北京航空航天大學 BEIHANG UNIVERSITY

Femtoscopy to unveil the nature of exotic hadrons

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Zhi-Wei Liu, Jun-Xu Lu, **LSG*,** PRD 107(2023)074019

Zhi-Wei Liu, Jun-Xu Lu, Ming-Zhu Liu, **LSG*,** PRD 108(2023)L031503

Zhi-Wei Liu, Jun-Xu Lu, Ming-Zhu Liu, **LSG*,** 2404.18607

Ming-Zhu Liu, Ya-Wen Pan, Zhi-Wei Liu, Tian-Wei Wu, Jun-Xu Lu, **LSG*,** 2404.06399

Contents

- ☞ **Brief introduction: exotic states and femtoscopy**
- ☞ **Femtoscopic correlation functions (CFs)—general features**
- ☞ **Recent applications**
  - ➢ DK **CFs for** $D_{s0}^*(2317)$**—deeply bound state**
  - ➢ $\Sigma_c\bar{D}^*$ **CFs for** $P_c(4440)$ **and** $P_c(4457)$**—weakly bound states**
  - ➢ $D\bar{D}^*$ **and** $D\bar{D}_s^*$ **CFs for** $Z_c(3900)$ **and** $Z_{cs}(3985)$**—resonant states**
- ☞ **Summary and outlook**

2003—the beginning of a new era in hadron physics

LEPS, 0301020
1186

$\Theta^{+}(1540)$

BaBar, 0304021
1003

$D_{s0}^{*}(2317)$

BABAR

Belle, 0309032
2586

X(3872)

Citations as of 2024.09.23

2003—the beginning of a new era in hadron physics

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BABAR

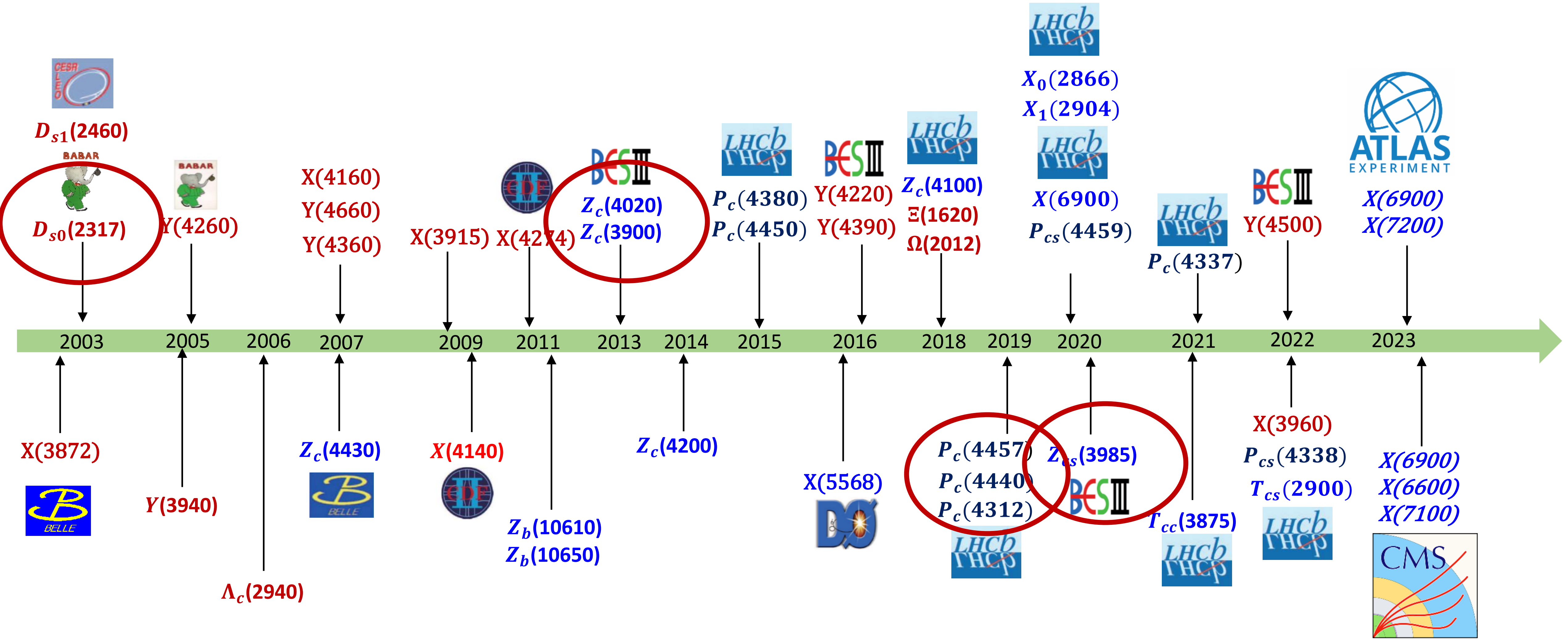
Belle, 0309032
2586

X(3872)

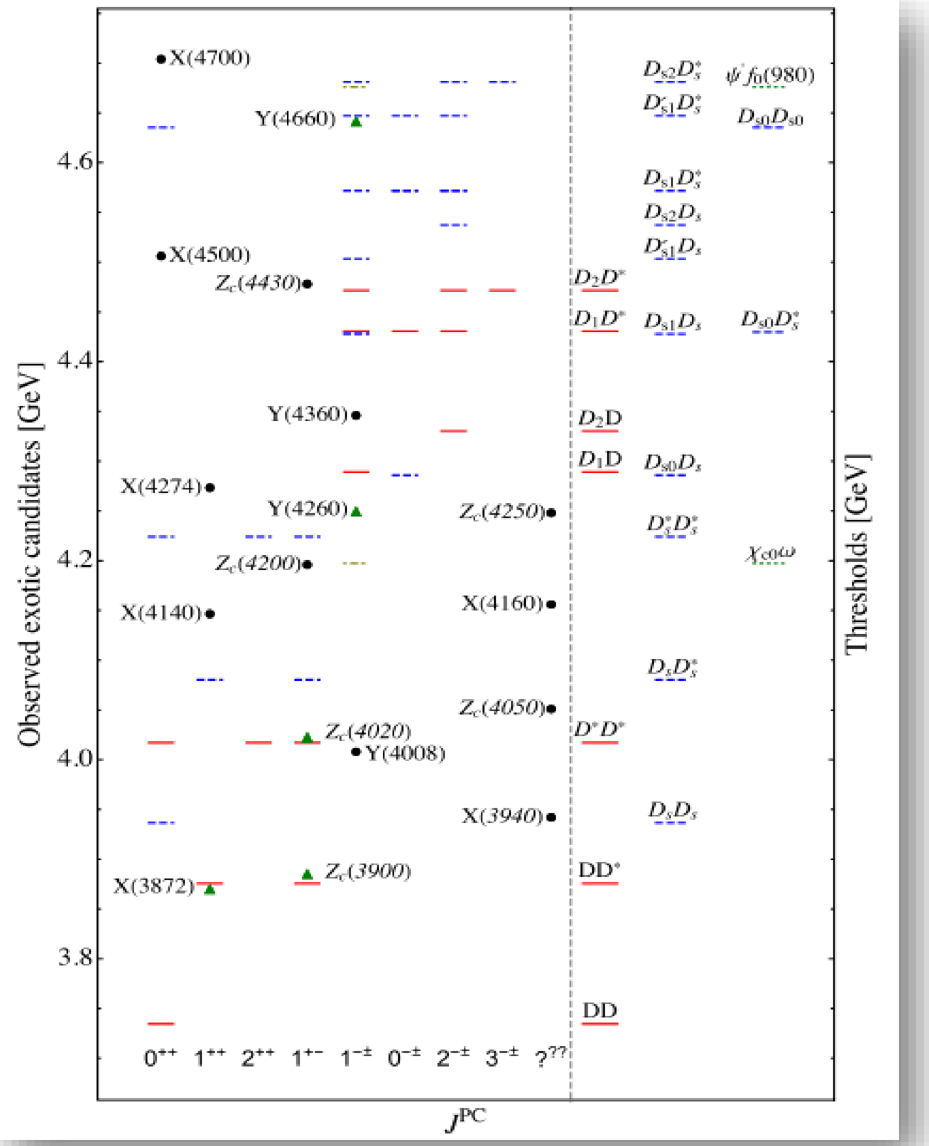
BELLE

Citations as of 2024.09.23

Many more exotic hadrons discovered



Many (if not all) of them close to thresholds—molecules



Feng-Kun Guo, Christoph Hanhart, Ulf-G. Meißner, Qian Wang, Qiang Zhao, Bing-Song Zou. Rev.Mod.Phys. 90 (2018) 015004

1153 citations as of 2024.07.10

Hua-Xing Chen, Wei Chen, Xiang Liu and Shi-Lin Zhu, *The hidden-charm pentaquark and tetraquark states,* Phys. Rept.639 (2016) 1

1109 citations as of 2024.07.10

Q: How to check the molecular picture?

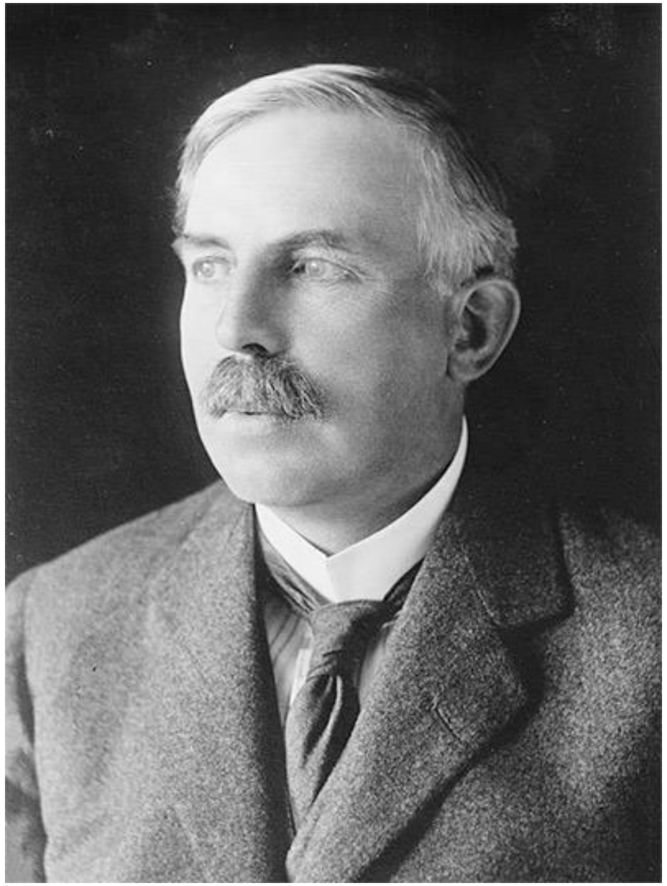
Q: How to check the molecular picture?

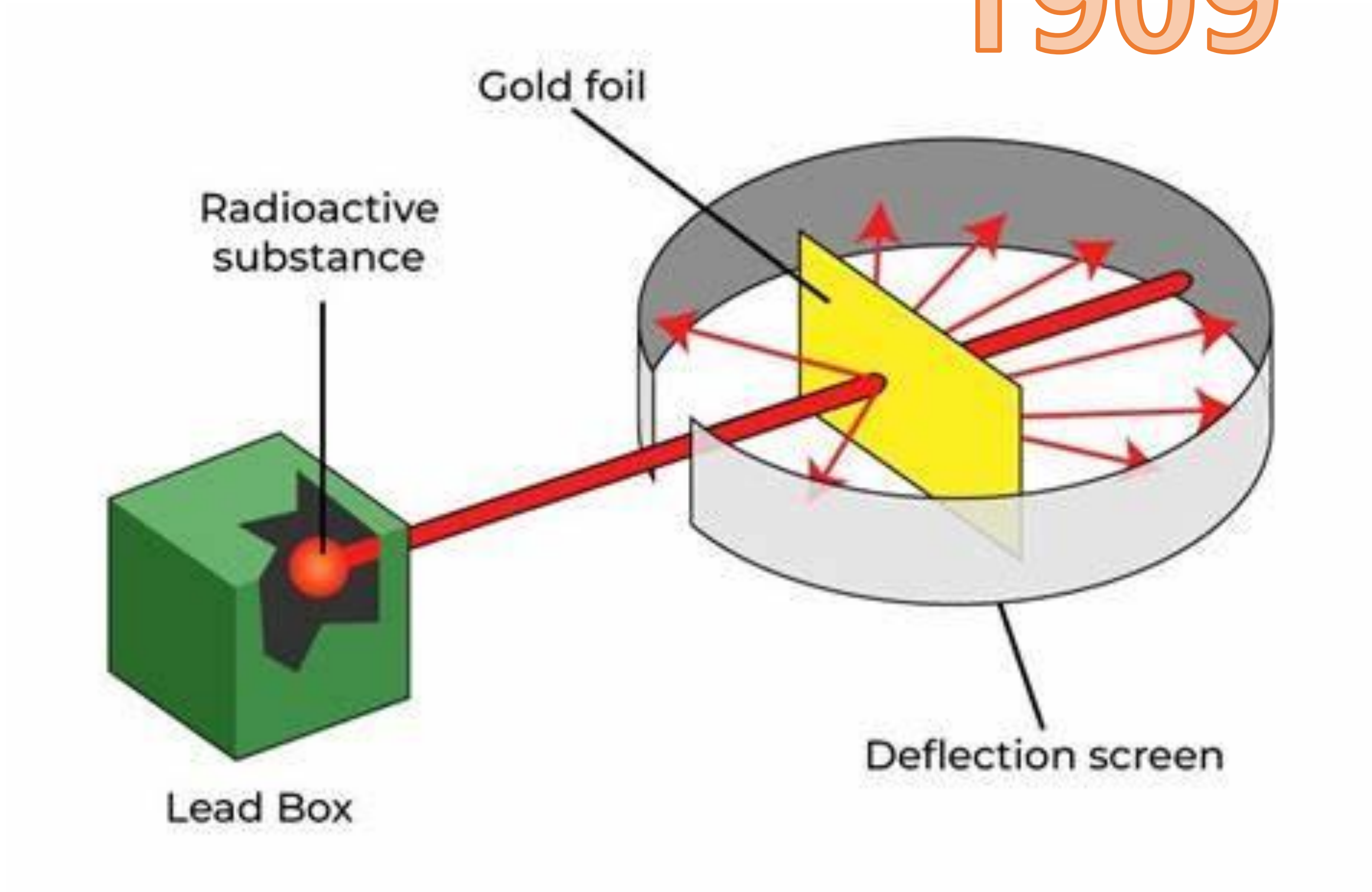
A: Measure the hadron-hadron interactions responsible for forming these hadronic molecules

