Nuclear Lattice Effective Field Theory

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Facility for Rare Isotope Beams
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Nuclear Lattice EFT Collaboration

The 23rd International Conference on Few-Body Problems in Physics
Beijing, China
September 24, 2024

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NUCLEI
Nuclear Computational Low-Energy Initiative
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LEADERSHIP COMPUTING FACILITY

Other talks related to lattice effective field theory

Zhengyu Zhang
Three-body bound state of $DD^*\bar{K}$ system in lattice effective field theory
Tuesday, September 24, 3:50 PM, China Hall 2

Bing-Nan Lu
Nuclear binding energies from a lattice regulated chiral EFT
Wednesday, September 25, 12:00 noon, China Hall 3

Myungkuk Kim
Neutron Dripline with Nuclear Lattice EFT
Thursday, September 26, 11:50 AM, Han Hall

Hang Yu
Nucleons in a finite volume: from ground states to the continuum
Thursday, September 26, 12:10 PM, Han Hall

Shihang Shen
Structure of Low-Lying States in Carbon-12 Using Nuclear Lattice Effective Field Theory
Thursday, September 26, 5:30 PM, China Hall 3

Outline

Lattice effective field theory

Essential elements of nuclear binding

Pinhole algorithm

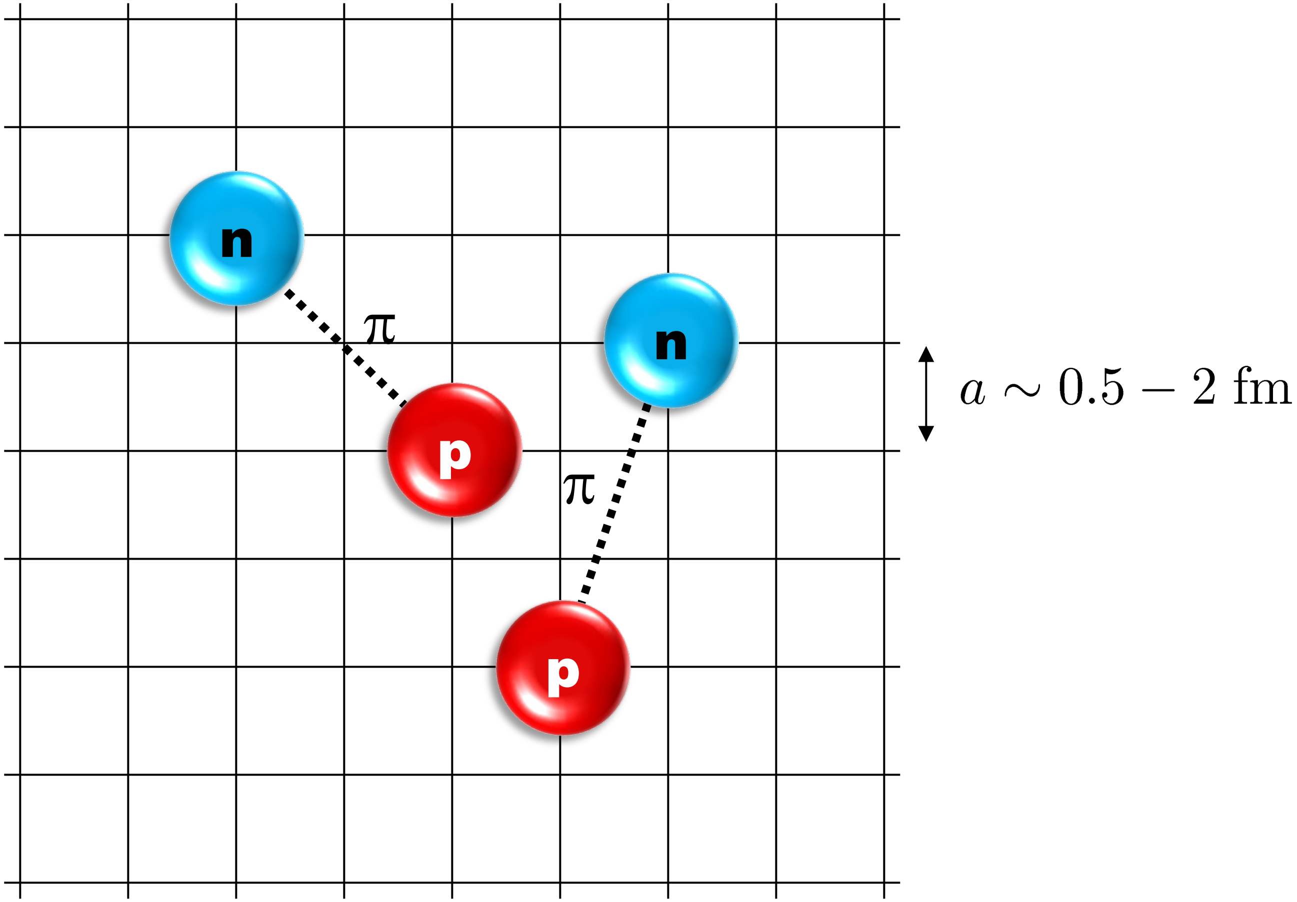
Emergent geometry and duality of ^{12}C

Wavefunction matching

Superfluidity

Summary and outlook

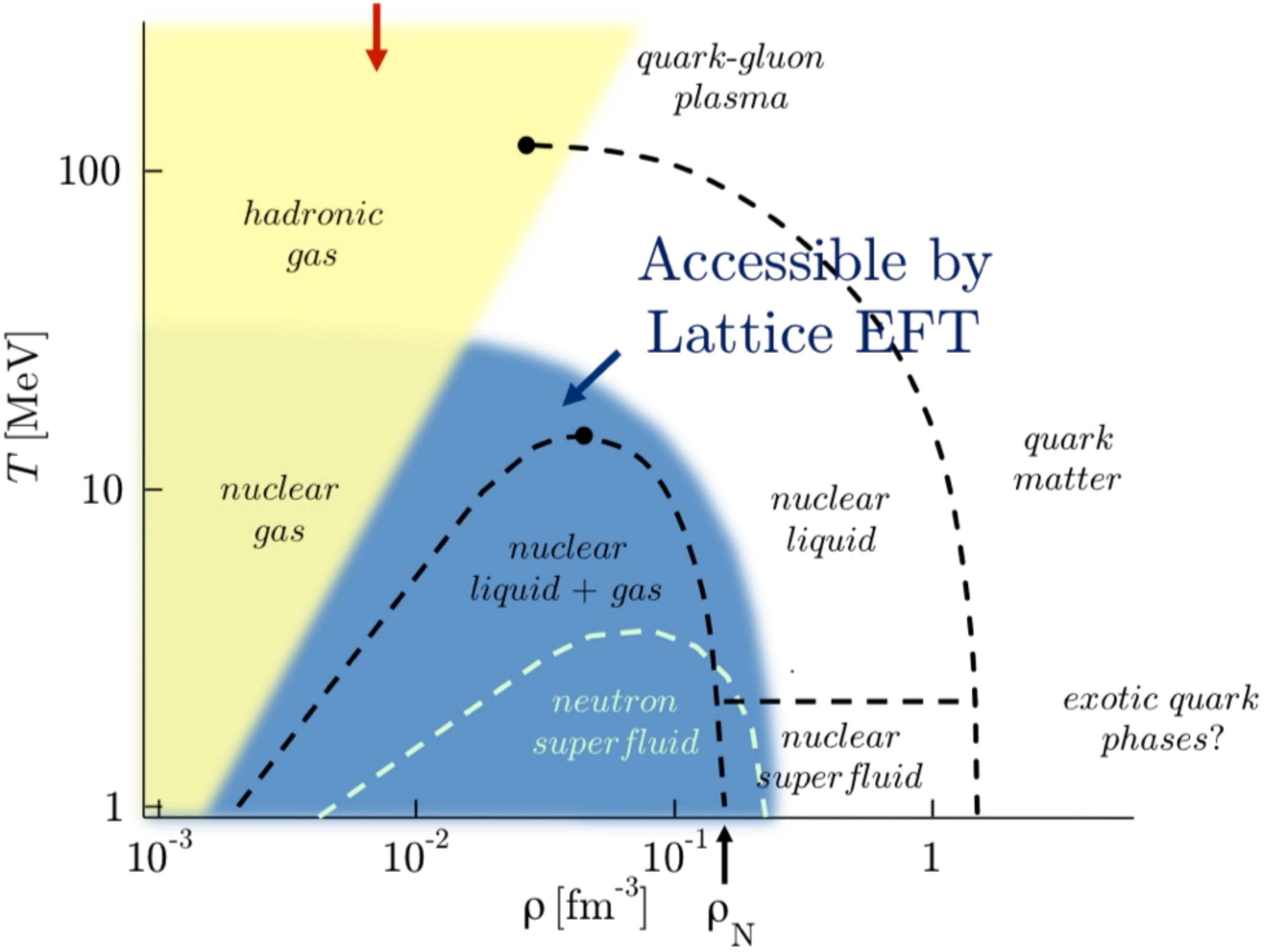
Lattice effective field theory



D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)

Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer

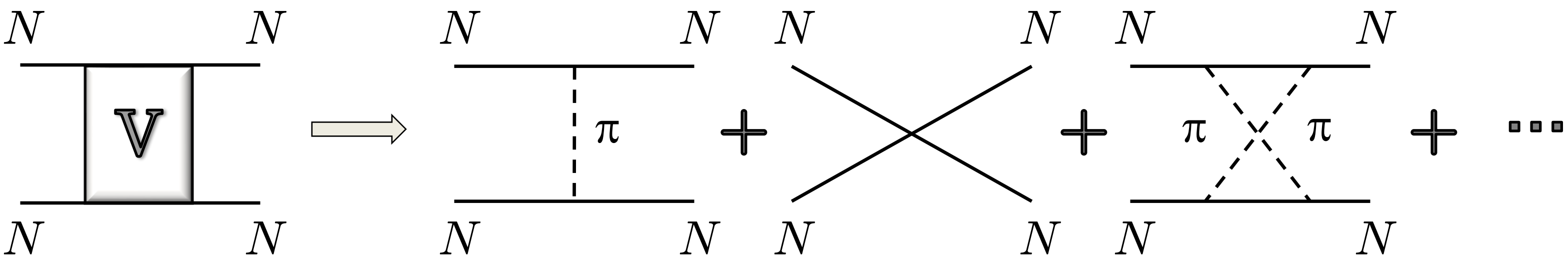
Accessible by
Lattice QCD



Accessible by
Lattice EFT

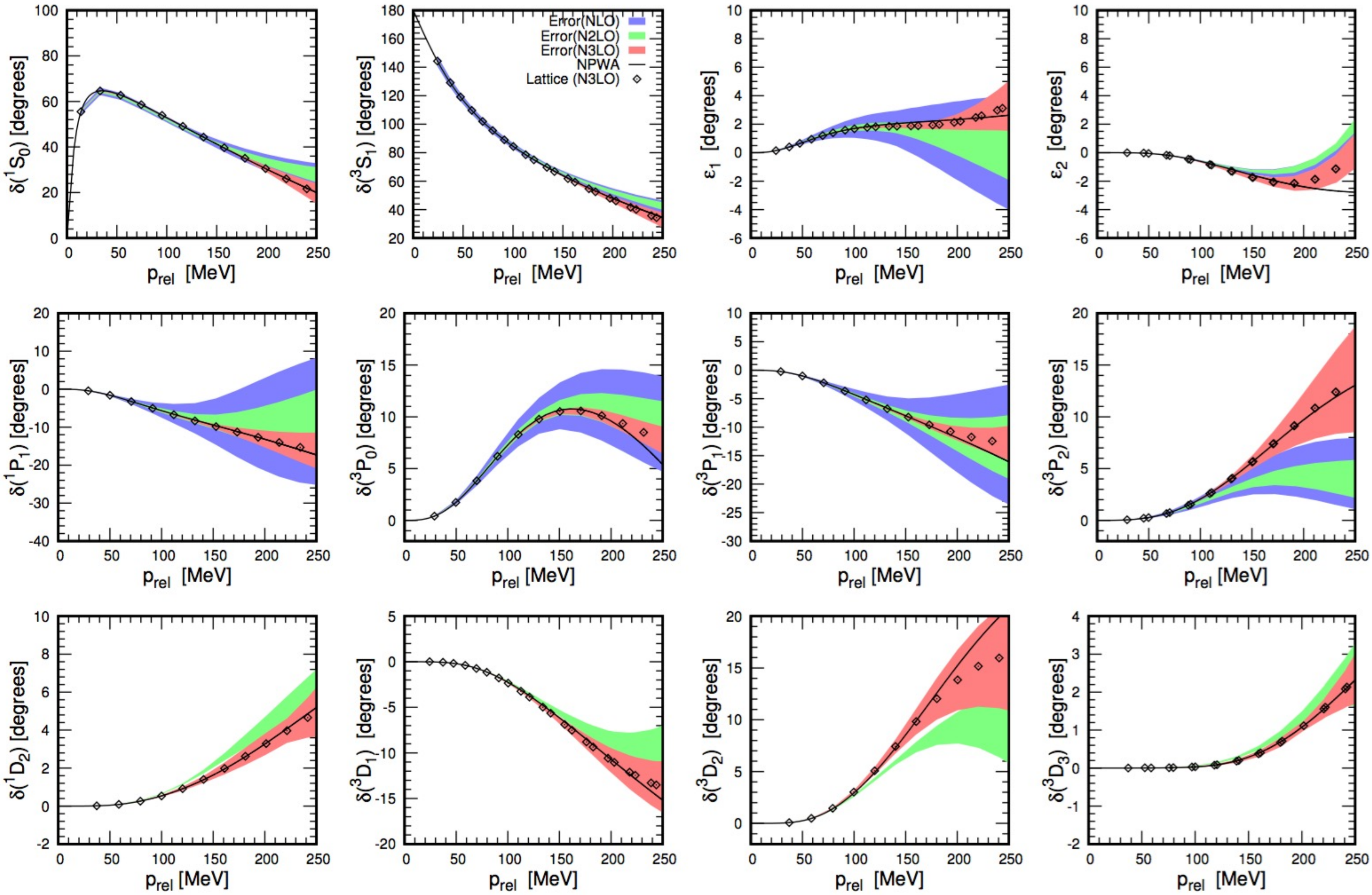
Chiral effective field theory

Construct the effective potential order by order



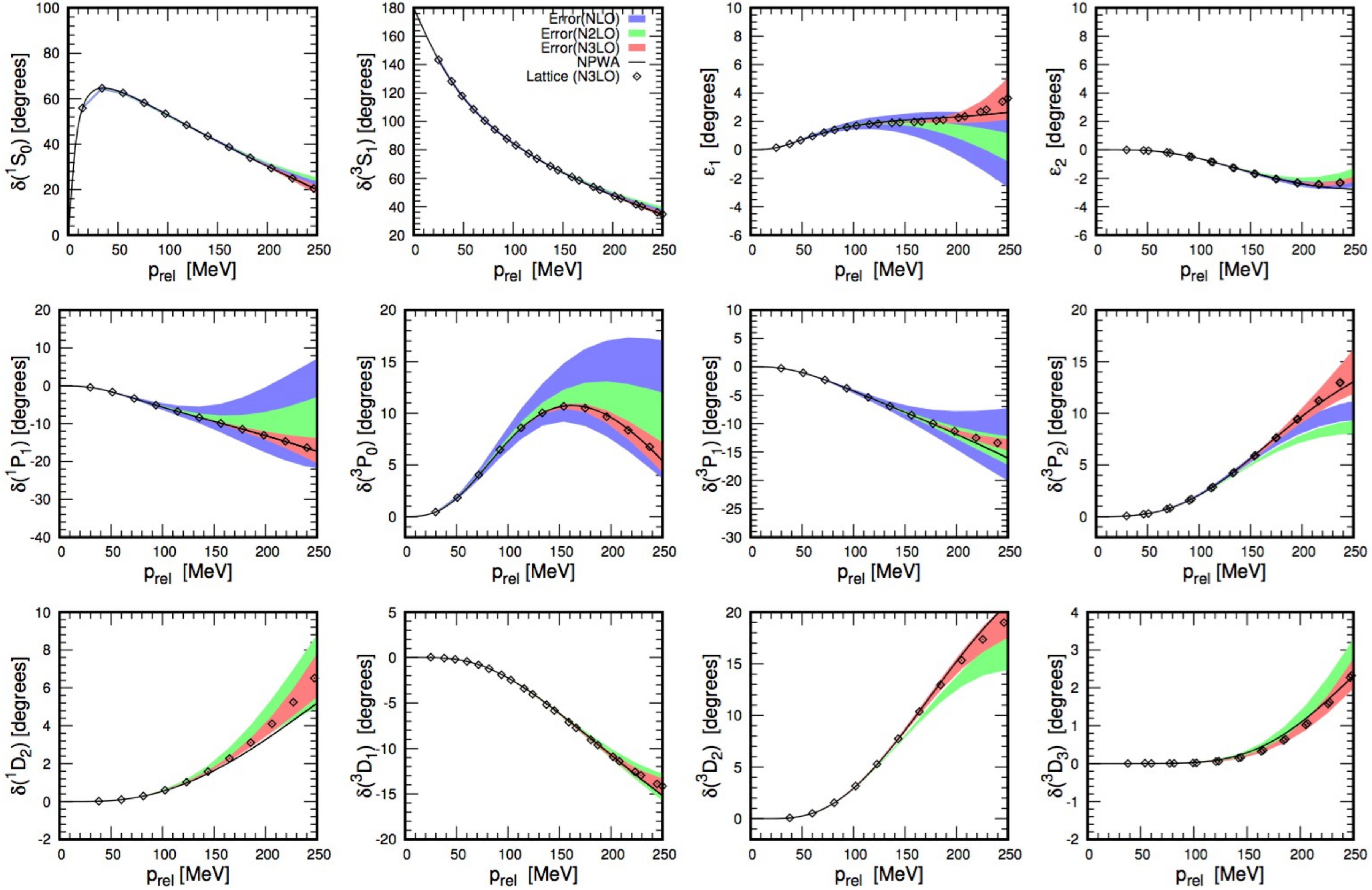
$$\underbrace{V^{\mathrm{OPEP}}\ \text{Contact interactions}}_{\text{Leading order (LO)}}\quad \underbrace{V^{\mathrm{TPEP}}}_{\text{Next-to-leading order (NLO)}}$$

$a = 1.315\,\mathrm{fm}$

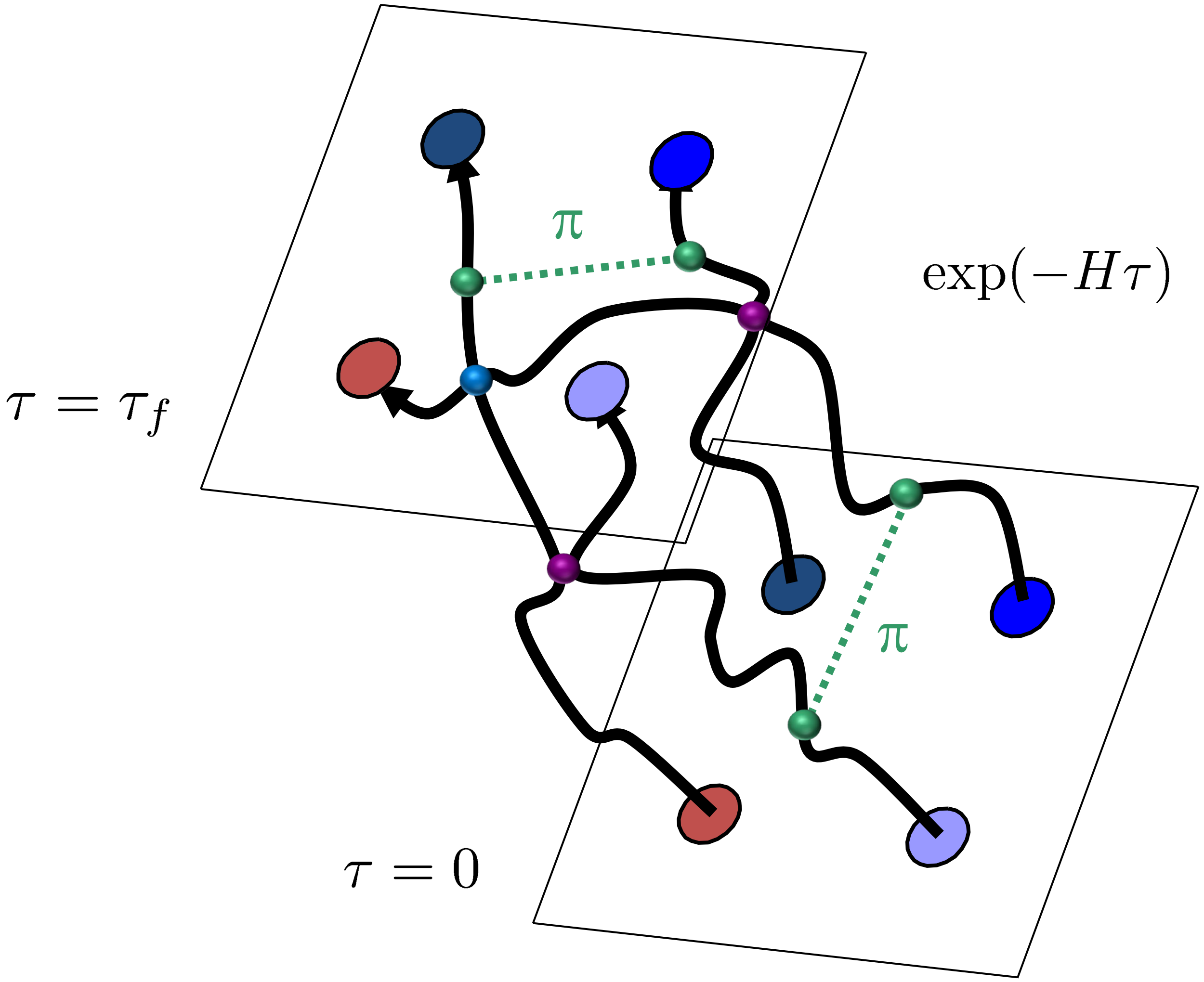


Li, Elhatisari, Epelbaum, D.L., Lu, Meißner, PRC 98, 044002 (2018)

$$a = 0.987\,\mathrm{fm}$$



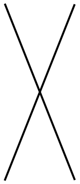
Euclidean time projection



Auxiliary field method

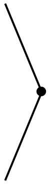
We can write exponentials of the interaction using a Gaussian integral identity

$$\exp\left[-\frac{C}{2}(N^\dagger N)^2\right]$$



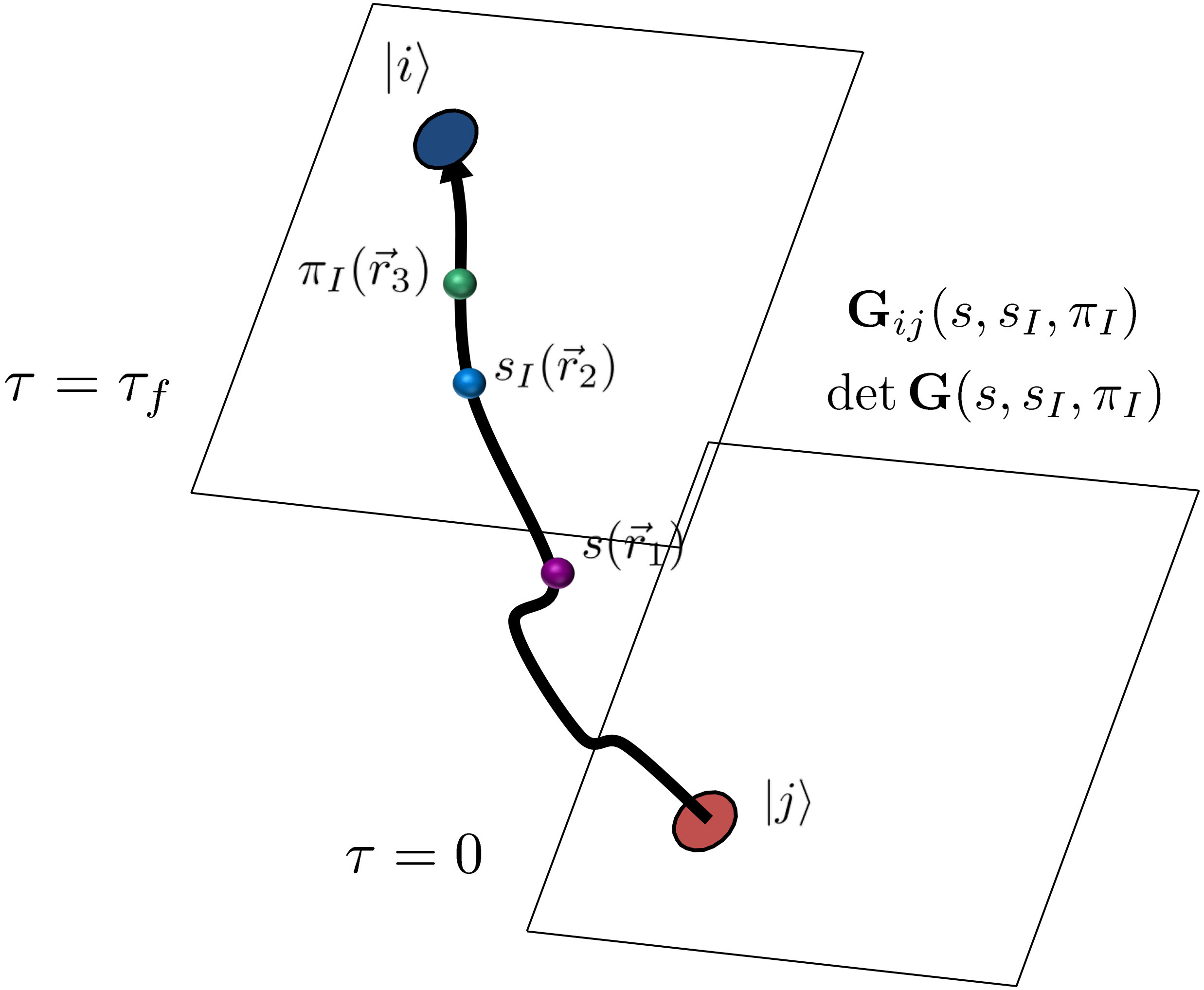
$$(N^\dagger N)^2$$

$$= \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} ds\, \exp\left[-\frac{1}{2}s^2 + \sqrt{-C}\, s(N^\dagger N)\right]$$



$$sN^\dagger N$$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.

