

QUARK MODEL WITH HIDDEN LOCAL SYMMETRY AND ITS APPLICATION TO THE MULTI QUARK SYSTEMS

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2306.03526
2307.16280

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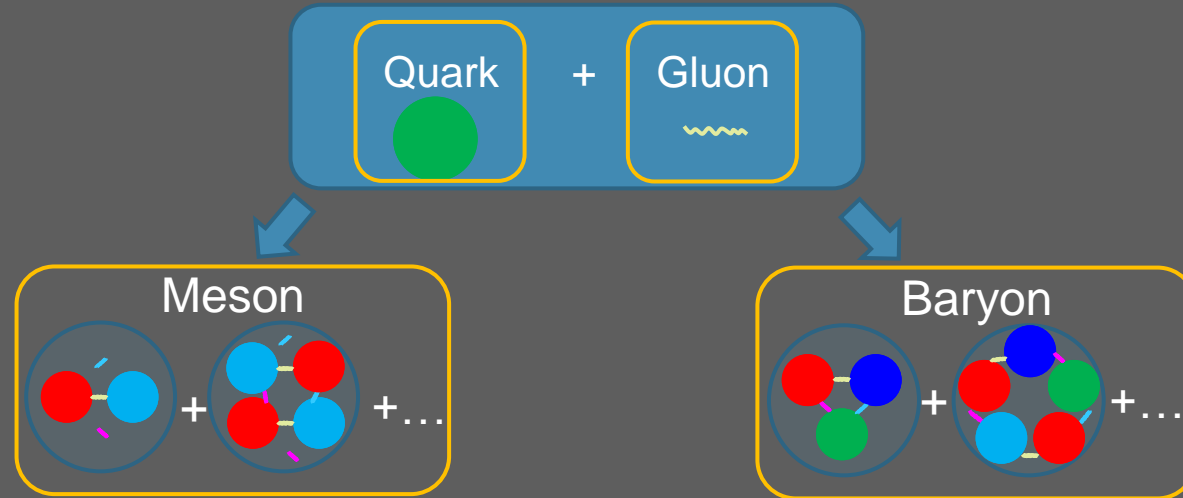
Outline

➤ Introduction

- ⊙ Chiral quark model with HLS
- ⊙ $SU2$ ground states + excited states
- ⊙ $SU3$ ground states
- ⊙ Summary

Introduction

Hadron are made by quarks and gluons

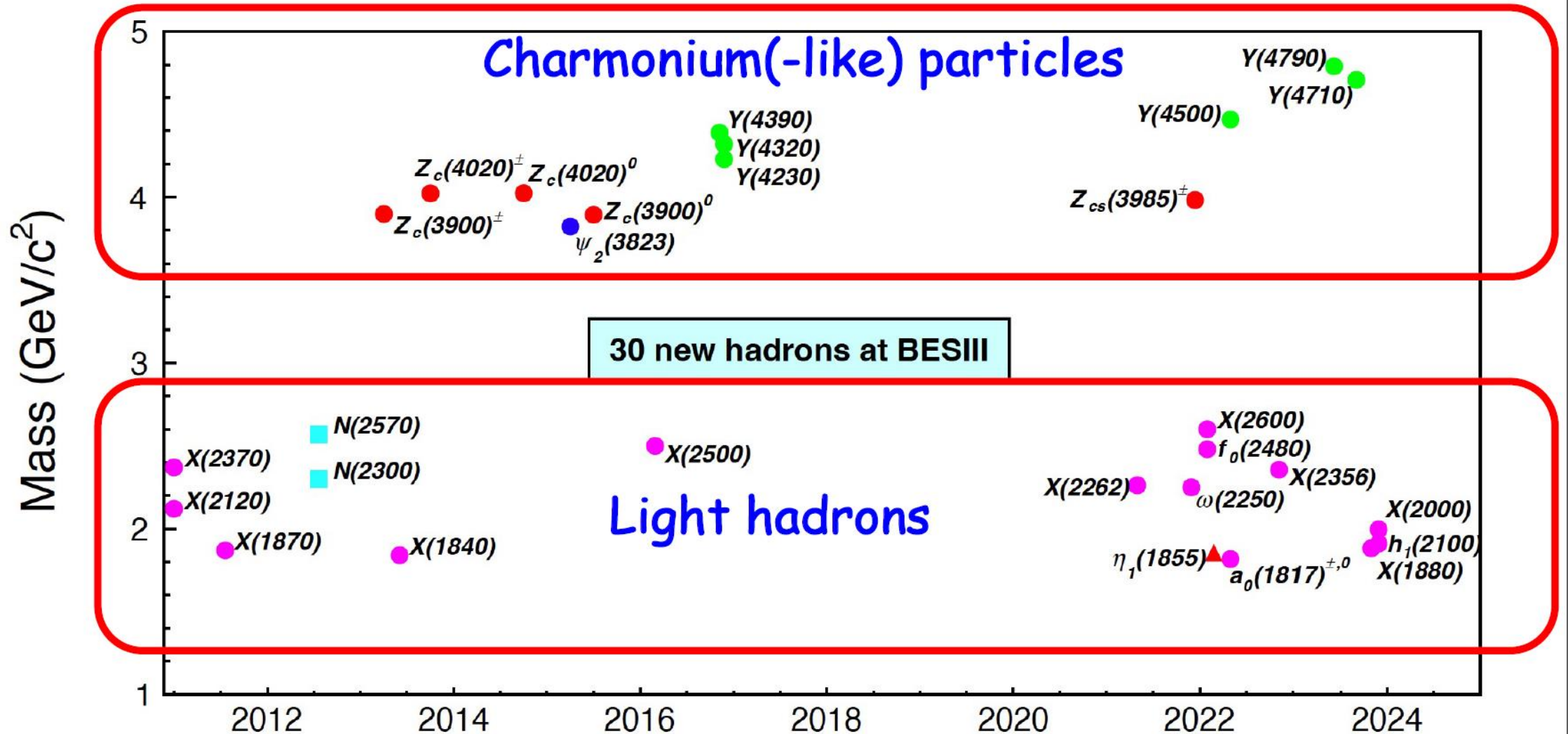


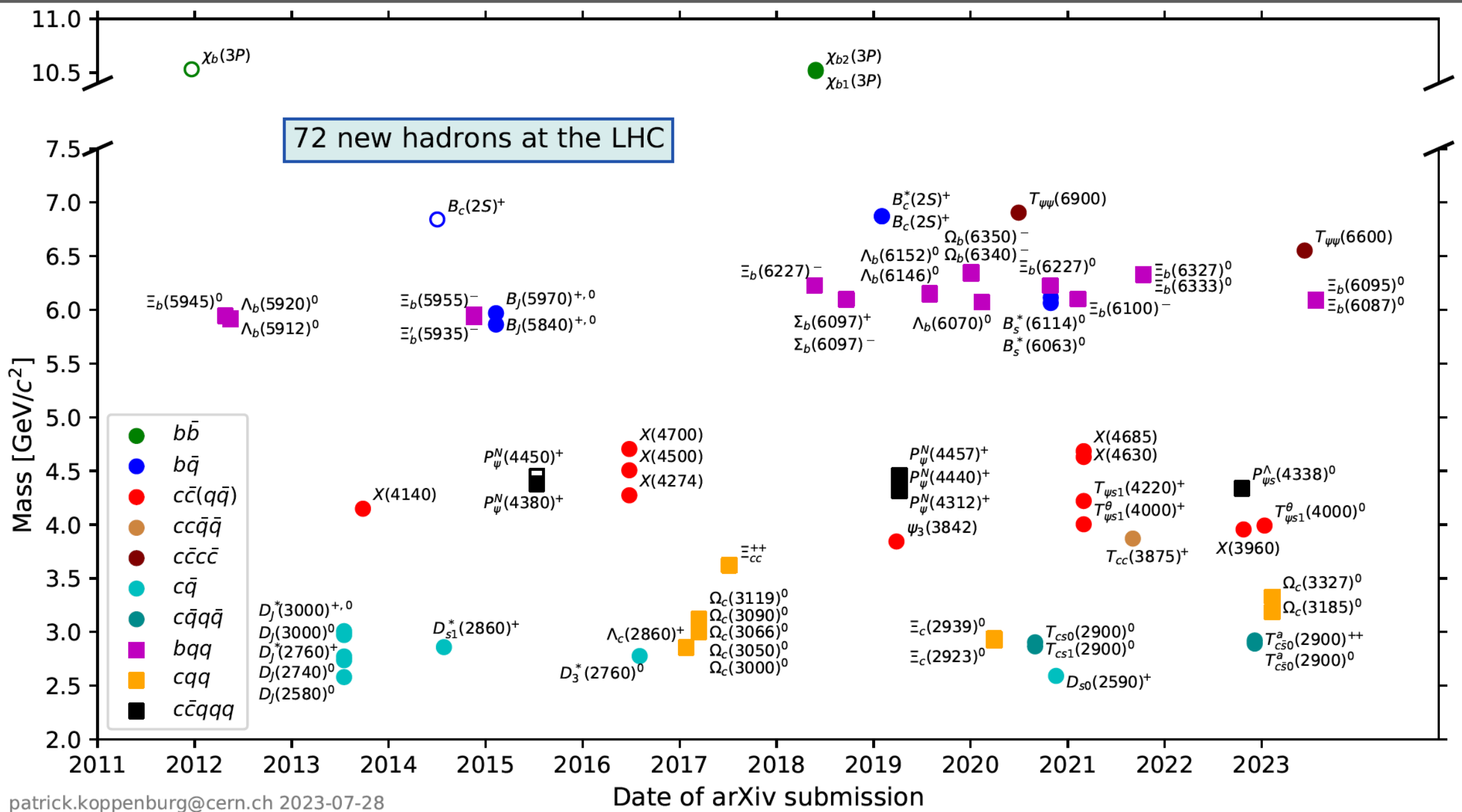
The dynamics of quarks and gluons are described by Quantum chromodynamics (QCD)

- QCD have two important features:
 - ◆ Color confinement
 - ◆ Asymptotic freedom
- In low energy region the perturbative calculation for QCD is impossible, alternatively:
 - ◆ Lattice QCD (non-perturbative calculation)
 - ◆ Effective models (chiral perturbation theory, quark model, etc...)

