Quark confinement in multiquark systems

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Phys. Rev. D 106, 096005, Phys. Rev. D 108 (2023) 7, L071501, arXiv: 2404.01238

Few-Body Problems in Physics (FB23), Beijing, 2024/09/25

Outline

- Background: Classical Quark model and exotic hadrons.

- Investigation of the confinement mechanisms:

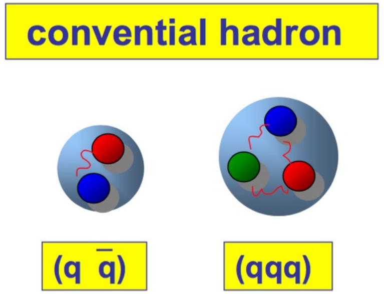
✓ Sum of two-body confinements

✓ Novel string-like confinement model.

- Summary.

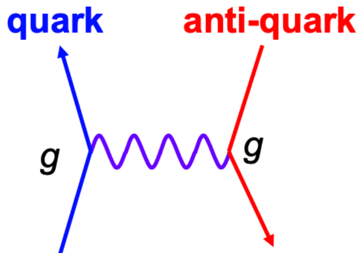
Classical Quark model

- Classical Quark model (QM):

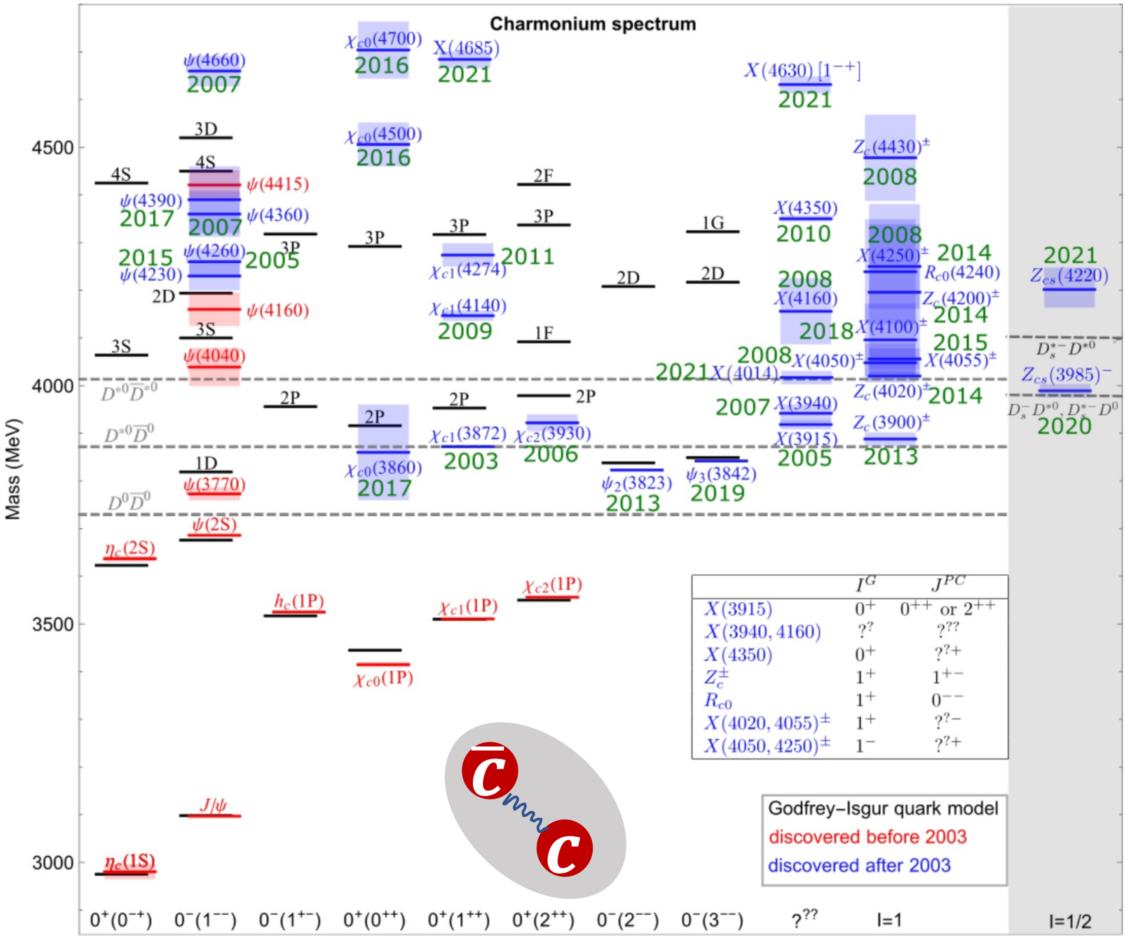


- Quark interactions: one gluon exchange + linear confinement

E. Eichten, et al. Phys. Rev. Lett. 34, 369



$$E \sim \sigma r$$



- Successfully for ground and low-lying excited states.

- Struggles with excited states above two-hadron threshold

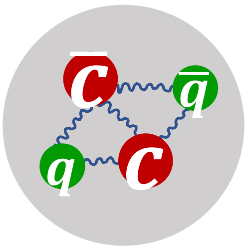
Exotic Hadrons

- Exotic states: beyond CQM

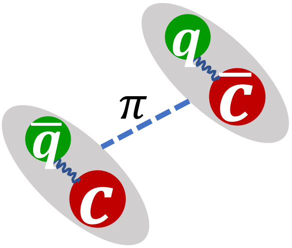
- Candidates for Multiquark Hadrons ($N \geq 4$)

- **What are they?**

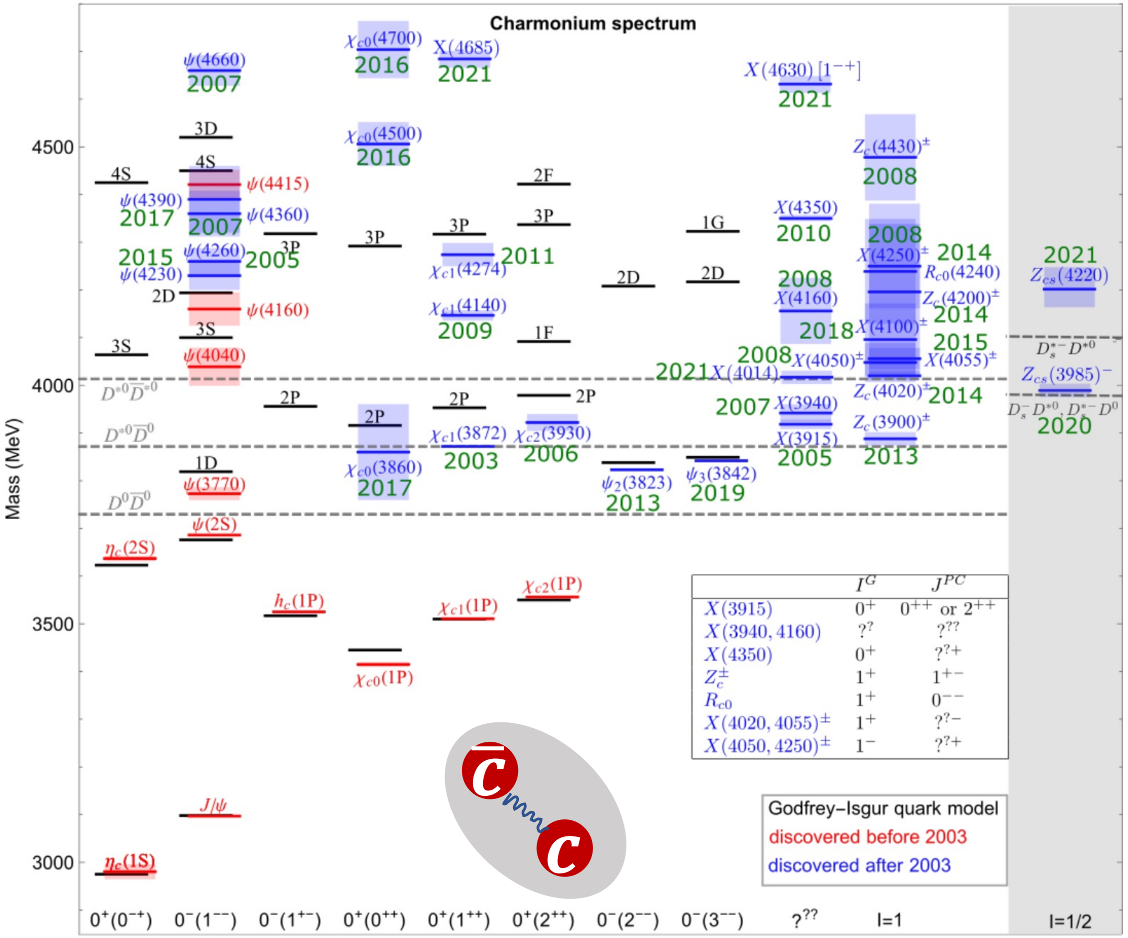
✓ ***Different Confinement configurations***



Tetraquark



Hadronic Molecule



	I^G	J^{PC}
$X(3915)$	0^+	0^{++} or 2^{++}
$X(3940, 4160)$	$?^?$	$?^{??}$
$X(4350)$	0^+	$?^{?+}$
$Z_c^{\pm}$	1^+	1^{+-}
R_{c0}	1^+	0^{--}
$X(4020, 4055)^{\pm}$	1^+	$?^{?-}$
$X(4050, 4250)^{\pm}$	1^-	$?^{?+}$

✓ Coupling to thresholds play an essential role

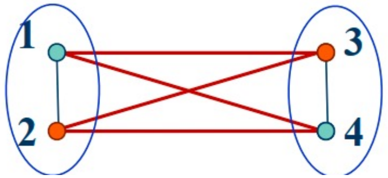
$D_{s0}^*(2317)$ &X(3872)@2003,…, P_c @2019, X(6900)@2020, T_{cc}^+@2021

- **Focus: Confinement Mechanism of tetraquark + coupling to two-hadron scattering states.**

Conventional quark model Hamiltonian

- Hamiltonian:

$$H = \sum_i \left(m_i + \frac{\boldsymbol{p}_i^2}{2m_i} \right) - T_G + \sum_{i<j} \frac{\left(\lambda_i \cdot \lambda_j\right)}{4} V_{ij}$$



- Nonrelativistic quark model: color-Coulomb+ linear+ hyperfine

- QM1: $$V_{ij}(\boldsymbol{r}) = \frac{\alpha_s}{r_{ij}} - \frac{3}{4} b r_{ij} - \frac{2\pi\alpha_s}{3m_i m_j} \left(\frac{\sigma}{\sqrt{\pi}} \right)^3 e^{-\sigma^2 r_{ij}^2} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

T Barnes , et al. Phys. Rev. D 72 (2005) 054026

- QM2(AL1): $$V_{ij}(\boldsymbol{r}) = -\frac{3}{4} \left(\frac{\kappa}{r} + \lambda r - \Lambda + \frac{2\pi\kappa'}{3m_i m_j} \frac{\exp(-r^2/r_0^2)}{\pi^{3/2} r_0^3} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right)$$

$$r_0(m_i, m_j) = A\left(\frac{2m_i m_j}{m_i + m_j}\right)^{-B}$$

Silvestre-Brac, Few-Body Syst. 20, 1 (1996)

- QM1 and QM2 reproduce the similar mass spectrum for $T_{cc\bar{c}\bar{c}}$.

parameter		Mass spectrum (MeV)			
		$^{2S+1}L_J$	Meson	EXP	THE
α_s	0.5461	$^1S_0$	η_c	2983.9	2984
b [GeV^2]	0.1452	$^3S_1$	J/ψ	3096.9	3092
m_c [GeV]	1.4794	$^3P_0$	χ_{c0}	3414.7	3426
σ [GeV]	1.0946	$^3P_1$	χ_{c1}	3510.7	3506
		$^1P_1$	$h_c(1P)$	3525.4	3516
		$^3P_2$	χ_{c2}	3556.2	3556
		$^1S_0$	$\eta_c(2S)$	3637.5	3634
		$^3S_1$	$\psi(2S)$	3686.1	3675
		$^3S_1$	$\psi(3S)$	4039.0	4076
		$^3S_1$	$\psi(4S)$	4421.0	4412

PDG

Conventional quark model

- Four body system: *two independent color singlet states are allowed*

$$\mathbf{3}\otimes\mathbf{3}\otimes\overline{\mathbf{3}}\otimes\overline{\mathbf{3}}=2\times\mathbf{1}\oplus 4\times\mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