Direct verifications of hadron-hadron interactions

- Rutherford scattering experiment

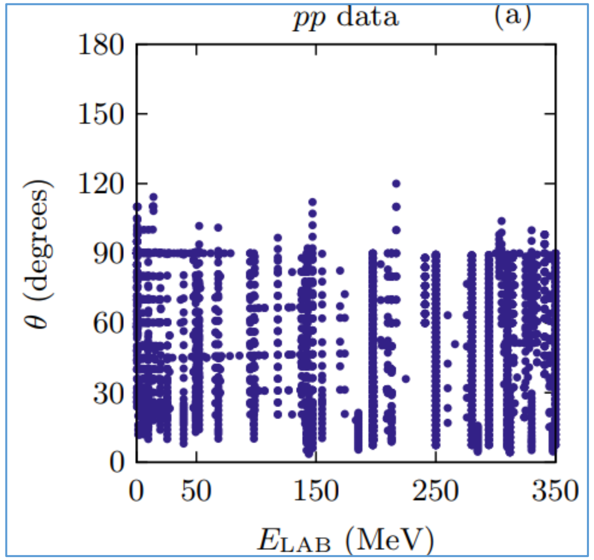
1909

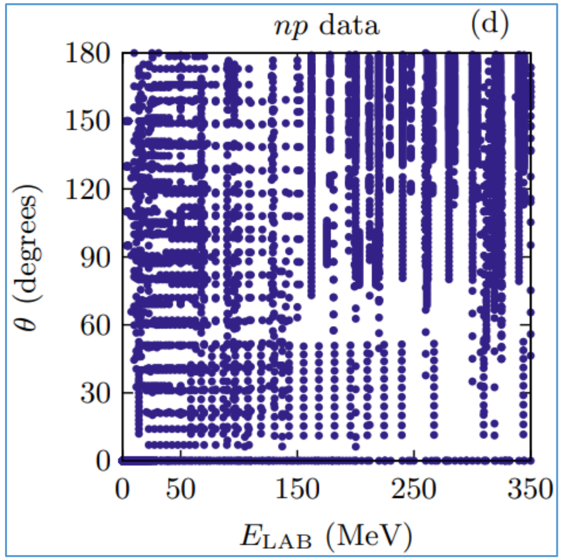




Direct verifications of hadron-hadron interactions

- **For stable hadrons, scattering experiments are routinely employed in studying their interactions**

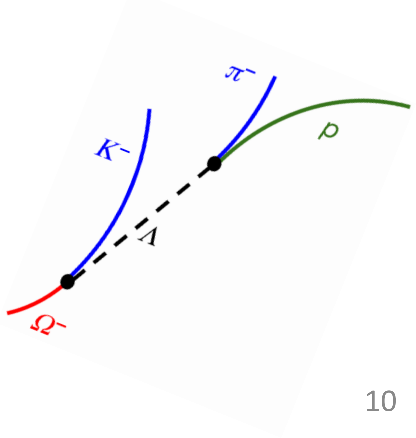




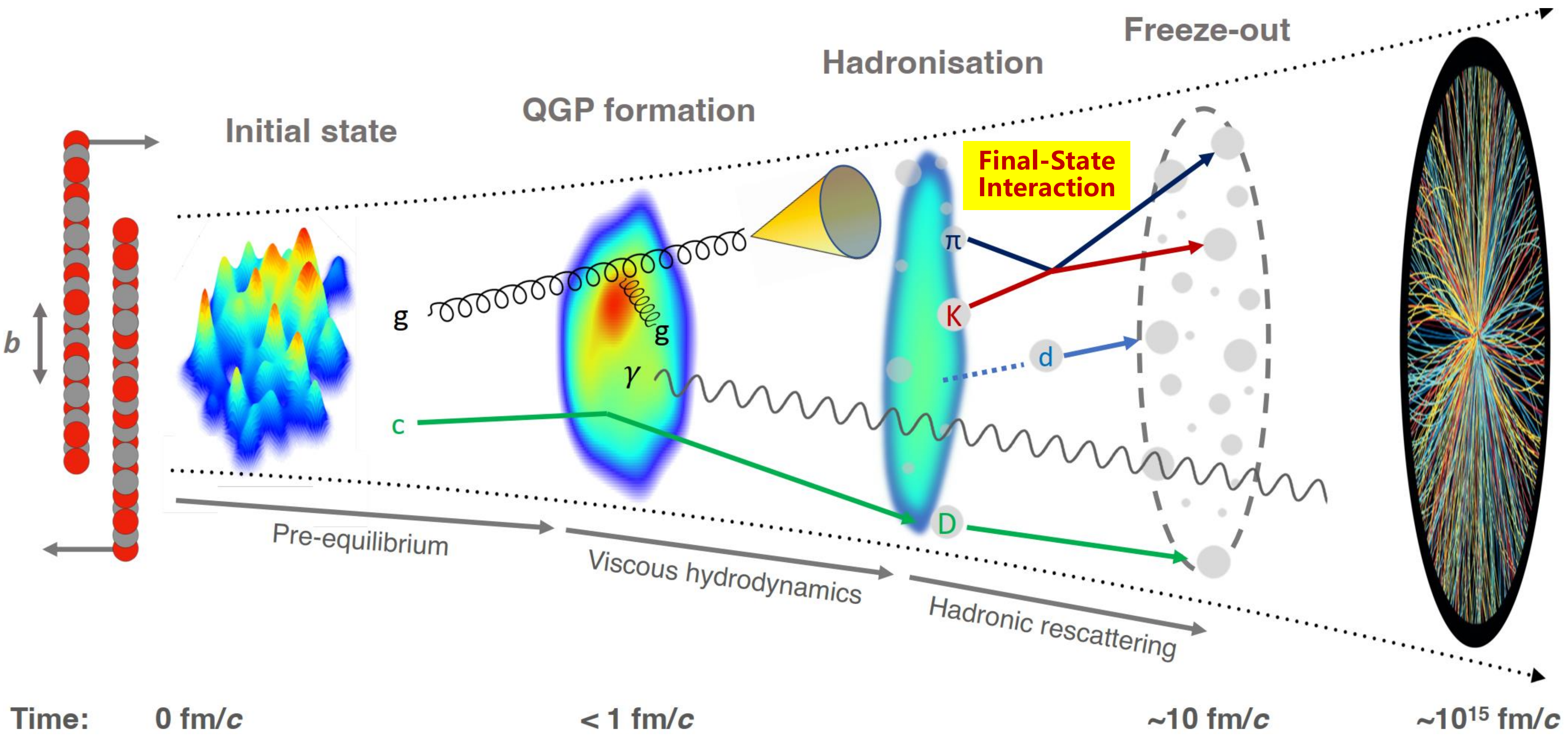
Phys.*Rev.C* 89 (2014) 064006

There exist abundant data for nucleon-nucleon scattering (8125)

- **For unstable particles, direct scattering experiments are difficult or impossible!**

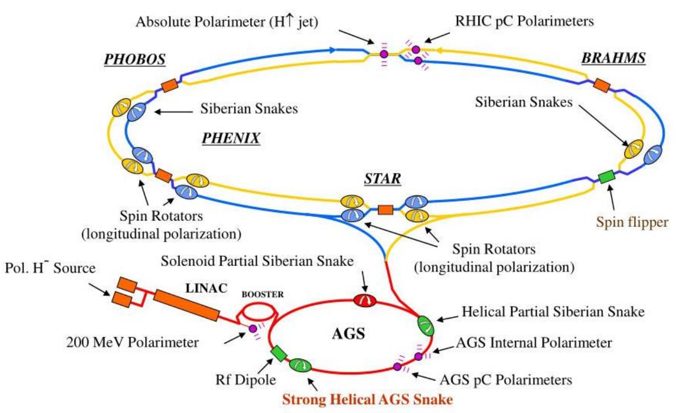


New probe—femtoscopic correlation functions

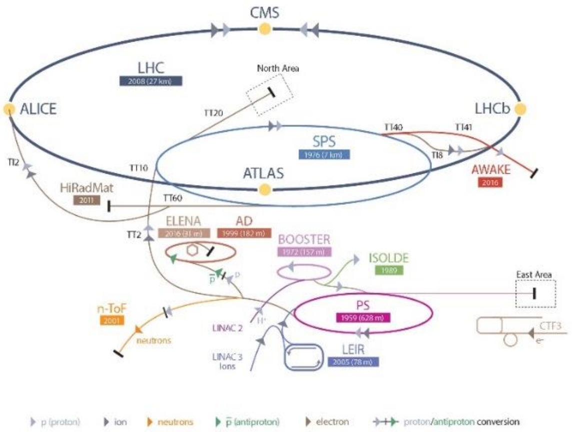


New probe—femtoscopic correlation functions

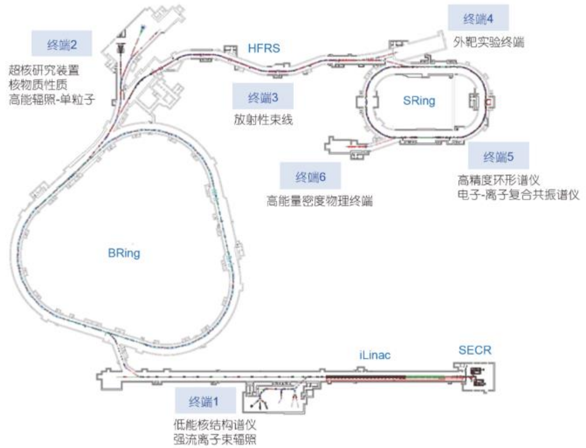
Brookhaven National Laboratory RHIC Relativistic Heavy Ion Collider



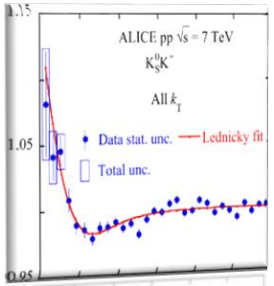
Large Hadron Collider

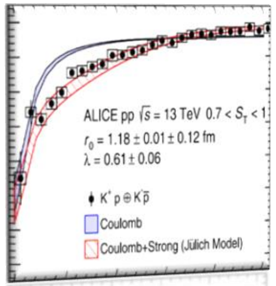


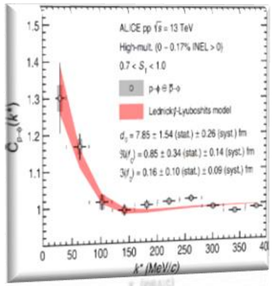
HIAF 强流重离子加速器装置 High Intensity heavy-ion Accelerator Facility

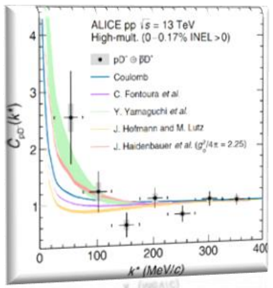


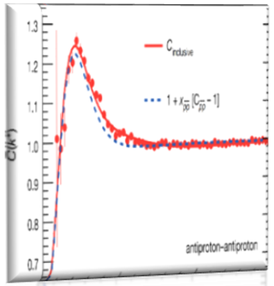
$K^0_S K^\pm$	$K^\pm p$	ϕp	$D^- p$	$\bar{p}\,\bar{p}$	$\Lambda\Lambda$	$\Xi^- p$	$\Omega^- p$
$p+p, \sqrt{s} = 7$ TeV	$p+p, \sqrt{s} = 5, 7, 13$ TeV	$p+p, \sqrt{s} = 13$ TeV	$p+p, \sqrt{s} = 13$ TeV	Au + Au, $\sqrt{s} = 200$ MeV	Au + Au, $\sqrt{s} = 200$ MeV	p + Pb, $\sqrt{s} = 5.02$ TeV	$p+p, \sqrt{s} = 13$ TeV

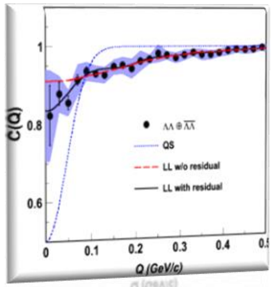


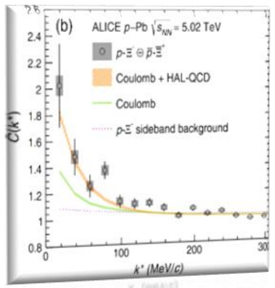


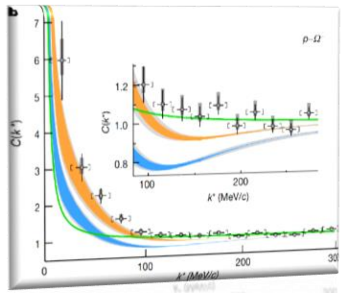












ON-GOING

ALICE Collaboration, Phys. Lett. B **790** (2019) 22

ALICE Collaboration, Phys. Rev. Lett. **124** (2020) 092301

ALICE Collaboration, Phys. Rev. Lett. **127** (2021) 172301

ALICE Collaboration, Phys. Rev. D **106** (2022) 052010

STAR Collaboration, Nature **527** (2015) 345

STAR Collaboration, Phys. Rev. Lett. **114** (2015) 022301

ALICE Collaboration, Phys. Rev. Lett. **123** (2019) 112002

ALICE Collaboration, Nature **588** (2020) 232

Contents

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- ☞ **Femtoscopic correlation functions (CFs)—general features**
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  - ➢ DK CFs for $D_{s0}^*(2317)$—**deeply bound state**
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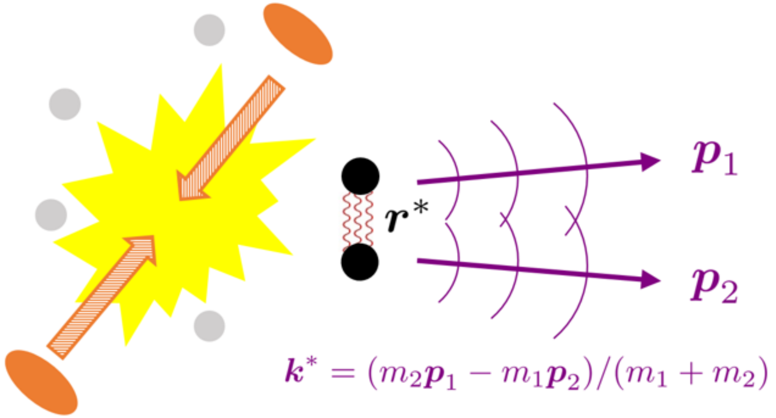
  - ➢ $D\bar{D}^*$ and $D\bar{D}_s^*$ CFs for $Z_c(3900)$ and $Z_{cs}(3985)$—**resonant states**
- ☞ **Summary and outlook**

