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SOUTH CHINA NORMAL UNIVERSITY

# The three-hadron $DD^*K$ system on the lattice EFT

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The 23rd International Conference on Few-Body Problems in Physics
Beijing, September 22-27, 2024

Zhenyu Zhang, Xin-Yue Hu, Guangzhao He, Jun Liu, Jia-Ai Shi, Bing-Nan Lu, and Qian Wang, arXiv: 2409.01325.

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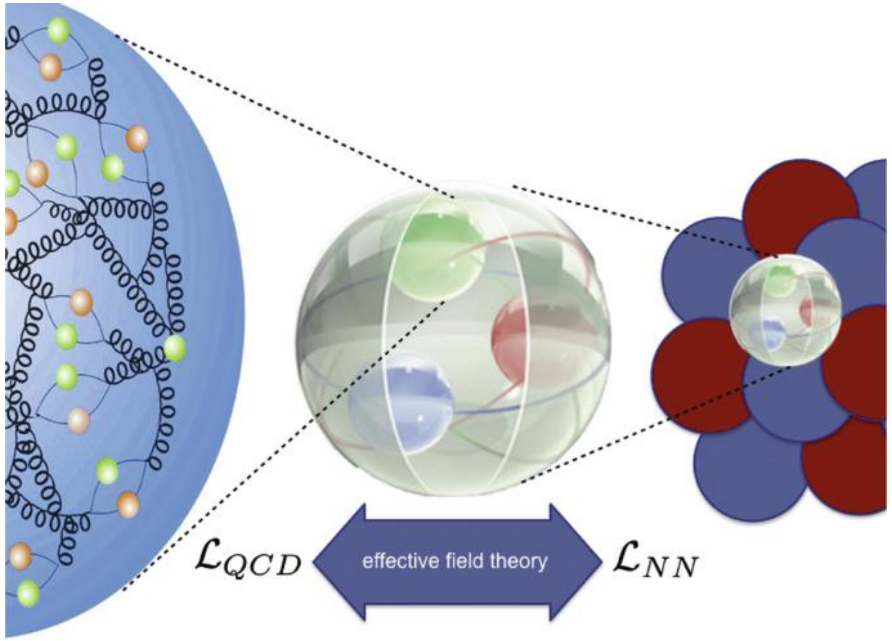
01

# Introduction

# Introduction to LEFT




**Lattice EFT = Chiral EFT+ Lattice + Monte Carlo**

(1) EFT description of hadron interactions
(contact terms + pion exchange potential)

(2) The degrees of freedom on the lattice are hadrons

(3) Lattice spacing a ≈ 1fm
(~ chiral symmetry breaking scale)

**Solving low-energy many-body problems!**

| | LQCD | LEFT |
|---|---|---|
| degree of freedom | quarks & gluons | hadrons |
| lattice spacing | ~0.1fm | ~1fm |
| dispersion relation | relativistic | non-relativistic |
| continuum limit | ✓ | × |
| model | Lagrangian | Hamiltonian |
| solver | path integral | Schrödinger equation |


[1] Dean Lee, Prog. Part. Nucl. Phys. 63, 117 (2009).
[2] Lähde, Meißner, "Nuclear Lattice Effective Field Theory", Springer (2019).

# Applications of LEFT

## Applications in Nuclear Physics

(1) Neutron-proton scattering [Li et al., Phys. Rev. C, 98, 044002(2018)]

- Phase shifts are more accurate at $N^3LO$ interaction

(2) Nuclear binding [Elhatisari et al., Phys. Rev. Lett., 117, 132501(2016)]

- The quantum phase transition of alpha particles depend on nuclear binding

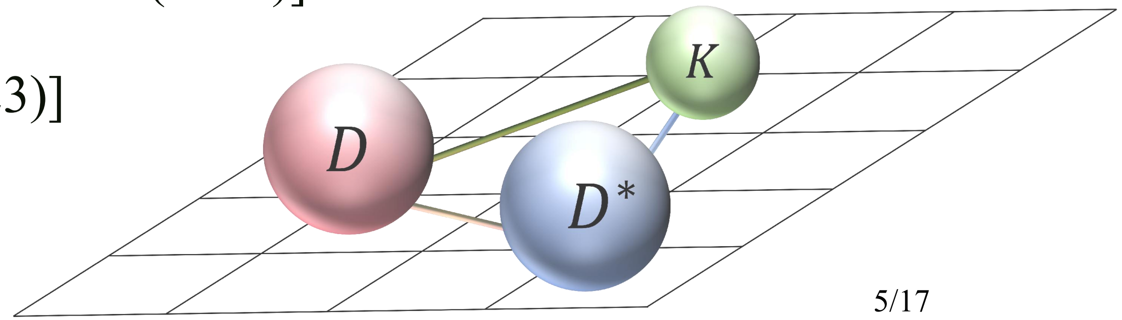
(3) Alpha–alpha scattering [Elhatisari et al., Nature, 528, 111(2015)]

(4) Nuclear thermodynamics [Lu et al., Phys. Rev. Lett., 125, 192502(2020)]

(5) Properties of nuclei [Lu et al., Phys. Lett. B, 797, 134863(2019)]

(6) Hoyle state [Shen et al., Nat. Commun., 14, 2777(2023)]

## Applications in Exotic Hadron ???



# Heavy-flavor three-body system

(1) Gaussian expansion method (GEM)

(2) QCD sum rule (QCDSR)

(3) Born-Oppenheimer approximation (BO)

(4) Fixed center approximation (FCA)

(5) Faddeev equation (F) et al.