✓ Diquark-antidiquark: (QQ)-$(\bar{Q}\bar{Q})$:

$$\bar{3}_c\otimes 3_c=1_c \text{ and } 6_c\otimes\bar{6}_c=1_c.$$

✓ Meson-Meson: $(Q\bar{Q})$-$(Q\bar{Q})$: singlet + hidden color states

$$|\mathbf{1}\rangle\equiv|(Q_1\bar{Q}_3)_\mathbf{1}(Q_2\bar{Q}_4)_\mathbf{1}\rangle,$$
$$|\mathbf{8}\rangle\equiv|(Q_1\bar{Q}_3)_\mathbf{8}(Q_2\bar{Q}_4)_\mathbf{8}\rangle,$$

Or

$$|\mathbf{1}\rangle\equiv|(Q_1\bar{Q}_3)_\mathbf{1}(Q_2\bar{Q}_4)_\mathbf{1}\rangle,$$
$$|\mathbf{1}'\rangle\equiv|(Q_1\bar{Q}_4)_\mathbf{1}(Q_2\bar{Q}_3)_\mathbf{1}\rangle,$$

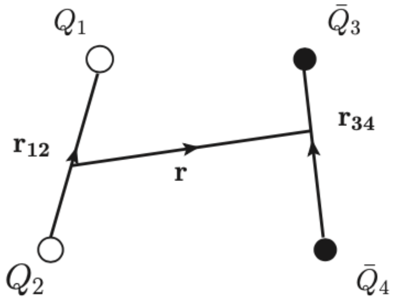
Either two of them are equivalent

- *Not orthogonal*

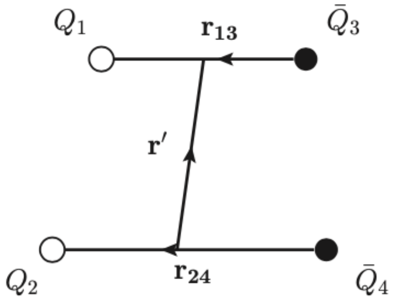
$$\langle\mathbf{1}'|\mathbf{1}\rangle=\frac{1}{3}$$

Gaussian expansion method

- 4-body Jacobi coordinates



Compact diquark-antidiquark



Scattering channel (2 color singlet mesons)

- Gaussian expansion method: solving Few-body problem: E. Hiyama, et al. Prog. Part. Nucl. Phys. 51 223-307

$$\psi_{JM}=\sum_{C=a,b}\sum_{\alpha}A_{12}A_{34}\sum_{\alpha}\mathscr{B}_{\alpha}^{(C)}\underbrace{\xi_{C}^{(C)}}_{\text{Color}}\underbrace{\eta_{I}^{(C)}}_{\text{flavor}}\left[\underbrace{\left[\phi_{nl}\left(\boldsymbol{r}_C\right)\otimes\phi_{NL}\left(\boldsymbol{R}_C\right)\otimes\phi_{\nu\lambda}\left(\boldsymbol{\rho}_C\right)\right]_{J_L}}_{\text{Gaussian function}}\otimes\underbrace{\chi_{S}^{(C)}}_{\text{spin}}\right]_{JM}$$

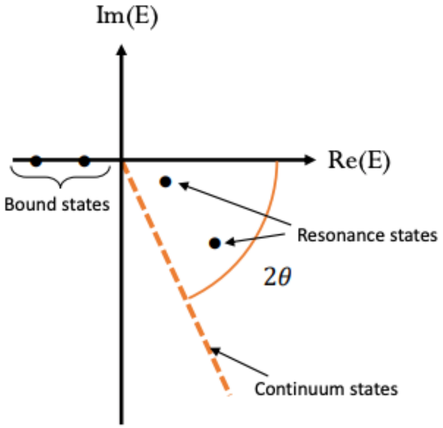
$$\phi_{n_a l_a}(r_{12},\beta_a)=\left\{\frac{2^{l_a+2}(2\nu_{n_a})^{l+3/2}}{\sqrt{\pi}(2l_a+1)!!}\right\}^{1/2}r_{12}^{l_a}e^{-\nu_{n_a}r_{12}^2}$$

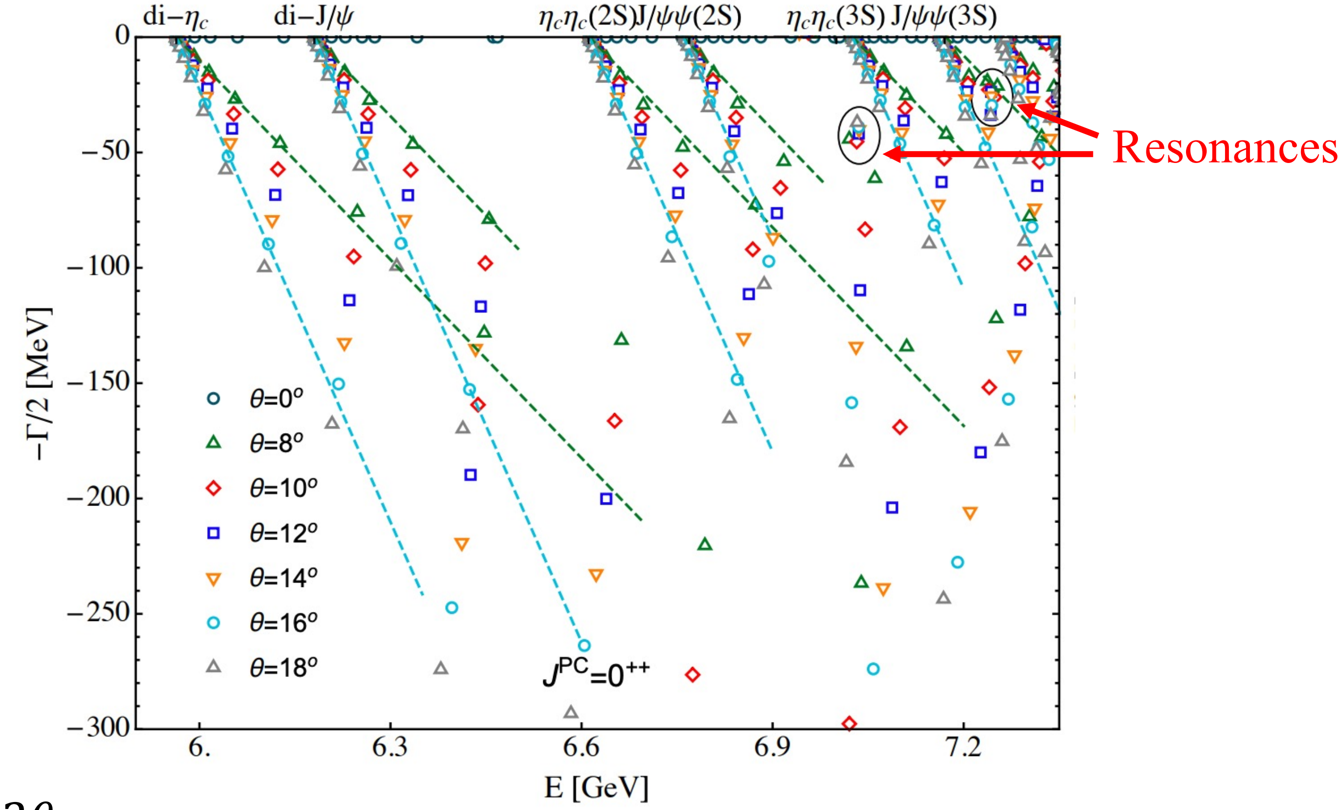
Complex scaling method

- Complex scaling method: identification of resonant and bound states.

$$\boldsymbol{r} \to \boldsymbol{r}e^{i\theta}, \quad \boldsymbol{q} \to \boldsymbol{q}e^{-i\theta}$$

$$H_\theta \Phi_\theta = E_\theta \Phi_\theta,$$





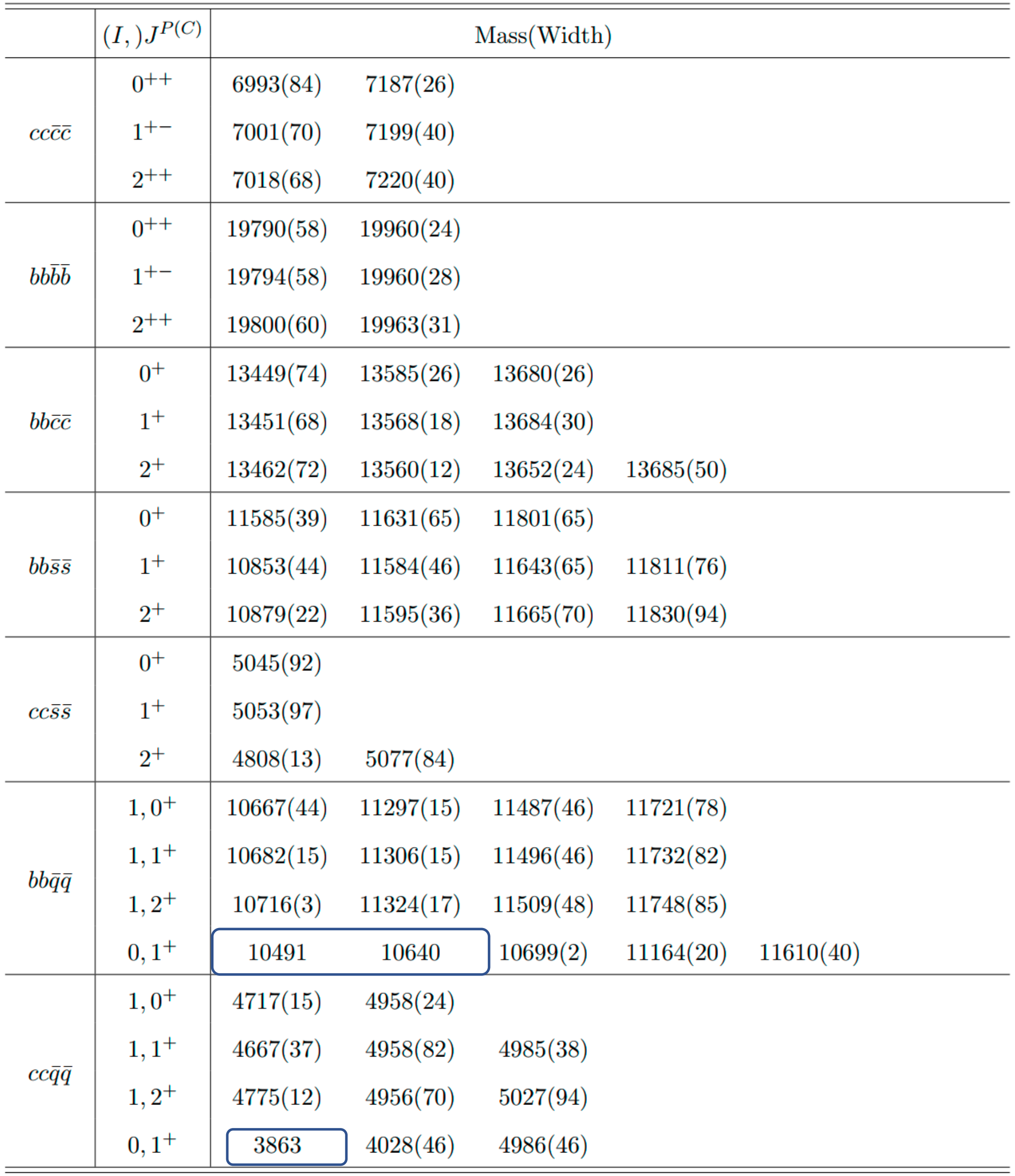
- Bound states & resonances: independent of θ

- Scattering state: along continuum line and rotate with 2θ

Phys. Rev. D 106, 096005

S.Aoyama et al. PTP. 116, 1 (2006).
T. Myo et al. PPNP. 79, 1 (2014)
N. Moiseyev, Physics reports 302, 212 (1998)

