

- ϕ -N bound state in the unitary coupled-channel approximation

- **Bao-Xi Sun (孙宝玺)**

- **Beijing University of Technology**

- **2024.09.24**

-

1. ϕ -N bound state?

1. H. Gao et al., PRC, 63, 2201(R), 2001

QCD van der Waals potential

2. F. Huang et al., PRC, 73, 025207, 2006

Chiral quark model, intermediate scalar meson is dominant

3. B. X. Sun et al., CTP, 75, 055301, 2023

Bethe-Salpeter equation in the unitary coupled-channel approximation

4. S. Acharya et al., PRL, 127, 172301, 2021

The ϕ -p attractive interaction by ALICE Collaboration

1. Content

- 1. Bethe-Salpeter equation in the unitary coupled-channel approximation
- 2. Including vector mesons in the Lagrangian with the hidden gauge symmetry.
- 3. The attractive ϕ - p interaction supplied by ALICE collaboration.
- 4. The ϕ -proton bound state at 1949 MeV

2. Bethe-Salpeter equation

Unitarity: The unitarity of the scattering amplitude must be conserved

$$S^+ S = 1$$

Since $S = I - iT$,

The T-amplitude can be obtained by solving the Bethe-Salpeter equation in the on-shell approximation:

$$T = V + VGT = (1 - VG)^{-1} V.$$

2. Bethe-Salpeter equation

V denotes the interaction vertex

G represents the loop function, in the dimensional regularization scheme, the loop function takes a

explicit form:

$$G_i(\sqrt{s}) = \frac{2M_i}{(4\pi)^2} \left\{ a_i(\mu) + \log \frac{m_i^2}{\mu^2} + \frac{M_i^2 - m_i^2 + s}{2s} \log \frac{M_i^2}{m_i^2} \right\} \\ + \frac{2M_i}{(4\pi)^2} \cdot \frac{Q_i(\sqrt{s})}{\sqrt{s}} \left[\log(s - (M_i^2 - m_i^2) + 2\sqrt{s}Q_i(\sqrt{s})) + \log(s + (M_i^2 - m_i^2) + 2\sqrt{s}Q_i(\sqrt{s})) \right. \\ \left. - \log(-s + (M_i^2 - m_i^2) + 2\sqrt{s}Q_i(\sqrt{s})) - \log(-s - (M_i^2 - m_i^2) + 2\sqrt{s}Q_i(\sqrt{s})) \right]$$

with the three-momentum in the center of mass system

$$Q_i(\sqrt{s}) = \frac{\sqrt{(s - (M_i + m_i)^2)} \sqrt{(s - (M_i - m_i)^2)}}{2\sqrt{s}}$$

2. Bethe-Salpeter equation

The decay widths of intermediate rho and K* mesons are taken into account in the loop function:

$$\tilde{G}(\sqrt{s}) = \frac{1}{N} \int_{(m_1-2\Gamma_1)^2}^{(m_1+2\Gamma_1)^2} d\tilde{m}_1^2 \left(-\frac{1}{\pi} \right) \text{Im} \frac{1}{\tilde{m}_1^2 - m_1^2 + im_1\tilde{\Gamma}_1(\tilde{m}_1)} \times G(s, \tilde{m}_1^2, M^2),$$

with

$$N = \int_{(m_1-2\Gamma_1)^2}^{(m_1+2\Gamma_1)^2} d\tilde{m}_1^2 \left(-\frac{1}{\pi} \right) \text{Im} \frac{1}{\tilde{m}_1^2 - m_1^2 + im_1\tilde{\Gamma}_1(\tilde{m}_1)}.$$

3. Hidden Gauge Symmetry

The interaction of the vector meson with the baryon octet can be involved according to the hidden gauge symmetry:

U. G. Meissner, *Phys. Rept.* 161, 213 (1988).

M. Bando et al., *Phys. Rev. Lett.* 54, 1215 (1985).

M. Bando et al., *Phys. Rep.* 164, 217 (1988).

M. Harada and K. Yamawaki, *Phys. Rep.* 381, 1 (2003).

H. Nagahiro et al., *Phys. Rev. D* 79, 014015 (2009).

S. Sarkar, B. X. Sun, E. Oset and M. J. Vicente Vacas, *EPJA* 44, 431 (2010)

E. Oset and A. Ramos, *EPJA* 44, 445 (2010)

3. Hidden Gauge Symmetry

Therefore, the vertex of three vector mesons can be obtained with the Lagrangian

$$L_{3V} = ig \langle (V^\mu \partial_\nu V_\mu - \partial_\nu V_\mu V^\mu) V^\nu \rangle,$$