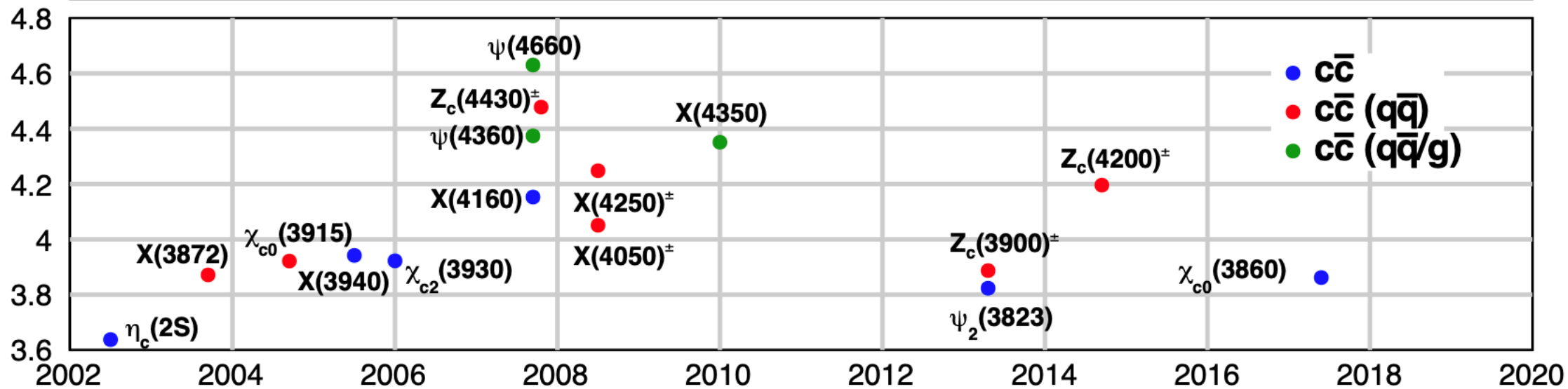
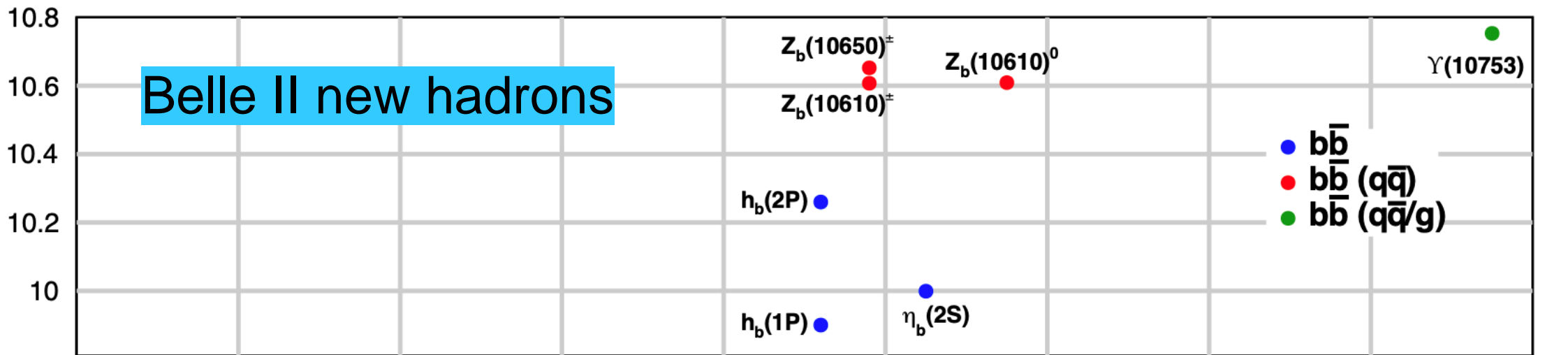
New resonant structures at BESIII

From Prof. Shuang-Shi Fang





Mass [GeV]



Data of arXiv submission

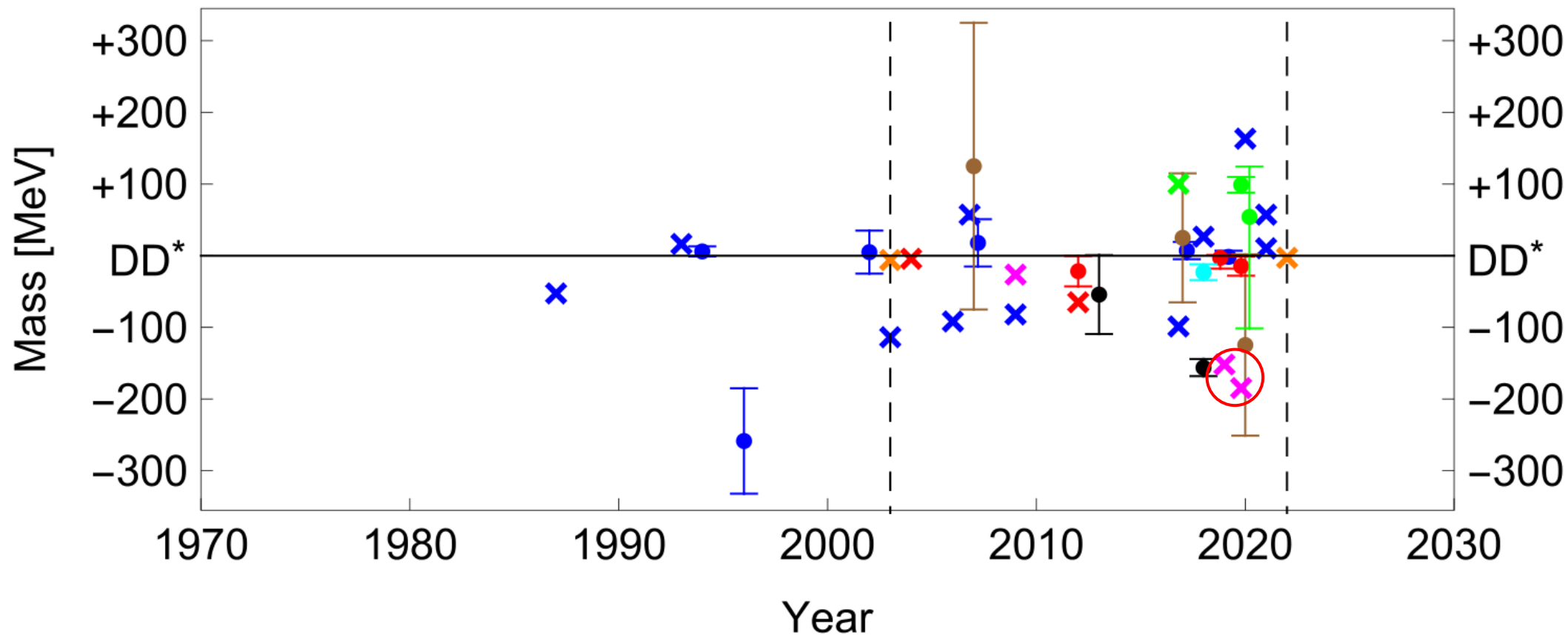


Figure 42. Theoretical predictions on the mass of the doubly charmed tetraquark state $cc\bar{u}\bar{d}$ with $(I)J^P = (0)1^+$, with uncertainties (error bars) and without uncertainties (crosses), calculated based on the compact tetraquark picture through various quark models [454, 500–515] (blue), QCD sum rules [462, 516, 517] (brown), heavy quark symmetry [467–469] (green), and others [461, 466] (black), as well as those calculated through the hadronic molecular picture [477, 479, 518–520] (red), the quark model considering the mixture of the meson-meson and diquark–antidiquark structures [481–483] (magenta), and lattice QCD [499] (cyan). The two dashed lines with orange crosses denote the $\chi_{c1}(3872)$ ($X(3872)$) first observed by Belle in 2003 [30] and the T_{cc}^+ recently observed by LHCb in 2021 [42, 43].

