强 子 共 振 态 理 论

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伯尔尼大学

乔治华盛顿大学

TRR110: NSFC Grant No. 12070131001, DFG Project-ID 196253076

DOE: DE-SC0016582, DE-AC05-06OR23177, DE-FG02-95ER40907

DFG: Heisenberg Programme (project number: 532635001)

NSF: PHY-2012289

第二十三届少体问题会议 北京 2024年9月23日

THEORY OF HADRON RESONANCES

MAXIM MAI

UNIVERSITY OF BERN (MAIN)

THE GEORGE WASHINGTON UNIVERSITY

TRR110: NSFC Grant No. 12070131001, DFG Project-ID 196253076

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23rd Few-Body Conference Beijing 23.09.2024

HADRON SPECTRUM

Mostly excited states

≈ 100 mesons

≈ 50 baryons (****)

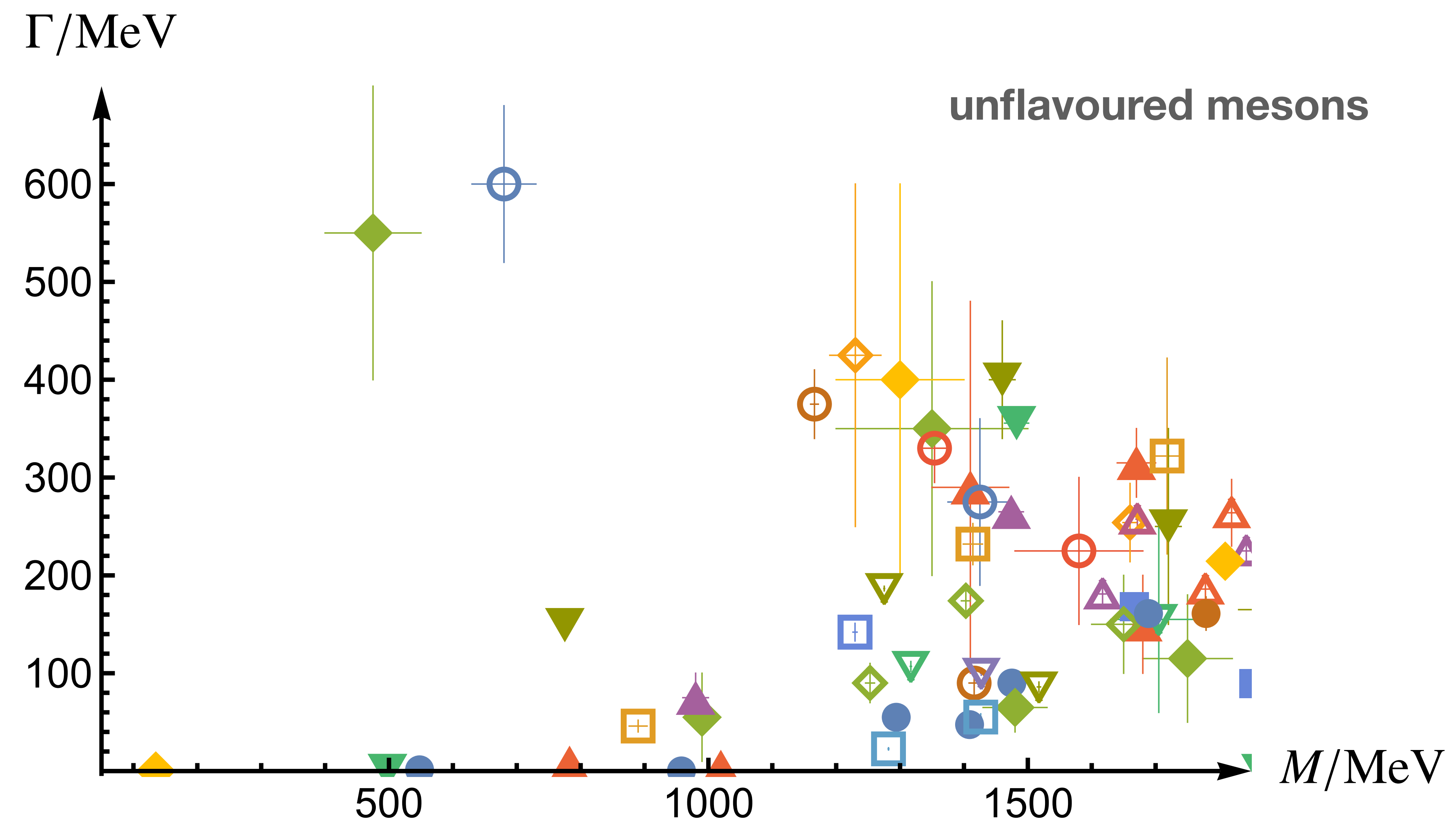
Key questions

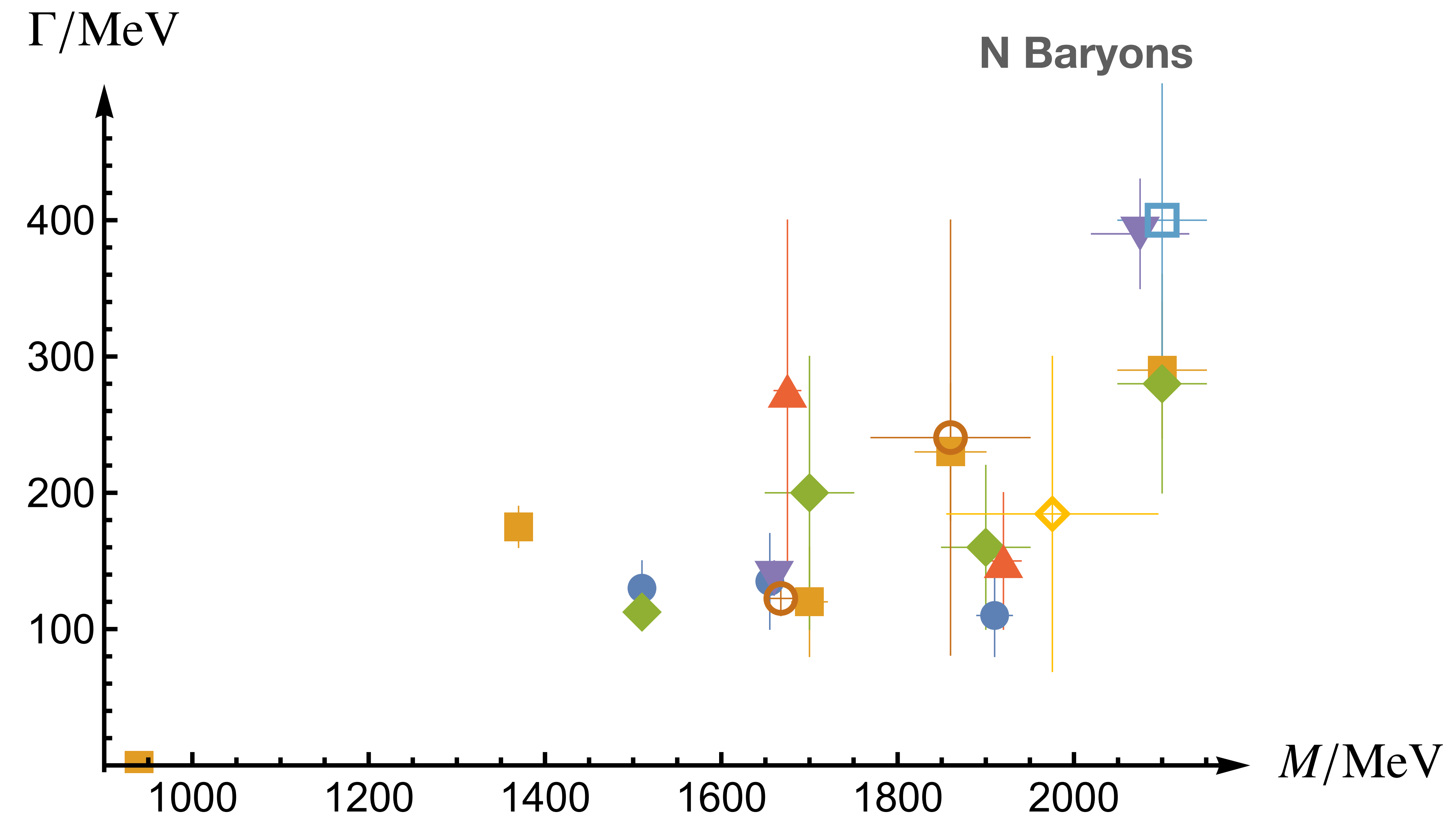
“what is the pattern of these states?“

>> More experiments & cross-channel models <<

“how are they formed?”

*>> change the probe (photon, **heavy pions—lattice QCD**) <<*



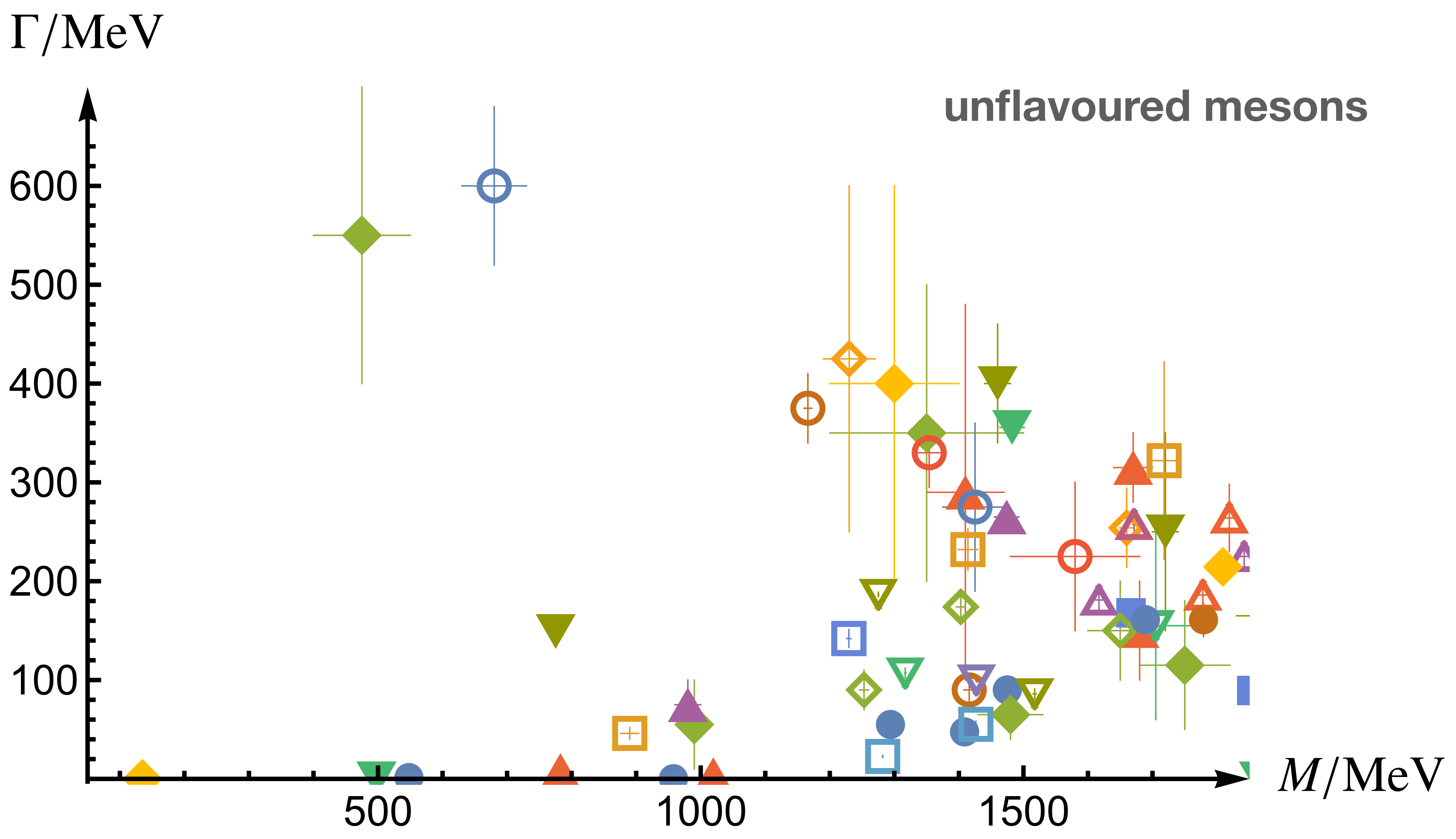


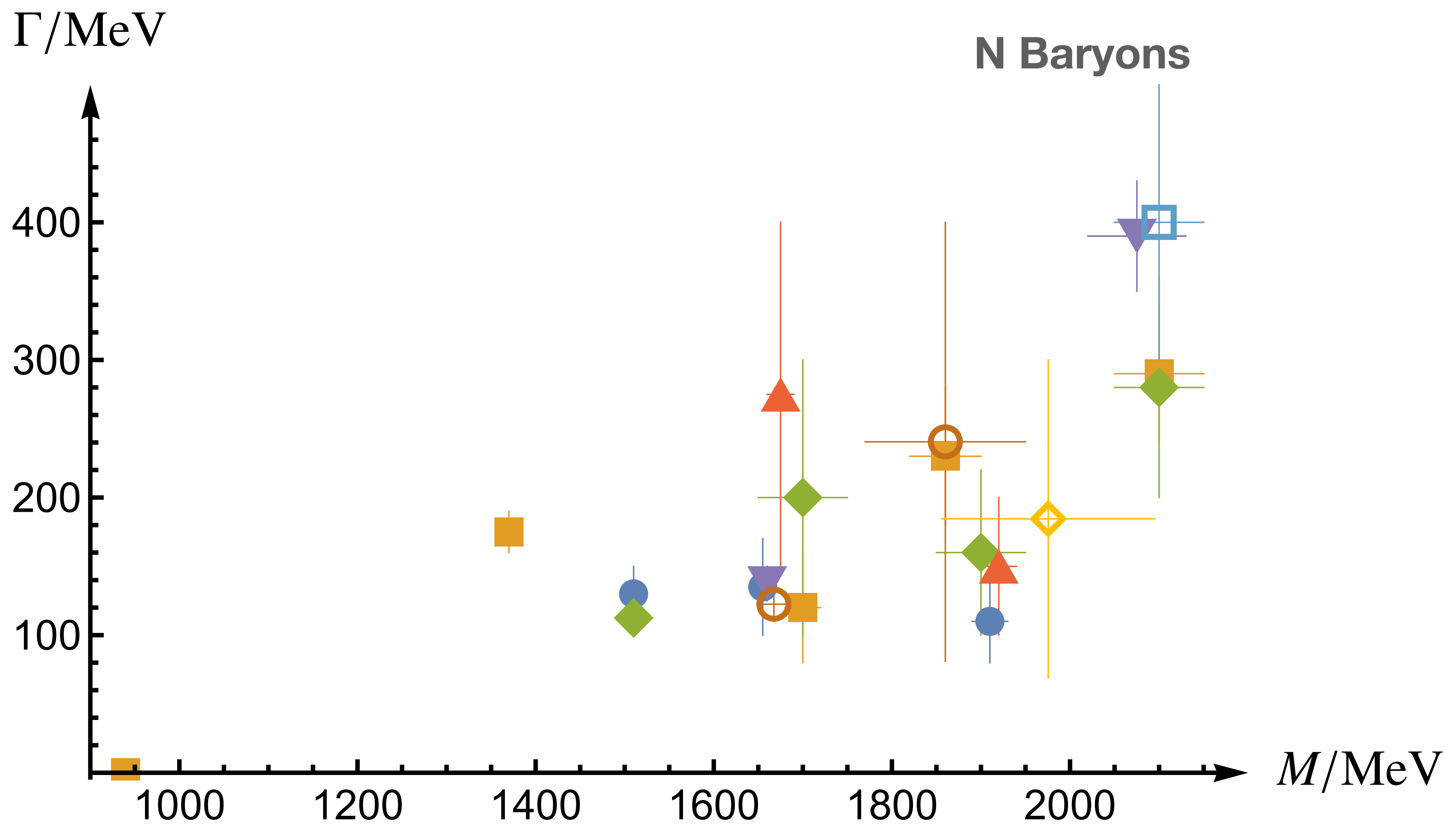
Talks: Masahiko Iwasaki — Bingran He — Shuqi Sheng — Yuhao Wang — Zhenwei Yang…

HADRON SPECTRUM

Theoretical tools

- Quark models
- Functional methods
- Dynamical coupled-channel models
- Chiral EFT
- Lattice QCD
- …





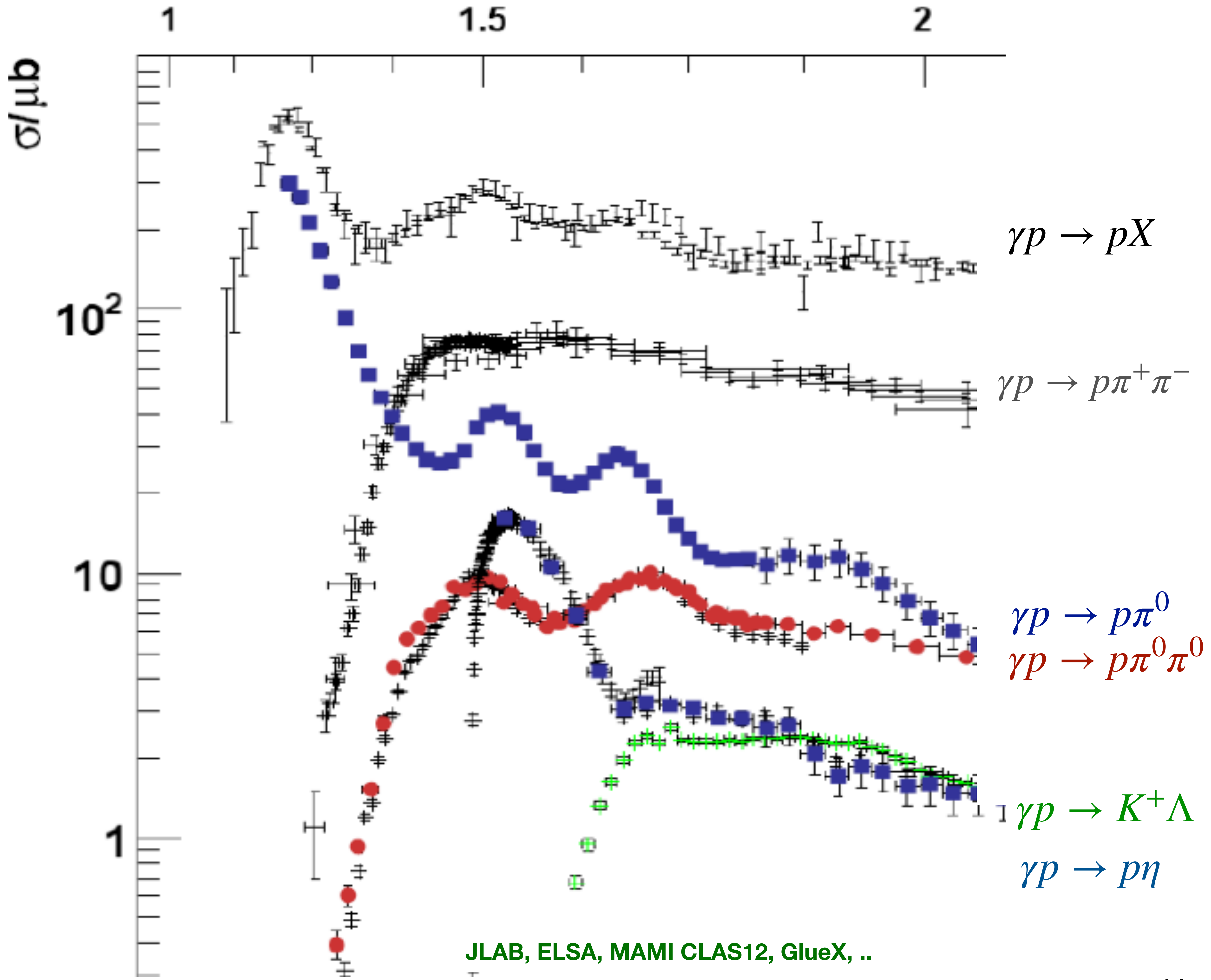
Review: Eichmann/Sanchis-Alepuz/Alkofer/Fischer Prog.Part.Nucl.Phys. 91 (2016) 1-100
Review: Döring/Haidenbauer/Sato/MM PPNP in progress
Review: MM/Meißner/Urbach Phys.Rept. 1001 (2023) 1-6
Review: Chen/Chen/Liu/Liu/Zhu Rept.Prog.Phys. 86 (2023) 2
TALKS: Epelbaum — Bingran He — Ahmad Jafar Arifi — 思危 胡…

RESONANCE PARAMETER

Physical input

- many available data and ongoing experiments
- resonances ⇒ increased interaction rates

