双重味矢量介子B_c^*半轻衰变研究

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目录

一、研究动机

B_c^*

- $B = 1, C = 1$
- $n^{2s+1}L_J = 1^3S_1\ , J^P = 1^-$
- $m_{B_c^*} < m_B + m_D$
- $m_{B_c^*} - m_{B_c} \approx 50\text{Mev}$

- B_c^* 介子通过强相互作用衰变是严格禁闭的，B_c^* 介子主要通过$B_c^* \to B_c\gamma$过程发生衰变。

- B_c^* 介子弱衰变分为三类:
  1) b夸克衰变
  2) c夸克衰变
  3) b夸克和c夸克湮灭

- 目前还没有B_c^*介子相关的实验测量结果。

一、研究动机

- Nonleptonic $B_c^* \to BP, BV, B^*P, \psi P, \eta_c P, \eta_c V$ decays using naïve factorization approach, CPC (2023) 013110

- Nonleptonic $B_c^* \to BP, BV$ decays using QCD factorization approach, PRD 95 (2017) 7, 074032

- Nonleptonic $B_c^* \to \psi(1S, 2S)P, \eta_c(1S, 2S)$ decays using perturbative QCD approach, PRD 95 (2017) 3, 036024

- Semileptonic $B_c^* \to B\ell\nu, B_s\ell\nu, D\ell\nu, \eta_c\ell\nu, B^*\ell\nu, B_s^*\ell\nu, D^*\ell\nu, J/\psi\ell\nu$,decays with Bethe-Salpeter method, JPG45(2018)115001

- Semileptonic $B_c^* \to \eta_c\ell\nu, J/\psi\ell\nu$ decays with heavy quark symmetry, PRD98(2018)036004

- Semileptonic $B_c^* \to J/\psi\ell\nu$ decays, Advances in high energy physics (2020)3079670

二、理论框架

在标准模型中，通过$b \to q_1 \ell \bar{\nu}$或$c \to q_2 \ell^+ \nu$ 过程发生的$B_c^* \to P(V)\ell\bar{\nu}$衰变的低能有效哈密顿量写为：

$$
\begin{aligned}
\mathcal{H}_{\rm eff} &= \frac{G_F}{\sqrt{2}} V_{q_1 b}[\bar{q}_1 \gamma_\mu (1-\gamma_5) b][\bar{\ell}\gamma^\mu (1-\gamma_5)\nu_\ell] \\
&+ \frac{G_F}{\sqrt{2}} V_{cq_2}^* [\bar{q}_2 \gamma_\mu (1-\gamma_5) c][\bar{\nu}_\ell \gamma^\mu (1-\gamma_5)\ell],
\end{aligned}
$$

以$b \to c\ell\bar{\nu}$过程为例，则$B_c^* \to M\ell\bar{\nu}$ (M表示P或V)的矩阵元的平方为

$$
\begin{aligned}
|\mathcal{M}(\bar{B}_c^* \to M\ell\bar{\nu}_\ell)|^2 &= \frac{G_F^2 |V_{cb}|^2}{2} |\langle M|\bar{c}\gamma_\mu(1-\gamma_5)b|\bar{B}_c^*\rangle \, \bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell|^2 \\
&\equiv \frac{G_F^2 |V_{cb}|^2}{2} L_{\mu\nu} H^{\mu\nu} .
\end{aligned}
$$

二、理论框架

插入极化矢量的完备集

$$\sum_{m,n} \bar{\epsilon}_\mu(m)\bar{\epsilon}^*_\nu(n) g_{mn} = g_{\mu\nu} \, ,$$

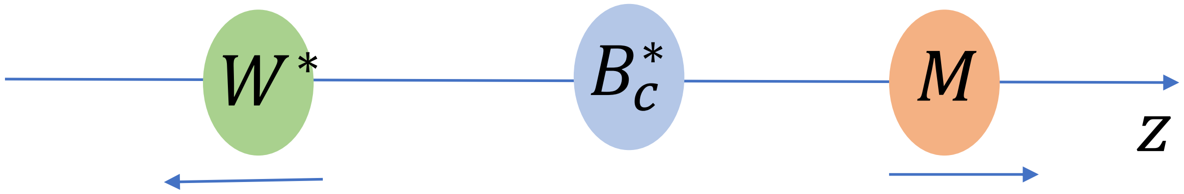
$L_{\mu\nu}H^{\mu\nu}$可以写为如下形式:

$$L_{\mu\nu}H^{\mu\nu} = \sum_{m,m',n,n'} L(m,n)H(m',n')g_{mm'}g_{nn'} \, ,$$

$L(m,n) = L^{\mu\nu}\bar{\epsilon}_\mu(m)\bar{\epsilon}^*_\nu(n)$和$\mathrm{H}(m,n) = H^{\mu\nu}\bar{\epsilon}^*_\mu(m)\bar{\epsilon}_\nu(n)$是洛仑兹不变的。

二、理论框架

强子部分运动学



$$p^{\mu}_{B_c^*} = (m_{B_c^*}, 0, 0, 0)$$

$$p^{\mu}_{M} = (E_M, 0, 0, |\vec{p}|)$$

$$q^{\mu} = (q^0, 0, 0, -|\vec{p}|)$$

$$\bar{\epsilon}^{\mu}(t) = \frac{1}{\sqrt{q^2}}(q_0, 0, 0, -|\vec{p}|)$$

$$\bar{\epsilon}^{\mu}(0) = \frac{1}{\sqrt{q^2}}(|\vec{p}|, 0, 0, -q_0)$$

$$\bar{\epsilon}^{\mu}(\pm) = \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0)$$

$$\epsilon^{\mu}_1(0) = (0, 0, 0, 1)$$

$$\epsilon^{\mu}_1(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$

$$\epsilon^{\mu}_2(0) = \frac{1}{m_M}(|\vec{p}|, 0, 0, E_M)$$

$$\epsilon^{\mu}_2(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$

强子螺旋度振幅(M for P)

$$H_{\lambda_{B^*}\lambda_{W^*}} = H_\mu(\lambda_{B^*})\,\bar{\epsilon}^{*\mu}(\lambda_{W^*})$$

$$\langle M(p_M)|\bar{c}\gamma_\mu b|\bar{B}_c^*(\epsilon_1, p_{B_c^*})\rangle = -\frac{2iV(q^2)}{m_{B_c^*}+m_M}\varepsilon_{\mu\nu\alpha\beta}\epsilon_1^\nu p_M^\alpha p_{B_c^*}^\beta$$

$$\langle M(p_M)|\bar{c}\gamma_\mu\gamma_5 b|\bar{B}_c^*(\epsilon_1, p_{B_c^*})\rangle = 2m_{B_c^*}A_0(q^2)\frac{\epsilon_1\cdot q}{q^2}q_\mu + (m_{B_c^*}+m_M)A_1(q^2)\left(\epsilon_{1\mu} - \frac{\epsilon_1\cdot q}{q^2}q_\mu\right)$$
$$+A_2(q^2)\frac{\epsilon_1\cdot q}{m_{B_c^*}+m_M}\left[P_\mu - \frac{m_{B_c^*}^2 - m_M^2}{q^2}q_\mu\right]$$

The amplitudes $H_{0t}(q^2), H_{00}(q^2), H_{\pm\mp}(q^2)$ with $\lambda_{B_c^*} = \lambda_M - \lambda_{W^*}$ survive.

二、理论框架

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强子螺旋度振幅(M for V)

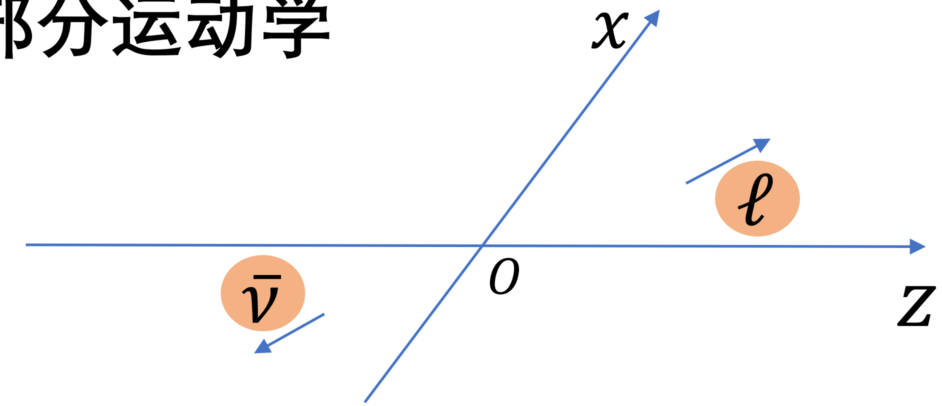
$$H_{\lambda_{W^*}\lambda_{B_c^*}\lambda_M} = \langle M(p_M,\lambda_M)|\bar{c}\gamma_\mu(1-\gamma_5)b|\bar{B}_c^*(p_{B_c^*},\lambda_{B_c^*})\rangle\,\bar{\epsilon}^{*\mu}(\lambda_{W^*})$$

