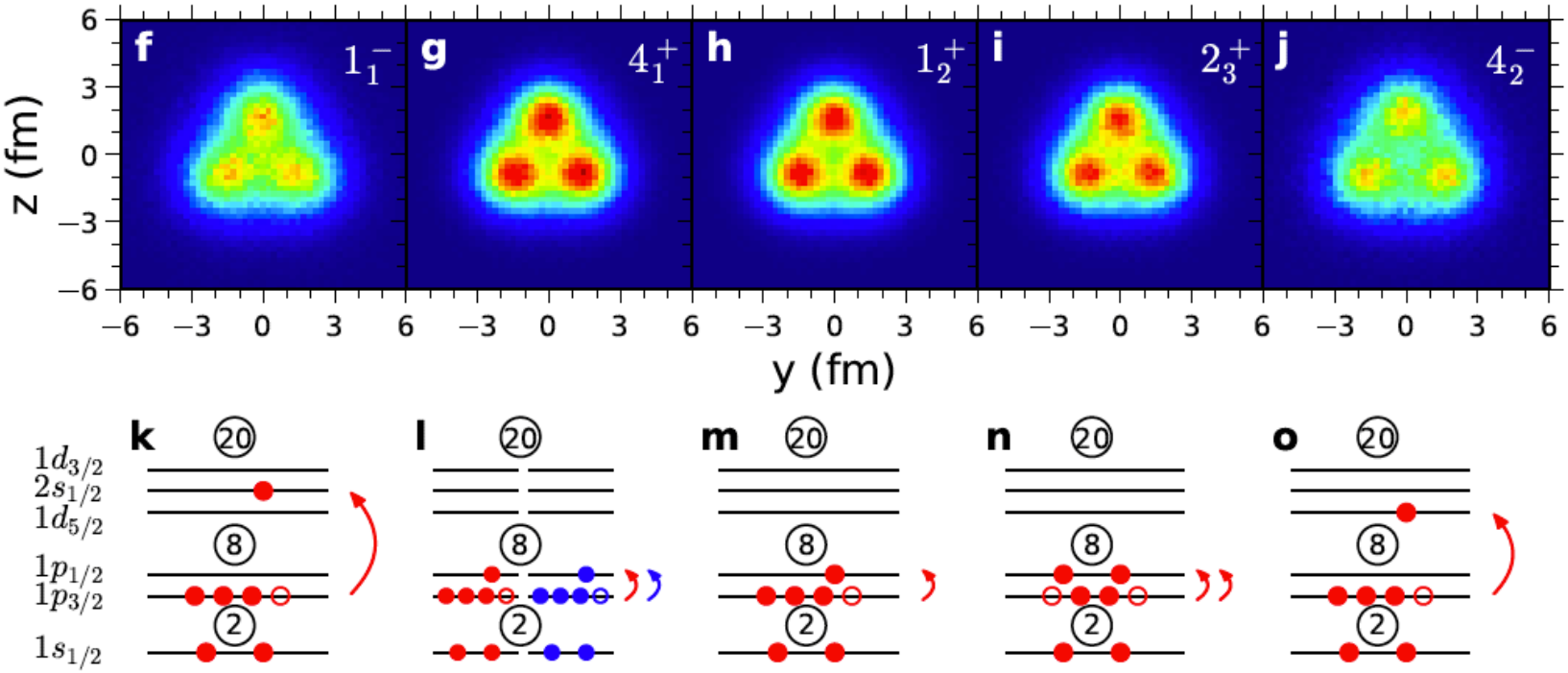
Monte Carlo
updates of auxiliary/pion fields

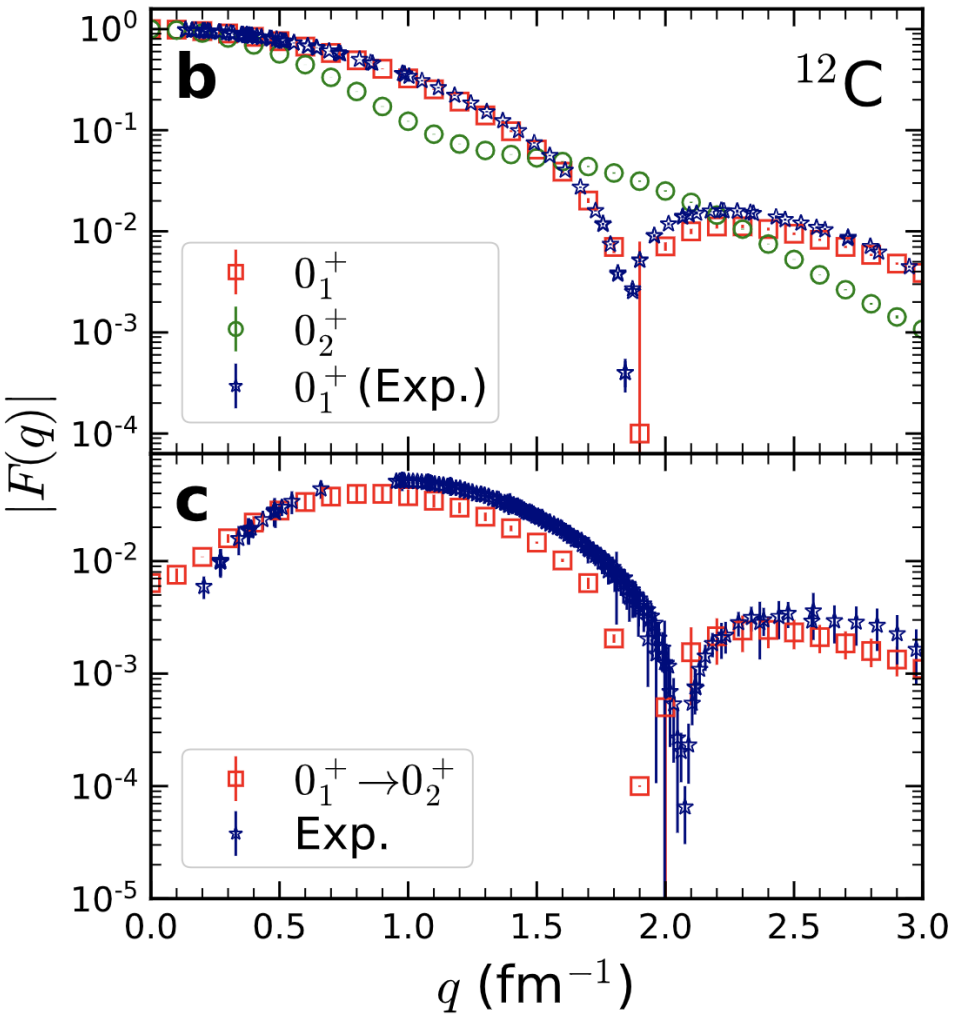
Elhatisari, Epelbaum, Krebs, Lähde, D.L., Li, Lu, Meißner, Rupak, PRL 119, 222505 (2017)

Emergent geometry and duality of ^{12}C

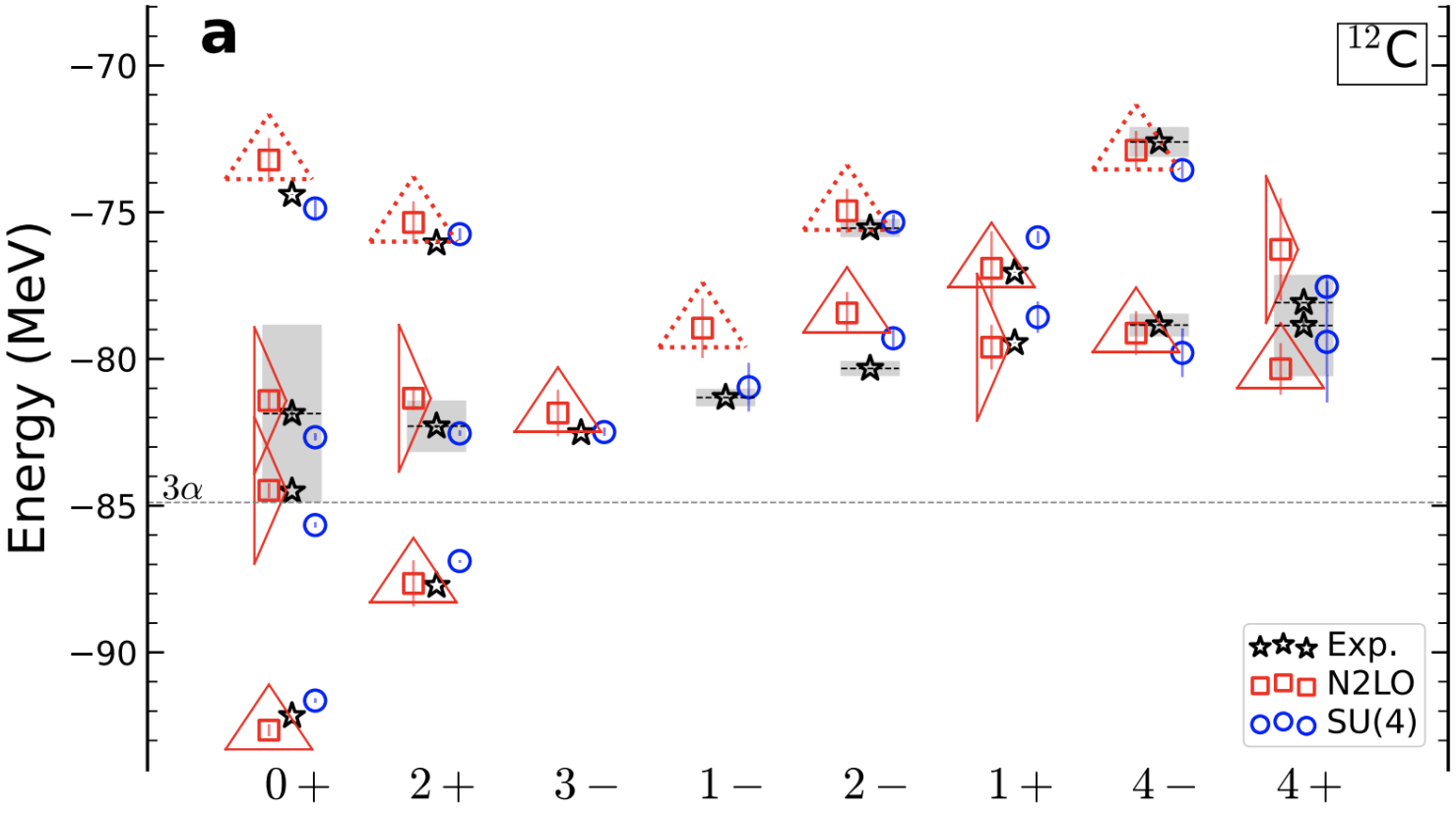


Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)


f
1^-_1
g
4^+_1
h
1^+_2
i
2^+_3
j
4^-_2
z (fm)
y (fm)
6
3
0
−3
−6
k
l
m
n
o
20
8
2
$1d_{3/2}$
$2s_{1/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$