  ➞ depends on the reaction-type



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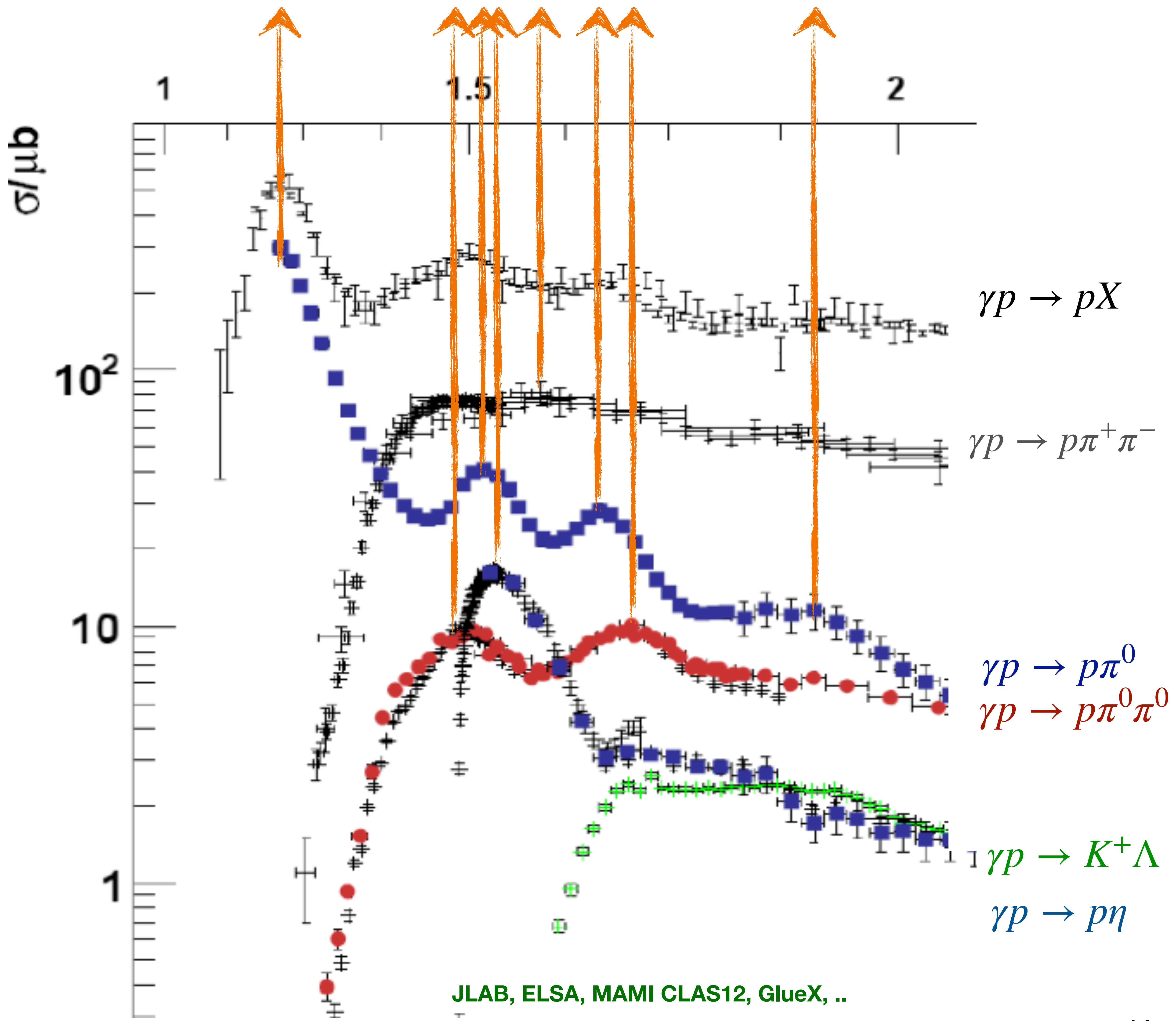
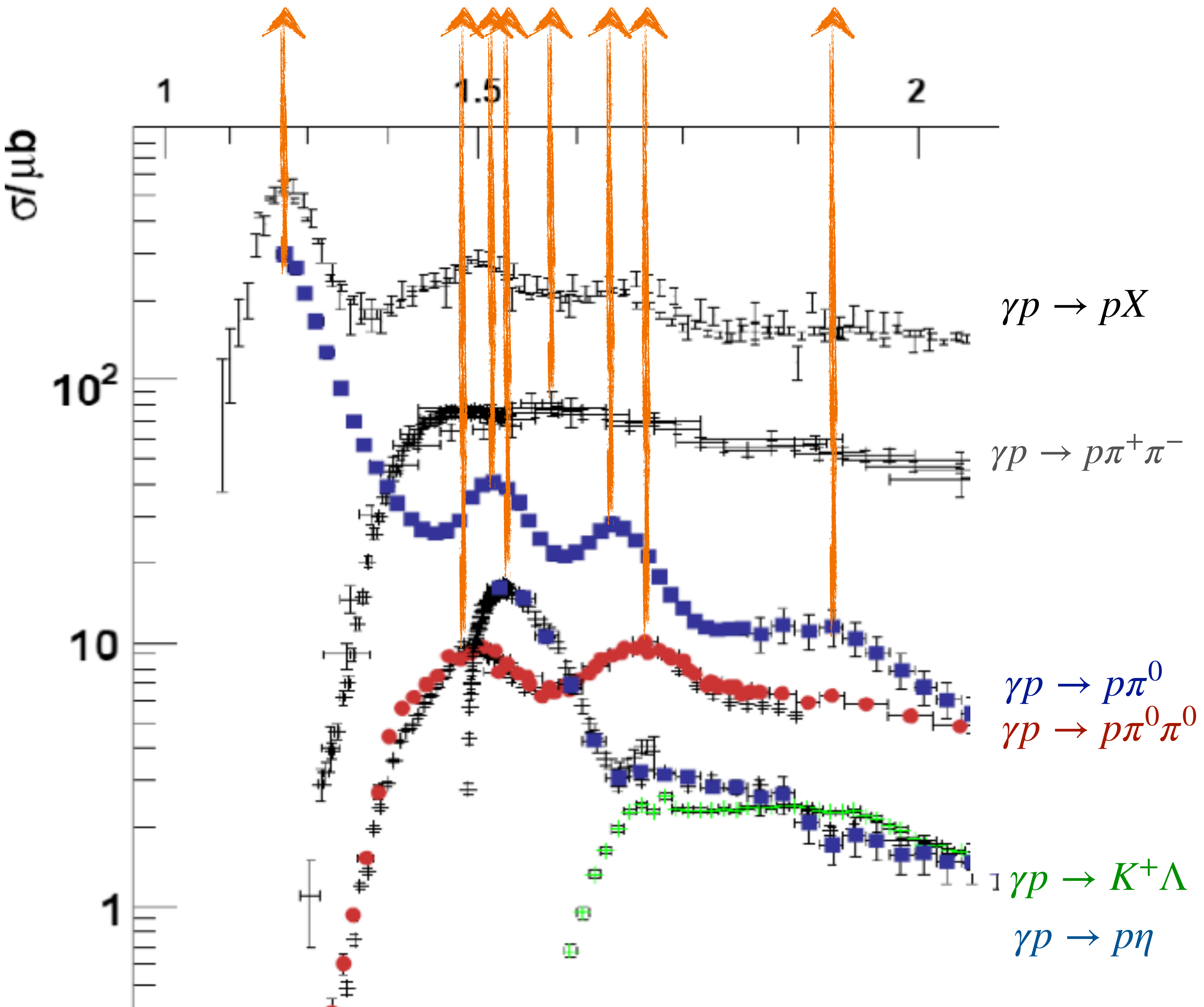


RESONANCE PARAMETER

B

need P ~~D~~ G=*“particle bump group”*



JLAB, ELSA, MAMI CLAS12, GlueX, ..

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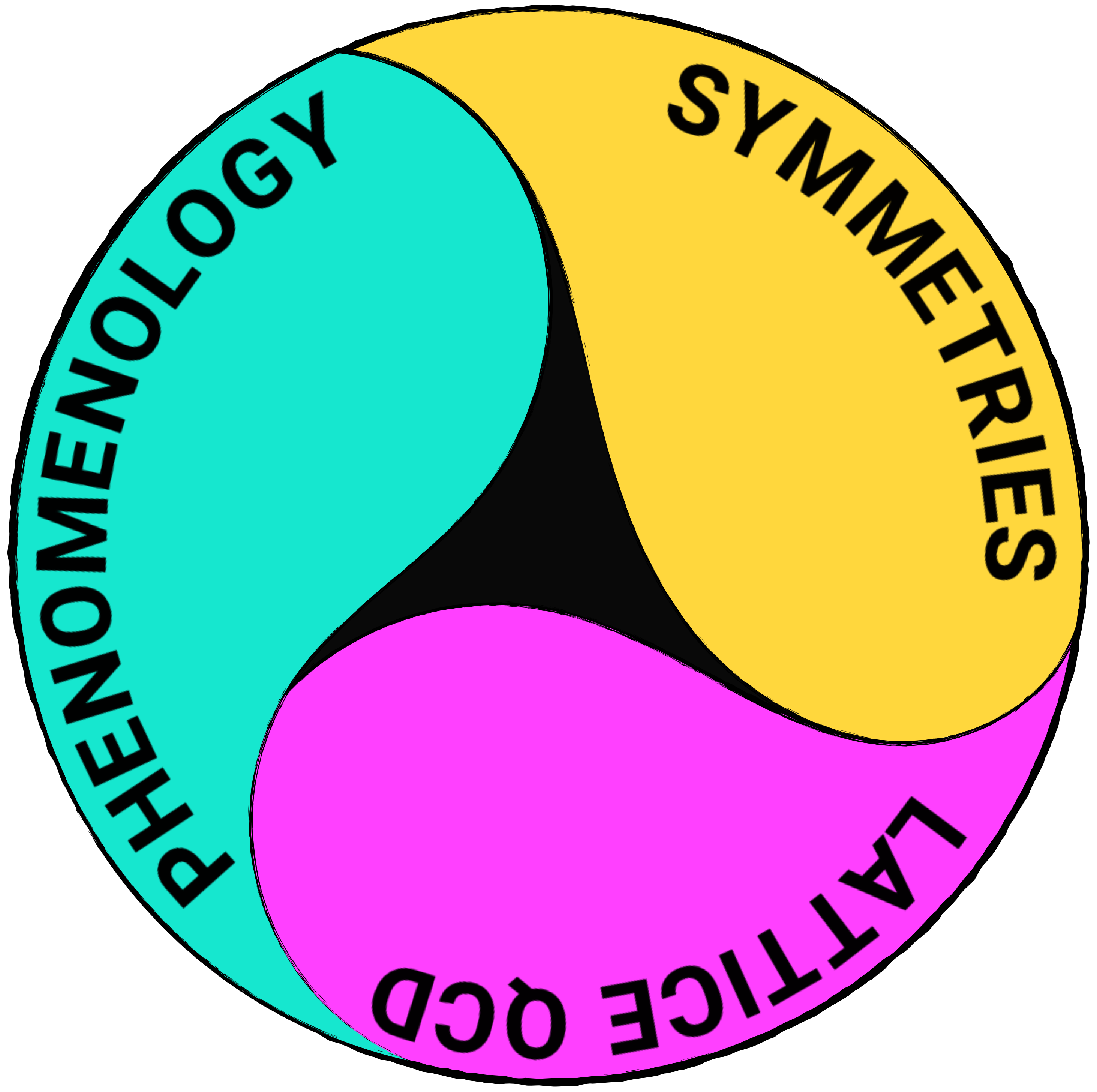
UNIVERSAL* RESONANCE PARAMETER



(*) Reaction independent

RESONANCE PARAMETER

S-matrix theory: ***transition amplitude***

- Unitarity/Analyticity/Crossing symmetry
- Poles on unphysical Riemann Sheets

Review: MM/Urbach/Meißner Phys.Rept. 1001 (2023) 1-66

RESONANCE PARAMETER

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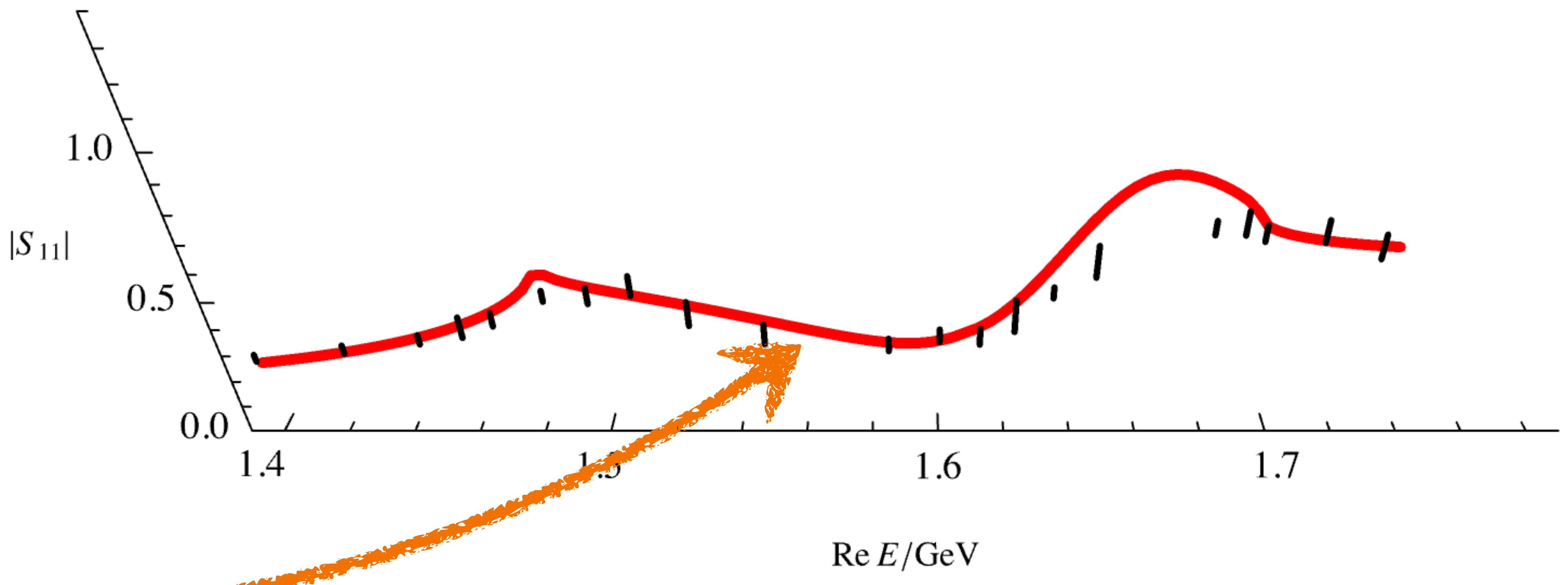
Boundary ($E \in \mathbb{R}$):

- Experiment
- Lattice QCD
- CHPT



Data: SAID: Phys. Rev. C 74 (2006) 045205
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RESONANCE PARAMETER

Analytic continuation to unphysical

Riemann Sheet

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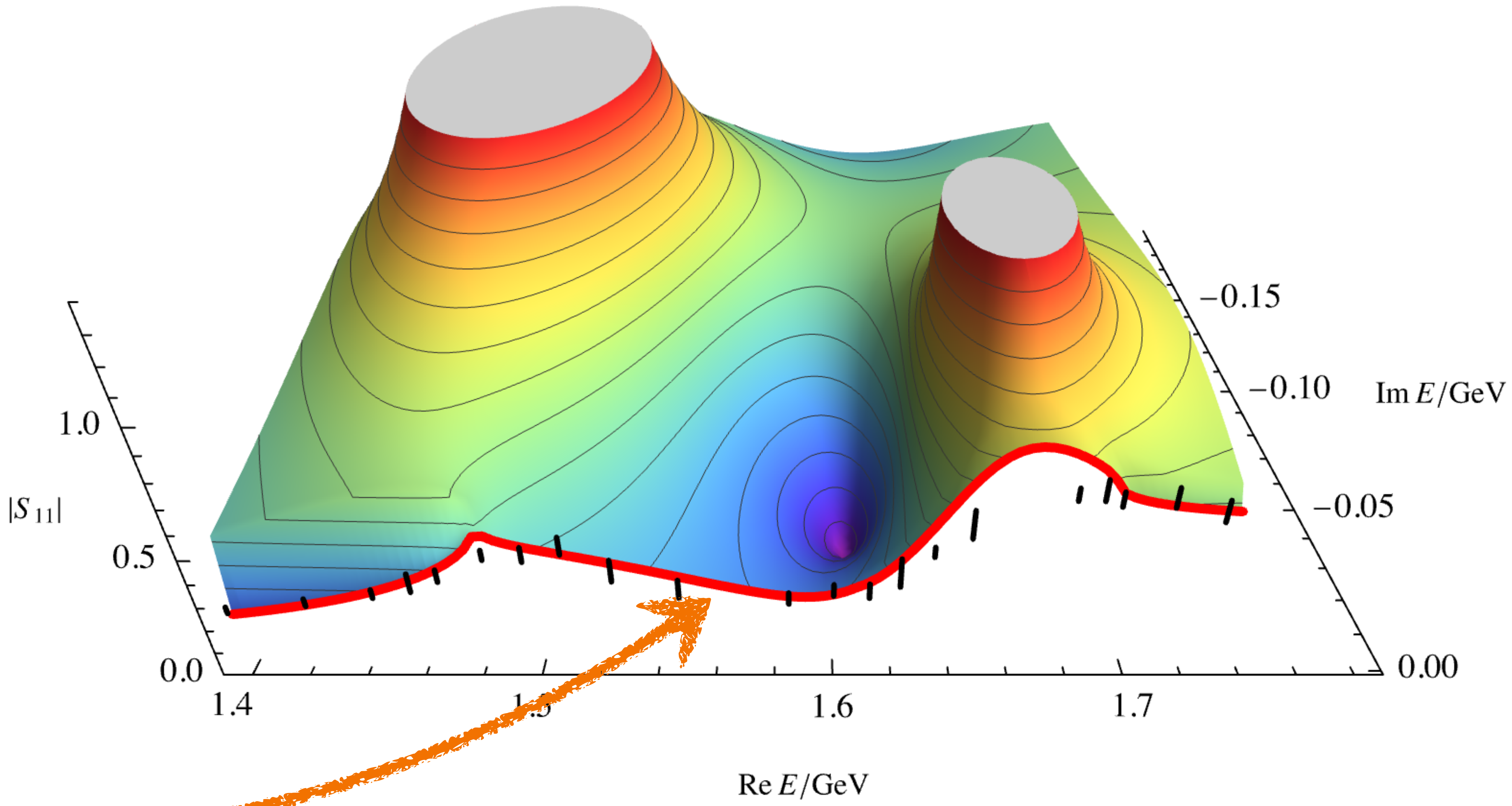

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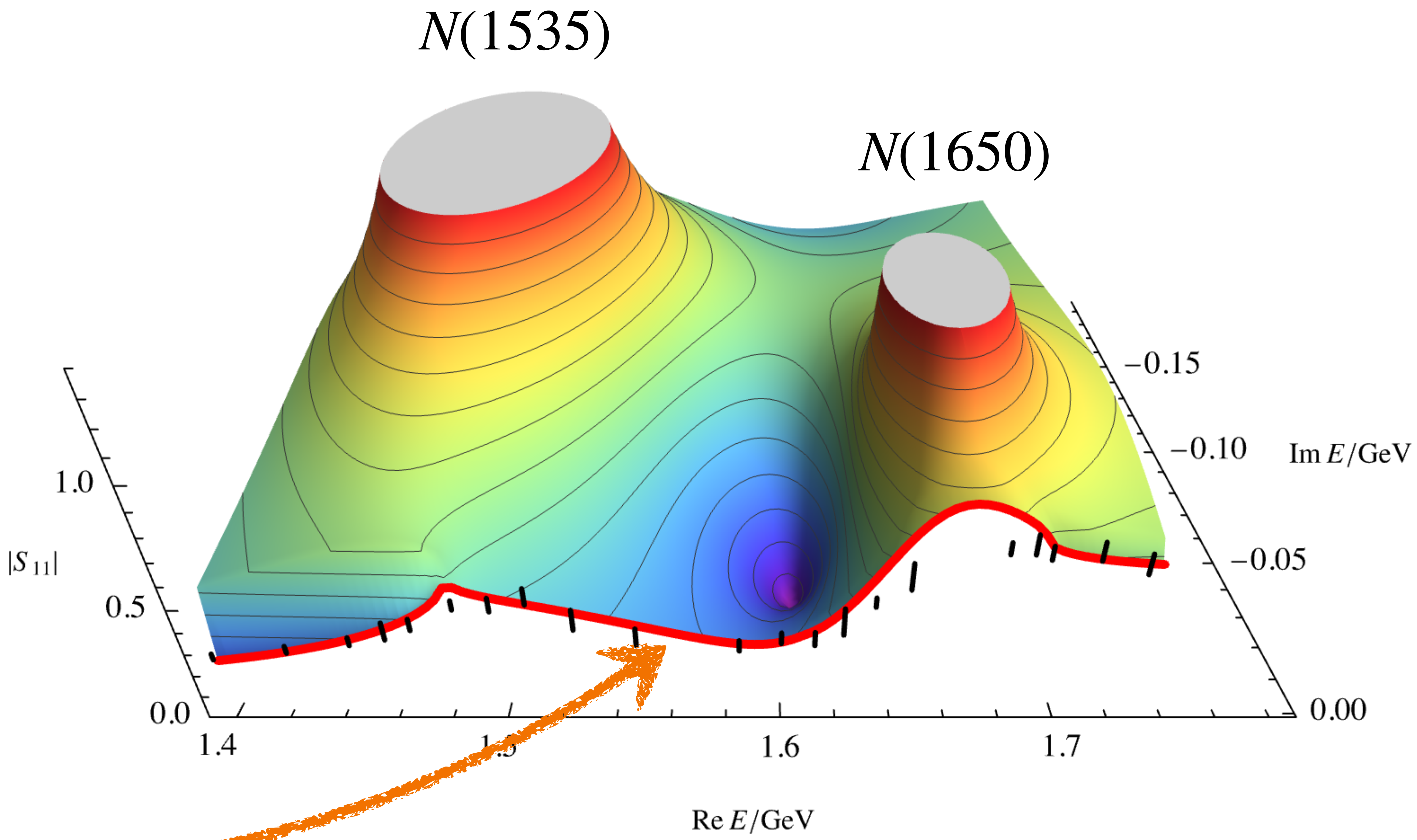

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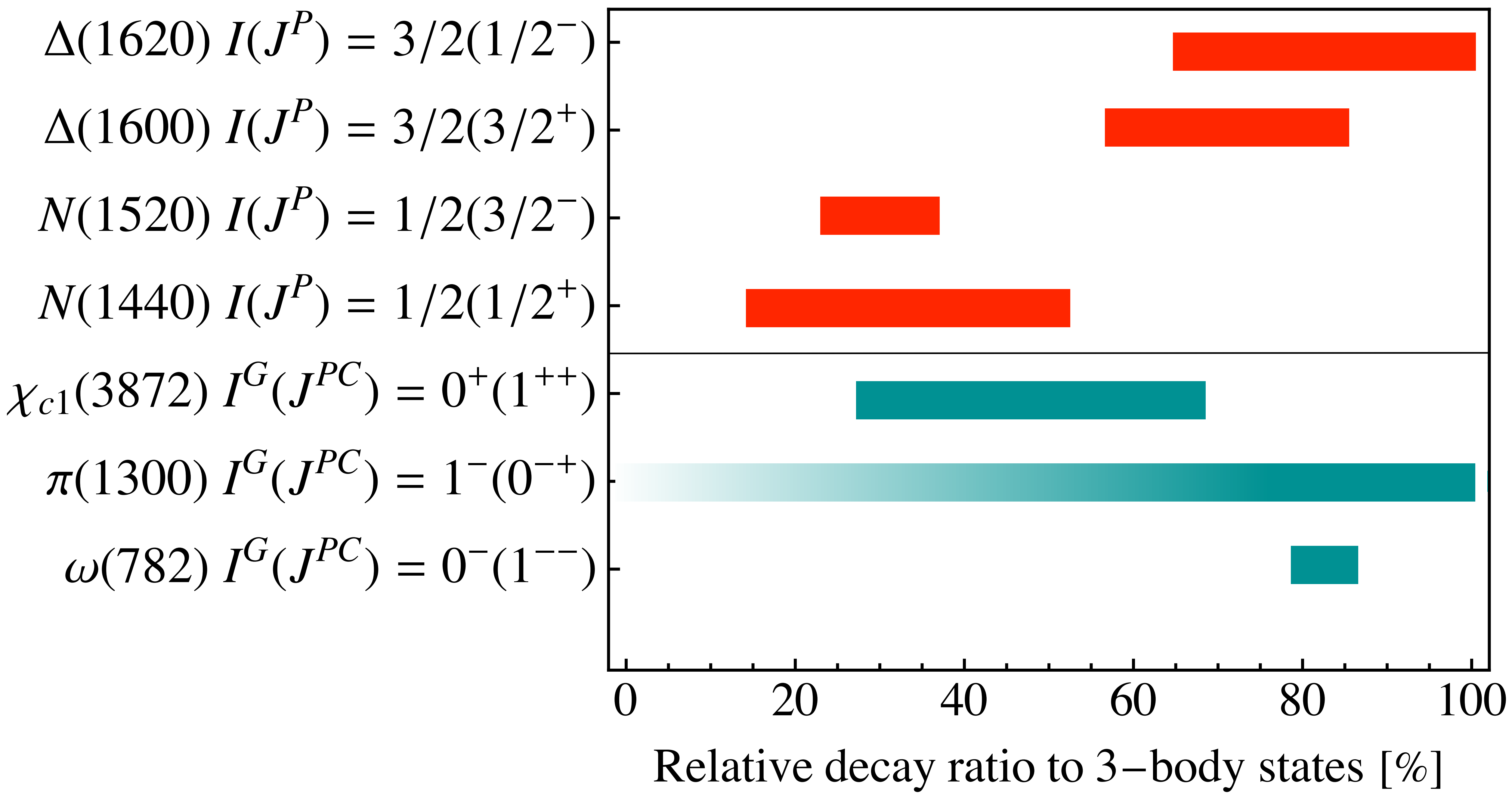
THREE-HADRON STATES