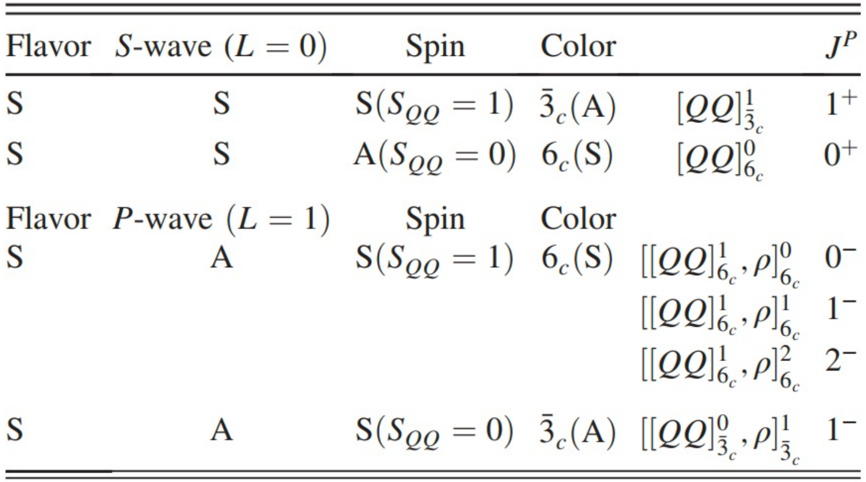
Tetraquark



	$(I,)J^{P(C)}$	Mass(Width)				
$cc\bar{c}\bar{c}$	0^{++}	6993(84)	7187(26)			
	1^{+-}	7001(70)	7199(40)			
	2^{++}	7018(68)	7220(40)			
$bb\bar{b}\bar{b}$	0^{++}	19790(58)	19960(24)			
	1^{+-}	19794(58)	19960(28)			
	2^{++}	19800(60)	19963(31)			
$bb\bar{c}\bar{c}$	0^{+}	13449(74)	13585(26)	13680(26)		
	1^{+}	13451(68)	13568(18)	13684(30)		
	2^{+}	13462(72)	13560(12)	13652(24)	13685(50)	
$bb\bar{s}\bar{s}$	0^{+}	11585(39)	11631(65)	11801(65)		
	1^{+}	10853(44)	11584(46)	11643(65)	11811(76)	
	2^{+}	10879(22)	11595(36)	11665(70)	11830(94)	
$cc\bar{s}\bar{s}$	0^{+}	5045(92)				
	1^{+}	5053(97)				
	2^{+}	4808(13)	5077(84)			
$bb\bar{q}\bar{q}$	$1,0^{+}$	10667(44)	11297(15)	11487(46)	11721(78)	
	$1,1^{+}$	10682(15)	11306(15)	11496(46)	11732(82)	
	$1,2^{+}$	10716(3)	11324(17)	11509(48)	11748(85)	
	$0,1^{+}$	10491	10640	10699(2)	11164(20)	11610(40)
$cc\bar{q}\bar{q}$	$1,0^{+}$	4717(15)	4958(24)			
	$1,1^{+}$	4667(37)	4958(82)	4985(38)		
	$1,2^{+}$	4775(12)	4956(70)	5027(94)		
	$0,1^{+}$	3863	4028(46)	4986(46)		

- $QQ\bar{Q}'\bar{Q}'(Q,Q'=c,b), QQ\bar{q}\bar{q}(q=u,d,s)$

✓ Constrained by Fermi-Dirac statistics: less No. of possible flavor-spin-color wave functions



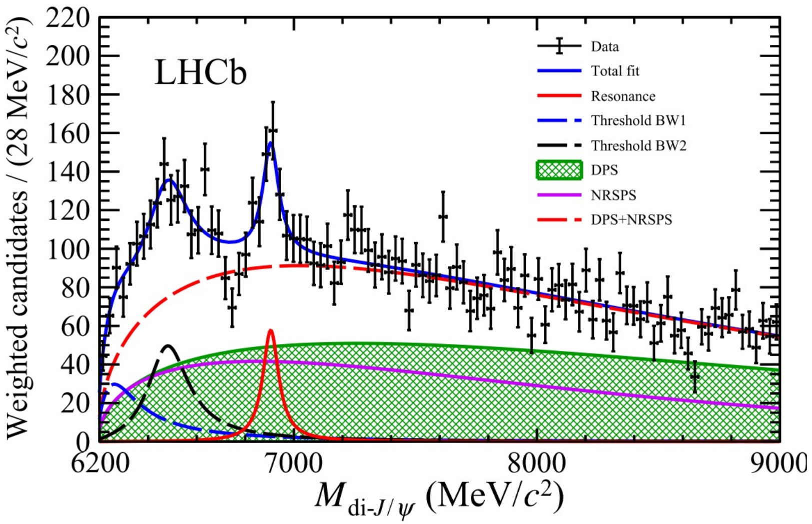
Flavor	S-wave $(L=0)$	Spin	Color		J^P
S	S	$\mathrm{S}(S_{QQ}=1)$	$\bar{3}_c(\mathrm{A})$	$[QQ]^1_{\bar{3}_c}$	1^+
S	S	$\mathrm{A}(S_{QQ}=0)$	$6_c(\mathrm{S})$	$[QQ]^0_{6_c}$	0^+
Flavor	P-wave $(L=1)$	Spin	Color		
S	A	$\mathrm{S}(S_{QQ}=1)$	$6_c(\mathrm{S})$	$[[QQ]^1_{6_c},\rho]^0_{6_c}$	0^-
				$[[QQ]^1_{6_c},\rho]^1_{6_c}$	1^-
				$[[QQ]^1_{6_c},\rho]^2_{6_c}$	2^-
S	A	$\mathrm{S}(S_{QQ}=0)$	$\bar{3}_c(\mathrm{A})$	$[[QQ]^0_{\bar{3}_c},\rho]^1_{\bar{3}_c}$	1^-

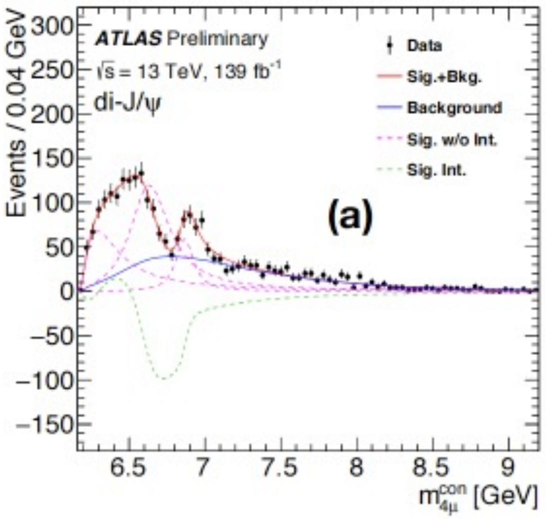
- 62 resonant states and 3 bound states

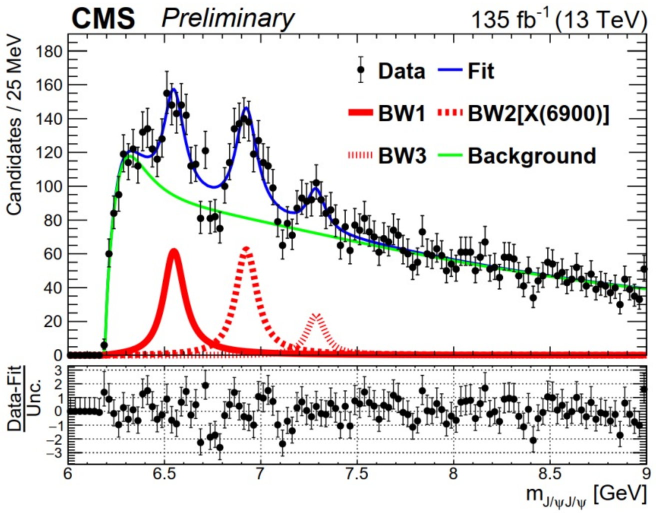
arXiv: 2404.01238

Experimental search for $T_{cc\bar{c}\bar{c}}$

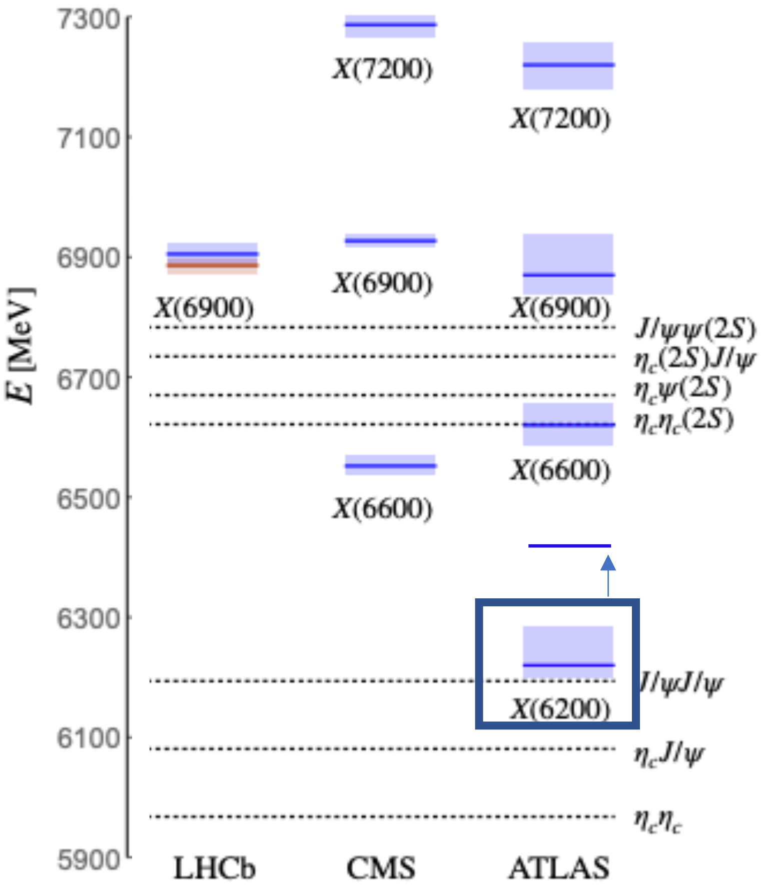
- Observation of structure $T_{cc\bar{c}\bar{c}}$ in di-J/ψ and $J/\psi\psi(2S)$ channel







J/ψ-J/ψ resonances observed in experiments



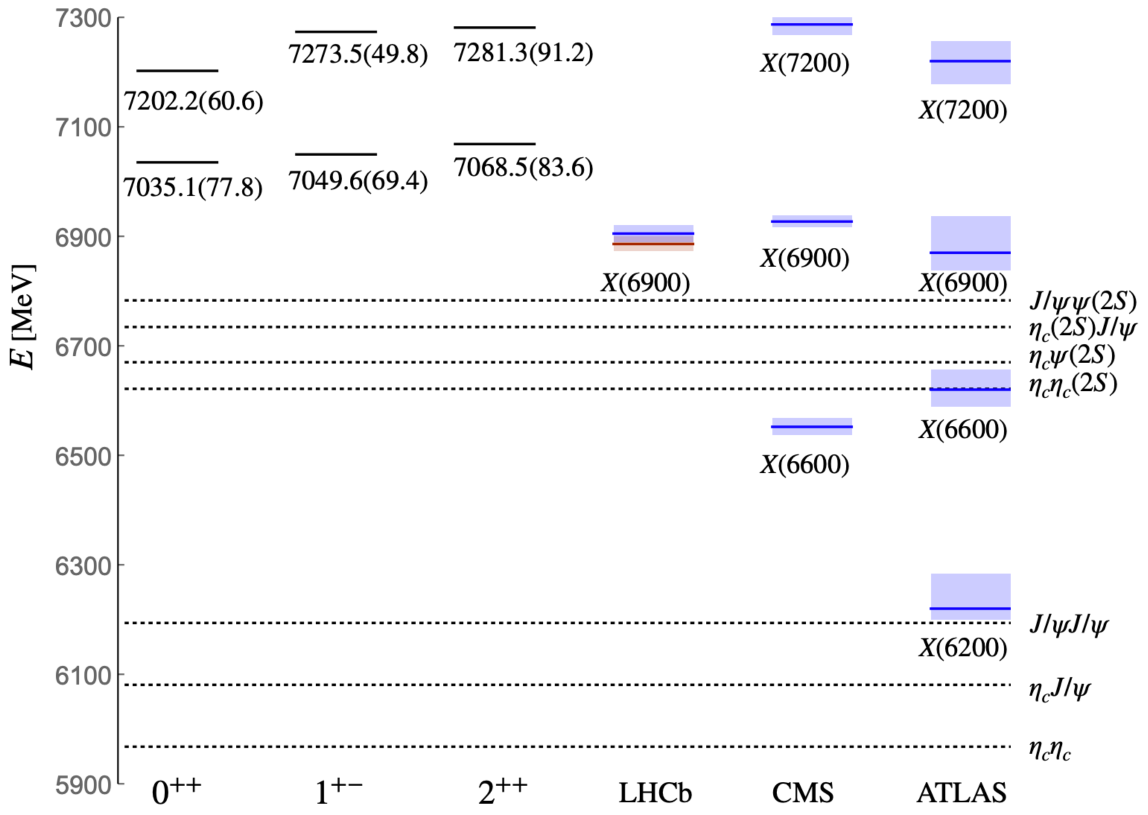
LHCb, Science Bulletin 65 (2020) 198 3.

CMS, Phys. Rev. Lett. 132, 111901.