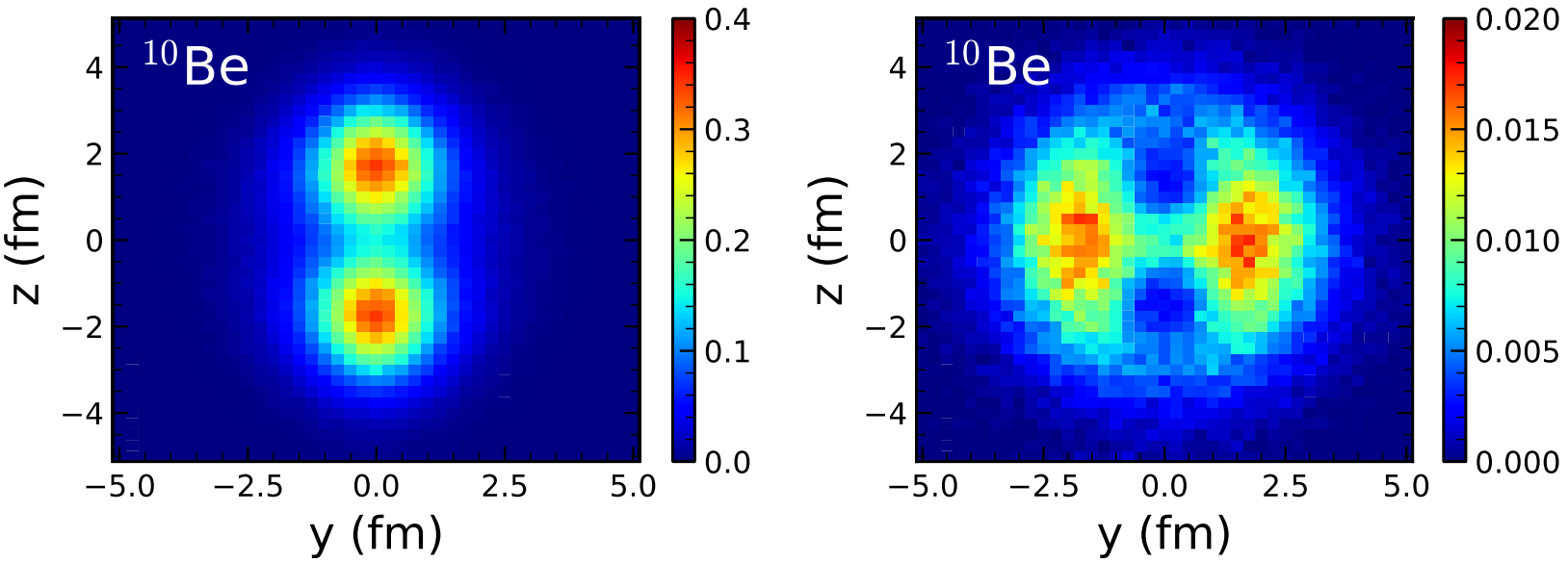


Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)



Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)

Seeing the structure of ^{10}Be



The left panel shows the intrinsic shape of the total nucleon density for ^{10}Be. The right panel shows the density distribution of the two neutrons furthest away from the protons in ^{10}Be

Shen, et al., work in progress

See Ulf's talk

Adiabatic projection method

The adiabatic projection method is a first principles method for scattering and reactions. Strategy is to divide the problem into two parts.

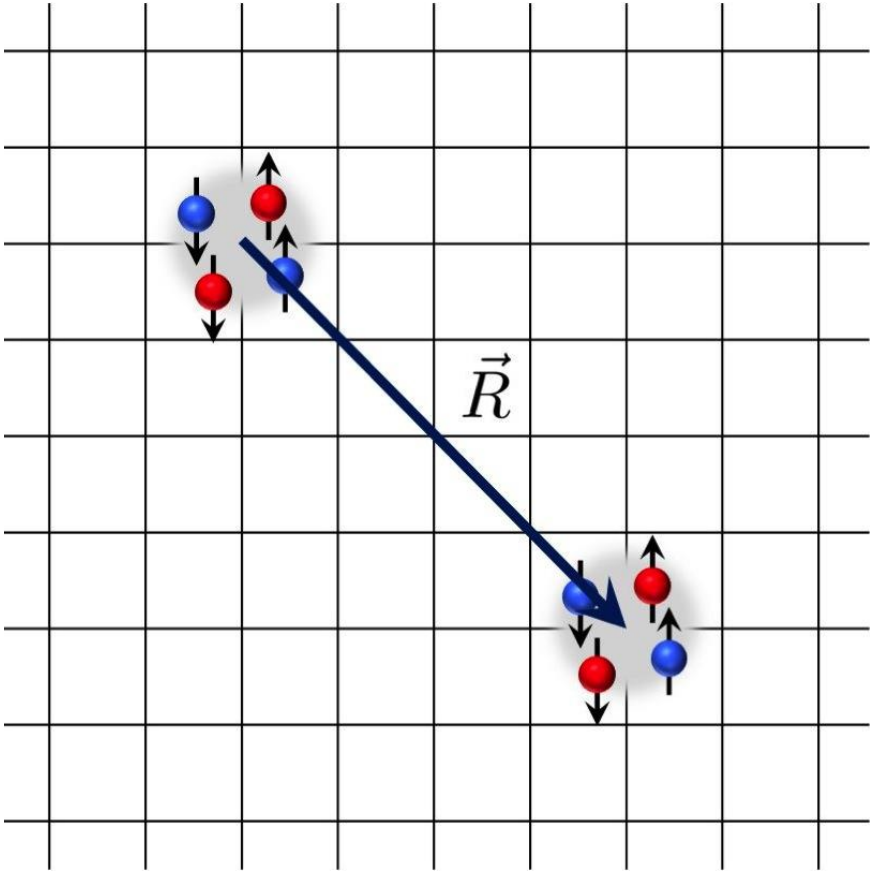
In the first part, use Euclidean time projection and lattice Monte Carlo to derive an ab initio low-energy cluster Hamiltonian, called the adiabatic Hamiltonian.

In the second part, we use the adiabatic Hamiltonian to compute scattering phase shifts or reaction amplitudes.

Elhatisari, D.L., Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, Nature 528, 111 (2015)

Start with localized cluster states for all possible separation vectors $\vec{R}$

$$|\vec{R}\rangle = \sum_{\vec{r}} |\vec{r}+\vec{R}\rangle_1 \otimes |\vec{r}\rangle_2$$

