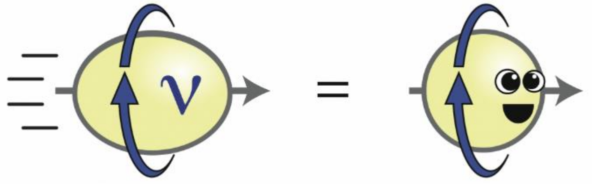
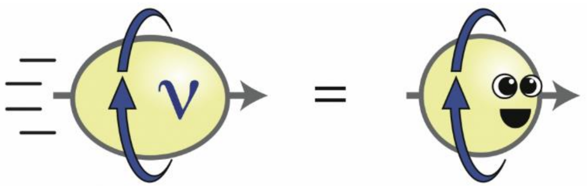
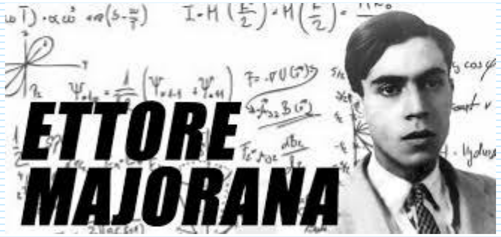


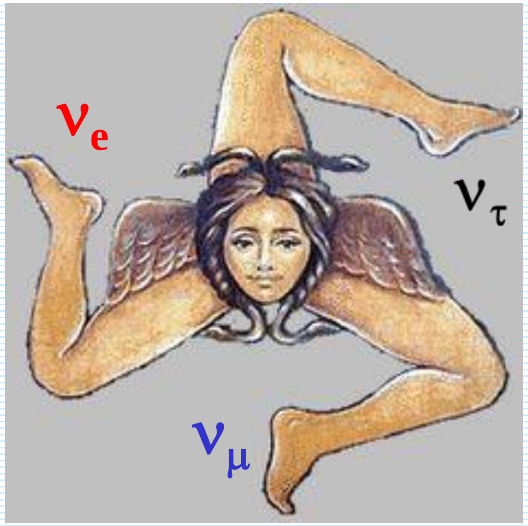
Institute of High Energy Physics, Chinese Academy of Sciences
Beijing, China - November 29 (Wen), 2023





Neutrino masses, oscillations and 0νββ decay

Fedor Šimkovic



OUTLINE

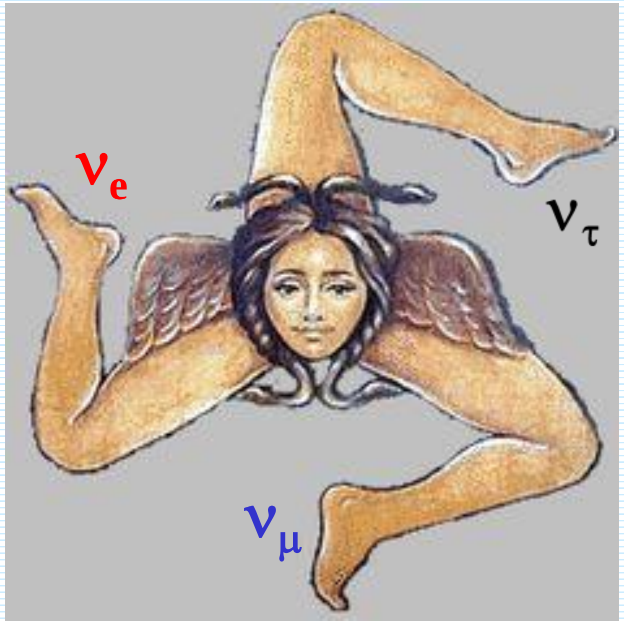
I. ν-oscillations
(standard approach -)
II. ν-oscillations as a single Feynman diagram
(QFT approach)
III. Oscillations of Quasi-Dirac neutrinos
IV. Neutrino-antineutrino oscillations
V. Oscillation of neutral atoms

After 93/67 years we know

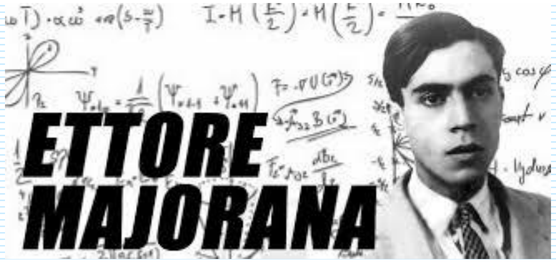
Fundamental ν properties

No answer yet

- 3 families of light (V-A) neutrinos: ν_e, ν_μ, ν_τ
- ν are massive: we know mass squared differences
- relation between flavor states and mass states (neutrino mixing)

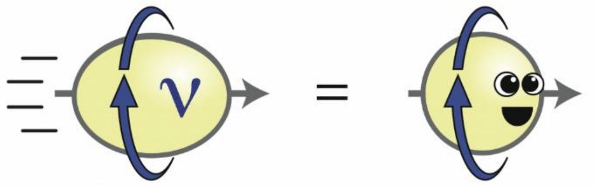


- Are ν Dirac or Majorana?
- Is there a CP violation in ν sector?
- Are neutrinos stable?
- What is the magnetic moment of ν?
- Sterile neutrinos?
- Statistical properties of ν? Fermionic or partly bosonic?



Currently main issue

Nature, Mass hierarchy,
CP-properties, sterile ν



The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{12} & s_{12} \\ 0 & -s_{12} & c_{12} \end{pmatrix}$$

$$\tilde{R}_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

3 neutrino masses, 2 mass squared differences

$$\delta m^2 = m_2^2 - m_1^2, \quad \Delta m^2 = m_3^2 - (m_1^2 + m_2^2)/2$$

$$U = R_{23} \tilde{R}_{13} R_{12}$$

3 mixing angles
CP-phase

$$|\nu_\alpha\rangle = \sum_{j=1}^{3} U^*_{\alpha j} |\nu_j\rangle$$

$$(\alpha = e,\ \mu, \tau)$$

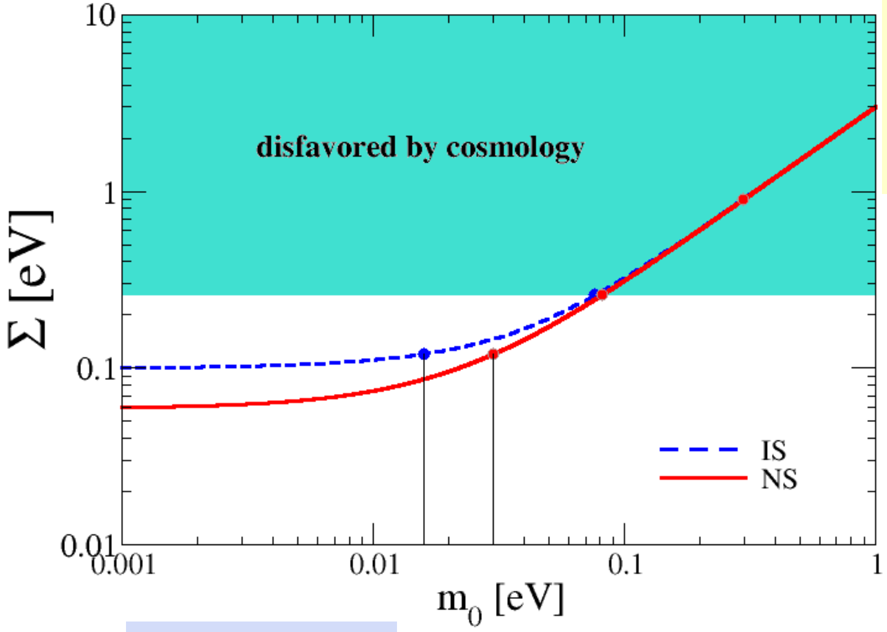
	best − fit	1σ	2σ	3σ
		Normal hierarchy (NH)		
$\delta m^2/10^{-5}$ eV^2	7.34	7.20-7.51	7.05-7.69	6.92-7.90
$\Delta m^2/10^{-3}$ eV^2	2.485	2.453-2.514	2.419-2.547	2.2389-2.578
$\sin^2\theta_{12}/10^{-1}$	3.05	2.92-3.19	2.78-3.32	2.65-3.47
$\sin^2\theta_{13}/10^{-2}$	2.22	2.14-2.28	2.07-2.34	2.01-2.41
$\sin^2\theta_{23}/10^{-1}$	5.45	4.98-5.65	4.54-5.81	4.36-5.95
δ/π	1.28	1.10-1.66	0.95-1.90	0-0.07 $\oplus$ 0.81-2.00
		Inverted hierarchy (IH)		
$\delta m^2/10^{-5}$ eV^2	7.34	7.20-7.51	7.05-7.69	6.92-7.91
-$\Delta m^2/10^{-3}$ eV^2	2.465	2.434-2.495	2.404-2.526	2.374-2.556
$\sin^2\theta_{12}/10^{-1}$	3.03	2.90-3.17	2.77-3.31	2.64-3.45
$\sin^2\theta_{13}/10^{-2}$	2.23	2.17-2.30	2.10-2.37	2.03-2.43
$\sin^2\theta_{23}/10^{-1}$	5.51	5.17-5.67	4.60-5.82	4.39-5.96
		$\oplus$ 5.31-6.10		
δ/π	1.52	1.37-1.65	1.23-1.78	1.09-1.90

Global neutrino oscillations analysis
(PRD 101, 116013 (2020))