$$\begin{aligned}
\langle M(\epsilon_2,p_M)|\bar{c}\gamma_\mu b|\bar{B}_c^*(\epsilon_1,p_{B_c^*})\rangle &= (\epsilon_1\cdot\epsilon_2^*)\left[-P_\mu V_1(q^2)+q_\mu V_2(q^2)\right]+\frac{(\epsilon_1\cdot q)(\epsilon_2^*\cdot q)}{m_{B_c^*}^2-m_M^2}\left[P_\mu V_3(q^2)-q_\mu V_4(q^2)\right]\\
&\quad -(\epsilon_1\cdot q)\,\epsilon_{2\mu}^* V_5(q^2)+(\epsilon_2^*\cdot q)\,\epsilon_{1\mu} V_6(q^2),\\
\langle M(\epsilon_2,p_M)|\bar{c}\gamma_5\gamma_\mu b|\bar{B}_c^*(\epsilon_1,p_{B_c^*})\rangle &= -\mathrm{i}\varepsilon_{\mu\nu\alpha\beta}\epsilon_1^\alpha\epsilon_2^{*\beta}\left[P^\nu A_1(q^2)-q^\nu A_2(q^2)\right]-\frac{\mathrm{i}\epsilon_2^*\cdot q}{m_{B_c^*}^2-m_M^2}\varepsilon_{\mu\nu\alpha\beta}\epsilon_1^\nu P^\alpha q^\beta A_3(q^2)\\
&\quad +\frac{\mathrm{i}\epsilon_1\cdot q}{m_{B_c^*}^2-m_M^2}\varepsilon_{\mu\nu\alpha\beta}\epsilon_2^{*\nu}P^\alpha q^\beta A_4(q^2)
\end{aligned}$$

The amplitudes $H_{0++}(q^2), H_{t++}(q^2), H_{-+0}(q^2), H_{0--}(q^2), H_{t--}(q^2), H_{+-0}(q^2)$,$H_{+0+}(q^2), H_{-0-}(q^2), H_{000}(q^2)$ and $H_{t00}(q^2)$, with $\lambda_{B_c^*} = \lambda_M - \lambda_{W^*}$ survive.

二、理论框架

轻子部分运动学



$$p_\ell^\mu = (E_\ell, |\vec{p}_\ell| \sin\theta, 0, |\vec{p}_\ell| \cos\theta)$$

$$p_{\nu_\ell}^\mu = (|\vec{p}_\ell|, -|\vec{p}_\ell| \sin\theta, 0, -|\vec{p}_\ell| \cos\theta)$$

$$\bar{\epsilon}^\mu(t) = (1, 0, 0, 0)$$

$$\bar{\epsilon}^\mu(0) = (0, 0, 0, 1)$$

$$\bar{\epsilon}^\mu(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)$$

$$h_{\lambda_\ell, \lambda_{\bar{\nu}_\ell}} = \bar{u}_\ell(\lambda_\ell)\gamma^\mu(1-\gamma_5)v_{\bar{\nu}}(\frac{1}{2})\bar{\epsilon}_\mu(\lambda_{W^*})$$

$$|h_{-\frac{1}{2},\frac{1}{2}}|^2 = 8(q^2 - m_\ell^2)$$

$$|h_{\frac{1}{2},\frac{1}{2}}|^2 = 8\frac{m_\ell^2}{2q^2}(q^2 - m_\ell^2)$$

二、理论框架

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(*M* for *P*) $$\frac{d\Gamma}{dq^2 d\cos\theta} = \frac{G_F^2|V_{pb}|^2}{(2\pi)^3}\frac{|\vec{p}|}{8m_{B_c^*}^2}\frac{1}{3}(1-\frac{m_\ell^2}{q^2})L_{\mu\nu}H^{\mu\nu}$$

$$\frac{d^2\Gamma[\lambda_\ell=-1/2]}{dq^2 d\cos\theta} = \frac{G_F^2|V_{cb}|^2|\vec{p}|}{256\pi^3 m_{B_c^*}^2}\frac{1}{3}q^2(1-\frac{m_\ell^2}{q^2})^2[(1-\cos\theta)^2H_{+-}^2+(1+\cos\theta)^2H_{-+}^2+2\sin^2\theta H_{00}^2]$$

$$\frac{d^2\Gamma[\lambda_\ell=1/2]}{dq^2 d\cos\theta} = \frac{G_F^2|V_{cb}|^2|\vec{p}|}{256\pi^3 m_{B_c^*}^2}\frac{1}{3}q^2(1-\frac{m_\ell^2}{q^2})^2\frac{m_\ell^2}{q^2}[\sin^2\theta(H_{+-}^2+H_{-+}^2)+2(H_{t0}-\cos\theta H_{00})^2]$$

(*M* for *V*) $$\frac{d^2\Gamma}{dq^2 d\cos\theta} = \frac{G_F^2|V_{cb}|^2}{(2\pi)^3}\frac{|\vec{p}|}{8m_{B_c^*}^2}\frac{1}{3}(1-\frac{m_\ell^2}{q^2})^2L_{\mu\nu}H^{\mu\nu}$$

$$\frac{d^2\Gamma[\lambda_\ell=-1/2]}{dq^2 d\cos\theta} = \frac{G_F^2|V_{cb}|^2|\vec{p}|}{256\pi^3 m_{B_c^*}^2}\frac{1}{3}q^2(1-\frac{m_\ell^2}{q^2})^2[(1-\cos\theta)^2(H_{+0+}^2+H_{+-0}^2)+(1+\cos\theta)^2(H_{-0-}^2+H_{-+0}^2)+2\sin^2\theta(H_{0++}^2+H_{0--}^2+H_{000}^2)]$$

$$\frac{d^2\Gamma[\lambda_\ell=1/2]}{dq^2 d\cos\theta} = \frac{G_F^2|V_{cb}|^2|\vec{p}|}{256\pi^3 m_{B_c^*}^2}\frac{1}{3}q^2(1-\frac{m_\ell^2}{q^2})^2\frac{m_\ell^2}{q^2}[\sin^2\theta(H_{+0+}^2+H_{+-0}^2+H_{-0-}^2+H_{-+0}^2+2(H_{t++}-\cos\theta H_{0++})^2\text{-}2(H_{t--}-\cos\theta H_{0--})^2+2(H_{t00}-\cos\theta H$$

${}^{d\Gamma}/_{dq^2}, \Gamma, R, A_\theta, A_\lambda$等物理可观测量。

三、数值结果 形状因子

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Regular Article - Theoretical Physics

Revisiting the form factors of $P \to V$ transition within the light-front quark models

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Abstract We investigate the self-consistency and Lorentz covariance of the covariant light-front quark model (CLF QM) via the matrix elements and form factors ($\mathcal{F} = g$, $a_\pm$ and f) of $P \to V$ transition. Two types of correspondence schemes between the manifest covariant Bethe–Salpeter approach and the light-front quark model are studied. We find that, for $a_-(q^2)$ and $f(q^2)$, the CLF results obtained via $\lambda = 0$ and $\pm$ polarization states of vector meson within the traditional type-I correspondence scheme are inconsistent with each other; and moreover, the strict covariance of the matrix element is violated due to the non-vanishing spurious contributions associated with noncovariance. We further show that such two problems have the same

tice calculations [2], vector meson dominance model [3,4], perturbative QCD with some nonperturbative inputs [5,6], QCD sum rules [7,8] and light-front quark models (LF QMs) [9–13]. The traditional LF QM, i.e., the so-called standard light-front quark model (SLF QM), proposed by Terentev and Berestetsky [9,10] is a relativistic quark model based on the LF formalism [14] and LF quantization of QCD [15]. It provides a conceptually simple and phenomenologically feasible framework for the determination of form factor, decay constant and distribution amplitude et al., which are further applied to phenomenological researches [16–39]. However, in the SLF QM, the Lorentz covariance of the matrix element is lost since it contains a spurious dependence on the orien-

Form factors of $V' \to V''$ transition within the light-front quark models

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ABSTRACT: In this paper, we calculate the matrix element and form factors of vector-to-vector ($V' \to V''$) transition within the standard light-front (SLF) and covariant light-front (CLF) quark models (QMs), and investigate the self-consistency and Lorentz covariance of the CLF QM within two types of correspondences between the manifest covariant Bethe-Salpeter approach and the LF approach. The zero-mode and valence contributions to the form factors of $V' \to V''$ transition in the CLF QM and their relation to the SLF results are analyzed, and the main conclusions obtained via the decay constants of vector and axial-vector mesons and the form factors of $P \to V$ transition in the previous works are confirmed again. Furthermore, we present our numerical predictions for the form