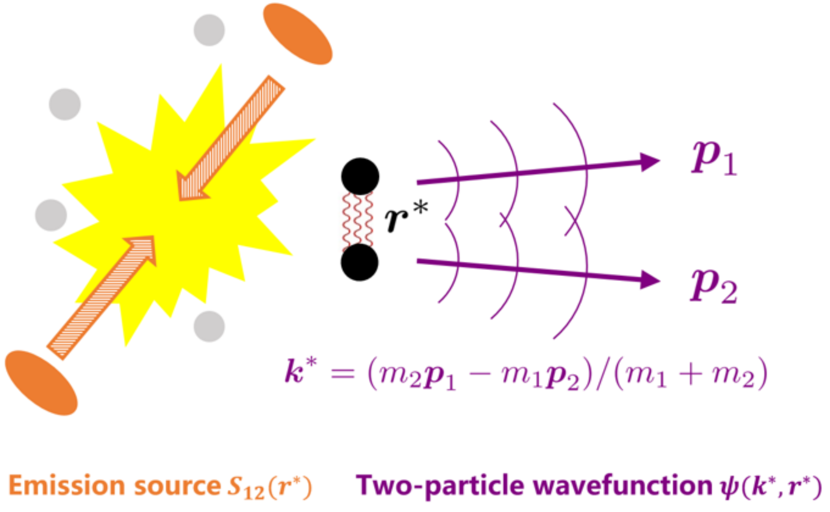
Femtoscopic correlation functions (CFs)



Emission source $S_{12}(r^*)$ Two-particle wavefunction $\psi(k^*, r^*)$

Femtoscopic correlation functions (CFs)

$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{\mathrm{P}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{\mathrm{P}(\boldsymbol{p}_1) \cdot \mathrm{P}(\boldsymbol{p}_2)}$$



Exp. measurement

mixed-event technique

$$C(k) = \xi(k) \frac{N_{\text{same}}(k)}{N_{\text{mixed}}(k)}$$

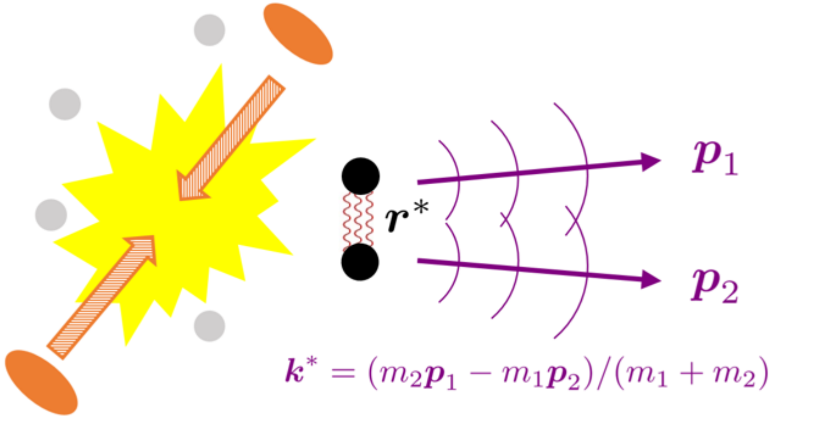
N_{same}: the same event distributions

N_{mixed}: the mixed event distributions

ξ: the corrections for experimental effects

Femtoscopic correlation functions (CFs)

$$C(\boldsymbol{p}_1,\boldsymbol{p}_2) = \frac{\mathrm{P}(\boldsymbol{p}_1,\boldsymbol{p}_2)}{\mathrm{P}(\boldsymbol{p}_1)\cdot\mathrm{P}(\boldsymbol{p}_2)}$$



Emission source $S_{12}(\boldsymbol{r}^*)$ Two-particle wavefunction $\psi(\boldsymbol{k}^*,\boldsymbol{r}^*)$

Exp. measurement

mixed-event technique

$$C(k) = \xi(k)\frac{N_{\mathrm{same}}(k)}{N_{\mathrm{mixed}}(k)}$$

N_{same}: the same event distributions

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Theo. description

Koonin–Pratt formula

$$C(k) = \int S_{12}(\boldsymbol{r})\ |\psi(\boldsymbol{k},\boldsymbol{r})|^2\ \mathrm{d}\boldsymbol{r}$$

spacial structure

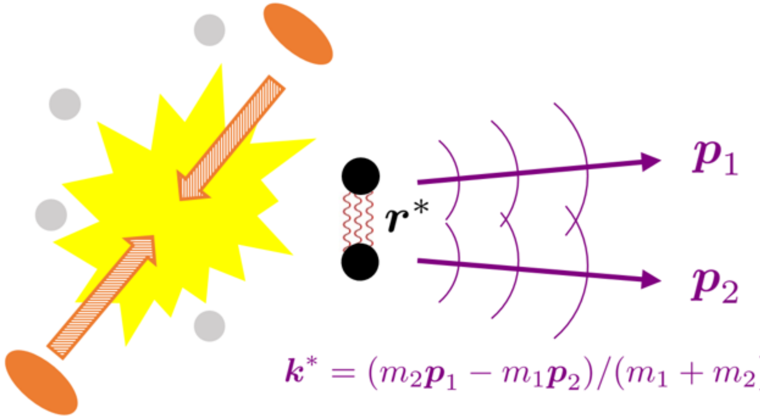
final-state interactions

quantum statistics effects

coupled-channel effects

Femtoscopic correlation functions (CFs)

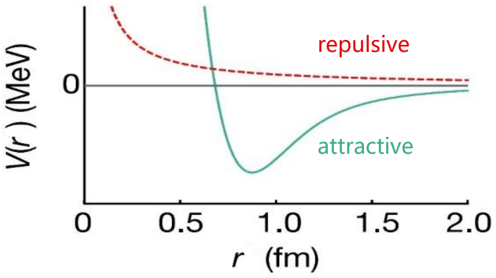
$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) = \frac{\mathrm{P}(\boldsymbol{p}_1, \boldsymbol{p}_2)}{\mathrm{P}(\boldsymbol{p}_1) \cdot \mathrm{P}(\boldsymbol{p}_2)}$$



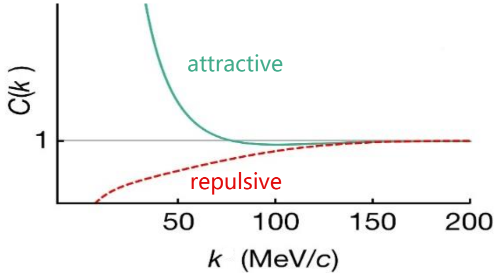
$$\boldsymbol{k}^* = (m_2\boldsymbol{p}_1 - m_1\boldsymbol{p}_2)/(m_1 + m_2)$$

Emission source $S_{12}(\boldsymbol{r}^*)$ Two-particle wavefunction $\psi(\boldsymbol{k}^*, \boldsymbol{r}^*)$

Interacting potential



Correlation function



Schrödinger equation

Two-particle wavefunction $\psi(\boldsymbol{k}, \boldsymbol{r})$

Exp. measurement

mixed-event technique

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N_{same}: the same event distributions

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Koonin–Pratt formula

$$C(k) = \int S_{12}(\boldsymbol{r})\, |\psi(\boldsymbol{k}, \boldsymbol{r})|^2\, \mathrm{d}\boldsymbol{r}$$

spacial structure

final-state interactions

quantum statistics effects

coupled-channel effects

Basic Properties

$C(k)$
- >1 if the interaction is attractive
- =1 if there is no interaction
- <1 if the interaction is repulsive

Femtoscopic correlation functions (CFs)

S. E. Koonin, Phys. Lett. B **70** (1) (1977) 43
A. Ohnishi, Nucl. Phys. A **954** (2016) 294

Koonin–Pratt (KP) formula

$$C(k) = \int S_{12}(r)\ |\Psi(\boldsymbol{r},\boldsymbol{k})|^2\ \mathrm{d}\boldsymbol{r}$$

Only S-waves
$$C(k) \simeq 1 + \int_0^\infty 4\pi r^2 \mathrm{d}r\ S_{12}(r)\ \left[|\psi_0(r,k)|^2 - |j_0(kr)|^2\right]$$

- **Common static and spherical Gaussian source**

$$S_{12}(r) = \exp[-r^2/(4R^2)]/(2\sqrt{\pi}R)^3$$

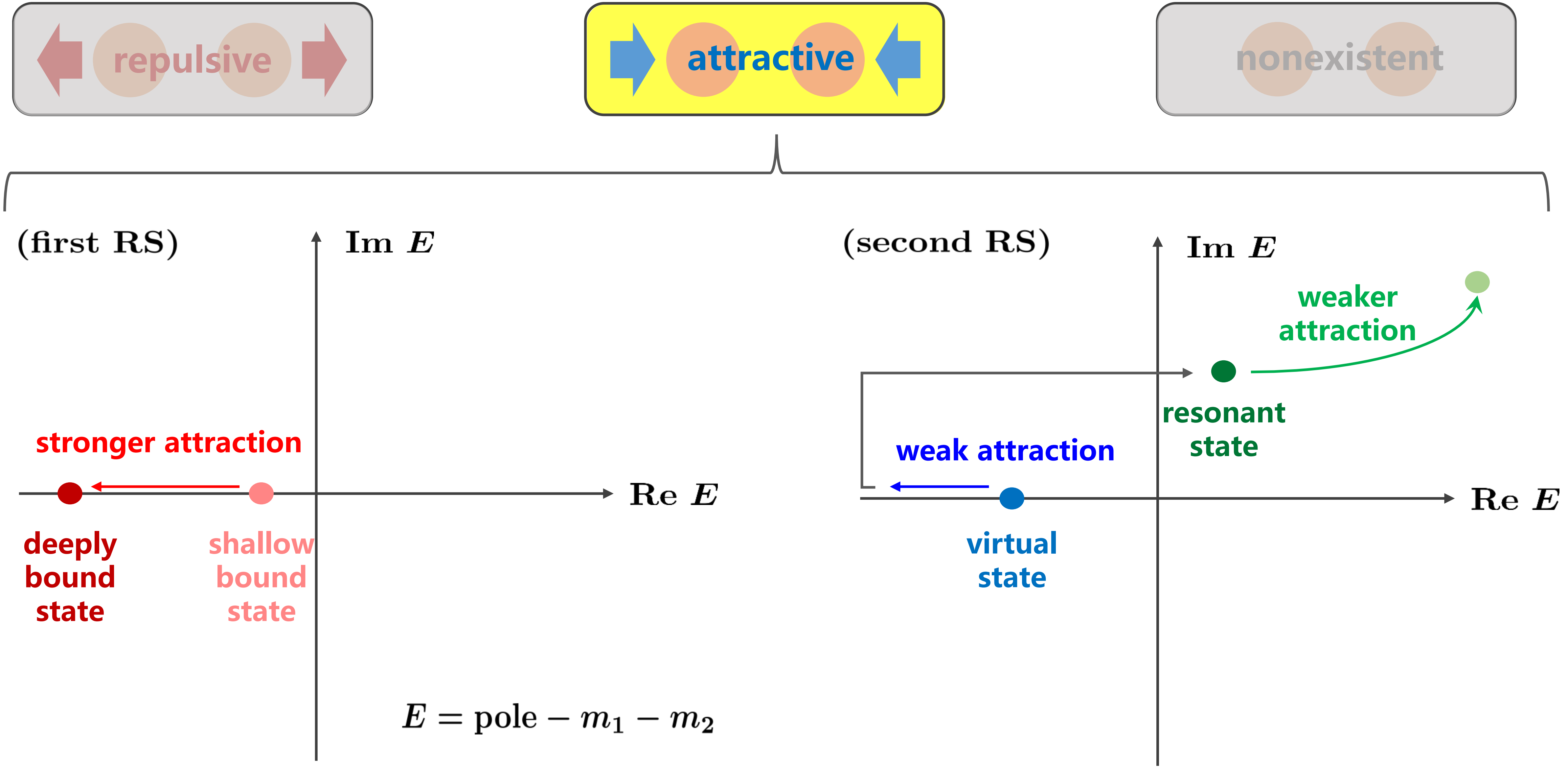
- **Obtain the scattering wave function once the interaction is known**
  - **Schrödinger equation**