Meson exchange in nuclear force:

- $\pi(138)$ **Long-ranged** tensor force
- $\sigma(500)$ **intermediate-ranged**, attractive central force plus LS force
- $\omega(782)$ **short-ranged**, repulsive central force plus strong LS force
- $\rho(770)$ **short-ranged** tensor force, opposite to pion

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- ◎ $SU3$ ground states
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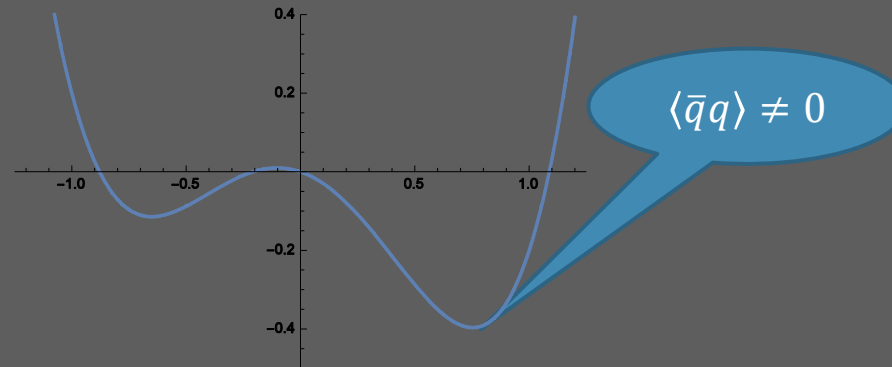
The chiral symmetry

The chiral symmetry:



Spontaneously breaking of chiral symmetry:

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$



The effective theory based on chiral symmetry:

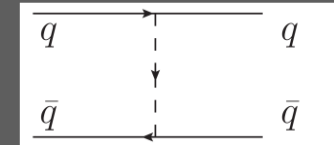
- Nonlinear sigma model
- Chiral perturbation theory

Chiral quark model

Naïve quark model:

- Quark mass term
- Kinetic term
- Color confinement potential (CON)
- One gluon exchange (OGE)

gluon exchange

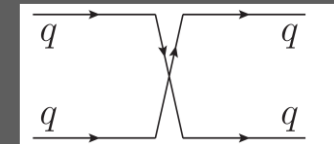
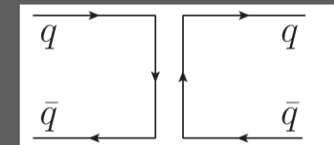


- Gell-Mann, M., 1964, Phys. Lett. 8, 214.
- Zweig, G., 1964, CERN Reports No. 8182/TH. 401 and No. 8419/TH. 412).
- N. Isgur, G. Karl, Phys.Lett.B 72 (1977) 109.

The Nambu–Goldstone boson exchange:

- Chiral symmetry is spontaneously broken
- Pseudoscalars (π , K , η) are the Nambu–Goldstone (NG) bosons of chiral symmetry breaking
- Scalar meson σ as the chiral partner of NG bosons

meson exchange



- K. Shimizu, Phys. Lett. B 148, 418-422 (1984)
- Z.Y. Zhang, Y.W. Yu, P.N. Shen, L.R. Dai, A. Faessler, U. Straub, Nucl. Phys. A 625 (1997) 59.
- J. Vijande, F. Fernandez, A. Valcarce, J. Phys. G 31, 481(2005)

Chiral quark model

pseudoscalar + vector meson exchange + CON

- Quark mass term
- Kinetic term
- Color confinement potential (CON)
- Pseudo-scalar + vector meson

- L. Y. Glozman and D. O. Riska, Phys. Rept. 268, 263-303 (1996).
- L. Y. Glozman, Nucl. Phys. A 663, 103-112 (2000).

Scalar + pseudoscalar + vector meson exchange + CON + OGE

- Quark mass term
- Kinetic term
- Color confinement potential (CON)
- One gluon exchange (OGE)
- Scalar + pseudoscalar + vector meson

- L. R. Dai, Z. Y. Zhang, Y. W. Yu and P. Wang, Nucl. Phys. A 727, 321-332 (2003).
- Bing-Ran He, Masayasu Harada, Bing-Song Zou 2306.03526, 2307.16280

Incorporate the vector meson contribution

The hidden local symmetry:

M. Bando, T. Kugo, K. Yamawaki.
Phys.Rept. 164 (1988) 217-314
M. Harada, K. Yamawaki.
Phys.Rept. 381 (2003) 1-233

$$U = \xi_L^\dagger \xi_R = e^{2i \frac{\pi(x)}{f_\pi}}$$

$$\xi_{L,R} \rightarrow h(x) \xi_{L,R} \cdot g_{L,R}^\dagger$$

$$h(x)^\dagger h(x) = 1$$

$$\xi_{L,R} = e^{i \frac{V(x)}{f_V}} e^{\mp i \frac{\pi(x)}{f_\pi}}$$

$$h(x) \in H_{\text{local}}, \quad g_{L,R} \in G_{\text{global}}$$

- The transformation for U do not changes, which seems that the freedom of vector meson is “hidden”

$$[SU(N_f)_L \times SU(N_f)_R]_{\text{global}} \times [SU(N_f)_V]_{\text{local}} \rightarrow [SU(N_f)_V]_{\text{global}}$$

- Hidden local symmetry is an extension of chiral perturbation theory
- Hidden local symmetry is a systematic way to include pseudo-scalar mesons (π, K, η, η') and vector mesons (ρ, K^*, ω, ϕ)

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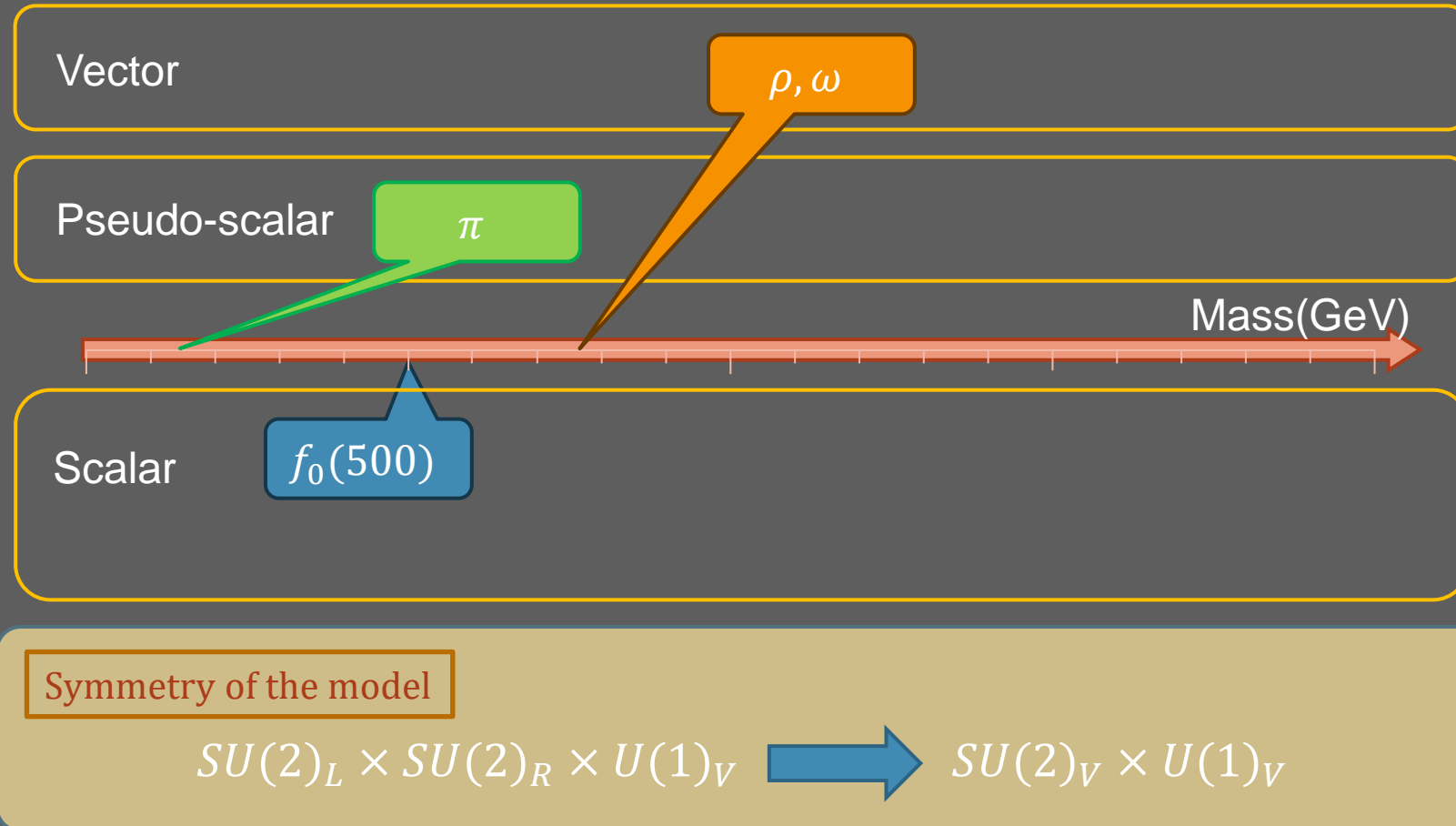
The Hamiltonian

$$H = \sum_{i=1} \left(m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1} (V_{ij}^{\text{CON}} + V_{ij}^{\text{OGE}} + V_{ij}^{\sigma} + V_{ij}^{\pi} + V_{ij}^{\omega} + V_{ij}^{\rho})$$