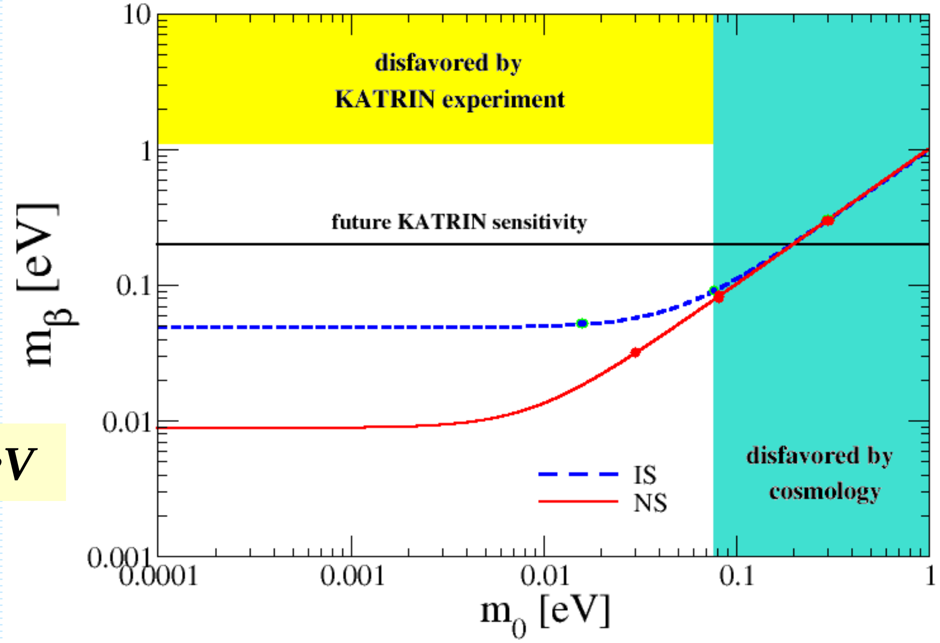
Cosmological measurements

$\Sigma = m_1 + m_2 + m_3$

Σ < 0.90 eV
< 0.26 eV (Planck coll.)
< 0.12 eV



Tritium β-decay

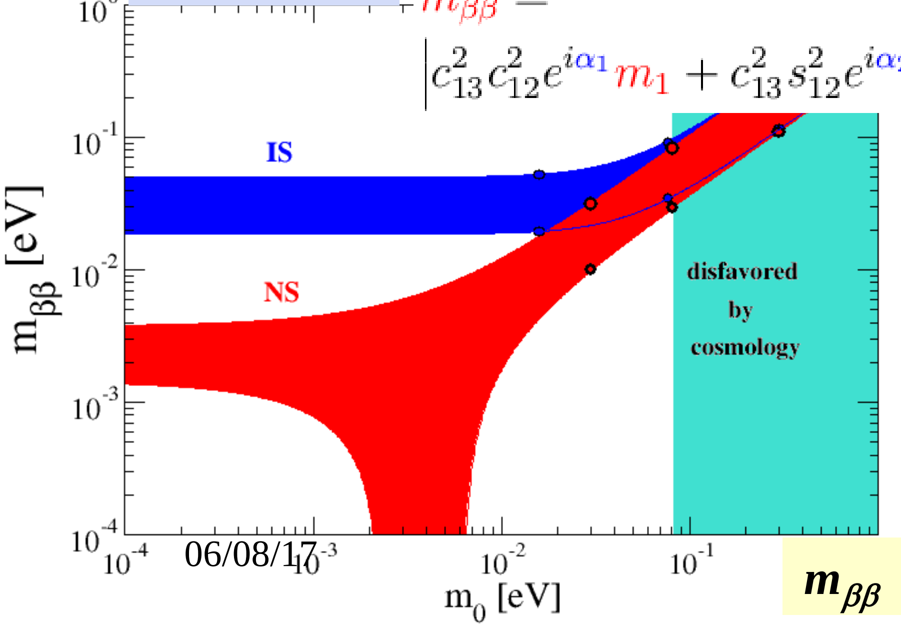


m_β < 1.1 eV

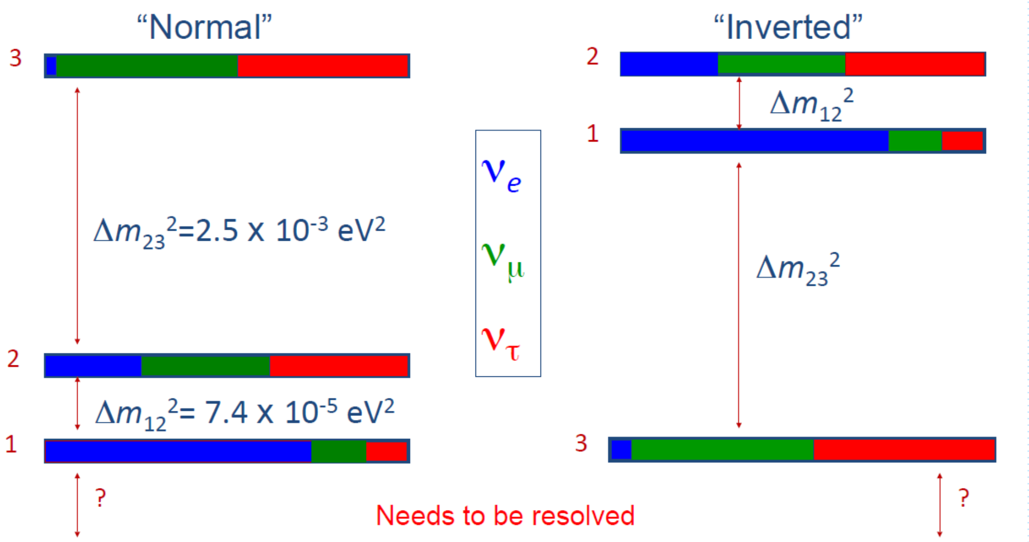
$$m_\beta = \sqrt{c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2}$$

0νββ-decay

$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3 \right|$$



$m_{\beta\beta}$ < 36-156 meV



Neutrino Mass Term

Dirac

$$L^{D}_{\text{mass}} = -\sum_{\alpha\beta} \overline{\nu}_{\alpha R}\, M^{D}_{\alpha\beta}\, \nu_{\beta_L} + H.c.$$

$$= -\sum_{k=1}^{3} m_k \overline{\nu}_k \nu_k$$

$\alpha, \beta = e, \mu, \tau,$ $\quad V^{\dagger}\, M^{D}\, U = M^{D}_{\text{diag}}$

Majorana

$$L^{M}_{\text{mass}} = \frac{1}{2} \sum_{\alpha\beta} \nu^{T}_{\alpha L} C^{\dagger}\, M^{L}_{\alpha\beta}\, \nu_{\beta_L} + H.c.$$

$$= \frac{1}{2} \sum_{k=1}^{3} m_k \nu^{T}_{k} C^{\dagger} \nu_k$$

$\alpha, \beta = e, \mu, \tau$

$M^{L}_{\alpha\beta} = M^{L}_{\beta\alpha}$ $\quad U^{T}\, M^{M}\, U = M^{M}_{\text{diag}}$

Dirac-Majorana

$$L^{D+M}_{\text{mass}} = -\sum_{\alpha\beta} \overline{N}_{\alpha R}\, M^{D+M}_{\alpha\beta}\, N_{\beta_L} + H.c.$$

$$= -\sum_{k=1}^{6} m_k \overline{\nu}_k \nu_k$$

$N = \begin{pmatrix} \nu_L \\ \nu^{C}_{R} \end{pmatrix}, \quad \alpha, \beta = e, \mu, \tau, s_1, s_2, s_3$

$$U^{T}\, M^{D+M}\, U = M^{D+M}_{\text{diag}}$$

Bruno Pontecorvo
Inverse beta processes and non-conservation of lepton charge
Zhur. Eksptl'. i Teoret. Fiz. 34, 247 (1958)

Neutrino oscillations
(Quantum Mechanics Approach)

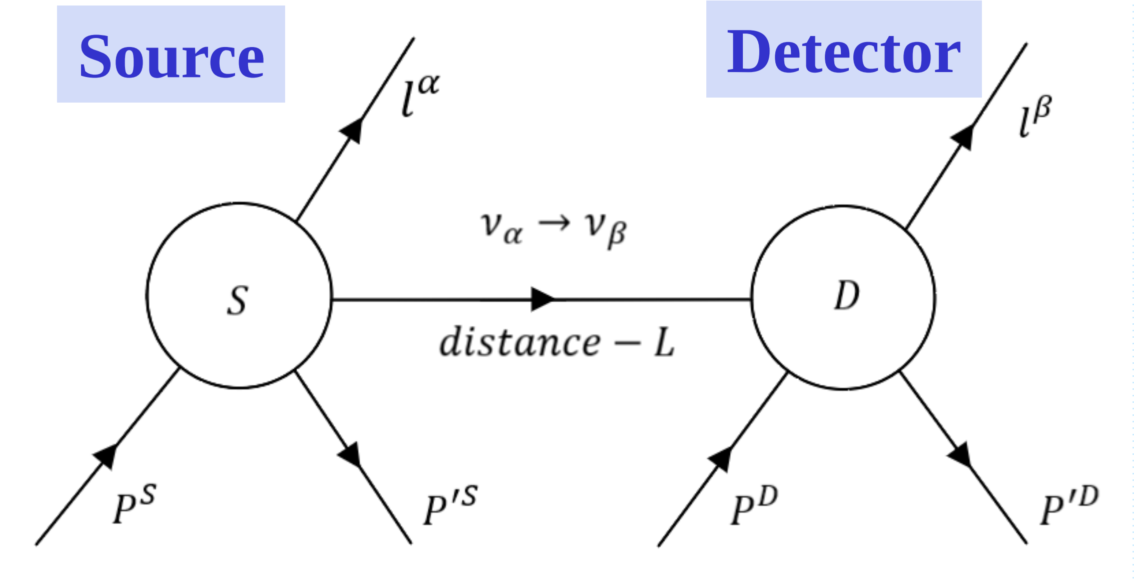
Massive neutrinos and neutrino oscillations

S. M. Bilenky
Joint Institute of Nuclear Research, Dubna, Union of Soviet Socialist Republics

S. T. Petcov
Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 1784 Sofia, People's Republic of Bulgaria

The theory of neutrino mixing and neutrino oscillations, as well as the properties of massive neutrinos (Dirac and Majorana), are reviewed. More specifically, the following topics are discussed in detail: (i) the possible types of neutrino mass terms; (ii) oscillations of neutrinos (iii) the implications of *CP* invariance for the mixing and oscillations of neutrinos in vacuum; (iv) possible varieties of massive neutrinos (Dirac, Majorana, pseudo-Dirac); (v) the physical differences between massive Dirac and massive Majorana neutrinos and the possibilities of distinguishing experimentally between them; (vi) the electromagnetic properties of massive neutrinos. Some of the proposed mechanisms of neutrino mass generation in gauge theories of the electroweak interaction and in grand unified theories are also discussed. The lepton number nonconserving processes $\mu \to e\gamma$ and $\mu \to 3e$ in theories with massive neutrinos are considered. The basic elements of the theory of neutrinoless double-β decay are discussed as well. Finally, the existing data on neutrino masses, oscillations of neutrinos, and neutrinoless double-β decay are briefly reviewed. The main emphasis in the review is on the general model-independent results of the theory. Detailed derivations of these are presented.

Rev. Mod. Phys. 59, 671 (1987) , cca 1000 citations (inspire hep)



$$S \to S' + \ell_\alpha^+ + \nu_\alpha$$
$$\nu_\alpha \to \nu_\beta$$
$$\nu_\beta + D \to D' + \ell_\beta^-$$

$$\Gamma_{osc} = \int \frac{d\Phi_\nu(E_\nu)}{dE_\nu} \frac{\mathcal{P}_{\alpha\beta}(E_\nu, L)}{4\pi L^2} \sigma(E_\nu)\, dE_\nu$$

Process is governed by the oscillation probability

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^{3} U_{\alpha j}^* U_{\beta j} e^{-i\, m_j^2\, L/(2E_\nu)} \right|^2$$

Neutrino oscillations (QM) – concept of oscillation probability
(Propagation of neutrinos described by a plane wave)

Evolution of flavor eigenstates

$$|\nu_\alpha^{\rm fl}\rangle = \sum_k U^*_{\alpha k}|\nu_k^{\rm mass}\rangle \quad \Rightarrow \quad |\nu_\alpha^{\rm fl}\rangle = \sum_k U^*_{\alpha k} e^{-i\Phi_k}|\nu_k^{\rm mass}\rangle$$

$$\Phi_k = E_k t - p_k x$$

Oscillation phase

$$\Delta\Phi = \Phi_j - \Phi_k = \Delta E t - p x$$

Same momentum prescription

$$\Delta p = 0$$

$$E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2p}$$

$$\Delta E \simeq \frac{m_2^2 - m_1^2}{2p} \equiv \frac{\Delta m^2}{2p}$$

$$\Delta\Phi = \frac{\Delta m^2}{2p} t$$

$L \approx t$ **Time-to-space conversion**

Same energy prescription

$$\Delta E = 0$$

$$p_k = \sqrt{E^2 - m_k^2} \simeq E - \frac{m_k^2}{2E}$$

$$-\Delta p \equiv p_1 - p_2 \simeq \frac{\Delta m^2}{2E}$$

$$\Delta\Phi = \frac{\Delta m^2}{2E} x$$

No time-to-space conversion necessary

Standard phase $\Rightarrow$

$$l_{\rm osc} = \frac{4\pi p}{\Delta m^2} =\simeq 2.48\ m\ \frac{p\ (MeV)}{\Delta m^2\ (eV^2)}$$

An example:

$$\pi^+ + n \to \mu^+ + e^- + p$$

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \nu_\mu \to \nu_e, \quad \nu_e + n \to p + e^-$$

Hamiltonian:

$$R^{\pi^+ n \to \mu^+ + p + e^-} = \int \frac{d\Gamma^{\pi \to \mu^+ \nu_\mu}(E_\nu)}{dE_\nu} \frac{P_{\nu_\mu \nu_e}(E_\nu)}{4\pi L^2} \sigma^{\nu_e n \to p e^-}(E_\nu)\, dE_\nu$$

Oscillation probability:

$$P_{\nu_\mu \nu_e}(E_\nu) = \left| \sum_{k=1}^{3} U^*_{\mu k} U_{ek} e^{-i m_k^2 L/(2E_\nu)} \right|^2$$

Energy distribution of ν_μ in π-decay

$$\pi \to \mu + \nu_\mu$$

$$P_\pi \equiv (0, m_\pi), \quad P_\mu \equiv (\mathbf{p}_\mu, E_\mu), \quad P_\nu \equiv (\mathbf{p}_\nu, E_\nu)$$

$$P_\pi = P_\mu + P_\nu \quad P_\mu^2 = m_\mu^2, \quad P_\nu^2 = 0$$

Neutrino mass assumed to be zero

Energy distribution of emitted ν_μ in π-decay

Hamiltonian:

$$H^{\pi-\mu\nu} = \frac{G_\beta}{\sqrt{2}}\, \overline{\nu}_\mu(x)\gamma^\rho(1-\gamma_5)\mu(x)\ \frac{f_\pi}{\sqrt{2}}\partial_\rho\Phi^+(x) + H.c.$$

S-matrix:

$$\langle f|S^{(1)}|i\rangle = (2\pi)^4\delta(P_\pi - P_\mu - P_\nu)\ \langle f|T|i\rangle$$

$$\langle f|T|i\rangle = -i\frac{G_\beta}{\sqrt{2}}\ \frac{f_\pi}{\sqrt{2}}\Phi_\pi(0)\ m_\mu\ \overline{v}(P_\nu)(1+\gamma_5)u(P_\mu)$$