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + V\psi = E\psi$$

  - **Lippmann-Schwinger equation**

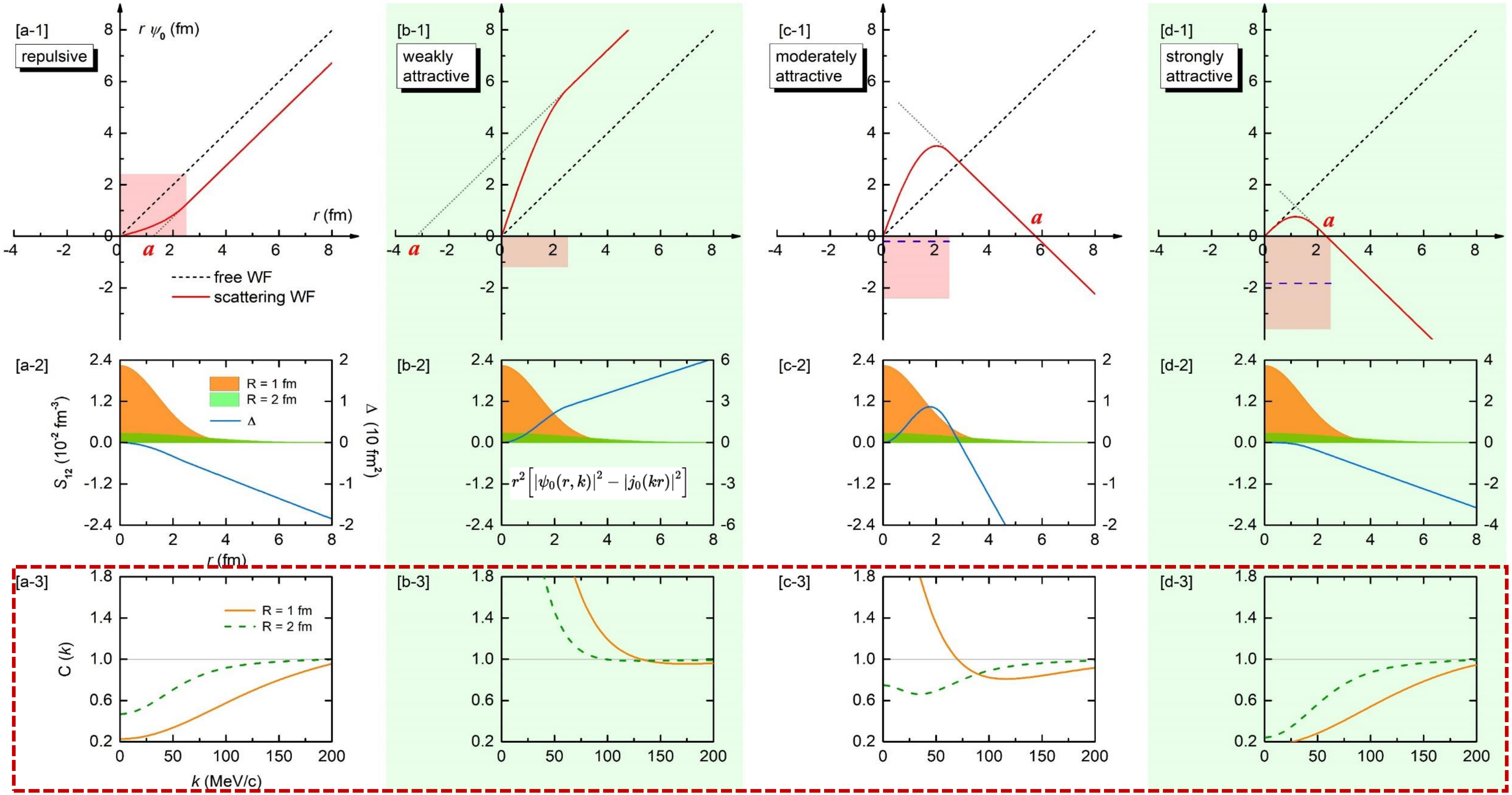
$$T = V + VGT \quad \Longrightarrow \quad |\psi\rangle = |\phi\rangle + GT|\phi\rangle$$

Classification of hadron-hadron interactions



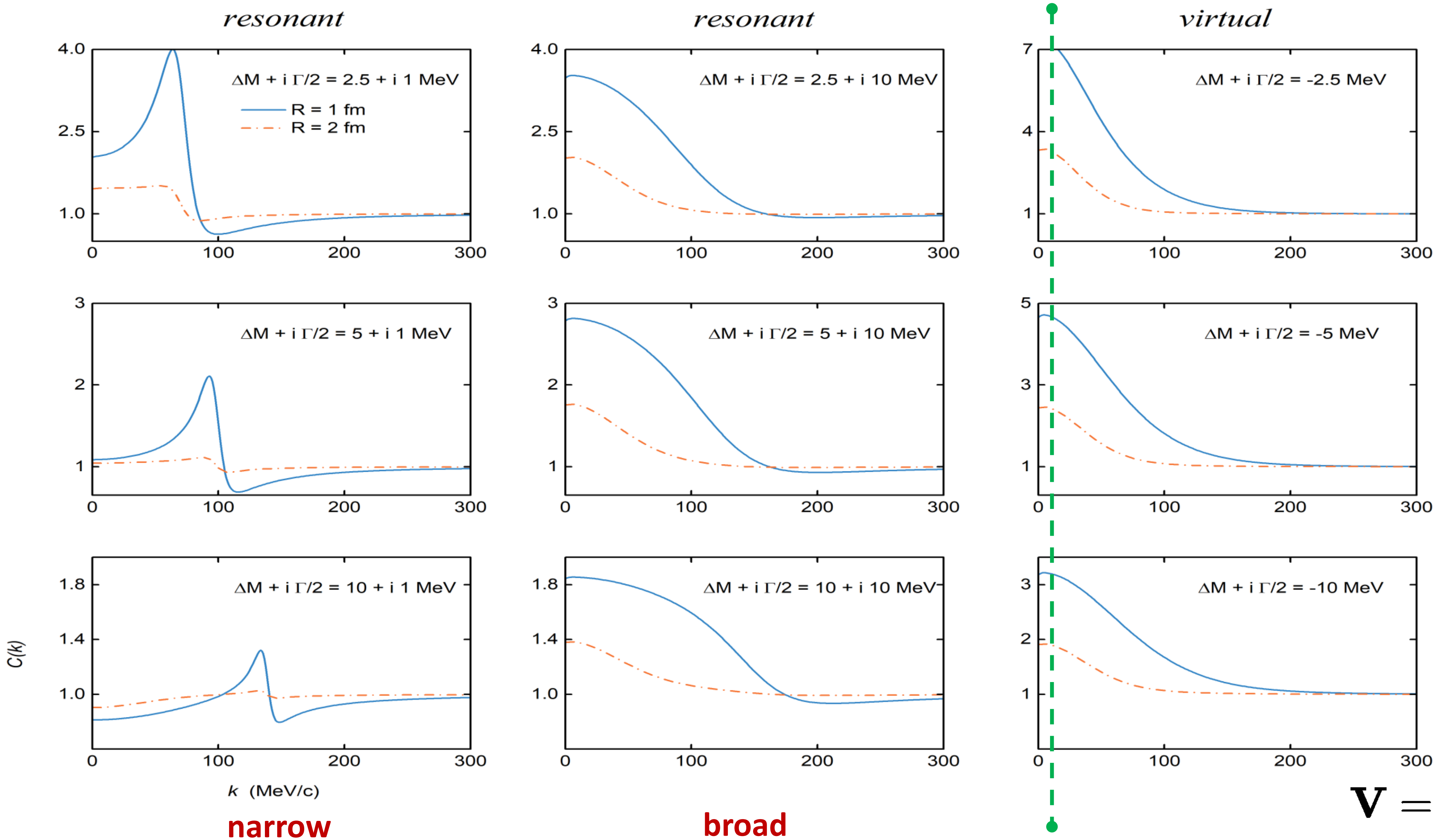
CFs in the presence of bound states

*Zhi-Wei Liu, Jun-Xu Lu and **LSG****, *PRD 107, 074019 (2023)*



CFs in the presence of resonant and virtual states

*Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and **LSG*,*** *2404.18607*



narrow **broad**

$$\mathbf{V = a + b \cdot k^2}$$

Contents

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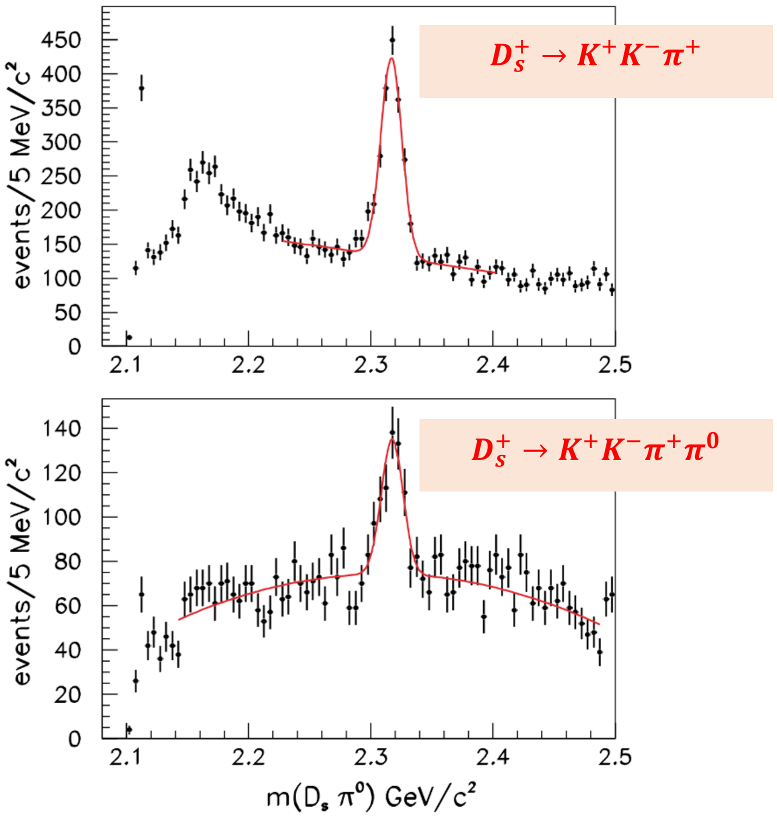
Mysterious exotic hadron $D_{s0}^*(2317)$—1003 citations

$M = 2317.8 \pm 0.6$ and $\Gamma < 3.8$ MeV



BABAR, PRL90 (2003) 242001

- **160 MeV lower than the quark model predictions – difficult to be understood as a conventional charm-strange meson**
- $m(D_{s1}) - m(D_{s0}^*) \approx m(D^*) - m(D)$
  F. K. Guo, C. Hanhart, U.-G. Meißner, PRL.102 (2009) 242004
- **DK scattering length from LQCD favors molecular scenario**
  L. Liu, K. Orginos, F. K. Guo, C. Hanhart, U.-G. Meißner, PRD87 (2013) 014508

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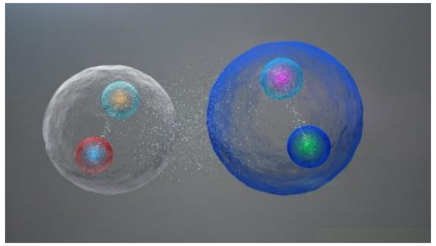
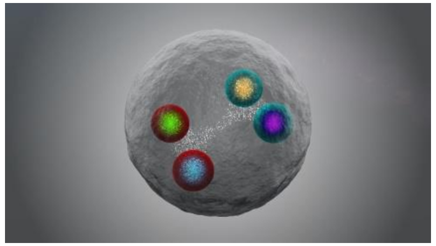


