Spinodal decomposition

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References

Supercritical Fluids
Fundamentals and Applications

Nuclear spinodal fragmentation

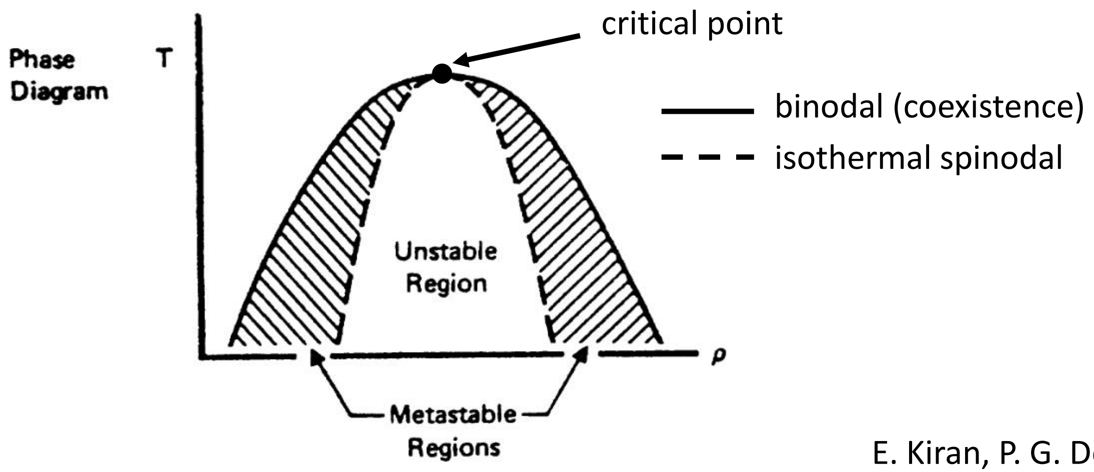
Philippe Chomaz[a], Maria Colonna[b], Jørgen Randrup[c,*]

Spinodal decomposition during the hadronization stage at RHIC?