3-HADRON STATES

Most known states have large 3-body content

- $\omega(782) \to \pi\pi\pi$
- $a_1(1260) \to \pi\pi\pi$
- $N(1440) \to \pi\pi N$
- $X(3872) \to DD\pi$



Beyond Standard Model searches (τ-EDM/…)

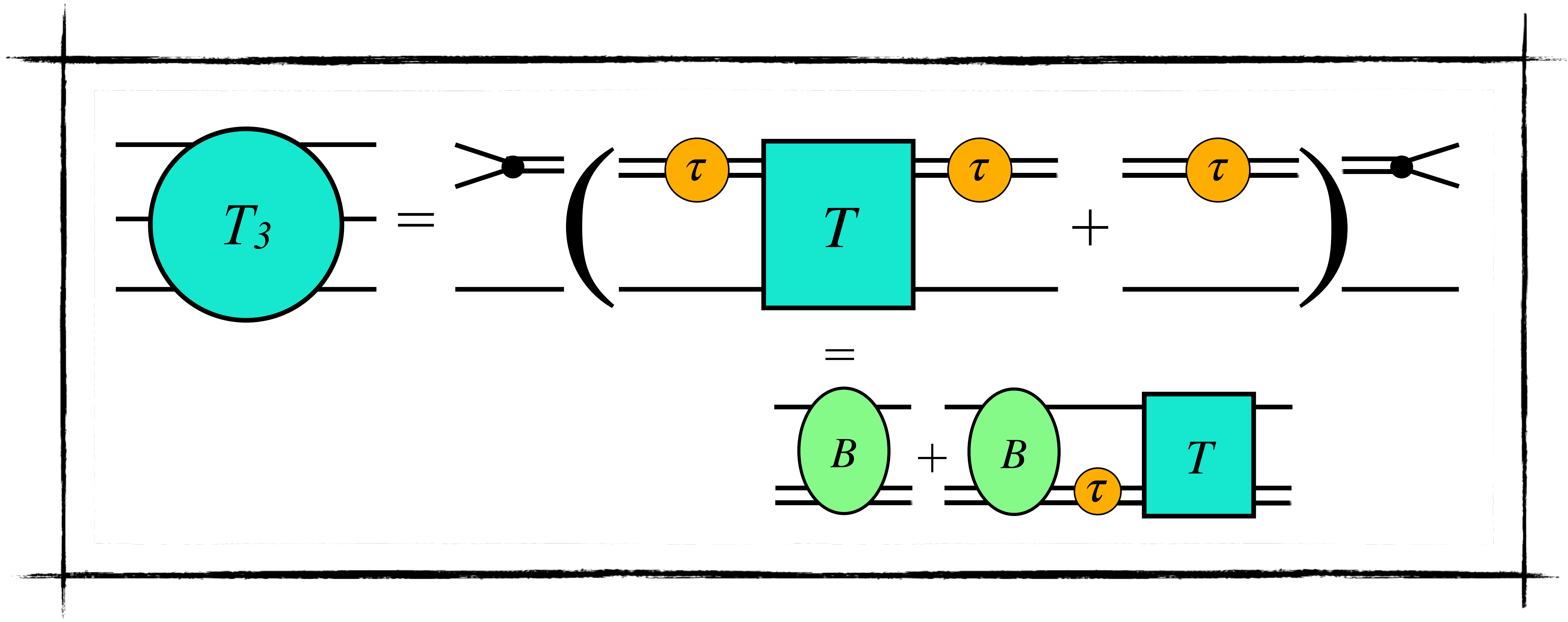
Exotic states of matter

GlueX@JLAB; COMPASS@CERN;

Singularity structure, long-range forces …

Talks: E.Epelbaum — A.Rusetsky — X.Zhang — J.Wu — Z.Zhang — A. Nefediev

TRANSITION AMPLITUDE



“Infinite Volume Unitarity” — IVU formalism

- Three-body scattering amplitude
  MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)
  Related: Hansen/Sharpe(2014), Wunderlich et al. JHEP 08 (2019); Jackura et al. Eur.Phys.J.C 79 (2019);
- Express 3-body through 2+1 system

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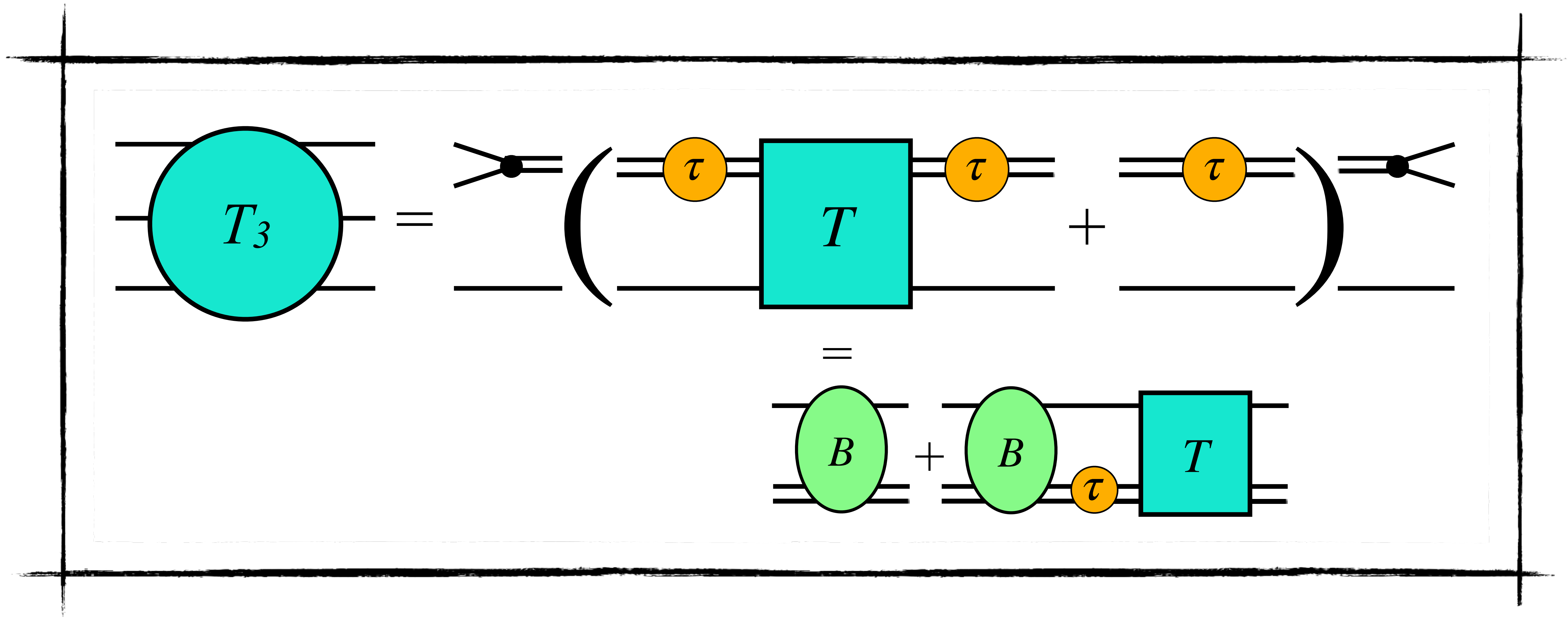

$$\langle \dots | T_3 - T_3^\dagger | \dots \rangle \xleftrightarrow{\text{3b-Unitarity}} i \int \langle \dots | T_3 T_3^\dagger | \dots \rangle$$

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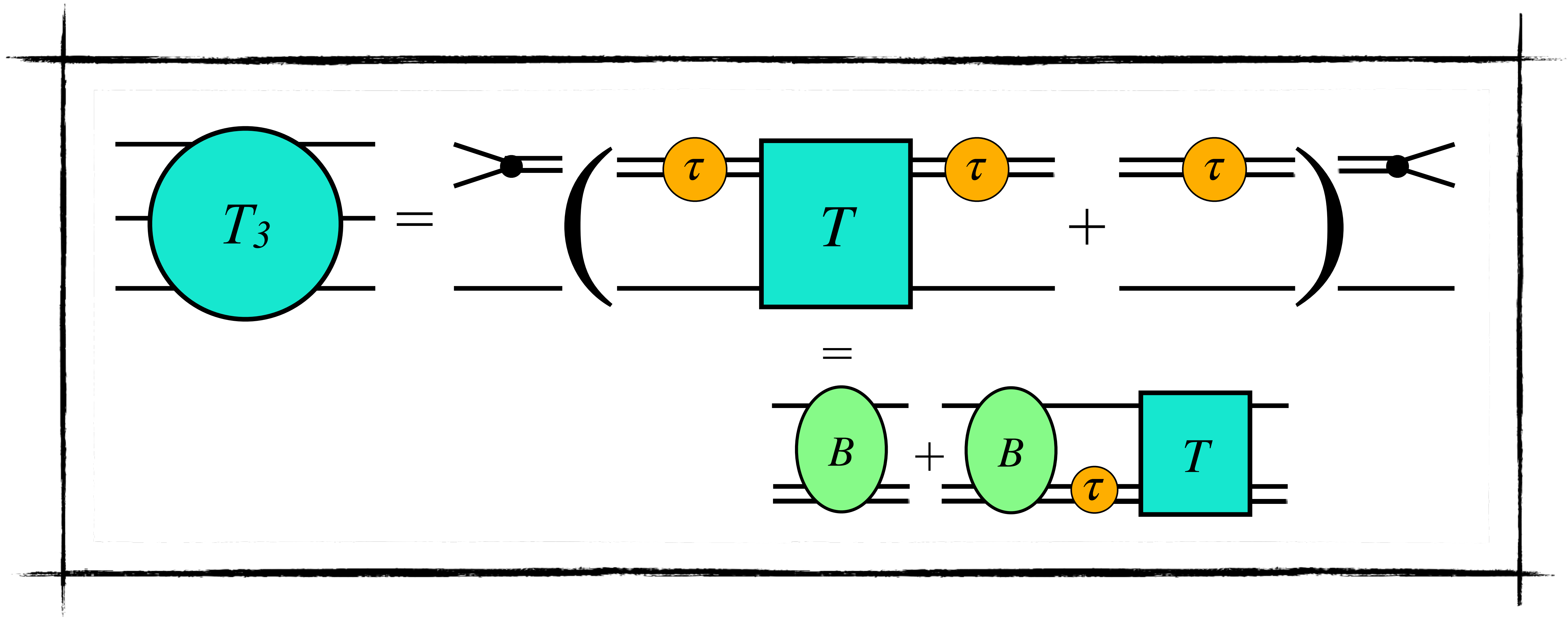

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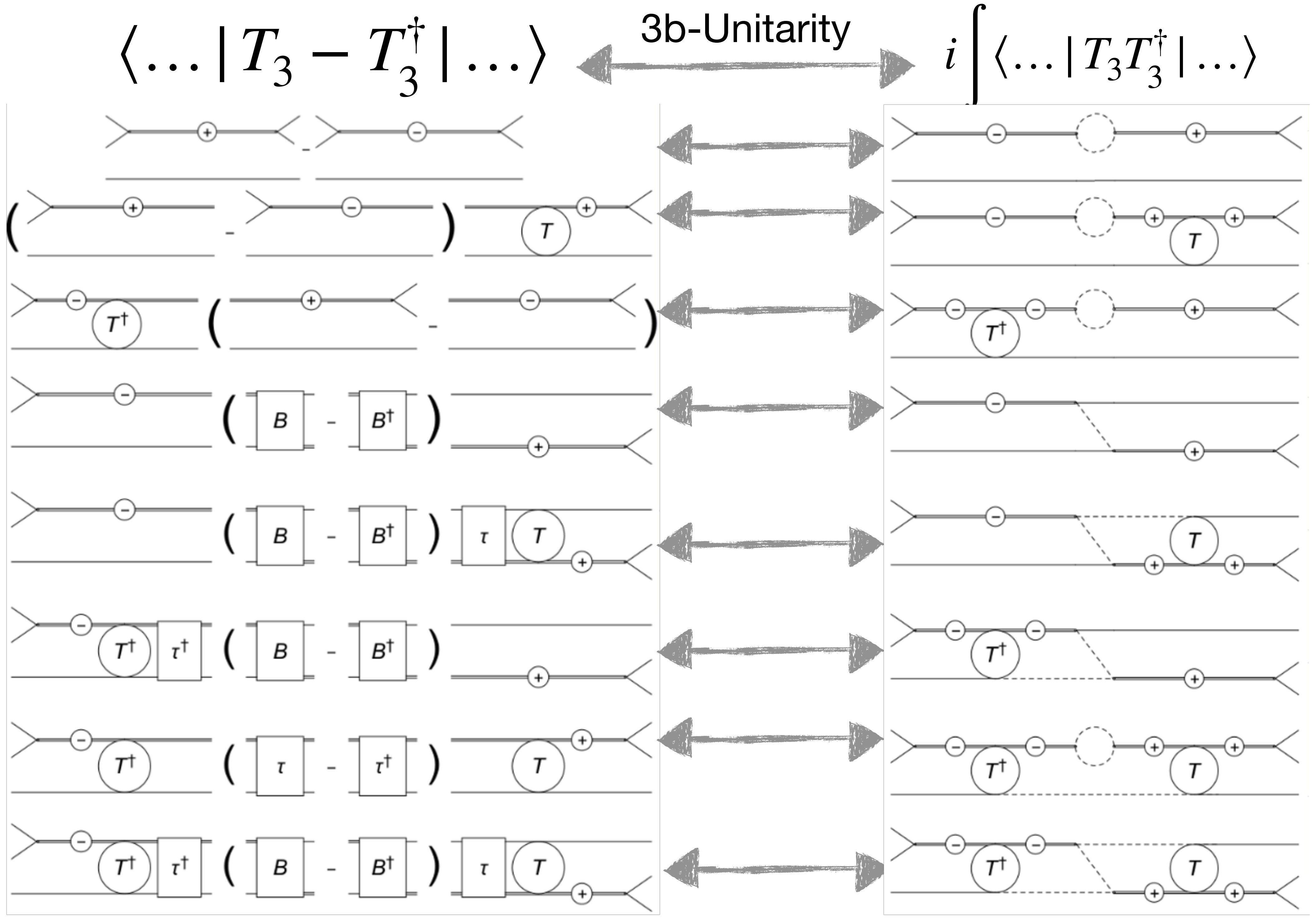

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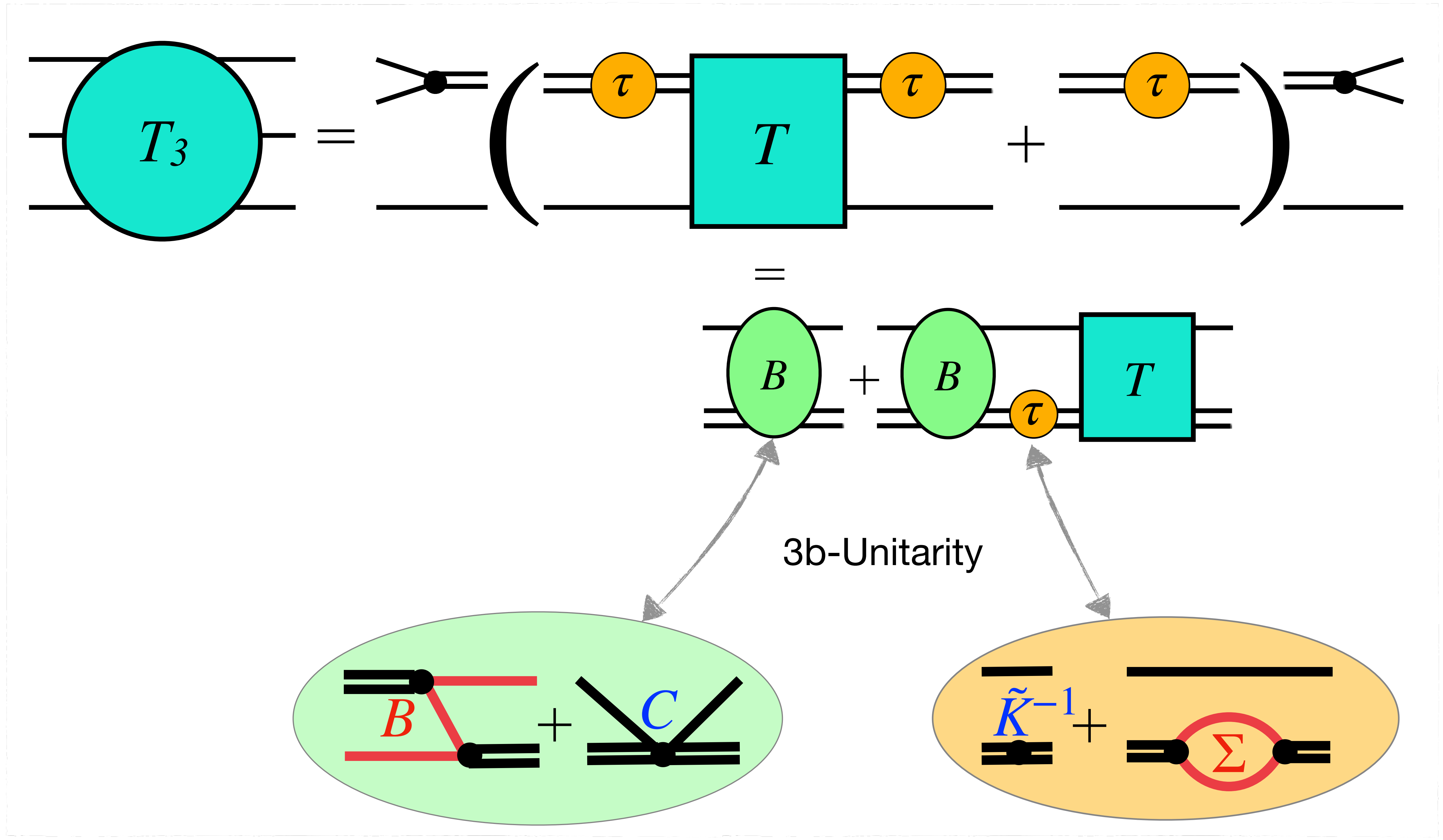

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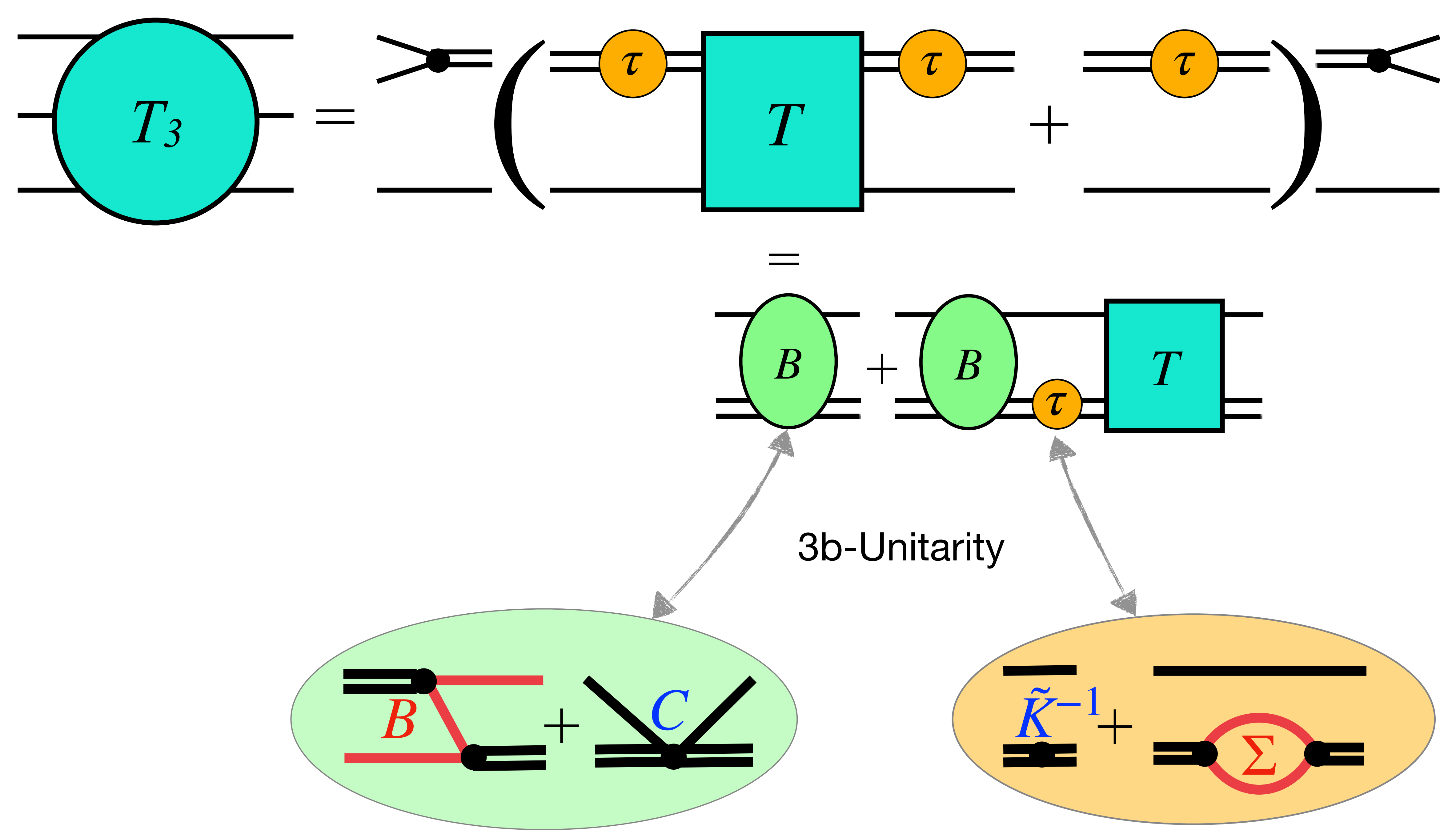