**Three-body problem** ⇒ **Quasi-two-body system solving**

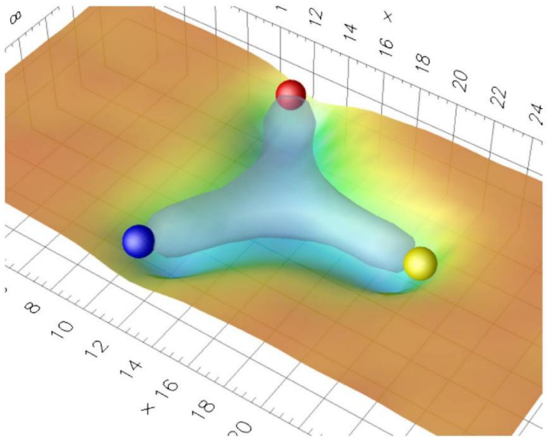
TABLE XXXIX. Summary for heavy-flavor three-body states. Energies are in units of MeV.

| Components | $I(J^P)$ | Results (Method) | Decay modes |
|---|---|---|---|
| $DNN$ | $\frac{1}{2}(0^-)$ | BS $\sim 3500-15i$ (FCA, V) [836] | $\Lambda_c\pi^- p, \Lambda_c p$ [836] |
| $NDK, ND\bar{K}, ND\bar{D}$ | $\frac{1}{2}(\frac{1}{2}^+)$ | BS $\sim 3050, 3150, 4400$ (FCA) [837] | † |
| $DD^*N$ | $\frac{1}{2}(\frac{1}{2}^+, \frac{3}{2}^+)$ | BS $\sim 4773.2, 4790.7$ (GEM) [838] | $T_{cc}p, DDp+\pi(\gamma), \Xi_{cc}+\pi(\gamma)$, charmed baryon + charmed meson [838] |
| $DD^*N$ | $\frac{3}{2}(-)$ | difficult to form bound states (GEM) [838] | † |
| $DK\bar{K}$ | $\frac{1}{2}(0^-)$ | $D$-like state $\sim 2845.5$ (FCA) [821], $D$-like state $\sim 2900$ (QCDSR, $\chi$F) [839] | $\pi\pi D$ [821] |
| $DKK$ | $\frac{1}{2}(0^-)$ | no bound state (FCA) [821] | † |
| $\bar{D}\bar{K}\Sigma_c$ | $1(\frac{1}{2}^+)$ | BS $\sim 4738.6$ (GEM) [840] | $D\Xi', D_s\Sigma_c$ [840] |
| $D^{(*)}$ multi $\rho$ | ... | several $D_J^{(*)}$ states (FCA) [841, 842] | † |
| $\rho D\bar{D}$ | $0(?), 1(?)$ | BS $\sim 4241-10i$, $[4320-13i, 4256-14i]$ (FCA) [843] | † |
| $DDK$ | $\frac{1}{2}(0^-)$ | BS $\sim 4162$ (GEM) [273], $4140$ ($\chi$F) [819], $4160$ (FV) [820] | $DD_s^*, D^*D_s$ [826] |
| $D\bar{D}K$ | $\frac{1}{2}(0^-)$ | BS $\sim 4181.2$ (GEM) [822], $4191$ (FCA) [825] | $D_s\bar{D}^*, J/\psi K$ [822] |
| $DD^*K$ | $\frac{1}{2}(1^-)$ | BS $\sim 4317.9$ (BO) [823] | † |
| $D\bar{D}^*K$ | $\frac{1}{2}(1^-)$ | BS $\sim 4294.1$ (GEM) [822], $4317.9$ (BO) [823], $4307$ (FCA) [824] | $D_s^{(*)}\bar{D}^{(*)}, J/\psi K^*$ [823, 844] |
| $D^*D^*\bar{K}^*$ | $\frac{1}{2}(0^-, 1^-, 2^-)$ | $4845-40i$, BS $\sim [4850-46i, 4754-50i]$, (FCA) [845] $[4840-43i, 4755-50i]$ | $D^*D^*\bar{K}^*$, $D^*D^{(*)}\bar{K}^*$, $[D^*D^*\bar{K}^*, D^*D^{(*)}\bar{K}^*]$ [845] |
| $\bar{D}\bar{D}^*\Sigma_c$ | $1(\frac{1}{2}^+, \frac{3}{2}^+)$ | BS $\sim 6292.3, 6301.5$ (GEM) [829] | $J/\psi p\bar{D}^{(*)}, \bar{T}_{cc}\Lambda_c\pi$ [829] |
| $J/\psi K\bar{K}$ | $0(1^-)$ | $Y(4260)\sim 4150-45i$ ($\chi$F) [481] | † |
| $DDD^*$ | $\frac{1}{2}(1^-)$ | BS $\sim 5742.2$ (GEM) [833] | $DDD\pi(\gamma)$[833] |
| $DD^*D^*$ | $\frac{1}{2}(0^-, 1^-, 2^-)$ | several loosely bound states (GEM) [834] | charmed mesons + ... [834] |
| $D^*D^*D^*$ | $\frac{1}{2}(0^-, 1^-, 2^-, 3^-)$ | several loosely bound states (GEM)[834] | charmed mesons + ... [834] |
| | $\frac{1}{2}(0^-, 1^-, 2^-)$ | BS $\sim 5790.9-49.8i, 5990.2, 5989.4$ (FCA) [835] | |
| $D^*D^*D^{(*)}$ | $\frac{3}{2}(-)$ | difficult to form bound states (GEM) [834] | † |
| $D^*D^*\bar{D}$ | $\frac{1}{2}(2^-)$ | BS $\sim 5879$ (F) [846] | † |
| $D^*D^*\bar{D}^*$ | $\frac{1}{2}(3^-)$ | BS $\sim 6019$ (F) [846] | † |
| $\Omega_{ccc}\Omega_{ccc}\Omega_{ccc}$ | $?(\frac{3}{2}^+)$ | no bound state (GEM) [847] | † |
| $\Xi_{cc}\Xi_{cc}\bar{K}$ | $\frac{1}{2}(0^-)$ | BS $\sim 7641.8$ (GEM) [848] | † |

