Fate of critical fluctuations in an interacting hadronic medium using maximum entropy distributions

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Model

Baseline model: ideal hadron resonance gas (HRG)

the equilibrium distribution function:

$$f^0_{i,k}(T,\mu_i) = \frac{1}{(2\pi\hbar)^3}\exp\left(-(E_{i,k}-\mu_i)/T\right)$$

↓

particle density:

$$n_i(T,\mu_i) = \frac{e^{\mu_i/T}}{2\pi^2\hbar^3} m^2 T \boxed{K_2(m_i/T)}$$

Bessel function of the second kind

↓

cumulants:

$$\kappa_{1,i} = N_i = V n_i \qquad \kappa_{n,i} = VT^3 \left.\frac{\partial^{n-1}(n_i/T^3)}{\partial(\mu_i/T)^{n-1}}\right|_T$$

Model

Critical mode fluctuations: HRG + critical mode

$$f_{i,k} = f^0_{i,k} + \delta f_i^{\text{critical}}$$

$$\delta f_i^{\text{critical}} = -\delta\sigma \frac{g_i}{T} \frac{f^0_{i,k}}{\gamma_{i,k}}$$

g_i : the strength of the coupling

$$\langle(\delta\sigma)^n\rangle_c = \left(\frac{T}{VH_0}\right)^{n-1} \frac{\partial^{n-1} M}{\partial h^{n-1}}\bigg|_r$$

$$\gamma_{i,k} = E_{i,k}/m_i$$

critical exponents:

$$M = M_0 R^\beta \theta\,,$$
$$r = R(1-\theta^2)\,,$$
$$h = H/H_0 = R^{\beta\delta}\tilde{h}(\theta)\,.$$

extended cumulants:

$$\kappa_n^{\text{net}} = \kappa_n^p + (-1)^n \kappa_n^{\bar{p}} + (-1)^n \langle(V\delta\sigma)^n\rangle_c (I_p - I_{\bar{p}})^n$$

$$\kappa_n^{\text{tot}} = \kappa_n^p + \kappa_n^{\bar{p}} + (-1)^n \langle(V\delta\sigma)^n\rangle_c (I_p + I_{\bar{p}})^n$$

$$I_i = \frac{g_i d_i}{T} \int \frac{d^3k}{(2\pi\hbar)^3} \frac{f^0_{i,k}}{\gamma_{i,k}}$$

Model

Method: Maximum entropy method $\xrightarrow{?}$ translate the cumulants into samples of (anti-)particles

(reconstruction distribution)

(Assumption: distributions in nature have a maximum information entropy)

$$S = -\sum_{x \in \Omega} P(x) \ln P(x)$$

$$\downarrow$$

$$P_\lambda(x) = Z_\lambda^{-1} \exp\left\{ \sum_{k=1}^{n} \lambda_k x^k \right\}$$

λ : Lagrange-multiplier

which is not exact physical distribution function of the particle number

Example:

\sqrt{s} = 15 GeV

↓ ME Sample

distribution function P(N)