Neutrino mass assumed to be zero

Differential decay rate:

$$d\Gamma = \sum_{\text{spin}} |\langle f|T|i\rangle|^2\ (2\pi)^4\delta(P_\pi - P_\mu - P_\nu)\ \frac{d\mathbf{p}_\mu}{(2\pi)^3}\ \frac{d\mathbf{p}_\nu}{(2\pi)^3}$$

Energy distribution is monoenergetic:

$$d\Gamma = \frac{1}{2\pi}G_\beta^2\ \left(\frac{f_\pi}{\sqrt{2}}\right)^2\ \frac{m_\mu^2}{m_\pi}\ \delta(E_\nu - E^0_\nu)p_\nu E_\nu dE_\nu$$

$$\text{with}\quad E^0_\mu = m_\pi(1 + m^2_\mu/m^2_\pi)/2$$

$\nu_e + n \to p + e^-$

Cross-section of reaction $\nu_e + n \to p + e^-$

Hamiltonian:

$$H^{\beta}(x) = \frac{G_{\beta}}{\sqrt{2}}\, \overline{e}(x)\gamma^{\rho}(1-\gamma_5)\nu_e(x)\; \overline{n}(x)\gamma_{\rho}(g_V - g_A\gamma_5)p(x) + H.c.$$

S-matrix:

$$\langle f|S^{(1)}|i\rangle = (2\pi)^4\delta(P_e + P_p - P_n - P_{\nu})\; \langle f|T|i\rangle$$

$$\langle f|T|i\rangle = -i\frac{G_{\beta}}{\sqrt{2}}\, \overline{u}(P_p)\gamma_{\rho}(g_V - g_A\gamma_5)u(P_n)\; \overline{u}(P_e)\gamma^{\rho}(1-\gamma_5)u(P_{\nu})$$

Differential cross section:

$$d\sigma = \frac{1}{j}\,\frac{1}{2E_{\nu}\;2E_n}\,\frac{1}{2}\sum_{\text{spin}}|\langle f|T|i\rangle|^2 \times$$
$$(2\pi)^4\delta(P_{\nu} + P_n - P_p - P_e)\;\frac{d\mathbf{p}_e}{2E_e\;(2\pi)^3}\;\frac{d\mathbf{p}_p}{2E_p\;(2\pi)^3}$$

Calculation of traces:

$$\sum_{\text{spin}} |\langle f|T|i\rangle|^2 = \left(\frac{G_\beta}{\sqrt{2}}\right)^2 64 \; \Big[\left(-g_V^2 + g_A^2\right) \; M_n M_p \; (P_\nu \cdot P_e) + (g_V - g_A)^2 (P_\nu \cdot P_p)(P_e \cdot P_n) + (g_V + g_A)^2 (P_\nu \cdot P_n)(P_e \cdot P_p)\Big]$$

Simplifications:

$$P_n = (M_n, 0), \quad \mathbf{p}_p = \mathbf{p}_\nu - \mathbf{p}_e, \quad M_p \simeq M_n \equiv M, \quad E_p \simeq M$$

$$\int d\Omega_e \sum_{\text{spin}} |\langle f|T|i\rangle|^2 = \left(\frac{G_\beta}{\sqrt{2}}\right)^2 4\pi \; 64 \; E_\nu E_e \; M^2 \; [g_V^2 + 3g_A^2].$$

Cross-section of the reaction $\nu_e + n \rightarrow p + e^-$:

$$d\sigma = \frac{1}{j} \frac{1}{\pi} G_\beta^2 \; [g_V^2 + 3g_A^2] \; \delta(E_\nu - E_e) \; p_e E_e dE_e$$

$$\sigma(E_\nu) = \frac{1}{\pi} G_\beta^2 \; [g_V^2 + 3g_A^2] \; p_e E_e$$

The production rate (standard approach)

$$\pi^+ + n \to \mu^+ + e^- + p$$

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \nu_\mu \to \nu_e, \quad \nu_e + n \to p + e^-$$

$$\Gamma^{\pi^+ n}_{osc} = \int \frac{d\Phi_\nu(E'_\nu)}{dE'_\nu} \frac{\mathcal{P}_{\nu_\mu\nu_e}(E'_\nu)}{4\pi L^2} \sigma(E'_\nu)\, dE'_\nu$$

$$= \frac{1}{2\pi^2} G^2_\beta \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m^2_\mu}{m_\pi} E^2_\nu \frac{P_{\nu_\mu\nu_e}(E_\nu)}{4\pi L^2} (g^2_V + 3g^2_A)\; p_e E_e$$

with

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left|\sum_{j=1}^{3} U^*_{\alpha j} U_{\beta j} e^{-i m^2_j L/(2E_\nu)}\right|^2$$

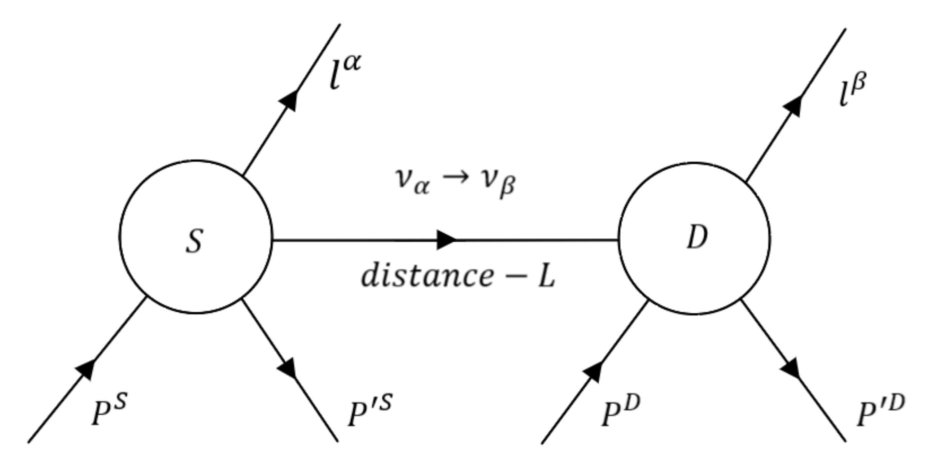
$$E_\nu = \frac{1}{2} m_\pi \left(1 - \frac{m^2_\mu}{m^2_\pi}\right) \simeq E_e$$

Neutrino oscillations as a single Feynman diagram

(within QFT, Walter Grimusapproach revisited)

e-Print: *2212.13635* [hep-ph]

$$S + D \rightarrow \ell_\alpha^+ + \ell_\beta^- + S' + D'$$



$$\langle f|S^{(2)}|i\rangle = -i \int d^4x_1 J_S^\mu(P'_S, P_S) e^{i(P_\alpha + P'_S - P_S)\cdot x_1} \times$$

$$\int d^4x_2 J_D^\mu(P'_D, P_D) e^{i(P_\beta + P'_D - P_D)\cdot x_2} \sum_{k=1}^{3} U^*_{\alpha k} U_{\beta k} \times$$

$$\overline{v}(P_\alpha;\lambda_\alpha)\gamma_\mu(1-\gamma_5)\ D(x_2 - x_1, m_k)\ (1-\gamma_5)\gamma_\nu u(P_\beta;\lambda_\beta)$$

The neutrino emission and detection are identified with the charged-Current vertices of a single second-order Feynman diagram for the underlying process, enclosing neutrino propagation between these two points.

$$D(x;m) = \theta(x_0) D^-(x;m) + \theta(-x_0) D^+(x;m),$$

$$D^\pm(x;m) = \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\mp(-\mathbf{q}\cdot\vec{\gamma} + \omega\gamma^0) + m}{2\omega} e^{\pm i(-\mathbf{q}\cdot\mathbf{x} + \omega x_0)}$$

Integration over time variables results in energy conservation and energy denominator

$$2\pi i \frac{\delta(E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S)}{\omega + E_\alpha + E'_S - E_S + i\varepsilon}.$$

Neutrino propagator

$$\int \frac{d\mathbf{q}}{(2\pi)^3} \frac{\not{p} + m_k}{2\omega(\omega + E_\alpha + E'_S - E_S + i\varepsilon)} e^{i\mathbf{q}\cdot(\mathbf{x}_2 - \mathbf{x}_1)}$$

$$\simeq \frac{1}{4\pi} \frac{e^{ip_k|\mathbf{x}_2 - \mathbf{x}_1|}}{|\mathbf{x}_2 - \mathbf{x}_1|} \left(\not{Q}_k + m_k\right) \simeq e^{i\mathbf{p}_k\cdot\mathbf{x}_D}\ e^{-i\mathbf{p}_k\cdot\mathbf{x}_S}\ \frac{e^{ip_k L}}{L} \left(\not{Q}_k + m_k\right)$$

$$Q_k \equiv (E_\nu, \mathbf{p}_k), \quad \mathbf{p}_k = p_k\,(\mathbf{x}_2 - \mathbf{x}_1)\,/|\mathbf{x}_2 - \mathbf{x}_1|, \quad p_k = \sqrt{E_\nu^2 - m_k^2}$$

$$E_\nu = E_S - E'_S - E_\alpha \ (source) = E_\beta + E'_D - E_D \ (detector)$$

Energy conservation

Amplitude

Momentum conservation at source

Momentum conservation at detector

Energy conservation

$$\langle f|S^{(2)}|i\rangle = (2\pi)^7 \delta(E_f - E_i)\sum_k U_{\alpha k}U^*_{\beta k}\ \frac{e^{ip_k L}}{4\pi L}\ \times$$

$$T_k^{\alpha\beta}\ \delta^3_{V_S}(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S)\ \delta^3_{V_D}(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_k)$$

with

$$E_f - E_i = E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S$$

$$T_k^{\alpha\beta} = J^\mu_S(P'_S, P_S) J^\nu_D(P'_D, P_D) \not{q}_k \gamma_\nu u(P_\beta;\lambda_\beta)\gamma_\nu u(P_\beta;\lambda_\beta)$$

Lepton current

Master Formula
(Fermi Golden Rule for a second-order process with on-shell intermediate state)

$$d\Gamma^{\alpha\beta}(L) = \sum_{km} U_{\alpha k} U^*_{\beta k} U_{\alpha m} U^*_{\beta m} \frac{e^{i(p_k - p_m)L}}{4\pi L^2} \times \mathcal{F}^{\alpha\beta}_{km}$$
$$\delta(\mathbf{p}_k + \mathbf{p}_\alpha + \mathbf{p}'_S - \mathbf{p}_S)\delta(\mathbf{p}_\beta + \mathbf{p}'_D - \mathbf{p}_D - \mathbf{p}_m)$$
$$\frac{(2\pi)^7}{4E_S E_D} \delta(E_\beta + E'_D - E_D + E_\alpha + E'_S - E_S) \times$$
$$\frac{1}{\hat{J}_S \hat{J}_D} \frac{d\mathbf{p}_\alpha}{2E_\alpha (2\pi)^3} \frac{d\mathbf{p}_\beta}{2E_\beta (2\pi)^3} \frac{d\mathbf{p}'_S}{2E'_S (2\pi)^3} \frac{d\mathbf{p}'_D}{2E'_D (2\pi)^3}$$

with

$$\mathcal{F}^{\alpha\beta}_{km} = 4\pi \sum_{\text{spin}} \frac{1}{2} \left(T^{\alpha\beta}_k \left(T^{\alpha\beta}_m\right)^* + T^{\alpha\beta}_m \left(T^{\alpha\beta}_k\right)^* \right)$$

$$\langle \Phi^{S,D}(\mathbf{P}_i) | \Phi^{S,D}(\mathbf{P}_k) \rangle = (2\pi)^3 \, 2E_k \, \delta^3_{V_{S,D}} (\mathbf{P}_i - \mathbf{P}_k)$$

Two normalization volumes:
i) source;
ii) Detector.

$$\delta^3_V(\mathbf{Q}_n - \mathbf{P}) \, \delta^3_V(\mathbf{Q}_m - \mathbf{P}) \simeq \frac{V}{(2\pi)^3} \frac{1}{2} \left(\delta^3_V(\mathbf{Q}_n - \mathbf{P}) + \delta^3_V(\mathbf{Q}_m - \mathbf{P}) \right)$$

An example:

$$\pi^+ + n \to \mu^+ + e^- + p$$

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \nu_\mu \to \nu_e, \quad \nu_e + n \to p + e^-$$