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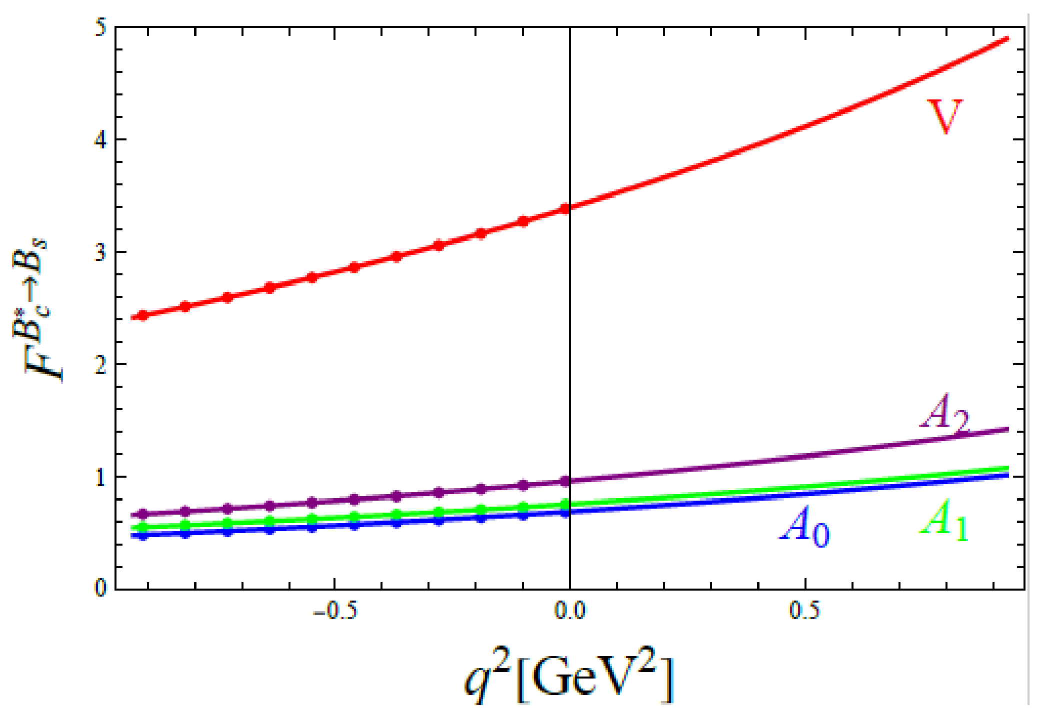
三、数值结果 形状因子



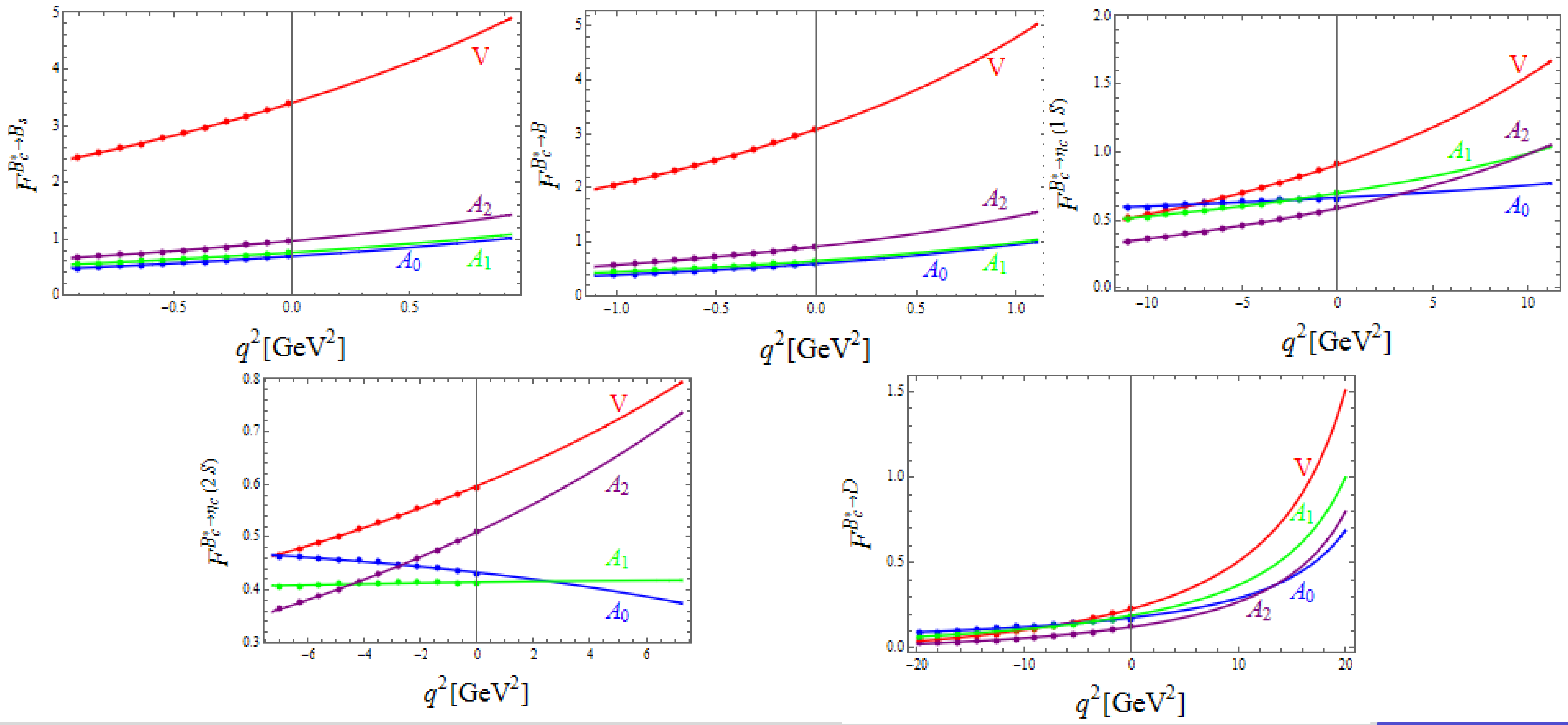
- 在类空区域$q^2=-q_\perp^2\leq 0$等间距地取数个点。
- 采用z –级数展开的BCL形式对上述点进行拟合(在N=1处截断)。

$$F(q^2)=\frac{F(0)}{1-q^2/m_{i,\mathrm{pole}}^2}\{1+\sum_{k=1}^{N}b_k\,[\mathrm{z}(\mathrm{q}^2,t_0)^k-\mathrm{z}(0,t_0)^k]\}$$

$$\mathrm{z}(q^2,t_0)=\frac{\sqrt{t_+-q^2}-\sqrt{t_+-t_0}}{\sqrt{t_+-q^2}+\sqrt{t_+-t_0}}$$