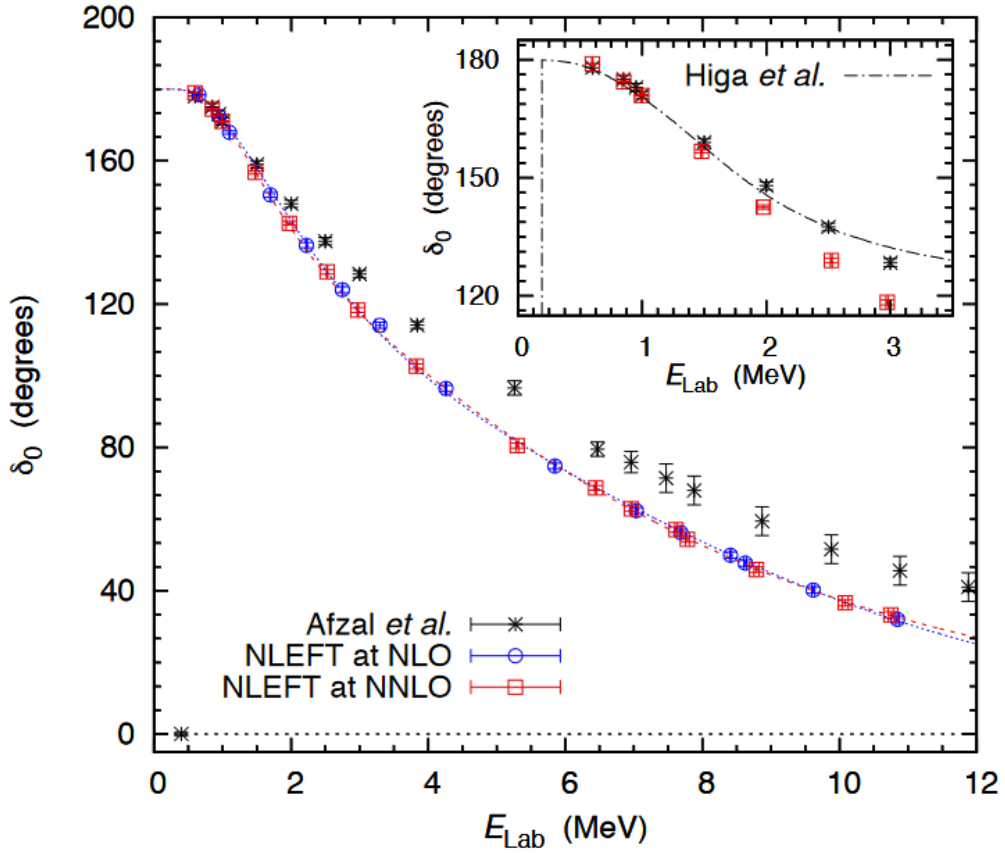


Evolve clusters with Euclidean time

$$|\vec{R}\rangle_\tau = \exp(-H\tau)|\vec{R}\rangle$$

Effective cluster-cluster Hamiltonian constructed from these states

$^4\mathrm{He} + {}^4\mathrm{He} \to {}^4\mathrm{He} + {}^4\mathrm{He}$



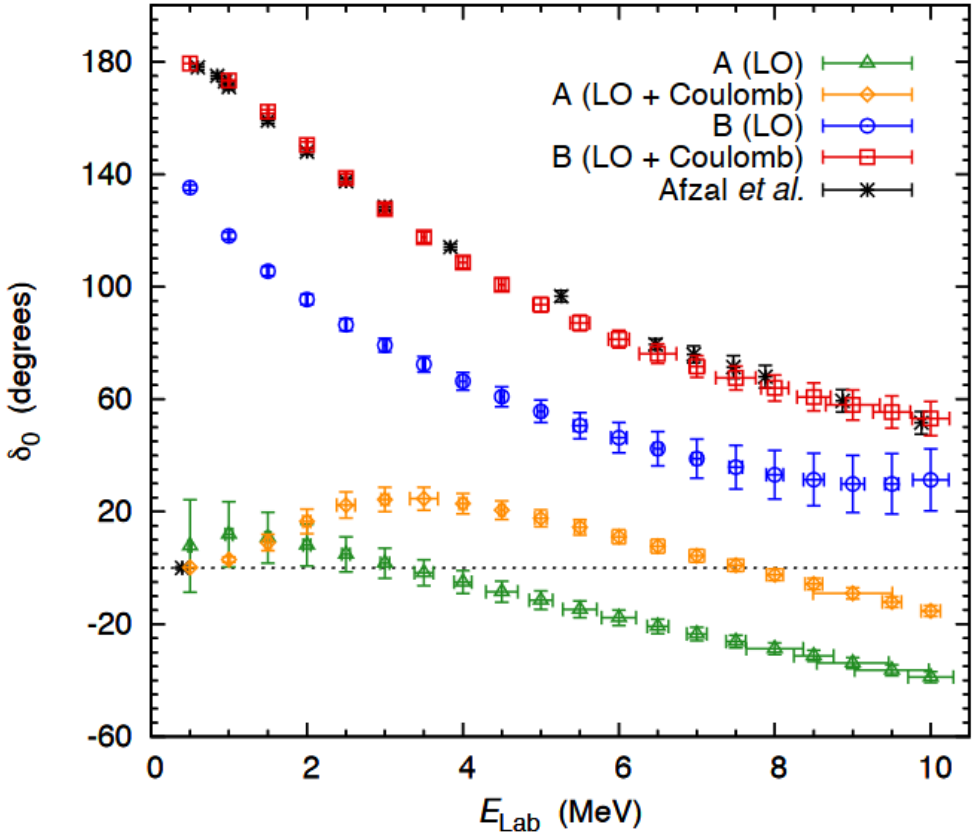
Afzal, Ahmad, Ali, RMP 41 247 (1969)

Higa, Hammer, van Kolck, NPA 809 171(2008)

Elhatisari, D.L., Rupak, Epelbaum, Krebs, Lähde, Luu, Meißner, Nature 528, 111 (2015)

A tale of two interactions

Two different LO interactions, A and B, that are nearly the same for up to four nucleon systems



Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, D.L., Rupak, PRL 117, 132501 (2016)

Nucleus	A (LO)	B (LO)	A (LO + Coulomb)	B (LO + Coulomb)	Experiment
^{8}Be	$-58.61(14)$	$-59.73(6)$	$-56.51(14)$	$-57.29(7)$	-56.591
^{12}C	$-88.2(3)$	$-95.0(5)$	$-84.0(3)$	$-89.9(5)$	-92.162
^{16}O	$-117.5(6)$	$-135.4(7)$	$-110.5(6)$	$-126.0(7)$	-127.619
^{20}Ne	$-148(1)$	$-178(1)$	$-137(1)$	$-164(1)$	-160.645

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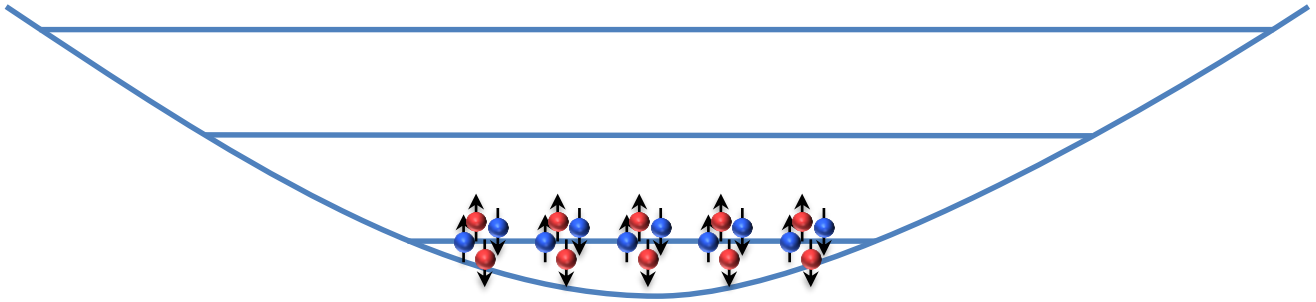
$$\frac{E_{^{8}\mathrm{Be}}}{E_{^{4}\mathrm{He}}} = 1.997(6)$$

$$\frac{E_{^{12}\mathrm{C}}}{E_{^{4}\mathrm{He}}} = 3.00(1)$$

$$\frac{E_{^{16}\mathrm{O}}}{E_{^{4}\mathrm{He}}} = 4.00(2)$$

$$\frac{E_{^{20}\mathrm{Ne}}}{E_{^{4}\mathrm{He}}} = 5.03(3)$$

Bose condensate of alpha particles!