[1] Ming-Zhu Liu et al., arXiv:2404.06399 (2024).

# The necessity of three-body force

## (1) **Direct three-body interactions**



| $\Lambda$ (MeV) | 350 | 375 | 400 | Exp |
|---|---|---|---|---|
| $c_E$ | 0.561 | 0.412 | 0.380 | |
| $E_{2NF}\left({}^{3}H\right)$ | -7.64(1) | -7.77(1) | -7.78(1) | -8.482 |
| $E_{2NF+3NF}\left({}^{3}H\right)$ | -8.483 | -8.483 | -8.483 | -8.482 |
| $E_{2NF}\left({}^{4}He\right)$ | -29.8(4) | -29.4(4) | -29.2(4) | -28.34 |
| $E_{2NF+3NF}\left({}^{4}He\right)$ | -29.0(4) | -28.6(4) | -28.4(4) | -28.34 |
| $E_{2NF}\left({}^{16}O\right)$ | -140.6(8) | -141.7(8) | -141.8(9) | -127.6 |
| $E_{2NF+3NF}\left({}^{16}O\right)$ | -127.3(8) | -128.1(8) | -128.1(8) | -127.6 |

- In three-quark systems
- LQCD calculate gluon flux-tube[1,2]
- The structure is **“Y” type**

- In nucleus systems
- LEFT calculate the binding energy[3]
- The results with 3NF close to Exp.

## (2) **Study of many-body systems**

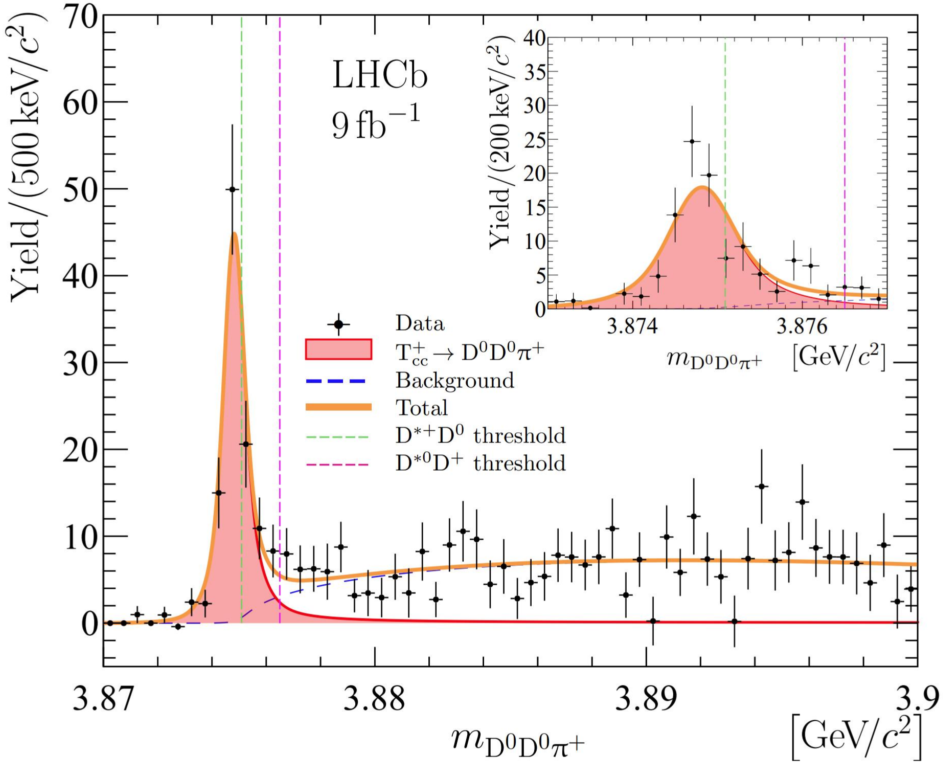
[1] H. Ichie et al., Nucl. Phys.A 721,C899-C902(2003). [2] F. Bissey et al., Phys.Rev.D 76,114512(2007).
[3] Bao-Ge Deng et al., Sci. Sin-Phys. Mech. Astron., 54,292009(2024).