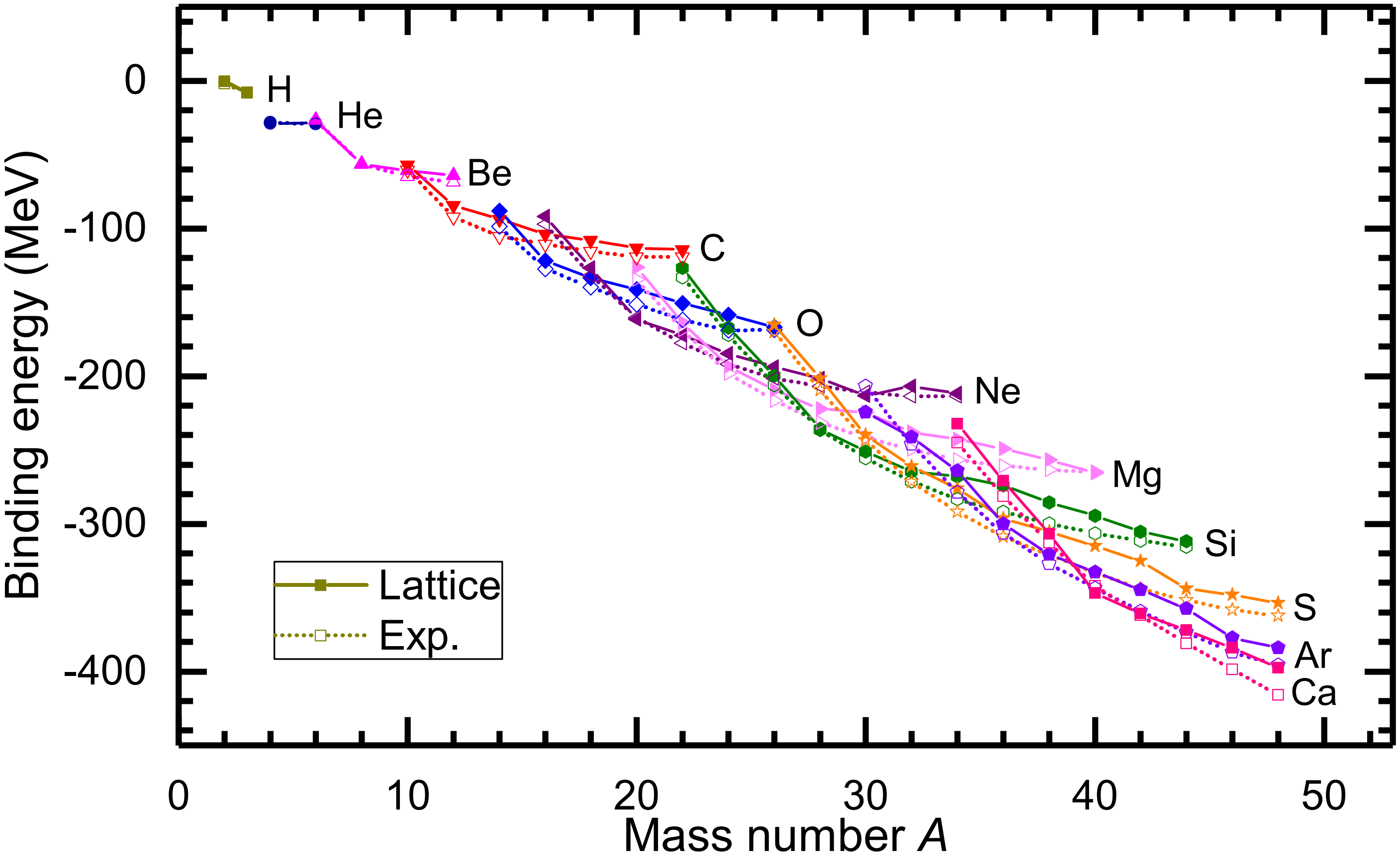


$\mathbf{G}_{ij}(s, s_I, \pi_I)$

$\det \mathbf{G}(s, s_I, \pi_I)$

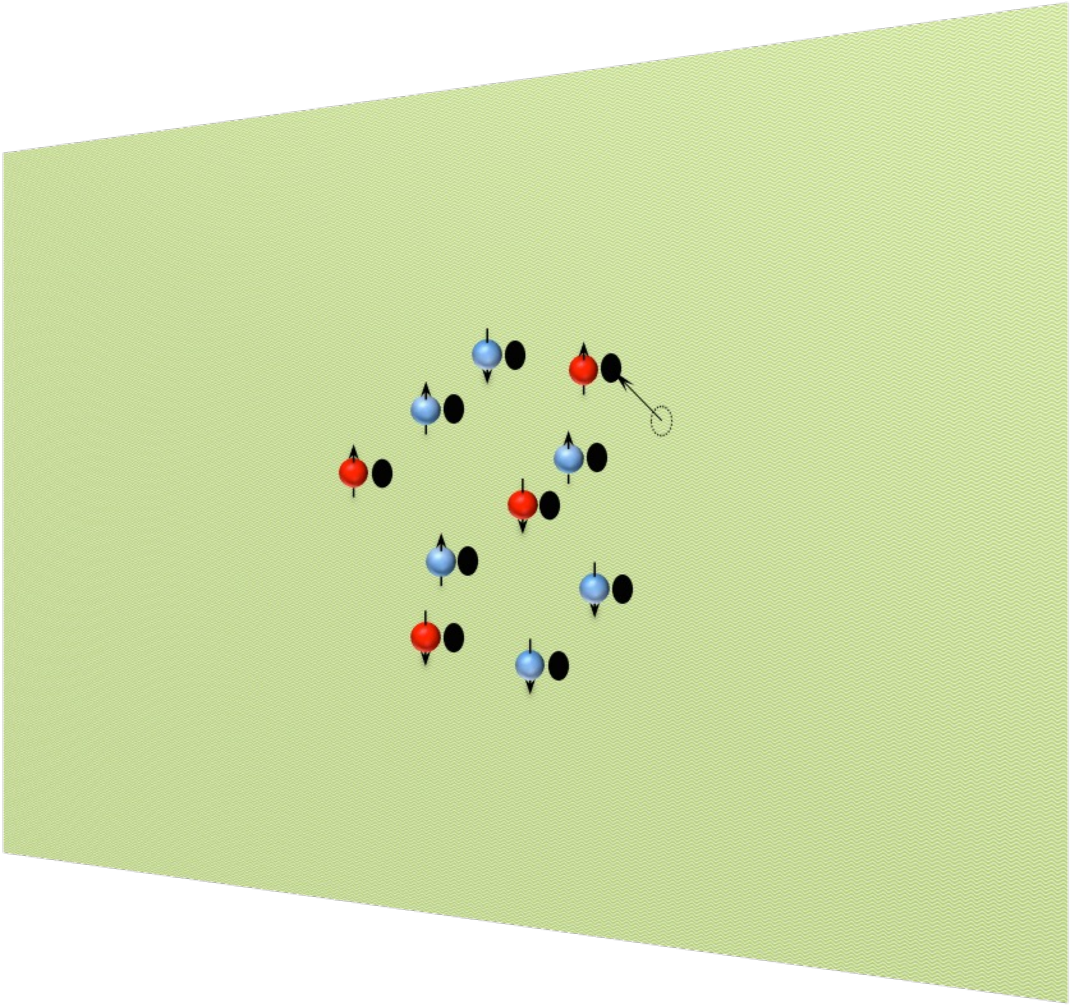
Essential elements for nuclear binding

$$H = H_{\rm free} + \frac{1}{2!}C_2 \sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^2 + \frac{1}{3!}C_3 \sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^3 + V_{\rm Coulomb}$$

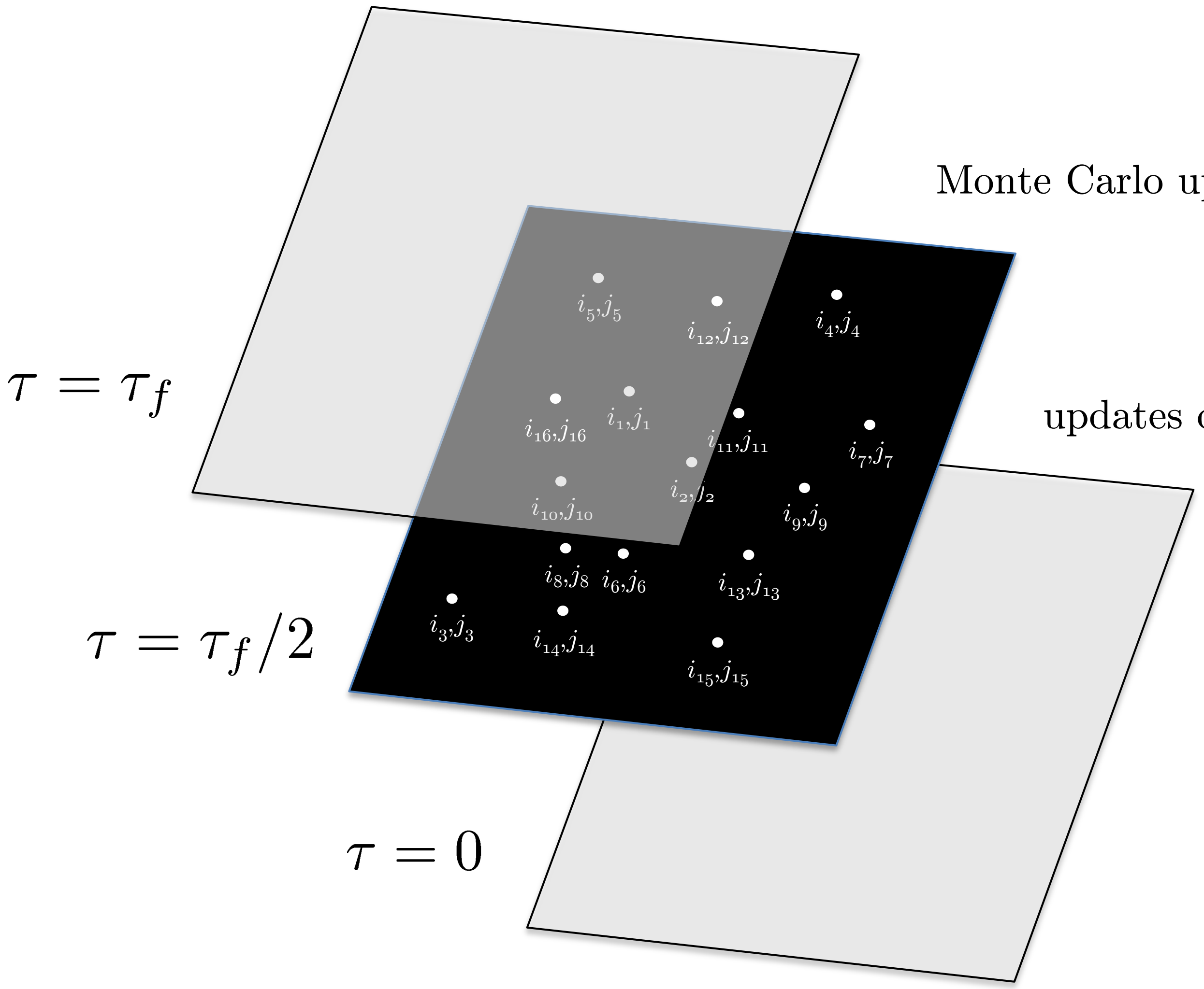


Lu, Li, Elhatisari, D.L., Epelbaum, Meißner, PLB 797, 134863 (2019)

Pinhole algorithm



Elhatisari, Epelbaum, Krebs, Lähde, D.L., Li, Lu, Meißner, Rupak, PRL 119, 222505 (2017)