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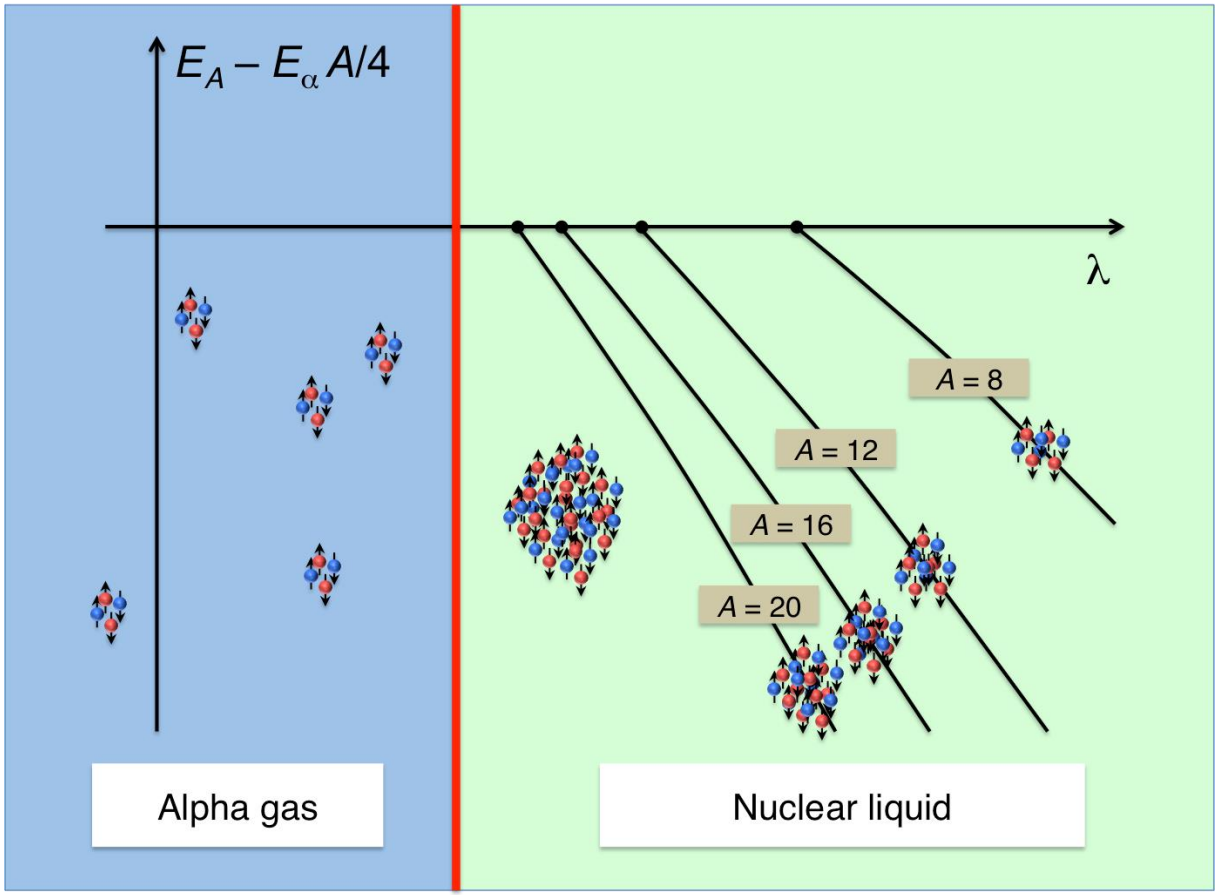
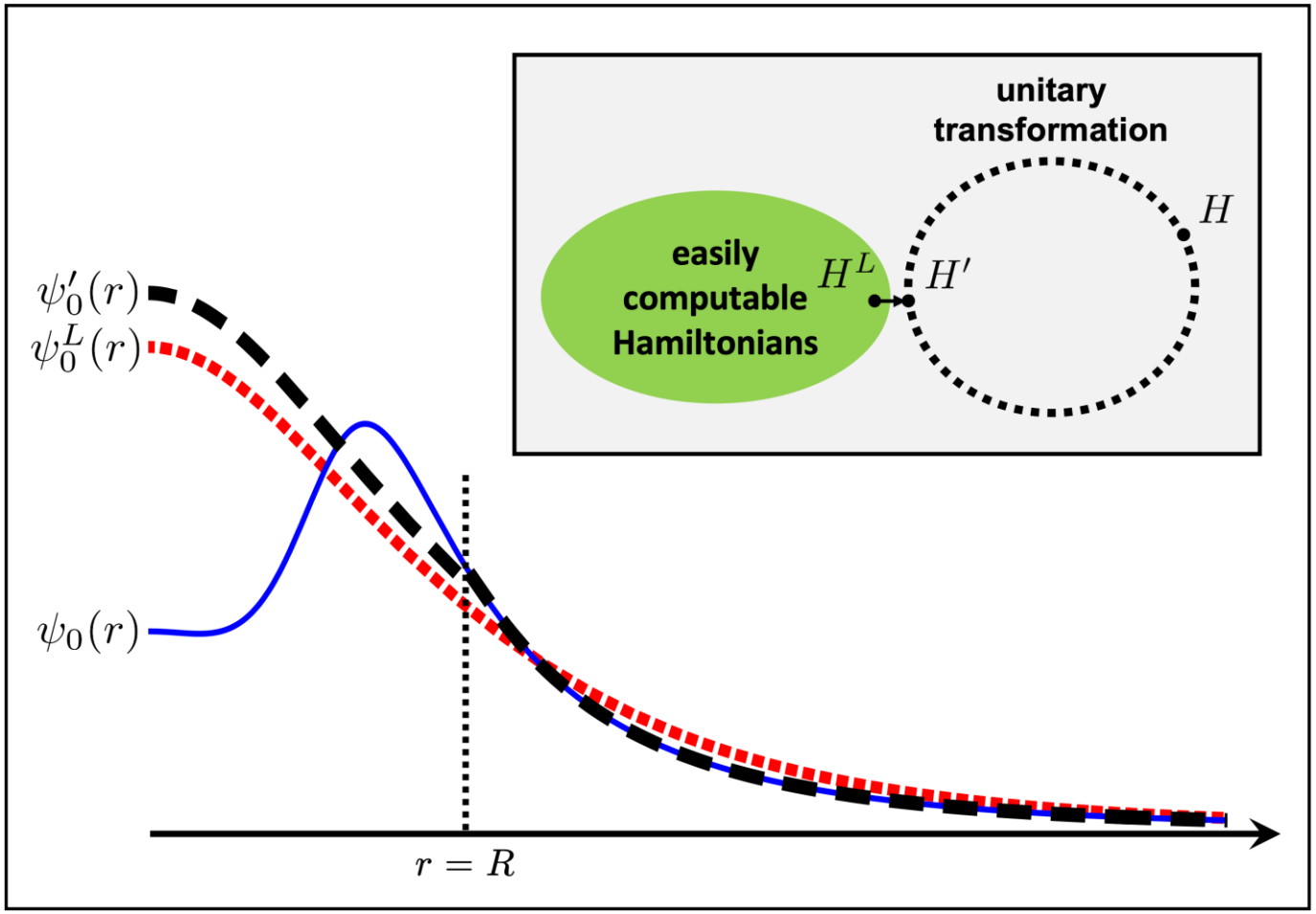


Elhatisari, Li, Rokash, Alarcon, Du, Klein, Lu, Meißner, Epelbaum, Krebs, Lähde, D.L., Rupak, PRL 117, 132501 (2016)