↓ SMASH

Evolution

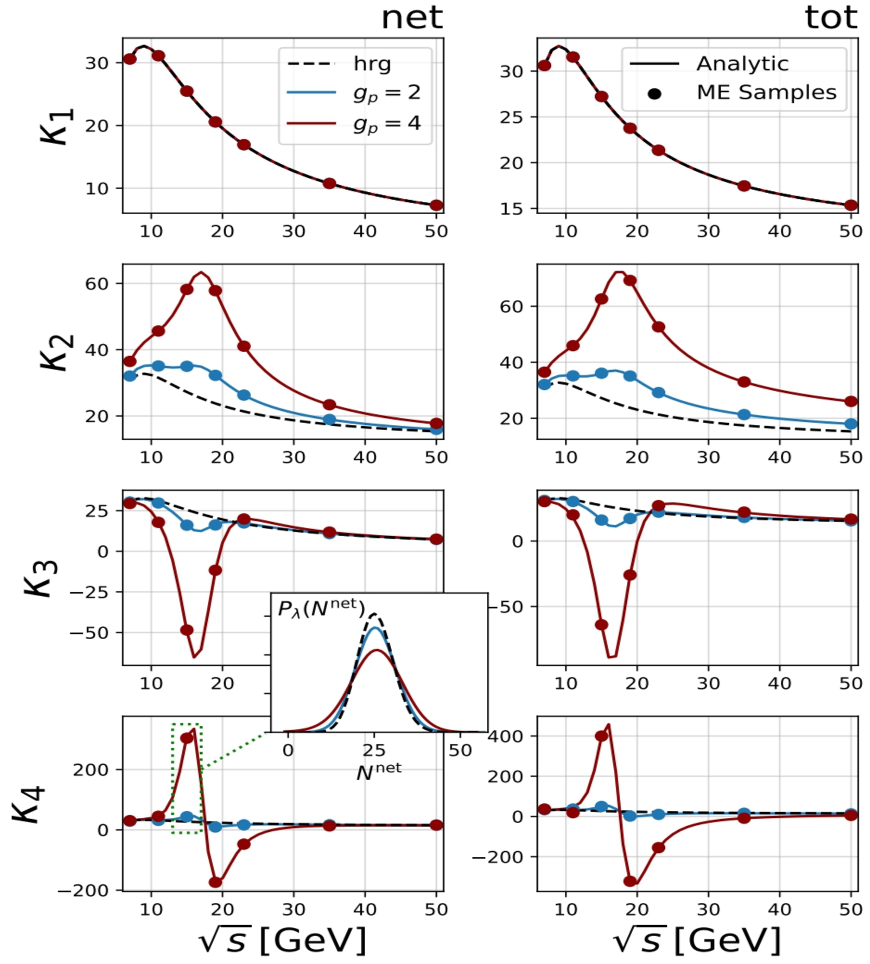

FIG. 3. Net (left column) and total (right column) proton number cumulants as a function of $\sqrt{s}$ up to fourth order (from top to bottom). The analytic HRG baseline (grey dashed) as well as the calculation including the critical mode with a coupling of $g_p = 2, 4$ (full blue and red) are presented. In addition, the results from the generated particle samples are presented as red circles. In addition, the reconstructed net proton number ME distributions are shown for $\sqrt{s} = 15\,\mathrm{GeV}$ plus the skellam distribution in the small subplot.

Results

tum distributions [43, 44]. The temperature and chemical potentials in the initial state are obtained from the freeze-out curve [32] which maps the beam energy $\sqrt{s}$ of a HIC

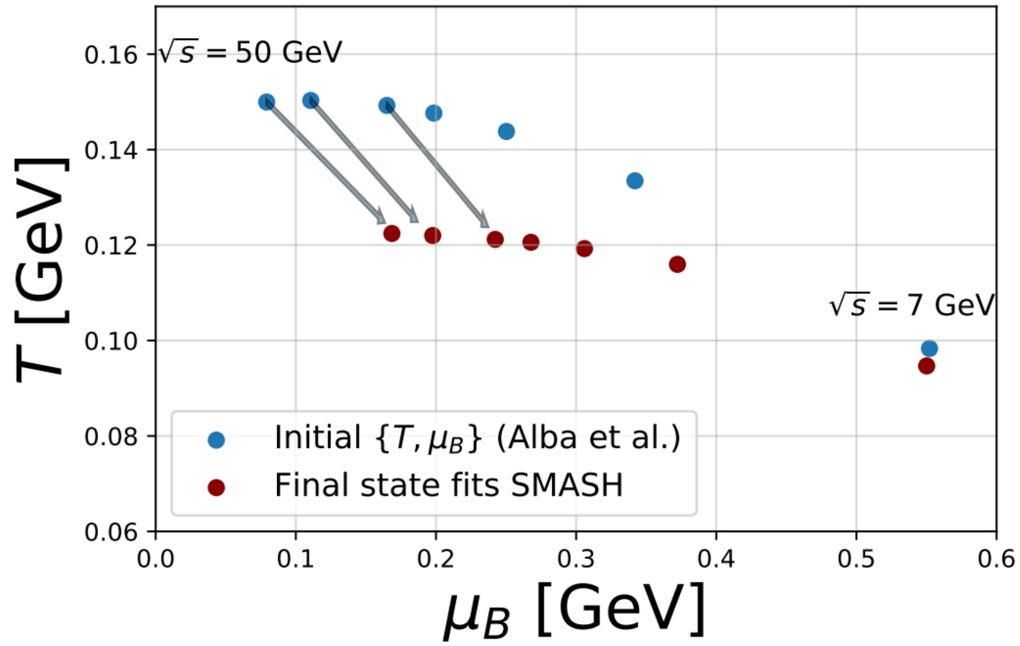

FIG. 4. Evolution of the temperature and baryon chemical potential of the expanding medium. The initial state values are shown as blue and the final state values as red points.