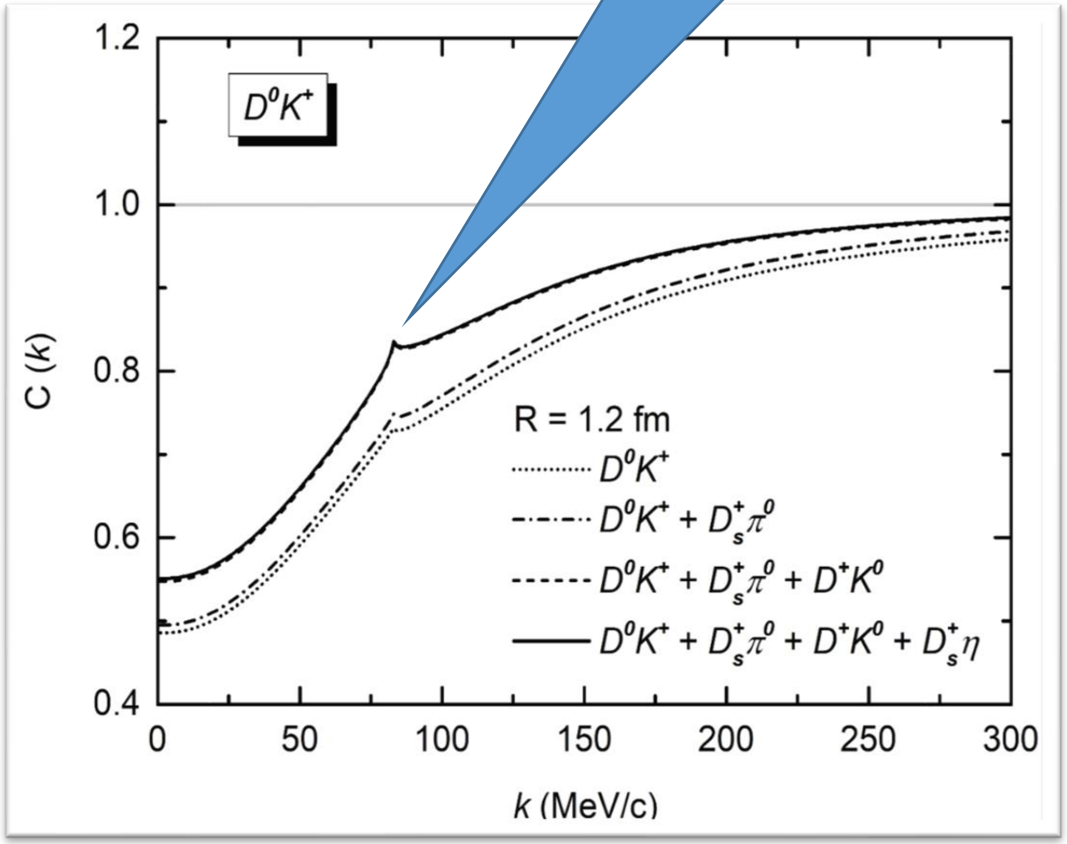
VS

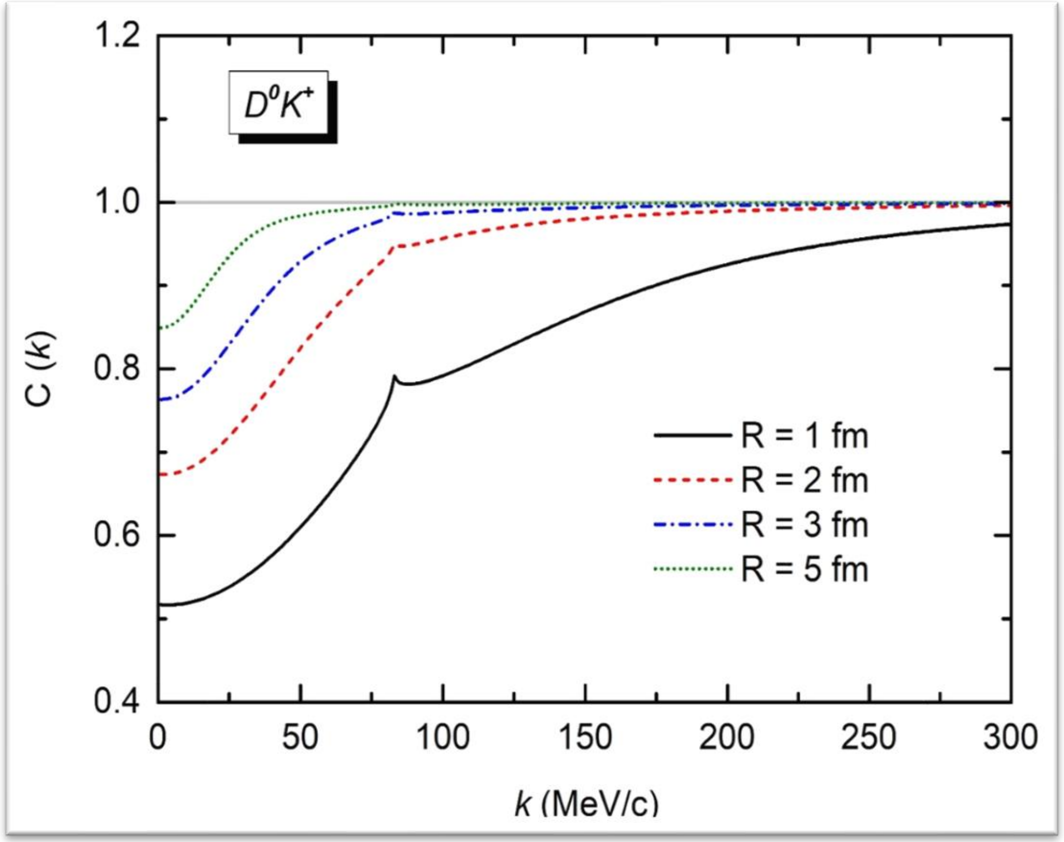


DK CFs and its source size dependence

$D^+K^0 \rightarrow D^0K^+$

Typical feature of deeply bound states





*Zhi-Wei Liu, Jun-Xu Lu and **LSG***, PRD107(2023)074019*

Confirmed by two subsequent studies

PHYSICAL REVIEW D **108**, 014020 (2023)

Femtoscopic signatures of the lightest *S*-wave scalar open-charm mesons

M. Albaladejo,[1,*] J. Nieves,[1,†] and E. Ruiz Arriola[2,‡]
[1]*Instituto de Física Corpuscular (centro mixto CSIC-UV), Institutos de Investigación de Paterna, C/Catedrático José Beltrán 2, E-46980 Paterna, Valencia, Spain*
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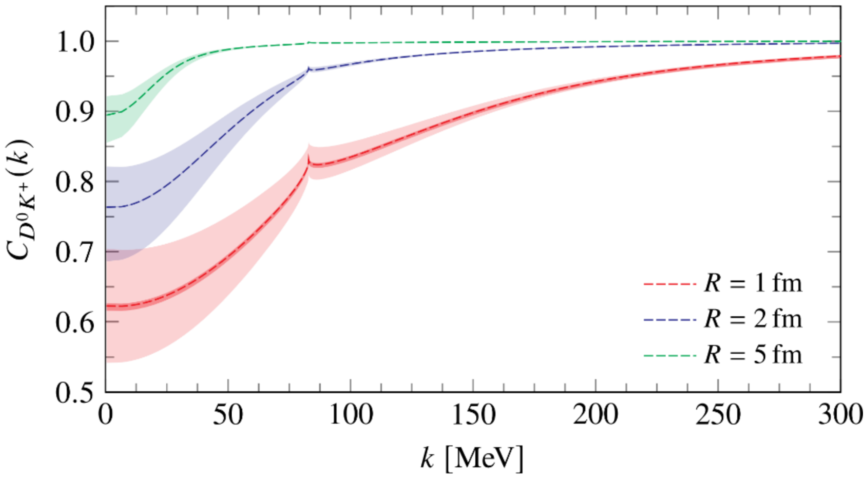
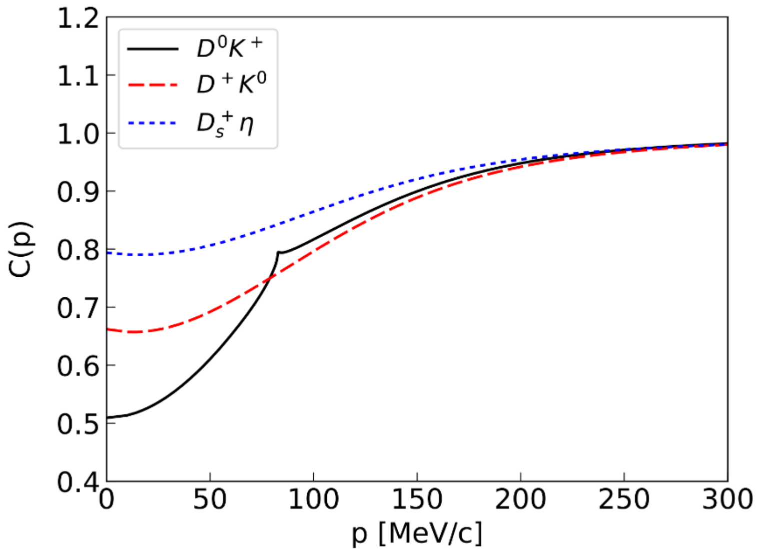
(Received 18 April 2023; accepted 26 June 2023; published 19 July 2023)





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Letter

Model independent analysis of femtoscopic correlation functions: An application to the $D^*_{s0}(2317)$

Natsumi Ikeno[a,b,*], Genaro Toledo[c], Eulogio Oset[d]
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[b] Cyclotron Institute, Texas A&M University, College Station, TX 77843, USA
[c] Instituto de Física, Universidad Nacional Autonoma de Mexico, AP20-364, Ciudad de Mexico 01000, Mexico
[d] Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia - CSIC, Institutos de Investigación de Paterna, Aptdo. 22085, 46071 Valencia, Spain

Contents

- ☞ **Brief introduction: exotic states and femtoscopy**
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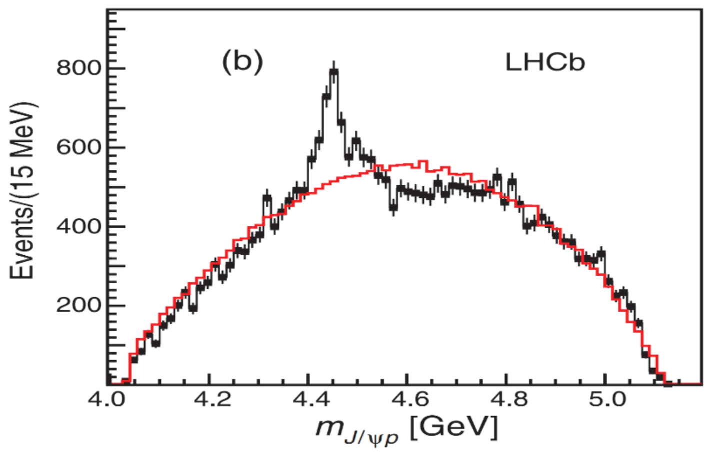
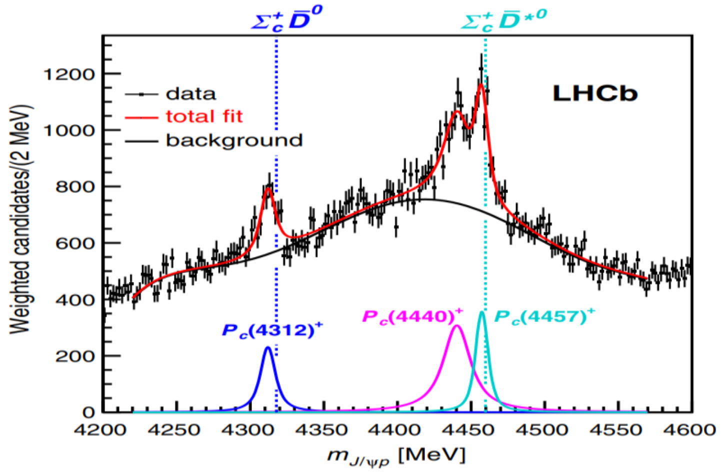
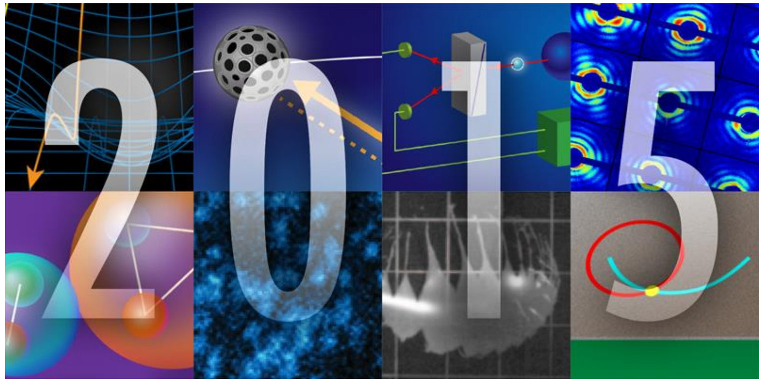
Pentaquark states $P_c(4440)$ & $P_c(4457)$—1798 citation



LHCb, PRL115 (2015) 072001



LHCb, PRL122 (2019) 222001



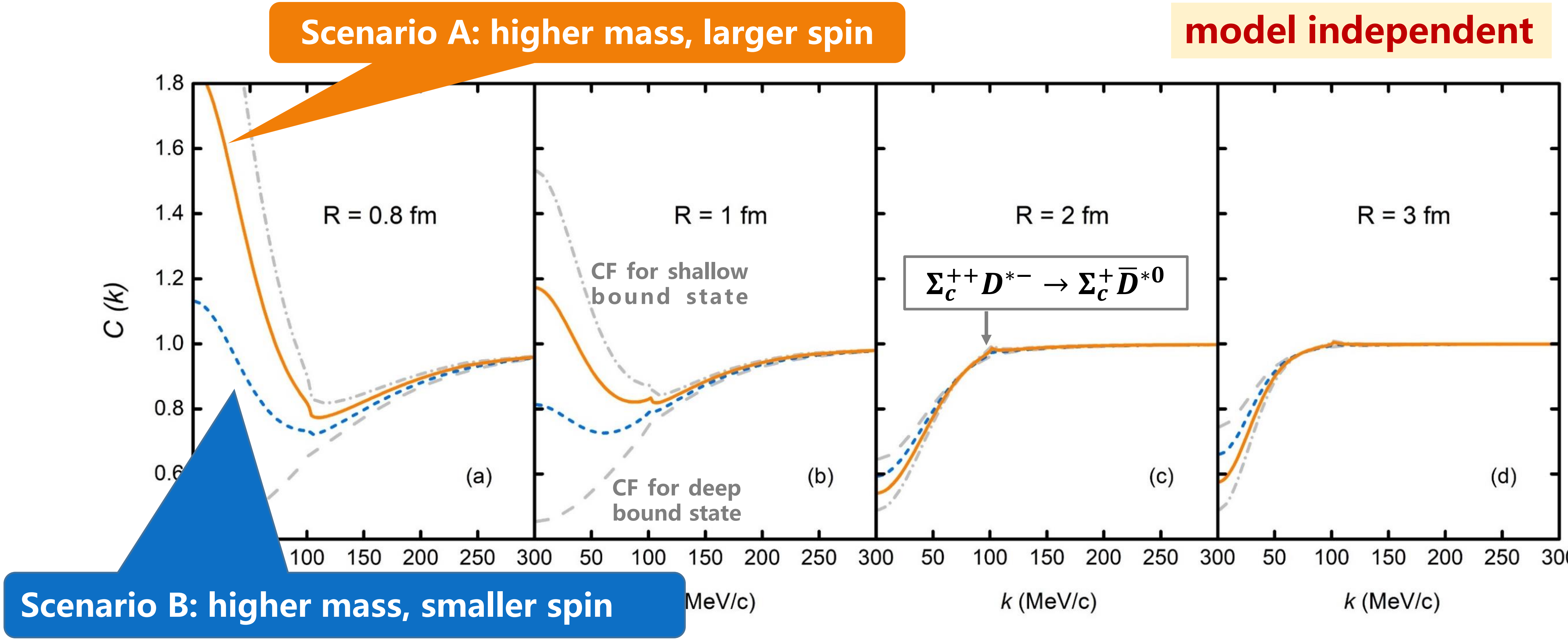
Pentaquark states • 2015 APS Highlights

How to distinguish the spins of $P_c(4440)$ and $P_c(4457)$?

