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Dispersive analysis of $\eta(1405/1475)$ on the recent BESIII decay $J/\psi \to \gamma K_0^S K_0^S \pi^0$

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Outline

- *Introduction & Motivation*
  - *Light-meson spectrum, status of $\eta(1405/1475)$, triangle singularity, S-matrix*
- *Muskhelishvili-Omnès Framework: two-body interactions*
  - *Dispersion relation (two-body unitarity), angular averages, analytical continuation*
- *Khrui-Trieman Framework: three-body interactions*
  - *Three-body unitarity, linearly-independence & KT solution basis*
- *Discussions*
  - *Monte-Carlo outputs, BESIII spectra*

Introduction

Introduction

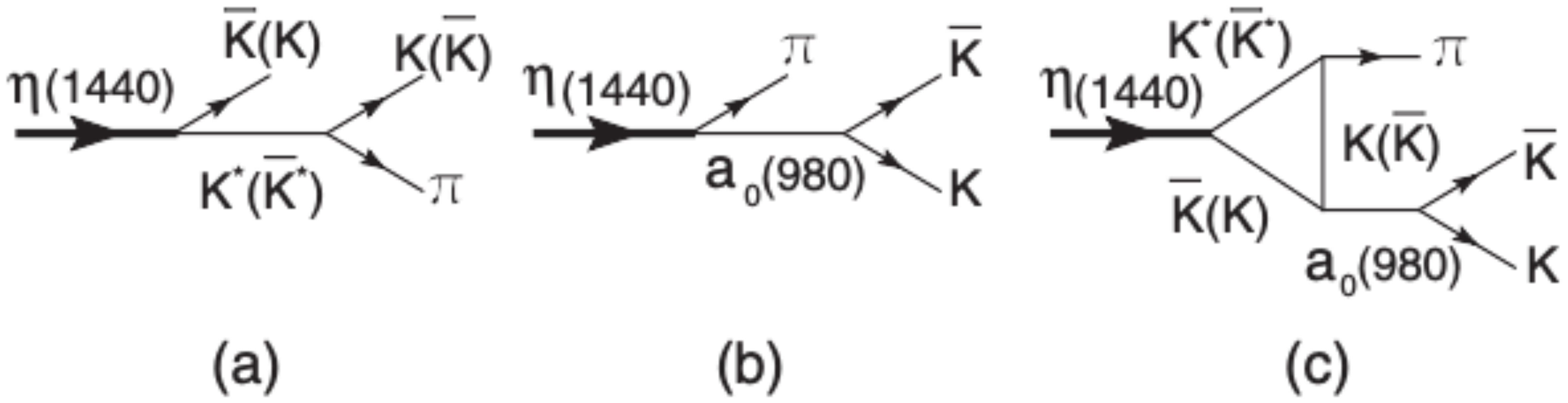
- The first radially-excited states of $\eta - \eta'$ are assigned to $\eta(1295) - \eta(1405/1475)$, among which one of the states is regarded as 0^{-+} glueball candidate.
- However, there are many puzzles:

1. Controversial observations: one state observed in $K\bar{K}\pi, \gamma V, \eta\pi\pi$ but two states observed (only) in other $K\bar{K}\pi$ final states;
2. LQCD simulation: the mass of 0^{-+} glueballs are calculated to be above 2GeV;
3. Supernumerary problem: one or two states? their natures?