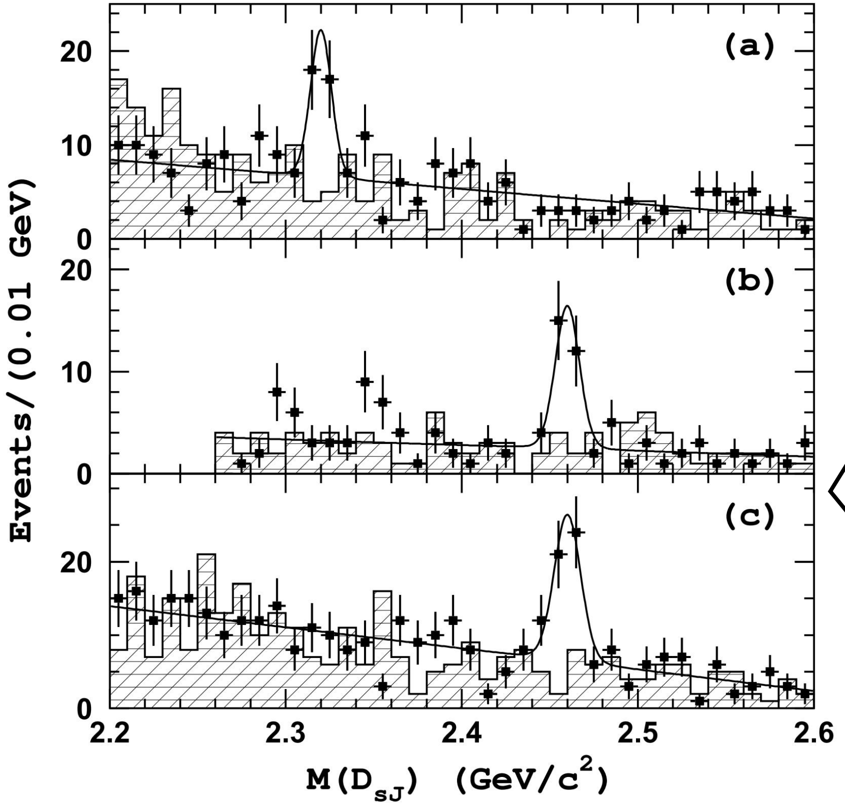
02

# Study the $DD^*K$ three-hadron system using LEFT

# $T_{cc}^{+}$, $D_{s0}^{*}(2317)$, $D_{s1}(2460)$ in experimental

Why study $DD^{*}K$ system?




Experimental proof of a bound state (resonance state) between two pairs of $DD^{*}K$ system.

- Discovery of the $T_{cc}^{+}$[1] on LHCb
- A threshold very close to $DD^{*}$
- $T_{cc}^{+}$ as $\boldsymbol{DD^{*}}$ **hadronic molecule**

- Discovery of $D_{s0}^{*}(2317)$, $D_{s1}(2460)$[2] on Belle
- Their mass difference is very close to the $D, D^{*}$
- $D_{s0}^{*}(2317)$, $D_{s1}(2460)$ as $\boldsymbol{DK}$, $\boldsymbol{D^{*}K}$ **hadronic molecule**
- LQCD calculation [3] supports this scenario

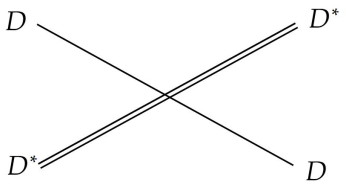

[1] R. Aaij et al. (LHCb), Nature Phys. 18, 751(2022). [2] P. Krokovny et al. (Belle), Phys. Rev. Lett. 91, 262002 (2003).
[3] L. Liu et al., Phys. Rev. D 87, 014508 (2013).

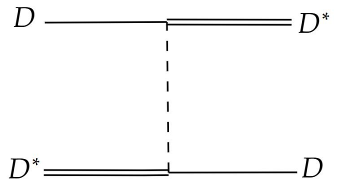
# Two-body interaction in $DD^*K$ system

By chiral effective field theory

**$DD^*$ interaction**

LO contact term and OPE




Deng et al., Phys. Rev. D 105, 054015(2022)

Ke et al., Phys. Rev. D 105, 114019(2022)

**Du et al., Phys. Rev. D 105, 014024(2022)**

Shi et al., Phys. Rev. D 106, 096012(2022)

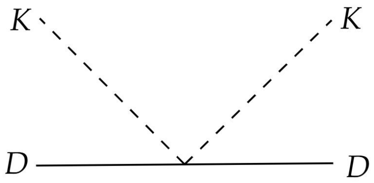
Chen et al., Phys. Lett. B 833, 137391(2022)

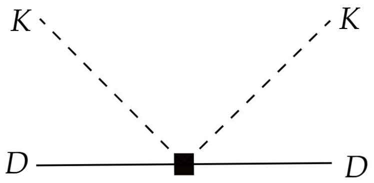
Liu et al., Phys. Rev. D 107, 054041(2023)