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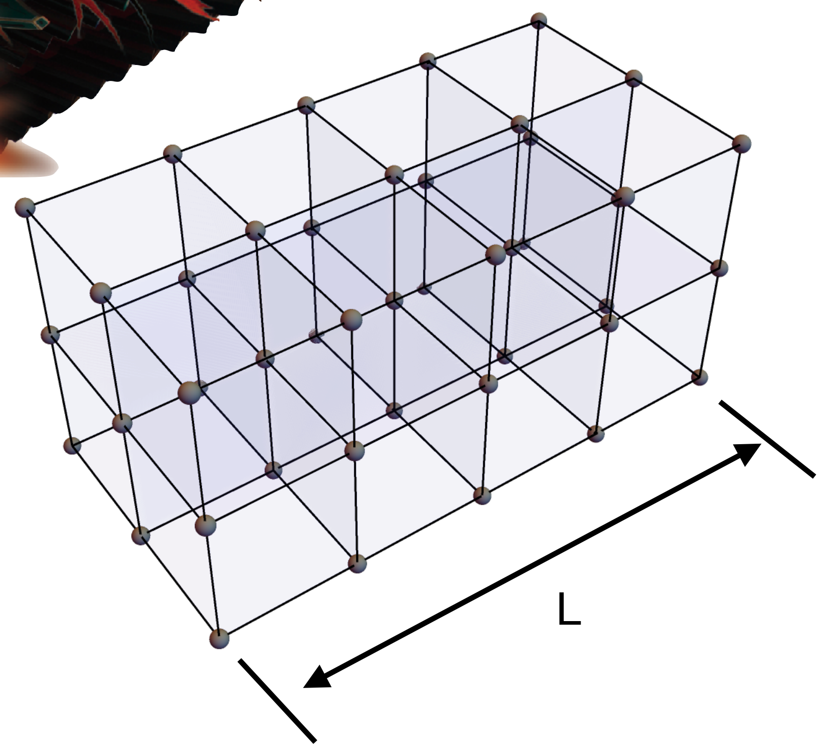
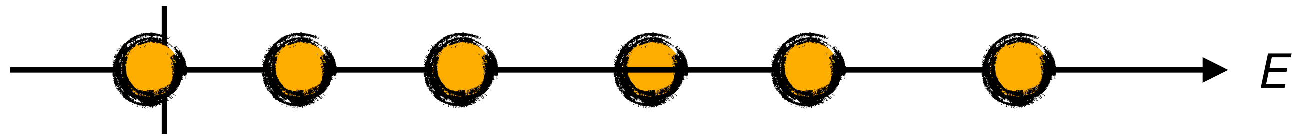
FINITE-VOLUME SPECTRUM



Lattice QCD — QCD Green's functions:

- Euclidean space-time
- unphysical pion mass
- **finite-volume**



Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, …
Reviews: Briceno/Dudek/Young (2017) Rev.Mod.Phys. 90 (2018); Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Doring/Rusetsky Eur.Phys.J.ST 230 (2021);
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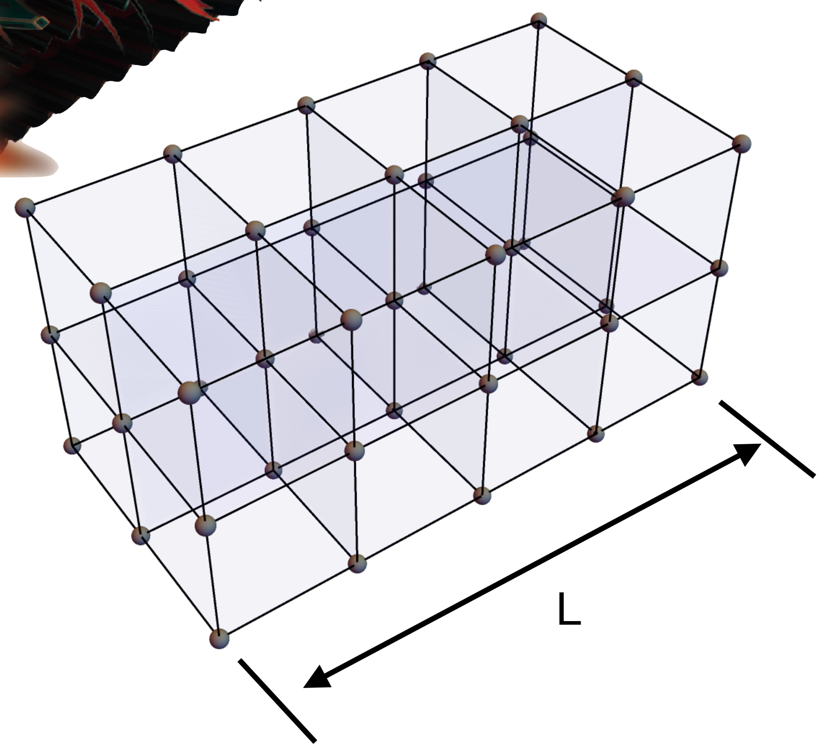

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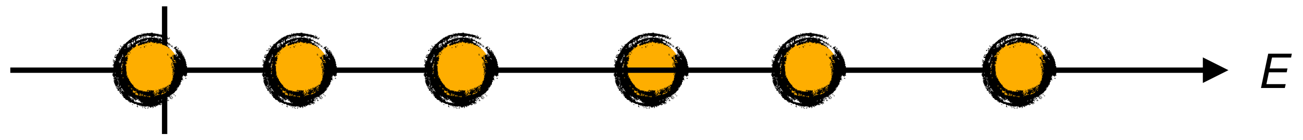
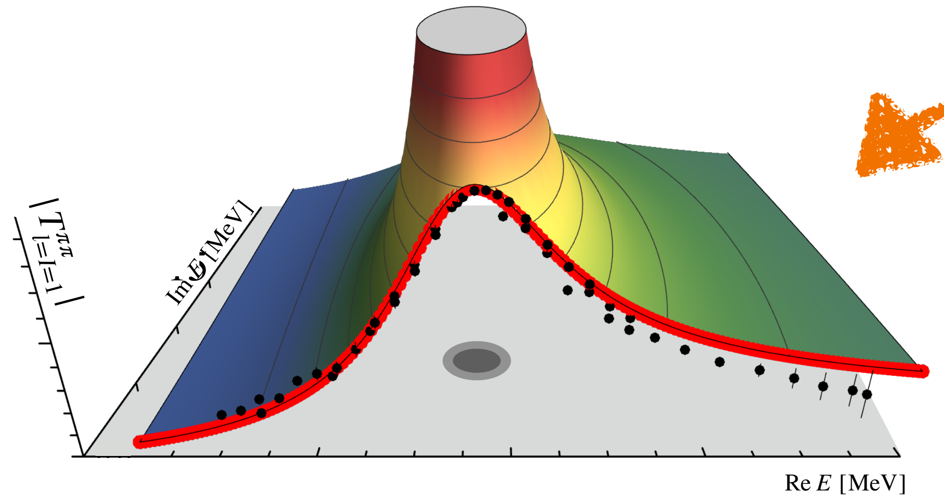
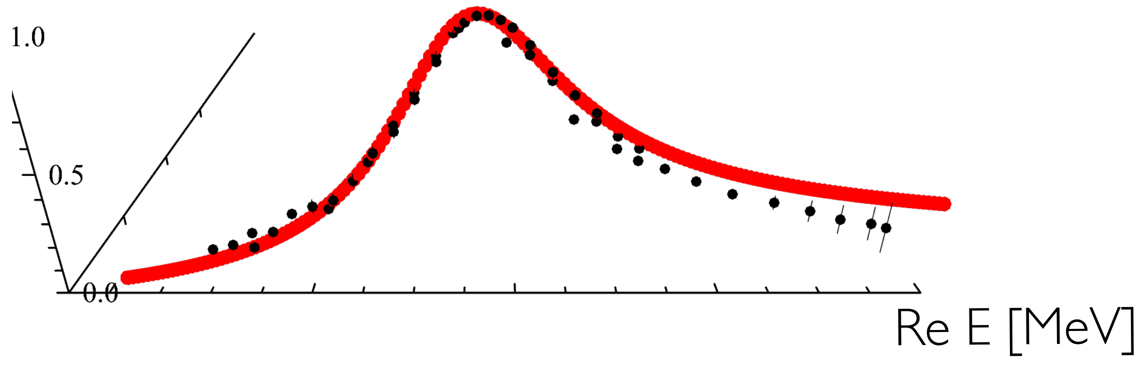


Quantization condition





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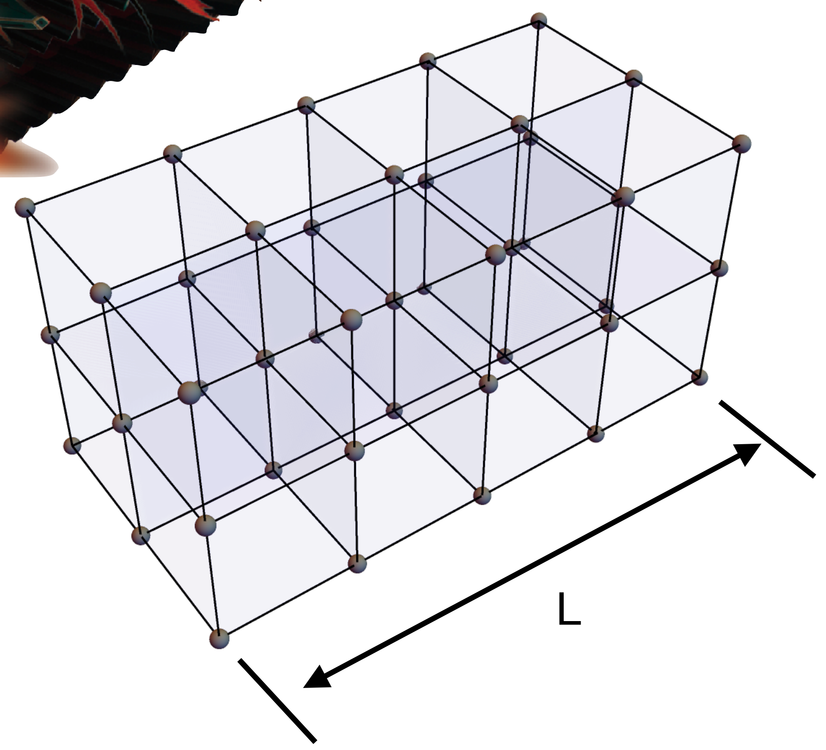

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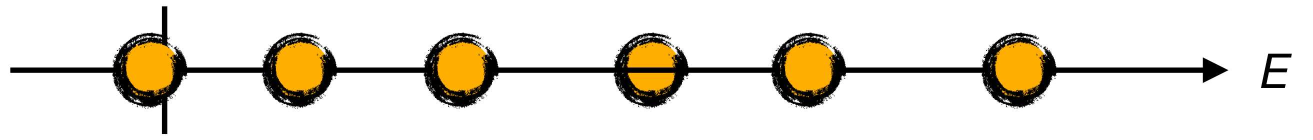


- on-shell states "feel" the box-size
- off-shell configurations decay exponentially $\sim e^{-ML}$
- unitarity separates those

"Finite Volume Unitarity" 3-body quantization condition

FVU

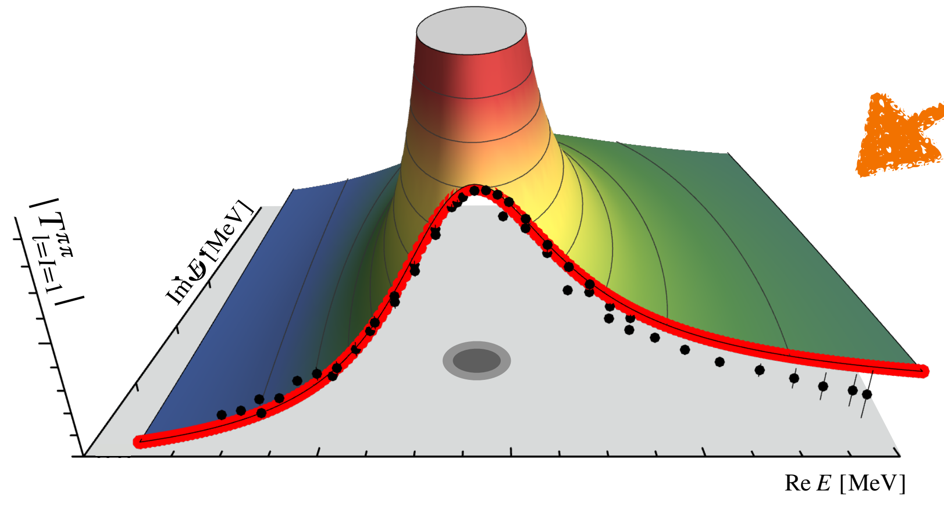
$$\det\left[\,2L^3E_{\mathbf{p}}\left(\tilde{K}^{-1}-\Sigma^L\right)-B-C\,\right]^{\Lambda}\equiv 0$$

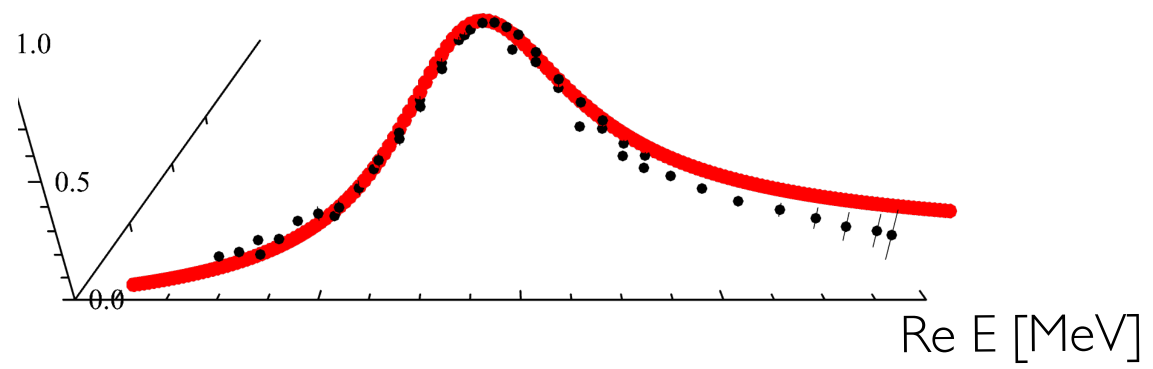
MM/Döring
***Eur.Phys.J.A* 53 (2017) 12, 240**

Soon as a python package: H.Yan+MM

- Alternatives:
  - RFT(Hansen/Sharpe 2014)
  - NREFT(Rusetsky/Hammer/Pang 2017)

Quantization condition





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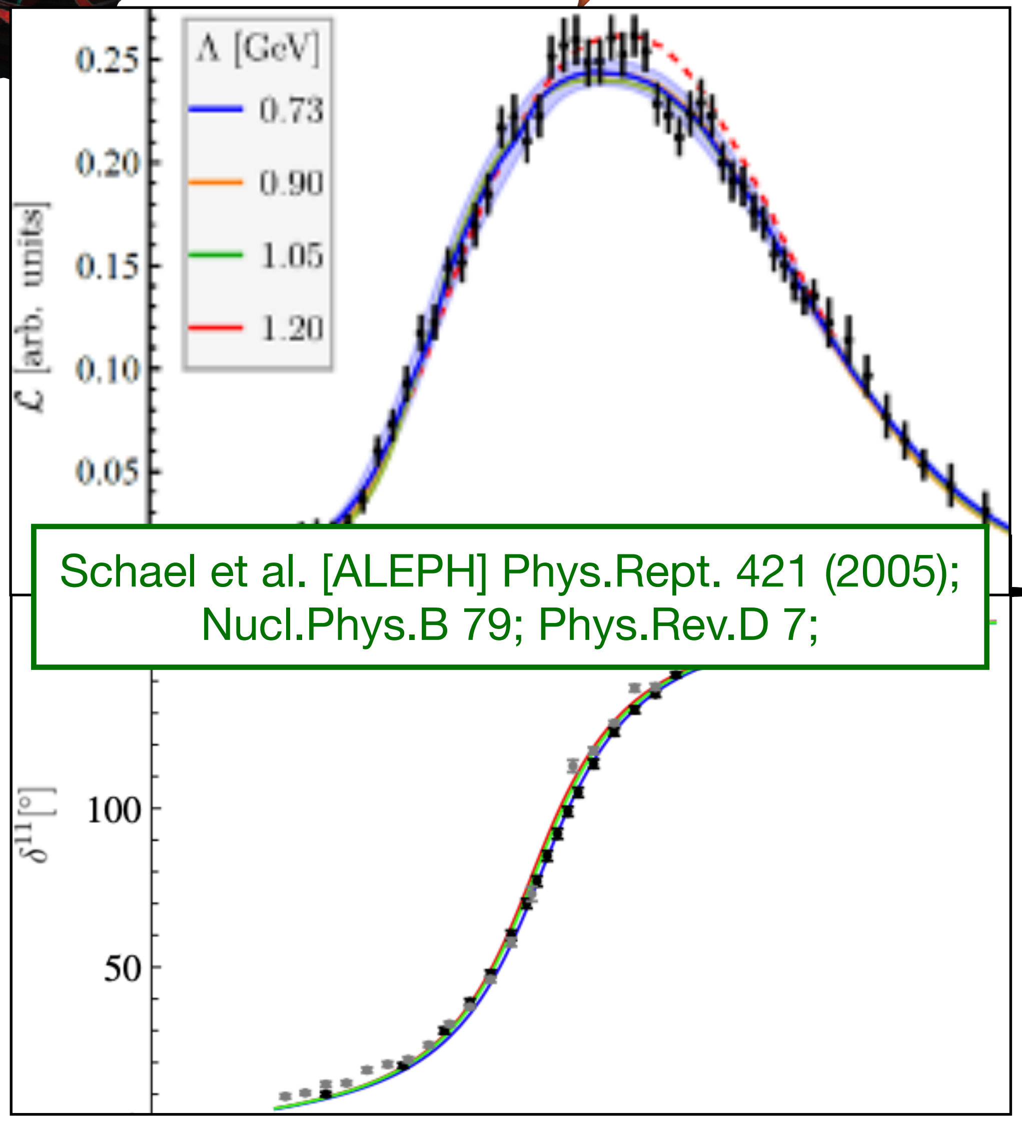
AXIAL-VECTOR MESON



Schael et al. [ALEPH] Phys.Rept. 421 (2005); Nucl.Phys.B 79; Phys.Rev.D 7;

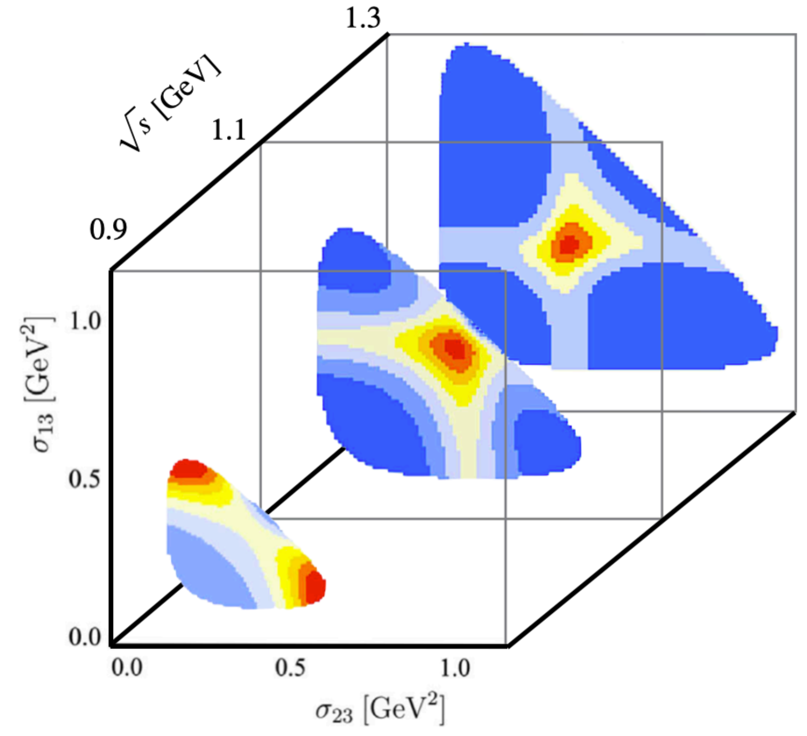
INPUT

EXPERIMENT

IVU

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}^{-1} - \Sigma} T^c$$

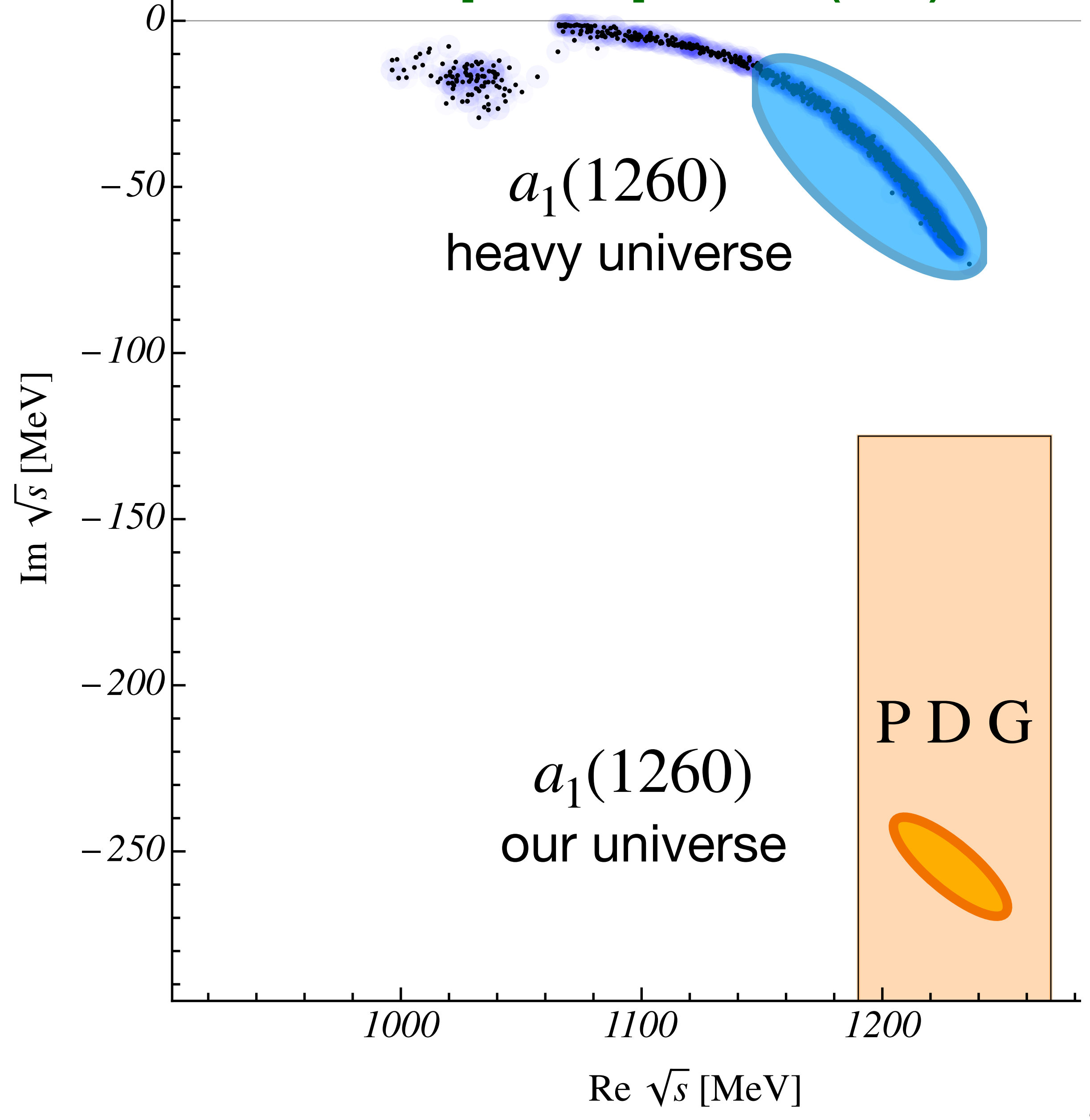
OUTPUT



Sadasivan/MM/... Phys.Rev.D 101 (2020);