- **Masses, invariant mass distributions, decays, magnetic momenta, production rates**
  M. Z. Liu, Y. W. Pan, F. Z. Peng, M. Sánchez S, LSG, A. Hosaka, M. P. Valderrama, PR122 (2019) 242001*
  M. L. Du, V. Baru, F. K. Guo, C. Hanhart, U. G. Meißner, J. A. Oller, Q. Wang, PRL124 (2020) 072001
  Y. H. Lin, B. S. Zou, PRD100 (2019) 056005
  M. W. Li, Z. W. Liu, Z. F. Sun and R. Chen, PRD104 (2021) 054016
  Q. Wu, D. Y. Chen, PRD100 (2019) 114002
- **Heavy antiquark diquark symmetry**
  Y. W. Pan, M. Z. Liu, F. Z. Peng, M. S. Sánchez, LSG, M. P. Valderrama, Phys. Rev. D 102 (2020) 011504*
- **Neural network-based approach**
  Z. Zhang, J. Liu, J. Hu, Q. Wang, U.-G. Meißner, Sci. Bull. 68 (2023) 981

OTHER

Spin-averaged $\Sigma_c\bar{D}^*$ CFs

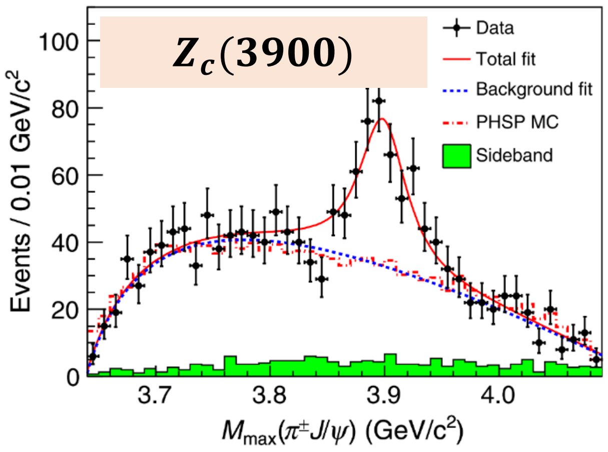


Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and ***LSG*****, PRD108(2023)L031503*

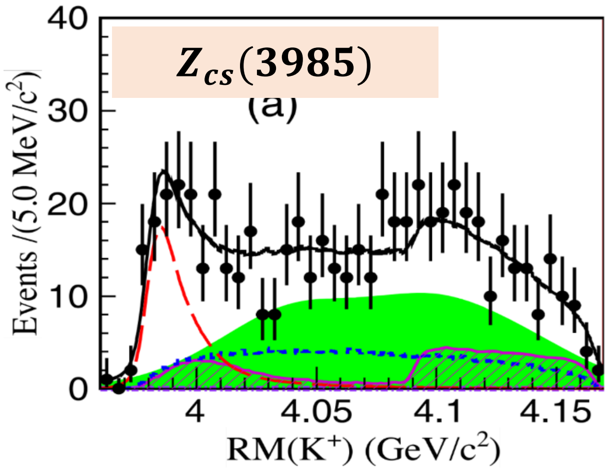
Contents

- ☞ **Brief introduction: exotic states and femtoscopy**
- ☞ **Femtoscopic correlation functions (CFs)—general features**
- ☞ **Recent applications**
  - ➢ DK CFs for $D_{s0}^{*}(2317)$—**deeply bound state**
  - ➢ $\Sigma_c\bar{D}^{*}$ **CFs for** $P_c(4440)$ **and** $P_c(4457)$—**weakly bound states**
  - ➢ $D\bar{D}^{*}$ and $D\bar{D}_s^{*}$ CFs for $Z_c(3900)$ and $Z_{cs}(3985)$—**resonant states**
- ☞ **Summary and outlook**

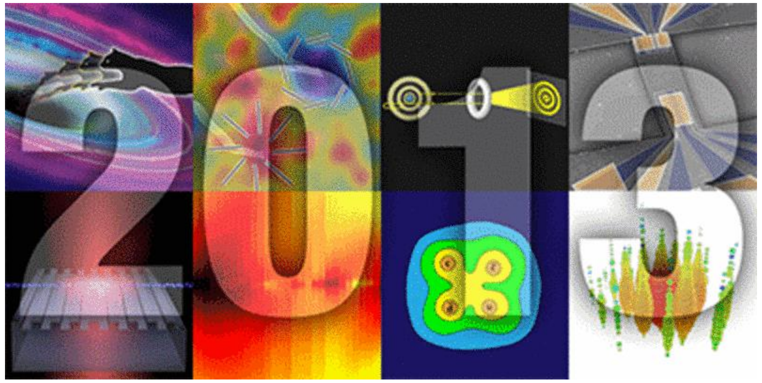
Tetraquark states $Z_c(3900)$ & $Z_{cs}(3985)$—1128 citations



BESIII, PRL110 (2013) 252001



BESIII, PRL126 (2021) 102001



Tetraquark states • 2013 APS Highlights

$Z_c(3900)$ & $Z_{cs}(3985)$: Resonant VS Virtual states

Particle Data Group, PTEP 2022 (2022) 083C01

M.-L. Du, M. Albaladejo, F.-K. Guo and J. Nieves, PRD 105 (2022) 074018

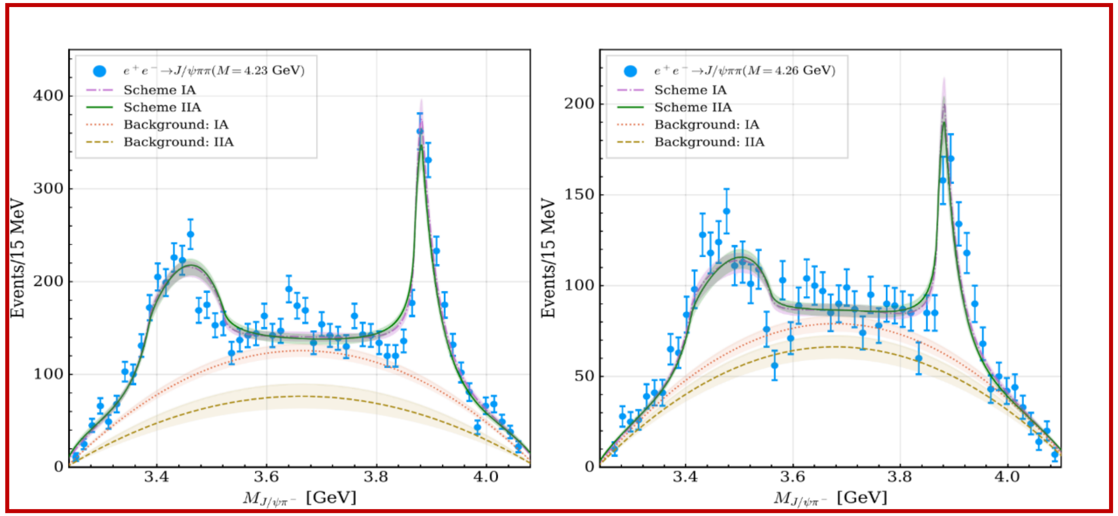
T. Ji, X.-K. Dong, M. Albaladejo, M.-L. Du, F.-K. Guo and J. Nieves, PRD106 (2022) 094002

L.-W. Yan, Z.-H. Guo, F.-K. Guo, D.-L. Yao and Z.-Y. Zhou, PRD109 (2024) 014026

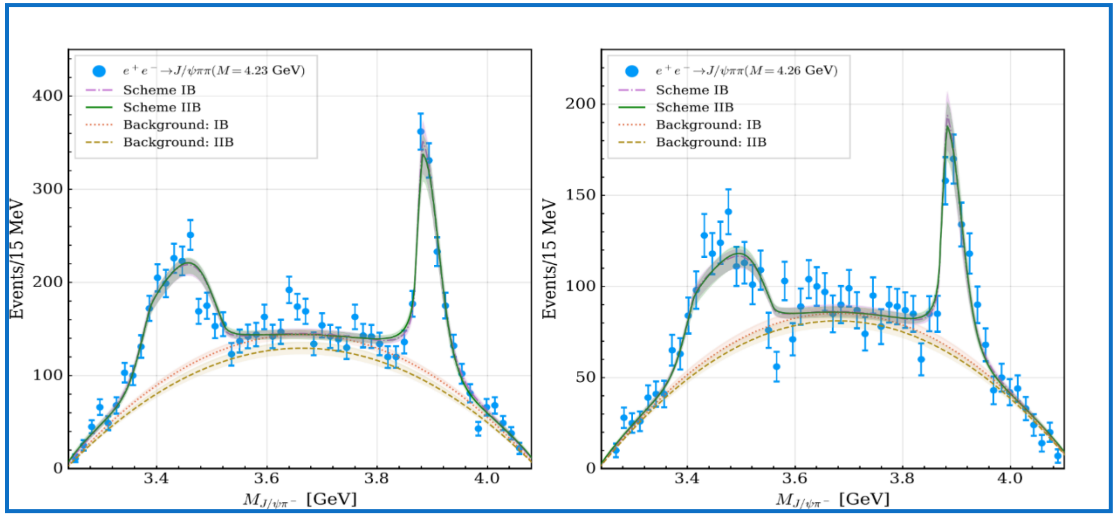
How to tell whether $Z_c(3900)$ and $Z_{cs}(3985)$ are resonant or virtual states ?

Invariant mass distributions fail to distinguish vir. or res.

Virtual state scenario



Resonant state scenario



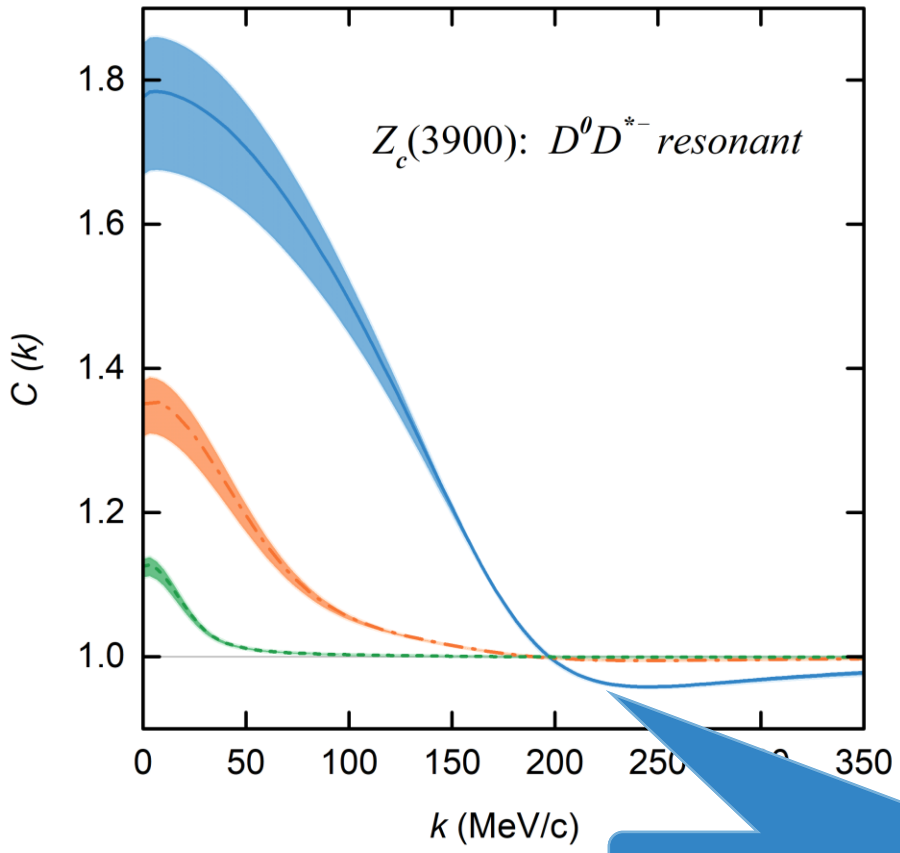
M.-L. Du, M. Albaladejo, F.-K. Guo, and J. Nieves, PRD105(2022)074018

Data are compatible with $Z_c(3900)/Z_{cs}(3985)$ as either a resonant or virtual state.