Wang et al., Phys. Rev. D 107, 094002(2023) **… …**

**$DK$ interaction**

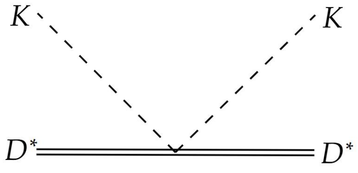
LO and NLO contact term

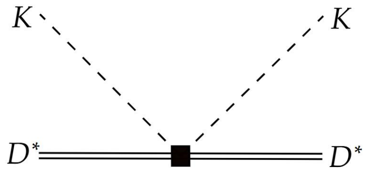



**$D^*K$ interaction**

LO and NLO contact term




Liu et al., Phys. Rev. D 79, 094026(2009)

Huang et al., Eur. Phys. J. C 83, 76(2023)

Geng et al., Phys. Rev. D 82, 054022(2010)

**Guo et al., Eur. Phys. J. A 40, 171-179(2009)**

Yao et al., J. High Energy Phys. 11, 058(2015)

Zhong et al., Phys. Rev. D 78, 014029(2008)

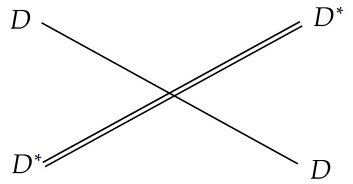
Wang, Phys. Rev. D 75, 034013(2007) … …

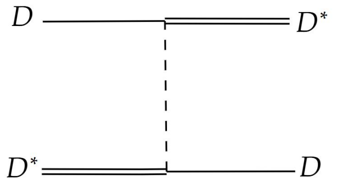
# Two-body interaction in $DD^*K$ system

By chiral effective field theory

**$DD^*$ interaction**[1]

LO contact term and OPE



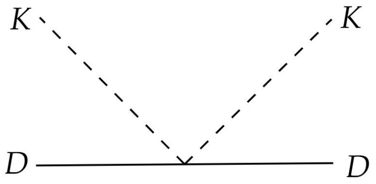



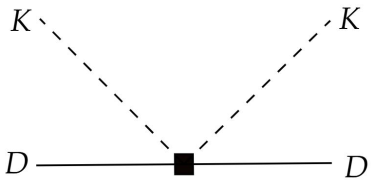

$$V^{\rm Con}_{DD^*} = v_0 \boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}^* \qquad V^{OPE}_{DD^*}(\boldsymbol{q}) = -\frac{3g^2}{4f_\pi^2}\frac{\boldsymbol{\epsilon}\cdot\boldsymbol{q}\,\boldsymbol{\epsilon}^*\cdot\boldsymbol{q}}{\boldsymbol{q}^2+\mu^2}$$

- One free parameter $v_0$

**$DK$ interaction**[2]

LO and NLO contact term



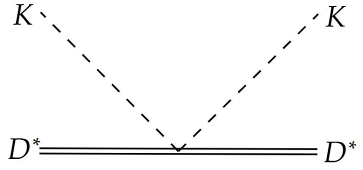

$$V^{DK}_{\rm LO}(p_i) = \frac{-1}{2f_\pi^2}(p_1\cdot p_2 + p_1'\cdot p_2' + p_1\cdot p_2' + p_2\cdot p_1')$$

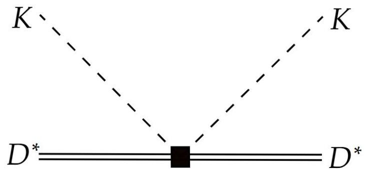
$$V^{DK}_{\rm NLO}(p_i) = -\frac{8M_K^2}{3f_\pi^2}h_1 + \frac{4}{f_\pi^2}(h_3 p_2\cdot p_2' + h_5(p_1\cdot p_2 p_1'\cdot p_2' + p_1\cdot p_2' p_2\cdot p_1'))$$

- One free parameter $h_3$
- Other parameters are determined by other physical processes

**$D^*K$ interaction**[2]

LO and NLO contact term




$$V^{D^*K}_{\rm NLO}(p_i) = (-\frac{8M_K^2}{3f_\pi^2}h_1^* + \frac{4}{f_\pi^2}(h_3^* p_2\cdot p_2' + h_5^*(p_1\cdot p_2 p_1'\cdot p_2' + p_1\cdot p_2' p_2\cdot p_1')))\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}^*$$

- One free parameter $h_3^*$

Hamiltonian:

$$H_{T_{cc}^+} = M_D + M_{D^*} + K_{DD^*} + f_{2B}(\boldsymbol{p}_i,\boldsymbol{p}_i')V^{\rm Con}_{DD^*} + V^{\rm OPE}_{DD^*},$$
$$H_{D_{s0}^*} = M_D + M_K + K_{DK} + f_{2B}(\boldsymbol{p}_i,\boldsymbol{p}_i')V_{DK},$$
$$H_{D_{s1}} = M_{D^*} + M_K + K_{D^*K} + f_{2B}(\boldsymbol{p}_i,\boldsymbol{p}_i')V_{D^*K},$$