Hamiltonians:

$$H^{\pi-\mu\nu}(x) = \frac{G_\beta}{\sqrt{2}} \, \overline{\nu}_\mu(x)\gamma^\rho(1-\gamma_5)\mu(x) \, \frac{f_\pi}{\sqrt{2}}\partial_\rho\Phi^+(x) + H.c.$$

$$H^\beta(x) = \frac{G_\beta}{\sqrt{2}} \, \overline{e}(x)\gamma^\rho(1-\gamma_5)\nu_e(x) \, \overline{n}(x)\gamma_\rho(g_V - g_A\gamma_5)p(x) + H.c.$$

Neutrino mixing:

$$\nu_\alpha = \sum_k U_{\alpha k} \, \nu_k$$

Second order process in weak interactions

$$S^{(2)} = (-i)^2 \left(\frac{G_\beta}{\sqrt{2}}\right)^2 \times$$

$$\int N\left[\overline{e}(x_2)\gamma^\rho(1-\gamma_5)\sum_k U_{ek}U^*_{\mu k}S_k(x_2-x_1)\gamma^\rho(1-\gamma_5)\mu(x_1)\right] \times$$

$$N\left[\overline{n}(x_2)\gamma_\rho(g_V - g_A\gamma_5)p(x_2) \; \Phi^+(x_1)\right] \; dx_2 \; dx_1$$

Neutrino propagator:

$$S_k(x_2 - x_1) = \int \frac{dP_k}{(2\pi)^4} e^{-iP\cdot(x_2-x_1)} S_k(p) = \int \frac{dP_k}{(2\pi)^4} e^{-iP\cdot(x_2-x_1)} \frac{\not{P}_k + m_k}{p^2 - m_k^2 + i\varepsilon}$$

$$= -i \int \frac{d\mathbf{p}}{(2\pi)^3} \left(e^{-iE_k(t_2-t_1)} e^{+i\mathbf{p}\cdot(\mathbf{x}_2-\mathbf{x}_1)} \frac{\not{P}_k + m_k}{2E_k} \Theta(t_2 - t_1) + e^{+iE_k(t_2-t_1)} e^{-i\mathbf{p}\cdot(\mathbf{x}_2-\mathbf{x}_1)} \frac{-\not{P}_k + m_k}{2E_k} \Theta(t_1 - t_2) \right)$$

$$\Rightarrow -i \int \frac{d\mathbf{p}}{(2\pi)^3} e^{-iP_k\cdot(x_2-x_1)} \frac{\not{P}_k + m_k}{2E_k} \Theta(t_2 - t_1)$$

with $\quad P_k \equiv (\mathbf{p}, E_k), \quad E_k = \sqrt{\mathbf{p}_k^2 + m_k^2}$

$t_2 > t_1$ is assumed (propagation forward in time)

S-matrix of the process:

$$\langle f|S^{(2)}|i\rangle = -\left(\frac{G_\beta}{\sqrt{2}}\right)^2 \sum_k U_{ek}U^*_{\mu k} \times$$
$$\overline{u}(P_e)\gamma^\sigma(1-\gamma_5)(\not{P}_k + m_k)\gamma^\rho(1-\gamma_5)v(P_\mu)\times$$
$$f_\pi(P_\pi)_\rho\,\overline{u}(P_p)\gamma_\sigma(g_V - g_A\gamma_5)u(P_n)\times$$
$$\int dx_1 \int dx_2\; e^{i(P_e+P_p-P_n)\cdot x_2} e^{i(P_\mu-P_\pi)\cdot x_1}\times$$
$$\int \frac{d\mathbf{p}}{(2\pi)^3} e^{-iP_k\cdot(x_2-x_1)}\frac{1}{2E_k}\Theta(t_2-t_1)$$

substitutions:

$$(t_1,t_2)\to(t_1,\tau=t_2-t_1)$$
$$(\mathbf{x}_1,\mathbf{t}_2)\to(\mathbf{x}_1,\mathbf{r}=\mathbf{x}_2-\mathbf{x}_1)$$

Integration over the time coordinates:

$$\int e^{i(E_e+E_p-E_n-E_k)t_2} e^{i(E_\mu+E_k-E_\pi)t_1}\Theta(t_2-t_1)dt_1dt_2$$
$$= 2\pi\delta\left(E_e+E_p-E_n+E_\mu-E_\pi\right)\lim_{\varepsilon\to 0}\frac{i}{E_\pi-E_\mu-E_k+i\varepsilon}$$

Integration over neutrino momentum (in the complex plane):

$$\int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot(\mathbf{x}_2-\mathbf{x}_1)} \frac{\not{P}+m_k}{2E_k(E_\pi - E_\mu - E_k + i\varepsilon)} \simeq \frac{1}{4\pi} \frac{e^{ip_k|\mathbf{x}_2-\mathbf{x}_1|}}{|\mathbf{x}_2-\mathbf{x}_1|}(\not{P}_k + m_k)$$

Energy conservation in the source and at the detector $E_k=E_\nu$

$$p_k = \sqrt{E_\nu^2 - m_k^2} = \sqrt{(E_\pi - E_\mu)^2 - m_k^2} = \sqrt{(E_e + E_p - E_n)^2 - m_k^2}$$

$$P_k \equiv (\mathbf{p}_k, E_\nu) \qquad \mathbf{p}_k = p_k(\mathbf{x}_2-\mathbf{x}_1)/|\mathbf{x}_2-\mathbf{x}_1|$$

x_S and x_D coordinates associated with the source and detector, respectively, are introduced

$$\mathbf{x}_2 - \mathbf{x}_1 = \mathbf{x}_D + \mathbf{L} - \mathbf{x}_S$$

$$L = |\mathbf{L}|, \quad L \gg |\mathbf{x}_S|, \quad L \gg |\mathbf{x}_D|$$

$$\frac{e^{ip_k|\mathbf{x}_2-\mathbf{x}_1|}}{|\mathbf{x}_2-\mathbf{x}_1|} \simeq e^{i\mathbf{p}_k\cdot\mathbf{x}_D}\, e^{-i\mathbf{p}_k\cdot\mathbf{x}_S}\, \frac{e^{ip_k L}}{L}$$

Lorentz invariant amplitude:

$$\langle f|S^{(2)}|i\rangle = -2\pi\delta\left(E_e + E_p - E_n + E_\mu - E_\pi\right)\; e^{-i(\mathbf{p}_e+\mathbf{p}_p-\mathbf{p}_n)\cdot\mathbf{L}} \left(\frac{G_\beta}{\sqrt{2}}\right)^2 \times$$

$$\sum_k U_{ek}U^*_{\mu k}\, \frac{e^{ip_k L}}{4\pi L}\, (2\pi)^3\delta\left(\mathbf{p}_k + \mathbf{p}_\mu - \mathbf{p}_\pi\right)\; (2\pi)^3\delta\left(\mathbf{p}_e + \mathbf{p}_p - \mathbf{p}_n - \mathbf{p}_k\right) \times$$

$$\overline{u}(P_e)\gamma^\sigma(1-\gamma_5)\left(\not{P}_k + m_k\right)\gamma^\rho(1-\gamma_5)v(P_\mu)\; \overline{u}(P_p)\gamma_\sigma(g_V - g_A\gamma_5)u(P_n)\; i f_\pi (P_\pi)_\rho$$

T-matrix introduced:

$$\langle f|S^{(2)}|i\rangle = 2\pi\delta\left(E_e + E_p - E_n + E_\mu - E_\pi\right) \times$$

$$\sum_k (2\pi)^3\delta\left(\mathbf{p}_k + \mathbf{p}_\mu - \mathbf{p}_\pi\right)\; (2\pi)^3\delta\left(\mathbf{p}_e + \mathbf{p}_p - \mathbf{p}_n - \mathbf{p}_k\right) \langle f|T_k|i\rangle$$

The differential production rate (Master formula):

$$d\Gamma = \frac{1}{2m_\pi}\frac{1}{2m_n}\; 2\pi\delta\left(E_e + E_p - E_n + E_\mu - E_\pi\right) \times$$

$$\frac{1}{2}\sum_{km}\left((2\pi)^3\delta\left(\mathbf{p}_k + \mathbf{p}_\mu - \mathbf{p}_\pi\right)\;(2\pi)^3\delta\left(\mathbf{p}_e + \mathbf{p}_p - \mathbf{p}_n - \mathbf{p}_m\right)\right.$$

$$\left. +(2\pi)^3\delta\left(\mathbf{p}_m + \mathbf{p}_\mu - \mathbf{p}_\pi\right)\;(2\pi)^3\delta\left(\mathbf{p}_e + \mathbf{p}_p - \mathbf{p}_n - \mathbf{p}_k\right)\right) \times$$

$$\frac{1}{2}\sum_{\mathrm{spin}}\langle f|T_k|i\rangle\;\left(\langle f|T_m|i\rangle\right)^* \frac{d\mathbf{p}_\mu}{2E_\mu\;(2\pi)^3}\;\frac{d\mathbf{p}_e}{2E_e\;(2\pi)^3}\;\frac{d\mathbf{p}_p}{(2E_p\;2\pi)^3}$$

$$\int d\Omega_e \, \mathrm{Tr}\left(\not{P}_e + m_e\right) \gamma^\sigma \not{P}_k \left(1-\gamma_5\right)\left(\not{P}_\mu - m_\mu\right) \not{P}_m \gamma^\delta \left(1+\gamma_5\right) \times$$

$$\mathrm{Tr}\left(\not{P}_p + m_p\right)\gamma_\sigma(g_V - g_A\gamma_5)\left(\not{P}_n + m_n\right)\gamma_\delta(g_V - g_A\gamma_5) \simeq 4\pi\; 64\; M^2 \times$$

$$\left(\left(g_V^2 + 3g_A^2\right) E_e E_\mu \left(E_\nu E_\nu + p_k p_m\right) + \left(g_A^2 - g_V^2\right) \frac{1}{2} E_e E_\nu \left(p_k + p_m\right)^2\right)$$

$$\simeq 4\pi\; 64\; M^2 \left(g_V^2 + 3g_A^2\right) E_e E_\mu \left(E_k E_m + p_k p_m\right)$$

$$m_\pi \ll m_p \simeq m_n \equiv M, \quad E_p \simeq E_n \simeq M$$

$$E_\nu = E_\pi - E_\mu = E_e + E_p - E_n \simeq E_e$$

Calculation of traces and assumed approximations:

The production rate (QFT approach):

$$\Gamma = \frac{1}{(2\pi)^3} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \times$$

$$\sum_{km} U_{ek} U_{\mu k}^* \, U_{ek}^* U_{\mu k} \frac{e^{i(p_m - p_k)L}}{L^2} \frac{1}{2}\left(1 + \frac{p_k p_m}{E_\nu^2}\right) \times$$

$$\left[g_V^2 + 3g_A^2\right] p_e E_e$$

A comparison of standard and QFT approaches (e-Print: 2212.13635 [hep-ph])

$$\pi^+ + n \to \mu^+ + e^- + p$$

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \nu_\mu \to \nu_e, \quad \nu_e + n \to p + e^-$$

Standard QM approach

$$\Gamma^{\pi^+ n}_{osc} = \int \frac{d\Phi_\nu(E'_\nu)}{dE'_\nu} \frac{\mathcal{P}_{\nu_\mu\nu_e}(E'_\nu)}{4\pi L^2} \sigma(E'_\nu)\, dE'_\nu$$

$$= \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{P_{\nu_\mu\nu_e}(E_\nu)}{4\pi L^2} (g_V^2 + 3g_A^2)\; p_e E_e$$

with

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \left| \sum_{j=1}^{3} U^*_{\alpha j} U_{\beta j} e^{-i m_j^2 L/(2E_\nu)} \right|^2$$

Question above entanglement of processes at the source and the detector

New QFT approach

$$\Gamma^{\pi^+ n}_{QFT} = \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}}\right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{\mathcal{P}^{QFT}_{\mu e}(E_\nu)}{4\pi L^2} (g_V^2 + 3g_A^2)\; p_e E_e$$

with

$$\mathcal{P}^{QFT}_{\alpha\beta}(E_\nu) = \frac{1}{2} \sum_{km} U_{\beta k} U^*_{\alpha k}\; U^*_{\beta k} U_{\alpha k}\; e^{i(p_m - p_k)L} \left(1 + \frac{p_k p_m}{E_\nu^2}\right)$$

Oscillation of six Quasi-Dirac neutrinos

Dirac-Majorana mass term

$$\mathcal{L}_m = \frac{1}{2}\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^C} \end{pmatrix} \mathcal{M} \begin{pmatrix} \nu_L^C \\ \nu_R \end{pmatrix} + H.c.$$

Mass matrix

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix}$$

M_D - 3x3 complex matrix (18 real numb.)
$M_{L,R}$ - 3x3 symmetric matrix (12 real numb.)
(42 parameters)

Diagonalization: 6x6 unitary mixing matrix (15 mixing angles plus 15 phases)

$$\mathcal{M} = \mathcal{U}^T\ \mathcal{M}_{\text{diag}}\ \mathcal{U}$$

$$m_i^\pm = \pm m_i + \epsilon_i$$

Product of 3 unitary matrices. A and S mix exclusively active and sterile neutrino flavors, each given by 3 angles and 3 phases.

$$\mathcal{U} = \mathcal{X} \cdot \mathcal{A} \cdot \mathcal{S}$$

$$\mathcal{A} \equiv \begin{pmatrix} U^T & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathcal{S} \equiv \begin{pmatrix} 1 & 0 \\ 0 & V^\dagger \end{pmatrix}$$

$$\mathcal{X} = \begin{pmatrix} 1 & X^\dagger \\ -X & 1 \end{pmatrix} + \mathcal{O}(X^2)$$

Parametrization of mass matrix

X given by 9 angles and 9 phases, small parameters.