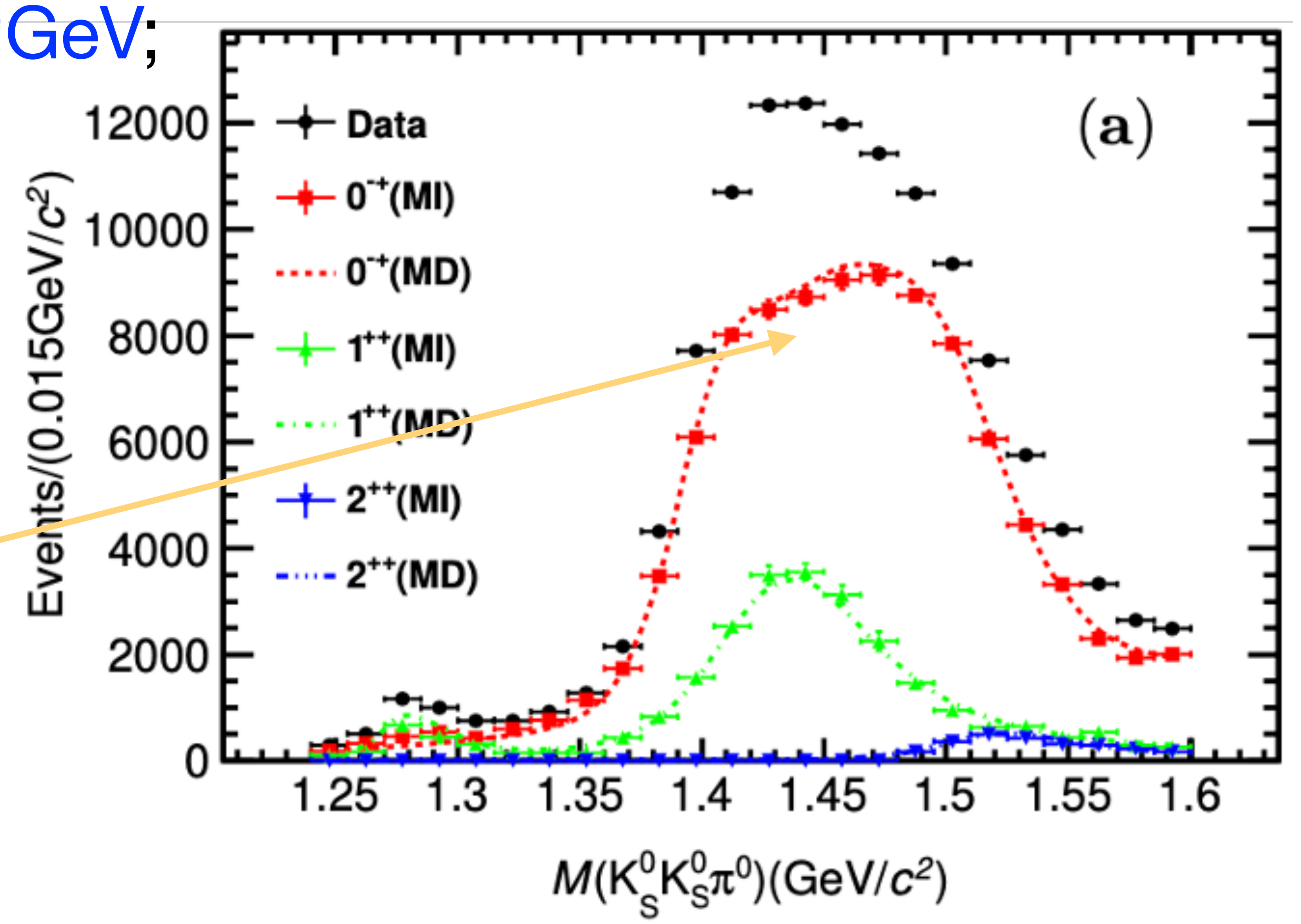
X(2370)?BESIII, PRL132.181901

BESIII, JHEP03(2023)121

Triangle singularity! Wu et al.,PRL.108.081803



Flat peak!



- Recently, BESIII collaboration reports the high-statistics $J/\psi \to \gamma K^0_S K^0_S \pi^0$ data and the data seem to favor the two-state scenario.