$C, \tilde{K}$

MM/Culver/... [GWQCD] PRL 127 (2021)



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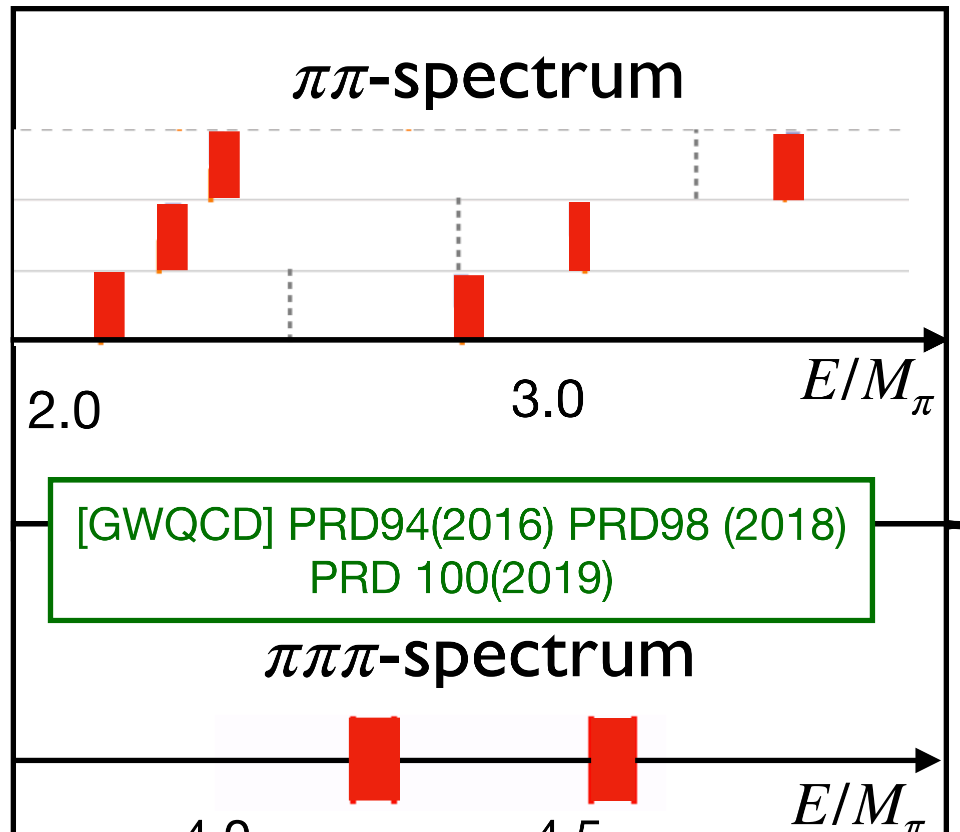

[GWQCD] PRD94(2016) PRD98 (2018) PRD 100(2019)

LATTICE QCD
$[M_\pi = 315\,\text{MeV}]$

INPUT

Lattice QCD

FVU

$$\det\left[\, 2L^3 E_{\mathbf{p}} \left(\tilde{K}^{-1} - \Sigma^L\right) - B - C \,\right]^\Lambda \equiv 0$$

AXIAL-VECTOR MESON*

Excited axial-vector meson: $a_1(1420)$

Observed by COMPASS/Belle in $\pi^-\pi^+\pi^-$ final state
COMPASS:2015kdx, Rabusov:2023tna

Creation mechanisms:

- Excited state of $a_1(1260)$
  COMPASS:2020yhb
- “Triangle singularity” $K^*(892)\bar{K} \xrightarrow{K} \pi f_0(980)$
  Mikhasenko:2015oxp **Review: Guo:2019twa**
  Related: Dai:2018hqb, Dai:2018rra, Liang:2019jtr, Jing:2019cbw, Du:2021zdg, Duan:2023dky, Wang:2016dtb, Nakamura:2023obk, Zhang:2024dth, Achasov:2022onn, Nakamura:2023hbt, arXiv:1609.04133 [hep-ph].
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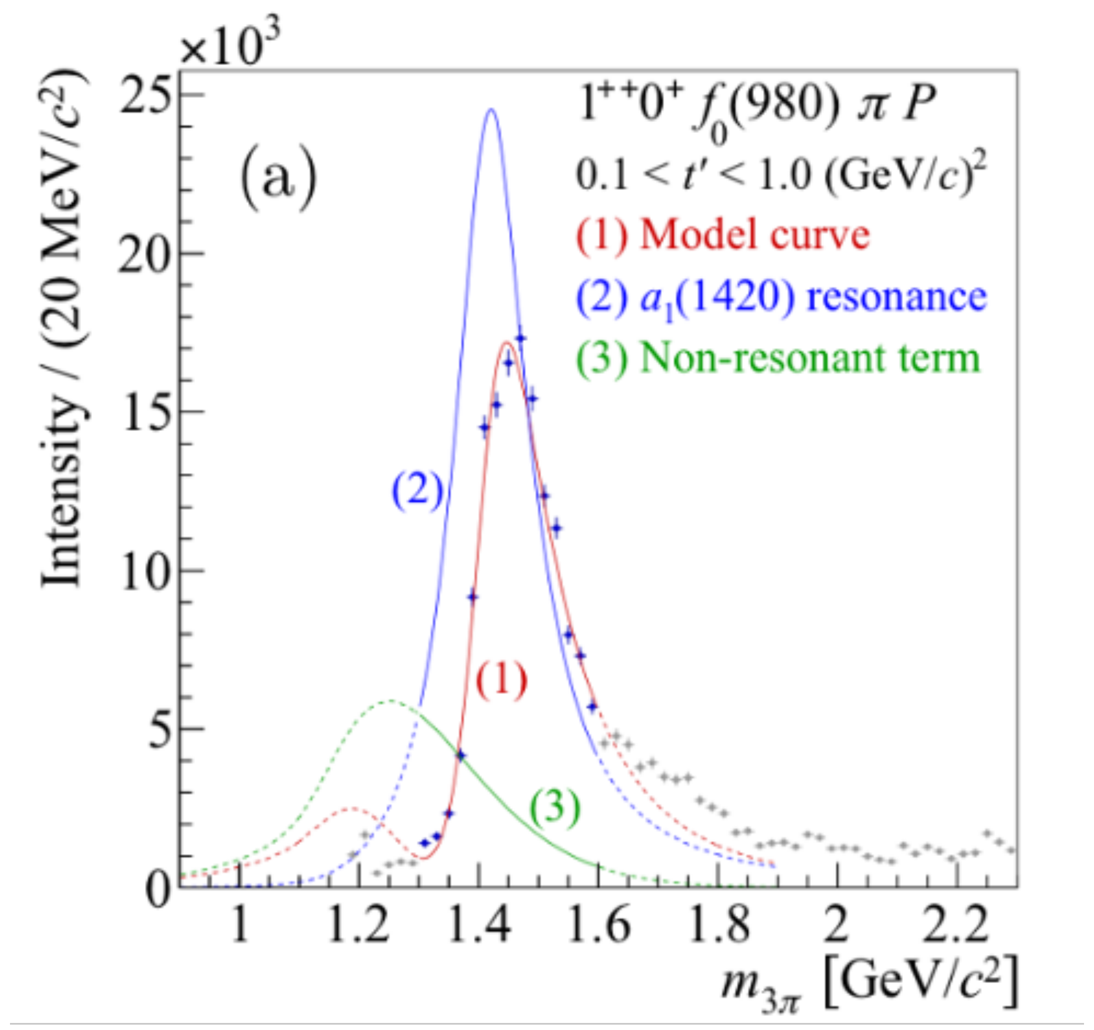

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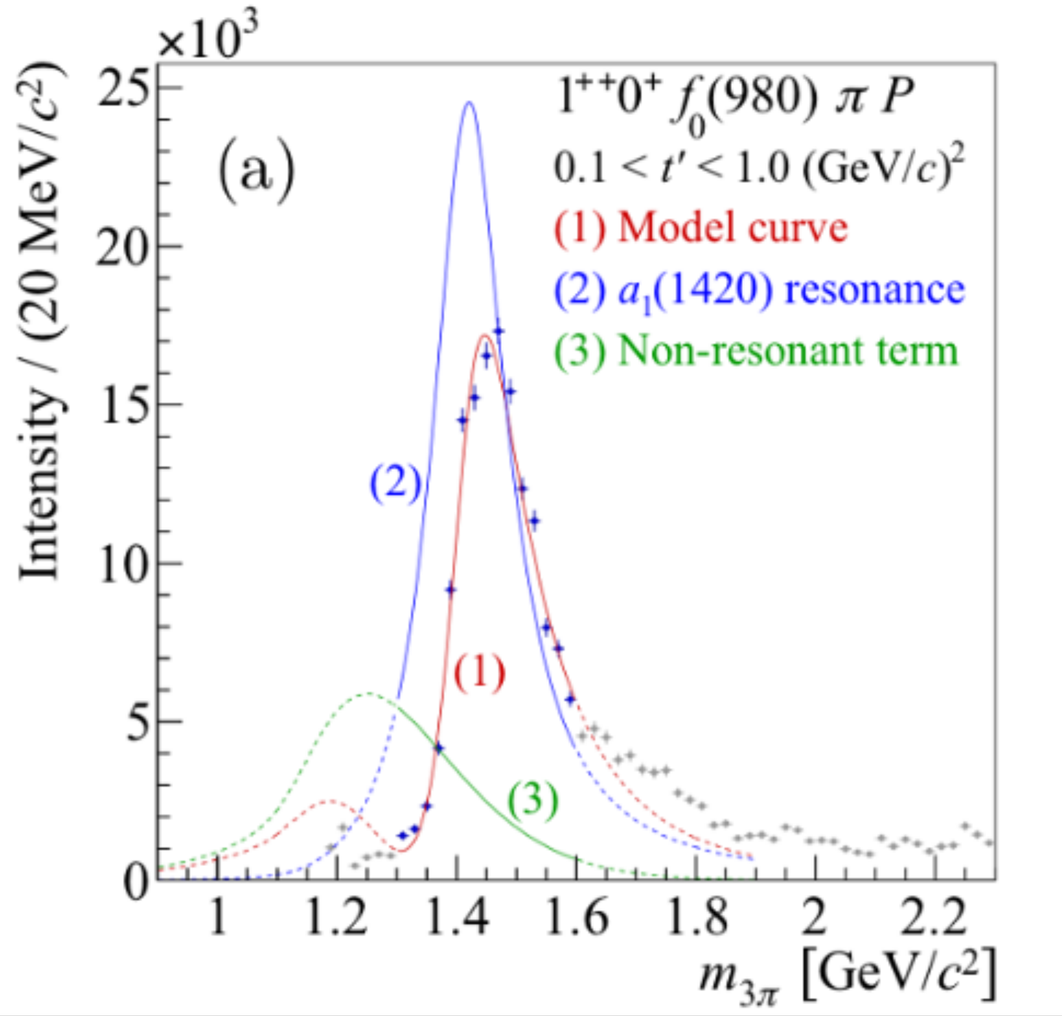

3b unitary formalism IVU

Ajay Sakthivasan/MM/Döring/Rusetsky 2407.17969 [hep-ph]

- formalism to incorporate both hypothesis
- full 2- and 3-body re-scattering
- **for now:** only kinematic/analytical properties (no spin)

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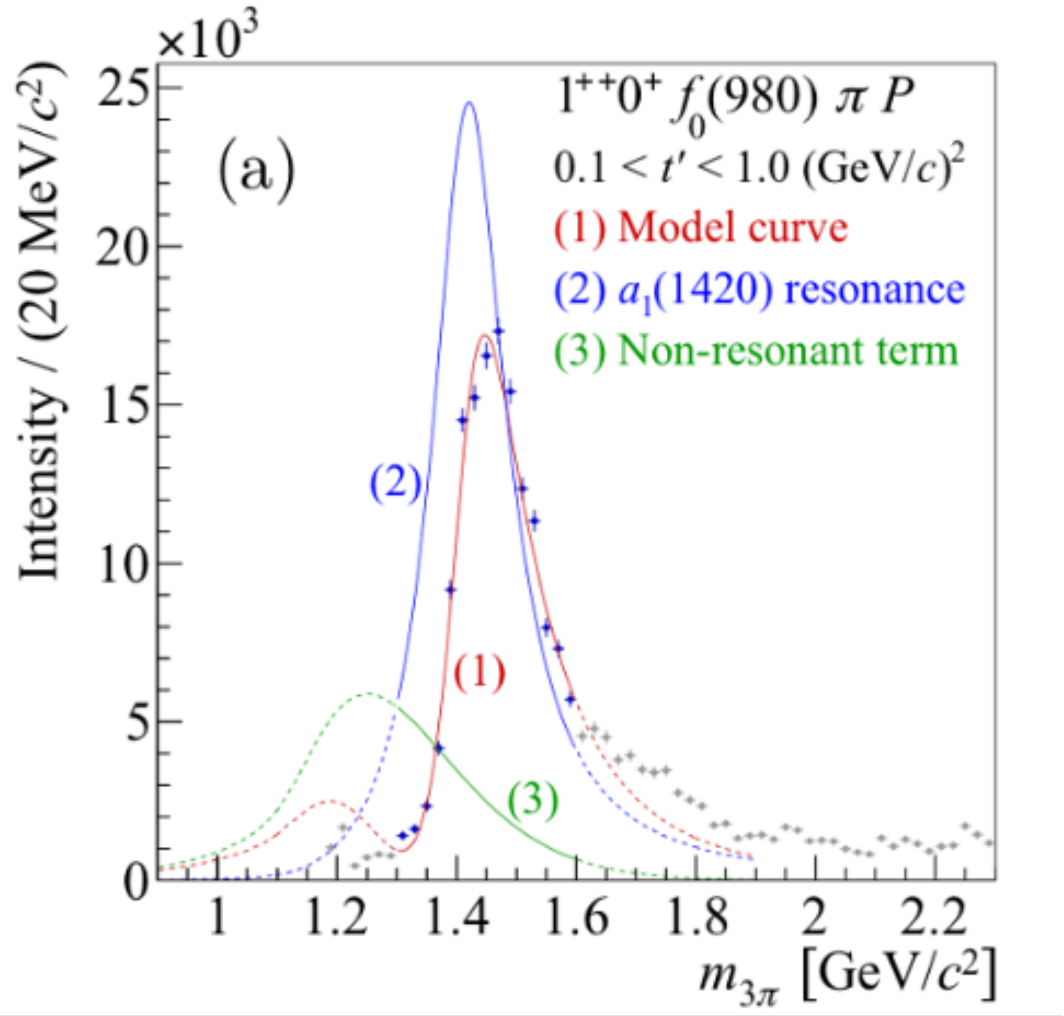

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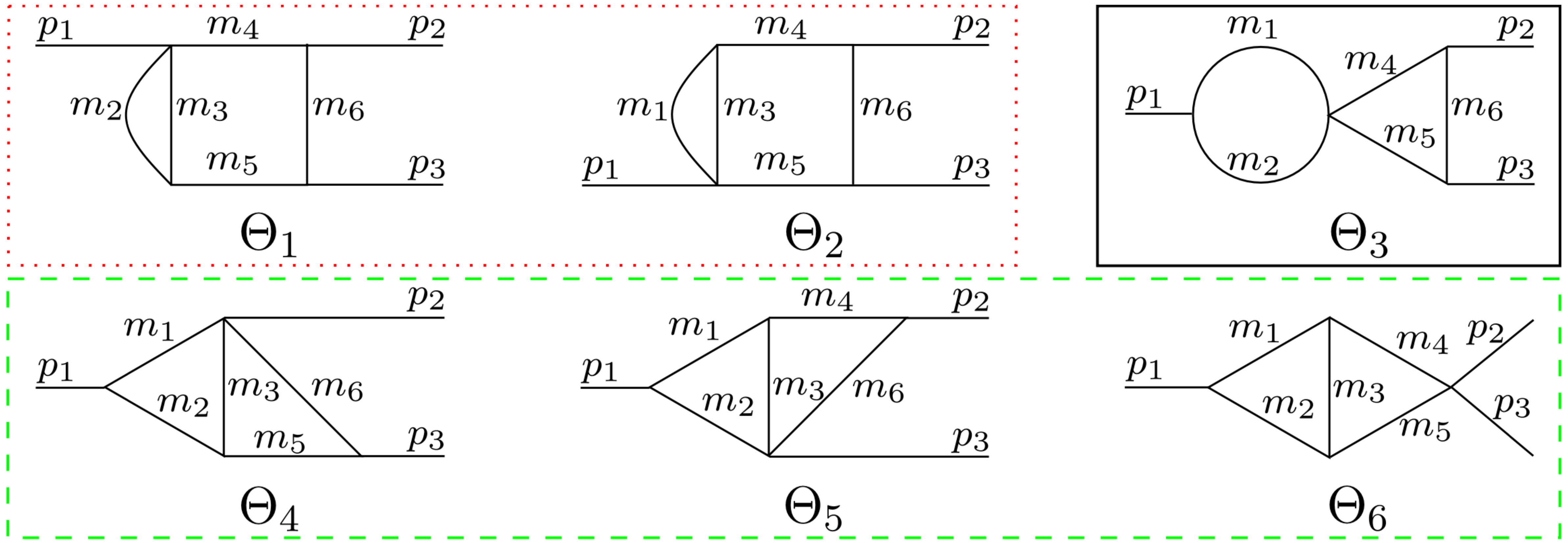

Landau equations

➡ singularity at the same place

➡ can become sub-leading

$$\alpha_i(q_i^2 - m_i^2) = 0\,, \qquad i = 1,\dots,N,$$
$$\sum_{i\in \text{loop}\, j} \alpha_i q_i^\mu(k_j) = 0\,, \qquad j = 1,\dots,L,$$

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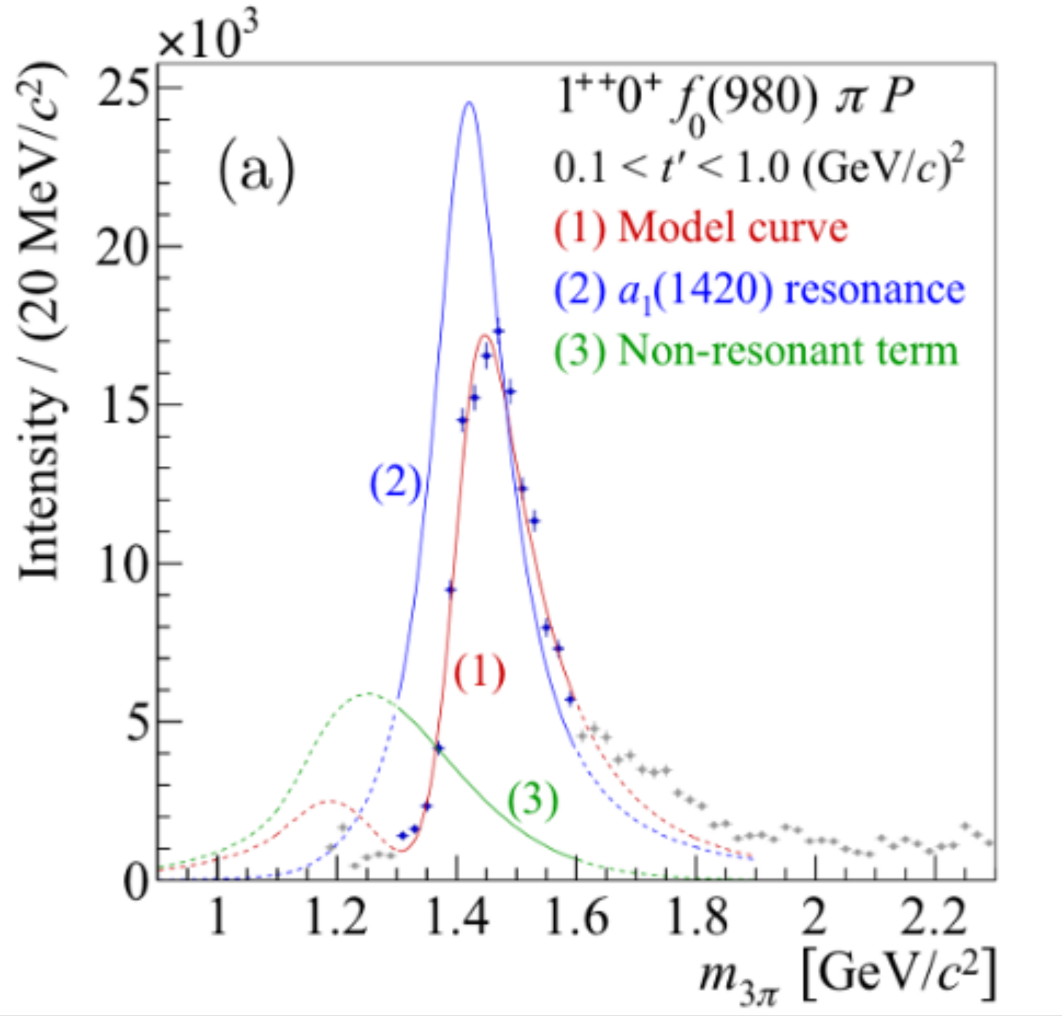
Excited axial-vector meson: $a_1(1420)$

Observed by COMPASS/Belle in $\pi^-\pi^+\pi^-$ final state
COMPASS:2015kdx, Rabusov:2023tna

Creation mechanisms:

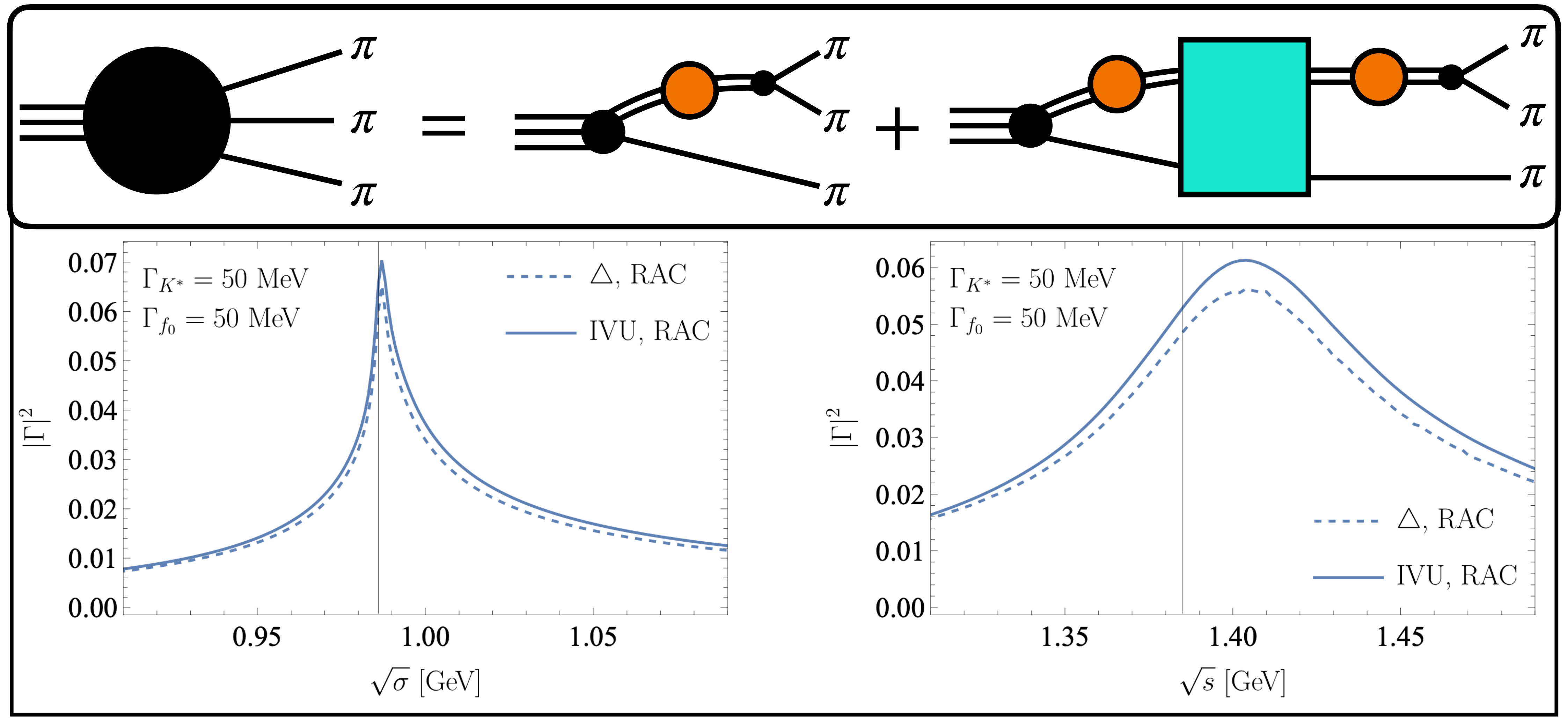
- Excited state of $a_1(1260)$
  COMPASS:2020yhb
- "Triangle singularity" $K^*(892)\bar{K} \xrightarrow{K} \pi f_0(980)$
  Mikhasenko:2015oxp **Review: Guo:2019twa**
  Related: Dai:2018hqb, Dai:2018rra, Liang:2019jtr, Jing:2019cbw, Du:2021zdg, Duan:2023dky, Wang:2016dtb, Nakamura:2023obk, Zhang:2024dth, Achasov:2022onn, Nakamura:2023hbt, arXiv:1609.04133 [hep-ph].
  Talks: J.J.Wu — Z.Zhang



3b unitary formalism IVU

Ajay Sakthivasan/MM/Döring/Rusetsky 2407.17969 [hep-ph]

- formalism to incorporate both hypothesis
- full 2- and 3-body re-scattering
- **for now:** only kinematic/analytical properties (no spin)



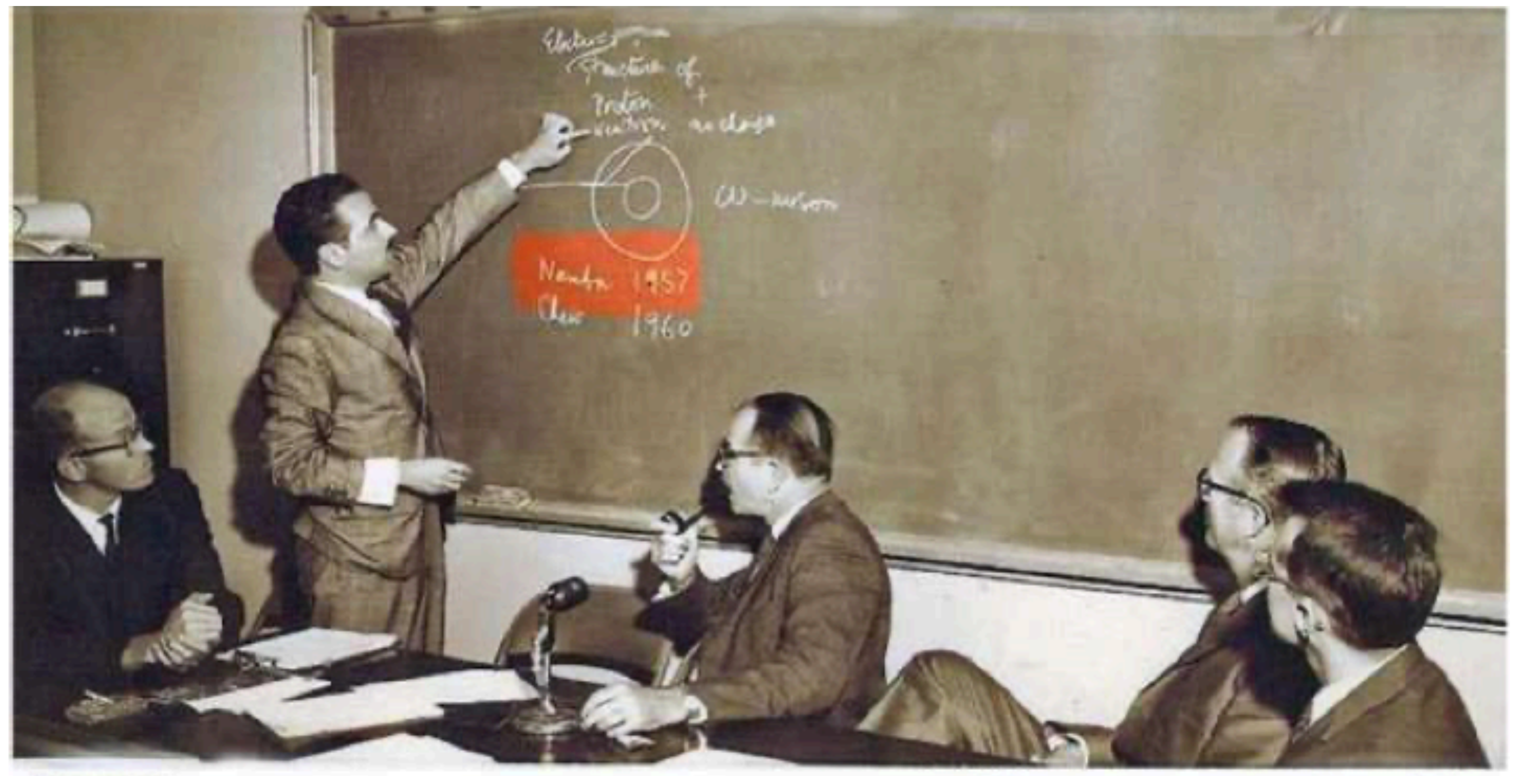
➡ Effect is small but distinguishable

➡ Add spin, fit to the line-shapes … in progress

VECTOR MESON

$\omega(782)$

- lightest hadron decaying into three particles
  Maglich/Alvarez/Rosenfeld/Stevenson *Phys.Rev.Lett.* 7 (1961) 178-182
- dominates the isoscalar response within the VMD picture of the photon-nucleon interactions
- generates the observed repulsion at < 1 fm in the one-boson-exchange picture of the N– N interaction
  Sakurai (1960); Erkelenz (1974); Brown and Jackson (1976);
  Barkov et al., 1985; Connell et al. (1997); Bazavov et al. (2021)



Stevenson Maglich MacMillan Alvarez Rosenfeld

PRESS/TV CONFERENCE ON DISCOVERY OF OMEGA MESON
Berkeley, August 31, 1961
Maglic', Alvarez, Rosenfeld & Stevenson, Phys. Rev. Lett. September 1, 1961

OVR

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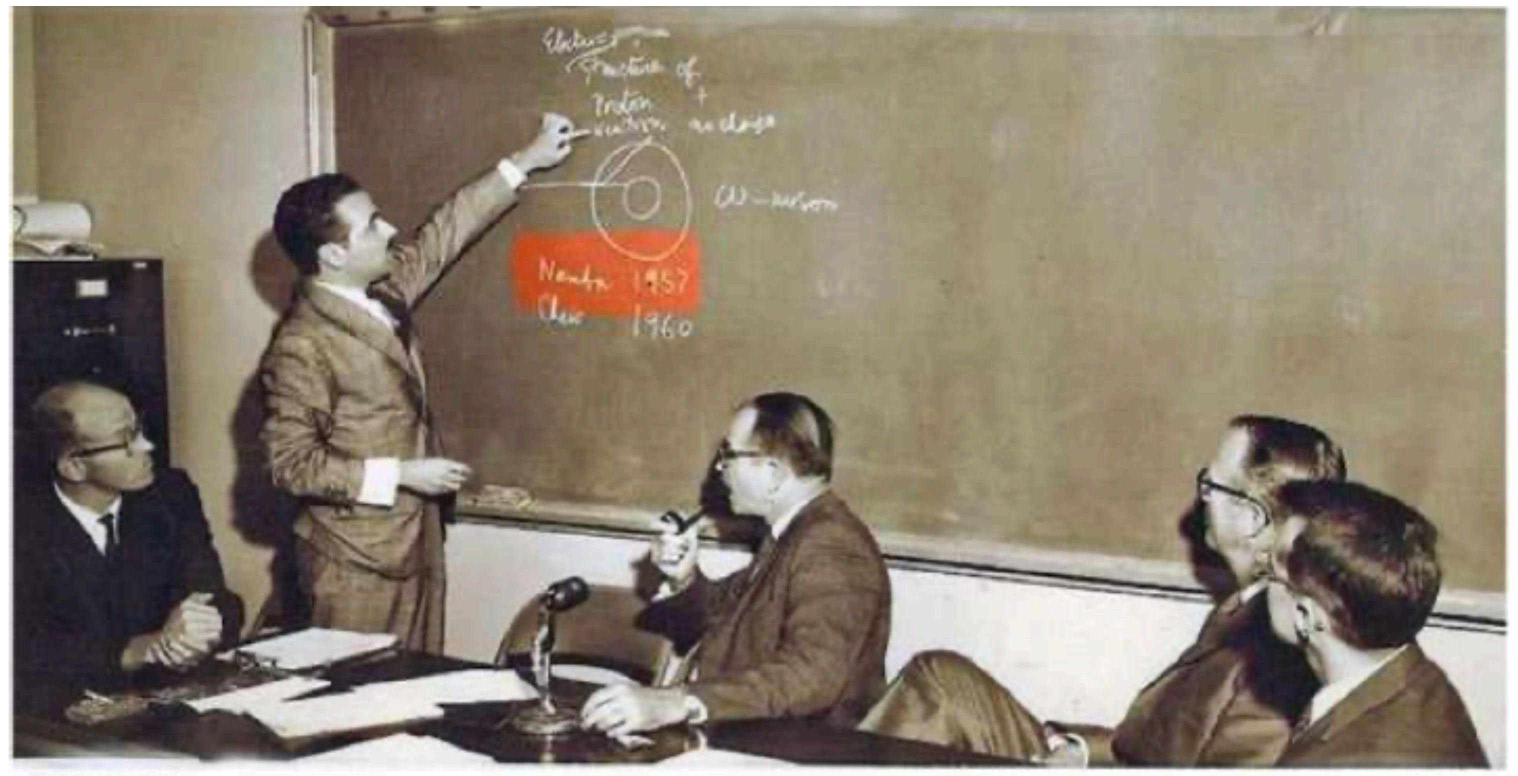

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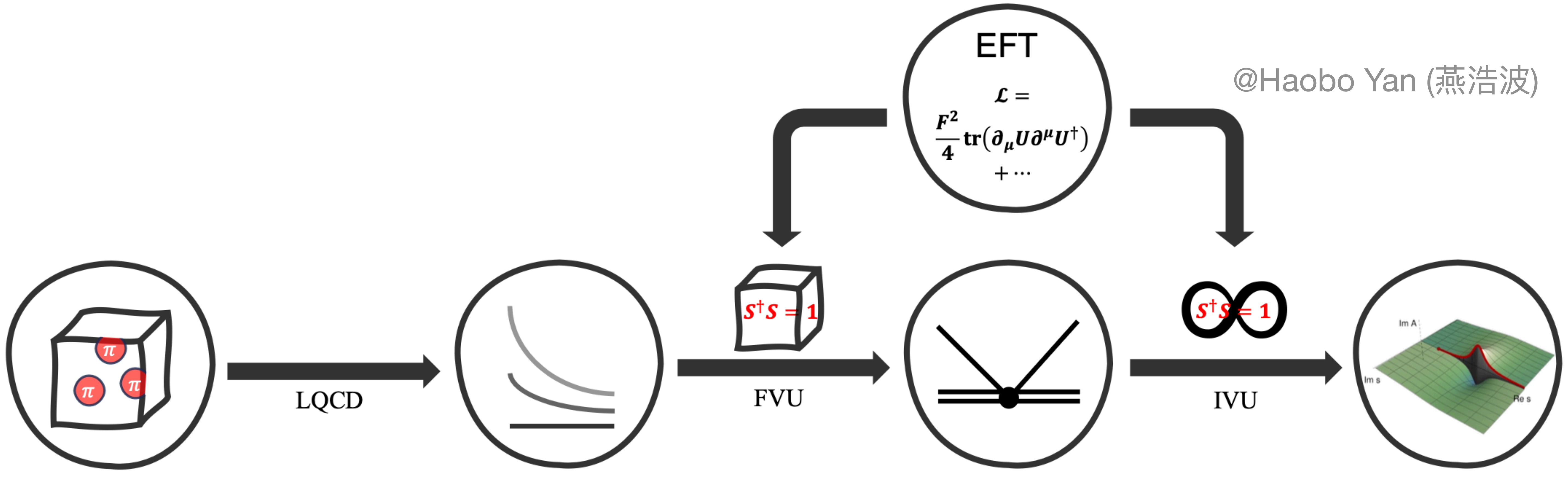
OVR

What can we learn from Lattice QCD?

Haobo Yan (燕浩波)/MM/Garofalo/Meißner/Lui/Liu/Urbach: 2407.16659 [hep-lat]

- two/three-body force
- pion-mass dependence
- KSFR/Universality relations/… in EFT
  Gell-Mann/Sharp/Wagner/Fujiwara/Kawarabayashi… **Review**: Meißner *Phys.Rept.* 161 (1988) 213

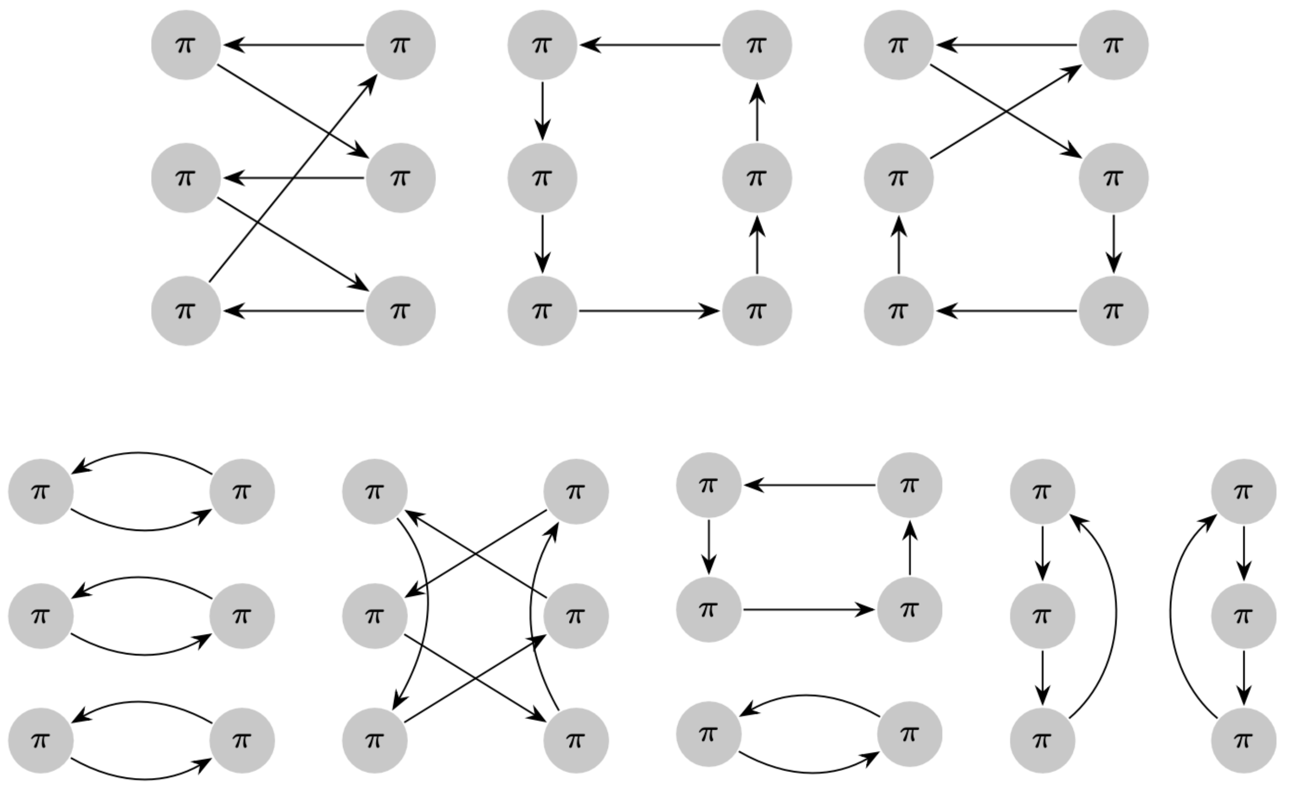


VECTOR MESON

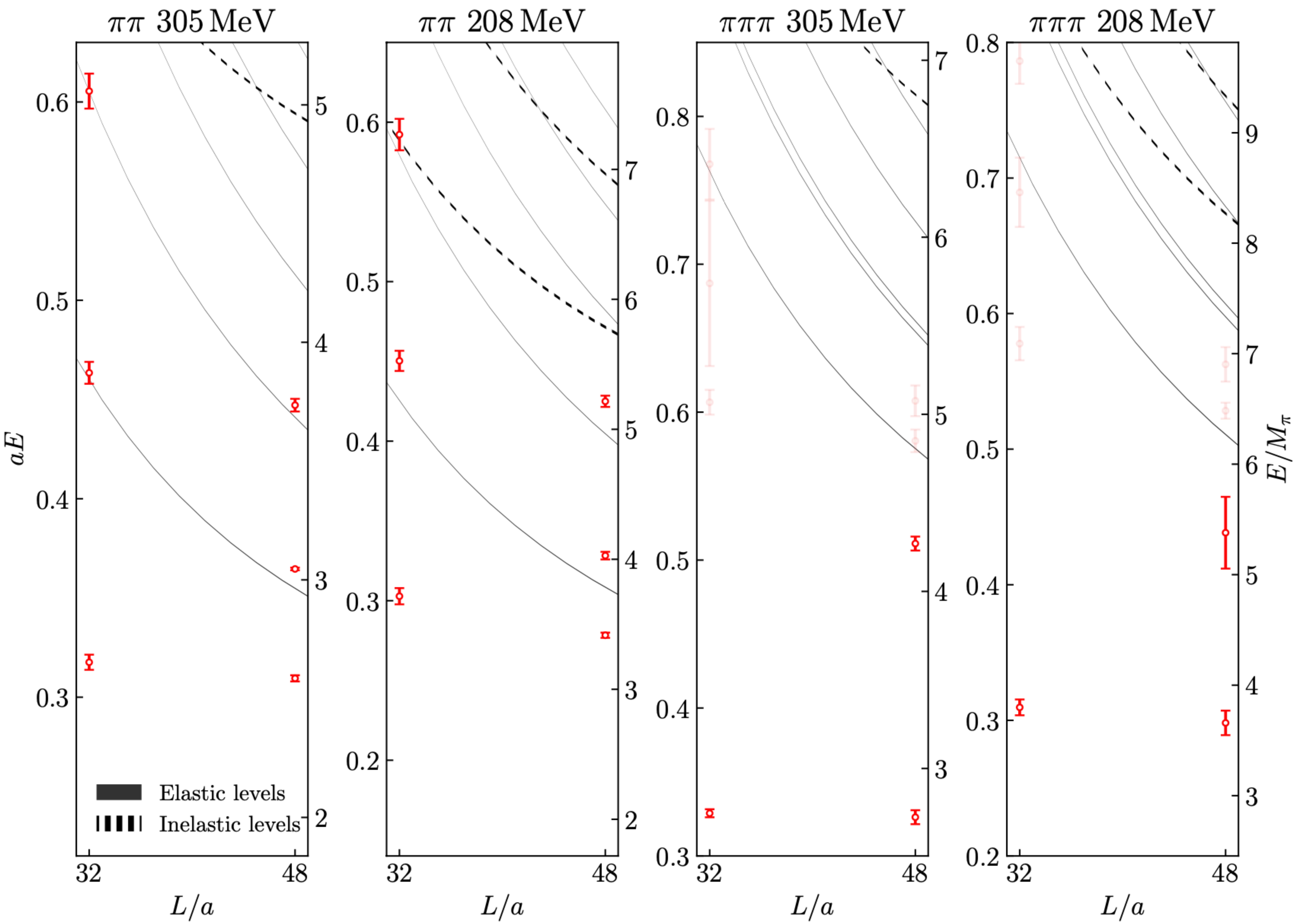
Lattice QCD setup

H.Yan/MM/Garofalo/Meißner/Lui/Liu/Urbach: 2407.16659 [hep-lat]

- $N_f = 2 + 1$ Clover fermions CLQCD, 2024
- 2/3 particle operators **OpTion** package @HaoboYan
- 2 pion masses ($\approx 210, 305$ MeV)
- 2 volumes ($L^3 = 32^3, 48^3$)



Finite-volume spectrum = Energy eigenvalues



VECTOR MESON

Mapping to infinite volume

H.Yan/MM/Garofalo/Meißner/Lui/Liu/Urbach: 2407.16659 [hep-lat]