$|M_{L,R}| \ll |M_D|$

Close to a restoration of 3 Dirac neutrinos

Unitary mixing matrix introduced

$$\mathcal{T} \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\mathcal{U} = \mathcal{X} \cdot \mathcal{T} \cdot \mathcal{A} \cdot \mathcal{S}$$

Expansion of mixing matrix in X

$$\mathcal{U} = \frac{1}{\sqrt{2}} \begin{pmatrix} (1+X^\dagger)U^T & -(1-X^\dagger)V^\dagger \\ (1-X)U^T & (1+X)V^\dagger \end{pmatrix} + \mathcal{O}(X^2)$$

Full restoration of Dirac neutrinos ($M_{L,R}=0$)

$$\mathcal{M}_{\rm diag}(\epsilon_i = 0) = \mathcal{U}^*(X=0)\ \mathcal{M}(M_{L,R}=0)\ \mathcal{U}^\dagger(X=0)$$

$$U^\dagger U = 1 = V^\dagger V$$

$$M_{\rm diag} = U^\dagger M_D V \equiv \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Simplified QD-mixing scenario

$$\mathcal{U}_{\rm QD} = \frac{1}{\sqrt{2}} \begin{pmatrix} U & U \\ -V^* & V^* \end{pmatrix}$$

Mixing of 3 active and 3 sterile neutrinos

Mixing of 6 Majorana neutrinos

$$\begin{pmatrix} \nu_\alpha \\ \nu_{\alpha'} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} U^* & U^* \\ -V & V \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_{i'} \end{pmatrix}$$

$$|\nu_\alpha\rangle = \frac{1}{\sqrt{2}} \sum_{i=1}^{3} U^*_{\alpha i} |\nu_i\rangle + \frac{1}{\sqrt{2}} \sum_{i'=1}^{3} U^*_{\alpha i'} |\nu_{i'}\rangle$$

$$|\nu_{\alpha'}\rangle = -\frac{1}{\sqrt{2}} \sum_{i=1}^{3} V_{\alpha' i} |\nu_i\rangle + \frac{1}{\sqrt{2}} \sum_{i'=1}^{3} V_{\alpha' i'} |\nu_{i'}\rangle$$

Time evolution of flavor states

$$|\nu_\alpha\rangle(t) = \frac{1}{\sqrt{2}} \sum_{i=1}^{3} U^*_{\alpha i} e^{-itE_i} |\nu_i\rangle + \frac{1}{\sqrt{2}} \sum_{i'=1}^{3} U^*_{\alpha i'} e^{-itE_{i'+3}} |\nu_{i'+3}\rangle$$

$$|\nu_{\alpha'}\rangle(t) = -\frac{1}{\sqrt{2}} \sum_{i=1}^{3} V_{\alpha' i} e^{-itE_i} |\nu_i\rangle + \frac{1}{\sqrt{2}} \sum_{i'=1}^{3} V_{\alpha' i'} e^{-itE_{i'+3}} |\nu_{i'+3}\rangle$$

Active-active
and
active-sterile
oscillation amplitudes

$$\langle \nu_\beta | \nu_\alpha(t) \rangle = \frac{1}{2} \sum_{i=1}^{3} U^*_{\alpha i} U_{\beta i} e^{-itE_i} + \frac{1}{2} \sum_{i'=1}^{3} U^*_{\alpha i'} U_{\beta i'} e^{-itE_{i'+3}}$$

$$\langle \nu_{\beta'} | \nu_\alpha(t) \rangle = -\frac{1}{2} \sum_{i=1}^{3} U^*_{\alpha i} V^*_{\beta' i} e^{-itE_i} + \frac{1}{2} \sum_{i'=1}^{3} U^*_{\alpha i'} V^*_{\beta' i'} e^{-itE_{i'+3}}$$

Oscillation probability for QD neutrinos (active-active)
(6x6 generalization of the PMNS matrix)

$$\mathcal{U}_{\mathrm{QD}} = \frac{1}{\sqrt{2}} \begin{pmatrix} U & U \\ -V^* & V^* \end{pmatrix}$$

$$m_i^{\pm} = \pm m_i + \epsilon \quad (\epsilon > 0)$$

3 Dirac masses and 1 universal Majorana mass splitting ε

Oscillation probabilities among active neutrinos

$$\begin{aligned} P_{\alpha\beta} = \; & \delta_{\alpha\beta} - \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2 \sin^2 \frac{m_i \epsilon}{E} L - \sum_{i>j=1}^{3} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \Big(\sin^2 \frac{\Delta m_{ij}^2 + 2\epsilon \Delta m_{ij}}{4E} L \\ & + \sin^2 \frac{\Delta m_{ij}^2 - 2\epsilon \Sigma m_{ij}}{4E} L + \sin^2 \frac{\Delta m_{ij}^2 + 2\epsilon \Sigma m_{ij}}{4E} L + \sin^2 \frac{\Delta m_{ij}^2 - 2\epsilon \Delta m_{ij}}{4E} L \Big) \\ & + \frac{1}{2} \sum_{i>j=1}^{3} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \Big(\sin \frac{\Delta m_{ij}^2 + 2\epsilon \Delta m_{ij}}{2E} L + \sin \frac{\Delta m_{ij}^2 - 2\epsilon \Sigma m_{ij}}{2E} L \\ & + \sin \frac{\Delta m_{ij}^2 + 2\epsilon \Sigma m_{ij}}{2E} L + \sin \frac{\Delta m_{ij}^2 - 2\epsilon \Delta m_{ij}}{2E} L \Big) \end{aligned}$$

Quasi-Dirac neutrinos and constraints on neutrino masses

The survival probability of electron antineutrinos

$$P_{\bar\nu_e\to\bar\nu_e}(\epsilon\neq 0) = P_{\bar\nu_e\to\bar\nu_e}(\epsilon = 0) - \frac{\epsilon^2 L^2}{E^2}\left[c_{13}^4 c_{12}^4 m_1^2 + c_{13}^4 s_{12}^4 m_2^2 + s_{13}^4 m_3^2\right]$$
$$-\frac{\epsilon^2 L^2}{4E^2}\left[4\, c_{13}^4 s_{12}^2 c_{12}^2 \Sigma m_{21}^2 \cos\frac{\Delta m_{21}^2 L}{2E} + 4 s_{13}^2 c_{13}^2 c_{12}^2 \Sigma m_{31}^2 \cos\frac{\Delta m_{31}^2 L}{2E}\right.$$
$$\left. +4\, s_{13}^2 c_{13}^2 s_{12}^2 \Sigma m_{32}^2 \cos\frac{\Delta m_{32}^2 L}{2E}\right] + \mathcal{O}(\epsilon^4)\,,$$

Tritium β-decay

$$m_\beta = \sqrt{m_1^2 c_{12}^2 c_{13}^2 + m_2^2 c_{13}^2 s_{12}^2 + m_3^2 s_{13}^2 + \epsilon^2}$$
$$= m_\beta^{(0)}\left(1 + \frac{1}{2}\left(\epsilon/m_\beta^{(0)}\right)^2 + \dots\right)$$

Cosmology

$$\frac{1}{2}\sum_{i=1}^{6}\left|\tilde{\mathcal{M}}_{ii}\right| = \sum_{i=1}^{3} m_i$$

0νββ-decay

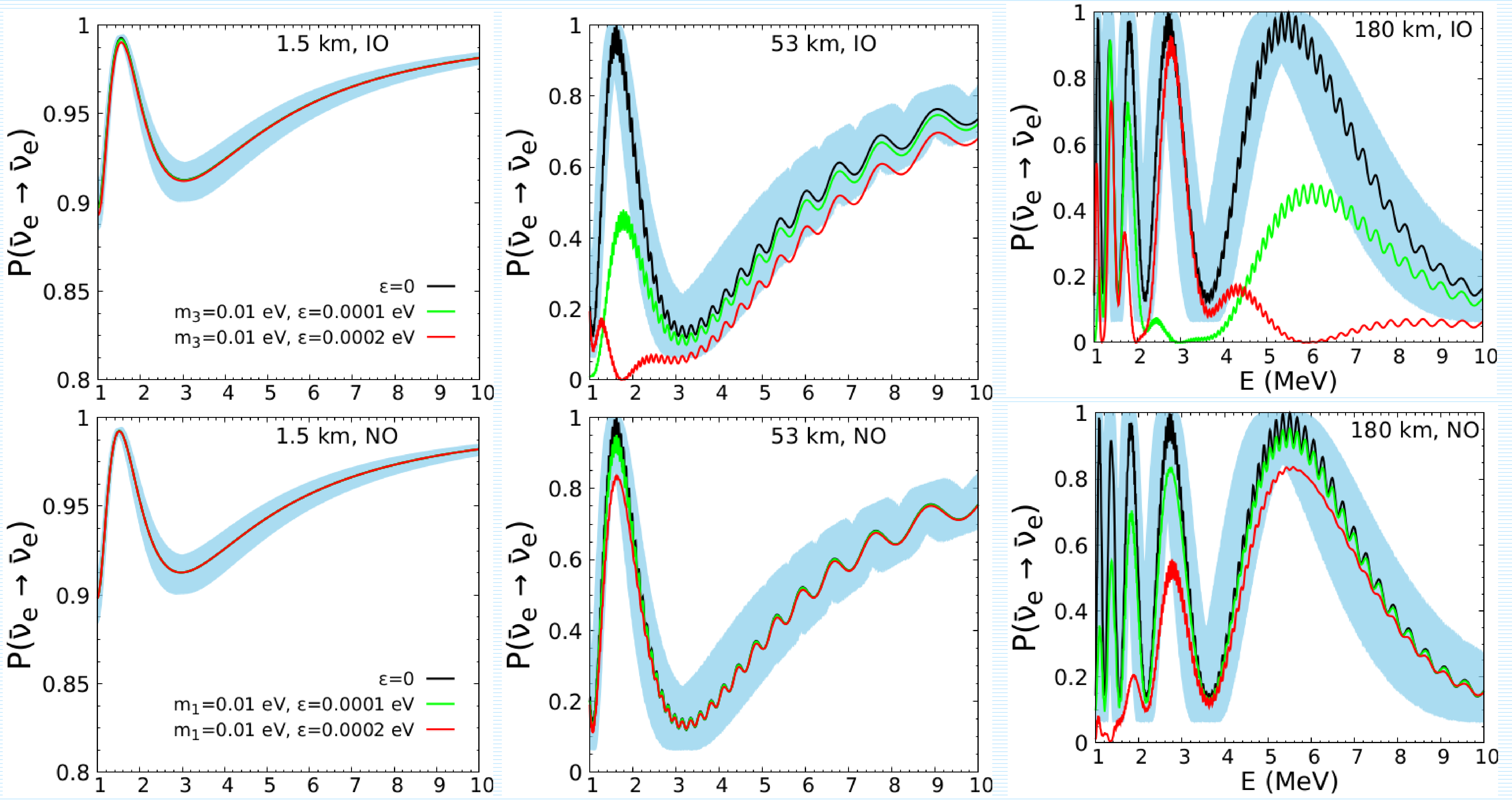
$$m_{\beta\beta} = \left[M_L\right]_{ee}$$
$$= \epsilon\,\left|c_{12}^2 c_{13}^2 + e^{2i\alpha_{21}} c_{13}^2 s_{12}^2 + e^{2i\alpha_{31}} s_{13}^2\right|$$

Restriction from Daya-Bay data (3σ):

Survival probabilities with non-zero ε are the same 3ν cases.

$$m_{\beta\beta} \lesssim 30\ \text{meV} \quad \text{for} \quad \text{NO}$$
$$\lesssim 1\ \text{meV} \quad \text{for} \quad \text{IO}$$