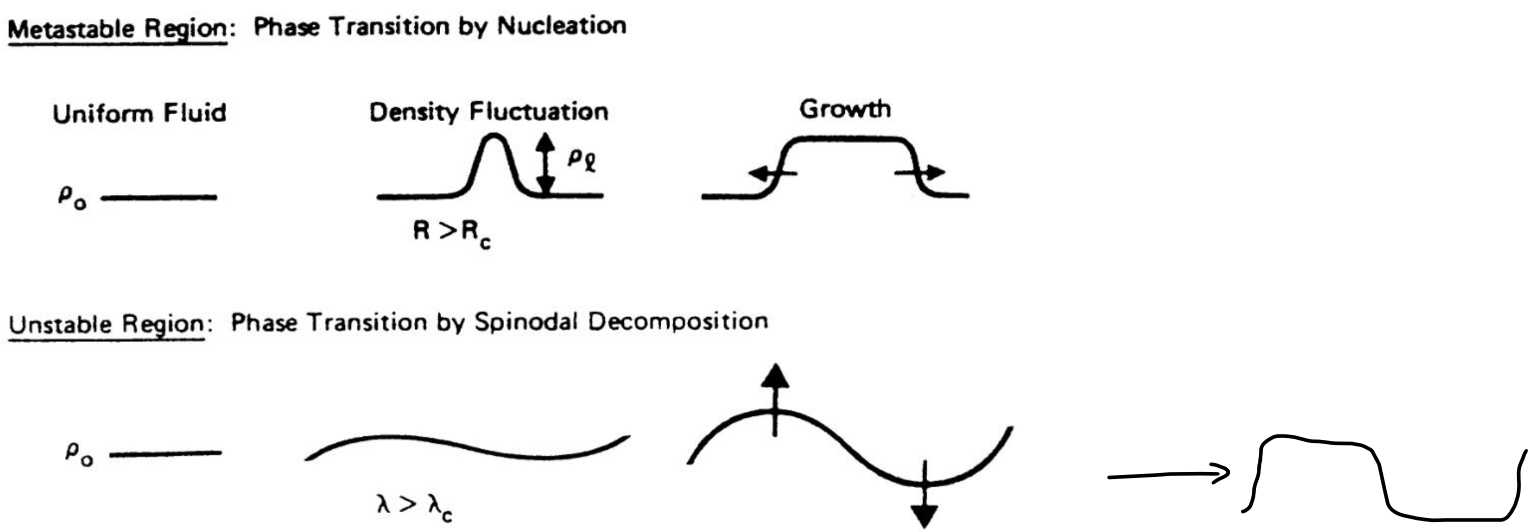
hep-ph/0308271

Jørgen Randrup

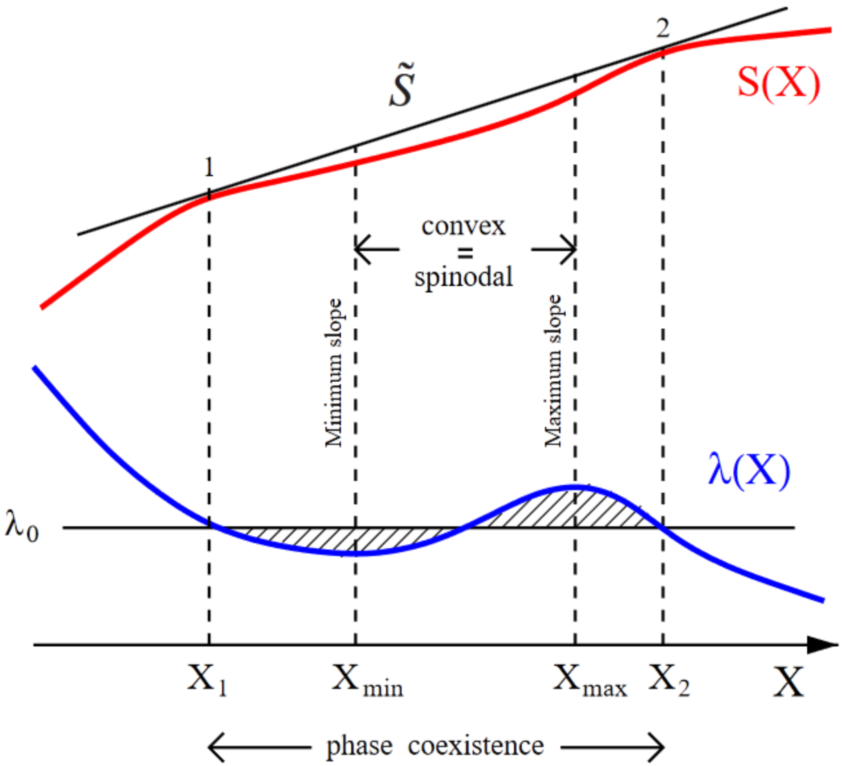
Two mechanisms of FOPT



E. Kiran, P. G. Debenedetti, C. J. Peters
Supercritical Fluid



Coexistence



$$S([\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots]) = \sum S_i(\boldsymbol{X}_i) \ ,$$

$$0 \doteq \sum_i \delta S_i(\boldsymbol{X}_i = \bar{\boldsymbol{X}}_i) = \sum_{i\ell} \left(\frac{\partial S_i}{\partial X_i^\ell}\right)_{\boldsymbol{X}_i = \bar{\boldsymbol{X}}_i} \delta X_i^\ell = \sum_{i\ell} \lambda_i^\ell \, \delta X_i^\ell \ ,$$

$$\lambda_i^\ell \ \equiv \ \partial S_i / \partial X_i^\ell \qquad \delta X^\ell = \sum\nolimits_i \delta X_i^\ell \doteq 0$$

e.g.

$$\lambda_i^E \equiv \frac{\partial S_i}{\partial E_i} = \frac{1}{T_i}, \quad \lambda_i^N \equiv \frac{\partial S_i}{\partial N_i} = -\frac{\mu_i}{T_i}, \quad \lambda_i^V \equiv \frac{\partial S_i}{\partial V_i} = \frac{P_i}{T_i} \ ,$$

$$\tilde{S} = N_1 \sigma(x_1) + N_2 \sigma(x_2) = \frac{N_1}{N} S(X_1) + \frac{N_2}{N} S(X_2) > S \ .$$

Maxwell construction

$$\int_{X_1}^{X_2} \mathrm{d}X \, (\lambda(X) - \lambda_0) \doteq 0 \ ,$$

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Spinodal instability

- bag model

$$p_\kappa(m) = \kappa\frac{T^4}{2\pi^2}\sum_{n=1}^{\infty}\frac{\kappa^n}{n^4}\left(\frac{nm}{T}\right)^2 K_2\!\left(\frac{nm}{T}\right) \asymp \frac{\pi^2}{90}T^4, \quad (1)$$

$$\varepsilon_\kappa(m) = \kappa\frac{T^4}{2\pi^2}\sum_{n=1}^{\infty}\frac{\kappa^n}{n^4}\left[3\left(\frac{nm}{T}\right)^2 K_2\!\left(\frac{nm}{T}\right) + \left(\frac{nm}{T}\right)^3 K_1\!\left(\frac{nm}{T}\right)\right] \asymp \frac{\pi^2}{30}T^4, \quad (2)$$