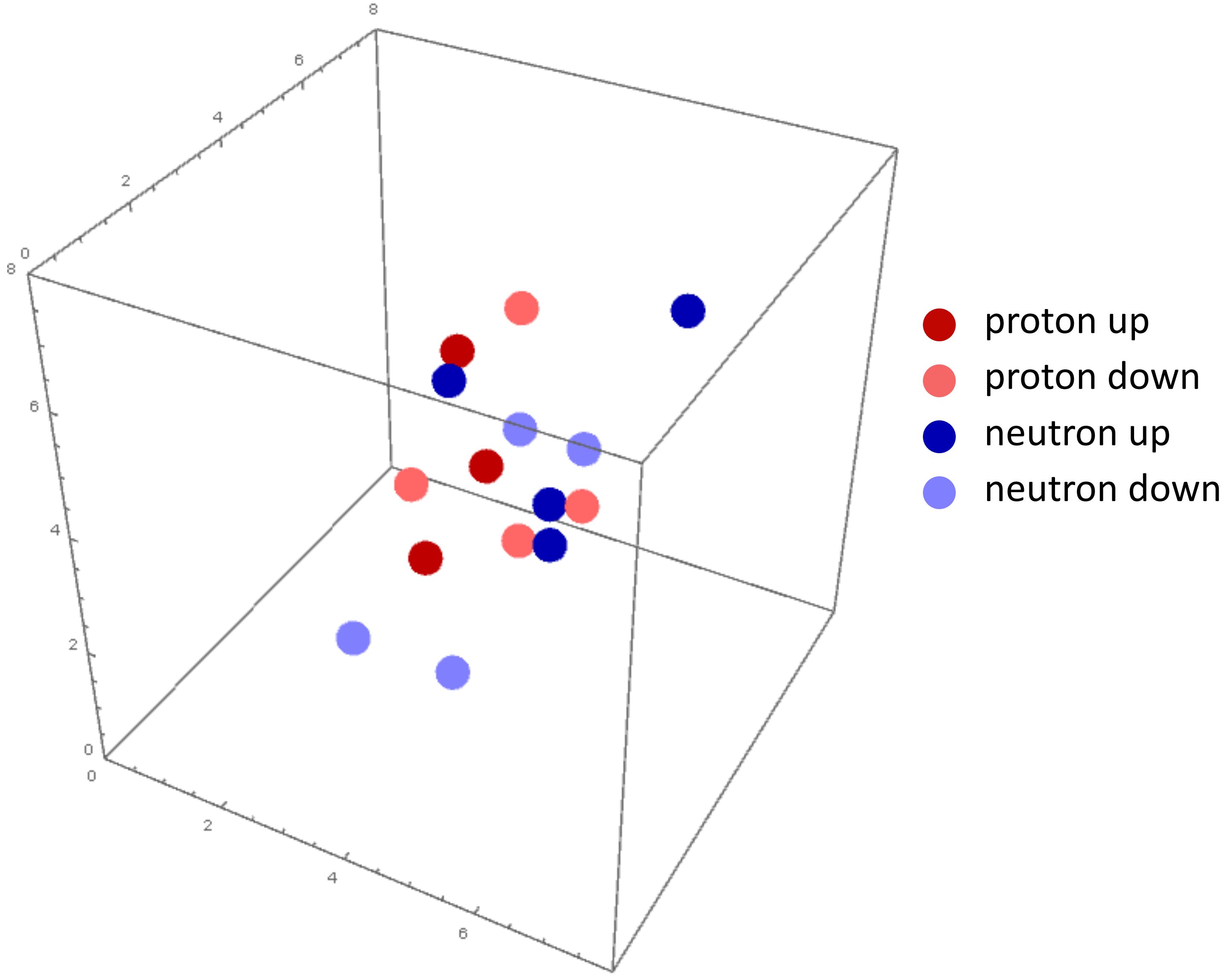


Monte Carlo updates of pinholes

Monte Carlo
updates of auxiliary/pion fields

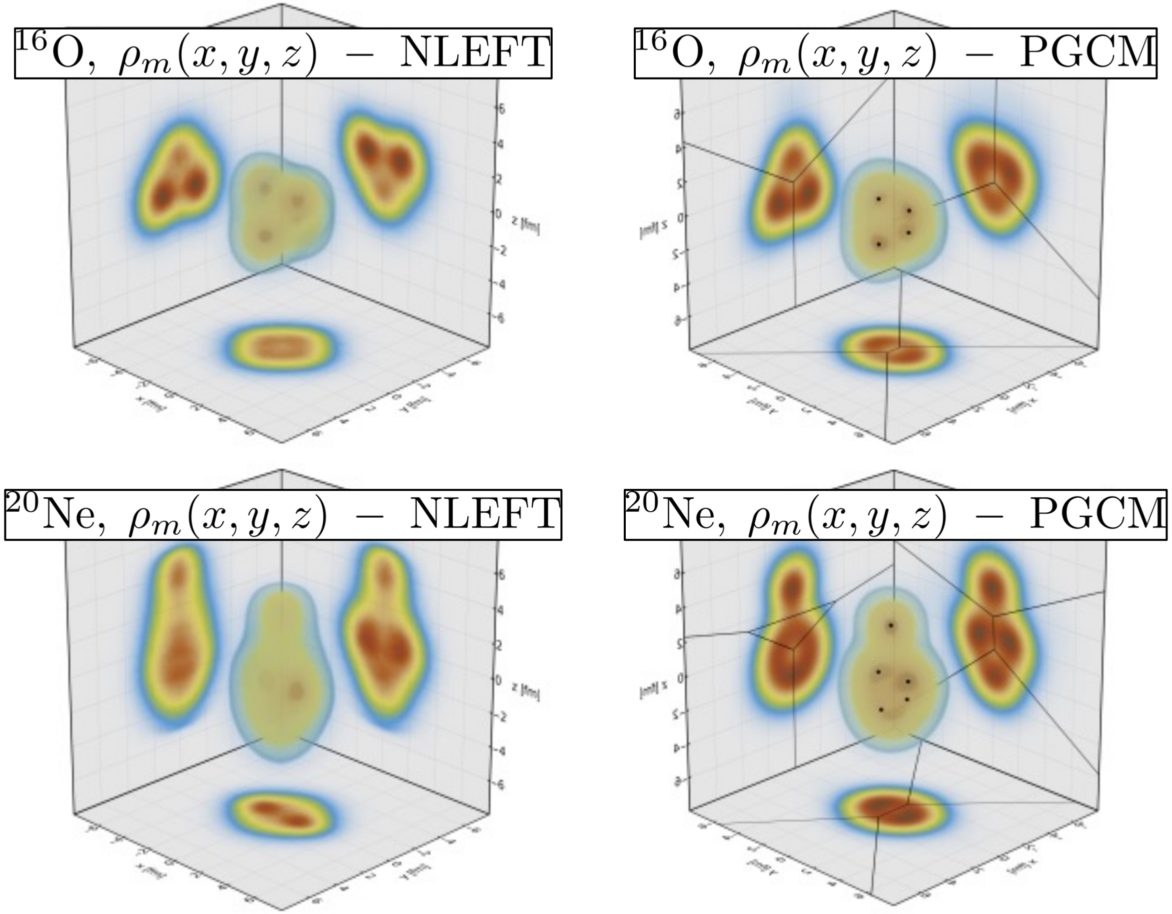
Elhatisari, Epelbaum, Krebs, Lähde, D.L., Li, Lu, Meißner, Rupak, PRL 119, 222505 (2017)

^{16}O



Summerfield, Lu, Plumberg, D.L., Noronha-Hostler, Timmins, Phys. Rev. C 104 L041901 (2021)

Relativistic heavy collisions: ^{16}O^{16}O versus ^{20}Ne^{20}Ne



^{16}O, $\rho_m(x,y,z)$ – NLEFT

^{16}O, $\rho_m(x,y,z)$ – PGCM

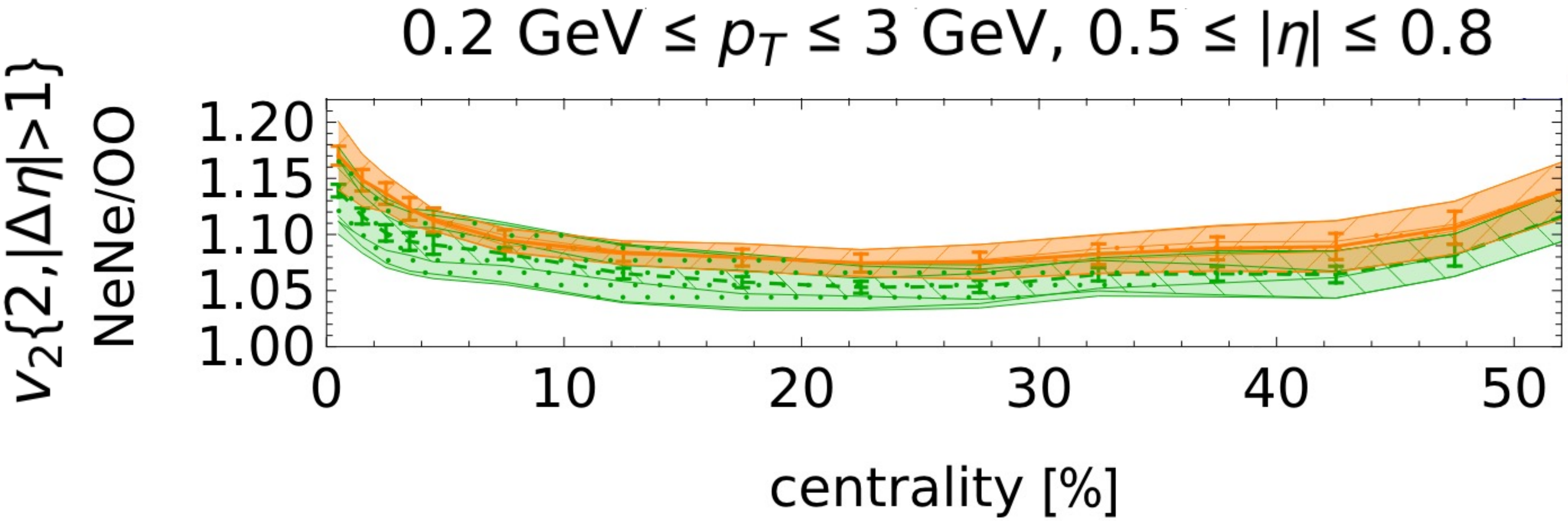
^{20}Ne, $\rho_m(x,y,z)$ – NLEFT

^{20}Ne, $\rho_m(x,y,z)$ – PGCM

Giacalone et al., arXiv:2402.05995

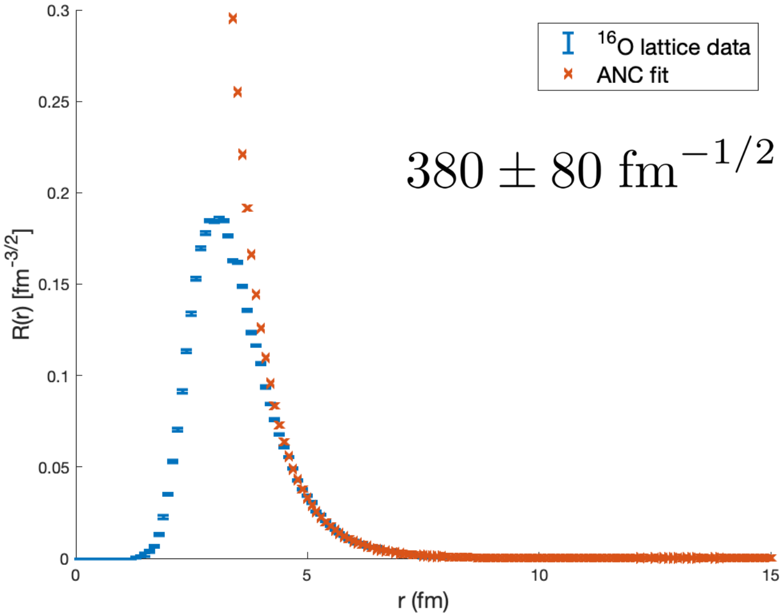
For the 1% most central events, the elliptic flow of ^{20}Ne^{20}Ne collisions relative to ^{16}O^{16}O collisions is enhanced by as much as