Quasi-Dirac neutrino oscillations at different distances



Nuovo Cim. 14, 322 (1937)

neutrino ↔ antineutrinos oscillations

Second order process with real intermediate neutrinos

$$S + D \to \ell_\alpha^+ + \ell_\beta^+ + S' + D'$$

$$S \to S' + \ell_\alpha^+ + \nu_\alpha, \;\; \nu_\alpha \to \overline{\nu}_\beta, \;\; \overline{\nu}_\beta + D \to D' + \ell_\beta^+$$

Amplitude proportional to ν–mass

$$T_k^{\alpha\beta} = J_S^\mu(P'_S, P_S) J_D^\nu(P'_D, P_D) \times \overline{v}(P_\alpha;\lambda_\alpha)\gamma_\mu(1-\gamma_5) m_k \gamma_\nu u(P_\beta;\lambda_\beta)$$

Replacement:

$$U_{\alpha k} \to U^*_{\alpha k}$$

$$U^*_{\beta m} \to U_{\beta m}$$

Oscillation probability

$$\mathcal{P}^{\mathrm{QFT}}_{\alpha\overline{\beta}}(E_\nu, L) \equiv |\langle \nu_\beta | \overline{\nu}_\alpha \rangle|^2 = \frac{(m^L_{\alpha\beta})^2}{E_\nu^2} = \left| \sum_{j=1}^{3} U^*_{\alpha j} U^*_{\beta j} \frac{m_j}{E_\nu} e^{-i m_j^2 L/(2E_\nu)} \right|^2$$

Particular process:

$$\pi^+ + p \to \mu^+ + e^+ + n$$

Production rate

$$\Gamma^{\pi^+ p}_{QFT} = \frac{1}{2\pi^2} G_\beta^2 \left(\frac{f_\pi}{\sqrt{2}} \right)^2 \frac{m_\mu^2}{m_\pi} E_\nu^2 \frac{P^{\mathrm{QFT}}_{\nu_\mu \overline{\nu}_e}(E_\nu, L)}{4\pi L^2} (g_V^2 + 3 g_A^2) \; p_e E_e$$

A connection of neutrino-antineutrino oscillation with 0νββ-decay

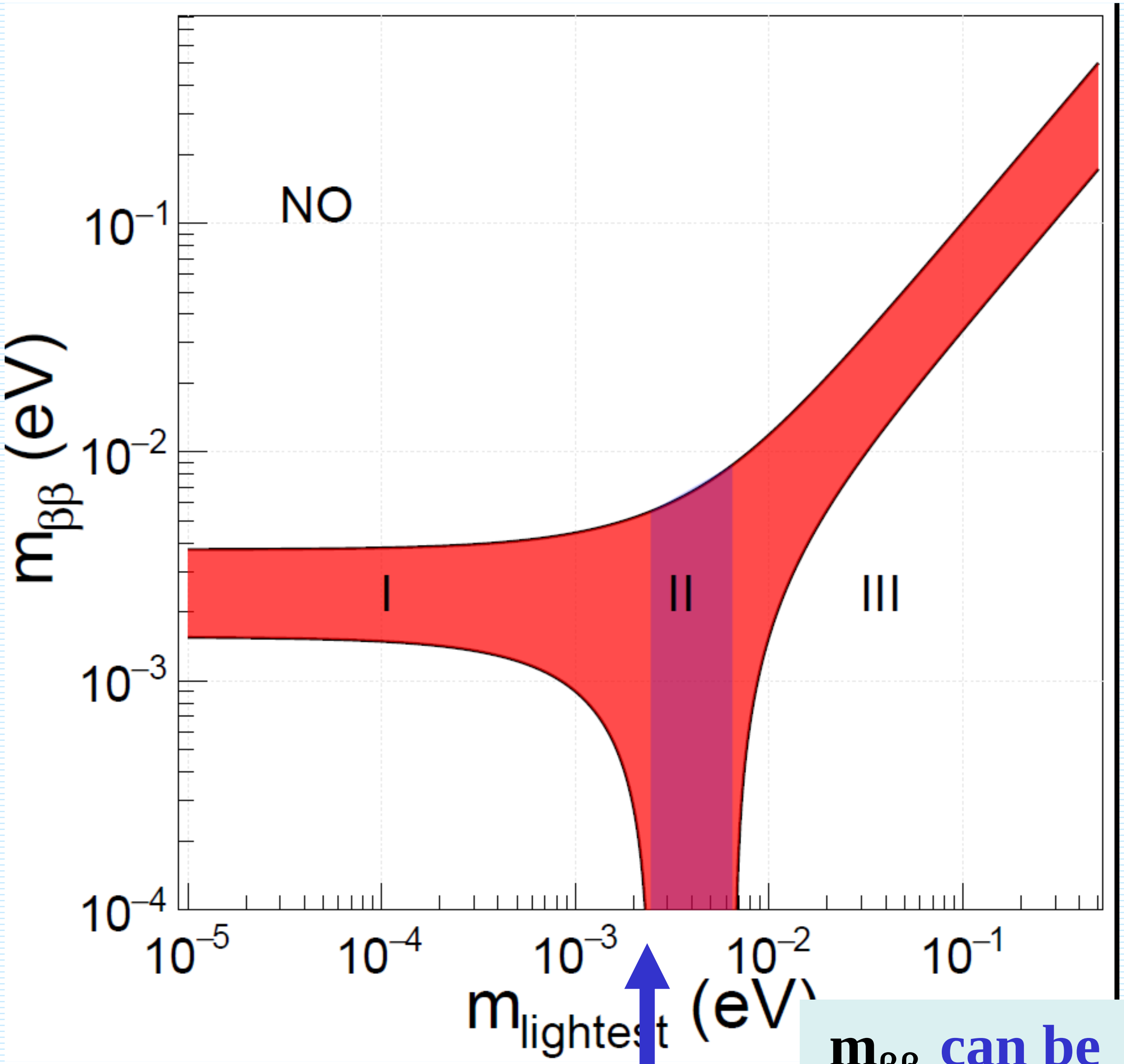
$$m^{L=0}_{ee} = m_{\beta\beta}$$

$$\mathcal{P}^{\rm QFT}_{e\bar{e}}(E_\nu, L=0) \equiv |\langle\nu_e|\overline{\nu}_e\rangle|^2 = \left|\sum_{k=1}^{3} U^*_{ek}U^*_{ek}\frac{m_k}{E_\nu}\right|^2 = \frac{(m^{L=0}_{ee})^2}{E^2_\nu} = \frac{(m_{\beta\beta})^2}{E^2_\nu}$$

$$m_{\beta\beta} = |\rho_1 e^{2i\phi_1} + \rho_2 e^{2i\phi_2} + \rho_3|$$

$$\rho_1 = c^2_{12}c^2_{13}m_1,\ \rho_2 = s^2_{12}c^2_{13}m_2,\ \rho_3 = s^2_{13}m_3$$

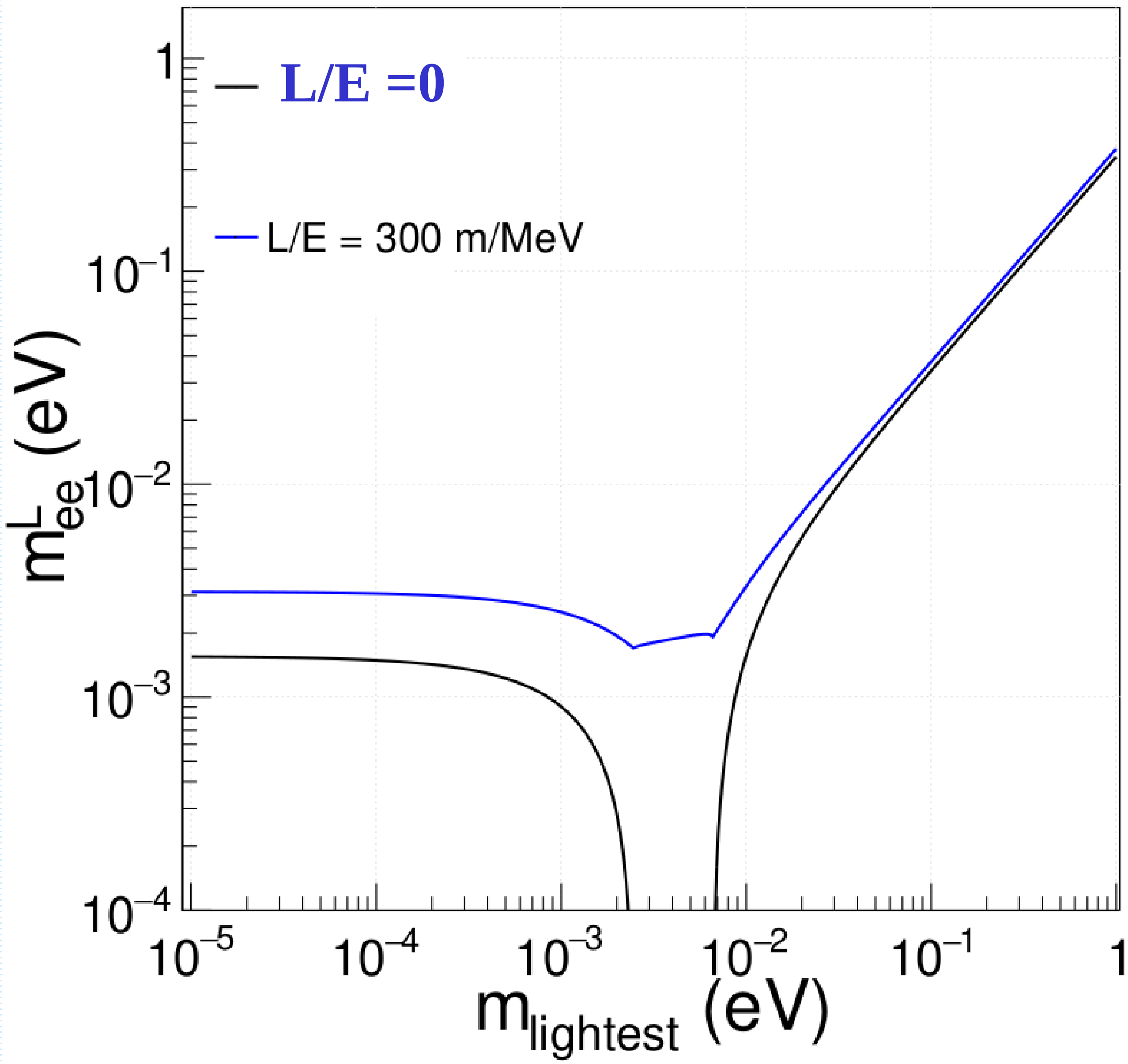
$$\min_{\phi_1,\phi_2} m_{\beta\beta} = \begin{cases} |\rho_2-\rho_3|-\rho_1, & \text{if } \rho_1 < |\rho_2-\rho_3| \quad : \text{region I}, \\ 0, & \text{if } |\rho_2-\rho_3| \le \rho_1 \le \rho_2+\rho_3 \quad : \text{region II}, \\ \rho_1-(\rho_2+\rho_3), & \text{if } \rho_2+\rho_3 < \rho_1 \quad : \text{region III}. \end{cases}$$