**EFT is applicable to physical processes where the $Q \ll \Lambda$**

Single-particle regulator[3] (equal to directly cutoff on lattice)

$$f_{2B}(\boldsymbol{p}_i,\boldsymbol{p}_i') = \prod_{i=1}^2 g_\Lambda(\boldsymbol{p}_i)g_\Lambda(\boldsymbol{p}_i')$$
$$g_\Lambda(\boldsymbol{p}) = \exp(-\boldsymbol{p}^6/2\Lambda^6)$$

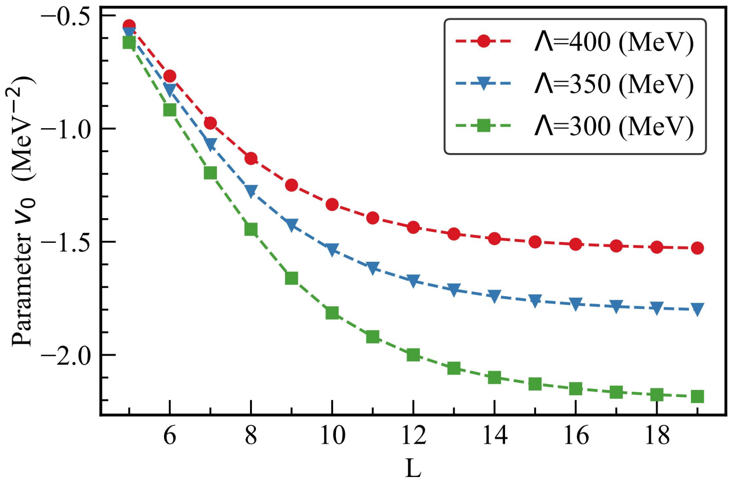
- Better **renormalization group** invariance



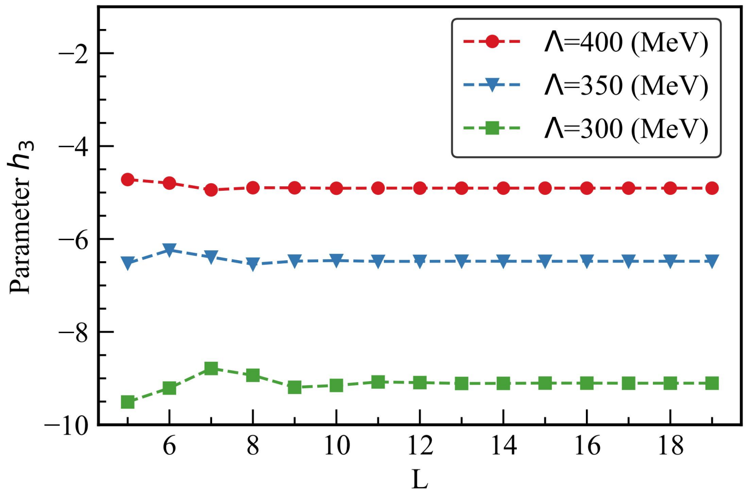
[1] M.-L. Du et al., Phys. Rev. D 105, 014024 (2022). [2] F.-K. Guo et al., Eur. Phys. J. A 40, 171 (2009). [3] B.-N. Lu et al., arXiv:2308.14559.

# Extract two-body interaction parameters

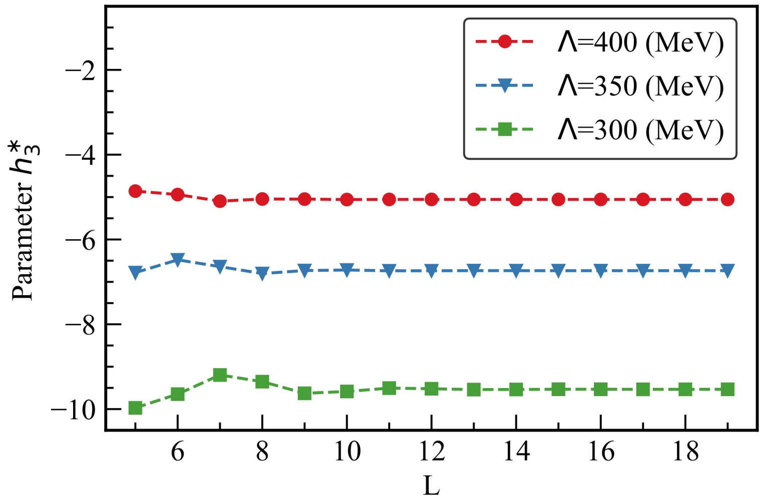
**$T_{cc}^{+}$** → **$DD^{*}$ interaction parameter**




**$D_{s0}^{*}(2317)$** → **$DK$ interaction parameter**




**$D_{s1}(2460)$** → **$D^{*}K$ interaction parameter**




- Use binding energy of $T_{cc}^{+}$, $D_{s0}^{*}(2317)$, $D_{s1}(2460)$ to extract parameters $v_0$, $h_3$, $h_3^{*}$
- Calculate on box size L=$5^3$~$19^3$ cubic lattice with N=2 bosons
- Set cutoff Λ =300, 350, 400 MeV, and lattice spacing a=1/200 MeV$^{-1}$ ≈0.99 fm.
- Long-range OPE makes convergence slower