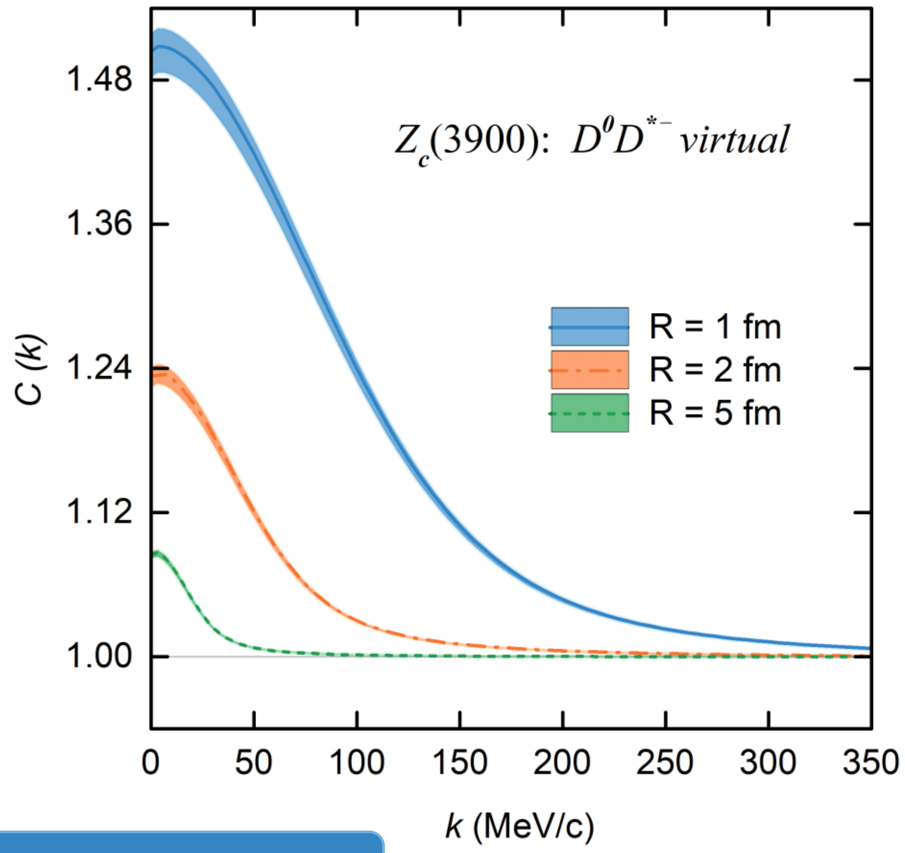
$D^0 D^{*-}$ CFs for $Z_c(3900)$

*Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and **LSG*,** 2404.18607*

Resonant state scenario



Virtual state scenario

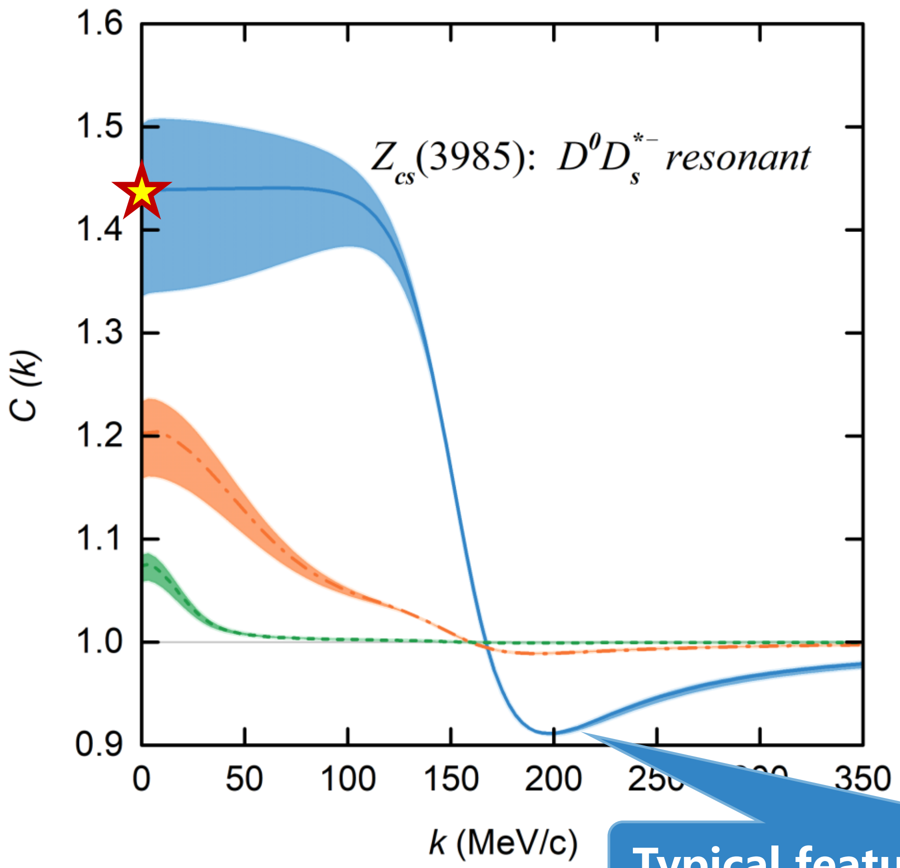


Typical feature of broad resonant state

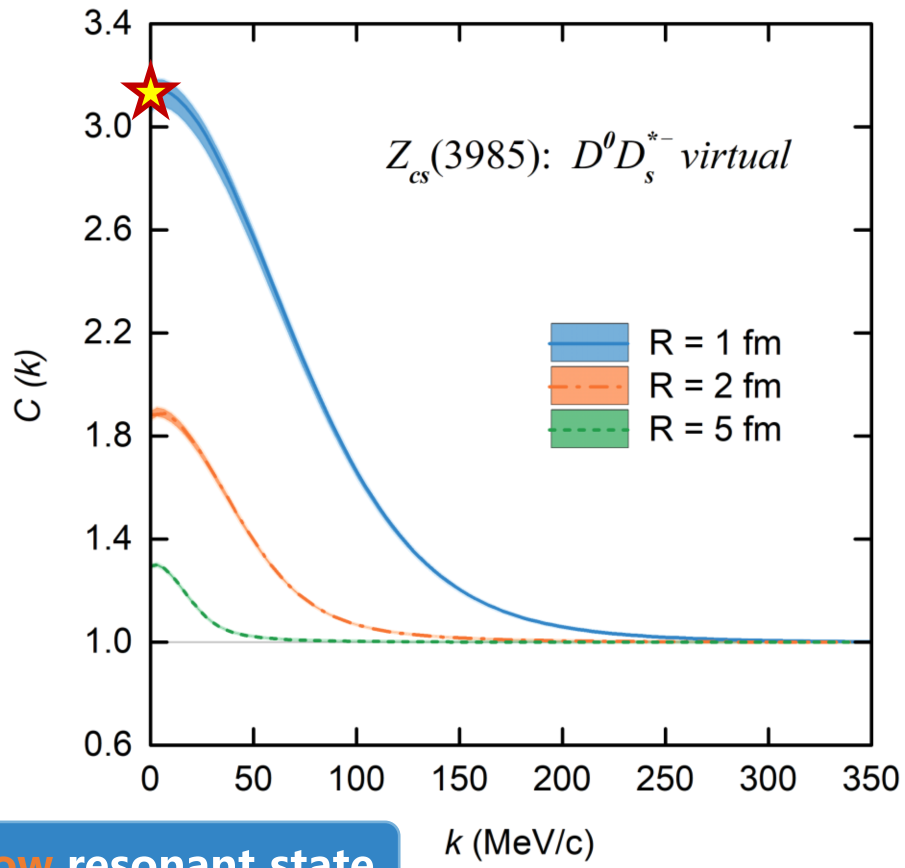
$D^0 D_s^{*-}$ CFs $Z_{cs}(3985)$

Zhi-Wei Liu, Ming-Zhu Liu, Jun-Xu Lu and **LSG***, 2404.18607

Resonant state scenario



Virtual state scenario



Typical feature of narrow resonant state

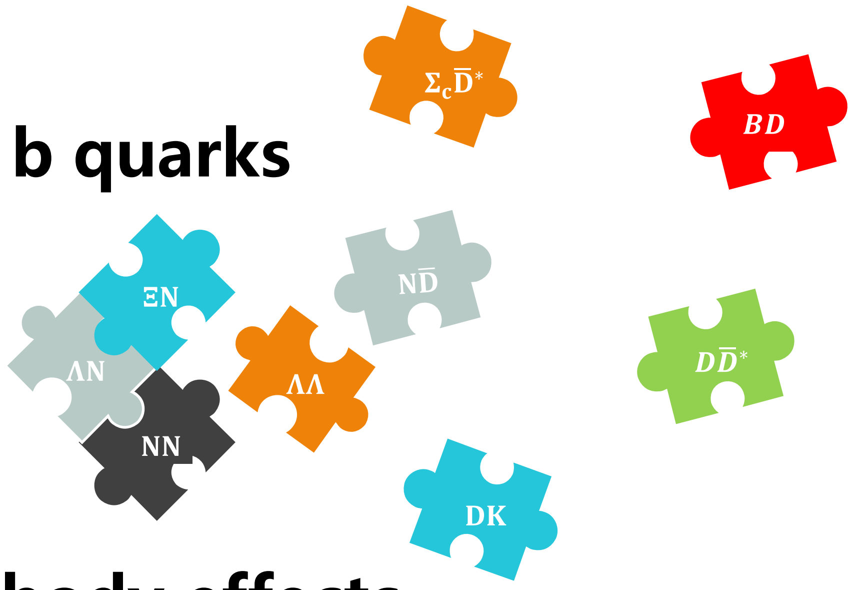
Contents

- ☞ **Brief introduction: exotic states and femtoscopy**
- ☞ **Femtoscopic correlation functions (CFs)—general features**
- ☞ **Recent applications**
  - ➢ DK CFs for $D_{s0}^*(2317)$**—deeply bound state**
  - ➢ $\Sigma_c\bar{D}^*$ **CFs for** $P_c(4440)$ **and** $P_c(4457)$**—weakly bound states**
  - ➢ $D\bar{D}^*$ and $D\bar{D}_s^*$ CFs for $Z_c(3900)$ and $Z_{cs}(3985)$**—resonant states**
- ☞ **Summary and outlook**

Summary and outlook

- **Femtoscopy offers high-precision tests of the strong interaction between pairs of (un)stable particles and can be valuable to decipher the nature of the many exotic hadrons discovered so far.**
  - ✓ DK correlation functions can be used to verify or refute the molecular picture of $D_{s0}^*(2317)$
  - ✓ $\Sigma_c\bar{D}^{(*)}$ correlation functions can be used to discriminate the spins of $P_c(4440)$ and $P_c(4457)$
  - ✓ $D\bar{D}^*/D\bar{D}_s^*$ correlation functions can tell whether $Z_c(3900)/Z_{cs}(3985)$ is a resonant or virtual state

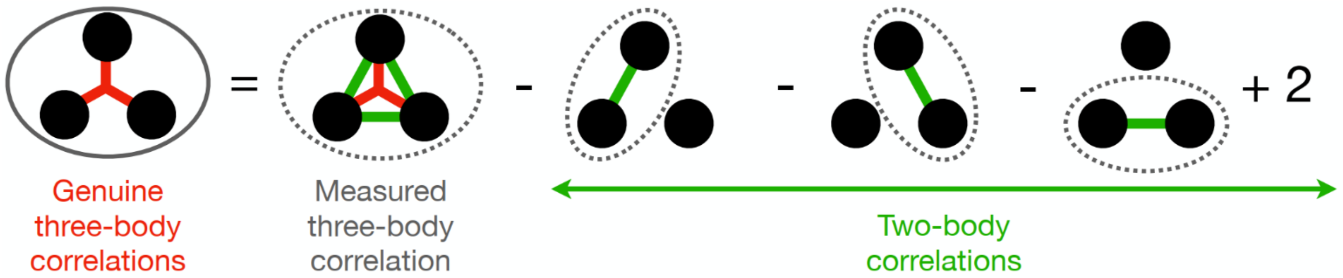
Summary and outlook



□ More two-hadron correlations involving s, c, b quarks

DD^*, I. Vidana, A. Feijoo, M. Albaladejo, J. Nieves, and E. Oset, PLB 846 (2023) 138201
DD^*, Y. Kamiya, T. Hyodo, and A. Ohnishi, EPJA 58 (2022) 131
BB^*, A. Feijoo, L. R. Dai, L. M. Abreu, and E. Oset, PRD 109 (2024) 016014
BD, H.P. Li, J.Y. Yi, C.W. Xiao, D.L. Yao, W.H. Liang, and E. Oset, CPC (2024)
……

□ Three-particle correlations — genuine three-body effects



ppp, $pp\Lambda$, ALICE Collaboration, Eur. Phys. J. A 59 (2023) 145
$ppK^{\pm}$, ALICE Collaboration, Eur. Phys. J. A 59 (2023) 298
ppp, A. Kievsky and et al., Phys. Rev. C 109 (2024) 034006

Thanks a lot for your attention!

(Image: CERN)

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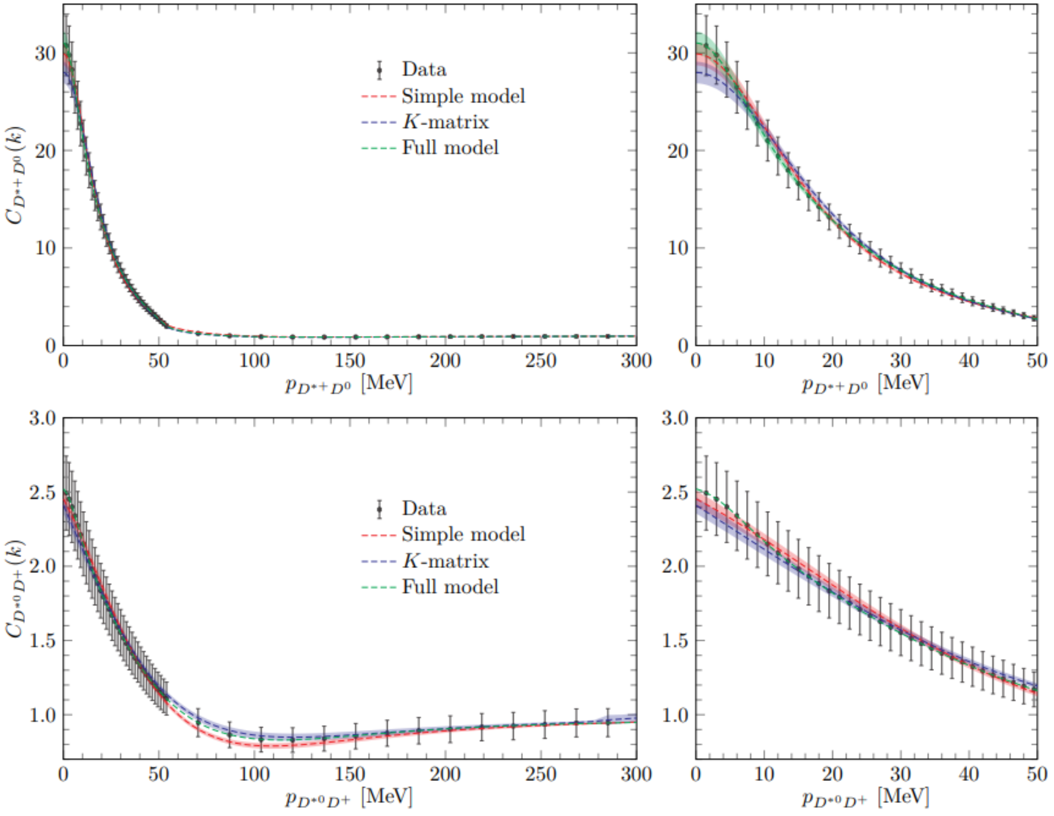
Inverse problem

- **One can also perform inverse studies and extract hadron-hadron interaction from the exp. CF data**