The coupling of the vector meson and the baryon octet comes from

$$L_{BBV} = g \left(\langle \bar{B} \gamma_\mu [V^\mu, B] \rangle + \langle \bar{B} \gamma_\mu B \rangle \langle V^\mu \rangle \right),$$

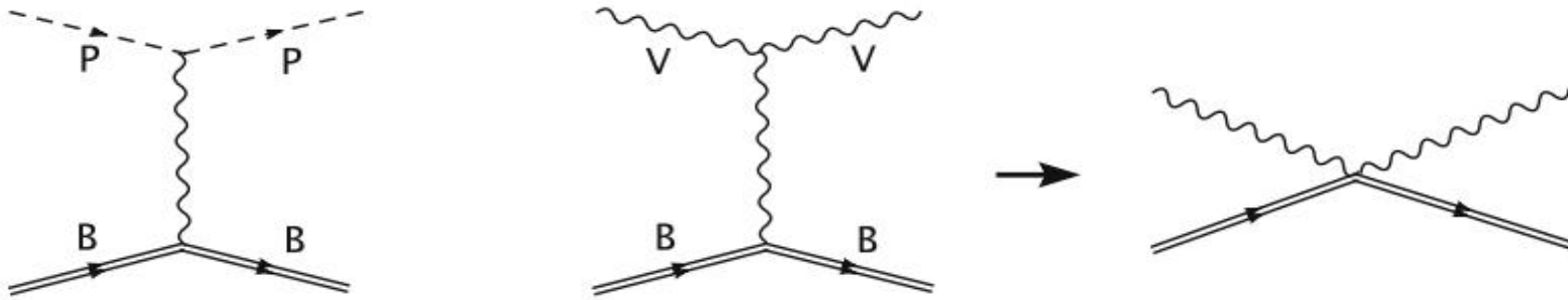
with

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho^0 + \omega & \sqrt{2}\rho^+ & \sqrt{2}K^{*+} \\ \sqrt{2}\rho^- & -\rho^0 + \omega & \sqrt{2}K^{*0} \\ \sqrt{2}K^{*-} & \sqrt{2}\bar{K}^{*0} & \sqrt{2}\phi \end{pmatrix}, \quad B = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix},$$

3. Hidden Gauge Symmetry

In analogy with the interaction of a pseudoscalar meson and a baryon, the vector-baryon interaction can be constructed similarly ,

S. Sarkar et al.,EPJA 44, 431 (2010)



and the kernel takes the form of

$$V_{ij} = -C_{ij} \frac{1}{4f^2} (k^0 + k'^0) \varepsilon \cdot \varepsilon'^*,$$

3. Hidden Gauge Symmetry

The coefficients C_{ij} for $S=0$ and $I=1/2$ are listed as

	ρN	ωN	ϕN	$K^* \Lambda$	$K^* \Sigma$
ρN	2	0	0	3/2	-1/2
ωN		0	0	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$
ϕN			0	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$
$K^* \Lambda$				0	0
$K^* \Sigma$					2

4. phi-proton interaction

S. Acharya et al., PRL, 127, 172301, 2021

The ALICE collaboration reports the experimental evidence on the attractive phi-proton interaction, and the real and imaginary parts of the scattering length read respectively,

$$\begin{aligned}\operatorname{Re}(f_0) &= 0.85 \pm 0.34(\text{stat}) \pm 0.14(\text{syst}) \text{ fm}, \\ \operatorname{Im}(f_0) &= 0.16 \pm 0.10(\text{stat}) \pm 0.09(\text{syst}) \text{ fm}.\end{aligned}$$

Apparently, the phi-p elastic scattering plays a dominant role, and the inelastic process is less important.