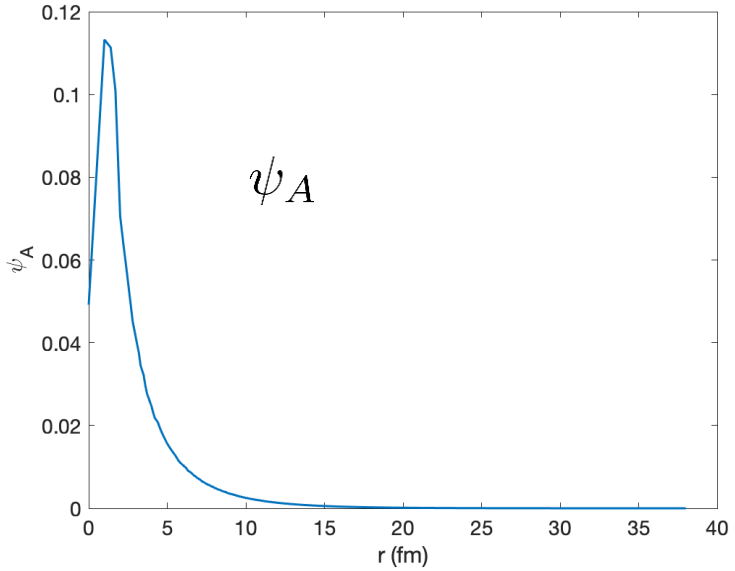
See Ulf's talk

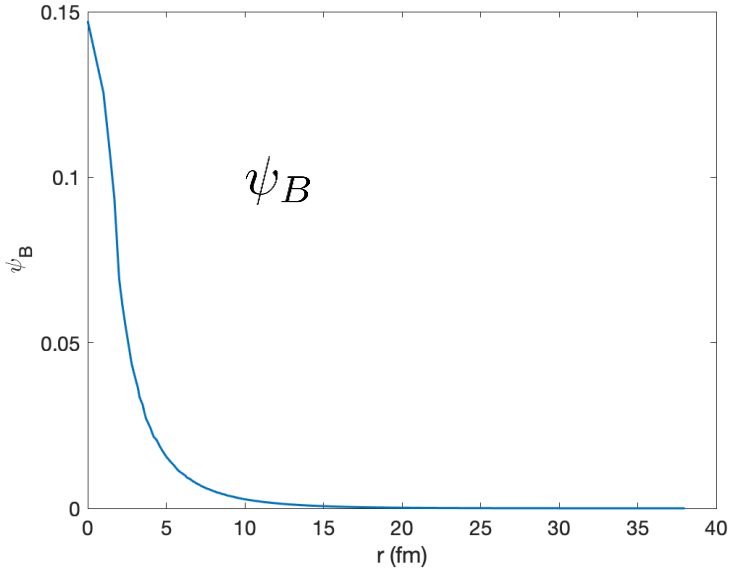
Wave function matching

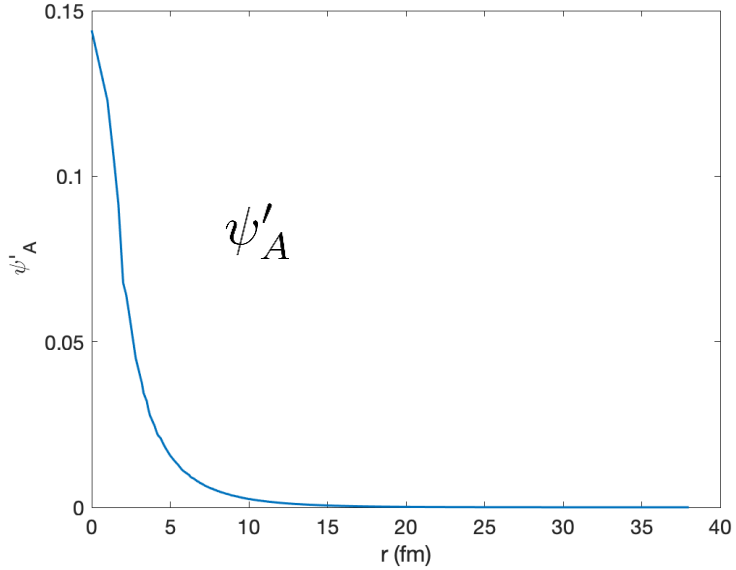


Elhatisari et al., Nature, in press

Ground state wave functions



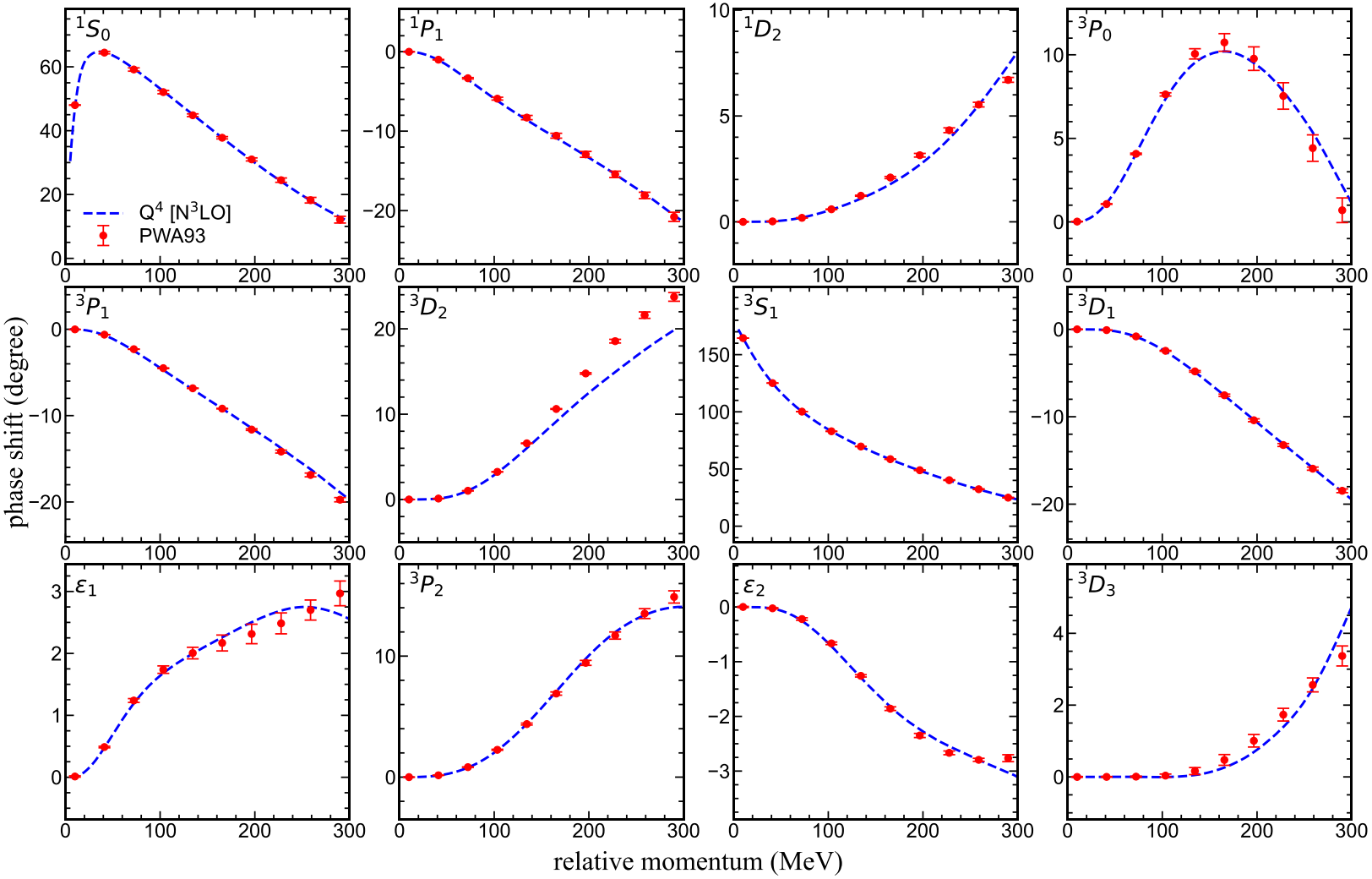




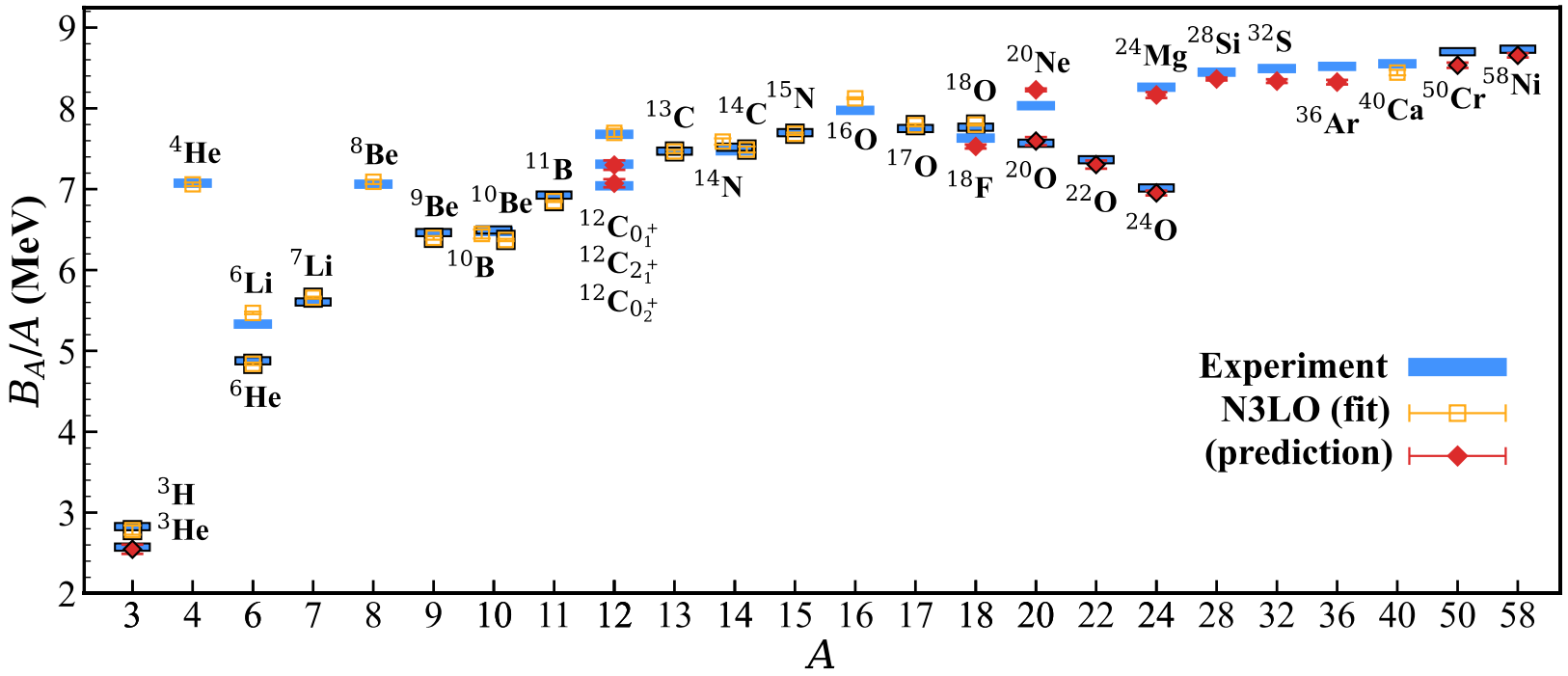
With wave function matching, we can now compute the eigenenergies starting from the eigenfunctions of H_B and using first-order perturbation theory.

		$R = 2.6$ fm
$E_{A,n} = E'_{A,n}$ (MeV)	$\langle\psi_{B,n}\|H_A\|\psi_{B,n}\rangle$ (MeV)	$\langle\psi_{B,n}\|H'_A\|\psi_{B,n}\rangle$ (MeV)
-1.2186	3.0088	-1.1597
0.2196	0.3289	0.2212
0.8523	1.1275	0.8577
1.8610	2.2528	1.8719
3.2279	3.6991	3.2477
4.9454	5.4786	4.9798
7.0104	7.5996	7.0680
9.4208	10.0674	9.5137
12.1721	12.8799	12.3163
15.2669	16.0458	15.4840