1.170(8)stat.(30)syst. for NLEFT
1.139(6)stat.(39)syst. for PGCM

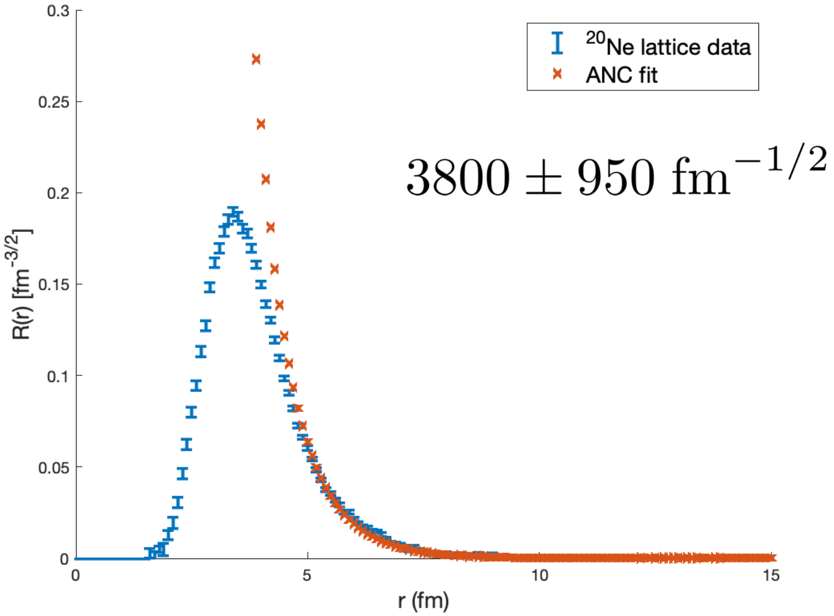


Giacalone et al., arXiv:2402.05995

Asymptotic normalization coefficients



$380 \pm 80 \text{ fm}^{-1/2}$

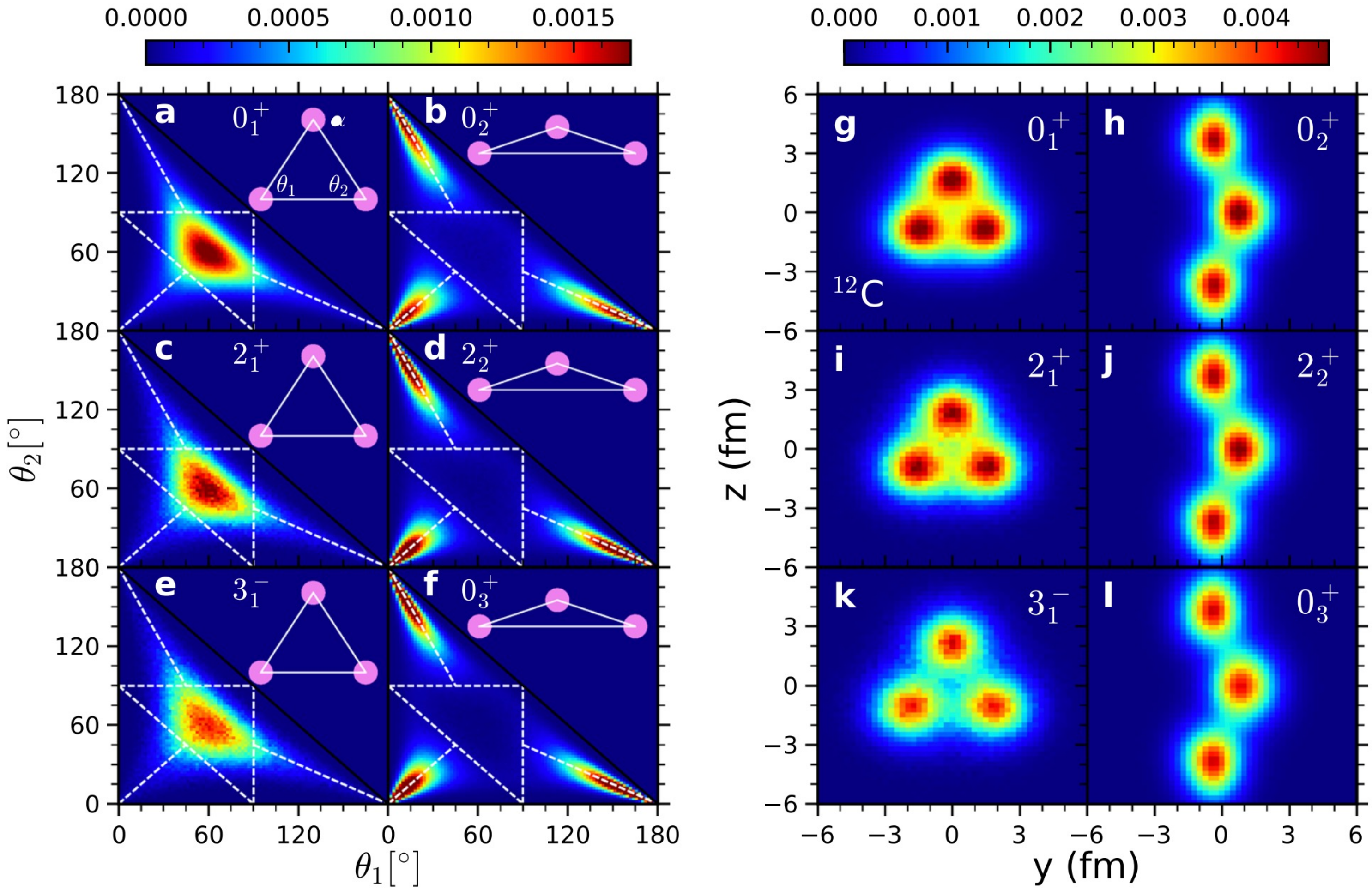


$3800 \pm 950 \text{ fm}^{-1/2}$

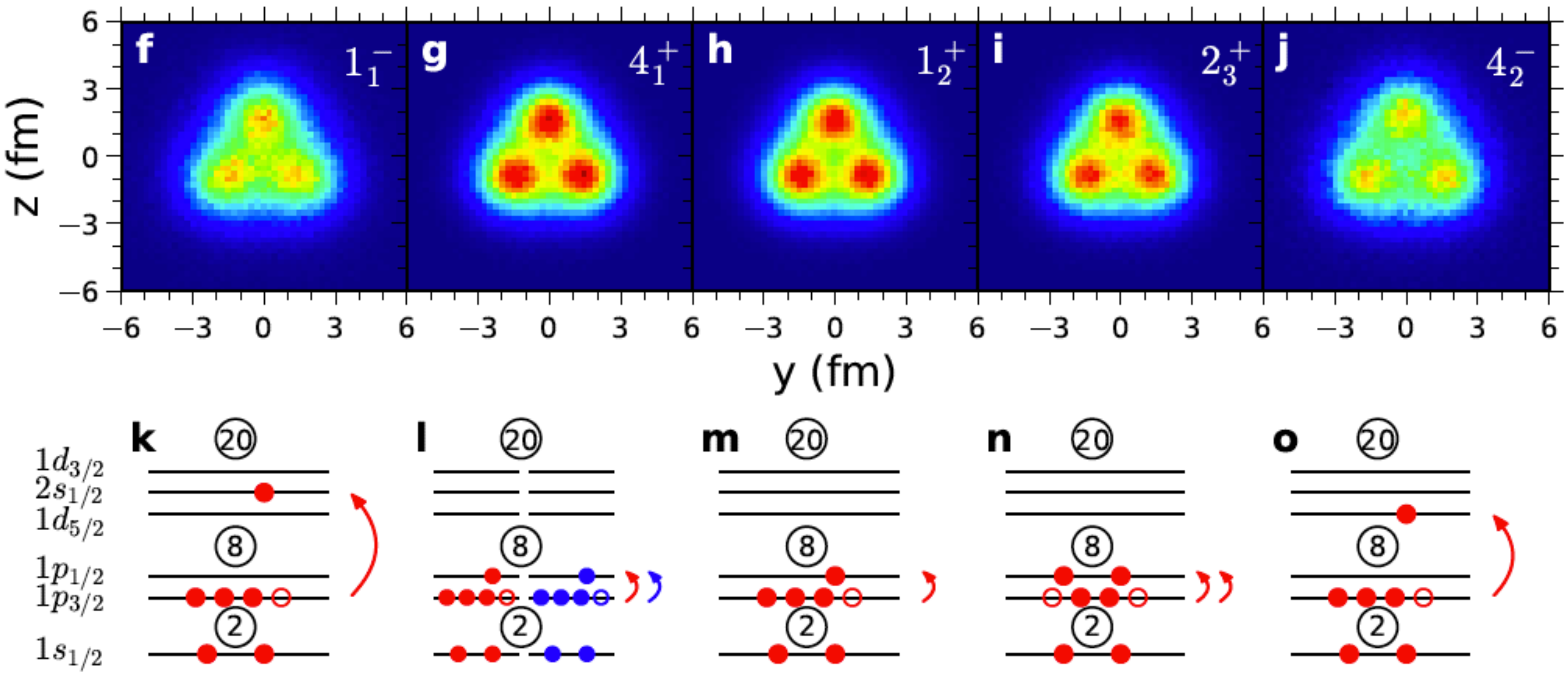
$$^{12}\mathrm{C}\left(^{20}\mathrm{Ne},^{16}\mathrm{O}\right)^{16}\mathrm{O}$$

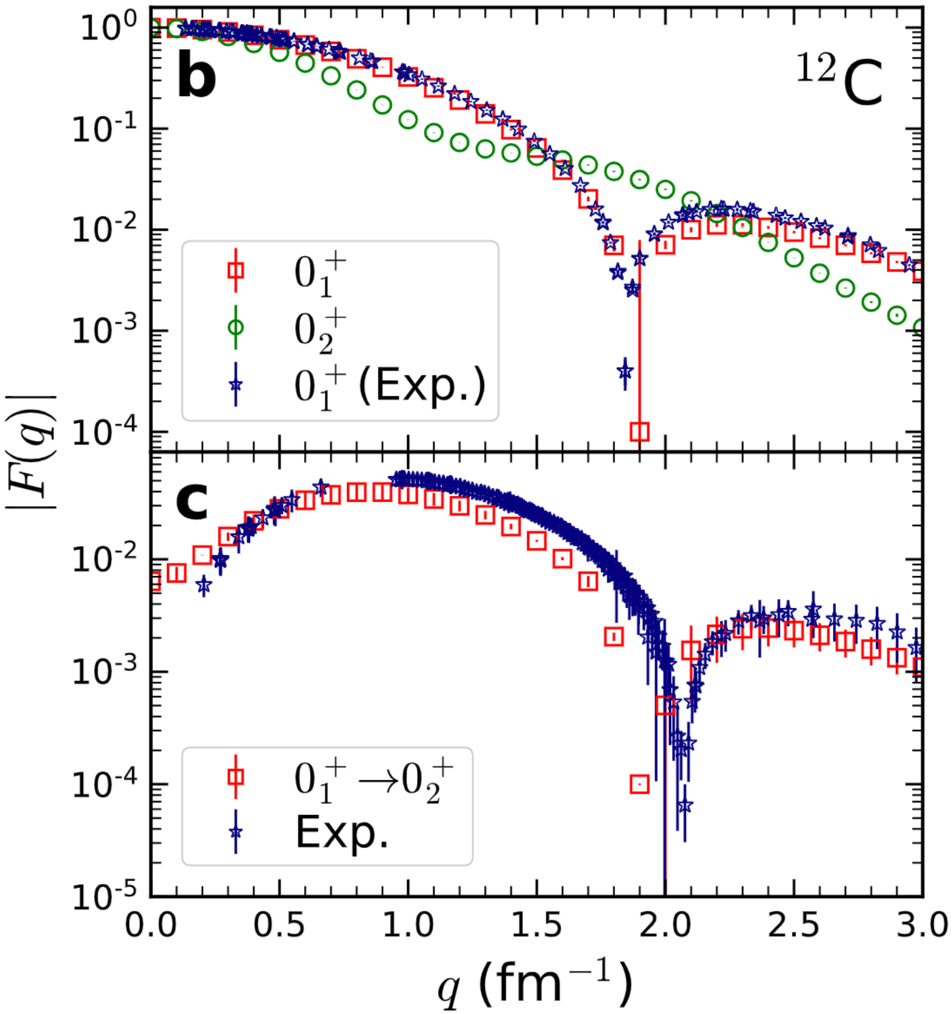
E. Harris et al., work in progress

Emergent geometry and duality of ^{12}C

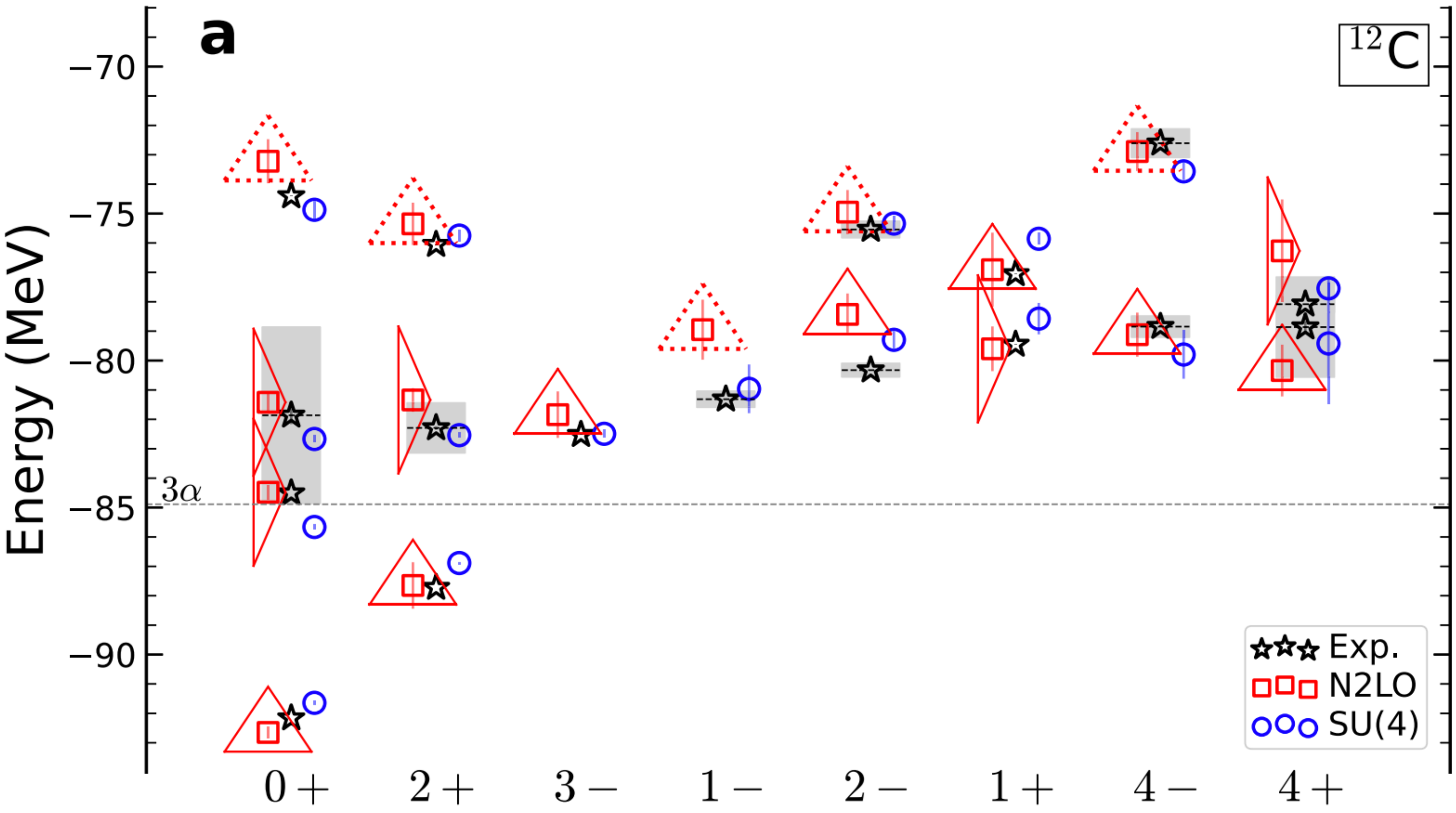


Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)


f
1_1^-
g
4_1^+
h
1_2^+
i
2_3^+
j
4_2^-
z (fm)
6
3
0
−3
−6
−6
−3
0
3
6
y (fm)
k
l
m
n
o
⑳
⑧
②
$1d_{3/2}$
$2s_{1/2}$
$1d_{5/2}$
$1p_{1/2}$
$1p_{3/2}$
$1s_{1/2}$