4. phi-proton interaction

S. Acharya et al., PRL, 127, 172301, 2021

The phi-p interaction take a Yukawa type of potential,

$$V(r) = -A \frac{e^{-\alpha r}}{r}, \quad \begin{aligned} A &= 0.021 \pm 0.009 \pm 0.006 \text{ (syst)}, \\ \alpha &= 65.9 \pm 38.0 \text{ (stat)} \pm 17.5 \text{ (syst)} \text{ MeV}^{-1}. \end{aligned}$$

It is different from the theoretical works where a phi-p bound state is predicted with the same kind of potential but

$$A = 1.25, \quad \alpha = 600 \text{ MeV}^{-1}.$$

H. Gao et al., PRC, 63, 022201(R), 2001

F. Huang et al., PRC, 73, 025207, 2006

4. phi-proton interaction

B. X. Sun et al., Commun. Theor. Phys.,75, 055301, 2023

In this work, the phi-p attractive interaction is included in the kernel of the Bethe-Salpeter equation in the unitary coupled-channel approximation. In the momentum space,

$$V(r) = -\frac{g_{\phi N}^2}{\vec{q}^2 + \alpha^2}, \quad A = \frac{g_{\phi N}^2}{4\pi}.$$

the three-momentum transfer is neglected in the calculation since the potential is only valid near the phi-p threshold. Moreover, a mass of the phi meson is added in the kernel when the BS equation is solved consistently.

5. Parameters

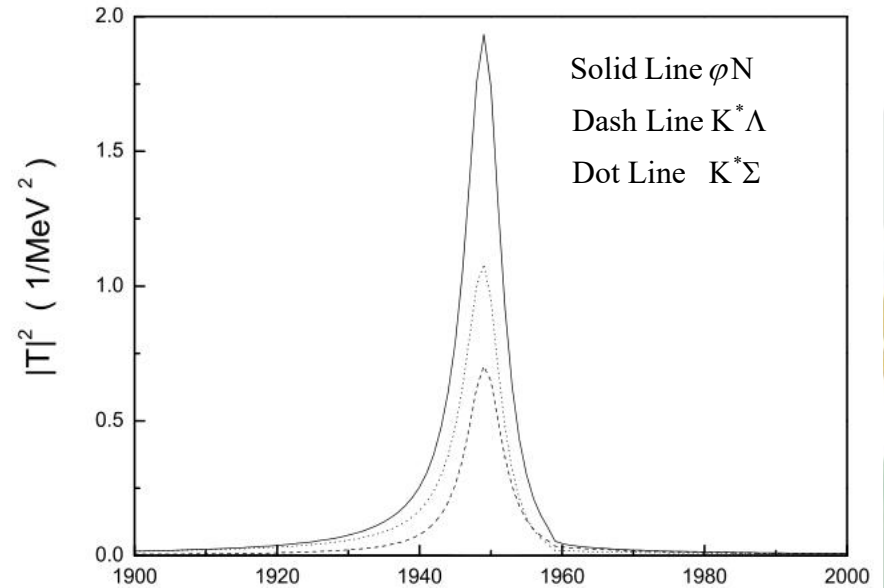
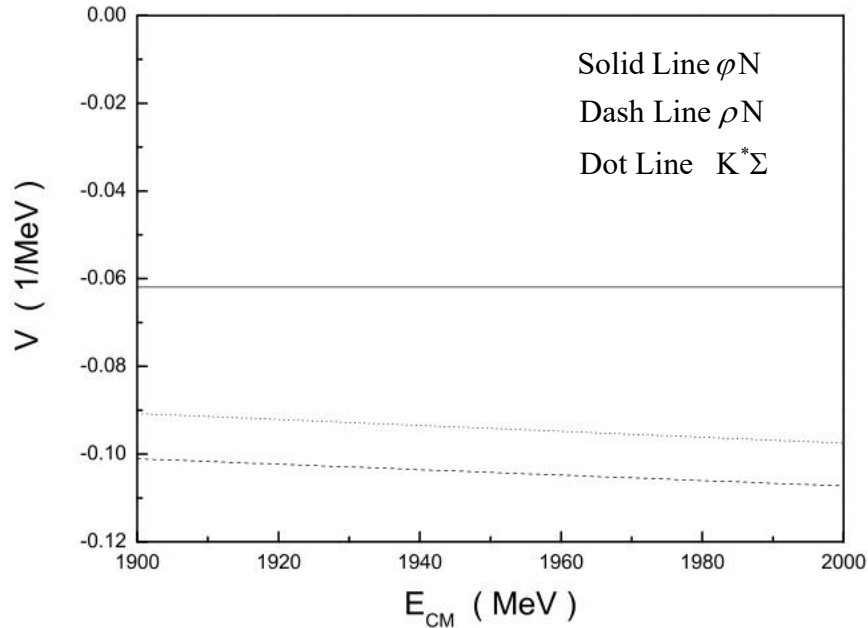
The phi-N scattering length can be written as

$$f_{\varphi N} = \frac{M_N}{8\pi\sqrt{s}} T_{\varphi N \rightarrow \varphi N}(\sqrt{s} = m_\varphi + M_N).$$