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Bing-Song Zou, 2306.03526

$$V_{ij}^v = \frac{\Lambda_v^2}{\Lambda_v^2 - m_v^2} \left\{ \frac{g_v^2}{4\pi} m_v \left[Y(m_v r) - \left(\frac{\Lambda_v}{m_v} \right) Y(\Lambda_v r) \right] + \frac{m_v^3}{m_i m_j} \left(\frac{g_v(2f_v + g_v)}{16\pi} + \frac{\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j (f_v + g_v)^2}{6 \cdot 4\pi} \right) \times \left[Y(m_v r) - \left(\frac{\Lambda_v}{m_v} \right)^3 Y(\Lambda_v r) \right] - \boldsymbol{S}_+ \cdot \boldsymbol{L} \frac{g_v(4f_v + 3g_v)}{8\pi} \frac{m_v^3}{m_i m_j} \times \left[G(m_v r) - \left(\frac{\Lambda_v}{m_v} \right)^3 G(\Lambda_v r) \right] - \boldsymbol{S}_{ij} \frac{(f_v + g_v)^2}{4\pi} \frac{m_v^3}{12m_i m_j} \times \left[H(m_v r) - \left(\frac{\Lambda_v}{m_v} \right)^3 H(\Lambda_v r) \right] \right\}$$

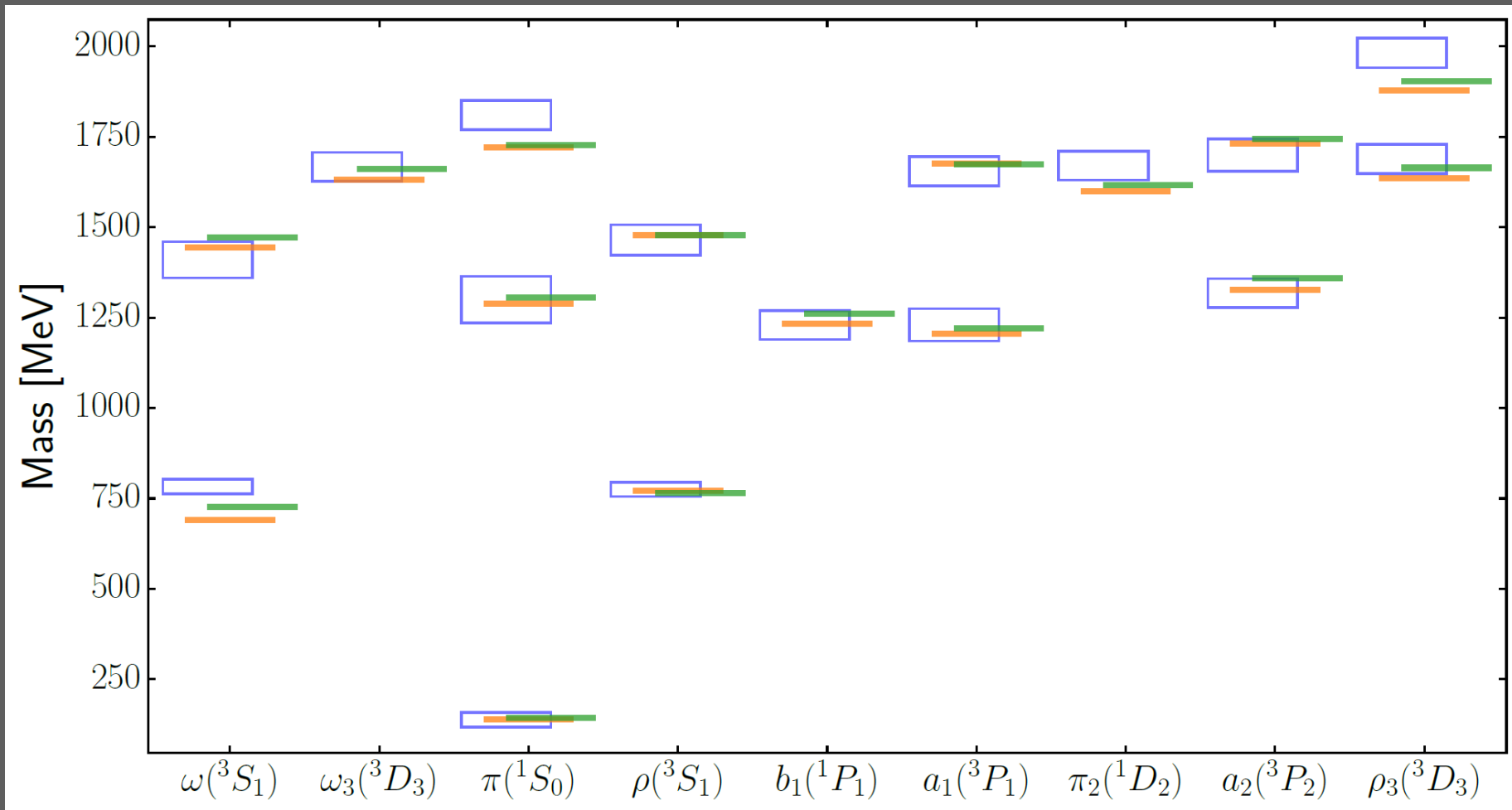
The exchanged mesons in $SU2$ model

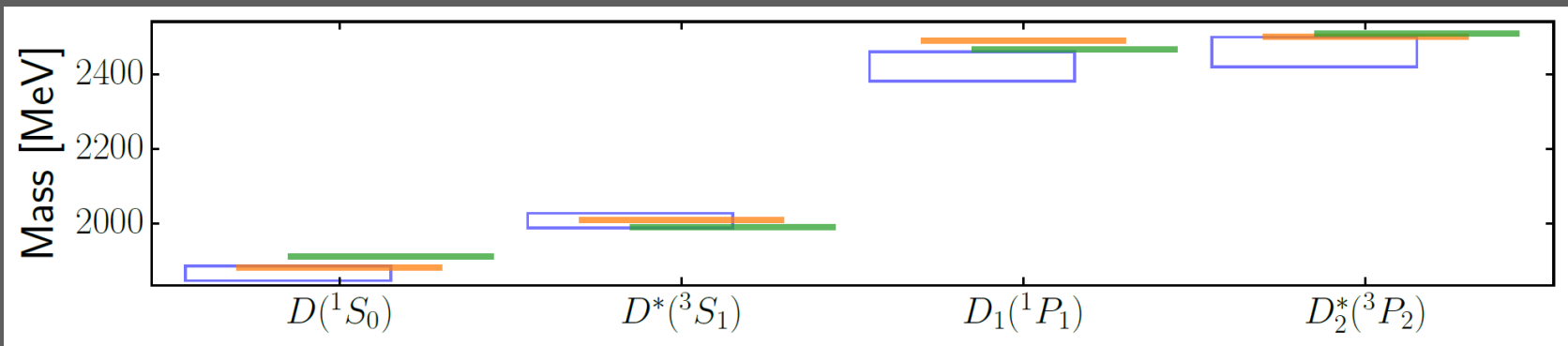
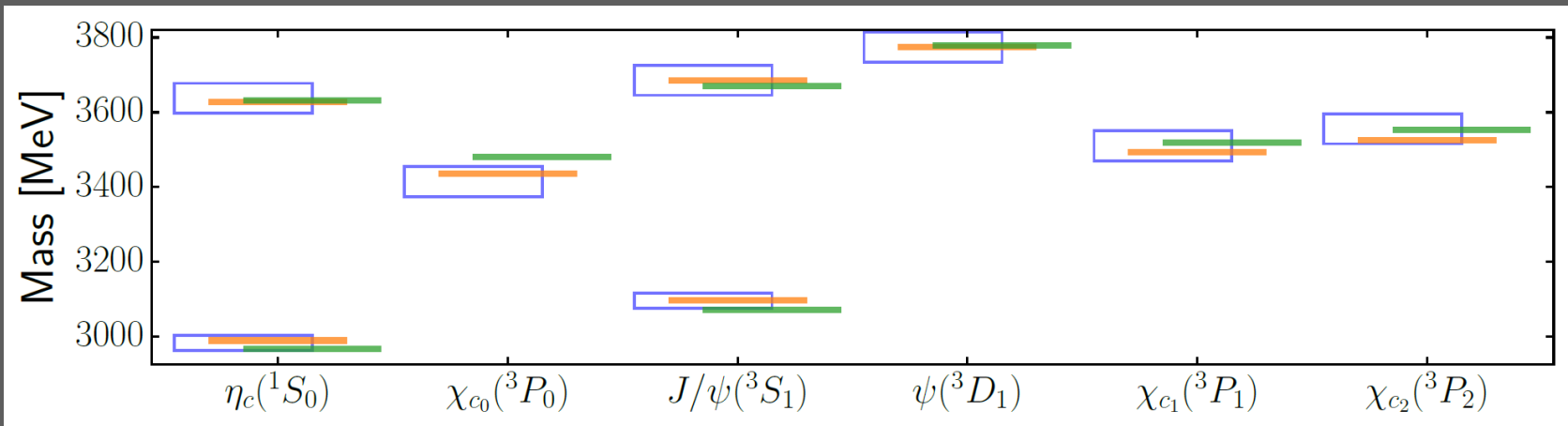