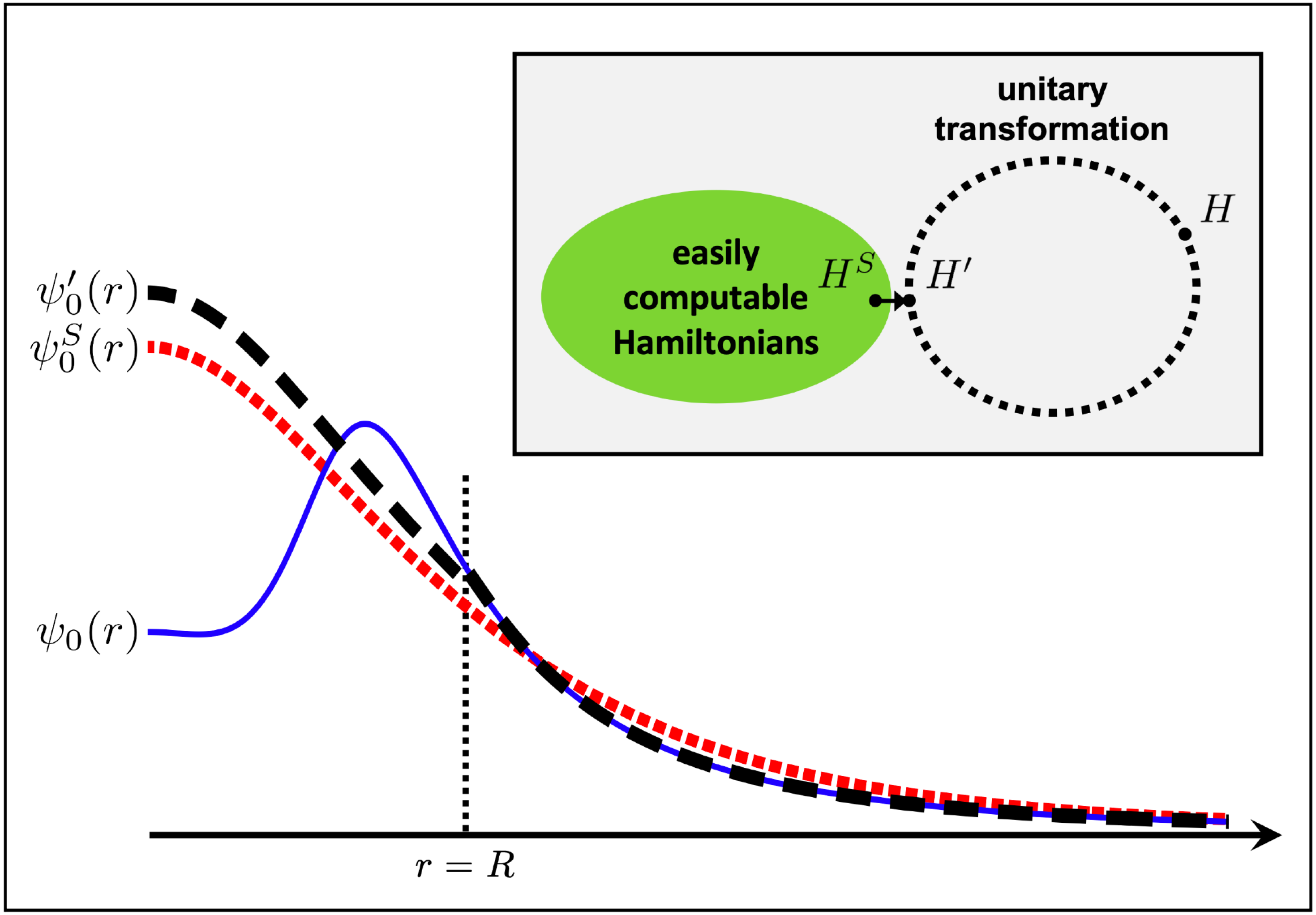


Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)



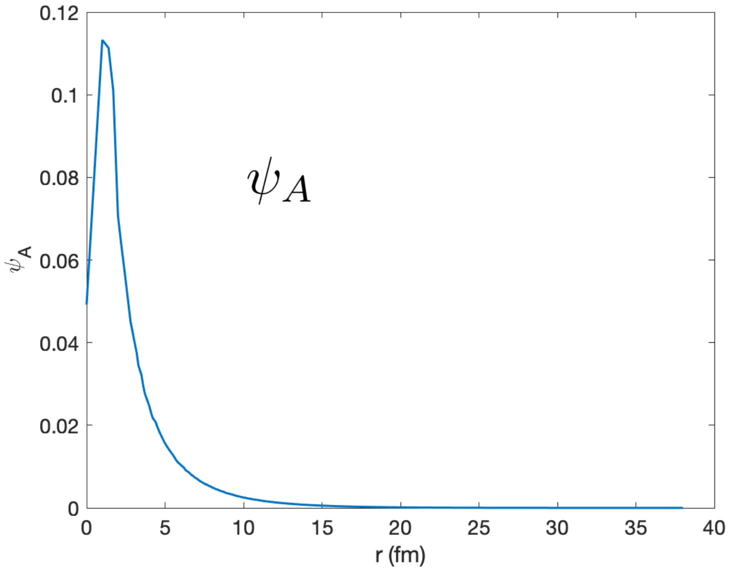
Shen, Elhatisari, Lähde, D.L., Lu, Meißner, Nature Commun. 14, 2777 (2023)

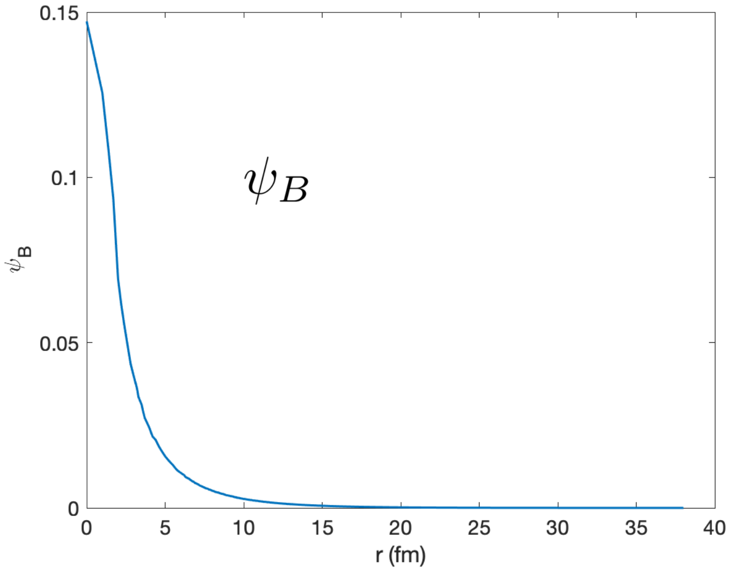
Wavefunction matching

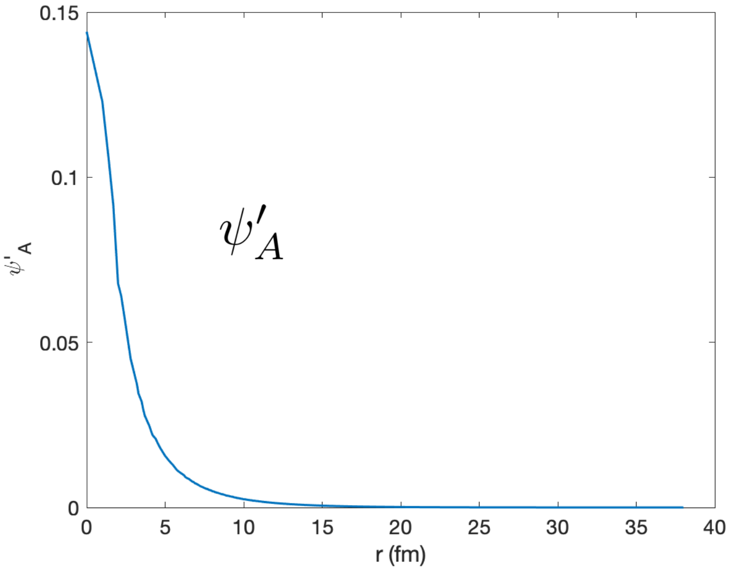


Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Ground state wavefunctions







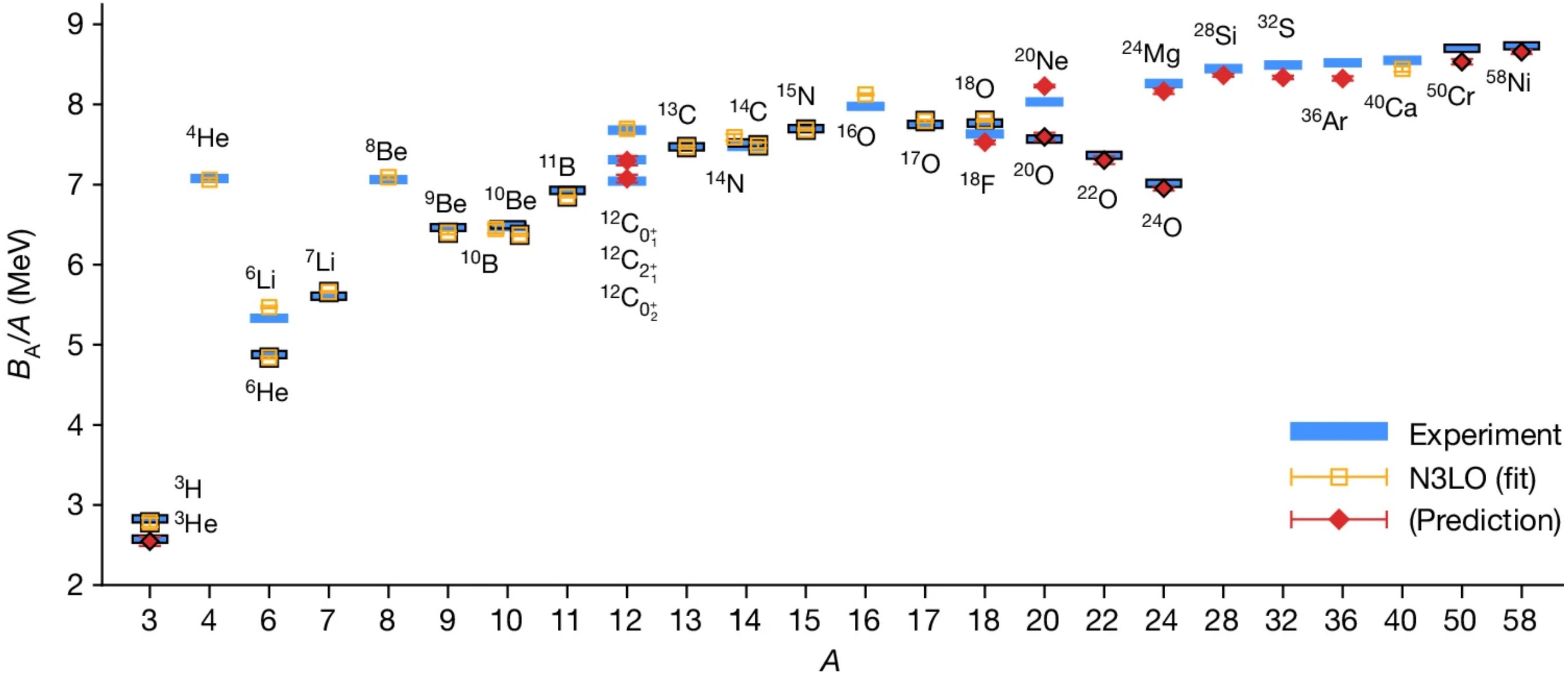
Try to compute the energies of H_A using the eigenfunctions of H_B and first-order perturbation theory. This doesn't work.