$\kappa = +$ for bosons and $\kappa = -$ for fermions

$$p_{\rm had}(T) = \sum_{i=\pi}^{\Delta} g_i p_{\kappa_i}(m_i)\ ,\ \varepsilon_{\rm had}(T) = \sum_{i=\pi}^{\Delta} g_i \varepsilon_{\kappa_i}(m_i)\ . \quad (3)$$

$i = \pi, K, \eta, \rho, \ldots, N, \Lambda, \Sigma, \Delta$

$$p_{\rm qgp}(T) = [g_g + \tfrac{7}{8}g_q]\frac{\pi^2}{90}T^4 + g_s p_-(m_s) - B\ , \quad (4)$$

$$\varepsilon_{\rm qgp}(T) = [g_g + \tfrac{7}{8}g_q]\frac{\pi^2}{30}T^4 + g_s \varepsilon_-(m_s) + B\ . \quad (5)$$

$g_g = 16$, $g_q = 24$, and $g_s = 12$.

$m_g, m_q = 0$, $\quad m_s = 150$ MeV

$T_c = 170$ MeV $\quad B^{1/4} = 250$ MeV hep-ph/0308271

Spinodal instability

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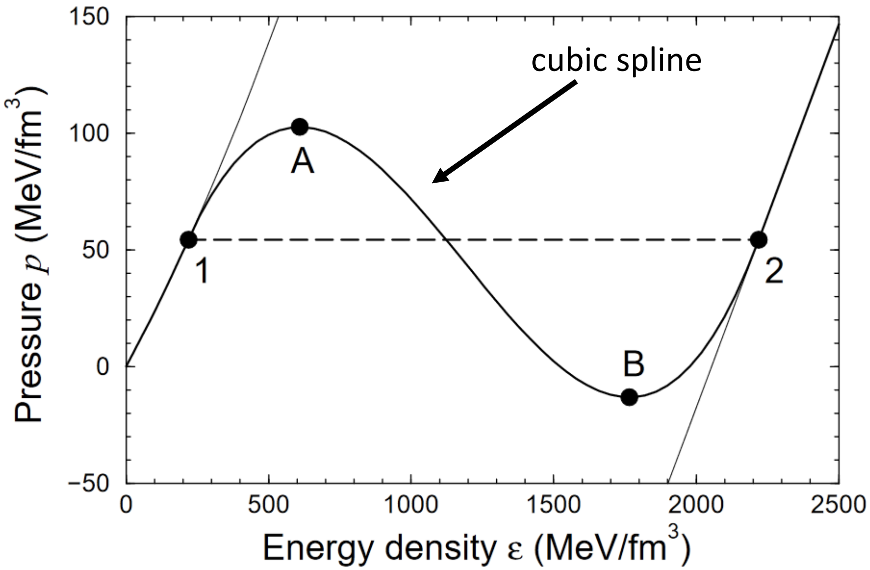

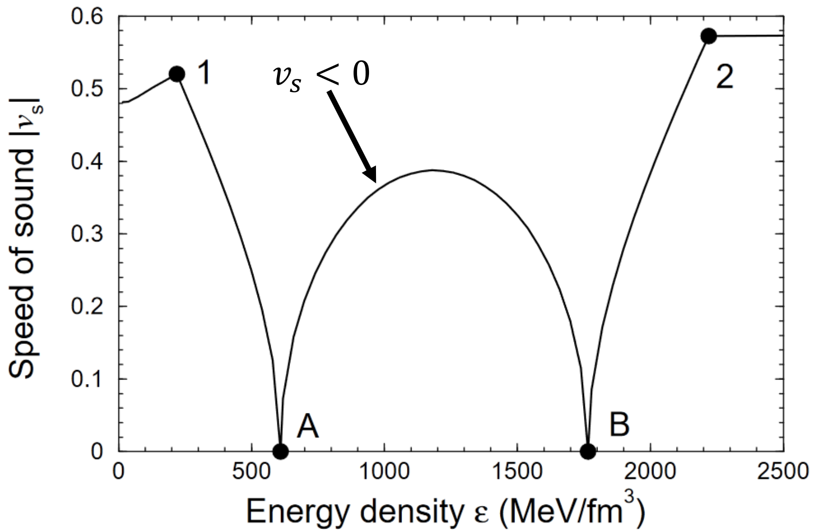

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p g^{\mu\nu}$$

$$\varepsilon(t,\boldsymbol{r}) = \varepsilon_0 + \delta\varepsilon(t,\boldsymbol{r}) \ , \ \delta\varepsilon(t,\boldsymbol{r}) \ll \varepsilon_0 \ .$$