E. B.-T. on behalf of the ATLAS Collaboration, https://agenda.infn.it/event/28874/ contributions/170298/.

ATLAS, Phys. Rev. Lett. 131 (2023) 15, 151902

$T_{cc\bar{c}\bar{c}}$



Phys. Rev. D 106, 096005

		M	Γ	Observable channels
LHCb model I [12]	$X(6900)$	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	di-J/ψ
LHCb model II [12]		$6886 \pm 11 \pm 11$	$168 \pm 33 \pm 69$	
CMS [14]	$X(6600)$	$6552 \pm 10 \pm 12$	$124 \pm 29 \pm 34$	di-J/ψ
	$X(6900)$	$6927 \pm 9 \pm 5$	$122 \pm 22 \pm 19$	
	$X(7200)$	$7287 \pm 19 \pm 5$	$95 \pm 46 \pm 20$	
ATLAS [15]	$X(6200)$	$6.22 \pm 0.05^{+0.04}_{-0.05}$	$0.31 \pm 0.12^{+0.07}_{-0.08}$	di-J/ψ
	$X(6600)$	$6.62 \pm 0.03^{+0.02}_{-0.01}$	$0.31 \pm 0.09^{+0.06}_{-0.11}$	
	$X(6900)$	$6.87 \pm 0.03^{+0.06}_{-0.01}$	$0.12 \pm 0.04^{+0.03}_{-0.01}$	
		$6.78 \pm 0.36^{+0.35}_{-0.54}$	$0.39 \pm 0.11^{+0.11}_{-0.07}$	$J/\psi\psi(2S)$
	$X(7200)$	$7.22 \pm 0.03^{+0.02}_{-0.03}$	$0.10^{+0.13+0.06}_{-0.07-0.05}$	

- ***1st pole VS X(6900):***

✓ 100 MeV higher mass & consistent decay width

- ***2nd pole: a candidate for X(7200).***

- ***Absence of the lower X(6600) state.***

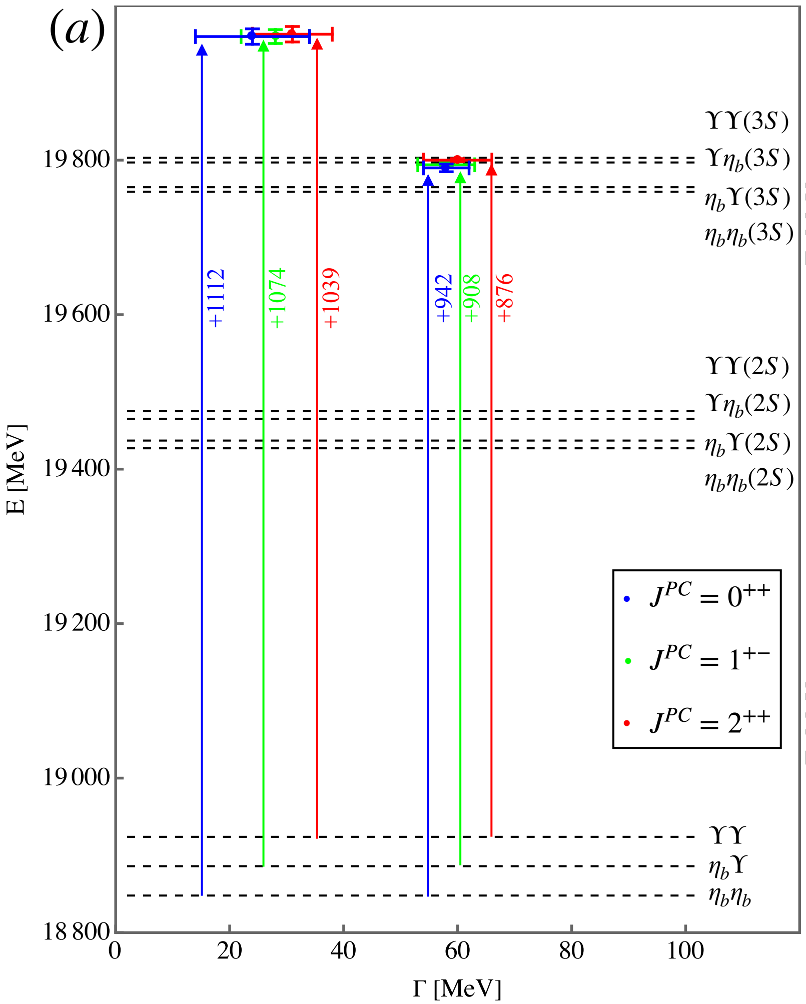
✓ a wide resonance asymptote will oscillate very strongly in the complex plane.

- The confinement mechanism~ br.

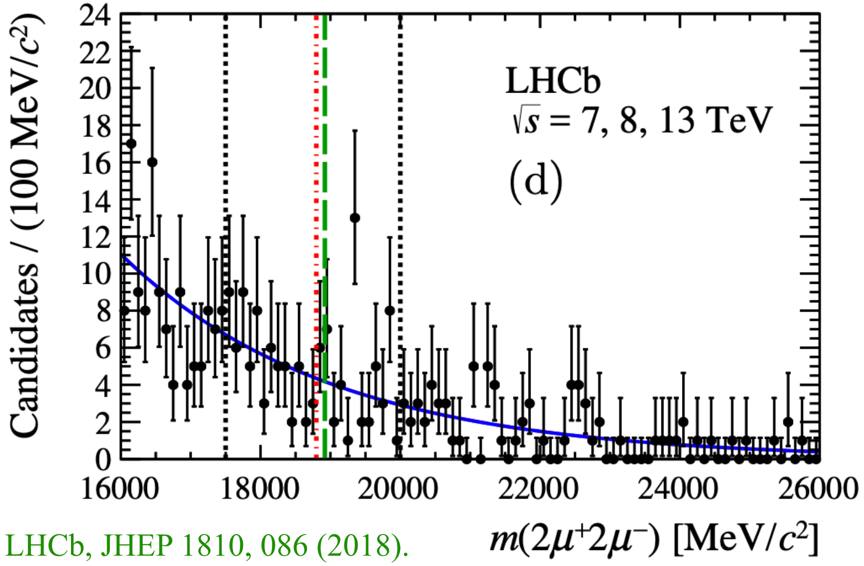
$$V=-\frac{3}{4}\sigma\sum_{i<j}(T_i\cdot T_j)r_{ij}$$

$T_{bb\bar{b}\bar{b}}$

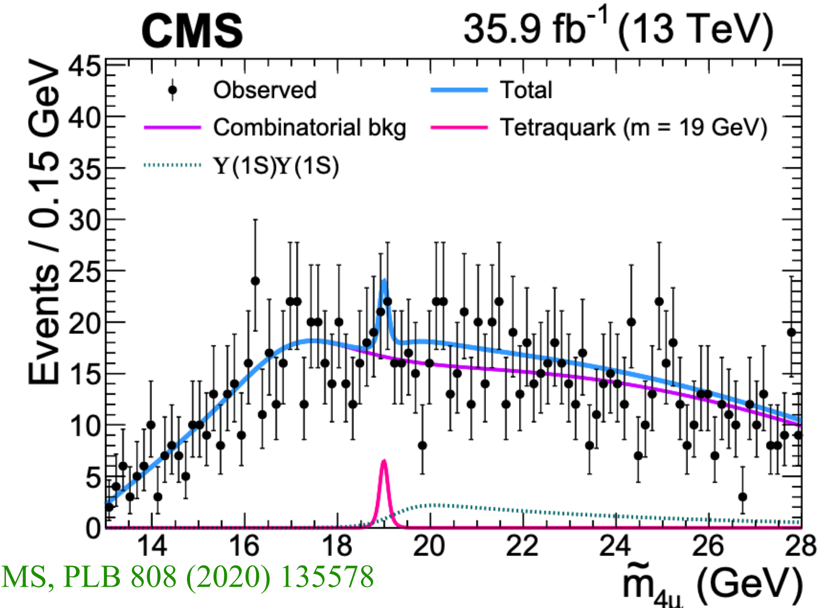
- ***Two resonances for $T_{bb\bar{b}\bar{b}}$.***



- ***No significant excess observed for $T_{bb\bar{b}\bar{b}}$.***

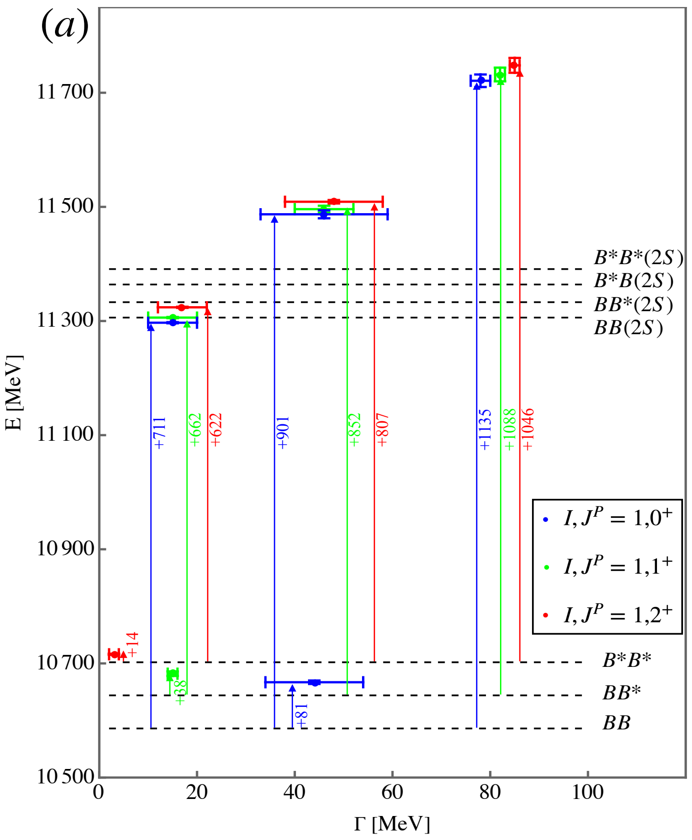


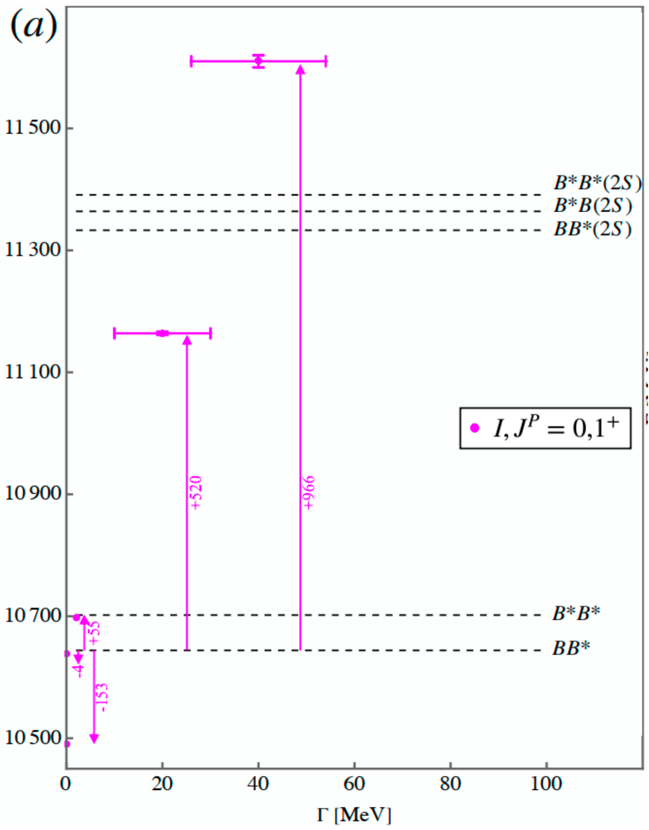
LHCb, JHEP 1810, 086 (2018).