- 3-body quantization condition

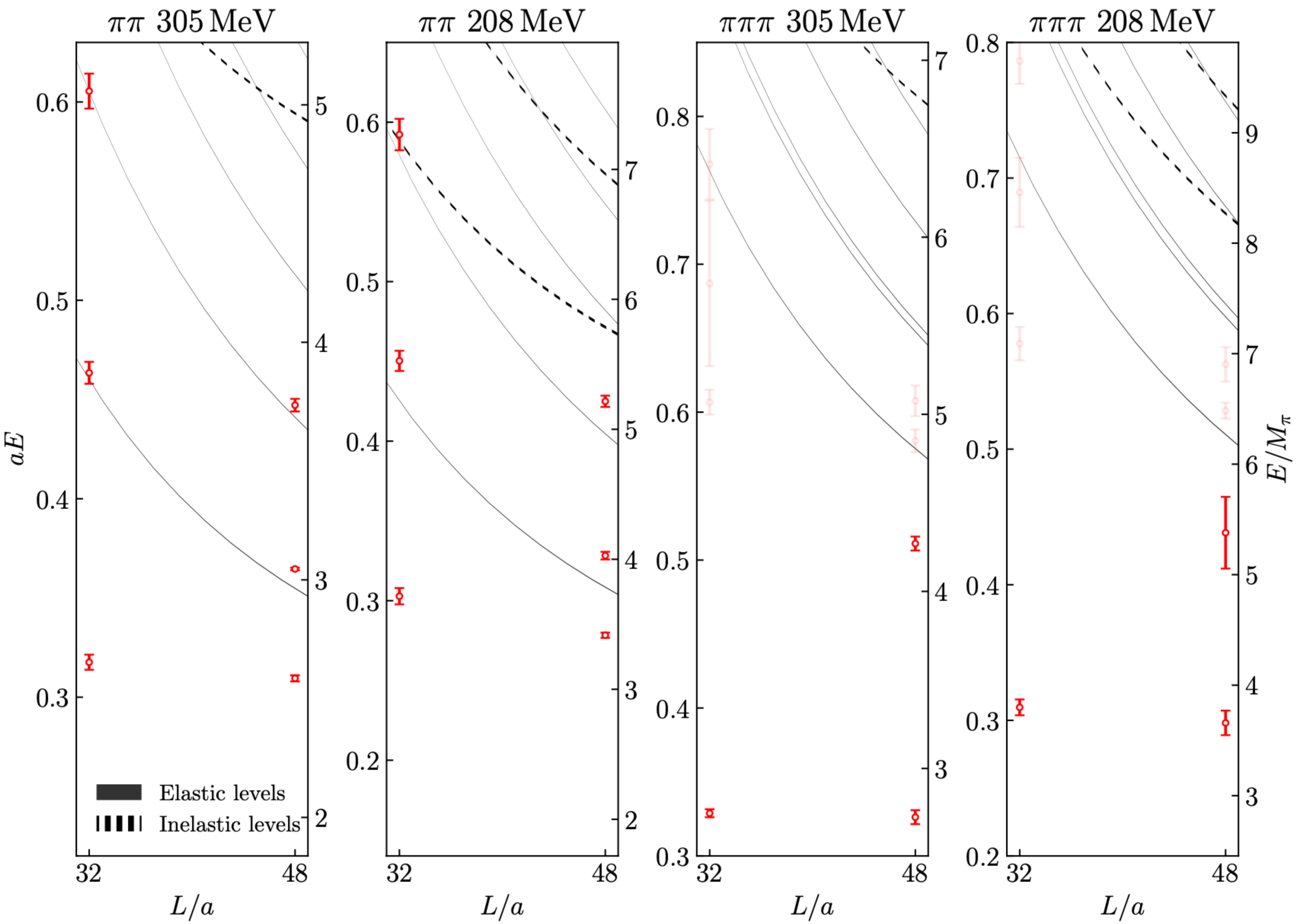
FVU

$$\det\left[\, 2L^3E_{\mathbf{p}}\left(\tilde{K}^{-1} - \Sigma^L\right) - B - C \,\right]^{\Lambda} \equiv 0$$

MM/Döring
Eur.Phys.J.A 53 (2017) 12, 240

- Volume-independent 2-, 3-body force $C, \tilde{K}$
  - **GENeric form**

Finite-volume spectrum = Energy eigenvalues



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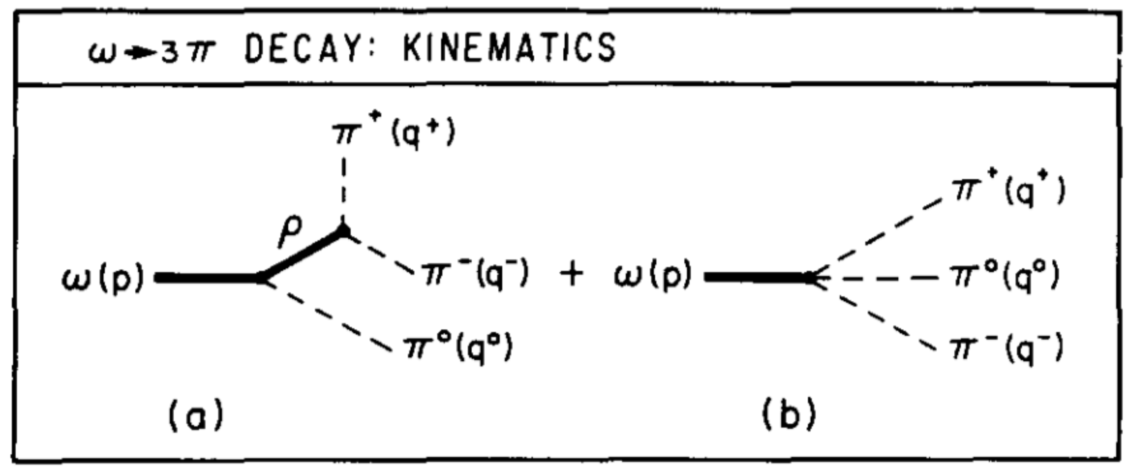
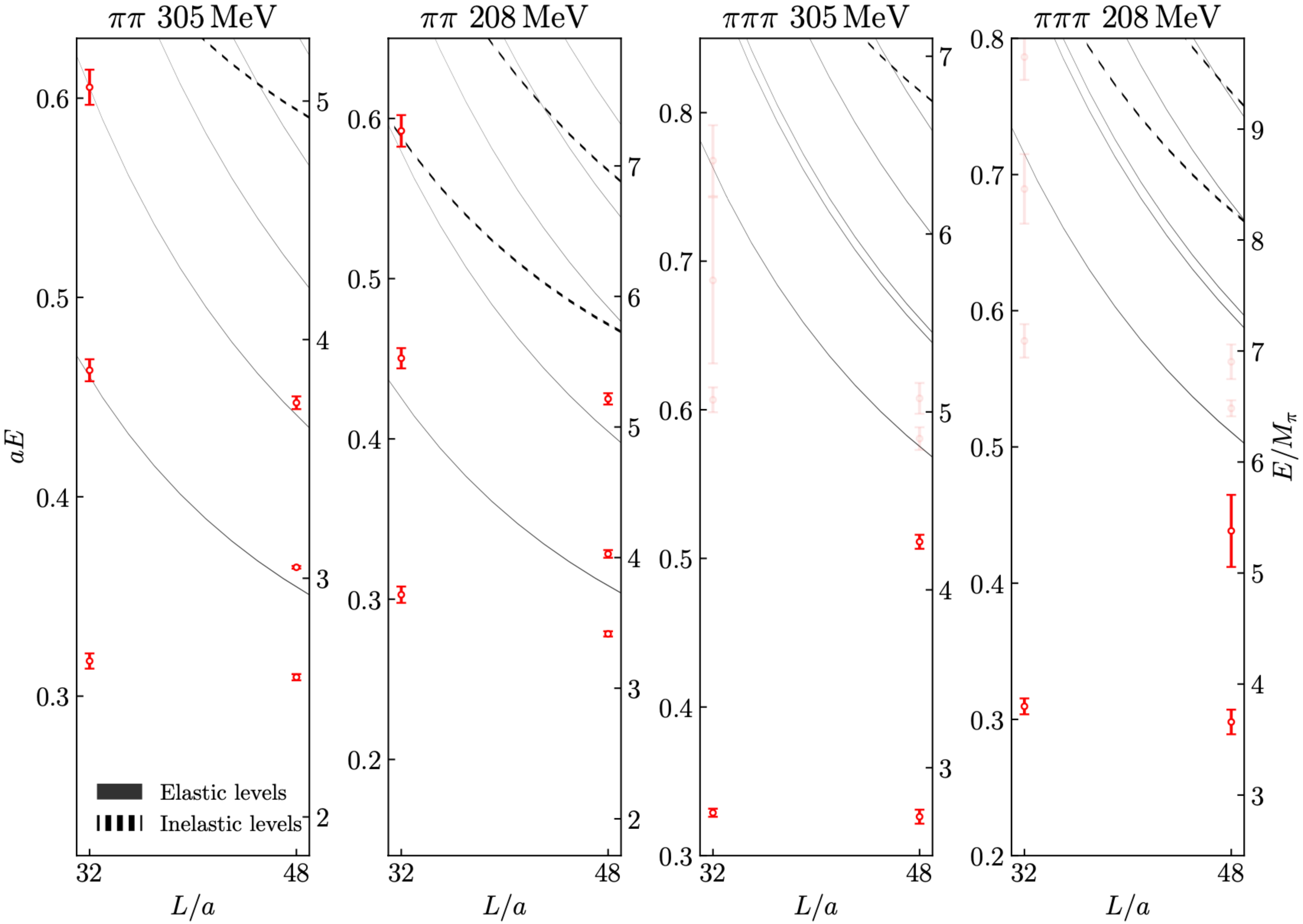
  - **EFT: saturation due to ρ, ω exchange**



Gell-Mann/Sharp/Wagner/Fujiwara/Kawarabayashi…
Review: Meißner *Phys.Rept.* 161 (1988) 213

Finite-volume spectrum = Energy eigenvalues



Maxim Mai

VECTOR MESON

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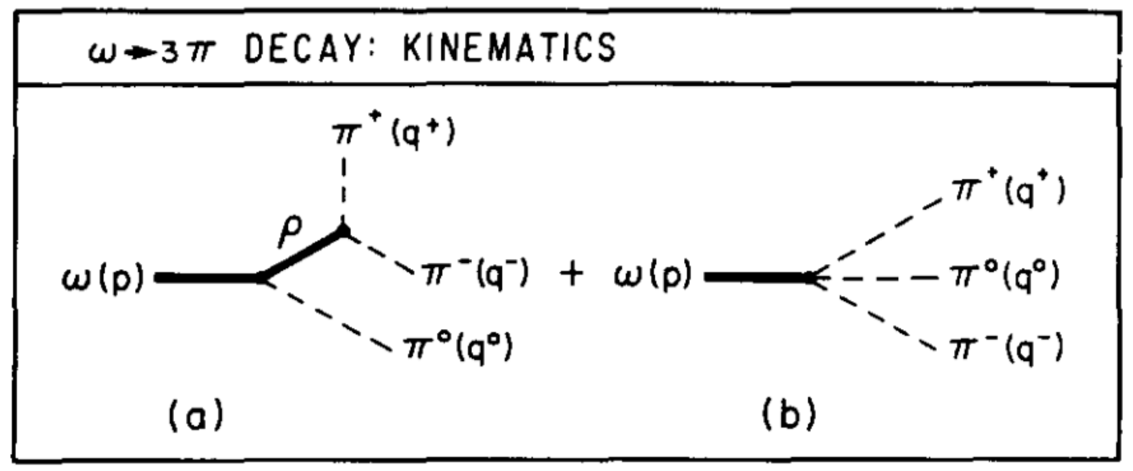

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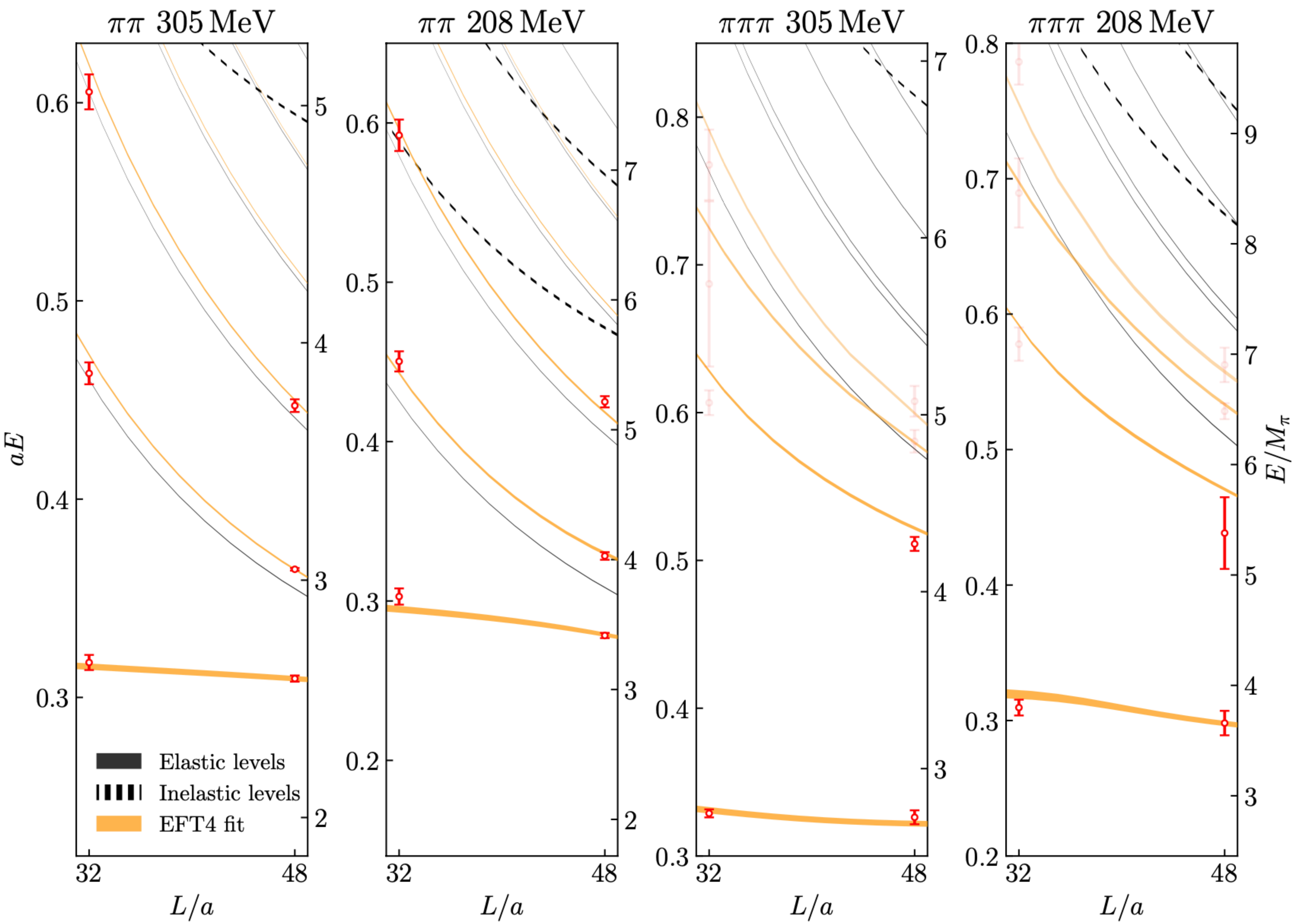

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MM/Döring
Eur.Phys.J.A 53 (2017) 12, 240

- Determine pole positions in infinite volume

IVU

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\frac{\left(B + C\right)}{2E_l}\frac{1}{\tilde{K}^{-1} - \Sigma}T^c$$

VECTOR MESON

Mapping to infinite volume

H.Yan/MM/Garofalo/Meißner/Lui/Liu/Urbach: 2407.16659 [hep-lat]

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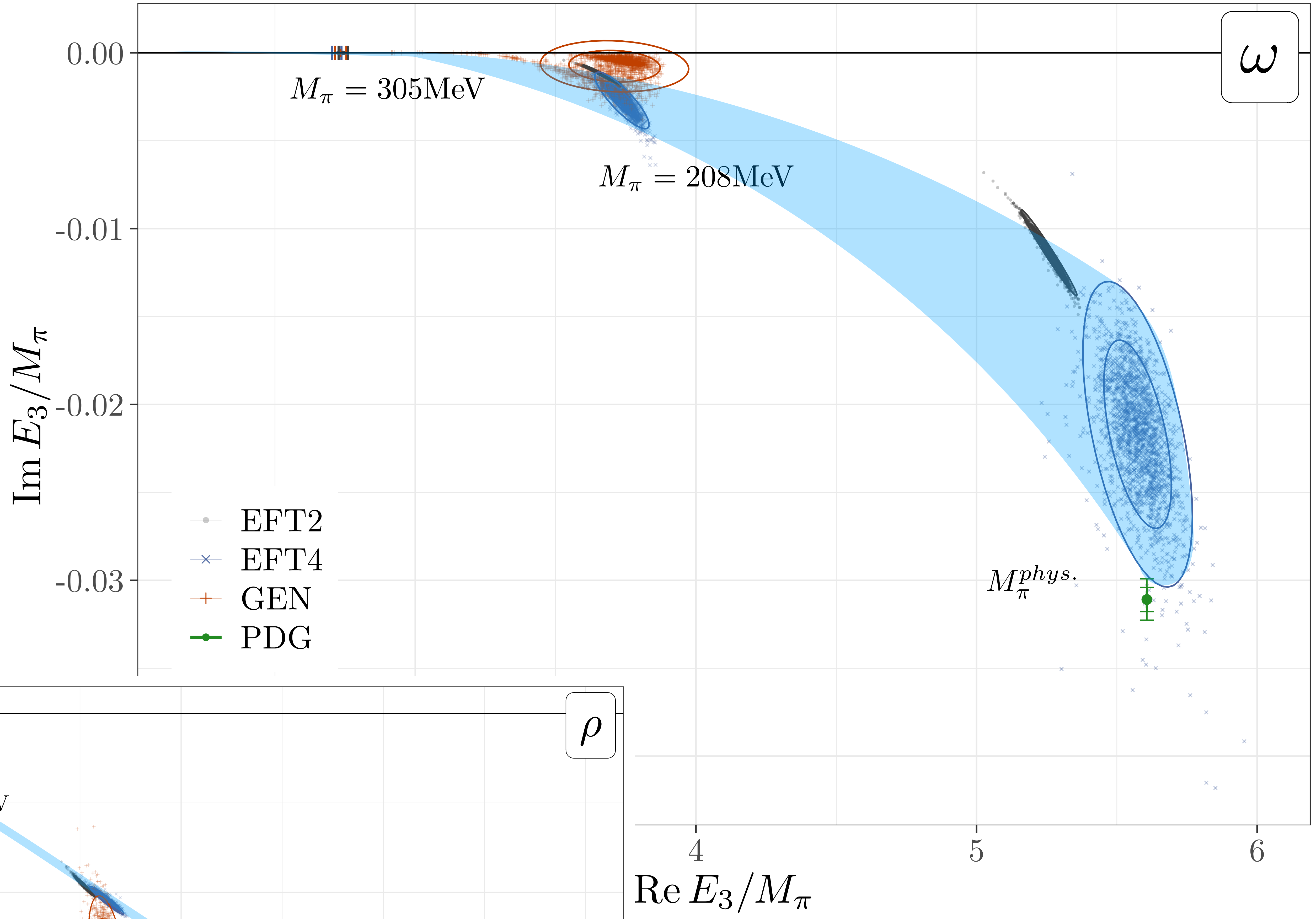
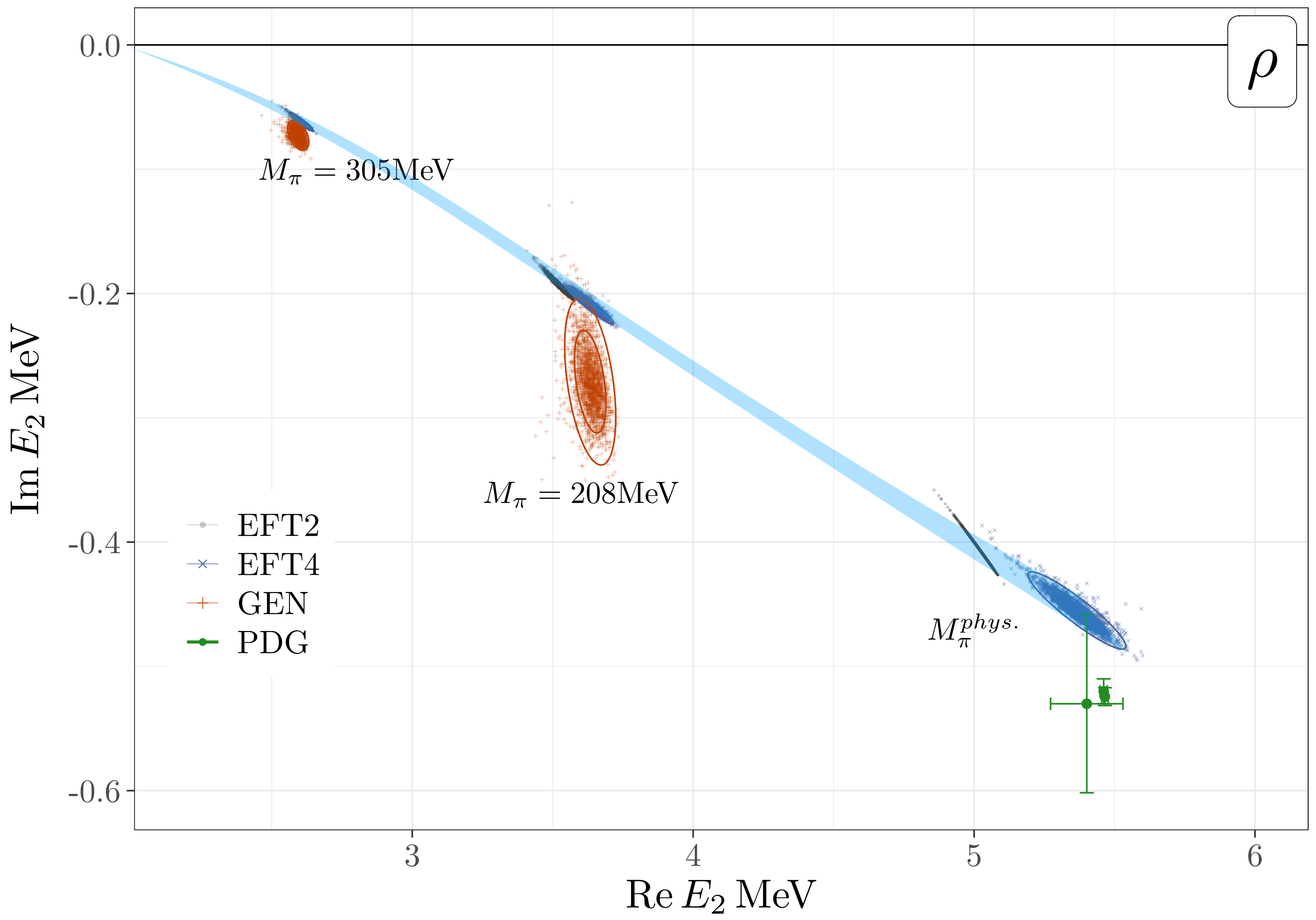