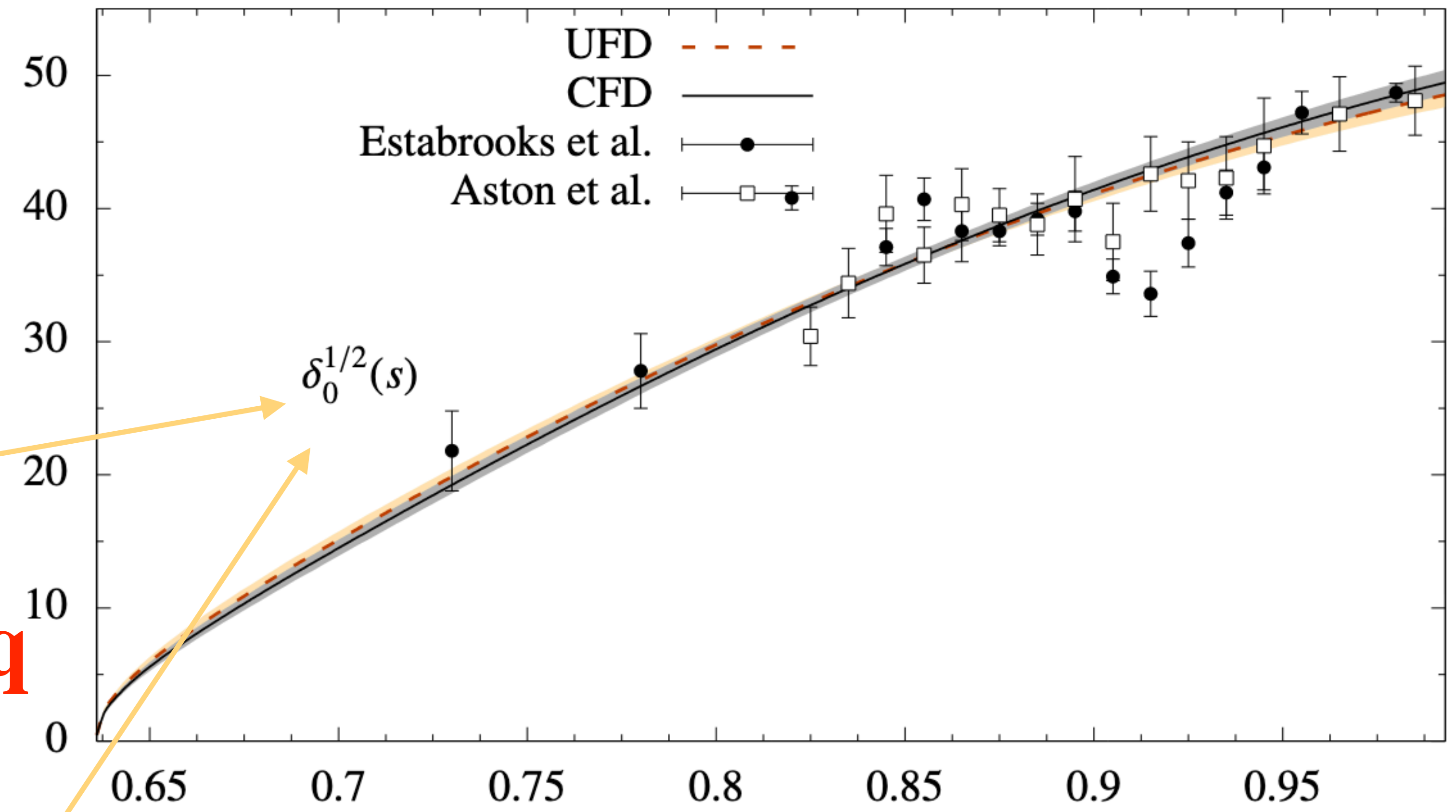
S.X.Nakamura et al., PRD.109.014021;PRD.107.L091505

A better understanding of 0^{-+} spectrum in $1.2 \sim 1.5$GeV is strongly desired!

Top-Down approach

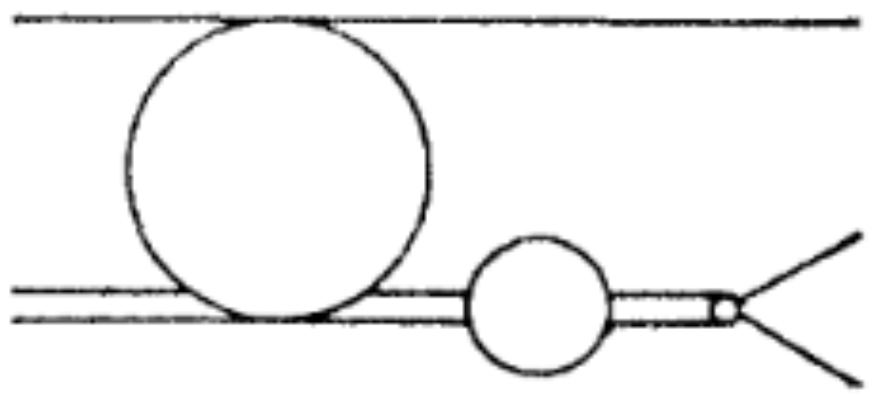
Isobar model (off-shell)