# $DD^*K$ three-body bound state

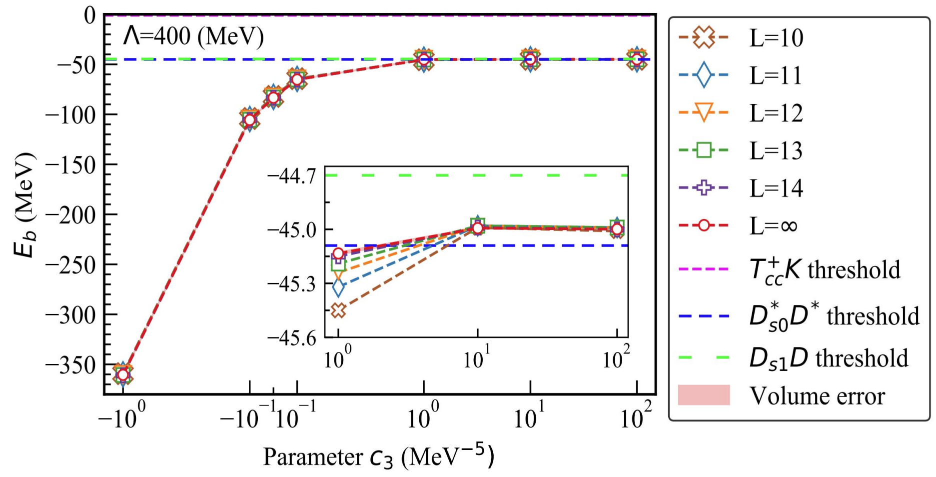
LO contact term

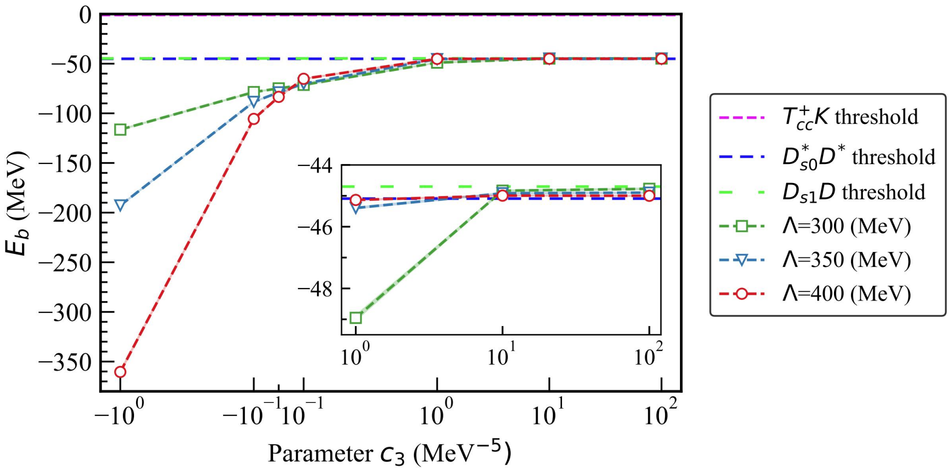
$DD^*K$ three-body lagrangian $\mathcal{L} = c_3\langle H\mathcal{D}_\mu H^\dagger H\mathcal{D}^\mu H^\dagger\rangle + c_3'\langle H\mathcal{A}_\mu H^\dagger H\mathcal{A}^\mu H^\dagger\rangle$

- The contribution of first term is leading

$DD^*K$ three-body forces $V_{DD^*K}(p_i) = \frac{c_3}{4f_\pi^2}(p_1\cdot p_3 + p_1\cdot p_3' + p_2\cdot p_3 + p_2\cdot p_3' + p_1'\cdot p_3 + p_1'\cdot p_3' + p_2'\cdot p_3 + p_2'\cdot p_3')\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}^*$

- One free parameter $c_3$




Mass and binding energies of the $DD^*K$ bound state (MeV)

| |
|---|
| This work (No three-body forces) |
| **$4292.39^{+4.36}_{-4.16}(79.06^{+4.36}_{-4.16})$** |
| Born-Oppenheimer approximation [2] |
| $4317.92^{+3.66}_{-4.32}(53.52^{+3.66}_{-4.32})$ |

- Calculate $DD^*K$ binding energy on box size L=$10^3$~$14^3$ cubic lattice with N=3 bosons
- Compare binding energy by sliding the strength of three-body force ($c_3$)
- **A clear three-body bound state** with bingding energy larger than 44 MeV
- Expand three-boson system in S-wave to infinite volume[1]

$$\frac{\Delta E}{E_T} = -(\kappa L)^{-3/2}\sum_{i=1}^{3} C_i \exp(-\mu_i\kappa L)$$

- **Consistent results** compared to other research



[1]Y. Meng, et al., Phys. Rev. D 98, 014508(2018). [2] Li Ma et al., Chin. Phys. C 43, 014102(2019).