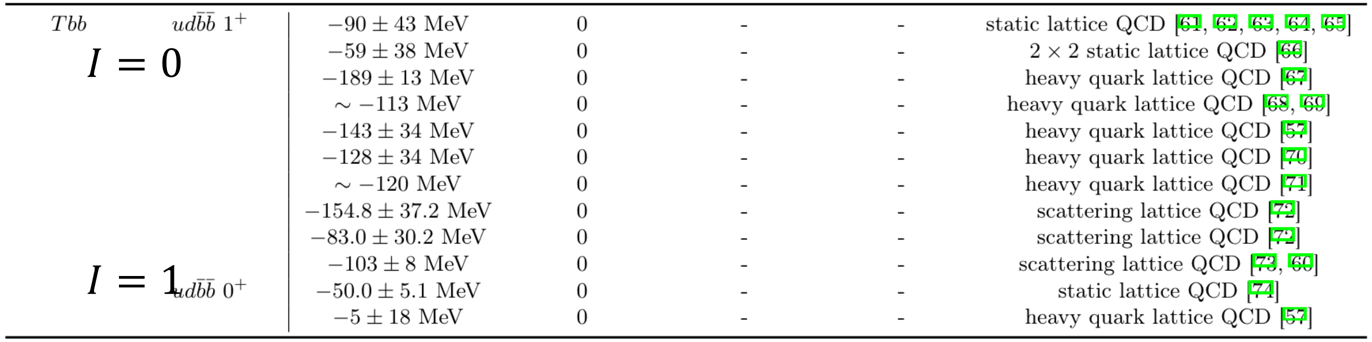
CMS, PLB 808 (2020) 135578

$T_{bb\bar{q}\bar{q}}$





Pedro Bicudo, Phys. Rept. 1039 (2023) 1-49



Tbb $I = 0$	$ud\bar{b}\bar{b}$ 1^+	-90 ± 43 MeV	0	-	-	static lattice QCD [61, 62, 63, 64, 65]
		-59 ± 38 MeV	0	-	-	2×2 static lattice QCD [66]
		-189 ± 13 MeV	0	-	-	heavy quark lattice QCD [67]
		~ -113 MeV	0	-	-	heavy quark lattice QCD [68, 69]
		-143 ± 34 MeV	0	-	-	heavy quark lattice QCD [57]
		-128 ± 34 MeV	0	-	-	heavy quark lattice QCD [70]
		~ -120 MeV	0	-	-	heavy quark lattice QCD [71]
		-154.8 ± 37.2 MeV	0	-	-	scattering lattice QCD [72]
		-83.0 ± 30.2 MeV	0	-	-	scattering lattice QCD [72]
		-103 ± 8 MeV	0	-	-	scattering lattice QCD [73, 60]
$I = 1$	$ud\bar{b}\bar{b}$ 0^+	-50.0 ± 5.1 MeV	0	-	-	static lattice QCD [74]
		-5 ± 18 MeV	0	-	-	heavy quark lattice QCD [57]

- I=1: 12 resonances, $\boldsymbol{V^{cm}}$**-repulsive**
- I=0: 2 bound states and 3 resonances. $\boldsymbol{V^{cm}}$**- attractive**

$I(J^P)$	E	ΔE	$\langle V^{ce}_{bb} \rangle$	$\langle V^{cm}_{\bar{q}\bar{q}} \rangle$
$0(1^+)$	10491	-153	-231.2	-214.3
$1(1^+)$	10682	+38	-102.7	+19.2

$$V^{cm} = \frac{1}{m_i m_j}\left\langle \frac{\lambda_i}{2}\frac{\lambda_j}{2}\boldsymbol{\sigma}_i\boldsymbol{\sigma}_j \right\rangle$$

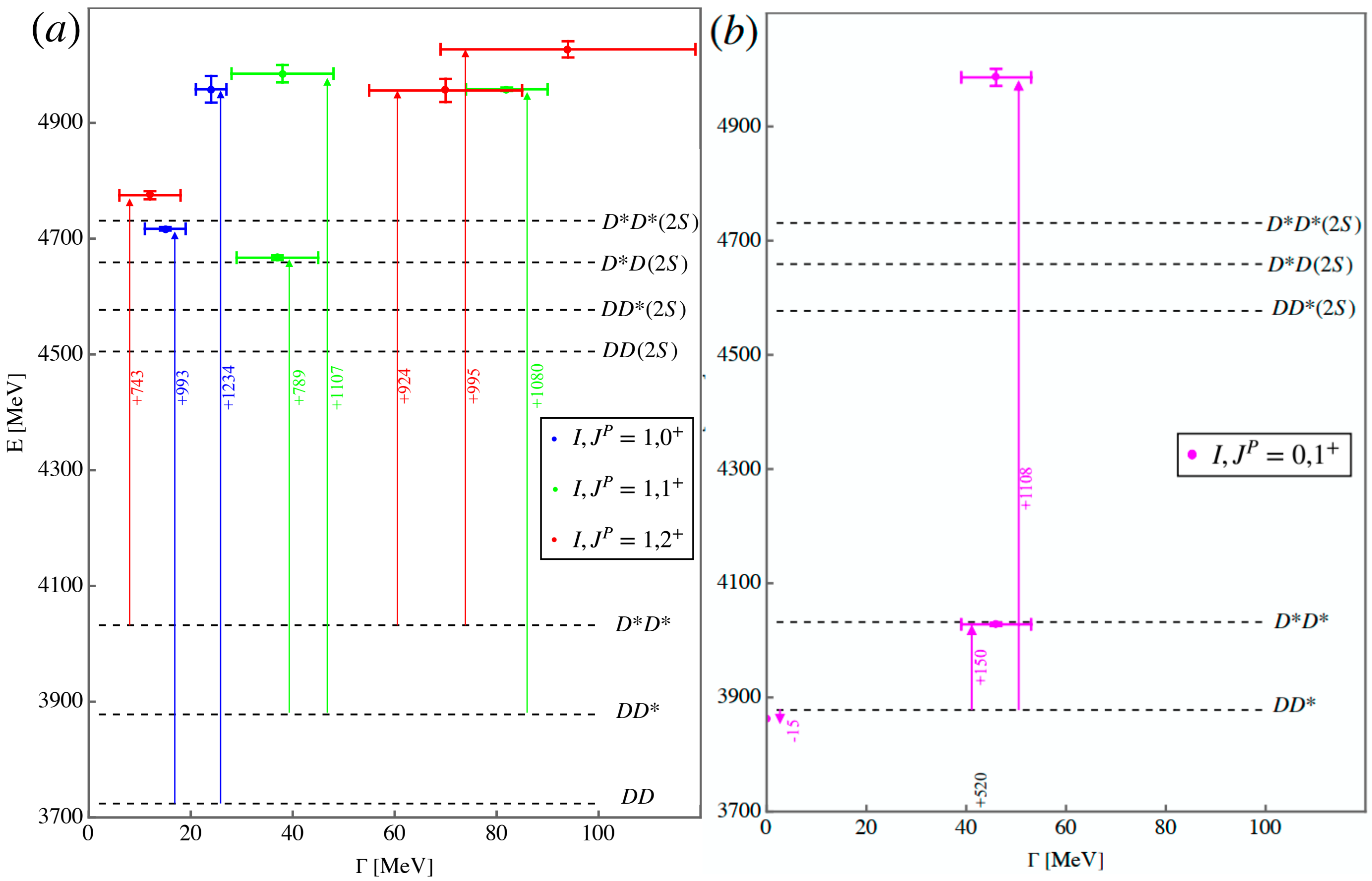
- 2 bound states with $I(J^P) = 0(1^+)$.

✓Lowest deeply bound state: $BB\gamma$

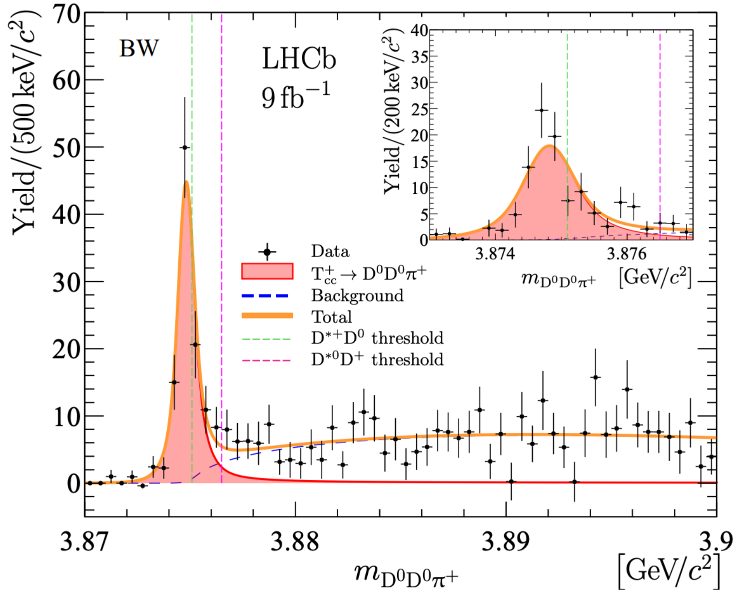
✓ Caution for second lower bound state:
The long-range interactions (e.g. π) for the near-threshold states

state	ΔE [MeV]	P_{di}	P_{MM^*}	$P_{M^*M^*}$
$bb\bar{q}\bar{q}$ $0(1^+)$	-153	63.9%	24.1%	12.0%
$bb\bar{q}\bar{q}$ $0(1^+)$	-4	1.9%	94.2%	3.9%

$T_{cc\bar{q}\bar{q}}$



- ***A loosely bound T_{cc}^{+} observed***



LHCb, Nature Commun. 13 (2022) 1, 3351

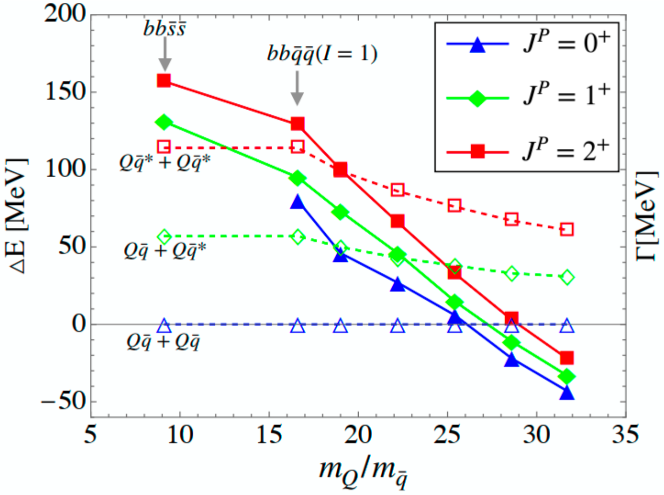
- The bound states with $I(J^P) = 0(1^+)$.

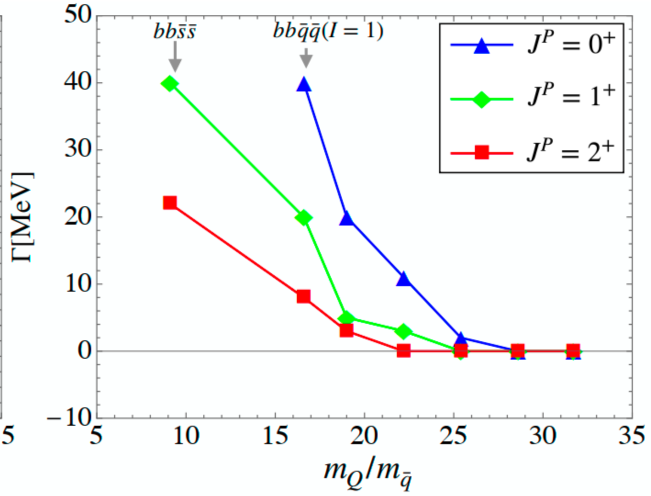
✓ $\delta m_U = -15$ MeV >>$\delta m_U^{exp} = -361 \pm 40$ keV → ***Experimental T_{cc}^{+} is not a tetraquark state***

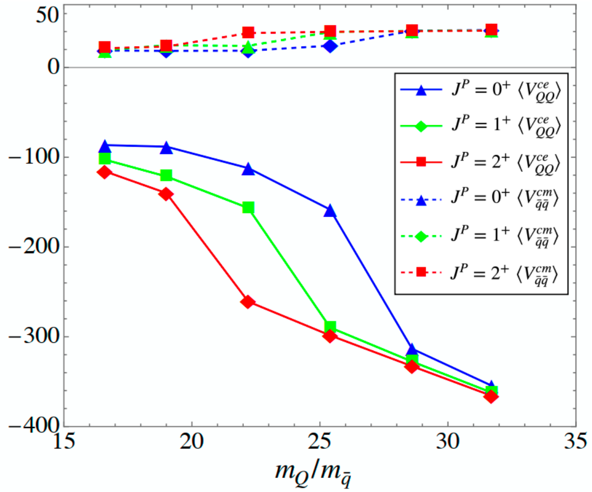
✓ Not allowed in $DD\pi$, but only $DD\gamma$

Mass ratio dependence

- $QQ\bar{q}\bar{q}(I=1)$: $\frac{m_Q}{m_{\bar{q}}}$ increasing, masses and decay widths of the resonances decrease → bound states.