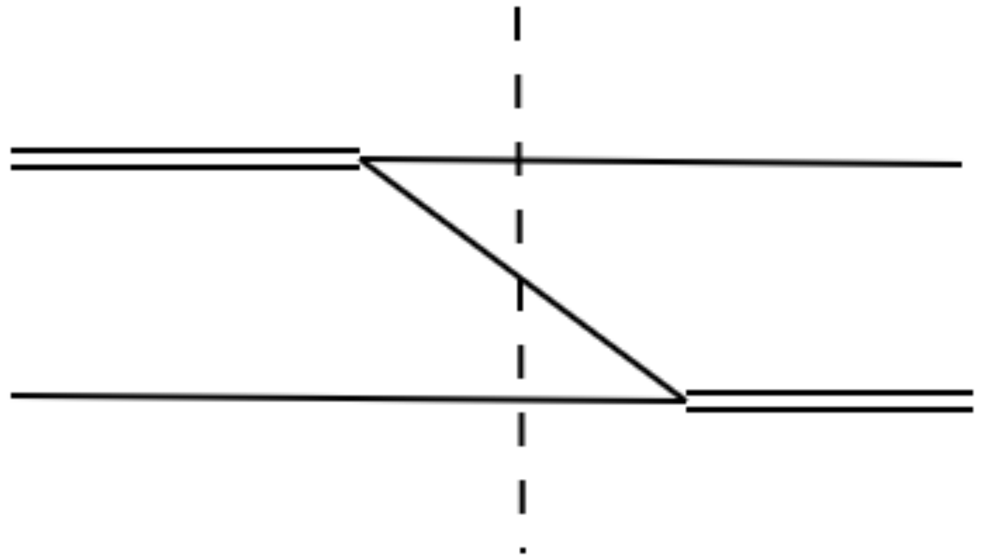
J.R.Peláez, j.physrep.2022.03.004

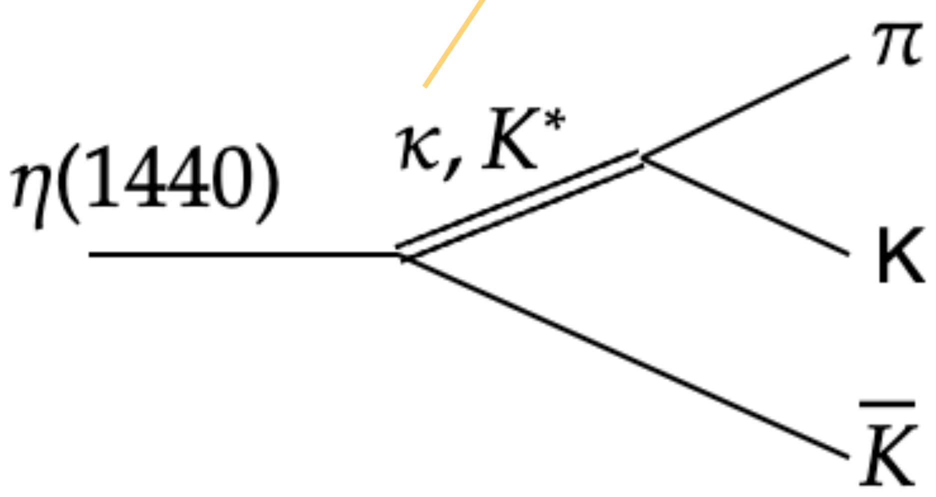


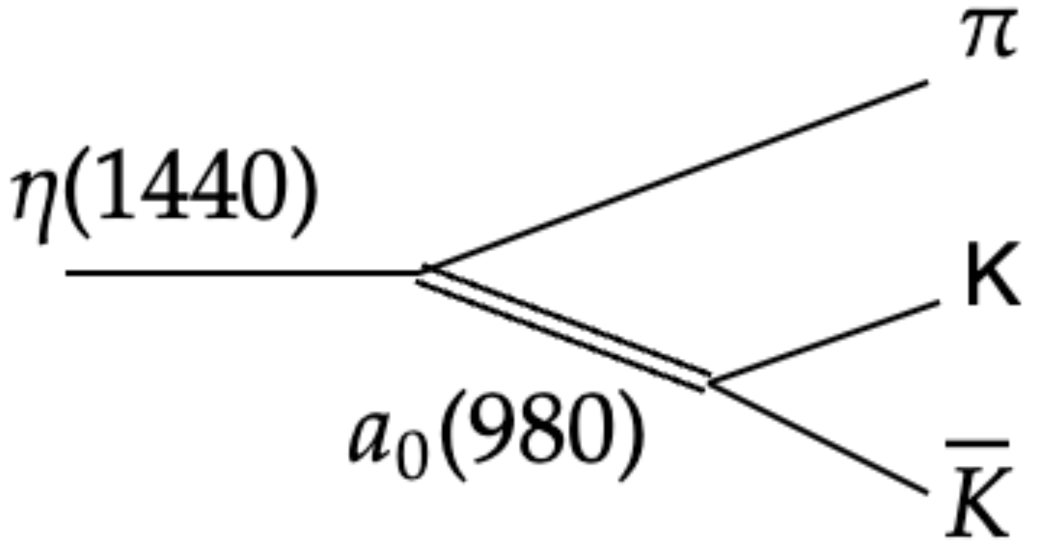
- ✓ Unitarity, Analyticity, Crossing symmetry
- ✗ may fail to describe two-body interactions properly
- ✗ hard to deal with off-shell cuts when truncated finitely