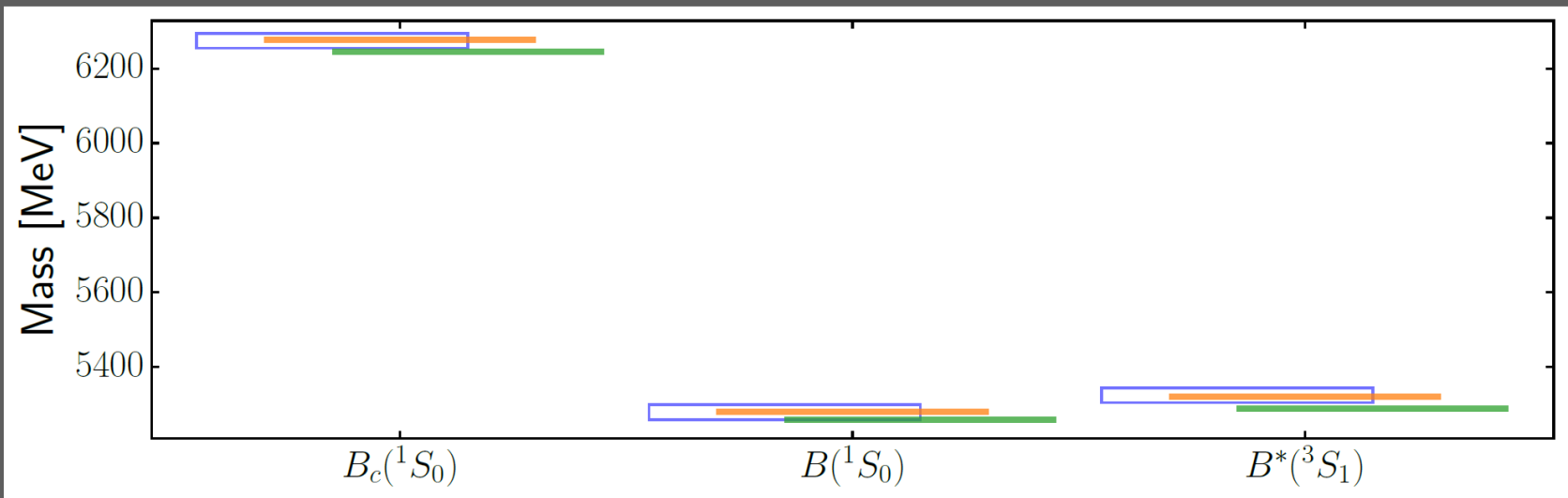
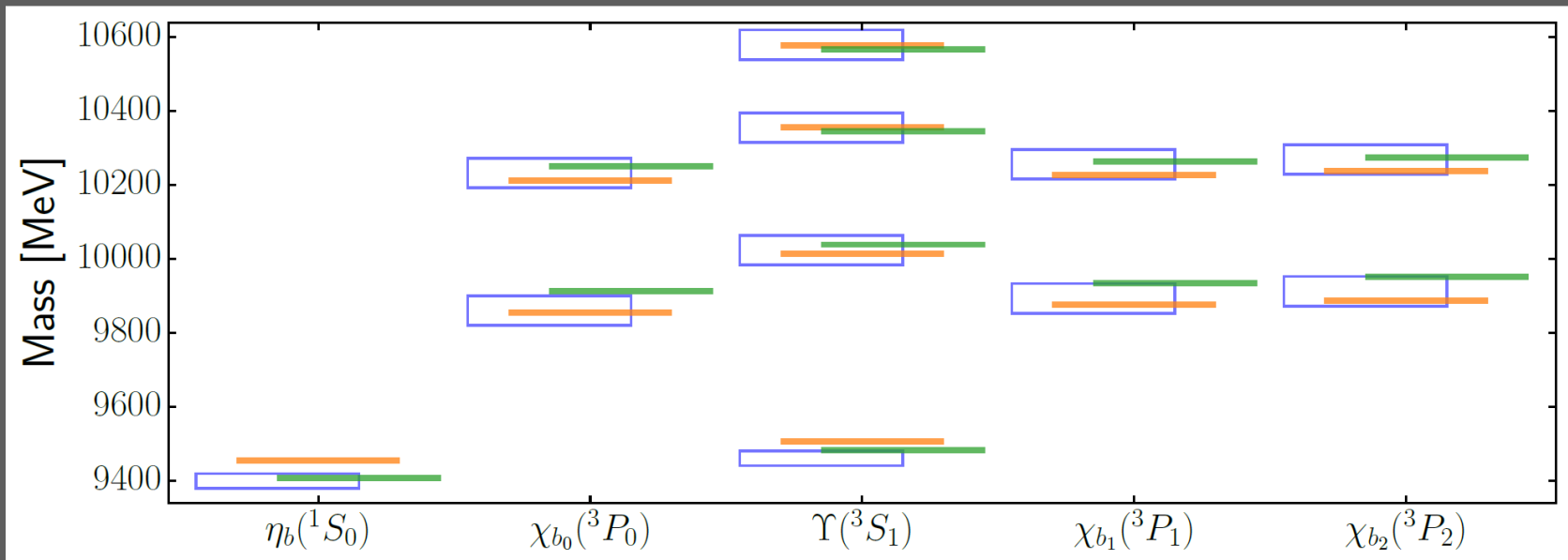
Wave functions

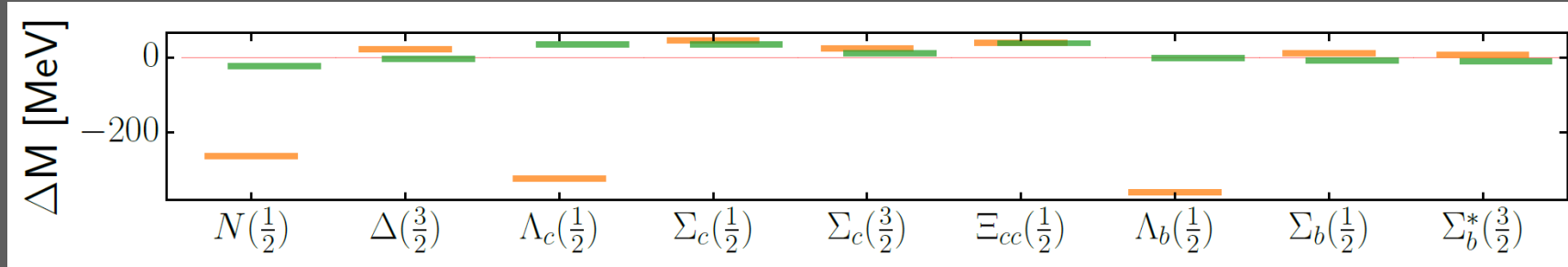
- Orbital (SO(3)): (ψ_L)
- Spin (SU(2)): (χ_S^σ)
- Flavor (SU(2)): (χ_I^f)
- Color (SU(3)): (χ^c)

$$\Psi_{JM_J IM_I}^{ijk} = \mathcal{A} \left[[\psi_L \chi_S^{\sigma i}]_{JM_J} \chi_I^{fj} \chi_k^c \right]$$









| | 1 | $\tau_i \tau_j$ | $\sigma_i \sigma_j$ | $\sigma_i \sigma_j \cdot \tau_i \tau_j$ |
|----------------------|-----|-----------------|---------------------|---|
| σ | -/- | | | |
| π | | | | +/- |
| a_0 | | -/+ | | |
| OGE | -/- | | +/+ | |
| CON | +/+ | | | |
| ω (This work) | +/- | | +/- | |
| ρ (This work) | | +/+ | | +/+ |

$qq/q\bar{q}$

| Channel | E_B | | Channel | E_B | |
|---|-------|-----|---|-------|-----|
| $[DD^*]_{1\otimes 1}$ | 6.2 | 39% | $[BB^*]_{1\otimes 1}$ | 0.5 | 12% |
| $[D^*D]_{1\otimes 1}$ | 6.2 | 39% | $[B^*B]_{1\otimes 1}$ | 0.5 | 12% |
| $[D^*D^*]_{1\otimes 1}$ | 83.1 | 5% | $[B^*B^*]_{1\otimes 1}$ | 31 | 32% |
| $[DD^*]_{8\otimes 8}$ | 383.1 | 0% | $[BB^*]_{8\otimes 8}$ | 253.6 | 0% |
| $[D^*D]_{8\otimes 8}$ | 383.1 | 0% | $[B^*B]_{8\otimes 8}$ | 253.6 | 0% |
| $[D^*D^*]_{8\otimes 8}$ | 337.3 | 0% | $[B^*B^*]_{8\otimes 8}$ | 233.3 | 0% |
| $[(cc)(\bar{q}\bar{q})^*]_{6\otimes \bar{6}}$ | 337.5 | 0% | $[(bb)(\bar{q}\bar{q})^*]_{6\otimes \bar{6}}$ | 233.7 | 0% |
| $[(cc)^*(\bar{q}\bar{q})]_{\bar{3}\otimes 3}$ | 120.3 | 17% | $[(bb)^*(\bar{q}\bar{q})]_{\bar{3}\otimes 3}$ | -37.8 | 44% |
| Mixed | -4.9 | | Mixed | -88.2 | |

| | r_{cc} | $r_{\bar{q}c}$ | $r_{\bar{q}\bar{q}}$ | $r_{\bar{q}b}$ | r_{bb} |
|----------|----------|----------------|----------------------|----------------|----------|
| T_{cc} | 1.56 | 1.24 | 1.70 | | |
| T_{bb} | | | 0.75 | 0.65 | 0.37 |