$E_{A,n} = E'_{A,n}$ (MeV)	$\langle \psi_{B,n} \vert H_A \vert \psi_{B,n} \rangle$ (MeV)
-1.2186	3.0088
0.2196	0.3289
0.8523	1.1275
1.8610	2.2528
3.2279	3.6991
4.9454	5.4786
7.0104	7.5996
9.4208	10.0674
12.1721	12.8799
15.2669	16.0458

Use wavefunction matching first to transform the Hamiltonian. Then the convergence of perturbation theory is much faster.

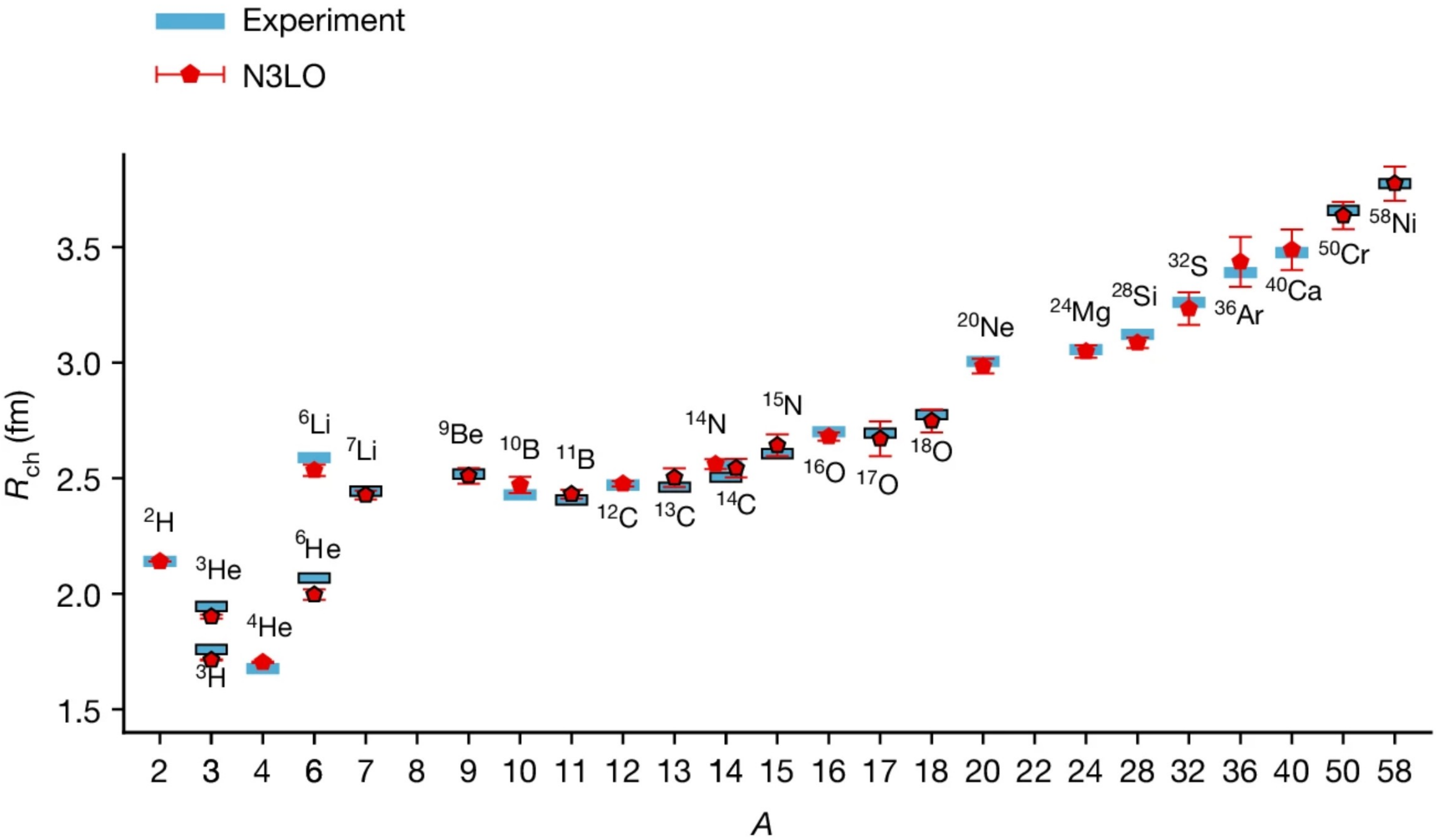
$E_{A,n} = E'_{A,n}$ (MeV)	$\langle \psi_{B,n} \vert H_A \vert \psi_{B,n} \rangle$ (MeV)	$\langle \psi_{B,n} \vert H'_A \vert \psi_{B,n} \rangle$ (MeV)
-1.2186	3.0088	-1.1597
0.2196	0.3289	0.2212
0.8523	1.1275	0.8577
1.8610	2.2528	1.8719
3.2279	3.6991	3.2477
4.9454	5.4786	4.9798
7.0104	7.5996	7.0680
9.4208	10.0674	9.5137
12.1721	12.8799	12.3163
15.2669	16.0458	15.4840

Binding energies



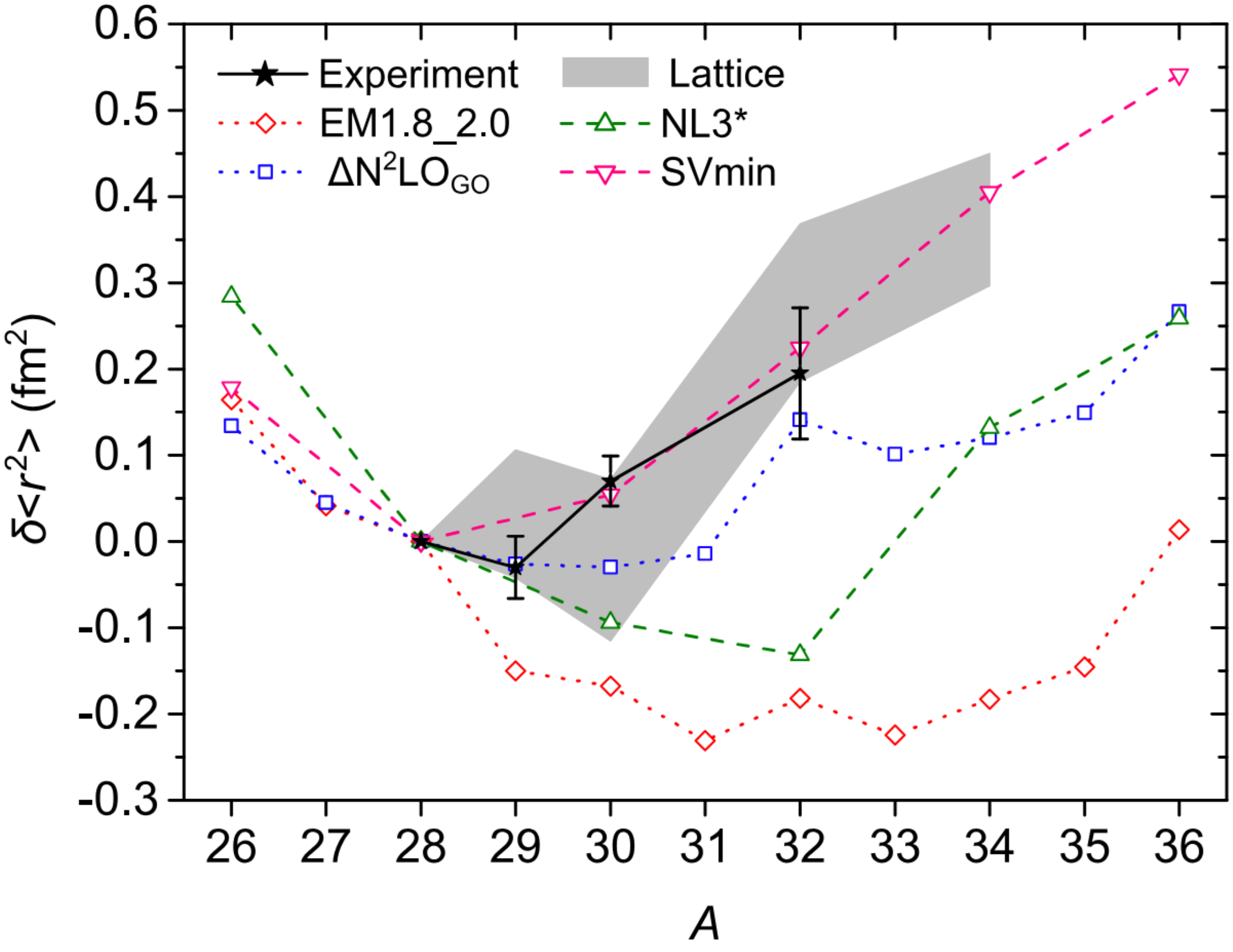
Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

Charge radii



Elhatisari, Bovermann, Ma, Epelbaum, Frame, Hildenbrand, Krebs, Lähde, D.L., Li, Lu, M. Kim, Y. Kim, Meißner, Rupak, Shen, Song, Stellin, Nature 630, 59 (2024)

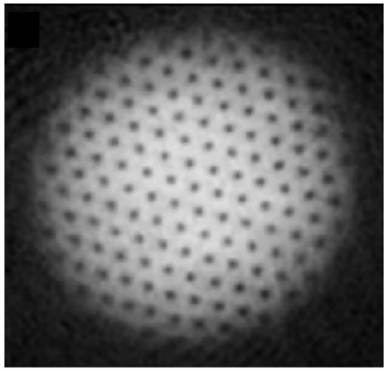
Charge radii of silicon isotopes



K. König et al., PRL 132, 162502 (2024)

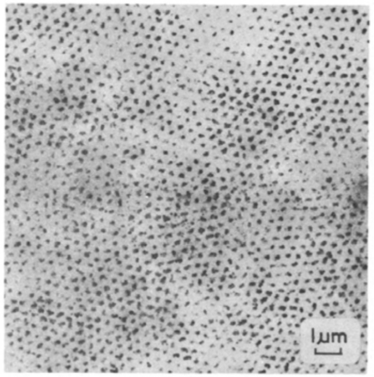
Superfluidity

BEC Theory



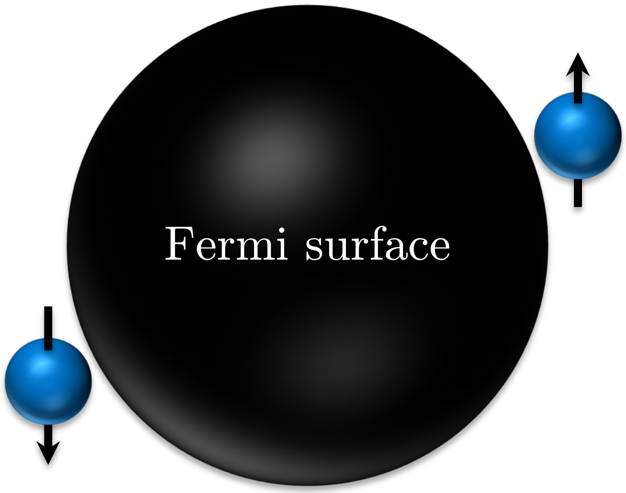
Ketterle, Zwierlein,
Ultracold Fermi Gases (2008)

BCS Theory



Essmann, Träuble,
Phys. Lett. A 27, 3 (1968)

Superfluid pairing is a few-body problem in a many-body environment



Off-diagonal long-range order

Bosonic superfluidity

$$\langle \Psi_0 | a^\dagger(\mathbf{r}) a(\mathbf{0}) | \Psi_0 \rangle$$

Fermionic superfluidity (S-wave)

$$\langle \Psi_0 | a_\downarrow^\dagger(\mathbf{r}) a_\uparrow^\dagger(\mathbf{r} + \Delta\mathbf{r}) a_\uparrow(\Delta\mathbf{r}) a_\downarrow(\mathbf{0}) | \Psi_0 \rangle$$