- 将拟合图像延拓至类时区域，即得到形状因子$F_i(q^2)$在整个运动学空间的结果。

三、数值结果 形状因子

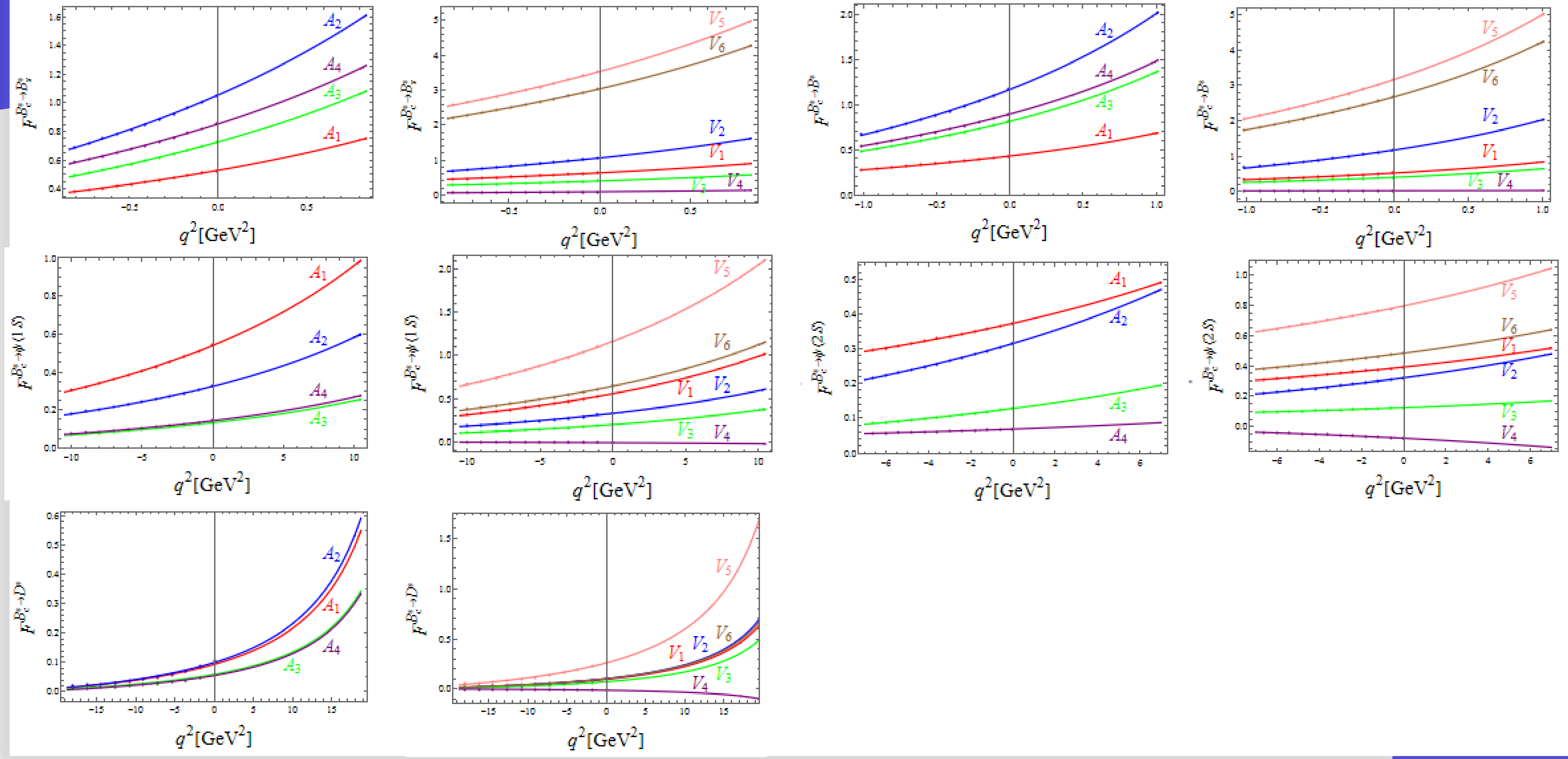

$F^{B_c^*\to B_s}$
V
A_2
A_0
A_1
q^2[GeV^2]
$F^{B_c^*\to B}$
V
A_2
A_0
A_1
q^2[GeV^2]
$F^{B_c^*\to \eta_c(1S)}$
V
A_2
A_1
A_0
q^2[GeV^2]
$F^{B_c^*\to \eta_c(2S)}$
V
A_2
A_1
A_0
q^2[GeV^2]
$F^{B_c^*\to D}$
V
A_1
A_0
A_2
q^2[GeV^2]

三、数值结果　形状因子

$\mathcal{F}$	$F(0)$	b_1	$\mathcal{F}$	$F(0)$	b_1	$\mathcal{F}$	$F(0)$	b_1
$V^{B_c^*\to D}$	$0.23^{+0.06}_{-0.07}$	$-10.8^{+0.23}_{-0.40}$	$V^{B_c^*\to\eta_c(1S)}$	$0.91^{+0.02}_{-0.01}$	$-9.41^{+0.05}_{-0.01}$	$V^{B_c^*\to\eta_c(2S)}$	$0.60^{+0.01}_{-0.02}$	$-4.69^{+0.83}_{-0.80}$
$A0^{B_c^*\to D}$	$0.18^{+0.05}_{-0.07}$	$-1.65^{+2.36}_{-3.80}$	$A0^{B_c^*\to\eta_c(1S)}$	$0.66^{+0.02}_{-0.02}$	$4.84^{+3.34}_{-5.27}$	$A0^{B_c^*\to\eta_c(2S)}$	$0.43^{+0.00}_{-0.01}$	$15.4^{+3.80}_{-6.36}$
$A1^{B_c^*\to D}$	$0.19^{+0.06}_{-0.07}$	$-6.25^{+1.29}_{-2.12}$	$A1^{B_c^*\to\eta_c(1S)}$	$0.70^{+0.03}_{-0.03}$	$-2.06^{+1.66}_{-2.65}$	$A1^{B_c^*\to\eta_c(2S)}$	$0.41^{+0.00}_{-0.01}$	$9.19^{+1.91}_{-3.42}$
$A2^{B_c^*\to D}$	$0.12^{+0.03}_{-0.04}$	$-10.3^{+0.34}_{-0.54}$	$A2^{B_c^*\to\eta_c(1S)}$	$0.59^{+0.02}_{-0.02}$	$-8.47^{+0.07}_{-0.14}$	$A2^{B_c^*\to\eta_c(2S)}$	$0.51^{+0.01}_{-0.01}$	$-9.63^{+0.60}_{-0.50}$
$V^{B_c^*\to B}$	$3.08^{+0.53}_{-0.59}$	$-89.4^{+20.7}_{-26.2}$	$V^{B_c^*\to B_s}$	$3.40^{+0.49}_{-0.54}$	$-83.2^{+20.7}_{-26.8}$			
$A0^{B_c^*\to B}$	$0.60^{+0.06}_{-0.05}$	$-104.1^{+26.9}_{-37.5}$	$A0^{B_c^*\to B_s}$	$0.69^{+0.05}_{-0.03}$	$-97.6^{+26.6}_{-37.5}$			
$A1^{B_c^*\to B}$	$0.65^{+0.08}_{-0.08}$	$-80.2^{+20.6}_{-27.2}$	$A1^{B_c^*\to B_s}$	$0.75^{+0.08}_{-0.07}$	$-75.6^{+20.7}_{-28.0}$			
$A2^{B_c^*\to B}$	$0.91^{+0.10}_{-0.06}$	$-114.4^{+26.0}_{-34.0}$	$A2^{B_c^*\to B_s}$	$0.96^{+0.06}_{-0.01}$	$-101.2^{+24.2}_{-31.2}$			

- $F_i(B_c^* \longrightarrow D)$的数值结果较小，初态较慢，末态较快，初末态波函数叠加较少。
- $F_i(B_c^* \longrightarrow \eta_c(1S)) > F_i(B_c^* \longrightarrow \eta_c(2S))$，由于$f_{\eta_c(1S)} > f_{\eta_c(2S)}$。

三、数值结果 形状因子


$F^{B_c^*\to B_s^*}$
A_2 A_4 A_3 A_1
V_5 V_6 V_2 V_1 V_3 V_4
$F^{B_c^*\to B^*}$
$F^{B_c^*\to\psi(1S)}$
$F^{B_c^*\to\psi(2S)}$
$F^{B_c^*\to D^*}$
$q^2[\mathrm{GeV}^2]$