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The Hamiltonian

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Bing-Song Zou, 2307.16280

$$V_{ij}^{\bar{\sigma}} = V_{ij}^{s=\bar{\sigma}, g_s=g_{\bar{\sigma}q}} \lambda_i^q \lambda_j^q + V_{ij}^{s=\bar{\sigma}, g_s=g_{\bar{\sigma}s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\eta} = V_{ij}^{p=\eta, g_p=g_{\eta q}} \lambda_i^q \lambda_j^q + V_{ij}^{p=\eta, g_p=g_{\eta s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\eta'} = V_{ij}^{p=\eta', g_p=g_{\eta'q}} \lambda_i^q \lambda_j^q + V_{ij}^{p=\eta', g_p=g_{\eta's}} \lambda_i^s \lambda_j^s,$$

$$\lambda^q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_{ij}^{\pi} = V_{ij}^{p=\pi, g_p=g_{\pi}} \sum_{a=1}^3 \lambda_i^a \lambda_j^a,$$

$$V_{ij}^K = V_{ij}^{p=K, g_p=g_K} \sum_{a=4}^7 \lambda_i^a \lambda_j^a,$$

$$V_{ij}^{\omega} = V_{ij}^{v=\omega, g_v=g_{\omega q}} \lambda_i^q \lambda_j^q + V_{ij}^{v=\omega, g_v=g_{\omega s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\phi} = V_{ij}^{v=\phi, g_v=g_{\phi q}} \lambda_i^q \lambda_j^q + V_{ij}^{v=\phi, g_v=g_{\phi s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\rho} = V_{ij}^{v=\rho, g_v=g_{\rho}} \sum_{a=1}^3 \lambda_i^a \lambda_j^a,$$

$$V_{ij}^{K^*} = V_{ij}^{v=K^*, g_v=g_{K^*}} \sum_{a=4}^7 \lambda_i^a \lambda_j^a$$

The coupling have relations based on SU_3 flavor symmetry:

$$g_{\eta s} = g_{\eta q} - \sqrt{3} \cos \theta_p g_{\pi}$$

$$g_{\eta'q} = -\cot \theta_p g_{\eta q} + \frac{1}{\sqrt{3} \sin \theta_p} g_{\pi}$$

$$g_{\eta's} = -\cot \theta_p g_{\eta q} + \frac{\cos \theta_p \cot \theta_p - 2 \sin \theta_p}{\sqrt{3}} g_{\pi}$$

$$g_{\pi} = g_K$$

$$g_{\omega s} = g_{\omega q} - g_{\rho}$$

$$g_{\phi q} = -\sqrt{\frac{1}{2}} (g_{\omega q} - g_{\rho})$$

$$g_{\phi s} = -\sqrt{\frac{1}{2}} (g_{\omega q} + g_{\rho})$$

$$g_{\rho} = g_{K^*}$$

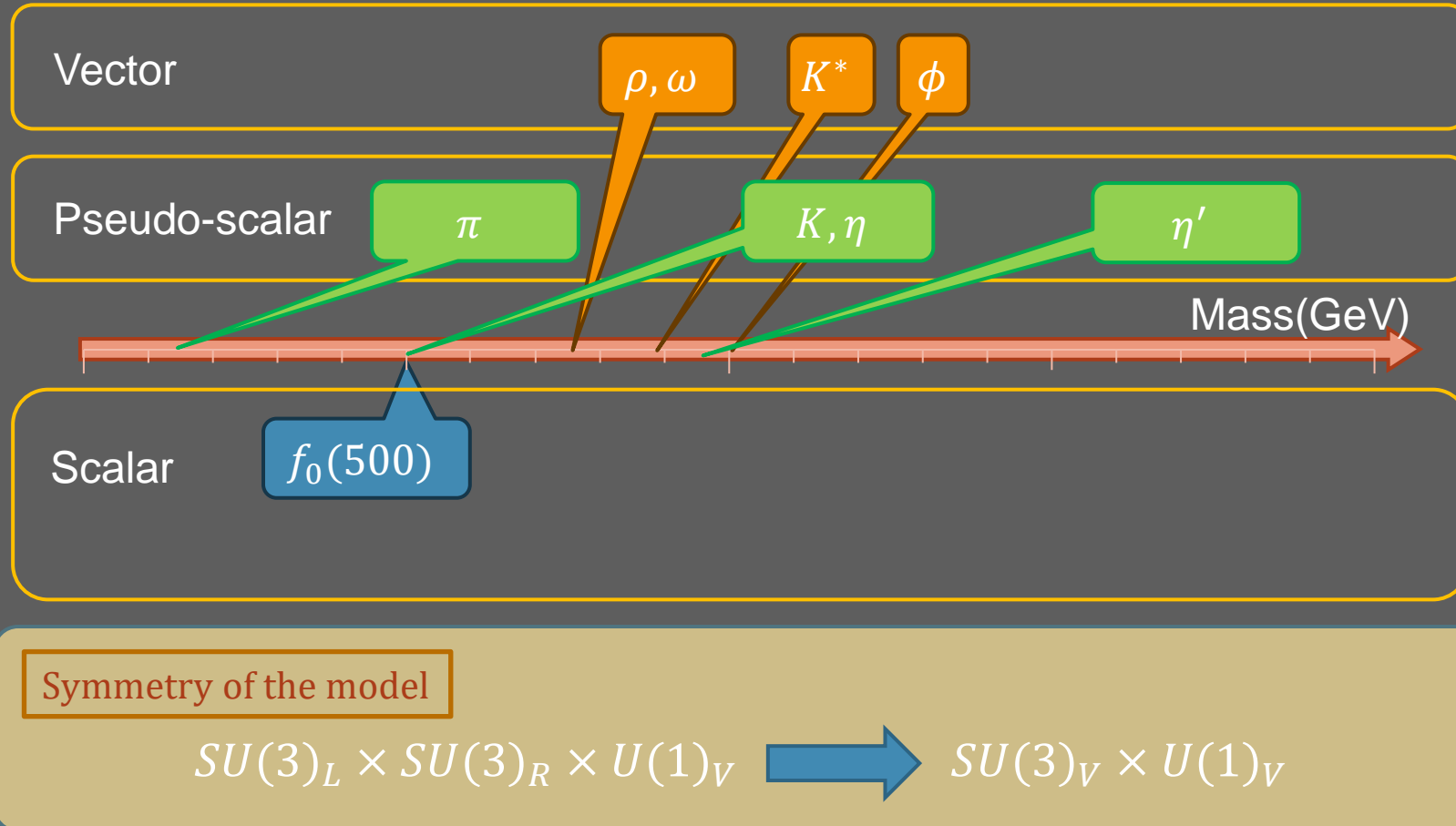
$$f_{\omega s} = f_{\omega q} - f_{\rho}$$