# First excited state of $DD^*K$ system

**Renormalization group invariance**

Same ground energy

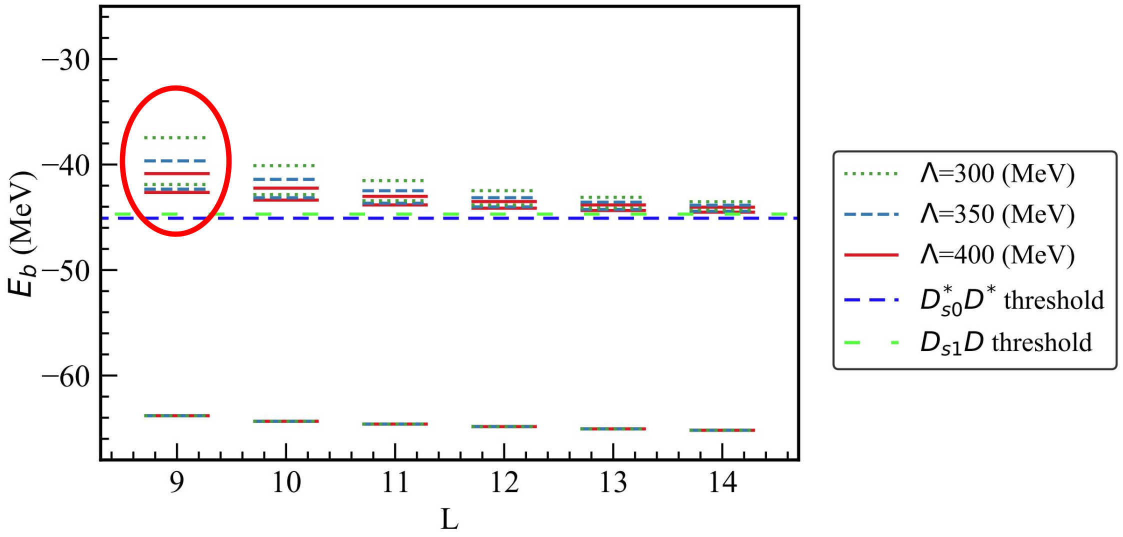
Different momentum cutoffs
Different parameters

Obtain the same excited states?

The values of the parameter $c_3$ on lattice with $\Lambda = 400, 350, 300$ MeV

| $\Lambda$ (MeV) | Parameter | $L$ = 9 | 10 | 11 | 12 | 13 | 14 | State |
|---|---|---|---|---|---|---|---|---|
| 400 | $c_3$ (MeV$^{-5}$) | 0.100 | 0.100 | 0.100 | 0.100 | 0.100 | 0.100 | Input |
| 350 | | 0.170 | 0.162 | 0.164 | 0.164 | 0.163 | 0.163 | Fitted |
| 300 | | 0.328 | 0.305 | 0.281 | 0.278 | 0.281 | 0.280 | Fitted |

$DD^*K$ excited states (**S-wave**)




- The three-body binding energy of the ground and the first excited state with the different cutoffs
- The first excited states with different cutoffs **coincide with each other** when the box size goes large
- Two close excited states correspond to the rho-type and lambda-type excitation in the quark model
- The quantum number is $1^-$: Use the standard angular momentum and parity projection technique[1]

$$|\Psi_A\rangle = \frac{d_n}{24}\sum_{i=1}^{24}\chi_n(\Omega_i)R(\Omega_i)|\Psi_0\rangle$$


[1]B.-N. Lu et al., Phys. Rev. D 90, 034507 (2014).

03

# Summary

# Summary

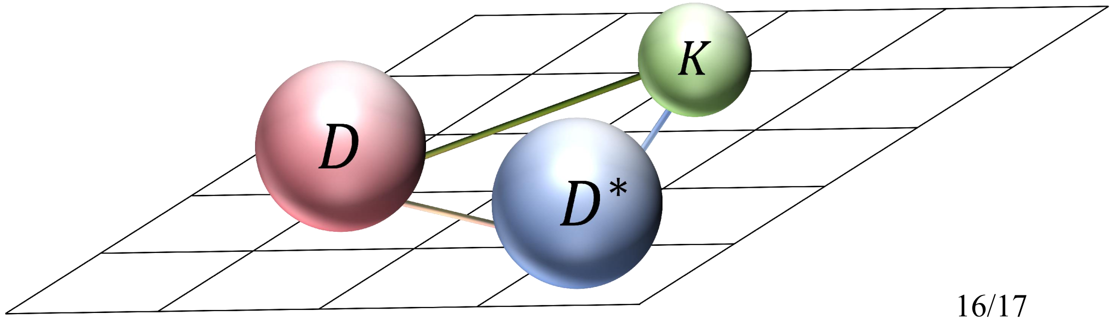
(1) We applied the lattice EFT to study exotic hadron.

- $DD^*K$ three-body system has a clear **bound state** with quantum number $J^P = 1^-$
- Binding energy is within the range of (-84, -44) MeV

(2) To check the renormalization group invariance of our framework

- The first excited states with different cutoffs **coincide with each other** when the box size goes large
- Splitting of the first excited state due to radial excitation between different constitents, like the $\rho$, $\lambda$ excitations when calculating the baryon excitation in the quark model.

(3) Lattice EFT, as an Ab initio calculation for many-body problem, is promising for the study of exotic hadrons.



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# THANKS !!!