N3LO chiral effective field theory interaction

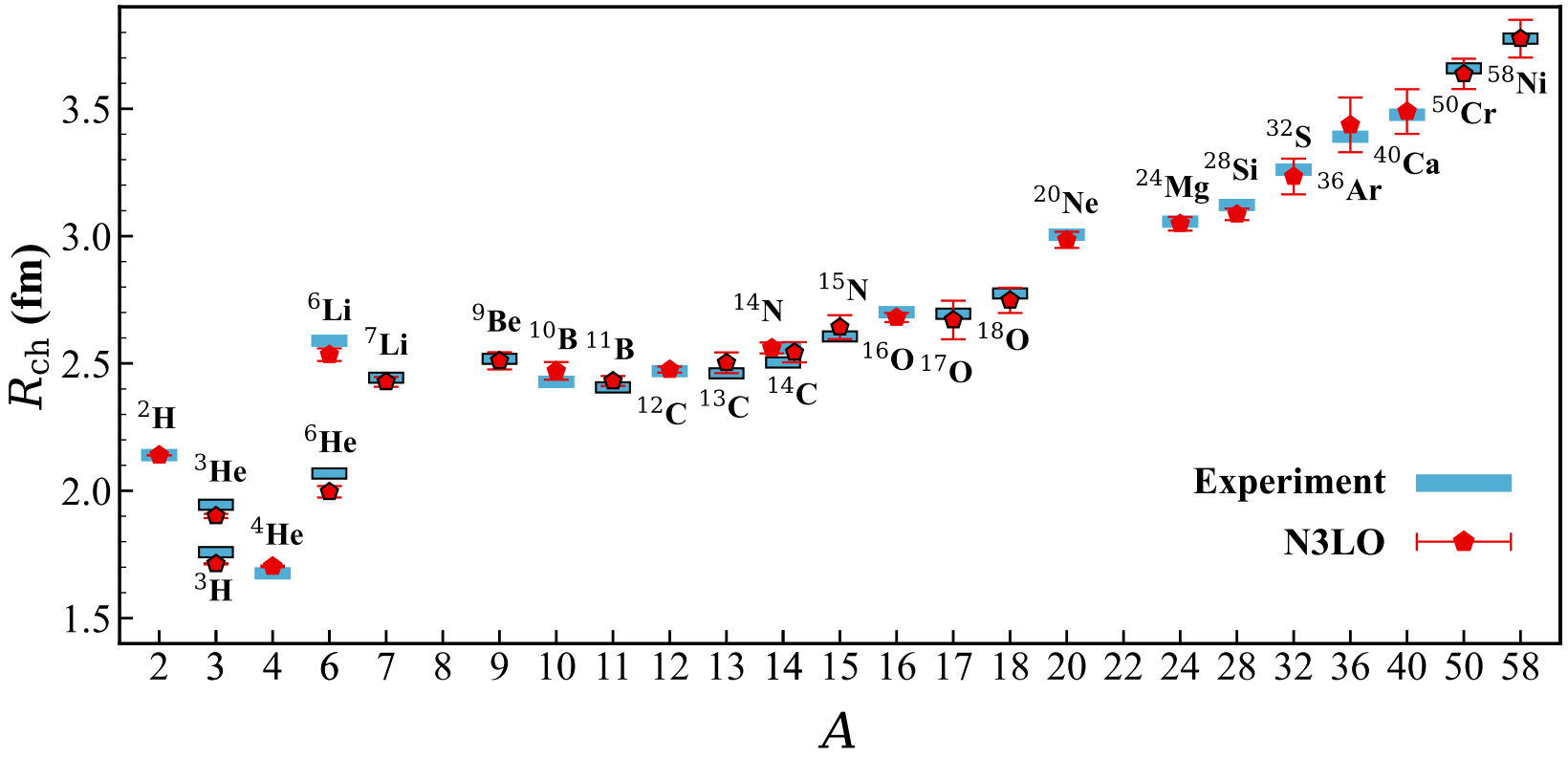


Binding energy per nucleon



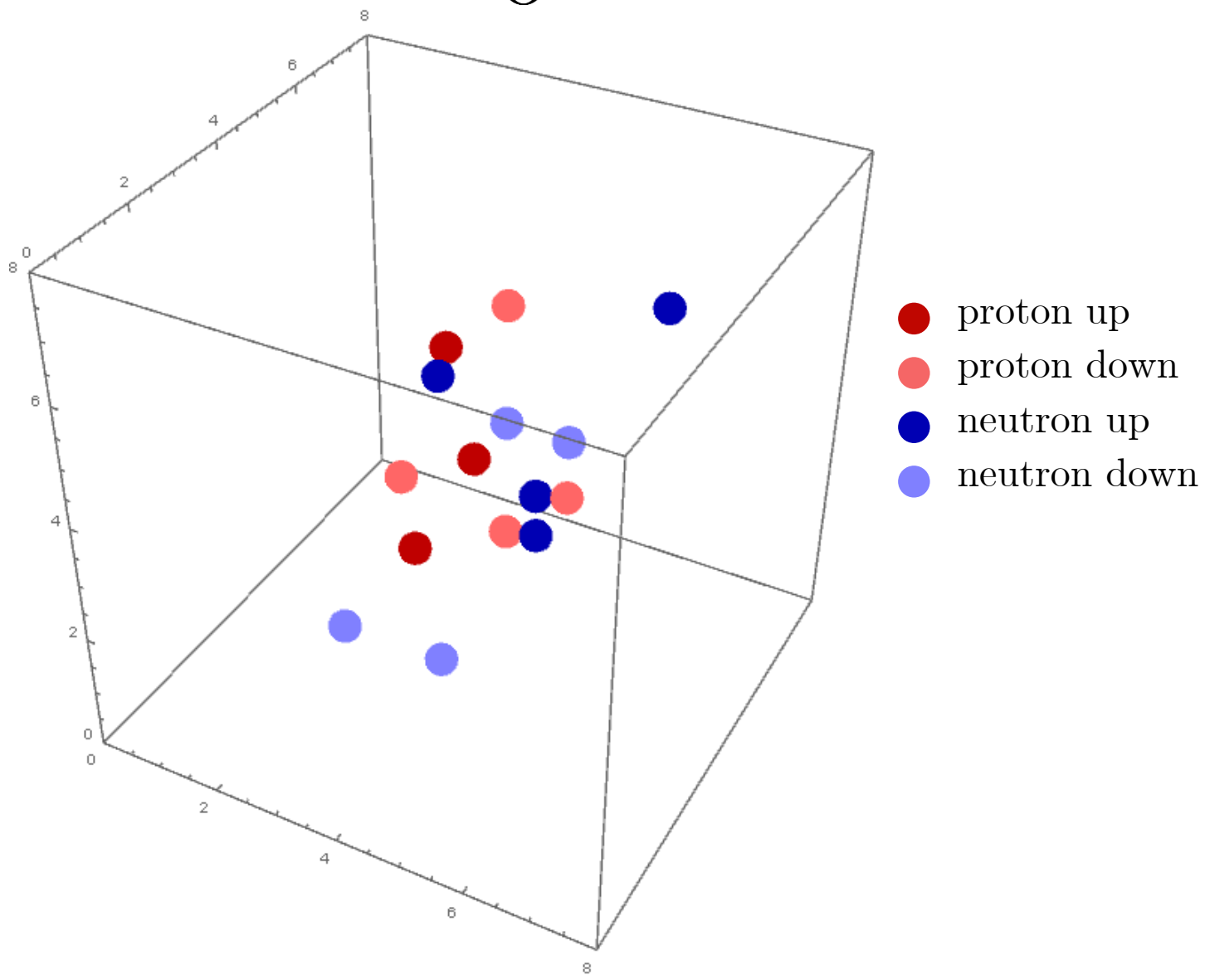
Elhatisari et al., Nature, in press

Charge radius



Elhatisari et al., Nature, in press

^{16}O



^{16}O^{16}O collisions at RHIC and LHC energies

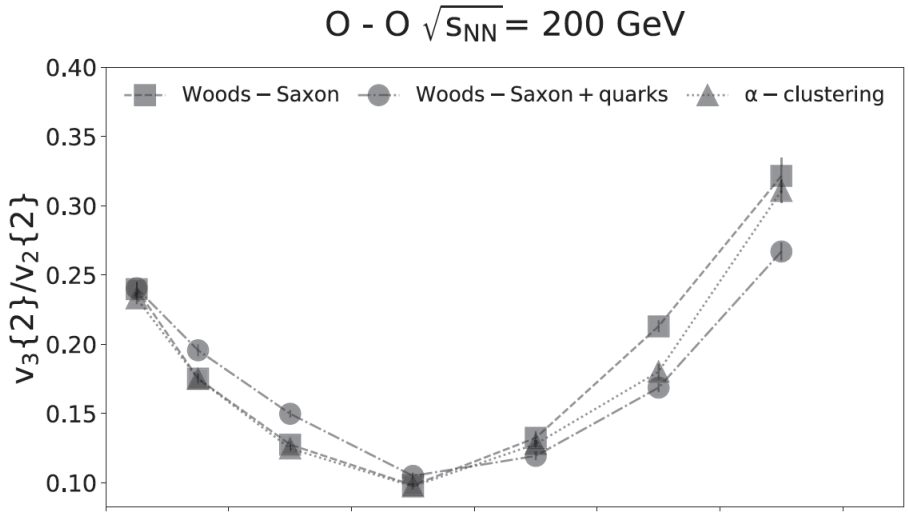
Summerfield, Lu, Plumberg, D.L., Noronha-Hostler, Timmins PRC 104, L041901 (2021)

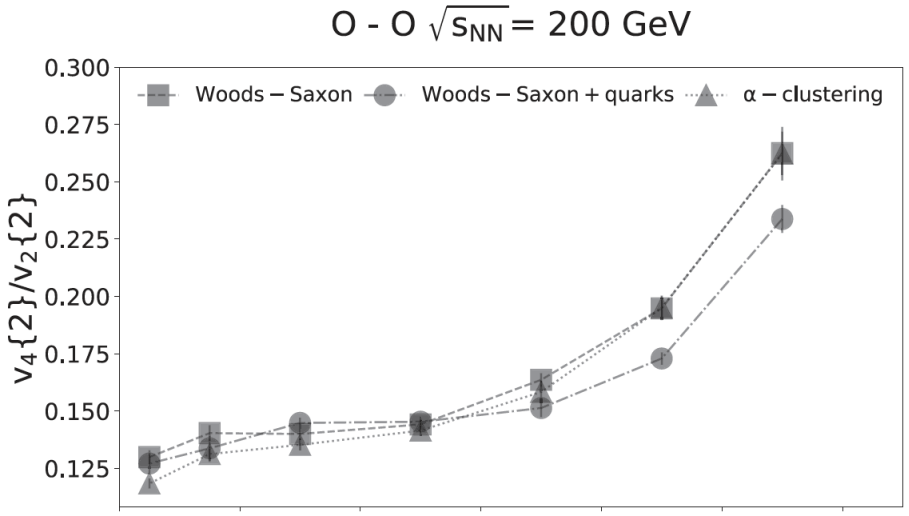
We use T$_{\mathrm{R}}$ENTo to generate the initial entropy distribution and compute the following cumulants of the flow harmonics v_n:

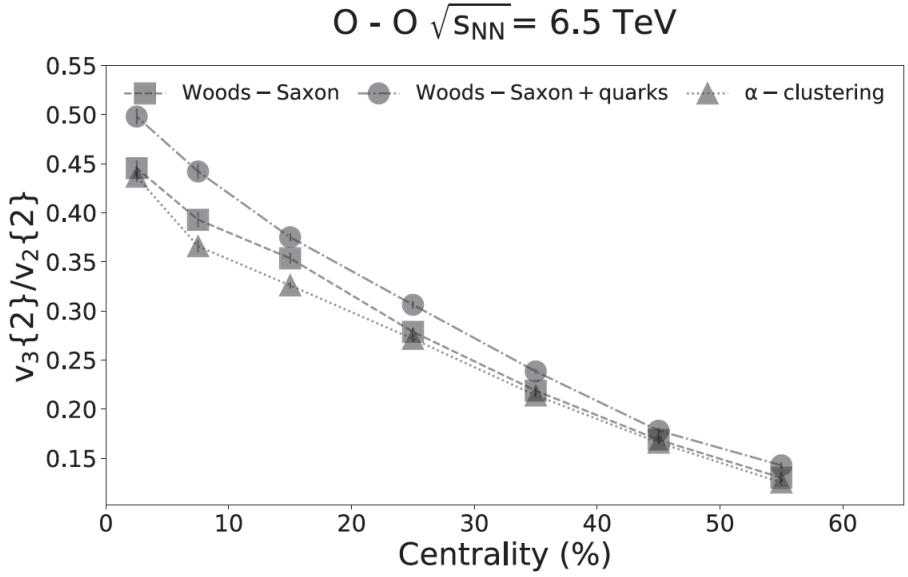
$$v_n\{2\} = \left[\left\langle v_n^2 \right\rangle\right]^{\frac{1}{2}}$$
$$v_n\{4\} = \left[2\left\langle v_n^2 \right\rangle^2 - \left\langle v_n^4 \right\rangle\right]^{\frac{1}{4}}$$