$$f_{\phi q} = -\sqrt{\frac{1}{2}} (f_{\omega q} - f_{\rho})$$

$$f_{\phi s} = -\sqrt{\frac{1}{2}} (f_{\omega q} + f_{\rho})$$

$$f_{\rho} = f_{K^*}$$

The exchanged mesons in $SU3$ model



Wave functions

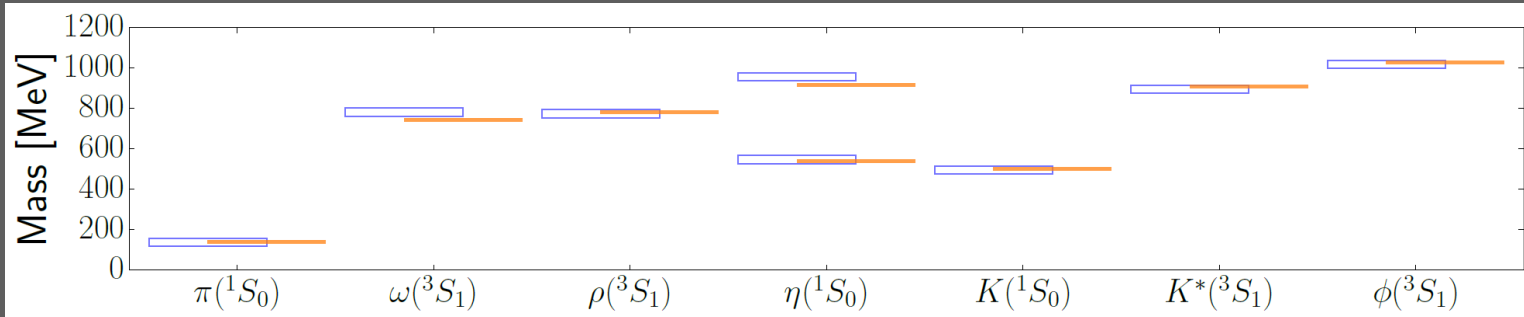
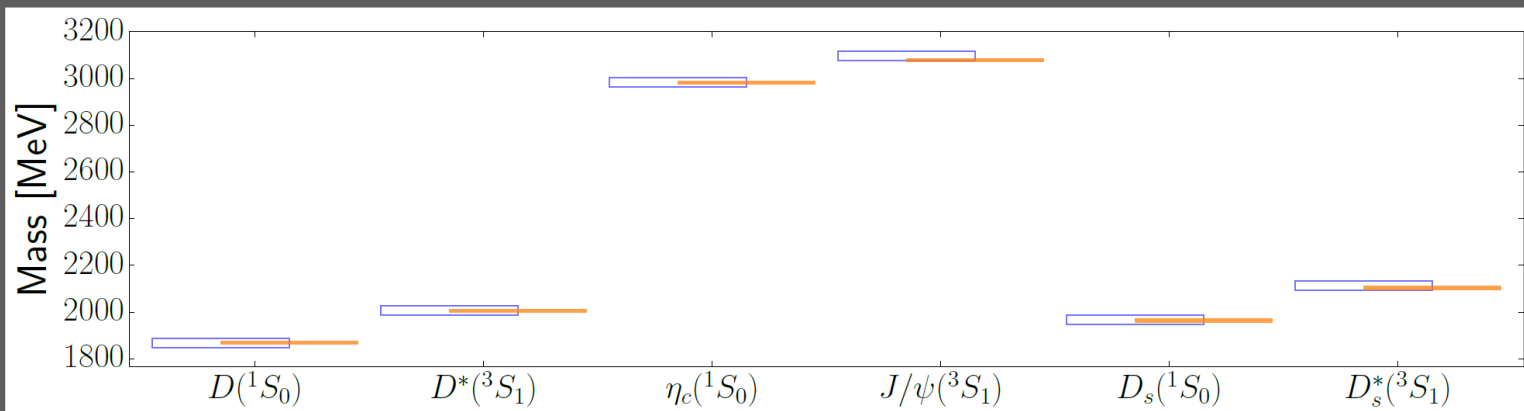
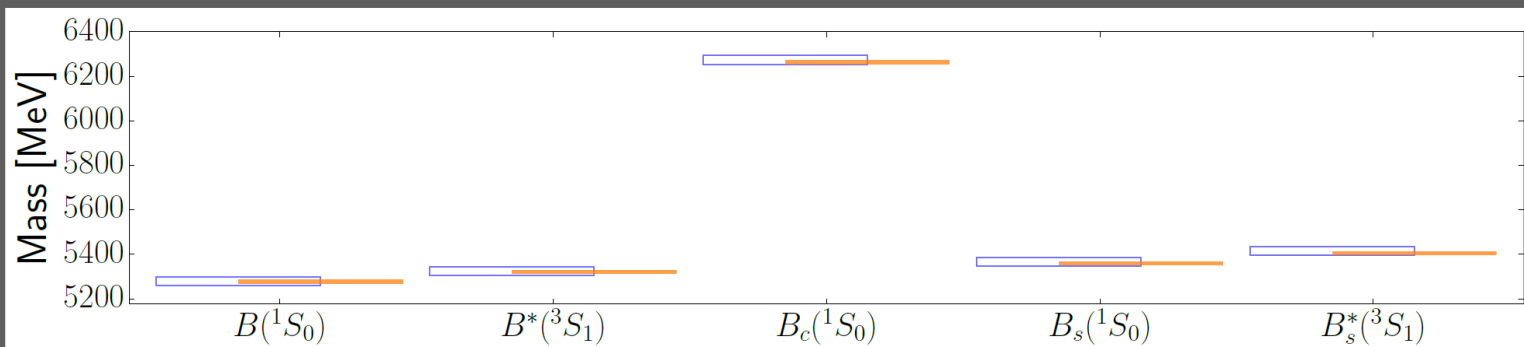
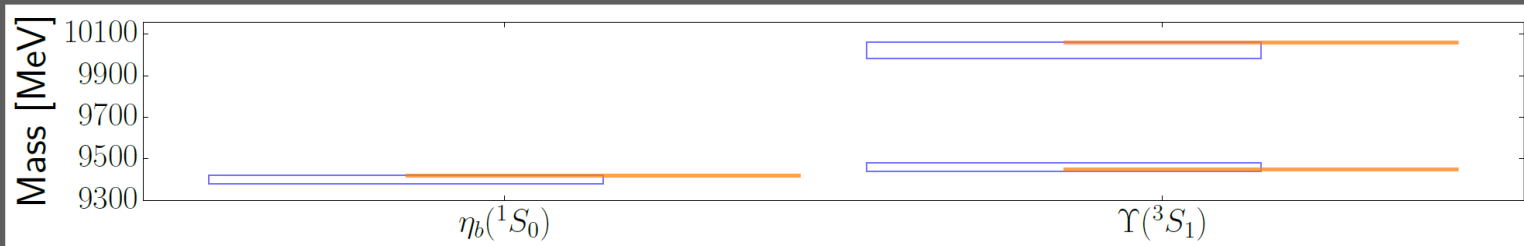
- Orbital (SO(3)): (ψ_L)
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- Color (SU(3)): (χ^c)

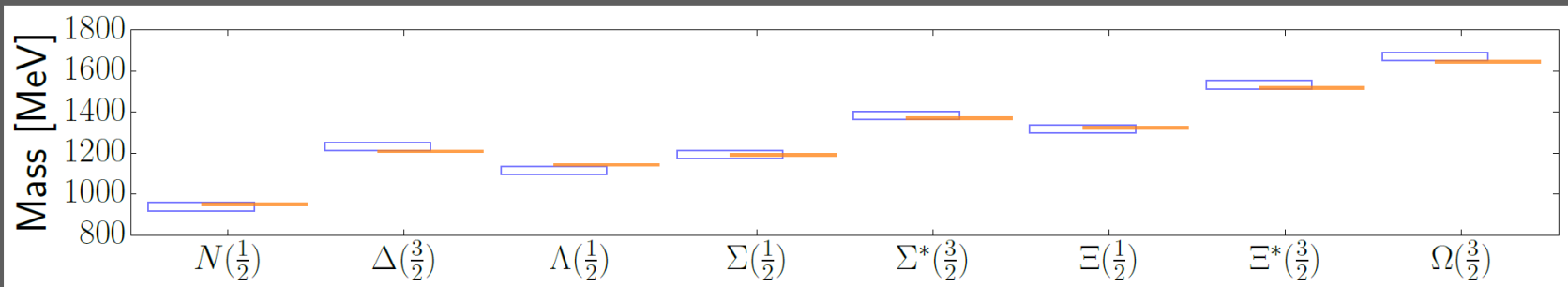
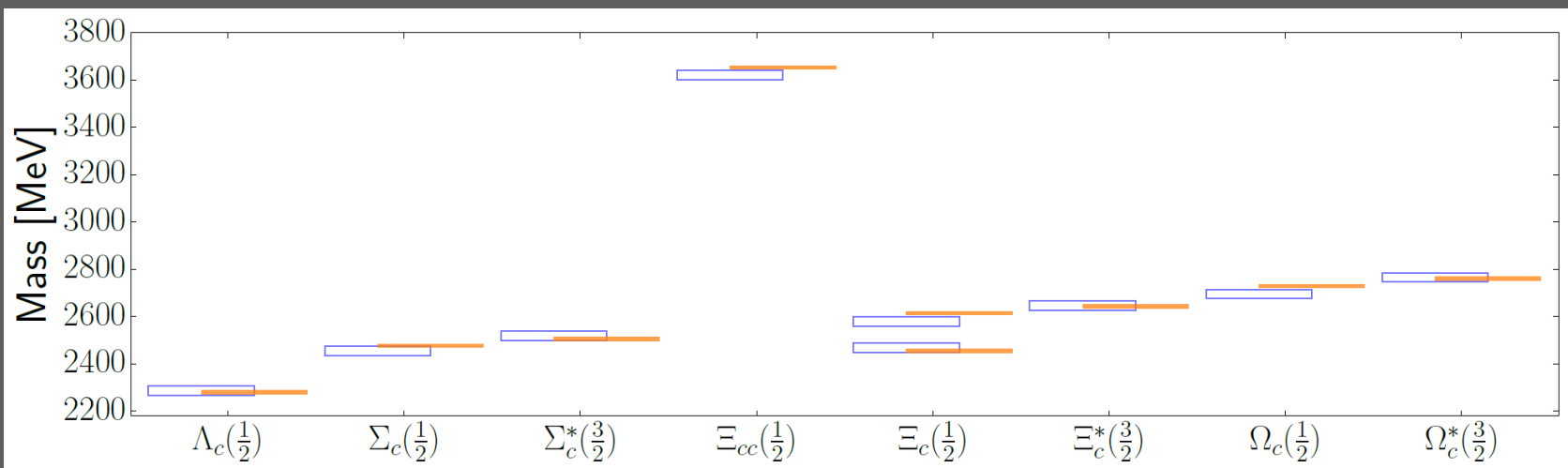
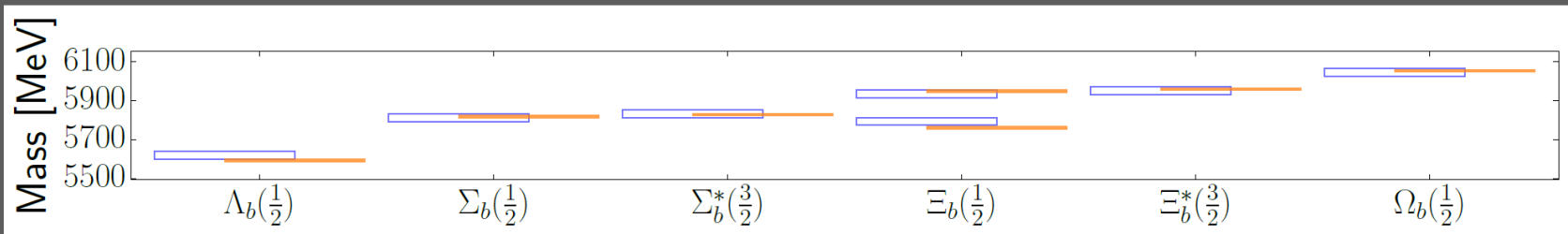
$$\Psi_{JM_J IM_I}^{ijk} = \mathcal{A} \left[[\psi_L \chi_S^{\sigma i}]_{JM_J} \chi_I^{fj} \chi_k^c \right]$$