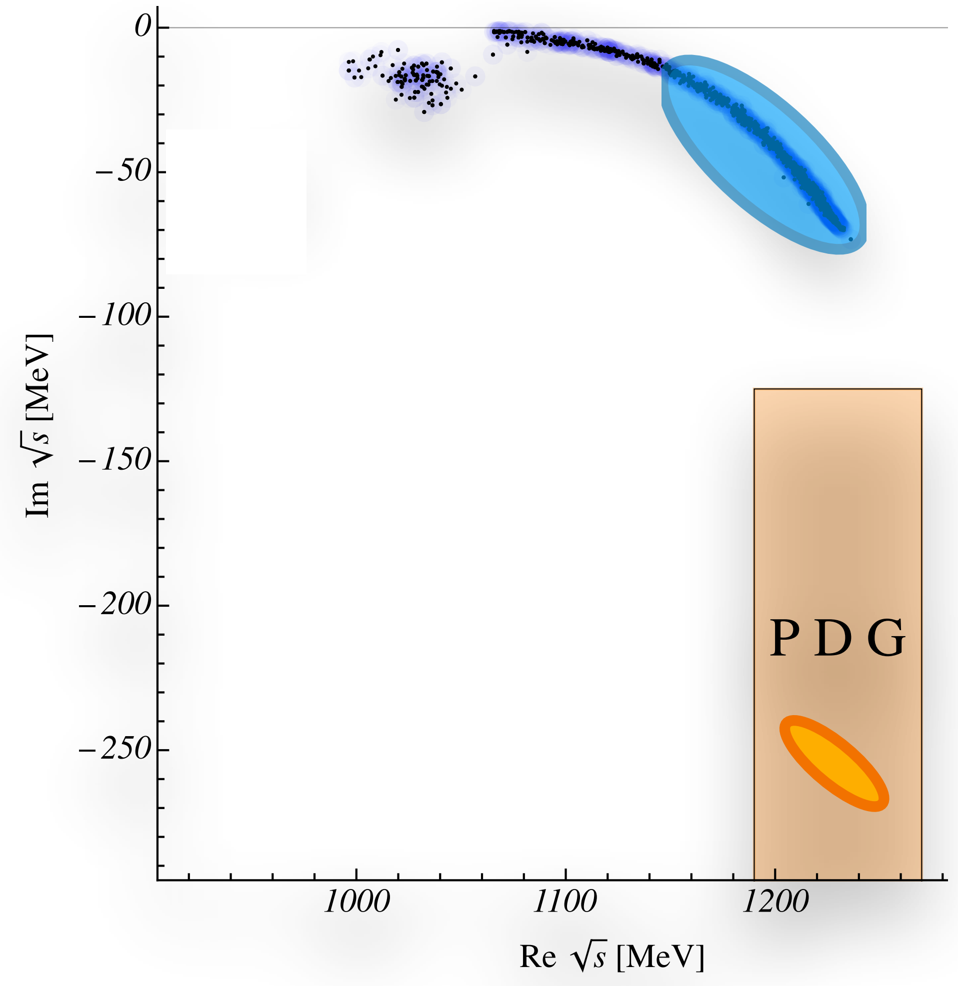
SUMMARY

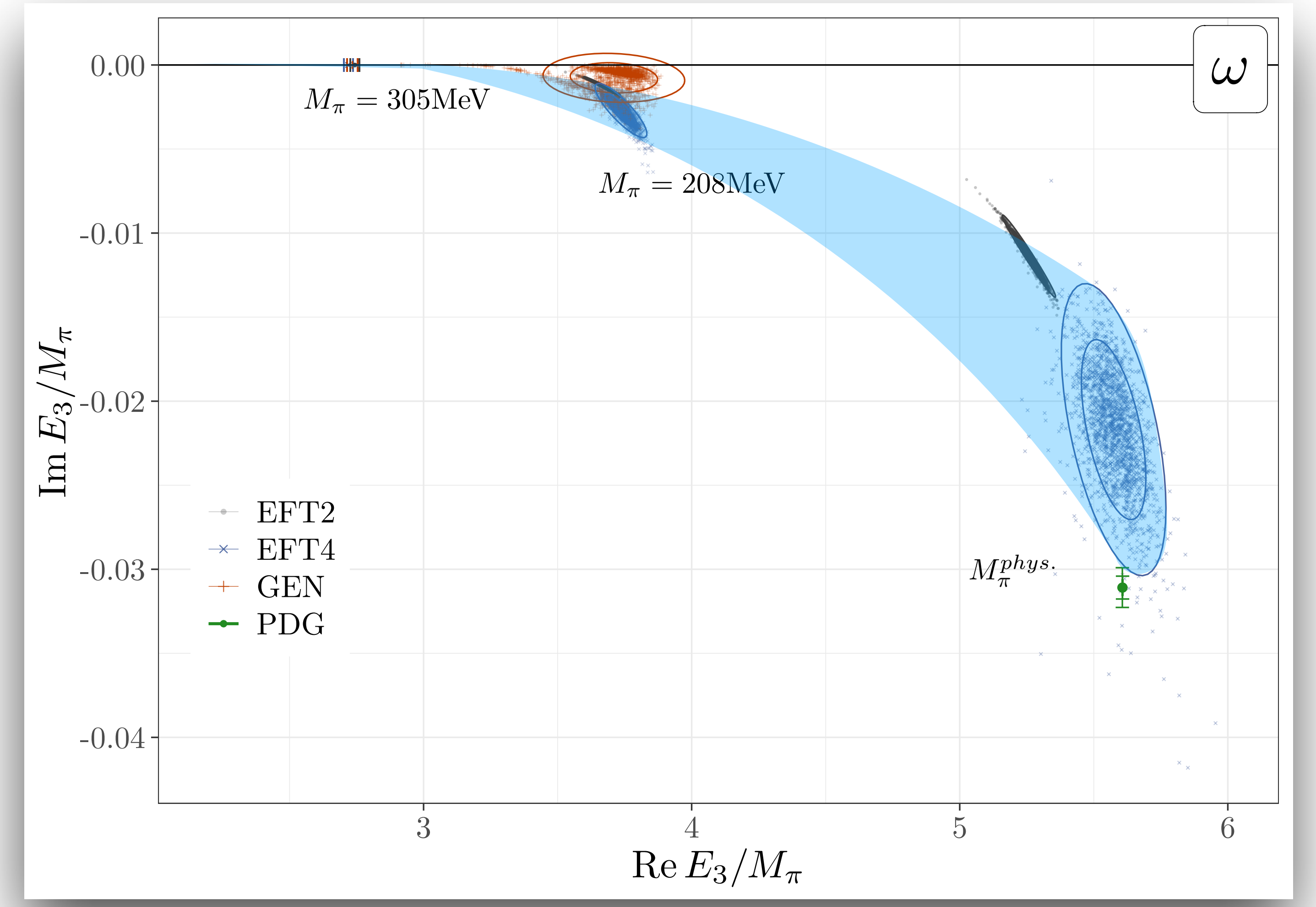
Synergetic approach to hadron spectrum

- **Lattice QCD:** ab-initio QCD calculations
- **EFTs:** quark-mass dependence, symmetries
- **(S-matrix/..):** 3-body Quantization condition

2-body systems/3-body systems

- $f_0(500), \rho(770), \ldots$ well established quark-mass dependence
- $N(1535), N(1650), \Lambda(1405), \Lambda(1380)$
- pilot results on $3\pi(I=2),\ \omega(782),\ a_1(1260)$
- chiral trajectories

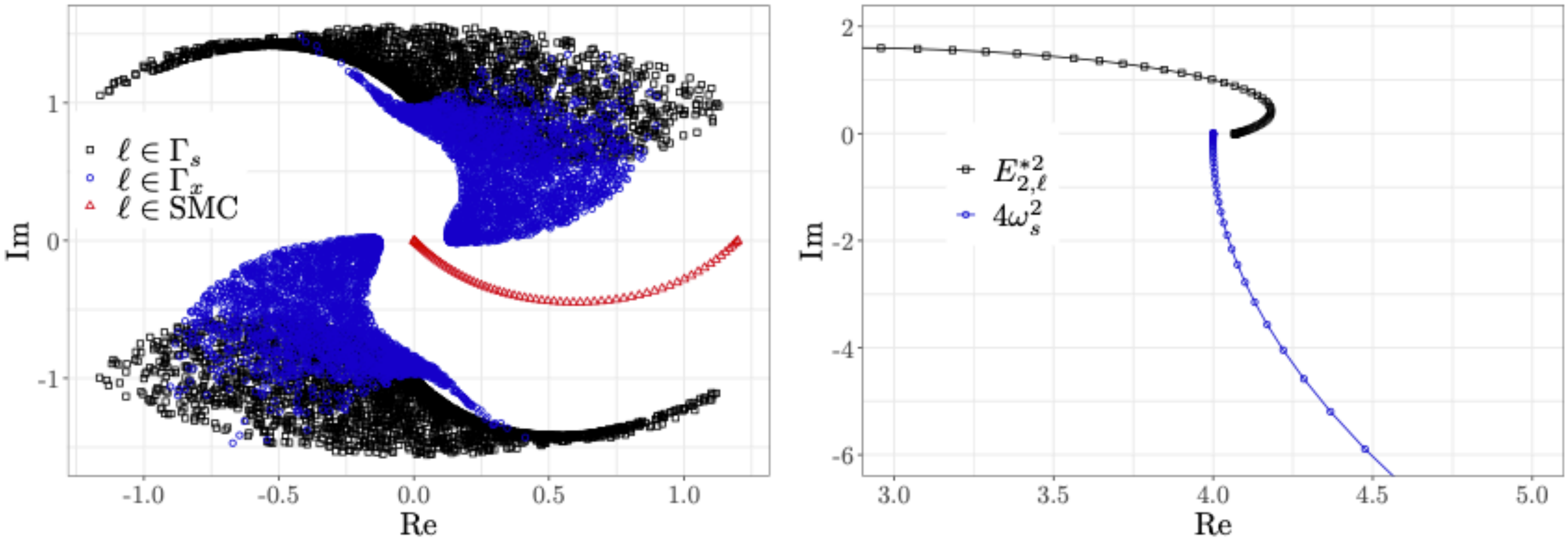




Outlook — it is just the beginning!

- $DD\pi$ – $N(1440)$ – …spin-exotics? — $a_1(1420)$
- systematic/statistics improvement
- EFT tests — Universality of $\omega \to 3\pi, \rho \to 2\pi$ coupling? — KSFR relation? …
- Cutoff treatment — Gradient flow?

谢谢



1
0
-1
-1.0
-0.5
0.0
0.5
1.0
Re
Im
$\ell \in \Gamma_s$
$\ell \in \Gamma_x$
$\ell \in \mathrm{SMC}$
2
0
-2
-4
-6
3.0
3.5
4.0
4.5
5.0
Re
Im
$E^{*2}_{2,\ell}$
$4\omega_s^2$

Current frontier: 3-body dynamics from LQCD

- ➥ 3-body Quantization Conditions[1]
  - ➥ RFT / FVU / NREFT
    - ➥ many perturbatively interacting systems are studied[2]

$$0 = \det\left(L^3\left(\tilde F/3 - \tilde F(\tilde K_2^{-1} + \tilde F + \tilde G)^{-1}\tilde F\right)^{-1} + K_{\mathrm{df},3}\right)$$ RFT

$$0 = \det\left(B_0 + C_0 - E_L\left(K^{-1}/(32\pi) + \Sigma_L\right)\right)$$ FVU

3-body force

2-body interaction

one-particle exchange

2-body self-energy

1) Rusetsky, Bedaque, Grießhammer, Sharpe, Meißner, Döring, Hansen, Davoudi, Guo....
Reviews:
Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019);
MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);
2) MM/Döring PRL122(2019); Blanton et al. PRL 124 (2020); Hansen et al. PRL 126 (2021); ....

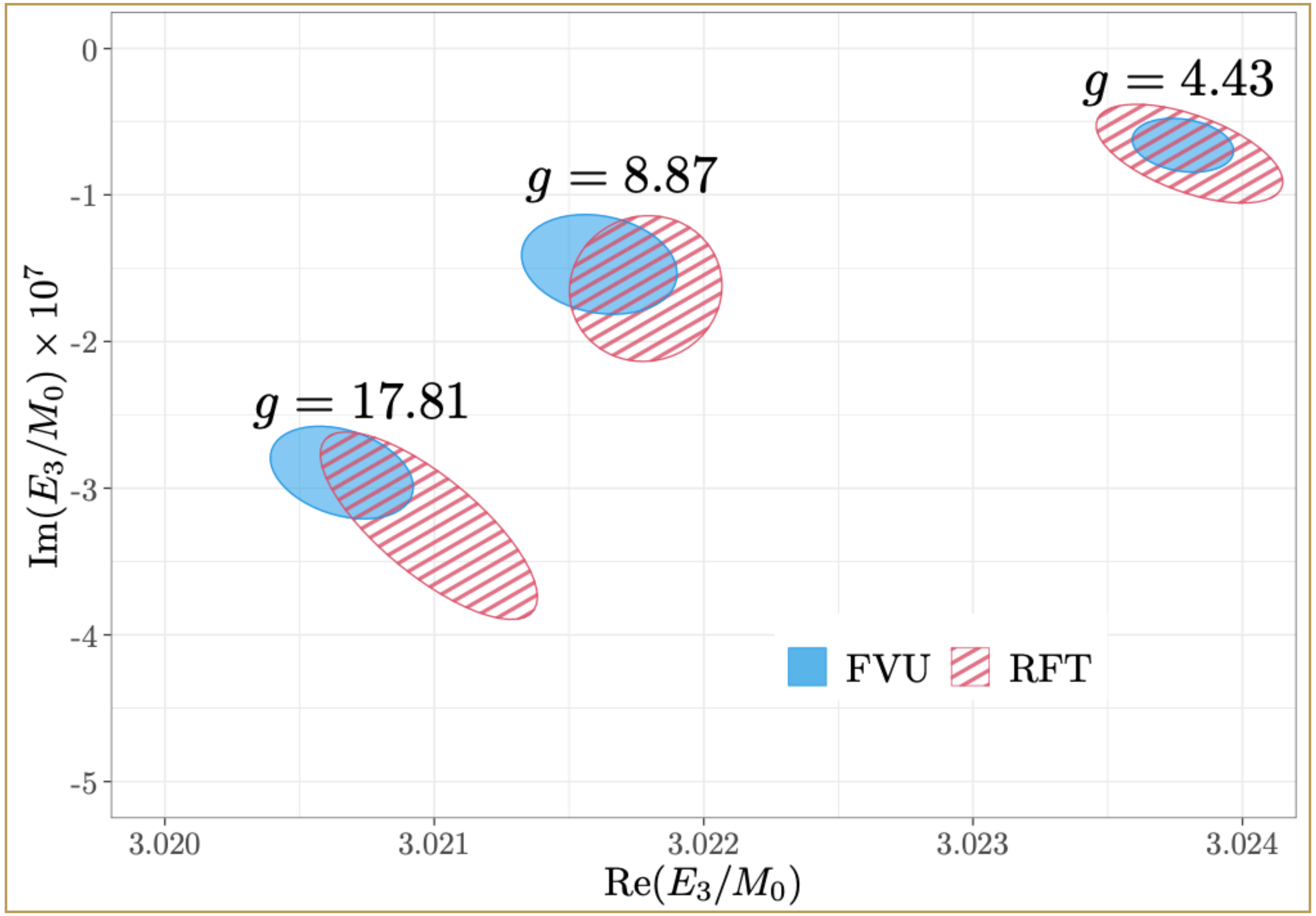
Pole positions

- FVU: complex energy-plane analysis[1]

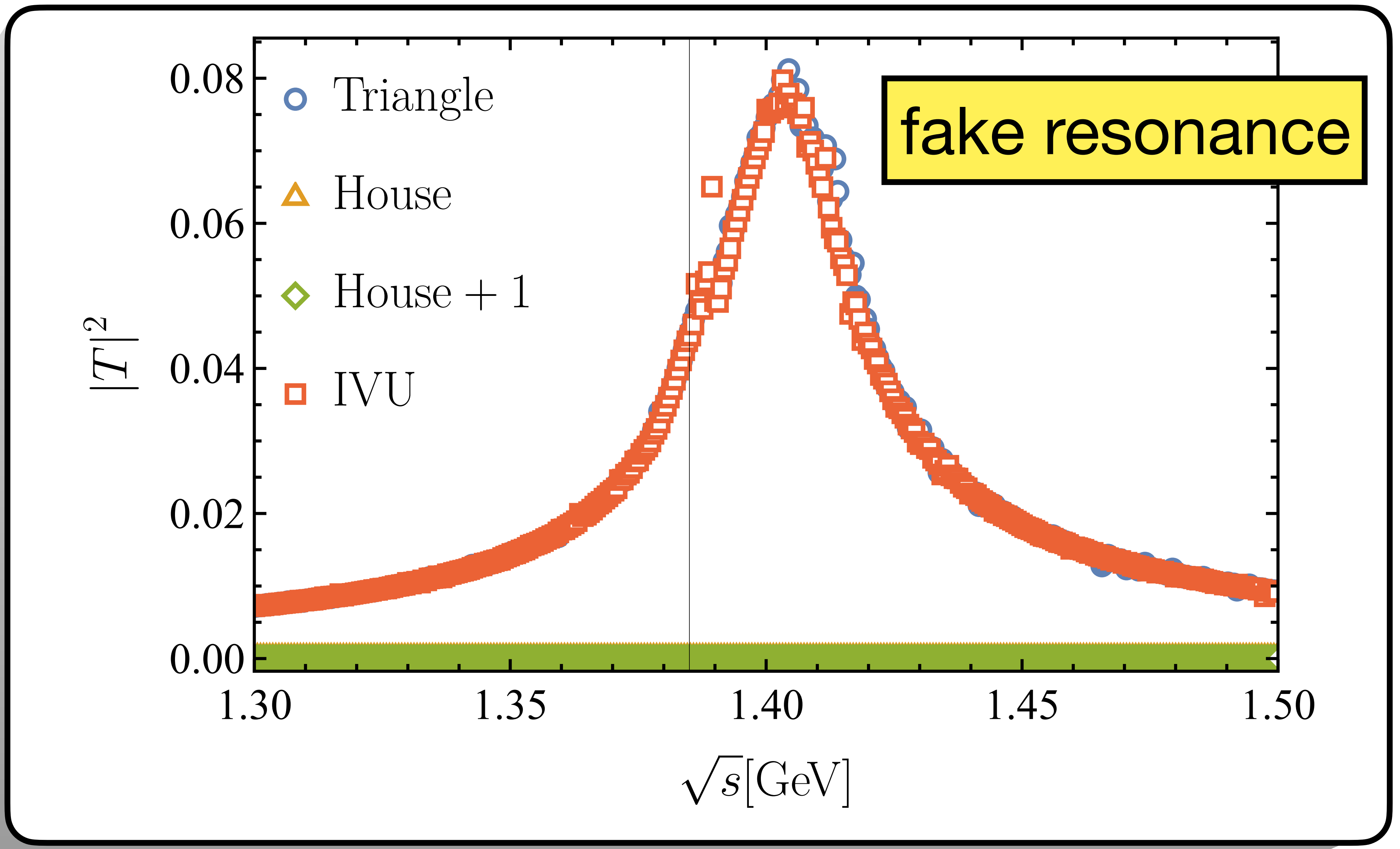
  -- resonance width grows ~ ***g²***

  -- avoided level crossing gap >> width

- Similarly from RFT with Breit-Wigner like approximation

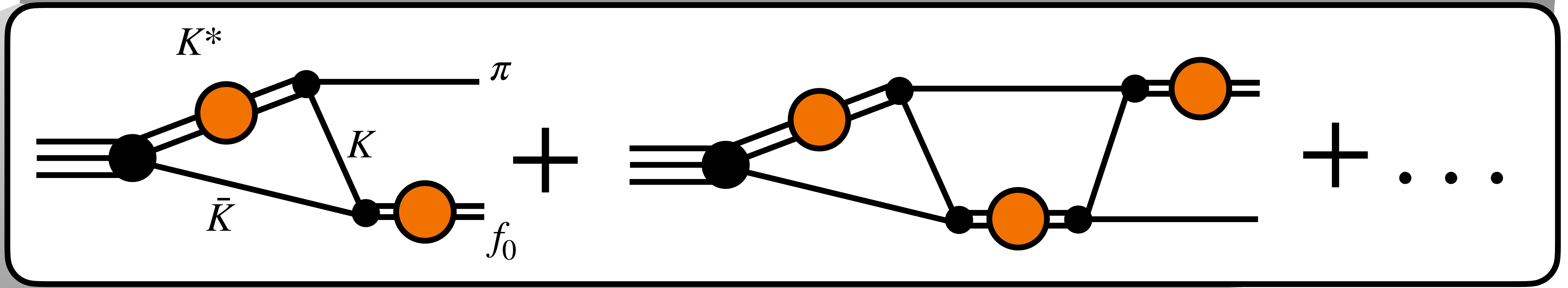


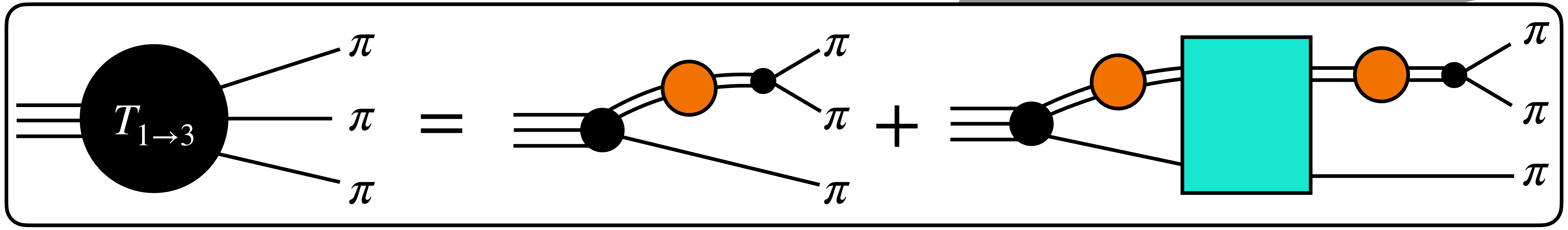
HILBERT'S HOTEL



"Infinite Volume Unitarity" — IVU formalism

- OPE: non-trivial analytic structure[1]
- Landau singularities
- Triangles[2] + Boxes+Boxes+…[3]





[1] Korpa/Lutz/Guo/Heo *Phys.Rev.D* 107 (2023) 3; Isken et al. 2309.09695; …
[2] Ketzer/Aceti/Dai/Oset/Mikhashenko/Bayar/Guo…
[3] Sakhtivasan/MM in preparation