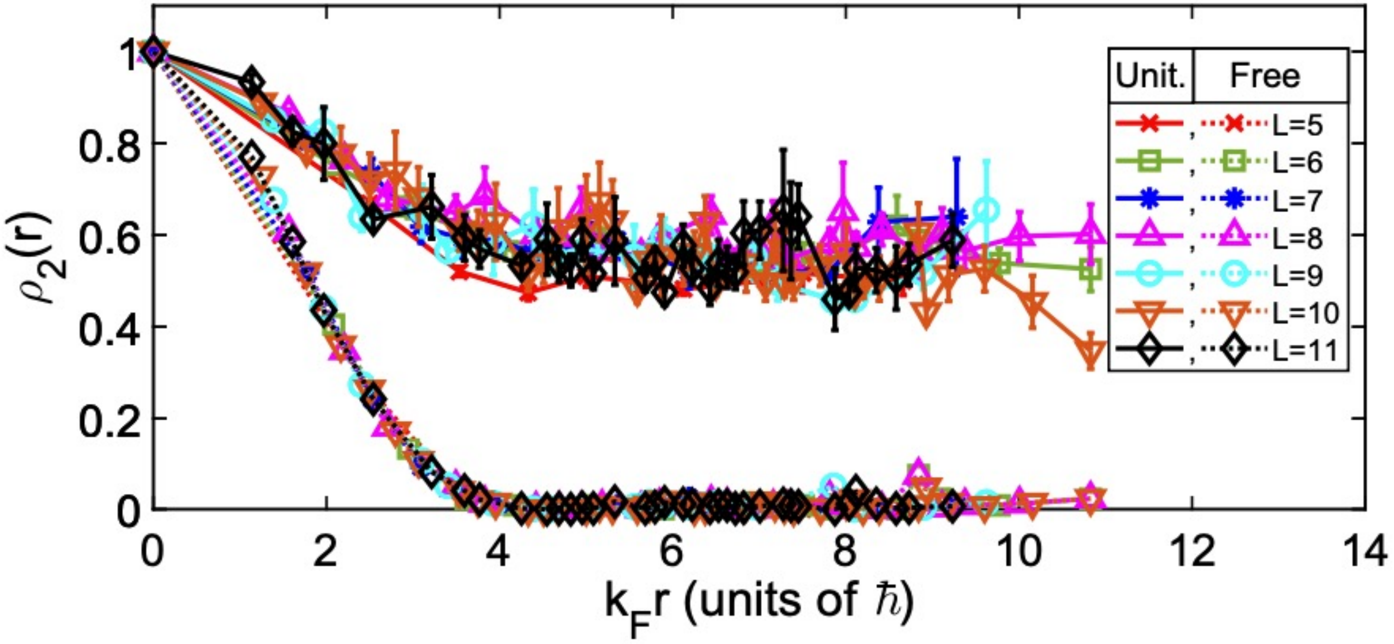
Fermionic superfluidity (P-wave)

$$\langle \Psi_0 | a_\uparrow^\dagger(\mathbf{r}) a_\uparrow^\dagger(\mathbf{r} + \Delta\mathbf{r}) a_\uparrow(\Delta\mathbf{r}) a_\uparrow(\mathbf{0}) | \Psi_0 \rangle$$

Yang, RMP **34**, 694 (1962)

Unitary limit

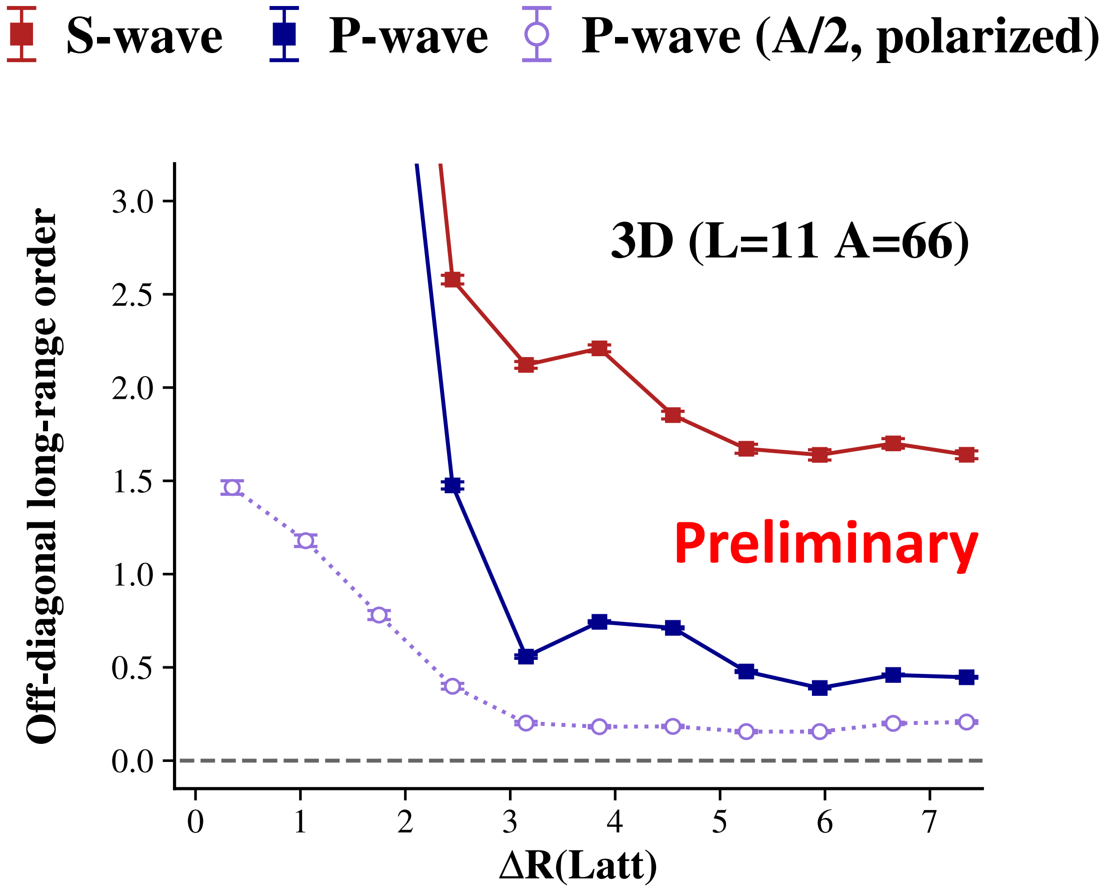
$$H = H_{\text{free}} + \tfrac{1}{2} C_2 \sum_{\mathbf{n}} \rho(\mathbf{n})^2$$



He, Li, Lu, D.L., Phys. Rev. A 101, 063615 (2020)

Multimodal superfluidity

$$H = H_{\text{free}} + \frac{1}{2} C_2 \sum_{\mathbf{n}} \tilde{\rho}(\mathbf{n})^2$$

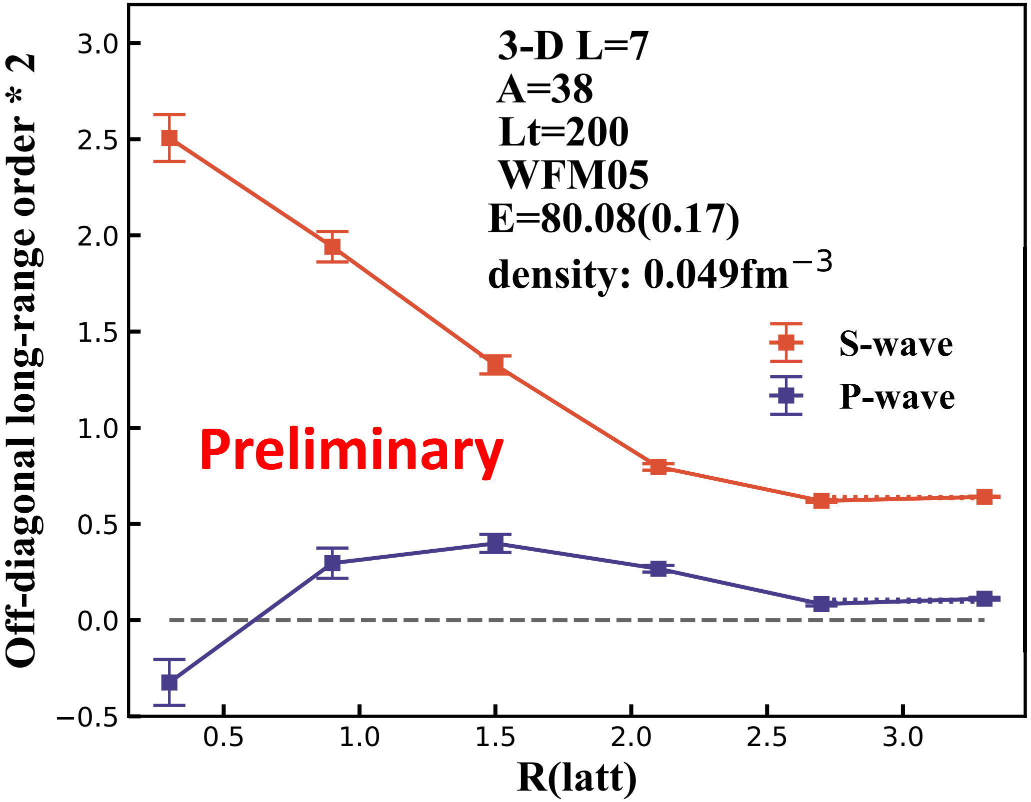


Preliminary

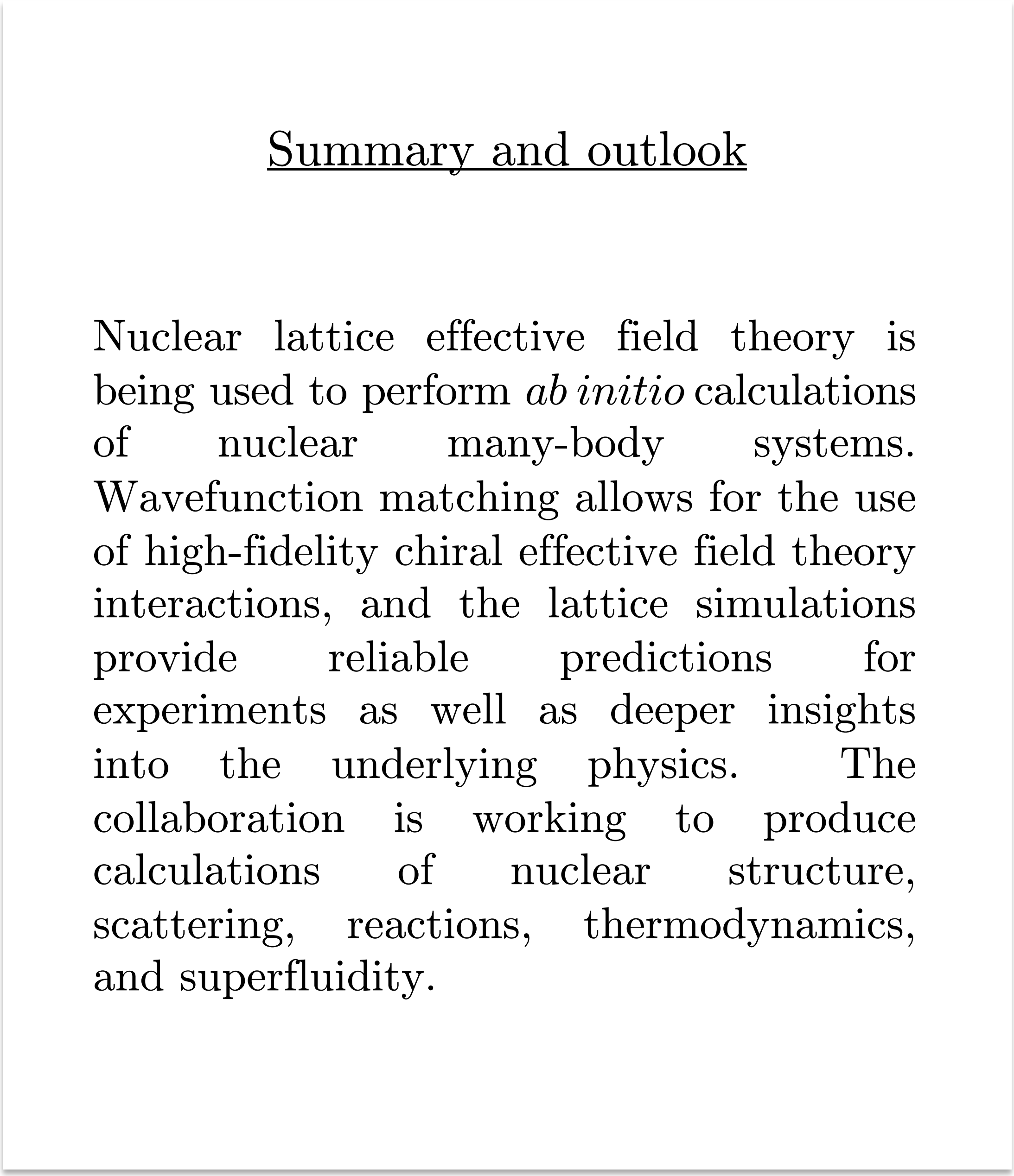
Ma *et al.*, work in progress

Multimodal superfluidity in neutron matter

Leading-order chiral EFT interaction



Ma *et al.*, work in progress



Summary and outlook

Nuclear lattice effective field theory is being used to perform *ab initio* calculations of nuclear many-body systems. Wavefunction matching allows for the use of high-fidelity chiral effective field theory interactions, and the lattice simulations provide reliable predictions for experiments as well as deeper insights into the underlying physics. The collaboration is working to produce calculations of nuclear structure, scattering, reactions, thermodynamics, and superfluidity.