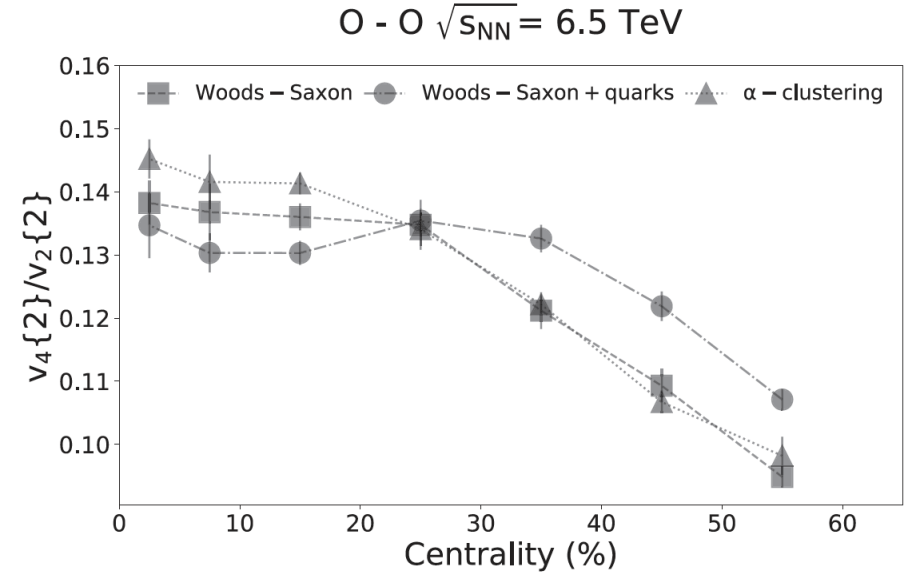
We first compute results taking the initial density as a Woods-Saxon potential with density, radius, and diffusive fitted to empirical values. We then consider the same Woods-Saxon potential, taking into account the quark substructure of the nucleons. Lastly, we consider using the nucleon distribution from the lattice effective field theory calculations. This will incorporate correlations such as alpha clustering.

Summerfield, Lu, Plumberg, D.L., Noronha-Hostler, Timmins PRC 104, L041901 (2021)





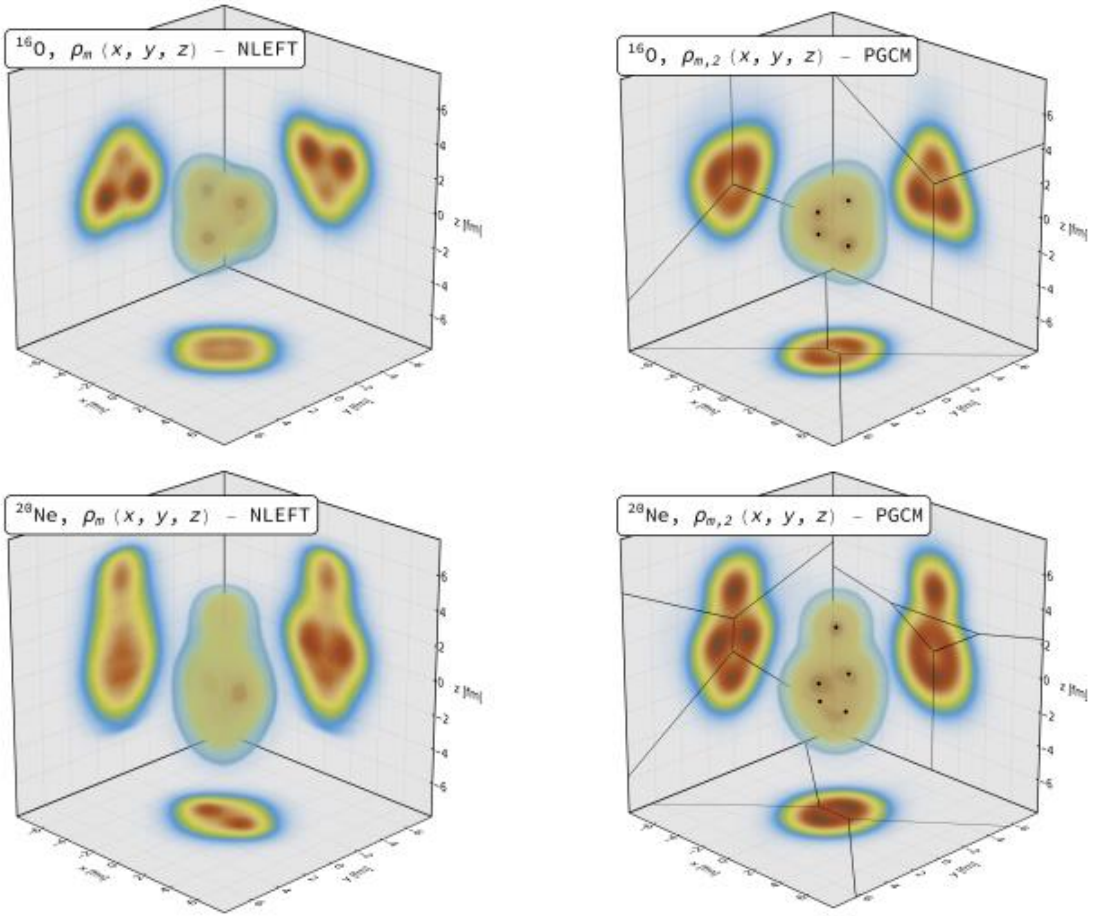




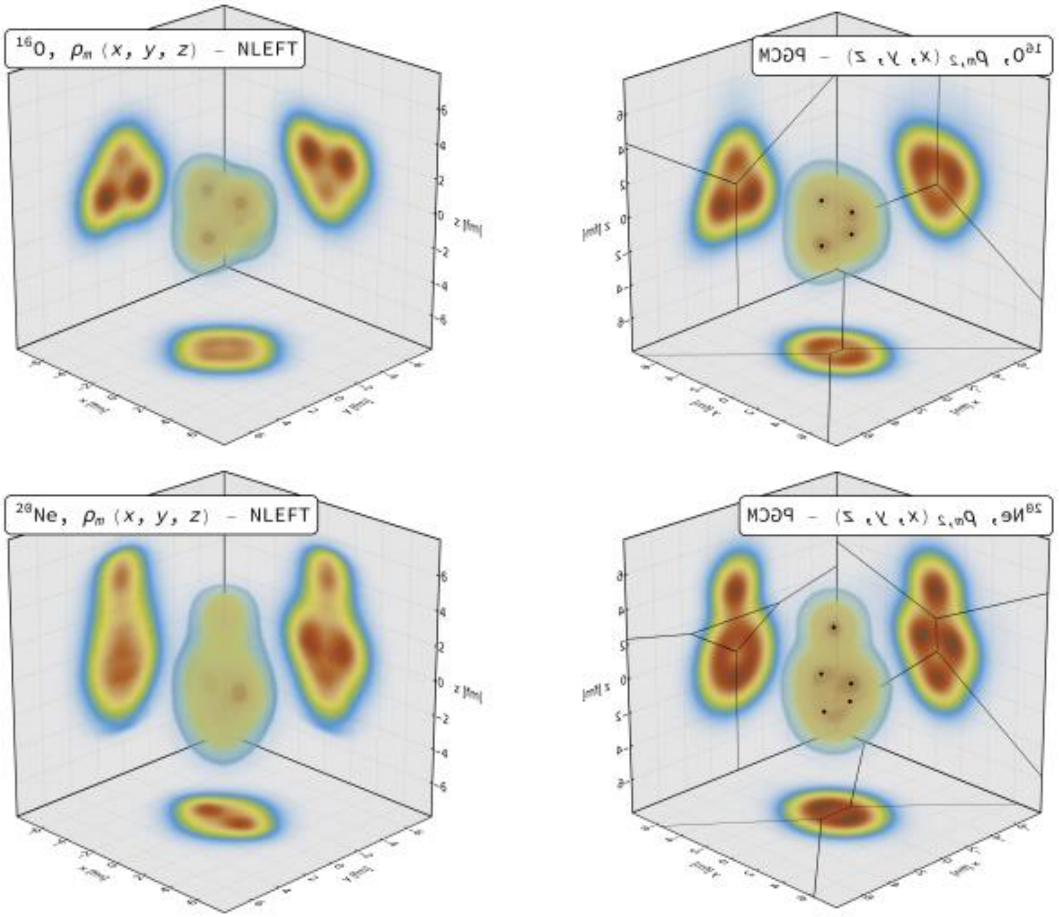
Summerfield, Lu, Plumberg, D.L., Noronha-Hostler, Timmins PRC 104, L041901 (2021)