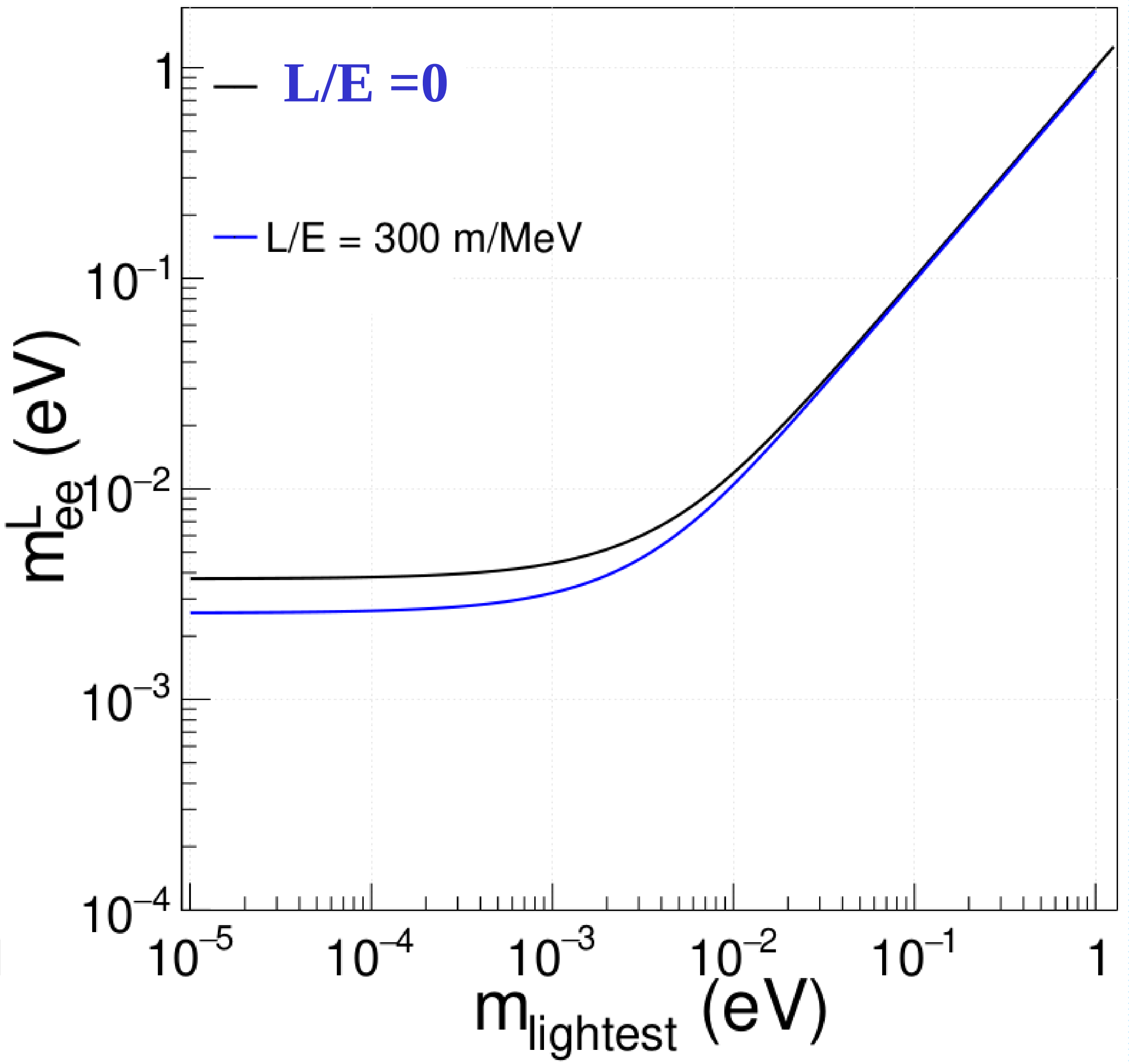


$m_{\beta\beta}$ can be strongly \suppressed (?!)

Dependence of m^{L}_{ee} on $m_{lightest}$ and L/E

$m^{L=0}_{ee} = m_{\beta\beta}$



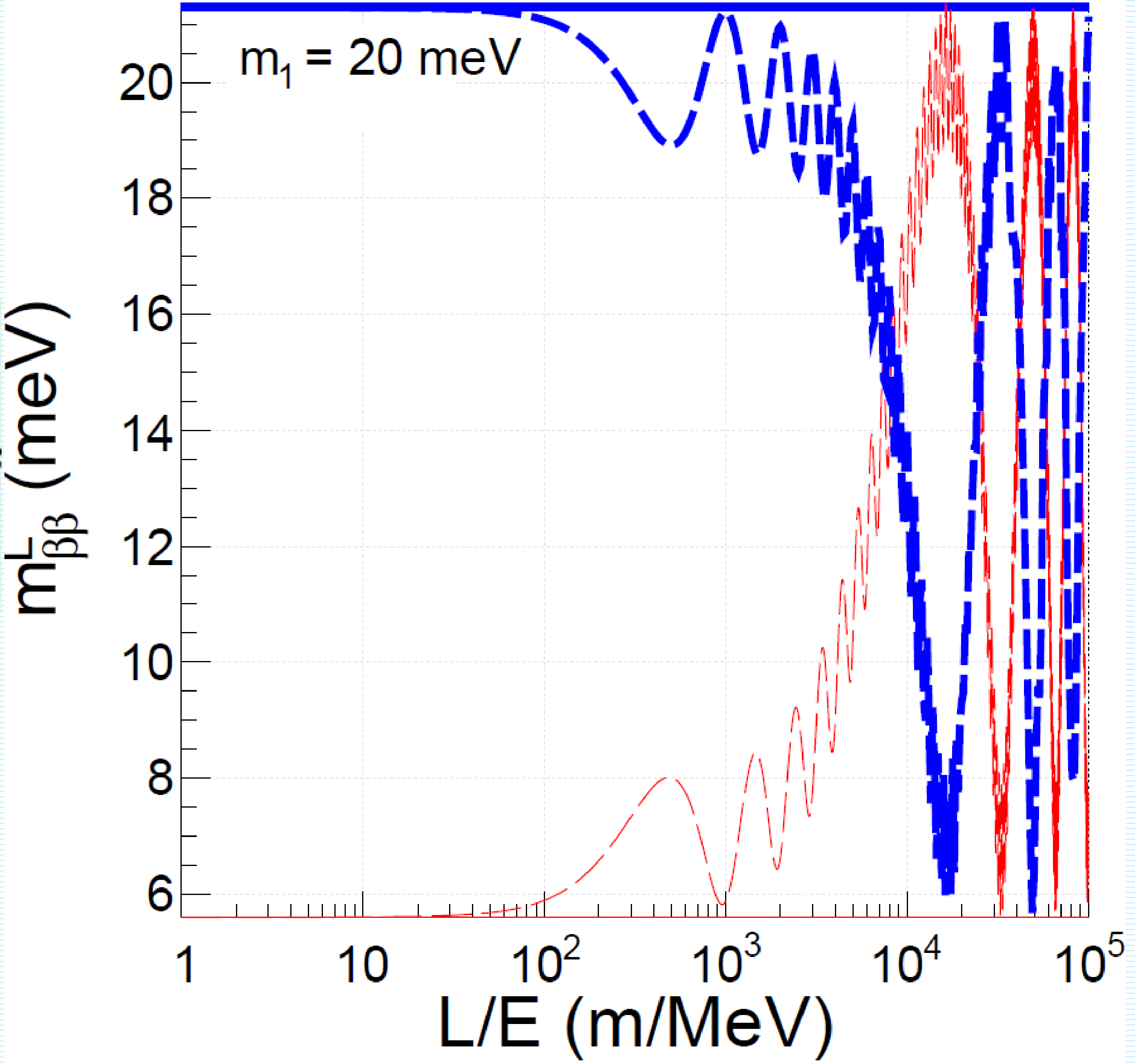


Dependence of m^L_{ee} on L/E for $m_{lightest}$ = 20 meV

Symmetry 14, 1383 (2022)

$$\mathcal{P}^{\rm QFT}_{\alpha\bar\beta}(E_\nu, L) \equiv |\langle \nu_\beta|\overline{\nu}_\alpha\rangle|^2 = \frac{(m^L_{\alpha\beta})^2}{E_\nu^2}$$
$$= \left|\sum_{j=1}^{3} U^*_{\alpha j}U^*_{\beta j}\frac{m_j}{E_\nu}e^{-im_j^2 L/(2E_\nu)}\right|^2$$

$$\mathcal{P}^{\rm QFT}_{\alpha\bar\beta}(E_\nu, L=0) = \frac{(m^{L=0}_{\alpha\beta})^2}{E_\nu^2} \equiv \frac{(m_{\alpha\beta})^2}{E_\nu^2}$$
$$= \frac{1}{E_\nu^2}\left|\sum_{j=1}^{3} U^*_{\alpha j}U^*_{\beta j} m_j\right|^2$$



Different types of Oscillations

$$H_{\rm flavor} = \frac{1}{2}(E_2 + E_1) + \frac{1}{2}(E_2 - E_1)\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

Oscillations of ν_l-$\nu_{l'}$
(lepton flavor)

$$H_{eff}^{K_0\overline{K_0}} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \Gamma_{12} \\ M_{12}^* - \Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of K_0-anti{K_0}
(strangeness)

$$H_{eff}^{n\bar{n}} = \begin{pmatrix} M & V^{BNV} \\ V^{BNV} & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of n-anti{n}
(baryon number)

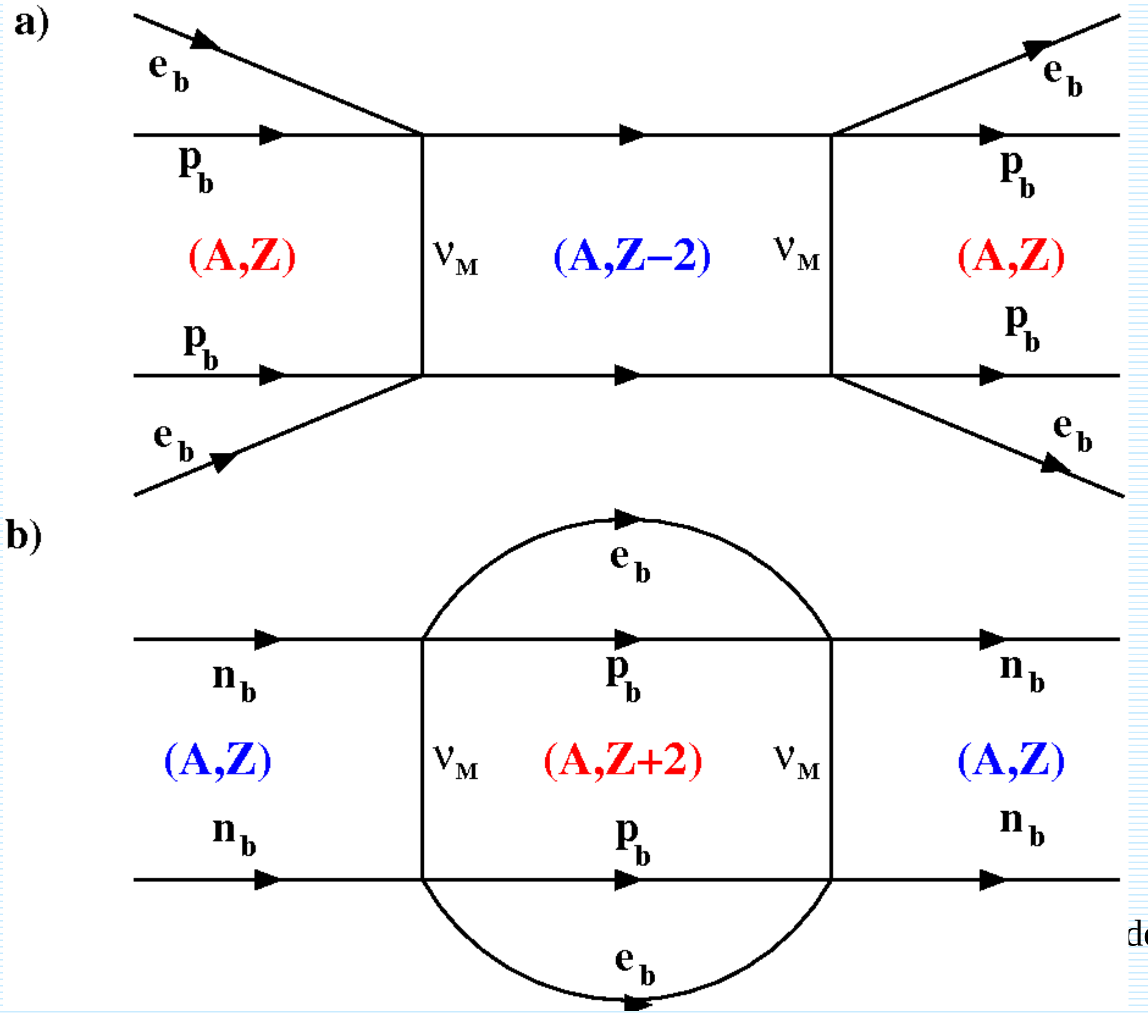
$$H_{eff}^{atom} = \begin{pmatrix} M_i & V^{LNV} \\ V^{LNV} & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of Atoms (OoA)
(total lepton number)

F.Š., M. Krivoruchenko, Phys.Part.Nucl.Lett. 6 (2009) 485.