$\mathcal{F}$	$F(0)$	b_1	$\mathcal{F}$	$F(0)$	b_1	$\mathcal{F}$	$F(0)$	b_1
$A_1^{B_c^*\to D^*}$	$0.09^{+0.03}_{-0.04}$	$-12.80^{+0.17}_{-0.35}$	$A_1^{B_c^*\to\psi(1S)}$	$0.54^{+0.02}_{-0.02}$	$-11.23^{+0.00}_{-0.04}$	$A_1^{B_c^*\to\psi(2S)}$	$0.37^{+0.00}_{-0.00}$	$-4.69^{+0.30}_{-0.33}$
$A_2^{B_c^*\to D^*}$	$0.10^{+0.03}_{-0.04}$	$-13.04^{+0.18}_{-0.37}$	$A_2^{B_c^*\to\psi(1S)}$	$0.32^{+0.04}_{-0.05}$	$-11.63^{+0.12}_{-0.13}$	$A_2^{B_c^*\to\psi(2S)}$	$0.32^{+0.02}_{-0.02}$	$-12.75^{+0.24}_{-0.25}$
$A_3^{B_c^*\to D^*}$	$0.06^{+0.01}_{-0.02}$	$-13.88^{+0.01}_{-0.12}$	$A_3^{B_c^*\to\psi(1S)}$	$0.13^{+0.01}_{-0.02}$	$-13.35^{+0.49}_{-0.64}$	$A_3^{B_c^*\to\psi(2S)}$	$0.13^{+0.01}_{-0.01}$	$-14.45^{+0.79}_{-1.07}$
$A_4^{B_c^*\to D^*}$	$0.06^{+0.02}_{-0.02}$	$-13.54^{+0.03}_{-0.14}$	$A_4^{B_c^*\to\psi(1S)}$	$0.14^{+0.01}_{-0.01}$	$-13.36^{+0.46}_{-0.59}$	$A_4^{B_c^*\to\psi(2S)}$	$0.07^{+0.00}_{-0.00}$	$-2.84^{+0.26}_{-0.69}$
$V_1^{B_c^*\to D^*}$	$0.10^{+0.03}_{-0.04}$	$-12.83^{+0.16}_{-0.34}$	$V_1^{B_c^*\to\psi(1S)}$	$0.56^{+0.03}_{-0.03}$	$-11.26^{+0.01}_{-0.02}$	$V_1^{B_c^*\to\psi(2S)}$	$0.39^{+0.00}_{-0.00}$	$-5.06^{+0.22}_{-0.21}$
$V_2^{B_c^*\to D^*}$	$0.10^{+0.03}_{-0.04}$	$-13.05^{+0.18}_{-0.36}$	$V_2^{B_c^*\to\psi(1S)}$	$0.33^{+0.04}_{-0.05}$	$-11.65^{+0.13}_{-0.14}$	$V_2^{B_c^*\to\psi(2S)}$	$0.32^{+0.02}_{-0.02}$	$-12.76^{+0.23}_{-0.23}$
$V_3^{B_c^*\to D^*}$	$0.07^{+0.02}_{-0.03}$	$-13.25^{+0.08}_{-0.22}$	$V_3^{B_c^*\to\psi(1S)}$	$0.20^{+0.02}_{-0.02}$	$-12.72^{+0.35}_{-0.44}$	$V_3^{B_c^*\to\psi(2S)}$	$0.12^{+0.01}_{-0.00}$	$-6.91^{+0.30}_{-0.52}$
$V_4^{B_c^*\to D^*}$	$-0.01^{+0.00}_{-0.00}$	$-16.72^{+0.53}_{-0.76}$	$V_4^{B_c^*\to\psi(1S)}$	$-0.01^{+0.01}_{-0.01}$	$-18.81^{+1.02}_{-0.52}$	$V_4^{B_c^*\to\psi(2S)}$	$-0.08^{+0.01}_{-0.01}$	$-25.82^{+0.23}_{-0.30}$
$V_5^{B_c^*\to D^*}$	$0.26^{+0.08}_{-0.10}$	$-12.49^{+0.21}_{-0.41}$	$V_5^{B_c^*\to\psi(1S)}$	$1.16^{+0.06}_{-0.07}$	$-10.87^{+0.02}_{-0.05}$	$V_5^{B_c^*\to\psi(2S)}$	$0.79^{+0.01}_{-0.01}$	$-4.65^{+0.23}_{-0.21}$
$V_6^{B_c^*\to D^*}$	$0.10^{+0.03}_{-0.04}$	$-12.30^{+0.24}_{-0.43}$	$V_6^{B_c^*\to\psi(1S)}$	$0.64^{+0.01}_{-0.01}$	$-10.35^{+0.17}_{-0.25}$	$V_6^{B_c^*\to\psi(2S)}$	$0.48^{+0.00}_{-0.00}$	$-5.02^{+0.14}_{-0.05}$
$A_1^{B_c^*\to B^*}$	$0.43^{+0.05}_{-0.03}$	$-107.6^{+24.5}_{-30.8}$	$A_1^{B_c^*\to B_s^*}$	$0.53^{+0.05}_{-0.03}$	$-101.1^{+22.4}_{-29.1}$			
$A_2^{B_c^*\to B^*}$	$1.17^{+0.13}_{-0.08}$	$-159.4^{+36.9}_{-49.7}$	$A_2^{B_c^*\to B_s^*}$	$1.05^{+0.08}_{-0.01}$	$-157.8^{+36.8}_{-52.2}$			
$A_3^{B_c^*\to B^*}$	$0.81^{+0.12}_{-0.10}$	$-140.3^{+30.9}_{-38.4}$	$A_3^{B_c^*\to B_s^*}$	$0.73^{+0.07}_{-0.04}$	$-138.1^{+30.8}_{-40.1}$			
$A_4^{B_c^*\to B^*}$	$0.89^{+0.16}_{-0.17}$	$-133.4^{+30.0}_{-37.4}$	$A_4^{B_c^*\to B_s^*}$	$0.85^{+0.13}_{-0.12}$	$-131.5^{+29.7}_{-38.6}$			
$V_1^{B_c^*\to B^*}$	$0.52^{+0.07}_{-0.08}$	$-108.0^{+24.5}_{-30.6}$	$V_1^{B_c^*\to B_s^*}$	$0.63^{+0.08}_{-0.08}$	$-101.1^{+22.3}_{-28.7}$			
$V_2^{B_c^*\to B^*}$	$1.17^{+0.14}_{-0.08}$	$-159.1^{+36.7}_{-49.4}$	$V_2^{B_c^*\to B_s^*}$	$1.06^{+0.09}_{-0.02}$	$-157.4^{+36.7}_{-51.9}$			
$V_3^{B_c^*\to B^*}$	$0.40^{+0.04}_{-0.02}$	$-107.0^{+21.9}_{-24.0}$	$V_3^{B_c^*\to B_s^*}$	$0.40^{+0.02}_{-0.00}$	$-106.6^{+22.2}_{-26.3}$			
$V_4^{B_c^*\to B^*}$	$0.02^{+0.06}_{-0.11}$	$17.0^{+249.6}_{-197.1}$	$V_4^{B_c^*\to B_s^*}$	$0.09^{+0.07}_{-0.13}$	$-120.7^{+173.3}_{-75.9}$			
$V_5^{B_c^*\to B^*}$	$3.16^{+0.53}_{-0.59}$	$-102.5^{+23.3}_{-29.0}$	$V_5^{B_c^*\to B_s^*}$	$3.53^{+0.52}_{-0.57}$	$-96.3^{+21.4}_{-27.6}$			
$V_6^{B_c^*\to B^*}$	$2.67^{+0.49}_{-0.58}$	$-101.9^{+23.7}_{-29.9}$	$V_6^{B_c^*\to B_s^*}$	$3.03^{+0.50}_{-0.57}$	$-94.9^{+21.5}_{-28.0}$			

三、数值结果 B_c^*总宽度

$$\Gamma_{tot}(B_c^*) \cong \Gamma(B_c^* \to B_c^* \gamma)$$

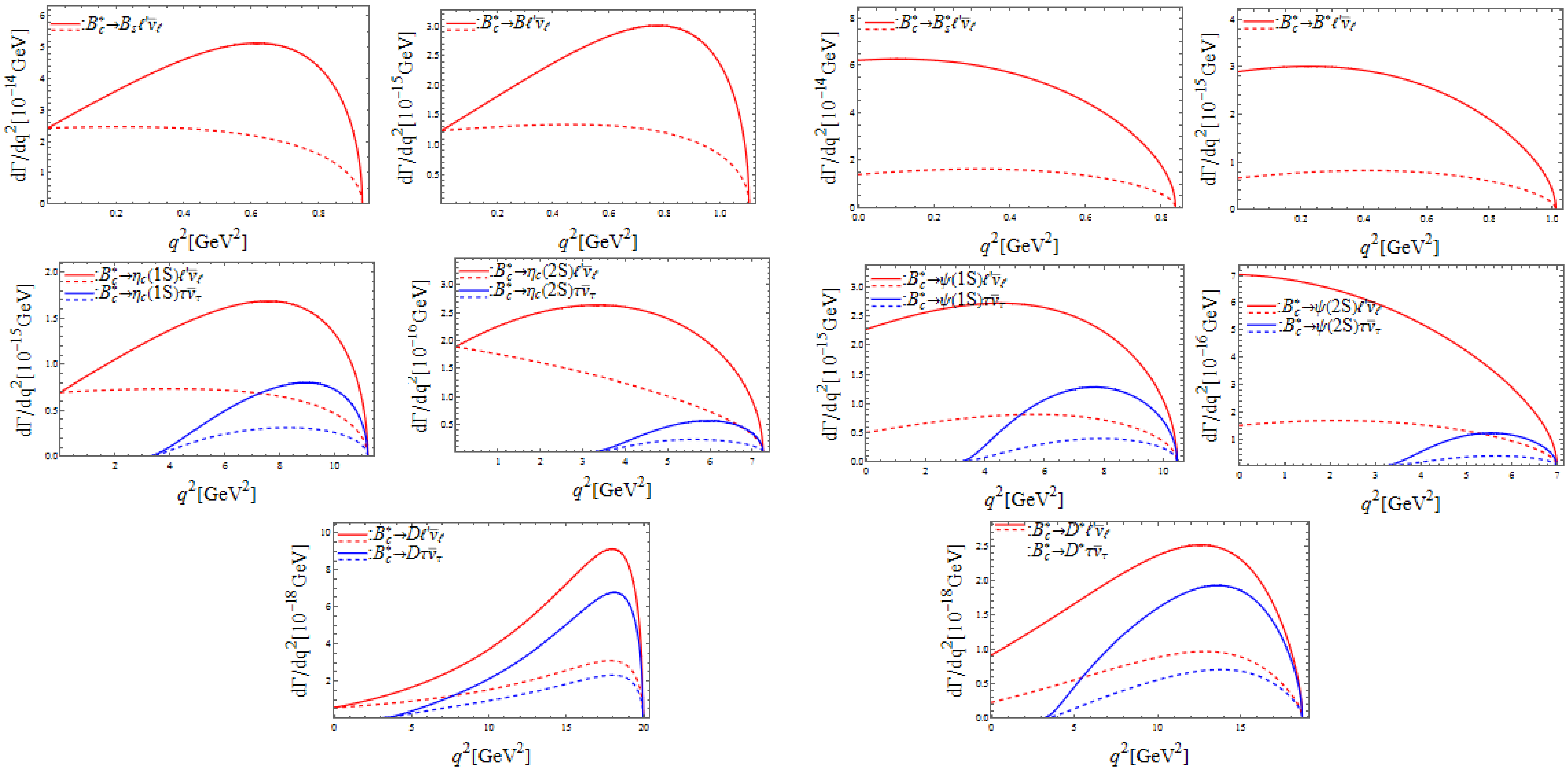
$$\Gamma(B^* \longrightarrow B\gamma) = \frac{\alpha}{3}\left[e_1 I(m_1, m_2, 0) + e_2 I(m_2, m_1, 0)\right]^2 \kappa_\gamma^3$$

$$I(m_1, m_2, q^2) = \int_0^1 \frac{dx}{8\pi^3}\int d^2k_\perp \frac{\psi(x, k'_\perp)\,\psi(x, k_\perp)}{x\,\tilde{M}_0\,\tilde{M}'_0} \times \left\{\mathscr{A} + \frac{2}{\mathscr{M}_0}\left[k_\perp^2 - \frac{(k_\perp \cdot q_\perp)^2}{q_\perp^2}\right]\right\}$$

$$\psi(x, k_\perp) = 4\frac{\pi^{3/4}}{\beta^{3/2}}\sqrt{\frac{\partial k_z}{\partial x}}\exp\left[-\frac{k_z^2 + k_\perp^2}{2\beta^2}\right]$$

$$\Gamma_{tot}(B_c^*) \cong \Gamma(B_c^* \to B_c^* \gamma) = 39^{+23}_{-17}\,\mathrm{eV}$$