Low efficiency of **q**



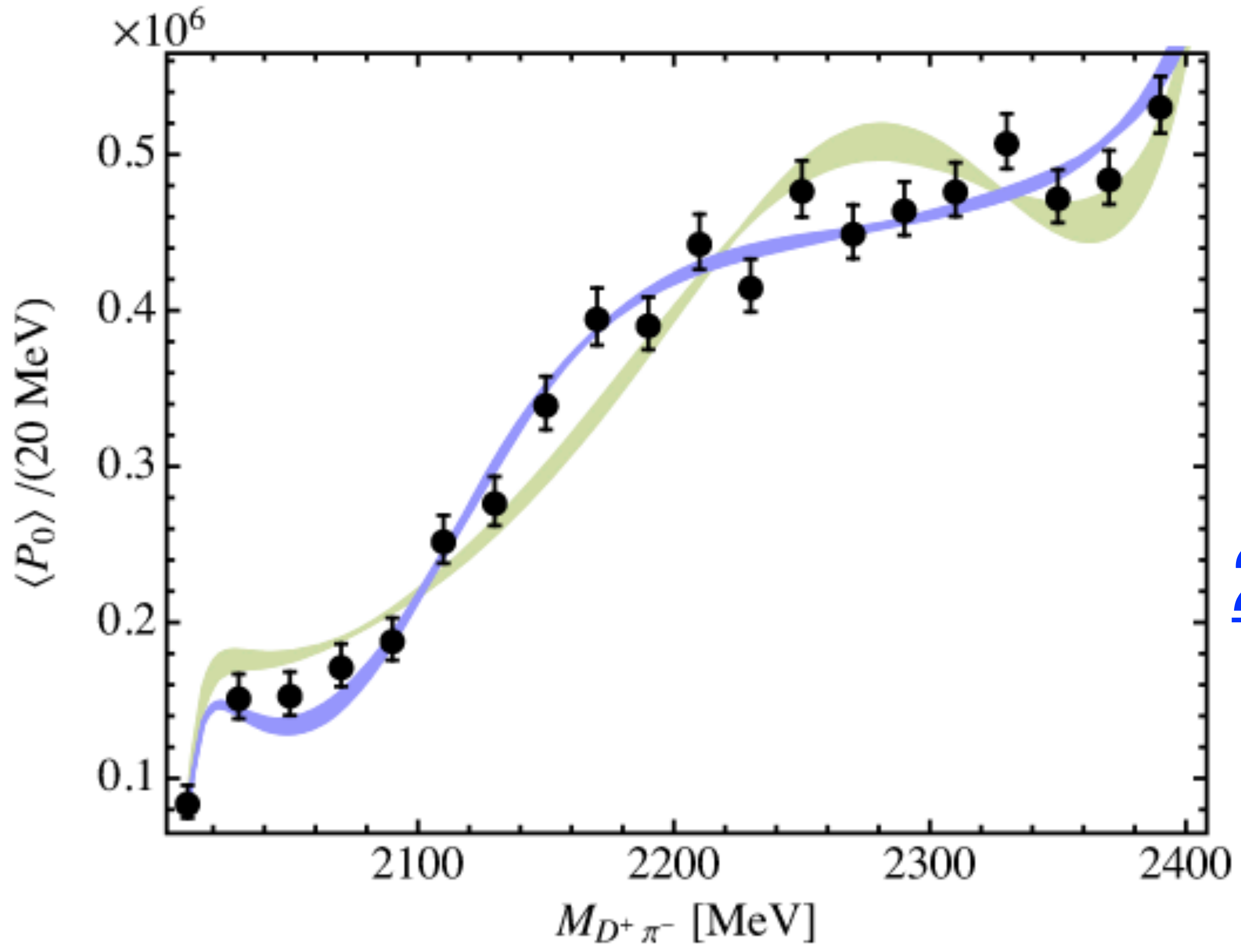