Inverse problem in femtoscopic correlation functions: The Tcc(3875)+ state,

Albaladejo , Feijoo , Vidaña , Nieves , and Oset, 2307.09873

$$C_{D^0D^{*+}}(p_{D^0}) = 1 + 4\pi\,\theta(\Lambda - p_{D^0}) \int_0^\infty dr r^2 S_{12}(r) \times \left\{ \left| j_0(p_{D^0}r) + T_{11}(s)\widetilde{G}_1(r;s) \right|^2 + \left| T_{12}(s)\widetilde{G}_2(r;s) \right|^2 - j_0^2(p_{D^0}r) \right\} , \quad (1)$$

$$C_{D^+D^{*0}}(p_{D^+}) = 1 + 4\pi\,\theta(\Lambda - p_{D^+}) \int_0^\infty dr r^2 S_{12}(r) \times \left\{ \left| j_0(p_{D^+}r) + T_{22}(s)\widetilde{G}_2(r;s) \right|^2 + \left| T_{12}(s)\widetilde{G}_1(r;s) \right|^2 - j_0^2(p_{D^+}r) \right\} , \quad (2)$$

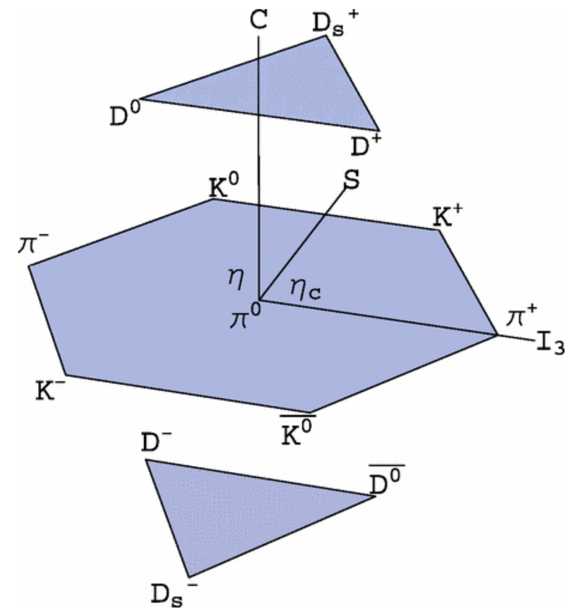
Weinberg-Tomozawa Interaction (leading order)

LO interaction between a NGB and a heavy pseudoscalar boson

$$\mathcal{L} = \frac{1}{4f_\pi^2}\left(\partial^\mu P[\Phi, \partial_\mu \Phi]P^\dagger - P[\Phi, \partial_\mu \Phi]\partial^\mu P^\dagger\right)$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$

$$P = \left(D^0, D^+, D_s^+\right)$$

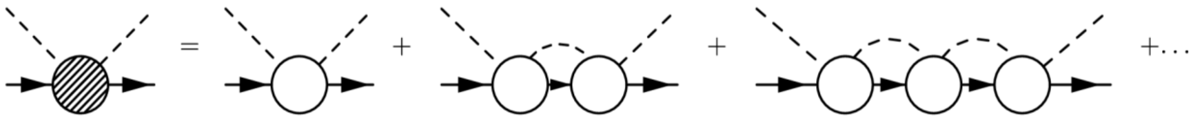


Weinberg-Tomozawa (WT) potential – parameter free

$$V_{\nu'\nu} = \frac{C_{\nu'\nu}}{4f_0^2}\left[(p_1 + p_2)^2 - (p_1 - p_4)\right] \qquad p_{1(3)} = (E_{1(3)}, \boldsymbol{p}^{(\prime)}), \quad p_{2(4)} = (\sqrt{s} - E_{1(3)}, -\boldsymbol{p}^{(\prime)})$$

Scattering wave function

- **Coupled-channel scat. eq.**



$$T_{\nu'\nu}(k',k) = V_{\nu'\nu} \cdot f_{\Lambda_F}(k',k) + \sum_{\nu''} \int_0^\infty \frac{\mathrm{d}k'' k''^2}{8\pi^2} \frac{V_{\nu'\nu''} \cdot f_{\Lambda_F}(k',k'') \cdot T_{\nu''\nu}(k'',k)}{E_{P,\nu''} E_{\Phi,\nu''} (\sqrt{s} - E_{P,\nu''} - E_{\Phi,\nu''} + i\epsilon)}$$

$$f_{\Lambda_F}(k',k) = \exp\left[-\left(\frac{k'}{\Lambda_F}\right)^2 - \left(\frac{k}{\Lambda_F}\right)^2 \right]$$

$M_{D_{s0}^*} = 2317.8$ MeV → $\Lambda_F = 1107$ MeV

- **S-wave scattering wave function (including off-shell effect)**

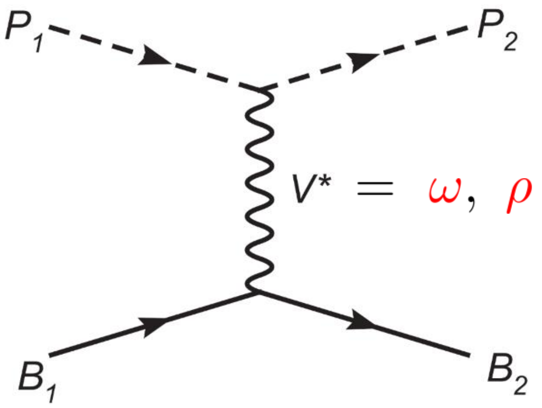
$$\psi_{\nu'\nu}(k,r) = \delta_{\nu'\nu} j_0(kr) + \int_0^\infty \frac{\mathrm{d}k' k'^2}{8\pi^2} \frac{T_{\nu'\nu}(k',k) \cdot j_0(k'r)}{E_{P,\nu'} E_{\Phi,\nu'} (\sqrt{s} - E_{P,\nu'} - E_{\Phi,\nu'} + i\epsilon)}$$

Light vector meson exchange interactions

◻ Interactions in the hidden local symmetry approach – parameter free

$$V_{\Sigma_c \bar{D}^{(*)}}^{I=\frac{3}{2}} = 2M_{\Sigma_c} M_{\bar{D}^{(*)}} \widetilde{\beta}_1 \widetilde{\beta}_2 g_V^2 \left(\frac{1}{m_\omega^2} + \frac{1}{m_\rho^2} \right)$$

$$V_{\Sigma_c \bar{D}^{(*)}}^{I=\frac{1}{2}} = 2M_{\Sigma_c} M_{\bar{D}^{(*)}} \widetilde{\beta}_1 \widetilde{\beta}_2 g_V^2 \left(\frac{1}{m_\omega^2} - \frac{2}{m_\rho^2} \right)$$



Isospin basis

↕

Charge basis

$$\left| \Sigma_c \bar{D}^{(*)}, I = \frac{3}{2}, I_3 = \frac{1}{2} \right\rangle = \sqrt{\frac{1}{3}} \left| \Sigma_c^{++} D^{(*)-} \right\rangle + \sqrt{\frac{2}{3}} \left| \Sigma_c^{+} \bar{D}^{(*)0} \right\rangle$$

$$\left| \Sigma_c \bar{D}^{(*)}, I = \frac{1}{2}, I_3 = \frac{1}{2} \right\rangle = \sqrt{\frac{2}{3}} \left| \Sigma_c^{++} D^{(*)-} \right\rangle - \sqrt{\frac{1}{3}} \left| \Sigma_c^{+} \bar{D}^{(*)0} \right\rangle$$

Two different spin assignments

☐ Interaction strengths

$$f_{\Lambda_F}(k', k) = \exp\left[-\left(\frac{k'}{\Lambda_F}\right)^2 - \left(\frac{k}{\Lambda_F}\right)^2\right]$$

$\Lambda_F = 1067$ MeV
deep bound state of $\Sigma_c \bar{D}^*$

itation: R.L. Workman et al. (Particle Data Group), Prog.Theor.Exp.Phys. **2022**, 083C01 (2022) and 2023 upda

$P_c(4440)^+$ Status: ∗

$P_c(4440)^+$ MASS

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
$\mathbf{4440.3 \pm 1.3^{+4.1}_{-4.7}}$	AAIJ	19W	LHCB	pp at 7, 8, 13 TeV

$\Lambda_F = 860$ MeV
shallow bound state of $\Sigma_c \bar{D}^*$

$P_c(4457)^+$ Status: ∗

$P_c(4457)^+$ MASS

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
$\mathbf{4457.3 \pm 0.6^{+4.1}_{-1.7}}$	AAIJ	19W	LHCB	pp at 7, 8, 13 TeV
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$4449.8 \pm 1.7 \pm 2.5$	[1] AAIJ	15P	LHCB	Repl. by AAIJ 19W

[1] Considering $P_c(4440)$ and $P_c(4457)$ as a single resonance.

CF for the shallow bound state is **significantly larger** than that for the deep bound

☐ Experimental CFs – spin-averaged

Scenarios A

$\mathbf{P_c(4440) : J^P = (1/2)^-}$ $\mathbf{P_c(4457) : J^P = (3/2)^-}$

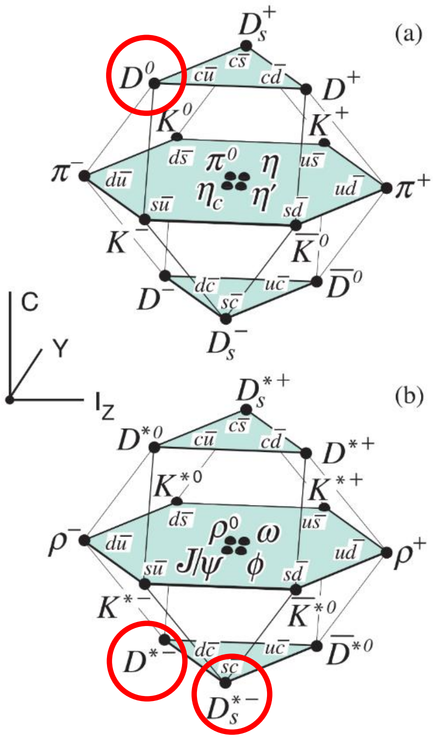
$$\bar{C} = \mathbf{1/3} \cdot C_{deep} + \mathbf{2/3} \cdot C_{shallow}$$

Scenarios B

$\mathbf{P_c(4440) : J^P = (3/2)^-}$ $\mathbf{P_c(4457) : J^P = (1/2)^-}$

$$\bar{C} = \mathbf{2/3} \cdot C_{deep} + \mathbf{1/3} \cdot C_{shallow}$$

General potential from EFTs



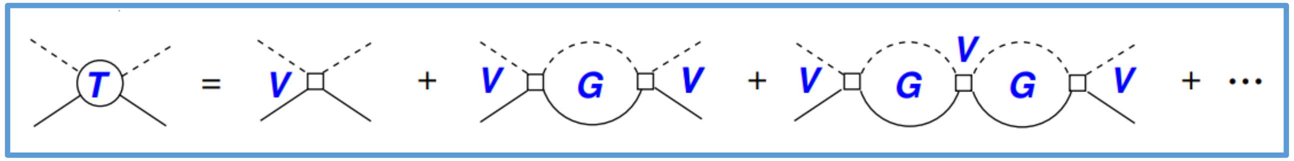
☐ Interaction between heavy pseudoscalar bosons

$$\mathrm{V} = \color{red}{\mathrm{a}} + \color{blue}{\mathrm{b}\cdot\mathrm{k}^2}, \qquad \mathrm{k} = \sqrt{[\mathrm{s}-(\mathrm{m}_1+\mathrm{m}+2)^2][\mathrm{s}-(\mathrm{m}_1-\mathrm{m}+2)^2]}/2\sqrt{\mathrm{s}}$$

- ➢ **energy-dependent potential –→ resonant state**
- ➢ **contact-range potential –→ bound or virtual state**

☐ Scattering equation – unitarity

$$\mathrm{T} = \mathrm{V} + \mathrm{VGT}$$ ⟷



Loop function G with cutoff regularization

$$\mathrm{G}(\sqrt{\mathrm{s}}) = \int_0^{|\boldsymbol{q}|<\mathrm{q}_{\max}} \frac{\mathrm{d}^3\mathrm{k}'}{(2\pi)^3} \frac{\mathrm{E}_1(\mathrm{k}')+\mathrm{E}_2(\mathrm{k}')}{2\mathrm{E}_1(\mathrm{k}')\mathrm{E}_2(\mathrm{k}')} \frac{1}{\sqrt{\mathrm{s}}^2-[\mathrm{E}_1(\mathrm{k}')+\mathrm{E}_2(\mathrm{k}')]^2+\mathrm{i}\varepsilon}, \qquad \mathrm{q}_{\max} \in [0.8, 1.2]\ \mathrm{GeV}$$

Interaction strengths determined by fitting to data

	Scenario	M [MeV]	Γ [MeV]	$m_1 + m_2$ [MeV]	a	b [MeV^{-2}]
$Z_c(3900)$	Res. [95]	3887.1	28.4	$D^0 D^{*-}$ (3875.1)	-101.68	-1380.60
	Vir. [27]	3796	0	$D^0 D^{*-}$ (3875.1)	-87.36	0
$Z_{cs}(3985)$	Res. [95]	3988	13	$D^0 D_s^{*-}$ (3977.04)	-84.17	-2894.16
	Vir. [27]	3967	0	$D^0 D_s^{*-}$ (3977.04)	-130.21	0

【95】 Particle Data Group, PTEP 2022,(2022)083C01
【27】 M.-L. Du, M. Albaladejo, F.-K. Guo, and J. Nieves, PRD105(2022)074018

◻ Correlation functions with on-shell approximation

$$\mathbf{C(k) = 1 + \int_0^\infty 4\pi r^2 dr\ S_{12}(r)\ \theta(q_{max} - k)\left[\left|j_0(kr) + T(\sqrt{s})\ \widetilde{G}(r, \sqrt{s})\right|^2 - |j_0(kr)|^2\right]}$$

$$\widetilde{\mathbf{G}}(\mathbf{r}, \sqrt{\mathbf{s}}) = \int_0^{|q|<\mathbf{q}_{max}} \frac{\mathbf{d^3k'}}{(\mathbf{2}\pi)^3} \frac{\mathbf{E_1(k') + E_2(k')}}{\mathbf{2E_1(k')E_2(k')}} \frac{\mathbf{j_0(k'r)}}{\sqrt{\mathbf{s}}^2 - [\mathbf{E_1(k') + E_2(k')}]^2 + \mathbf{i}\varepsilon}$$