三、数值结果


$B_c^*\to B_s\ell'\bar{\nu}_\ell$
$d\Gamma/dq^2[10^{-14}\mathrm{GeV}]$
$q^2[\mathrm{GeV}^2]$
$B_c^*\to B\ell'\bar{\nu}_\ell$
$d\Gamma/dq^2[10^{-15}\mathrm{GeV}]$
$q^2[\mathrm{GeV}^2]$
$B_c^*\to B_s^*\ell'\bar{\nu}_\ell$
$d\Gamma/dq^2[10^{-14}\mathrm{GeV}]$
$q^2[\mathrm{GeV}^2]$
$B_c^*\to B^*\ell'\bar{\nu}_\ell$
$d\Gamma/dq^2[10^{-15}\mathrm{GeV}]$
$q^2[\mathrm{GeV}^2]$
$B_c^*\to\eta_c(1S)\ell'\bar{\nu}_\ell$
$B_c^*\to\eta_c(1S)\tau\bar{\nu}_\tau$
$d\Gamma/dq^2[10^{-15}\mathrm{GeV}]$
$q^2[\mathrm{GeV}^2]$
$B_c^*\to\eta_c(2S)\ell'\bar{\nu}_\ell$
$B_c^*\to\eta_c(2S)\tau\bar{\nu}_\tau$
$d\Gamma/dq^2[10^{-16}\mathrm{GeV}]$
$q^2[\mathrm{GeV}^2]$
$B_c^*\to\psi(1S)\ell'\bar{\nu}_\ell$
$B_c^*\to\psi(1S)\tau\bar{\nu}_\tau$
$d\Gamma/dq^2[10^{-15}\mathrm{GeV}]$
$q^2[\mathrm{GeV}^2]$
$B_c^*\to\psi(2S)\ell'\bar{\nu}_\ell$
$B_c^*\to\psi(2S)\tau\bar{\nu}_\tau$
$d\Gamma/dq^2[10^{-16}\mathrm{GeV}]$
$q^2[\mathrm{GeV}^2]$
$B_c^*\to D\ell'\bar{\nu}_\ell$
$B_c^*\to D\tau\bar{\nu}_\tau$
$d\Gamma/dq^2[10^{-18}\mathrm{GeV}]$
$q^2[\mathrm{GeV}^2]$
$B_c^*\to D^*\ell'\bar{\nu}_\ell$
$B_c^*\to D^*\tau\bar{\nu}_\tau$
$d\Gamma/dq^2[10^{-18}\mathrm{GeV}]$
$q^2[\mathrm{GeV}^2]$

三、数值结果

Decay mode	This work	BS method
$B_c^* \to B\ell'\nu_{\ell'}$	$6.66^{+2.23+0.01+5.08}_{-1.72-0.01-2.52} \times 10^{-8}$	5.78×10^{-8}
$B_c^* \to B^*\ell'\nu_{\ell'}$	$6.39^{+2.48+0.01+4.86}_{-1.61-0.01-2.42} \times 10^{-8}$	1.59×10^{-7}
$B_c^* \to B_s\ell'\nu_{\ell'}$	$9.86^{+2.59+0.00+7.52}_{-1.96-0.00-3.73} \times 10^{-7}$	9.43×10^{-7}
$B_c^* \to B_s^*\ell'\nu_{\ell'}$	$1.09^{+0.30+0.00+0.83}_{-0.21-0.00+0.41} \times 10^{-6}$	2.47×10^{-6}
$B_c^* \to D\ell'\nu_{\ell'}$	$2.19^{+1.98+0.11+1.67}_{-1.42-0.09-0.83} \times 10^{-9}$	1.60×10^{-9}
$B_c^* \to D^*\ell'\nu_{\ell'}$	$8.96^{+14.2+0.43+6.82}_{-6.80-0.35-3.39} \times 10^{-10}$	4.61×10^{-9}
$B_c^* \to D\tau\nu_\tau$	$1.36^{+1.25+0.07+1.04}_{-0.88-0.05-0.52} \times 10^{-9}$	1.08×10^{-9}
$B_c^* \to D^*\tau\nu_\tau$	$5.11^{+8.65+0.25+3.90}_{-4.03-0.20-1.93} \times 10^{-10}$	3.22×10^{-9}
$B_c^* \to \eta_c(1S)\ell'\nu_{\ell'}$	$3.83^{+0.63+0.06+2.92}_{-0.45-0.11-1.45} \times 10^{-7}$	4.20×10^{-7}
$B_c^* \to \psi(1S)\ell'\nu_{\ell'}$	$6.30^{+0.76+0.11+4.80}_{-0.68-0.18-2.39} \times 10^{-7}$	1.13×10^{-6}
$B_c^* \to \eta_c(1S)\tau\nu_\tau$	$1.07^{+0.22+0.02+0.82}_{-0.14-0.03-0.41} \times 10^{-7}$	1.26×10^{-7}
$B_c^* \to \psi(1S)\tau\nu_\tau$	$1.64^{+0.22+0.03+1.25}_{-0.20-0.05-0.62} \times 10^{-7}$	3.13×10^{-7}
$B_c^* \to \eta_c(2S)\ell'\nu_{\ell'}$	$4.16^{+0.28+0.07+3.17}_{-0.25-0.12-1.57} \times 10^{-8}$	
$B_c^* \to \psi(2S)\ell'\nu_{\ell'}$	$9.28^{+0.19+0.16+7.07}_{-0.17-0.27-3.51} \times 10^{-8}$	
$B_c^* \to \eta_c(2S)\tau\nu_\tau$	$3.85^{+0.45+0.07+2.93}_{-0.32-0.11-1.45} \times 10^{-9}$	
$B_c^* \to \psi(2S)\tau\nu_\tau$	$7.90^{+0.27+0.13+6.02}_{-0.26-0.23-2.99} \times 10^{-9}$	

三、数值结果

$B_c^* \to D^* \ell' \nu_{\ell'}$	8.96×10^{-10}
$B_c^* \to B^* \ell' \nu_{\ell'}$	6.39×10^{-8}
$B_c^* \to \psi(1S) \ell' \nu_{\ell'}$	6.30×10^{-7}
$B_c^* \to B_s^* \ell' \nu_{\ell'}$	1.09×10^{-6}

$\mathrm{BR}(\mathrm{B}_c^* \to D^* \ell \bar{\nu})$	$V_{ub}=3.67 \times 10^{-3}$
$< \mathrm{BR}(B_c^* \to B^* \ell \bar{\nu})$	**$V_{cd}=0.225$**
$< \mathrm{BR}(B_c^* \to J/\psi \ell \bar{\nu})$	**$V_{cb}=4.14 \times 10^{-2}$**
$< \mathrm{BR}(B_c^* \to B_s^* \ell \bar{\nu})$	$V_{cs}=0.974$

- $B_c^* \to J/\psi \ell \bar{\nu}$的相空间远大于$B_c^* \to B^* \ell \bar{\nu}$的相空间
- $B_c^* \to P \ell \bar{\nu}$的过程也存在着相同的层级关系。

三、数值结果

$B_c^* \to \psi(1S)\ell'\nu_{\ell'}$	6.30×10^{-7}
$B_c^* \to \eta_c(1S)\ell'\nu_{\ell'}$	3.83×10^{-7}
$B_c^* \to \psi(2S)\ell'\nu_{\ell'}$	9.28×10^{-8}
$B_c^* \to \eta_c(2S)\ell'\nu_{\ell'}$	4.16×10^{-8}

$$BR(B_c^* \to \psi(1S)\ell\bar{\nu}) > BR(B_c^* \to \eta_c(1S)\ell\bar{\nu})$$
$$BR(B_c^* \to \psi(2S)\ell\bar{\nu}) > BR(B_c^* \to \eta_c(2S)\ell\bar{\nu})$$

- $B_c^* \to \psi(1S)\ell\bar{\nu}$的相空间与$B_c^* \to \eta_c(1S)\ell\bar{\nu}$的相空间相差不大

- $J = 0$　　$\eta_c(1S)$　$\eta_c(2S)$
  $J = 1$　　$\psi(1S)$　$\psi(2S)$

- 末态求和，$B_c^* \to \psi(1S)\ell\bar{\nu}$比$B_c^* \to \eta_c(1S)\ell\bar{\nu}$包含更多的态。

三、数值结果

$B_c^* \to \psi(1S)\ell'\nu_{\ell'}$	6.30×10^{-7}
$B_c^* \to \psi(2S)\ell'\nu_{\ell'}$	9.28×10^{-8}
$B_c^* \to \eta_c(1S)\ell'\nu_{\ell'}$	3.83×10^{-7}
$B_c^* \to \eta_c(2S)\ell'\nu_{\ell'}$	4.16×10^{-8}