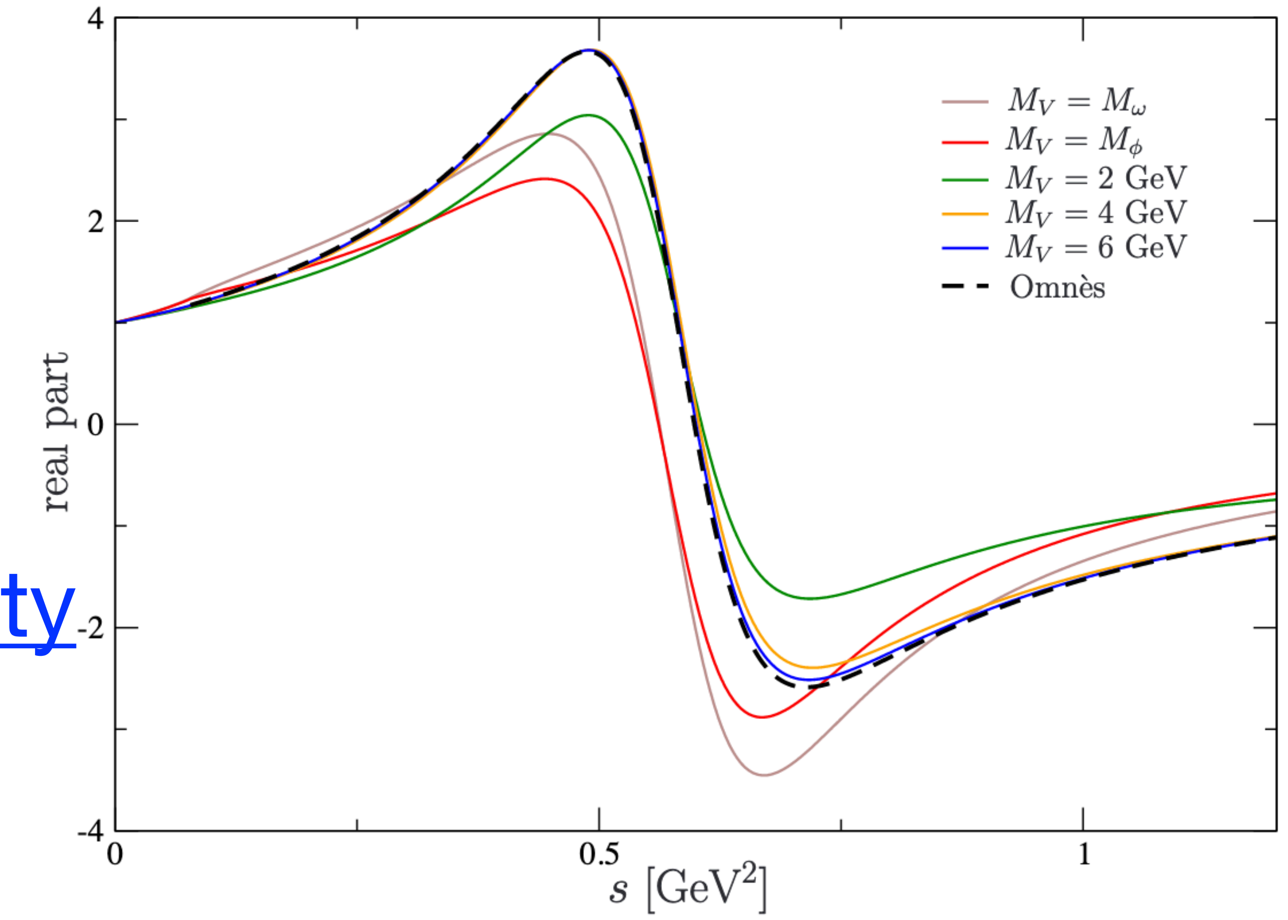
The inefficiency of isobar-model is manifesting in many aspects such as:

$B^- \to D^+\pi^-\pi^-$ Du et al., PRL126192001(2021)

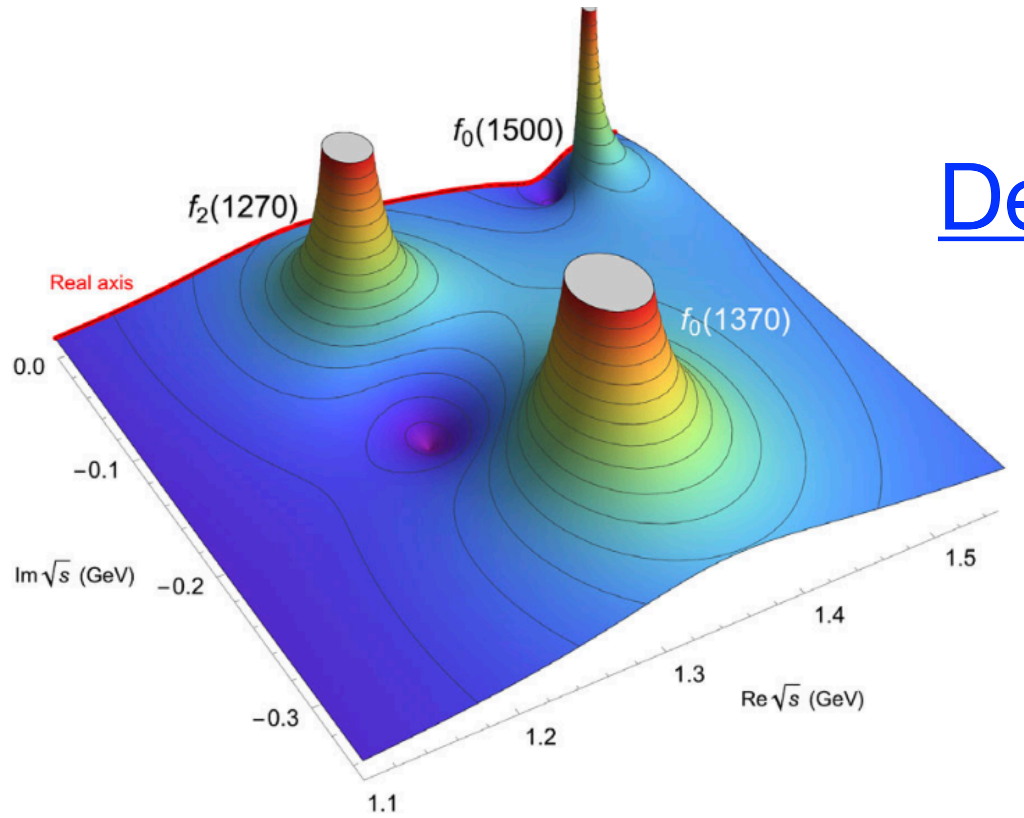


2&3-b unitarity

$V \to 3\pi$ F.Niecknig et al., EPJC722014(2012)