$$0 \approx \partial_t \varepsilon - \boldsymbol{\nabla}\cdot[(\varepsilon+p)\boldsymbol{v}] \approx \partial_t \delta\varepsilon - (\varepsilon_0 + p_0)\boldsymbol{\nabla}\cdot\boldsymbol{v}, \quad (14)$$

$$\boldsymbol{0} \approx \partial_t[(\varepsilon+p)\boldsymbol{v}] - \boldsymbol{\nabla}p \approx (\varepsilon_0+p_0)\partial_t \boldsymbol{v} - v_s^2\, \boldsymbol{\nabla}\delta\varepsilon, \quad (15)$$

$$\partial_t^2 \delta\varepsilon(t,\boldsymbol{r}) = \frac{\partial p_0}{\partial \varepsilon_0} \nabla^2 \delta\varepsilon(t,\boldsymbol{r}) \ . \qquad (16)$$

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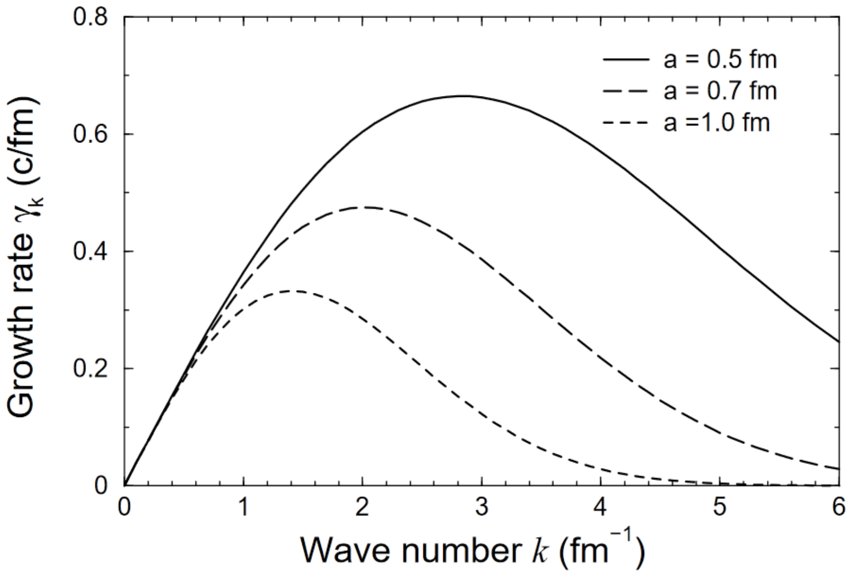
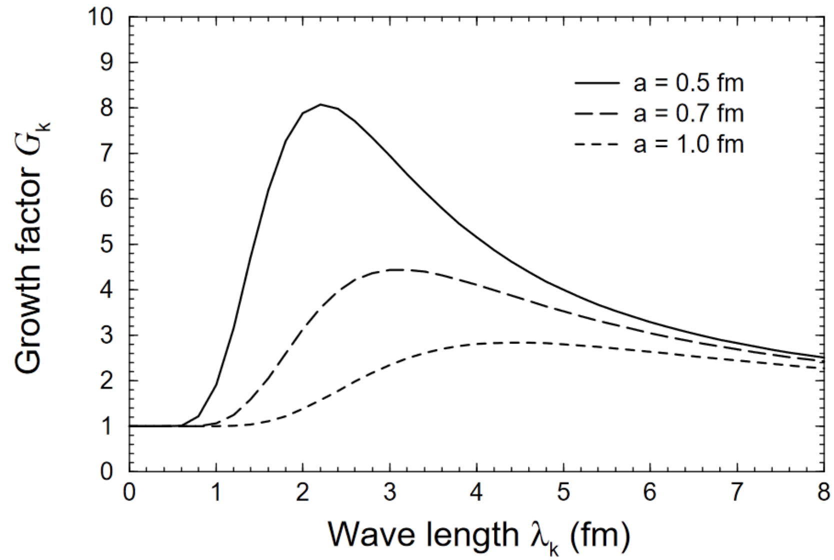
Spinodal instability