By fitting the experimental value of $f_{\varphi N}$, the subtraction constants are fixed as below:

$f_0(fm)$	$\mu(MeV)$	$a_{\rho N}$	$a_{\omega N}$	$a_{\varphi N}$	$a_{K^*\Lambda}$	$a_{K^*\Sigma}$
0.86+i0.19	630	-2.0	-2.0	-2.4	-1.9	-1.8

6. Results



The pole positions and couplings to different channels are listed as

	ρN	ωN	φN	$K^* \Lambda$	$K^* \Sigma$
1949-i3	0.0+i0.0	0.1+i0.0	2.1+i0.1	1.6+i0.3	1.8+i0.0
1969-283	0.1+i0.1	0.0+i0.2	0.1-i0.1	0.3-i0.4	0.2-i0.0

6. Results

The pole at $1949-i3\text{MeV}$ is about 10MeV lower than the $\phi\text{-N}$ threshold, so it can be regarded as a $\phi\text{-N}$ bound state, which couples strongly to the ϕN , $K^*\Lambda$ and $K^*\Sigma$ channels.

Pole	name	J^P	status	mass	width
1969-i283	N(1895)	1/2-	****	1890-1930	80-140
	N(1875)	3/2-	***	1850-1950	100-220

The pole at $1969-i283\text{MeV}$ might correspond to the N(1895) or N(1875) particle in the PDG data.

S. Navas et al.(PDG), PRD, 110, 030001, 2024

7. Discussion

In the momentum space,

$$V(r) = -\frac{g_{\varphi N}^2}{\vec{q}^2 + \alpha^2}, \quad A = \frac{g_{\varphi N}^2}{4\pi}.$$

the three-momentum transfer is neglected in the calculation since the potential is only valid near the φ - p threshold.

The pole positions are stable at the case of $\vec{q} \neq 0$.

Only the couplings to different channels change slightly.

7. Discussion

When the \vec{q} is neglected in the Yukawa potential, the ratio $g_{\varphi N}^2 / \alpha^2$ becomes critical in the calculation

1. B. X. Sun et al., CTP, 75, 055301, 2023,

$$g_{\varphi N}^2 / \alpha^2 \approx 6.0 \times 10^{-5} \text{ MeV}^{-2}.$$

2. H. Gao et al., PRC, 63, 2201(R), 2001,

$$g^2 / \alpha^2 \approx 4.4 \times 10^{-5} \text{ MeV}^{-2}.$$

$$A = 1.25, \quad \alpha = 600 \text{ MeV}.$$

3. F. Huang et al., PRC, 73, 025207, 2006,

$$g_{\sigma N}^2 / \alpha^2 \approx 2.4 \times 10^{-5} \text{ MeV}^{-2}.$$

8. Works on π -p bound state

E. Chizzali et al., Indication of a π - π bound state from a correlation function analysis, *PLB*, 848, 138358 (2024)

A. Feijoo et al., Relevance of the coupled channels in the π - π and ρ - π Correlation Functions, [[arXiv:2407.01128 \[hep-ph\]](https://arxiv.org/abs/2407.01128)].

A. Kuros et al., π - π bound state and completeness of quantum states, [[arXiv:2408.11941 \[hep-ph\]](https://arxiv.org/abs/2408.11941)].

L. M. Abreu et al., A study of the π -N correlation function, [[arXiv:2409.05170 \[hep-ph\]](https://arxiv.org/abs/2409.05170)].

9. Problem?

The resonance at 1969- i 283MeV is higher than the ϕ -N threshold, while the N(1895) and N(1875) particles are both under the ϕ -N threshold.

Hidden gauge symmetry is only reliable approximately.

Difference of the mass of vector mesons?

High-order correction?

.....Other reasons?

10. Summary

B. X. Sun et al., CTP, 75, 055301, 2023

1. Base on the attractive ϕ -N interaction announced by ALICE collaboration, a ϕ -N bound state is generated by solving the Bethe-Salpeter equation.
2. A correct ϕ -p scattering length is obtained in the calculation.
3. A ϕ -N resonance state is produced dynamically in the calculation.
4. The consistency with other theoretical works has been discussed.