- $\langle V^{ce} \rangle \sim \left\langle \frac{1}{r} \right\rangle \sim \alpha_s u_{ij}$ (reduced mass): $\langle V^{ce}_{QQ} \rangle$ is significant
- $\langle V^{cm} \rangle \sim \frac{1}{m_i m_j} \left\langle \frac{\lambda_i}{2} \frac{\lambda_j}{2} \sigma_i \sigma_j \right\rangle$: $\langle V^{cm}_{qq} \rangle$ is significant
- I=1:

✓ $\langle V^{ce}_{QQ} \rangle$ attractive and increasing with larger $\frac{m_Q}{m_{\bar{q}}}$

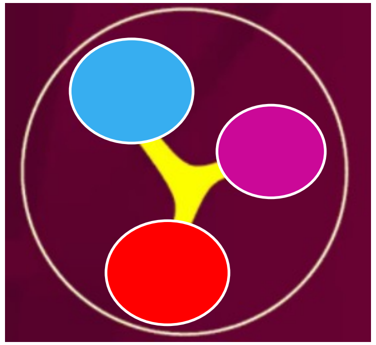
✓ $\langle V^{cm}_{qq} \rangle$ repulsive and slow rise

- I=0: attractive $\langle V^{ce}_{QQ} \rangle$ & $\langle V^{cm}_{qq} \rangle$, thus smaller $\frac{m_Q}{m_{\bar{q}}}$ (physical value) can form the bound state.

Investigation of the confinement mechanisms

- Conventional quark-quark confinement potential form $\bar{Q}Q$ meson: $V(r) \sim br$
- Application to baryons (qqq): $V = -\frac{3}{4}\sigma\sum_{i<j}(T_i \cdot T_j)r_{ij}$ (Δ-shape) + Y-shape?
- Direct application to $T_{Q_1Q_2\bar{Q}_3\bar{Q}_4}$: $V = -\frac{3}{4}\sigma\sum_{i<j}(T_i \cdot T_j)r_{ij}$

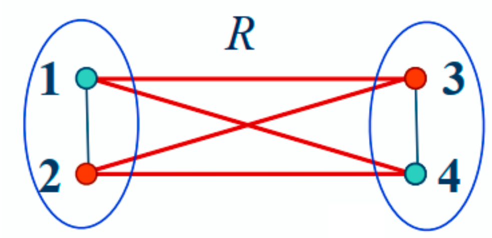
V. Dmitrasinovic et al., Eur. Phys. J. C 62, 383-397 (2009)



✓ Problem: ***long-range color van der Waals*** between color singlet mesons,

$$V_{\mathrm{cvdW}} = \frac{|\langle \mathbf{8}|V_{\mathrm{QM}}|\mathbf{1}\rangle|^2}{\Delta E} \propto -\frac{1}{R^3}$$

T. Appelquist, et al. Phys. Lett. B77, 405 (1978)

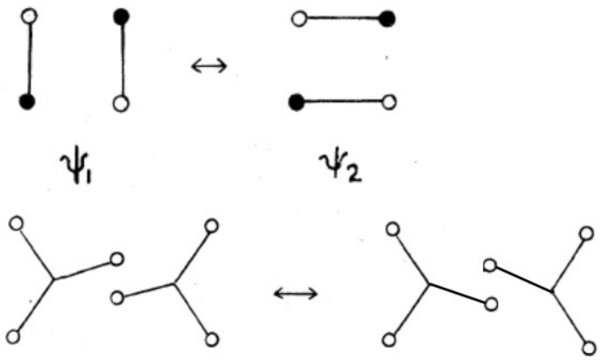


String confinement model

- “Reconnection of strings and quark matter”

$$V_{\text{string}} = \sigma \times \operatorname*{Min}_{\text{links}} \sum r_{\text{link}}$$

H. Miyazawa, PRD20, 2953 (1979).



- “String Flip-Flop” --Strings can make a transition to another spatial configuration when they touch each other.

- ***long-range color van der Waals*** between color singlet mesons disappear.

H. Miyazawa, PR D20, 2953 (1979)
N. Isgur, J. E. Paton, Phys. Lett. B 124, 247 (1983)
M. Oka, Phys. Rev. D 31, 2274 (1985).
J. Vijande, et. Al. Phys. Rev. D 85, 014019 (2012).

- The lattice QCD may choose the adiabatic potential of the configuration with the shortest string lengths to minimize the string tension energy – Flip-Flop model

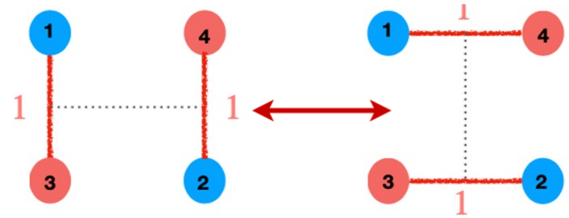
$$V_{\text{FF}} = \sigma \, \text{Min} \left[r_{13} + r_{24}, r_{14} + r_{23} \right].$$

F. Okiharu, et al. PRD72 (2005) 014505
C. Alexandrou . et al. Nucl. Phys. A 518, 723-751 (1990)
F. Okiharu .et al. J. Mod. Phys. 7, 774-789 (2016)

String Flip-Flop model

- The flip-flop potential model may not be satisfactory for color SU(3): choice of color configurations has some ambiguity

$$r_{13} + r_{24} = r_{14} + r_{23}$$



- $|\mathbf{1}\rangle$ and $|\mathbf{1}'\rangle$ are not smoothly connected in SU(3), because the overlap of $|\mathbf{1}\rangle$ and $|\mathbf{1}'\rangle$ is not complete.

only the $1/N_c$ part of $|\mathbf{1}\rangle$ can go directly to $|\mathbf{1}'\rangle$.

Hidden color channel automatically mixed

- The transition between two color configurations is dynamically generated, and the HC channel can be treated as an independent configuration.

Novel string-like confinement potential

- Three bases: States with different string configurations are orthogonal

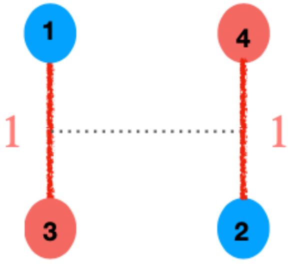
$$|\mathbf{1}\rangle\rangle \equiv |(Q_1 \to \bar{Q}_3)_{\mathbf{1}}(Q_2 \to \bar{Q}_4)_{\mathbf{1}}\rangle$$

$$|\mathbf{1}'\rangle\rangle \equiv |(Q_1 \to \bar{Q}_4)_{\mathbf{1}}(Q_2 \to \bar{Q}_3)_{\mathbf{1}}\rangle.$$

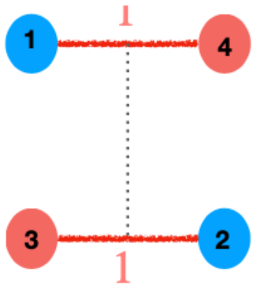
$$|\mathbf{hc}\rangle\rangle \equiv |(Q_1 \leftrightarrow Q_2)_{\bar{\mathbf{3}}} \leftarrow (\bar{Q}_3 \leftrightarrow \bar{Q}_4)_{\mathbf{3}}\rangle,$$

$$\langle\langle \mathbf{1}' | \mathbf{1} \rangle\rangle = 0.$$

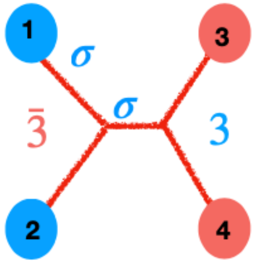
$$\langle\langle \mathbf{1} | \mathbf{hc} \rangle\rangle = \langle\langle \mathbf{1}' | \mathbf{hc} \rangle\rangle = 0.$$



$|1\rangle\rangle$



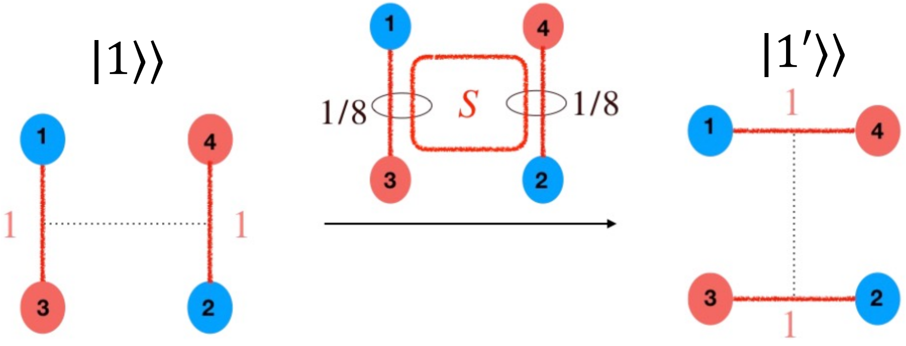
$|1'\rangle\rangle$



Hidden color

Phys. Rev. D.37.2431
Nucl. Phys. A 505, 655-669.
Prog. Theor. Phys. Suppl. 137, 21-42.

- Minimal surface area S: N-body force



$$V_{\mathrm{ST}} = \begin{pmatrix} \sigma(r_{13}+r_{24}) & \kappa e^{-\sigma S} & \kappa' e^{-\sigma S} \\ \kappa e^{-\sigma S} & \sigma(r_{14}+r_{23}) & -\kappa' e^{-\sigma S} \\ \kappa' e^{-\sigma S} & -\kappa' e^{-\sigma S} & \frac{\sigma}{4}[r_{13}+r_{24}+r_{14}+r_{23}+2(r_{12}+r_{34})] \end{pmatrix}$$

$\kappa' = \sqrt{8}\kappa$ confinement range: $a \sim 1/\sqrt{\sigma} \sim 0.45$ fm

$0 \leq \kappa' = \sqrt{8}\kappa \leq 2\sigma a$ $\kappa \leq 0.3\mathrm{GeV}$

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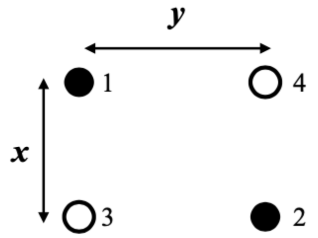
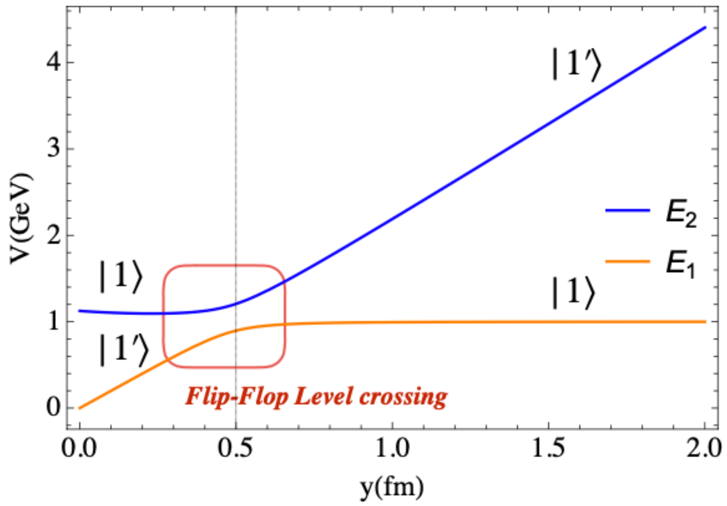
A toy model: Conventional QM VS String-like potential

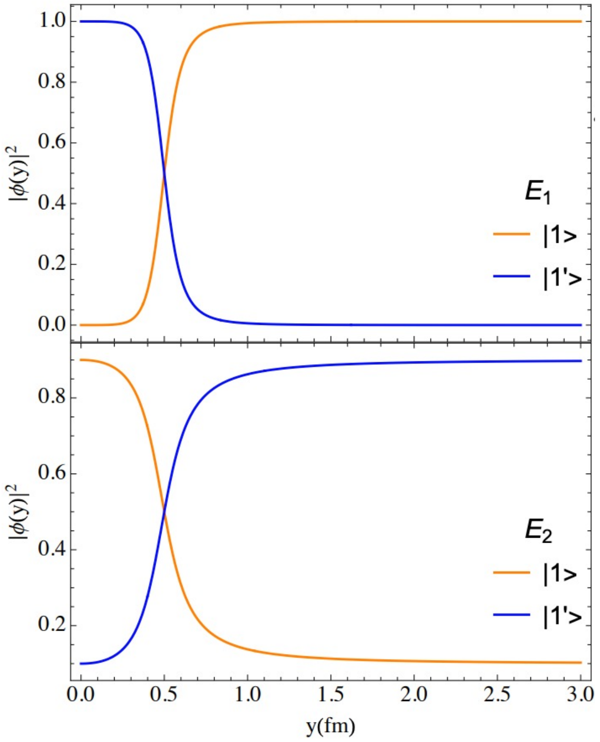
- Born-Oppenheimer (BO) potential: The quark positions are fixed.