$BR(B_c^* \to \psi(1S)\ell\bar{\nu}) > BR(B_c^* \to \psi(2S)\ell\bar{\nu})$

$BR(B_c^* \to \eta_c(1S)\ell\bar{\nu}) > BR(B_c^* \to \eta_c(2S)\ell\bar{\nu})$

- $B_c^* \to \eta_c(1S)\ell\bar{\nu}$的相空间与$B_c^* \to \eta_c(2S)\ell\bar{\nu}$的相空间大
- $f_{\eta_c(1S)} > f_{\eta_c(2S)}$，$B_c^* \to \eta_c(1S)$的形状因子均大于$B_c^* \to \eta_c(2S)$
- $B_c^* \to \psi(1S)\ell\bar{\nu}$与$B_c^* \to \psi(2S)\ell\bar{\nu}$对比也存在同样的情况。

三、数值结果

$B_c^* \to B\ell'\nu_{\ell'}$	6.66×10^{-8}
$B_c^* \to B^*\ell'\nu_{\ell'}$	6.39×10^{-8}
$B_c^* \to B_s\ell'\nu_{\ell'}$	9.86×10^{-7}
$B_c^* \to B_s^*\ell'\nu_{\ell'}$	1.09×10^{-6}
$B_c^* \to D\ell'\nu_{\ell'}$	2.19×10^{-9}
$B_c^* \to D^*\ell'\nu_{\ell'}$	8.96×10^{-10}

$$BR(B_c^* \to B\ell\bar{\nu}) \approx BR(B_c^* \to B^*\ell\bar{\nu})$$

$$BR(B_c^* \to B_s\ell\bar{\nu}) \approx BR(B_c^* \to B_s^*\ell\bar{\nu})$$

- $B_c^* \to B\ell\bar{\nu}$的相空间与$B_c^* \to B^*\ell\bar{\nu}$的相空间略大
- $B_c^* \to B$的形状因子均大于$B_c^* \to B^*$

$$BR(B_c^* \to D\ell\bar{\nu}) > BR(B_c^* \to D^*\ell\bar{\nu})$$

- 两者相空间大致相同
- $B_c^* \to D$的形状因子大于$B_c^* \to D^*$

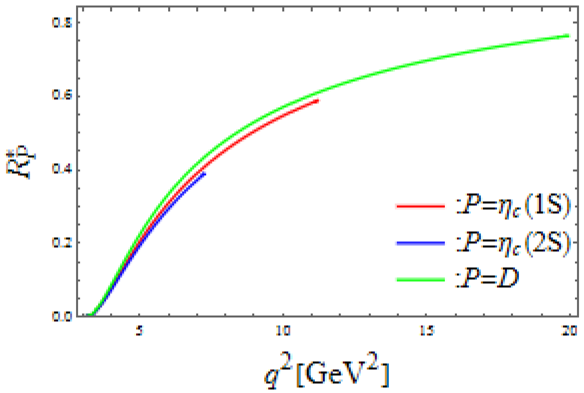
三、数值结果

- 考虑到LHCb实验上pp对撞的B_c^*介子产生截面为100nb, 以及未来的HL-LHC实验上将收集到总积分亮度为300fb^{-1}数据样本,预计实验上将收集到3×10^{10}个B_c^*事例。

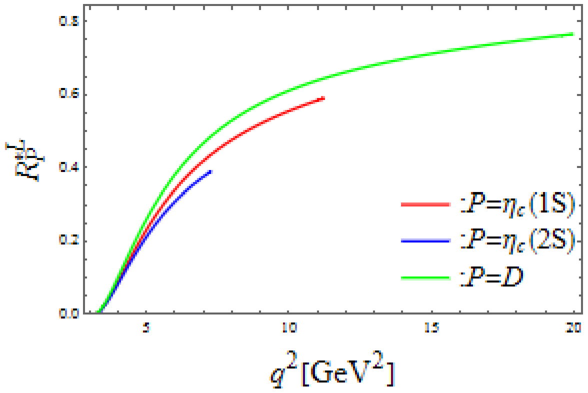
- 对于尚在筹建中的CEPC，考虑到$\mathcal{B}\left(z^0\to b\bar{b}\right)=15.12\%$和$f(b\to B_c^*)\sim 6\times 10^{-4}$,实验上通过$10^{13}$个$\mathrm{z}^0$可收集到$10^8$个$B_c^*$事例。

- 本文中的分支比均大于10^{-10}，均有望被将来的高能物理实验观测到，特别是$\mathcal{B}(B_c^*\to B_s^*\ell'\bar{\mathrm{v}})=1.09\times 10^{-6}$，有望被最先观测到。

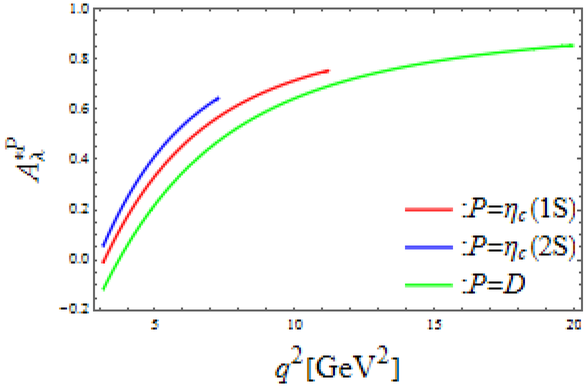
三、数值结果



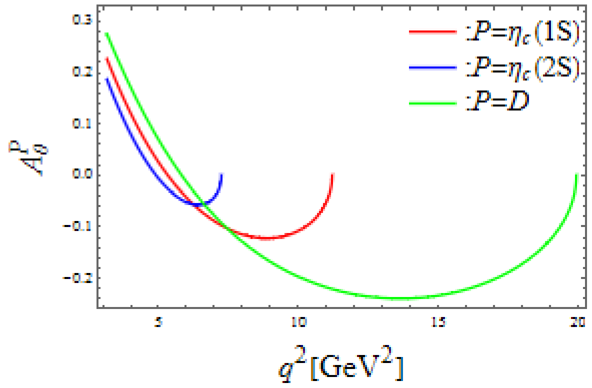
R^*_P
0.0
0.2
0.4
0.6
0.8
5
10
15
20
$q^2[\mathrm{GeV}^2]$
$:P=\eta_c(1S)$
$:P=\eta_c(2S)$
$:P=D$



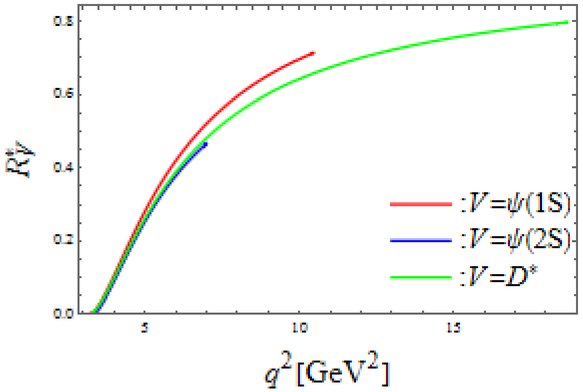
R^{*L}_P
0.0
0.2
0.4
0.6
0.8
5
10
15
20
$q^2[\mathrm{GeV}^2]$
$:P=\eta_c(1S)$
$:P=\eta_c(2S)$
$:P=D$



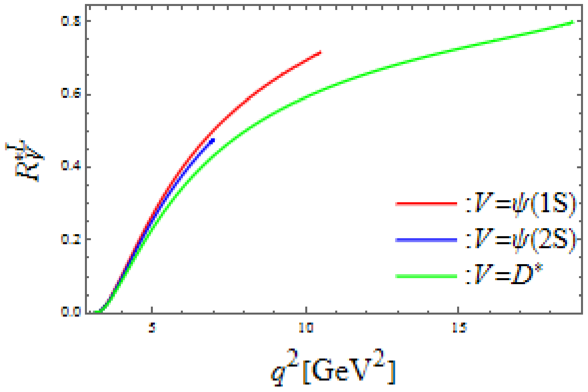
A^{*P}_λ
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
5
10
15
20
$q^2[\mathrm{GeV}^2]$
$:P=\eta_c(1S)$
$:P=\eta_c(2S)$
$:P=D$



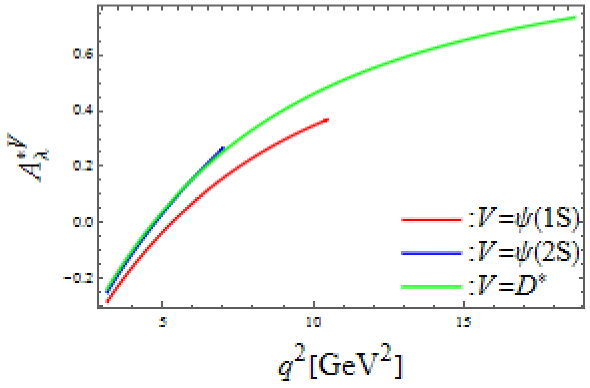
A^P_θ
-0.2
-0.1
0.0
0.1
0.2
0.3
5
10
15
20
$q^2[\mathrm{GeV}^2]$
$:P=\eta_c(1S)$
$:P=\eta_c(2S)$
$:P=D$