Consider finite range of interaction

$$p(\boldsymbol{r}_1) \approx \int d^3\boldsymbol{r}_2 \; g(|\boldsymbol{r}_1 - \boldsymbol{r}_2|) \; p(\varepsilon(\boldsymbol{r}_2)) \; . \qquad (17)$$

$$g(r) \sim \exp(-r^2/2a^2),$$

Consider finite traverse time in spinodal region

$$G_k = \exp(\Gamma_k)$$

$$\Gamma_k \equiv \int_{t_B}^{t_A} \gamma_k(\varepsilon(t))\, dt \approx \gamma_k(\tilde{\varepsilon})\, \Delta t_{\rm eff} \; .$$

$$t_B - t_A \approx 4\,{\rm fm}/c$$

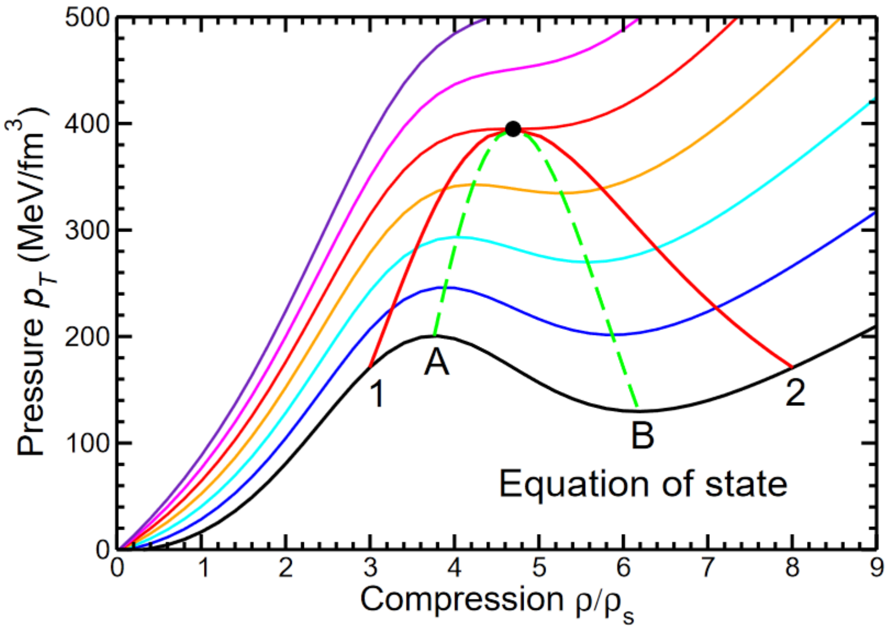
following work

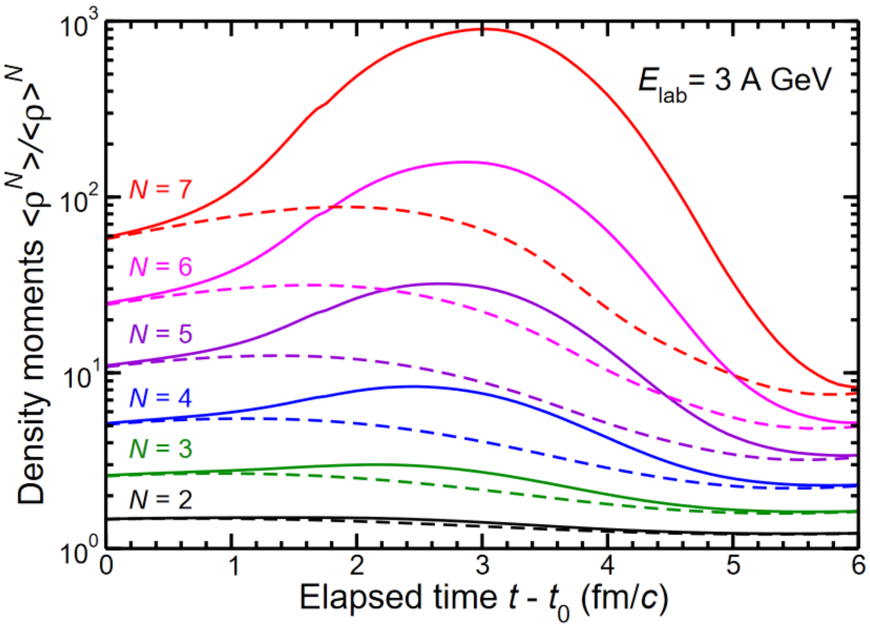
Phase transition dynamics for baryon-dense matter

Jørgen Randrup 0903.4736

Spinodal amplification of density fluctuations in fluid-dynamical simulations of relativistic nuclear collisions

Jan Steinheimer and Jørgen Randrup 1209.2462





Thanks for your listening!

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