Fixed $x = 0.5$ fm

$$\tilde{V}_{\mathrm{QM}}(x,y) =$$
$$\begin{pmatrix} 2\sigma x & \frac{2}{3}\sigma(x+y-\sqrt{x^2+y^2}) \\ \frac{2}{3}\sigma(x+y-\sqrt{x^2+y^2}) & 2\sigma y \end{pmatrix}$$





A toy model: Conventional QM VS String-like potential

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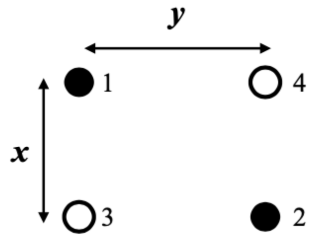
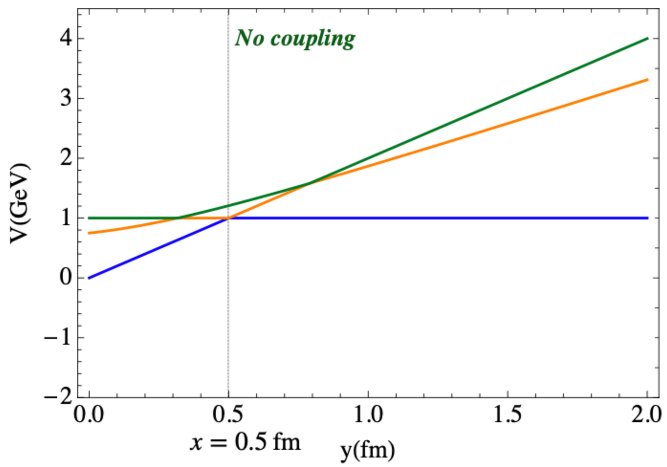
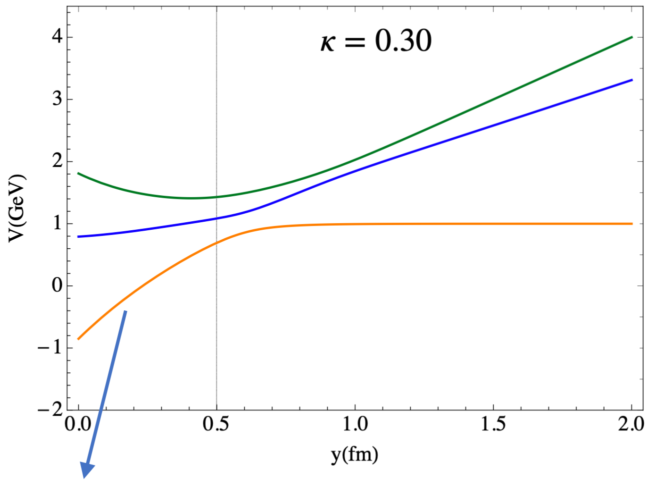
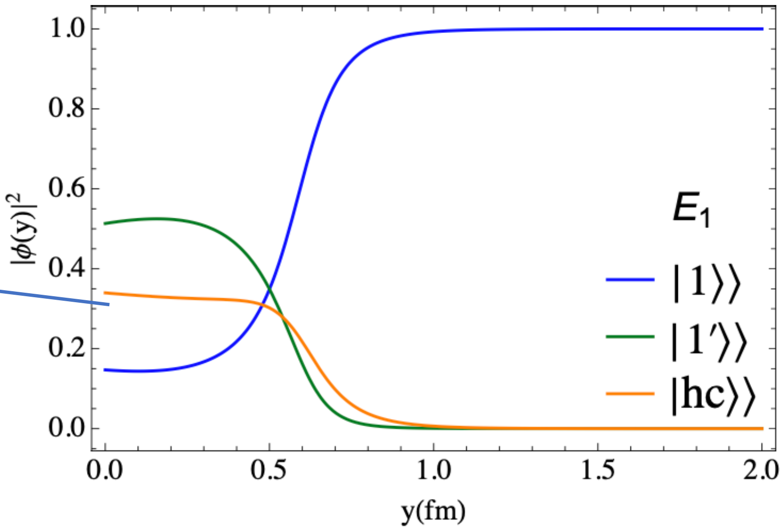


Fixed $x = 0.5$ fm







Mixing induced a strong attraction at short distances with important mixing of the hidden color (hc) state.

$$V_{\mathrm{ST}}(x,y) = \begin{pmatrix} 2\sigma x & \kappa e^{-\sigma xy} & \kappa' e^{-\sigma xy} \\ \kappa e^{-\sigma xy} & 2\sigma y & -\kappa' e^{-\sigma xy} \\ \kappa' e^{-\sigma xy} & -\kappa' e^{-\sigma xy} & \sigma(\frac{x+y}{2} + \sqrt{x^2+y^2}) \end{pmatrix}$$

Novel string-like potential: $T_{cc\bar{c}\bar{c}}$

- Application to the $T_{cc\bar{c}\bar{c}}$ and $T_{bb\bar{b}\bar{b}}$ states:
- Parameters are same as the conventional QM → reproduce the two meson thresholds
- ***Replace the linear confinement by the string-like confinement***

$$H = H_0 + \sum_{i,j} \frac{\lambda_i}{2} \cdot \frac{\lambda_j}{2} V_{\mathrm{SR}}\left(r_{ij}\right) + V_{\mathrm{ST}}$$

$$H_0 = \sum_{i=1}^{4} \frac{\mathbf{p}_i^2}{2m_i} + \sum_i m_i - T_G$$

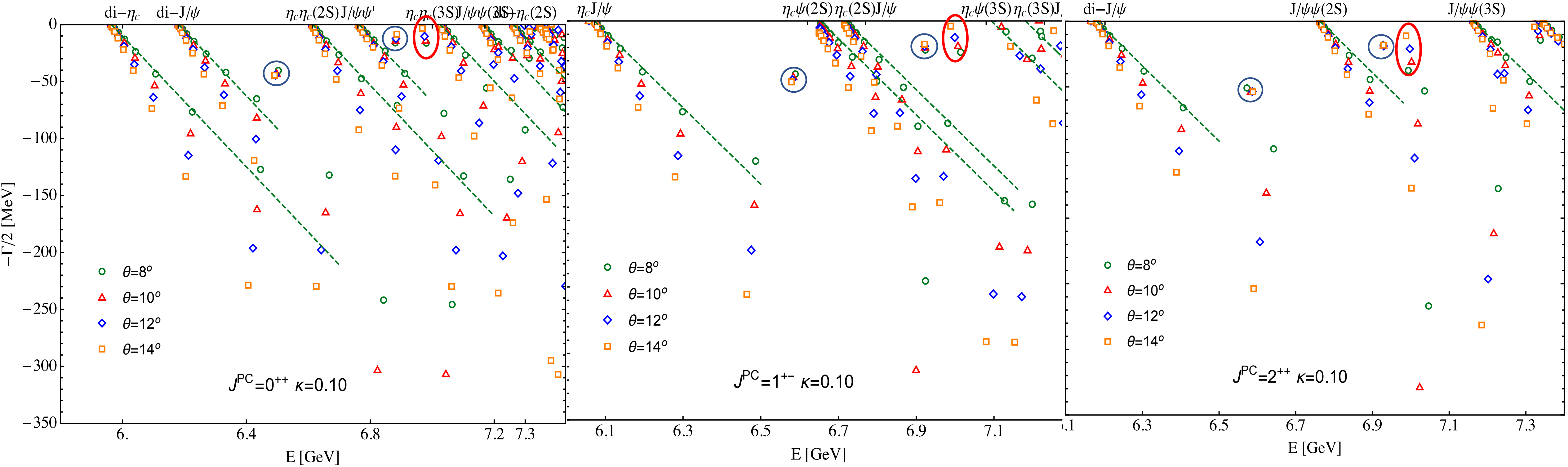
$$V_{\mathrm{SR}}\left(r_{ij}\right) = \frac{\alpha_s}{r_{ij}} - \frac{8\pi\alpha_s}{3m_i m_j}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r_{ij}^2} \mathbf{s}_i \cdot \mathbf{s}_j$$

$$V_{\mathrm{ST}} = \begin{pmatrix} \sigma\left(r_{13}+r_{24}\right) & \kappa e^{-\sigma S} & \kappa' e^{-\sigma S} \\ \kappa e^{-\sigma S} & \sigma\left(r_{14}+r_{23}\right) & -\kappa' e^{-\sigma S} \\ \kappa' e^{-\sigma S} & -\kappa' e^{-\sigma S} & \frac{\sigma}{4}\left[r_{13}+r_{24}+r_{14}+r_{23}+2\left(r_{12}+r_{34}\right)\right] \end{pmatrix}$$

with

$$S = \frac{1}{4}\left(r_{13}^2 + r_{24}^2 + r_{14}^2 + r_{23}^2\right) \longrightarrow \textit{N-body force}$$

Novel string-like potential: $T_{cc\bar{c}\bar{c}}$

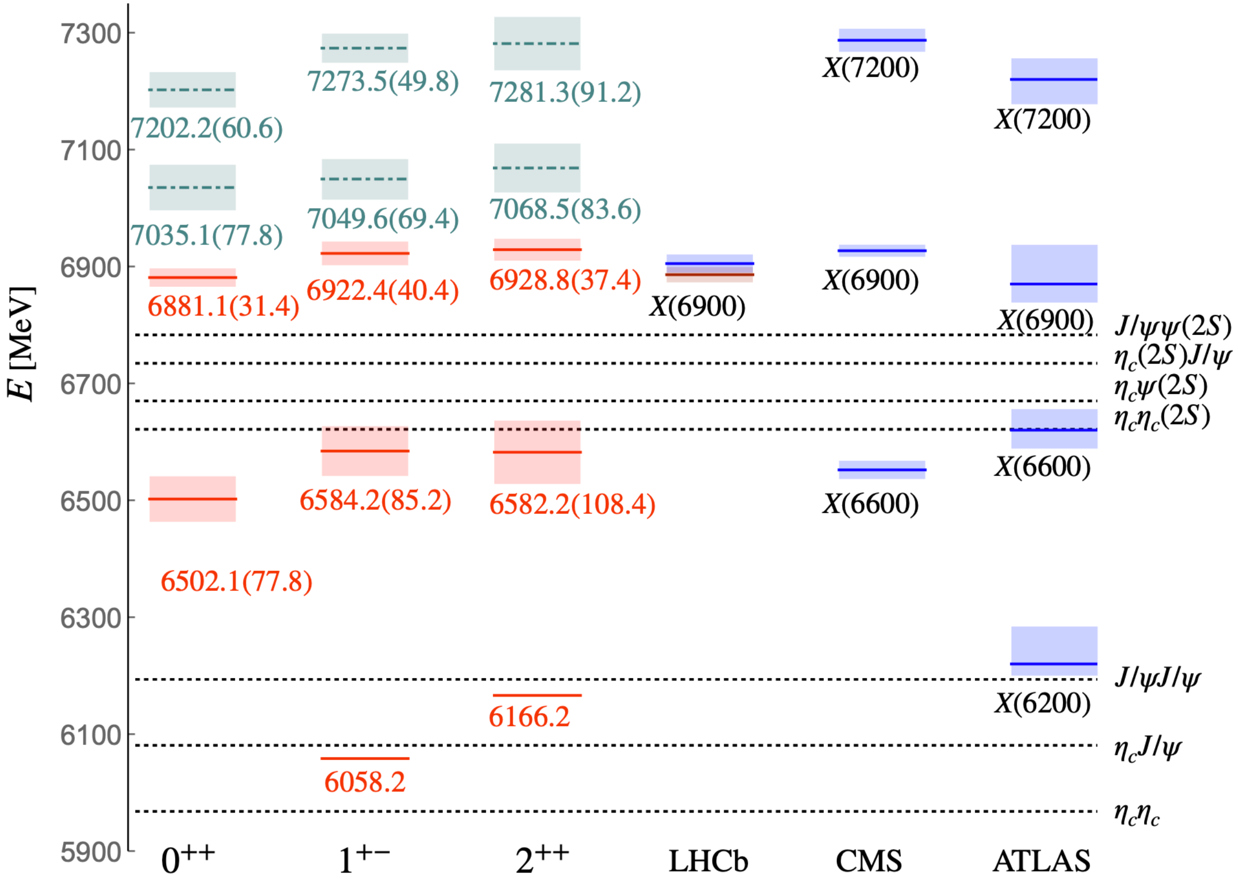


- *1st pole : a candidate for X(6600)*

- *2nd pole: a candidate for X(6900).*

- *A third pole at around 7.0 GeV?– convergency not good. For instance:* 0^{++}: *E=6980.4 MeV , Γ =29.0 MeV*