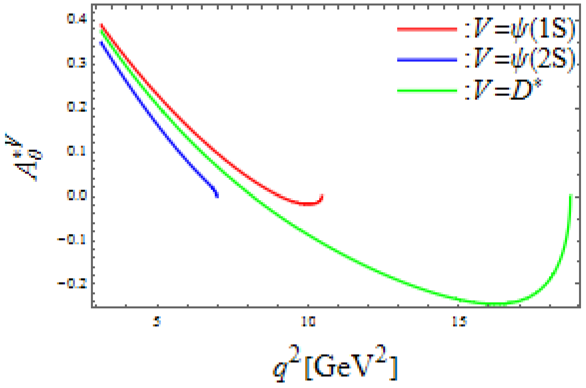
R^*_V
0.0
0.2
0.4
0.6
0.8
5
10
15
$q^2[\mathrm{GeV}^2]$
$:V=\psi(1S)$
$:V=\psi(2S)$
$:V=D^*$



R^{*L}_V
0.0
0.2
0.4
0.6
0.8
5
10
15
$q^2[\mathrm{GeV}^2]$
$:V=\psi(1S)$
$:V=\psi(2S)$
$:V=D^*$



A^{*V}_λ
-0.2
0.0
0.2
0.4
0.6
5
10
15
$q^2[\mathrm{GeV}^2]$
$:V=\psi(1S)$
$:V=\psi(2S)$
$:V=D^*$



A^{*V}_θ
-0.2
-0.1
0.0
0.1
0.2
0.3
0.4
5
10
15
$q^2[\mathrm{GeV}^2]$
$:V=\psi(1S)$
$:V=\psi(2S)$
$:V=D^*$

三、数值结果

Prediction for q^2 integrated observables A_λ, A_θ, R, R_L

Obs.	*Prediction*	*Obs.*	*Prediction*	*Obs.*	*Prediction*	*Obs.*	*Prediction*
$A_\lambda^{D^*}$	$0.432^{+0.255}_{-0.488}$	$A_\theta^{D^*}$	$-0.086^{+0.388}_{-0.247}$	R^{D^*}	$0.571^{+0.148}_{-0.088}$	$R_L^{D^*}$	$0.559^{+0.323}_{-0.098}$
$A_\lambda^{\psi(1S)}$	$0.215^{+0.063}_{-0.070}$	$A_\theta^{\psi(1S)}$	$0.077^{+0.027}_{-0.023}$	$R^{\psi(1S)}$	$0.260^{+0.006}_{-0.006}$	$R_L^{\psi(1S)}$	$0.275^{+0.009}_{-0.009}$
$A_\lambda^{\psi(2S)}$	$0.059^{+0.020}_{-0.021}$	$A_\theta^{\psi(2S)}$	$0.140^{+0.006}_{-0.006}$	$R^{\psi(2S)}$	$0.085^{+0.001}_{-0.002}$	$R_L^{\psi(2S)}$	$0.096^{+0.002}_{-0.002}$
A_λ^{D}	$0.759^{+0.035}_{-0.166}$	A_θ^{D}	$-0.188^{+0.108}_{-0.169}$	R^{D}	$0.621^{+0.049}_{-0.028}$	R_L^{D}	$0.601^{+0.142}_{-0.043}$
$A_\lambda^{\eta_c(1S)}$	$0.591^{+0.022}_{-0.032}$	$A_\theta^{\eta_c(1S)}$	$-0.082^{+0.020}_{-0.017}$	$R^{\eta_c(1S)}$	$0.280^{+0.011}_{-0.008}$	$R_L^{\eta_c(1S)}$	$0.239^{+0.015}_{-0.009}$
$A_\lambda^{\eta_c(2S)}$	$0.478^{+0.017}_{-0.024}$	$A_\theta^{\eta_c(2S)}$	$-0.021^{+0.017}_{-0.012}$	$R^{\eta_c(2S)}$	$0.092^{+0.005}_{-0.003}$	$R_L^{\eta_c(2S)}$	$0.070^{+0.005}_{-0.003}$

四、总结

- 使用螺旋度振幅展开的方法 双重味矢量介子B_c^* 半轻衰变进行了系统的研究。

- 对于非微绕输出入参数，形状因子和衰变宽度，均在CLFQM中进行了计算并给出了其在整个运动学区间的结果。

- 最后预言了部分物理可客观量（$\mathcal{B}$，$\mathcal{R}$，A_θ, A_λ）的结果, B_c^*介子通过带电夸克流发生的半轻衰变分支比均大于10^{-10}，特别是$\mathcal{B}(B_c^* \to B_s^* \ell' \bar{\nu}) = 1.09 \times 10^{-6}$，有望被最先观测到。

恳请各位老师批评指正！

三、数值结果

The SM prediction for the branching fractions of $B_c^* \to P(V)\ell\bar{\nu}$ decays

Decay mode	This work	Dipole	e index
$B_c^* \to B\ell'\nu_{\ell'}$	$6.66^{+2.23+0.01+5.08}_{-1.72-0.01-2.52} \times 10^{-8}$	6.43×10^{-8}	6.56×10^{-8}
$B_c^* \to B^*\ell'\nu_{\ell'}$	$6.39^{+2.48+0.01+4.86}_{-1.61-0.01-2.42} \times 10^{-8}$	6.33×10^{-8}	6.41×10^{-8}
$B_c^* \to B_s\ell'\nu_{\ell'}$	$9.86^{+2.59+0.00+7.52}_{-1.96-0.00-3.73} \times 10^{-7}$	9.48×10^{-7}	9.51×10^{-7}
$B_c^* \to B_s^*\ell'\nu_{\ell'}$	$1.09^{+0.30+0.00+0.83}_{-0.21-0.00+0.41} \times 10^{-6}$	1.09×10^{-6}	1.10×10^{-6}
$B_c^* \to D\ell'\nu_{\ell'}$	$2.19^{+1.98+0.11+1.67}_{-1.42-0.09-0.83} \times 10^{-9}$	6.33×10^{-10}	1.43×10^{-9}
$B_c^* \to D^*\ell'\nu_{\ell'}$	$8.96^{+14.2+0.43+6.82}_{-6.80-0.35-3.39} \times 10^{-10}$	4.03×10^{-10}	1.64×10^{-9}
$B_c^* \to D\tau\nu_\tau$	$1.36^{+1.25+0.07+1.04}_{-0.88-0.05-0.52} \times 10^{-9}$	2.93×10^{-10}	8.15×10^{-10}
$B_c^* \to D^*\tau\nu_\tau$	$5.11^{+8.65+0.25+3.90}_{-4.03-0.20-1.93} \times 10^{-10}$	1.34×10^{-10}	1.01×10^{-9}
$B_c^* \to \eta_c(1S)\ell'\nu_{\ell'}$	$3.83^{+0.63+0.06+2.92}_{-0.45-0.11-1.45} \times 10^{-7}$	3.48×10^{-7}	3.55×10^{-7}
$B_c^* \to \psi(1S)\ell'\nu_{\ell'}$	$6.30^{+0.76+0.11+4.80}_{-0.68-0.18-2.39} \times 10^{-7}$	6.43×10^{-7}	6.71×10^{-7}
$B_c^* \to \eta_c(1S)\tau\nu_\tau$	$1.07^{+0.22+0.02+0.82}_{-0.14-0.03-0.41} \times 10^{-7}$	9.27×10^{-8}	9.55×10^{-8}
$B_c^* \to \psi(1S)\tau\nu_\tau$	$1.64^{+0.22+0.03+1.25}_{-0.20-0.05-0.62} \times 10^{-7}$	1.67×10^{-7}	1.80×10^{-7}
$B_c^* \to \eta_c(2S)\ell'\nu_{\ell'}$	$4.16^{+0.28+0.07+3.17}_{-0.25-0.12-1.57} \times 10^{-8}$	3.83×10^{-8}	3.83×10^{-8}
$B_c^* \to \psi(2S)\ell'\nu_{\ell'}$	$9.28^{+0.19+0.16+7.07}_{-0.17-0.27-3.51} \times 10^{-8}$	8.73×10^{-8}	8.84×10^{-8}
$B_c^* \to \eta_c(2S)\tau\nu_\tau$	$3.85^{+0.45+0.07+2.93}_{-0.32-0.11-1.45} \times 10^{-9}$	3.32×10^{-9}	3.31×10^{-9}
$B_c^* \to \psi(2S)\tau\nu_\tau$	$7.90^{+0.27+0.13+6.02}_{-0.26-0.23-2.99} \times 10^{-9}$	6.76×10^{-9}	6.99×10^{-9}

Form factors of $P \to T$ transition within the light-front quark models

$$F = \frac{c}{1 - \frac{q^2}{M_{pole}^2}}\{1 + \mathrm{a}[\mathrm{z}(\mathrm{q}^2) - \mathrm{z}(0)]\}$$

Form factor of $V' \to V''$ transition within the light-front quark models

$$F = \frac{c}{1 - \frac{aq^2}{M_{pole}^2} + b\left(\frac{q^2}{M_{pole}^2}\right)^2}$$

Covariant light-front approach for B_c transition form factors

$$F = c * Exp[a * \frac{q^2}{M_{pole}^2} + b * \left(\frac{q^2}{M_{pole}^2}\right)^2]$$