chemical freeze-out

↓

kinetic freeze-out

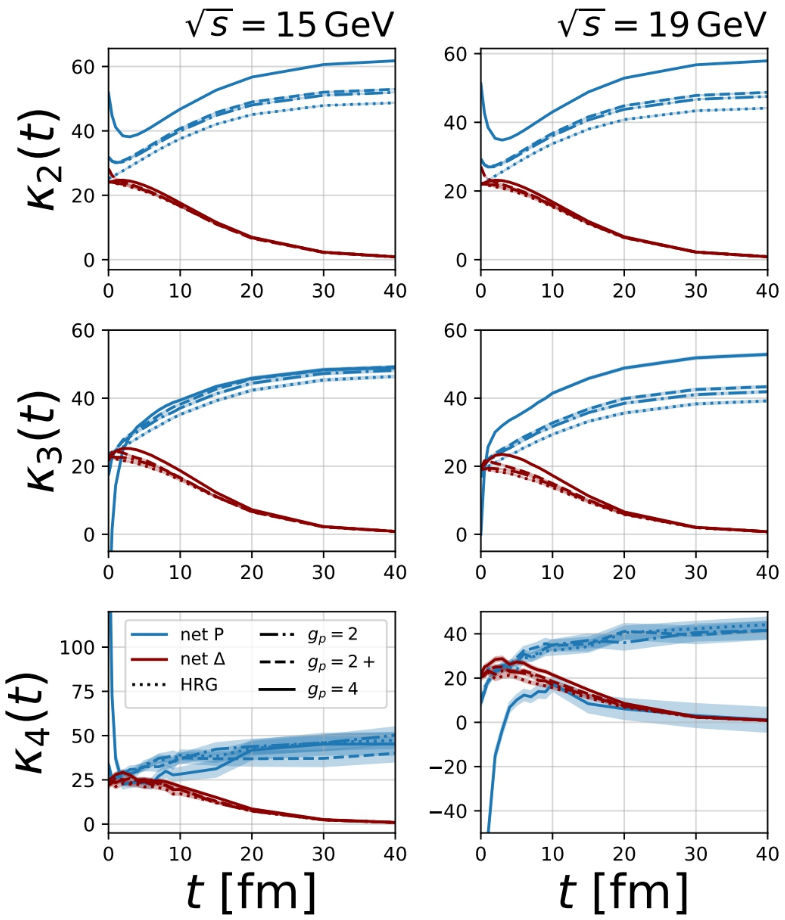

FIG. 6. Time evolution of the net proton (blue) and net delta (red) cumulants as a function of time. The second (upper row), third (center) and fourth cumulant (bottom row) are shown for two energies $\sqrt{s} = 15\,\mathrm{GeV}$ (left column) and $\sqrt{s} = 19\,\mathrm{GeV}$ (right column). The results are presented for initializing the system with the cases $g_p = 2$ (dashed-dotted line), $g_p = 4$ (straight line) as well as the case where more hadron species are coupled to the critical field $g_p = 2+$ (dashed line) and finally the HRG model (dotted line). A momentum cut of $0.3 < p < 2.0\,\mathrm{GeV}$ is included.