| | | | | |
|----------------|-----|-----------------------|---------------------|---|
| | 1 | $\lambda_i \lambda_j$ | $\sigma_i \sigma_j$ | $\sigma_i \sigma_j \cdot \lambda_i \lambda_j$ |
| $\bar{\sigma}$ | -/- | | | |
| η/η' | | | +/+ | |
| π/K | | | | +/- |
| ω/ϕ | +/- | | +/- | |
| ρ/K^* | | +/+ | | +/+ |
| OGE | -/- | | +/+ | |
| CON | +/+ | | | |

| | No vector J. Phys. G 31, 481(2005) | su2 vector 2306.03526 | su3 vector 2307.16280 |
|----------------|---------------------------------------|--------------------------|--------------------------|
| $\alpha_s(qq)$ | 0.536 | 0.880 | 0.456 |
| $\alpha_s(qs)$ | 0.479 | | 0.426 |
| $\alpha_s(qc)$ | 0.426 | 0.774 | 0.363 |
| $\alpha_s(qb)$ | 0.409 | 0.749 | 0.339 |
| $\alpha_s(ss)$ | 0.419 | | 0.388 |
| $\alpha_s(sc)$ | 0.360 | | 0.308 |
| $\alpha_s(sb)$ | 0.340 | | 0.279 |
| $\alpha_s(cc)$ | 0.288 | 0.510 | 0.205 |
| $\alpha_s(cb)$ | 0.260 | 0.447 | 0.168 |
| $\alpha_s(bb)$ | 0.223 | 0.366 | 0.128 |

$qq/q\bar{q}$

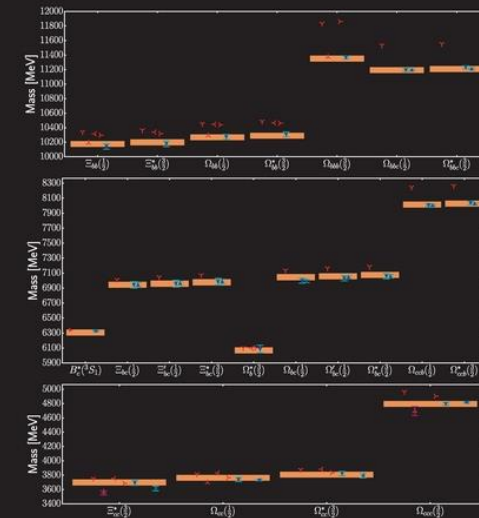
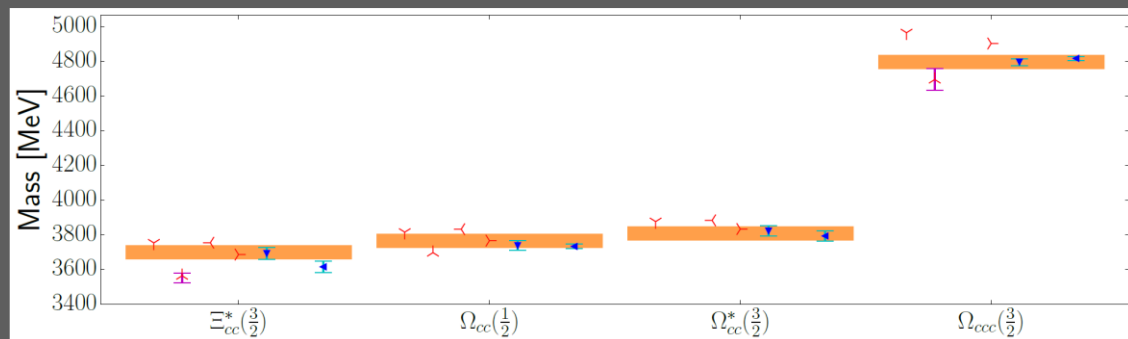
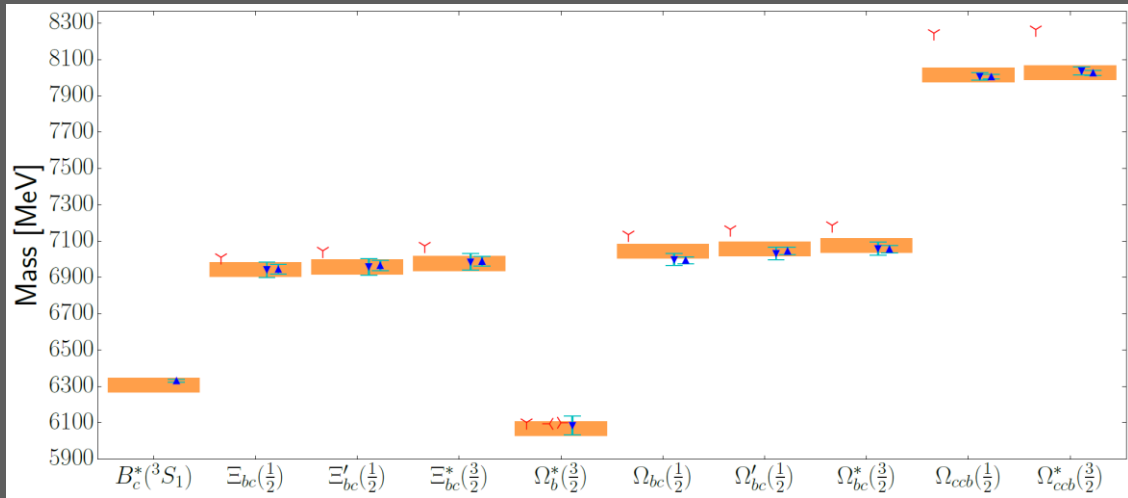
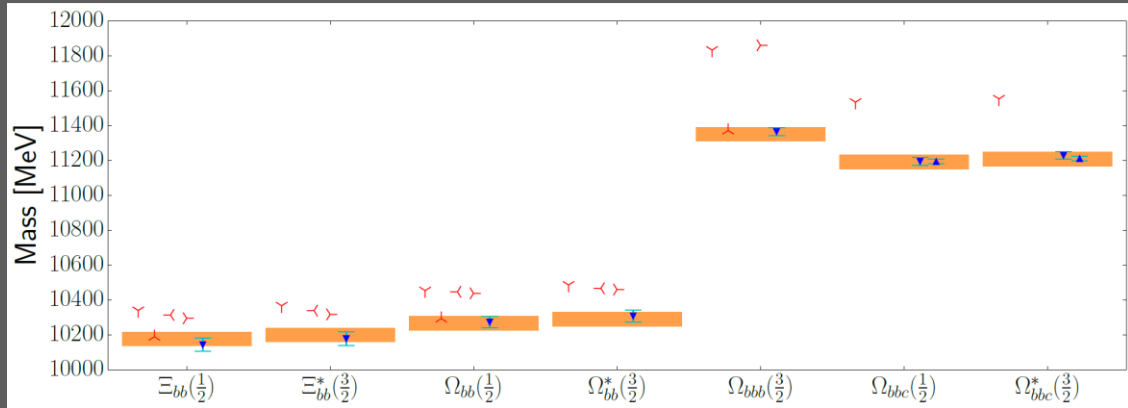




| Models | Meson χ^2 Err(the)=40MeV | Baryon χ^2 Err(the)=40MeV |
|---|---|--|
| Godfrey & Isgur (1985) | 5.3 (21 meson) | |
| Capstick & Isgur (1986) | | 3.78 (14 baryon) |
| Vijande & Fernandez & Valcarce (2005) | 9.5 (21 meson) | 305.8 (8 baryon, unpublished) |
| He & Harada & Zou 2307.16280 | 2.9 (21 meson) | 5.7 (24 baryon) |

$$\chi^2 = \sum_i \left(\frac{m_i(\text{the}) - m_i(\text{exp})}{\text{Err}_i(\text{sys})} \right)^2$$

$$\text{Err}(\text{sys}) = \sqrt{\text{Err}(\text{exp})^2 + \text{Err}(\text{the})^2}$$



Predicted mass spectrum of missing meson and baryons which have not been experimentally confirmed shown by orange line with 40 MeV error. The values of Ω_{bbb} shown here are shifted by -3000MeV .

From Bing-Ran He, Masayasu Harada & Bing-Song Zou on: Ground states of all mesons and baryons in a quark model with hidden local symmetry. Eur. Phys. J. C 83, 1159 (2023).



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- $SU3$ ground states
- **Summary**

- Quark model with hidden local symmetry gives a systematic description from light meson/baryon to heavy meson/baryon
- Quark model with hidden local symmetry gives correct interaction between qq and $q\bar{q}$, thus its easy to extend the model to describe multiquark states, e.g., tetraquark, pentaquark, ...
- Quark model with hidden local symmetry gives size messages and percentage of components, which could help us to identify the particles observed from experiments

Thank you for your attention!