$T_{cc\bar{c}\bar{c}}$ spectrum



$\kappa = 0.10$

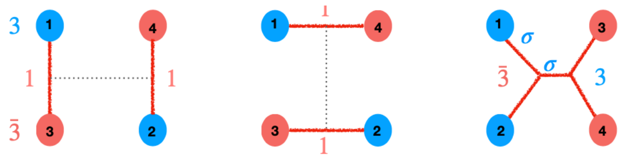
Phys. Rev. D 106, 096005 Phys. Rev. D 108 (2023) 7, L071501

- Conventional confinement: $V = -\frac{3}{4}\sigma\sum_{i<j}(T_i \cdot T_j)r_{ij}$

✓ ***1st pole -X(6900) &2nd pole-X(7200).***

✓ ***Absence of the lower $X(6600)$ state.***

- Novel string confinement: **N-body force**



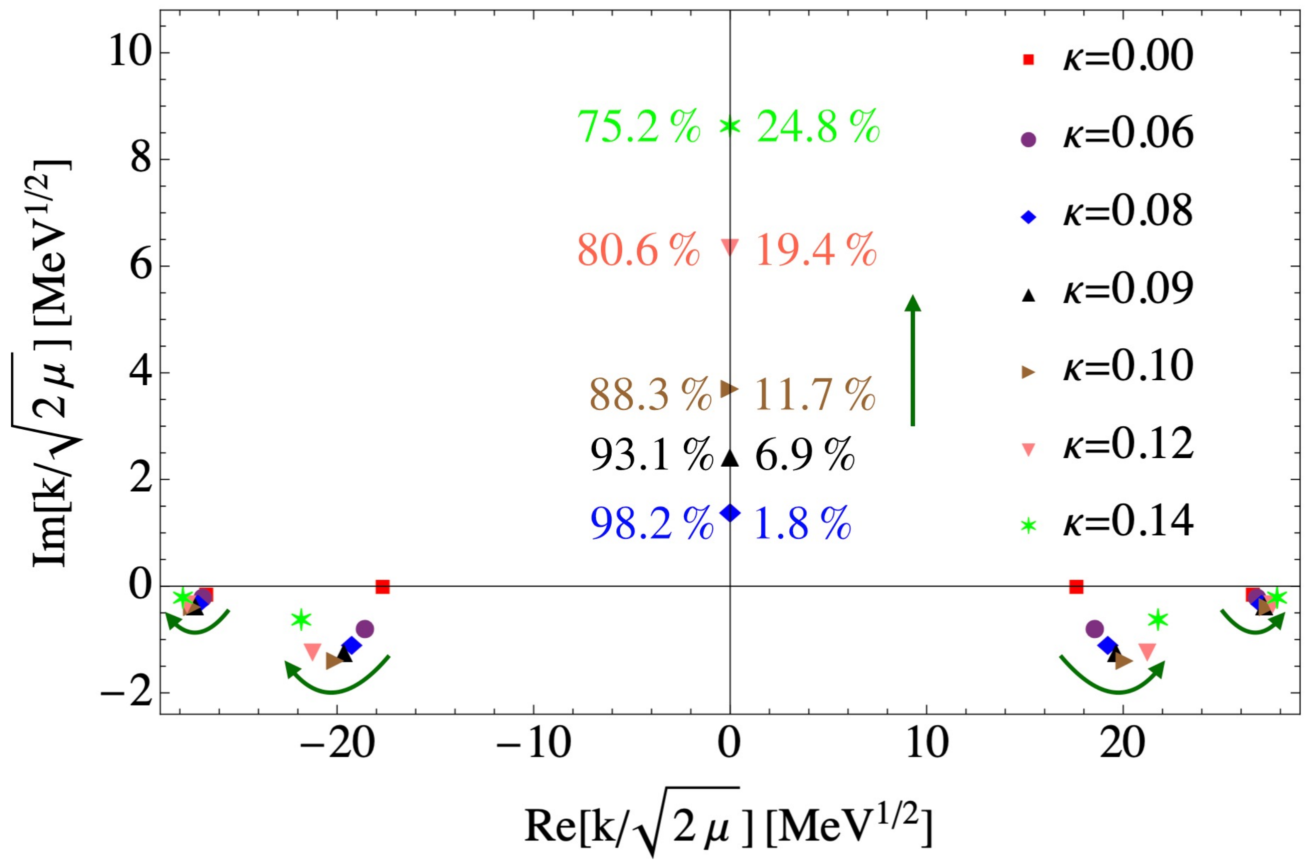
✓ Mixings of states induce a strong attraction.

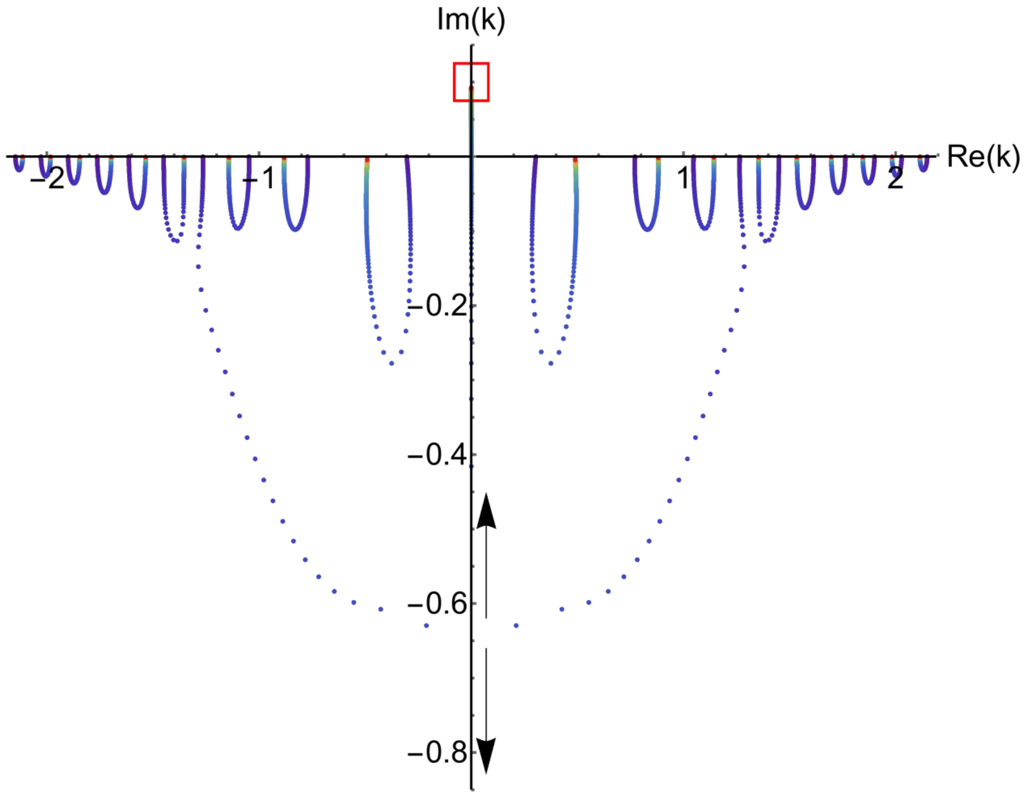
✓ ***A bound state appears.***

✓ Two candidates for ***X(6900) and X(6900).***

✓ ***X(7200) or X(7000)?***

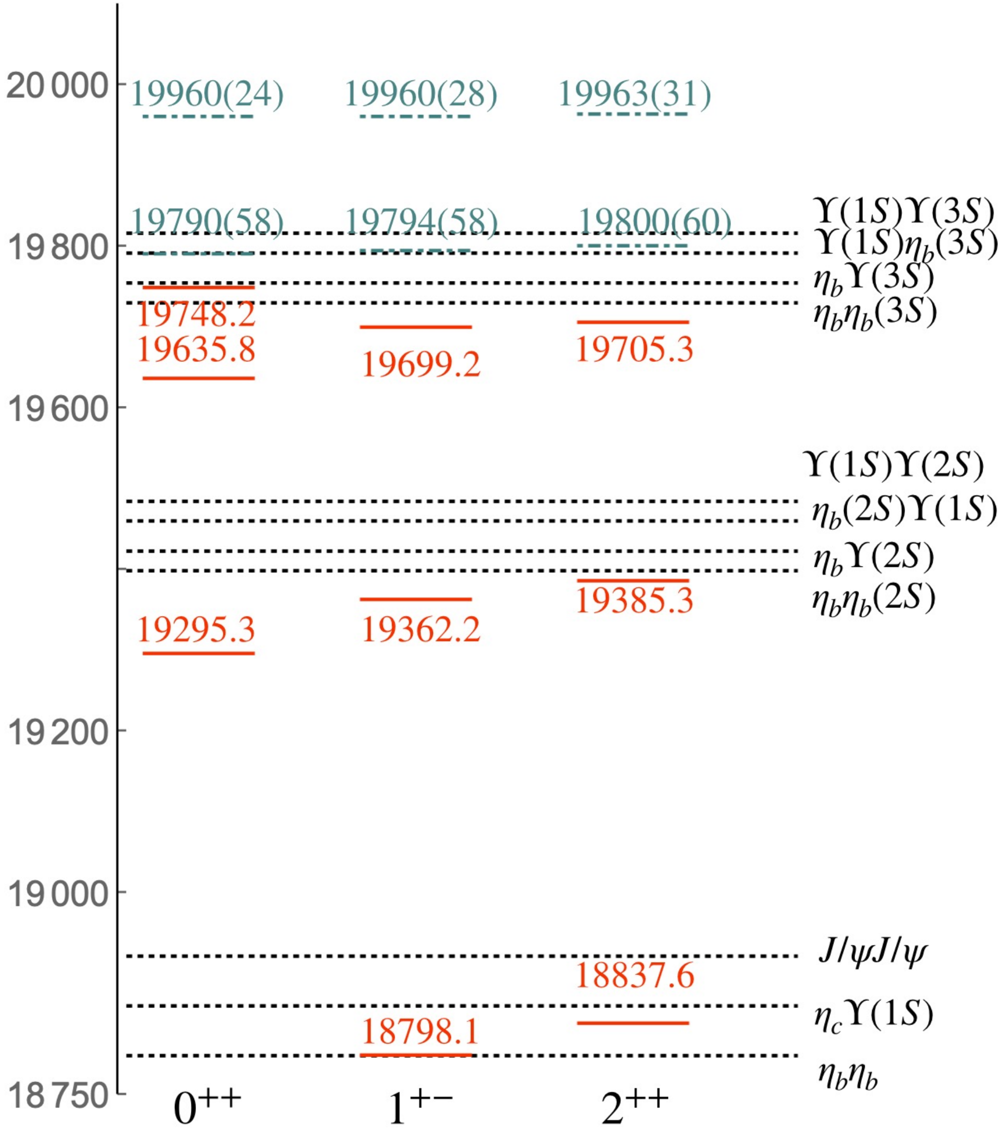
Pole trajectories





C. Hanhart et,al, Phys. Rev. D 106 (2022), 114003

$T_{bb\bar{b}\bar{b}}$ spectrum



Still on going

Summary

- The mass spectra of S-wave $QQ\bar{Q}'\,\bar{Q}'$ and $QQ\bar{q}\bar{q}$ tetraquark.

- TWO confinement potentials: $-\frac{3}{4}\sigma\Sigma_{i<j}(T_i \cdot T_j)r_{ij}$ VS $e^{-\frac{1}{4}(r_{13}^2+r_{24}^2+r_{14}^2+r_{23}^2)}$

- $-\frac{3}{4}\sigma\Sigma_{i<j}(T_i \cdot T_j)r_{ij}$: 3 bound states with $I(J^P) = 1^+$ and 62 low-lying resonant tetraquarks.

✓ ***A deep bound*** $T_{bb\bar{q}\bar{q}}$ $(BB\gamma)$ ***and a shallow*** $T_{bb\bar{q}\bar{q}}$ ***state; A bound*** $T_{cc\bar{q}\bar{q}}$ $(DD\gamma) \nrightarrow$ ***experimental*** T_{cc}^{+}

✓ $cc\bar{c}\bar{c}$: ***Absence of lower X(6600) & 1st pole -X(6900) &2nd pole-X(7200) .***

- General rule: Lager $\frac{m_Q}{m_{\bar{q}}}$, the easier to form the bound states

- $e^{-\frac{1}{4}(r_{13}^2+r_{24}^2+r_{14}^2+r_{23}^2)}$: $cc\bar{c}\bar{c}$ and $bb\bar{b}\bar{b}$

✓ $cc\bar{c}\bar{c}$: ***Additional bound state & 1st pole -X(6600)&2nd pole-X(6900) & Third pole at 7.0 GeV-X(7200)?***

✓$bb\bar{b}\bar{b}$: 2 bound state and 7 resonances.

Thank you for your attention!