Results

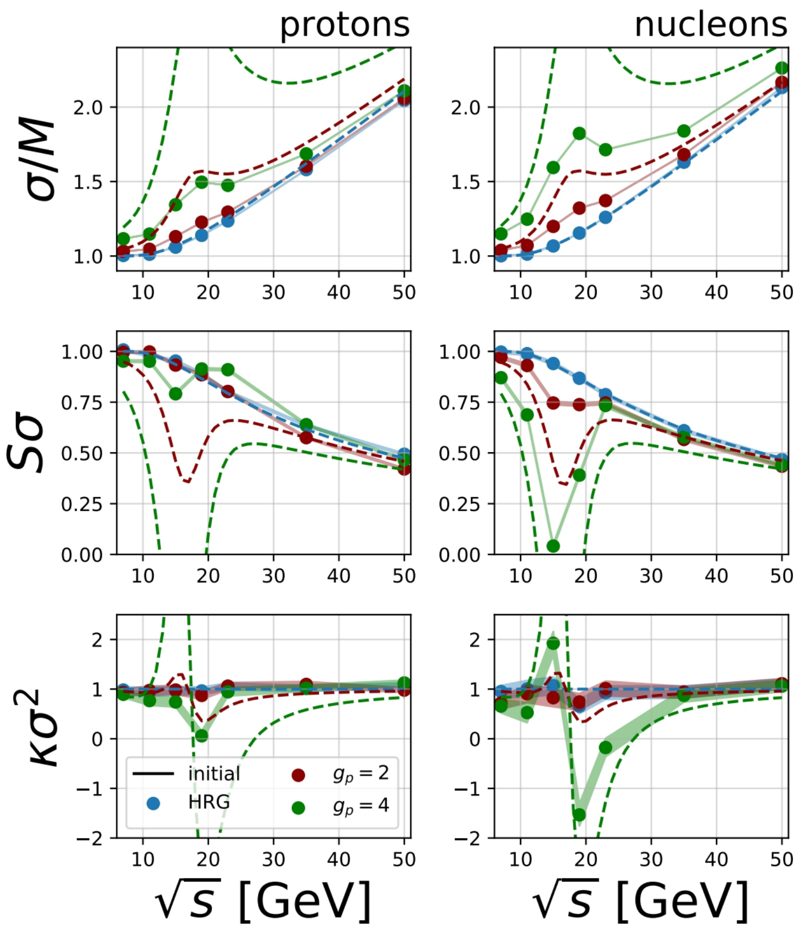

FIG. 9. Ratios of final state proton (upper row) and nucleon (lower row) cumulants as a function of $\sqrt{s}$. The scaled variance (left column) and skewness (center) kurtosis (right column) are shown. The results from SMASH are shown as points for the hrg initialization (blue) and coupling nucleons to the critical field with $g_p = 2$ (purple) and $g_p = 4$ (yellow) whereas the analytic results from the initial state are shown as dashed lines.

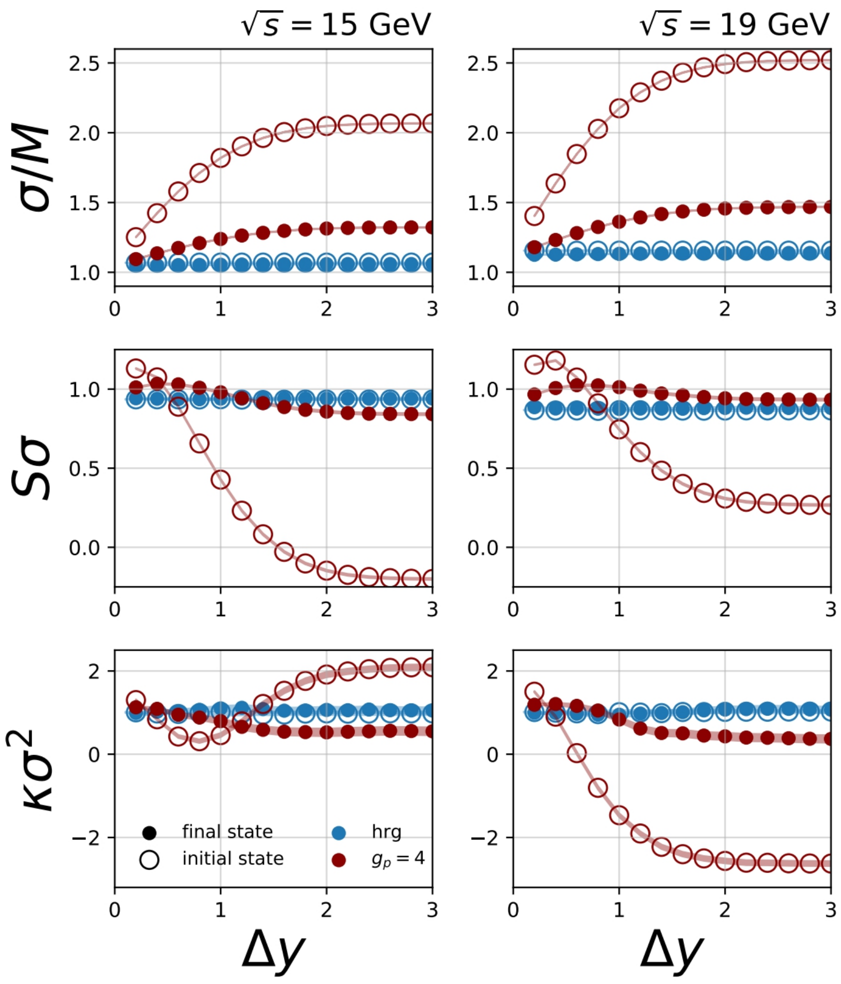

FIG. 10. Net proton scaled variance (top), skewness (center) and kurtosis (bottom) as a function of the rapidity window Δy for $\sqrt{s} = 15\,\mathrm{GeV}$ (left) and for $\sqrt{s} = 19\,\mathrm{GeV}$ (right). The results of the initial (open circles) and final state (closed circles) are shown for the HRG (blue) and $g_p = 4$ (red) case.

Thanks