See Govert's and Wilke's talks

^{16}O^{16}O versus ^{20}Ne^{20}Ne collisions



Giacalone et al., arXiv:2402.05995



Giacalone et al., arXiv:2402.05995

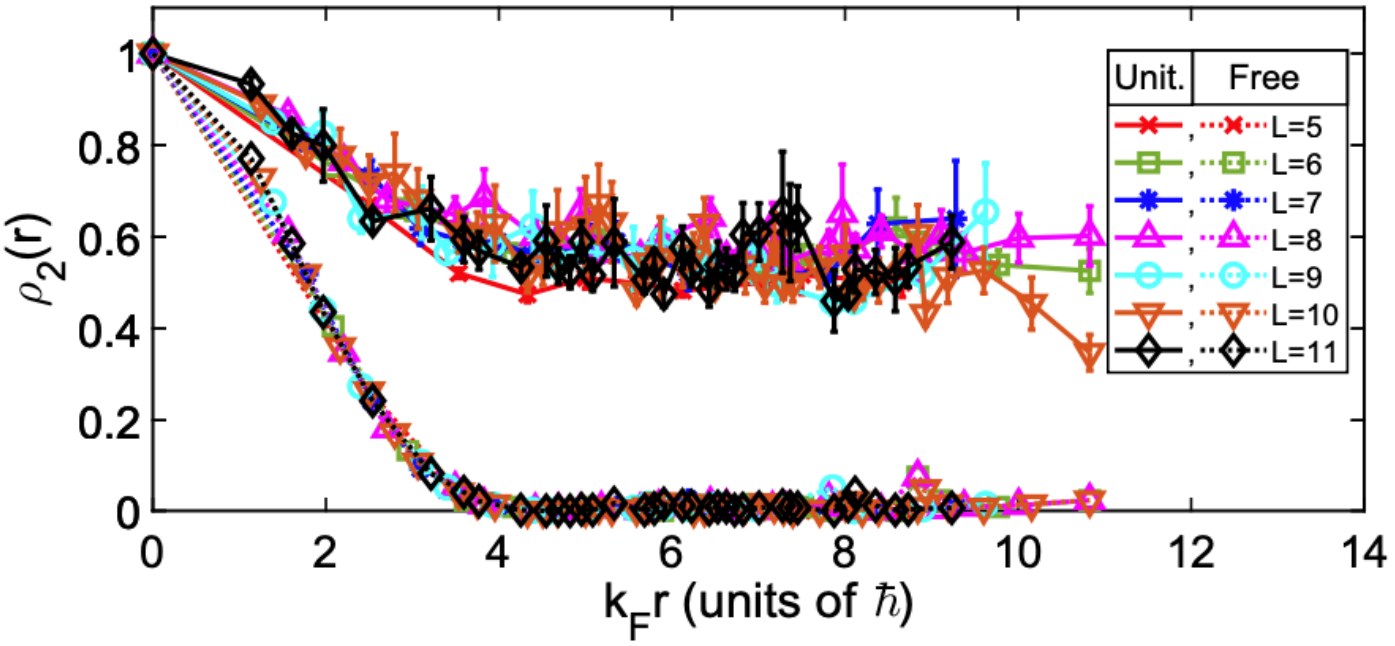
See Govert's and Wilke's talks

For the 1% most central events, the elliptic flow of ^{20}Ne^{20}Ne collisions is enhanced by as much as 1.170(8)stat.(30)syst. for NLEFT and 1.139(6)stat.(39)syst. for PGCM relative to ^{16}O^{16}O collisions.

Giacalone et al., arXiv:2402.05995

Superfluidity

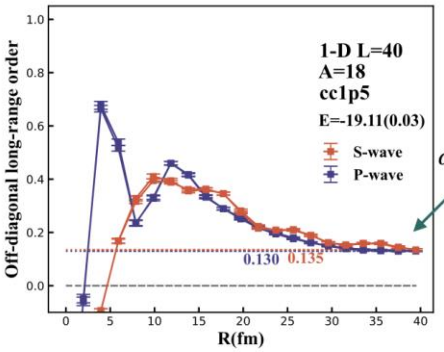
Ground state S-wave superfluid long-range order in the unitary limit



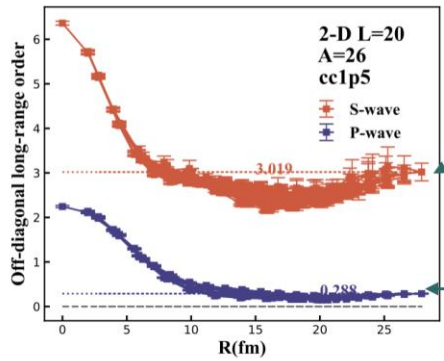
He, Li, Lu, D.L., Phys. Rev. A 101, 063615 (2020)

Multi-modal superfluidity; S-wave pairs and P-wave pairs

Off-diagonal long-range order

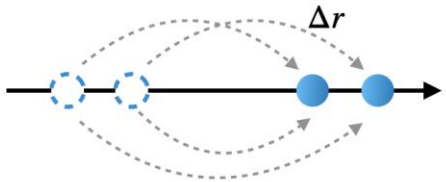


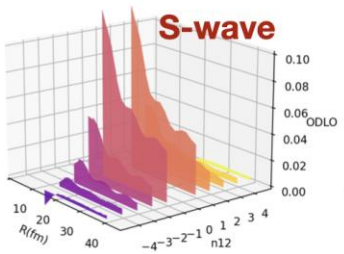
$$\alpha \approx \frac{0.13}{18/2} = 0.014$$



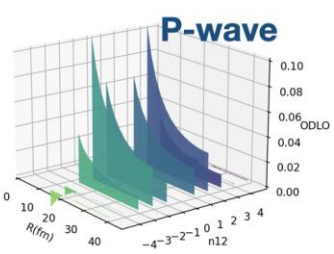
$$\alpha \approx \frac{3}{26/2} = 0.23$$

$$\alpha \approx \frac{0.29}{26/2} = 0.022$$





max @ |r_12|=1



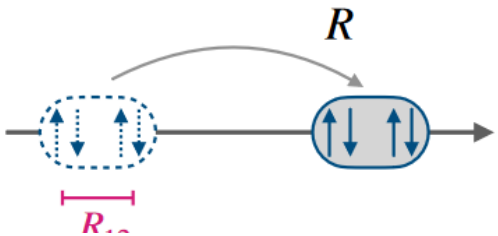
max @ |r_12|=2

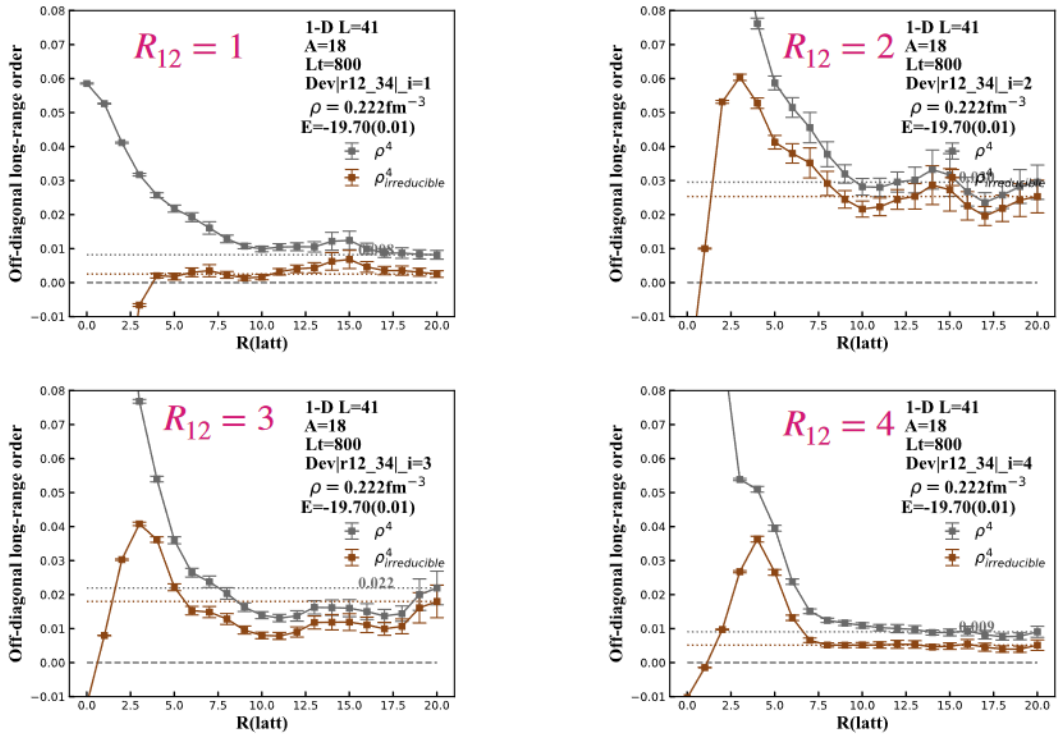
condensate fraction

$$\rho_2(\mathbf{r}_1', \mathbf{r}_2', \mathbf{r}_1, \mathbf{r}_2) = \alpha N/2 \cdot \phi^*(|\mathbf{r}_1' - \mathbf{r}_2'|)\phi(|\mathbf{r}_1 - \mathbf{r}_2|)$$

Ma et al., work in progress

Multi-modal superfluidity; S-wave pairs, P-wave pairs, and quartets





Ma et al., work in progress

Outlook

This talk started with an overview of lattice EFT and calculations of nuclear structure and scattering and emergent phenomena such as clustering and superfluidity. We then focused on how pinhole configurations provide *ab initio* data for initial states in relativistic heavy-ion collisions.

In a few years, it will be possible to do ab initio calculations of nuclear states across the nuclear chart using lattice EFT. We look forward to collaborating with our relativistic heavy-ion colleagues to explore this new field bringing together two different nuclear science communities.