Oscillations of neutral atoms

Oscillation of atoms due to total lepton number violation



In analogy with oscillations of n-anti{n} (baryon number violation)

$$(\mathcal{A},\mathcal{Z}) \Longleftrightarrow (\mathcal{A},\mathcal{Z}+2)^{**}$$
$$(\mathcal{A},\mathcal{Z}) \Longleftrightarrow (\mathcal{A},\mathcal{Z}-2)^{**}$$

$$n + n \leftrightarrow p + p + e_b^- + e_b^-$$

Effective Hamiltonian

$$H_{eff}^{atom} = \begin{pmatrix} M_i & V^{LNV} \\ V^{LNV} & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

F.Š., M. Krivoruchenko, Phys.Part.Nucl.Lett. 6 (2009) 485

Probability of oscillations of atoms

Effective Hamiltonian

$$H_{eff} = \begin{pmatrix} M_i & V \\ V & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

Effective Hamiltonian expressed with Pauli matrices

$$H_{eff} = \left(\frac{M_i + M_f}{2} - \frac{i}{4}\Gamma\right) \mathbf{1} + V\,\sigma_{\mathbf{1}} + \left(\frac{M_i - M_f}{2} - \frac{i}{4}\Gamma\right) \sigma_{\mathbf{3}}$$

Decomposition of evolution operator

$$e^{-iH_{eff}t} = e^{-i(M_i+M_f)t/2 - \Gamma t/4} \times \left(\cos\left(\sqrt{a^2+b^2}\; t\right) - i\frac{a\;\sigma_{\mathbf{1}} + b\;\sigma_{\mathbf{2}}}{\sqrt{a^2+b^2}} \sin\left(\sqrt{a^2+b^2}\; t\right)\right)$$

$$a = V\,t, \quad b = \left(\frac{M_i - M_f}{2} + \frac{i}{4}\Gamma\right) t$$

Probability of oscillation of atoms (initial atom is stable)

$$|<f|e^{-iH_{eff}t}|i>|^2 = \frac{V^2}{(M_i - M_f)^2 + \Gamma^2/4} \times \left(1 + e^{-\Gamma t} - 2e^{-\Gamma t/2}\cos\left[t\;(M_i - M_f)\right]\right)$$

Oscillations of stable atoms (Γ=0)

$$|<f|e^{-iH_{eff}t}|i>|^2 = \frac{4V^2}{(M_i - M_f)^2}\sin^2\left[t\ (M_i - M_f)/2\right]$$

Probability of oscillation of atoms
(both atoms are stable)

$$|<f|e^{-iH_{eff}t}|i>|^2 = V^2 t^2$$

For $(M_i - M_f)\ t << 1$

$$|<f|e^{-iH_{eff}t}|i>|^2 \approx \frac{V^2}{(M_i - M_f)^2}$$

For $(M_i - M_f)\ t >> 1$

$$^{164}_{68}Er \rightarrow {}^{164}_{66}Dy$$
$$(M_i - M_f) = 24.1\ keV$$

$$|<f|e^{-iH_{eff}t}|i>|^2 \leq 3\ 10^{-55}$$

Double electron capture (Γ≠0)
(resonant enhancement of atom)

Mass difference ~ keV

$$\Gamma^{r-0\nu\varepsilon\varepsilon} = \frac{V^2}{(M_i - M_f)^2 + \Gamma^2/4}\ \Gamma$$

Mass difference >> Γ

Resonant 0νεε-decay

0νεε capture rate

$$\Gamma^{0\nu ECEC}(J^\pi) = \frac{|V_{\alpha\beta}(J^\pi)|^2}{(M_i - M_f)^2 + \Gamma^2_{\alpha\beta}/4}\,\Gamma_{\alpha\beta}$$

β-decay Hamiltonian

$$\mathcal{H}^\beta(x) = \frac{G_\beta}{\sqrt{2}}\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)j_\mu(x) + \text{h.c.}$$

Potential

$$\begin{aligned} V_{\alpha\beta} &= i m_{\beta\beta}\left(\frac{G_\beta}{\sqrt{2}}\right)^2 \frac{1}{\sqrt{1+\delta_{\alpha\beta}}}\sum_{m_\alpha m_\beta} C^{JM}_{j_\alpha m_\alpha j_\beta m_\beta}\int d\vec{x}_1 d\vec{x}_2 \\ &\times \Psi_{\alpha m_\alpha}{}^T(\vec{x}_1)C\gamma^\mu\gamma^\nu(1-\gamma_5)\Psi_{\beta m_\beta}(\vec{x}_2)\int \frac{e^{-i\vec{q}\cdot(\vec{x}_1-\vec{x}_2)}}{2q_0}\frac{d\vec{q}}{(2\pi)^3} \\ &\times \sum_n \left[\frac{< A, Z-2|J_\mu(\vec{x}_1)|n >< n|J_\nu(\vec{x}_2)|A, Z >}{q_0 + E_n - M_i - \varepsilon_\beta}\right. \\ &\left. + \frac{< A, Z-2|J_\nu(\vec{x}_2)|n >< n|J_\mu(\vec{x}_1)|A, Z >}{q_0 + E_n - M_i - \varepsilon_\alpha}\right] - (\alpha \leftrightarrow \beta) \end{aligned} \qquad (2)$$

ν-mixing decay

$$\nu_{eL}(x) = \sum_{i=1}^{3} U_{ek}\chi_{kL}(x)$$

0νεε potential - approximations

The non-relativistic impulse approximation for nucleon current

$$J^\mu(0,\vec{x}) = \sum_{n=1}^{A} \tau_n^- [g_V g^{\mu 0} + g_A (\sigma_k)_n g^{\mu k}] \delta(\vec{x} - \vec{x}_n)$$

Closure approximation

$$E_n - M_i \Rightarrow < E > \approx 8\ MeV$$

$$\sum_n |n >< n| = 1$$

0νεε potential

$$V^{\alpha\beta}(J^\pi_f) = \frac{1}{4\pi}\, G^2_\beta m_{\beta\beta} \frac{g^2_A}{R} \sqrt{2J_f + 1} \mathcal{M}_{\alpha\beta}(J^\pi_f)$$

The atomic and nuclear parts are factorized

$$\mathcal{M}_{\alpha\beta}(J^\pi_f) \approx \mathcal{A}_{\alpha\beta}\ M^{0\nu}(J^\pi_f)$$

NME similar as those for 0νββ-decay

$$M^{0\nu}(0^+_f) = < 0^+_f \| \sum_{nm} \tau^-_n \tau^-_m h(r_{nm}) [-\frac{g^2_V}{g^2_A} + (\vec{\sigma}_n \cdot \vec{\sigma}_m)] \| 0^+_i >,$$

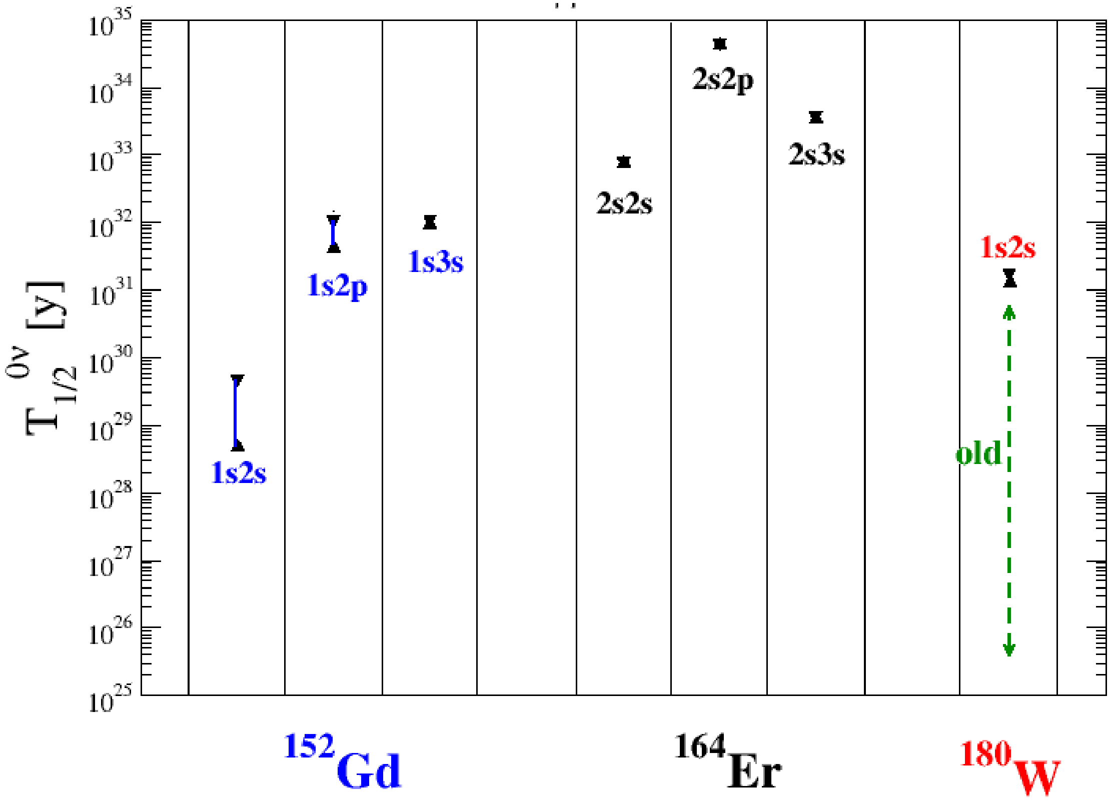
$$M^{0\nu}(0^-_f) = < 0^-_f \| \sum_{nm} \tau^-_n \tau^-_m h(r_{nm}) (\hat{r}_n - \hat{r}_m) \cdot [\frac{g_V}{g_A}(\vec{\sigma}_n - \vec{\sigma}_m) - i(\vec{\sigma}_n \times \vec{\sigma}_m)] \| 0^+_i >$$

$$h(r_{nm}) = \frac{2}{\pi} R \int_0^\infty j_0(q r_{nm}) \frac{q_0}{q_0 + < E > - m} dq.$$

Nucleus	$(n2jl)_a$	$(n2jl)_b$	E_a	E_b	E_C	Γ_{ab} (keV)	Δ (keV)	$T_{1/2}^{min}$ (y)	$T_{1/2}^{max}$ (y)
^{152}Gd	110	210	46.83	7.74	0.34	2.3×10^{-2}	-0.83 ± 0.18	4.7×10^{28}	4.8×10^{29}
	110	211	46.83	7.31	0.32	2.3×10^{-2}	-1.27 ± 0.18	4.2×10^{31}	1.1×10^{32}
	110	310	46.83	1.72	0.11	3.2×10^{-2}	-7.07 ± 0.18	9.4×10^{31}	1.1×10^{32}
^{164}Er	210	210	9.05	9.05	0.22	8.6×10^{-3}	-6.82 ± 0.12	7.5×10^{32}	8.4×10^{32}
	210	211	9.05	8.58	0.23	8.3×10^{-3}	-7.28 ± 0.12	4.2×10^{34}	4.6×10^{34}
	210	310	9.05	2.05	0.11	1.8×10^{-2}	-13.92 ± 0.12	3.5×10^{33}	3.9×10^{33}
^{180}W	110	110	63.35	63.35	1.26	7.2×10^{-2}	-11.24 ± 0.27	1.3×10^{31}	1.8×10^{31}

resonant 0νεε-decay half-lives

$m_{\beta\beta}$ =50 meV



Resonant 0νεε-decay half-lives are suppressed at least by 2 orders in magnitude when compared with 0νββ -decay half-lives for the same $m_{\beta\beta}$.

What is the nature of neutrinos?

The study of the 0νββ-decay is one of the highest-priority issues in particle and nuclear physics

$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$

Perturbation theory

$$\frac{1}{T^{0\nu}_{1/2}} = \left|\frac{m_{\beta\beta}}{m_e}\right|^2 G^{01}(E_0, Z) \left|M^{0\nu}\right|^2$$

- 2νββ-decay background can be a problem
- Uncertainty in NMEs factor ~2, 3
- $0^+ \rightarrow 0^+, 2^+$ transitions
- Large Q-value
- ^{76}Ge, ^{82}Se, ^{100}Mo, ^{130}Te, ^{136}Xe …
- Many exp. in construction, potential for observation in the case of inverted hierarchy (2025)

$e^- + e^- + (A,Z) \rightarrow (A,Z-2)^{**}$

Breit-Wigner form

$$\Gamma^{0\nu ECEC}(J^\pi) = \frac{|V_{\alpha\beta}(J^\pi)|^2}{(M_i - M_f)^2 + \Gamma^2_{\alpha\beta}/4}\,\Gamma_{\alpha\beta}$$

- 2νεε-decay is strongly suppressed
- NMEs need to be calculated
- $0^+ \rightarrow 0^+, 0^-, 1^+, 1^-$ transitions
- Small Q-value
- Q-value needs to be measured at least with 100 eV accuracy
- ^{152}Gd, looking for additional
- small experiments yet

謝謝