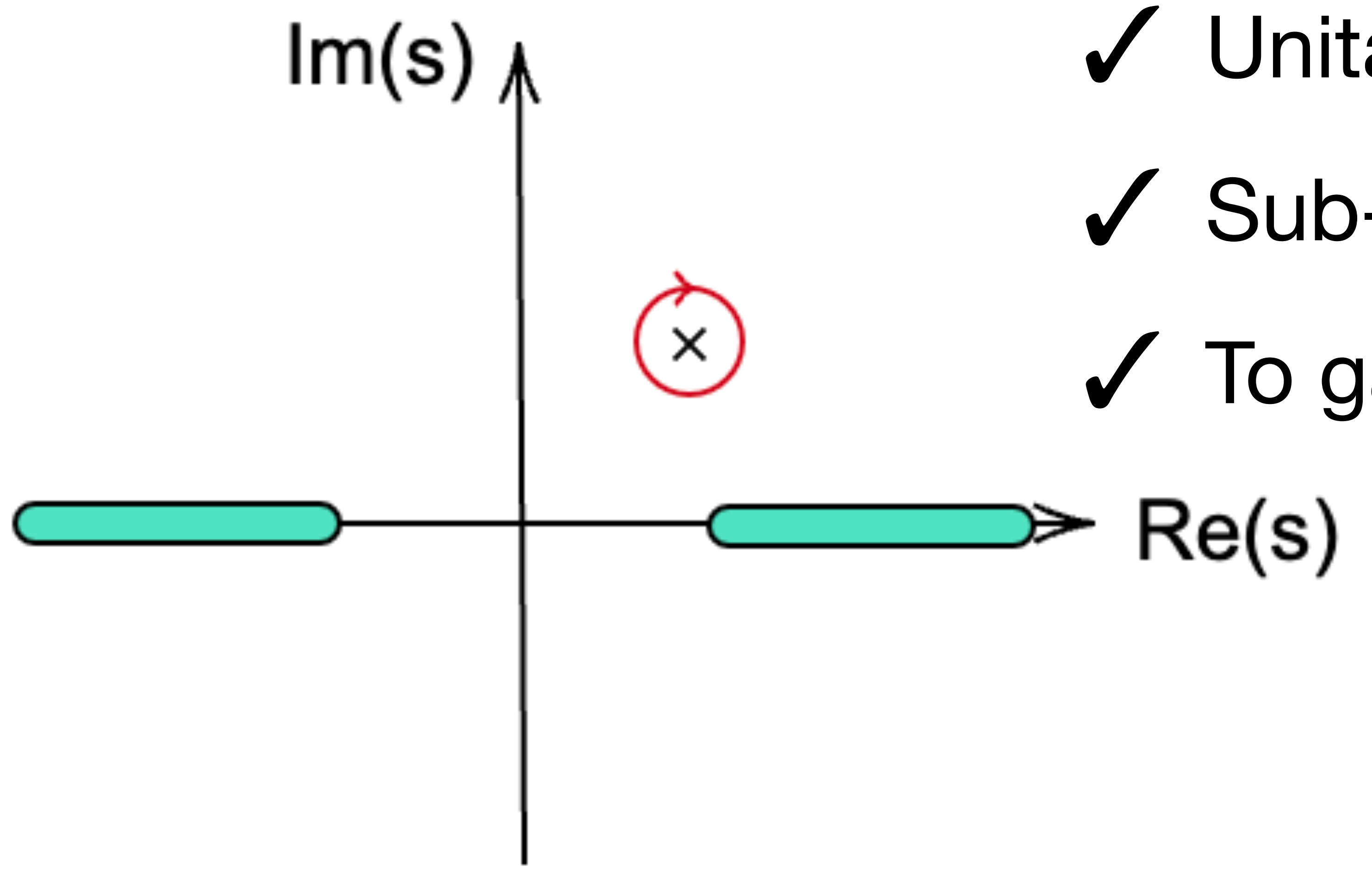


$f_0(1370)$ J.R.Peláez, PRL130051902(2023)



Deep pole

Bottom-Up approach



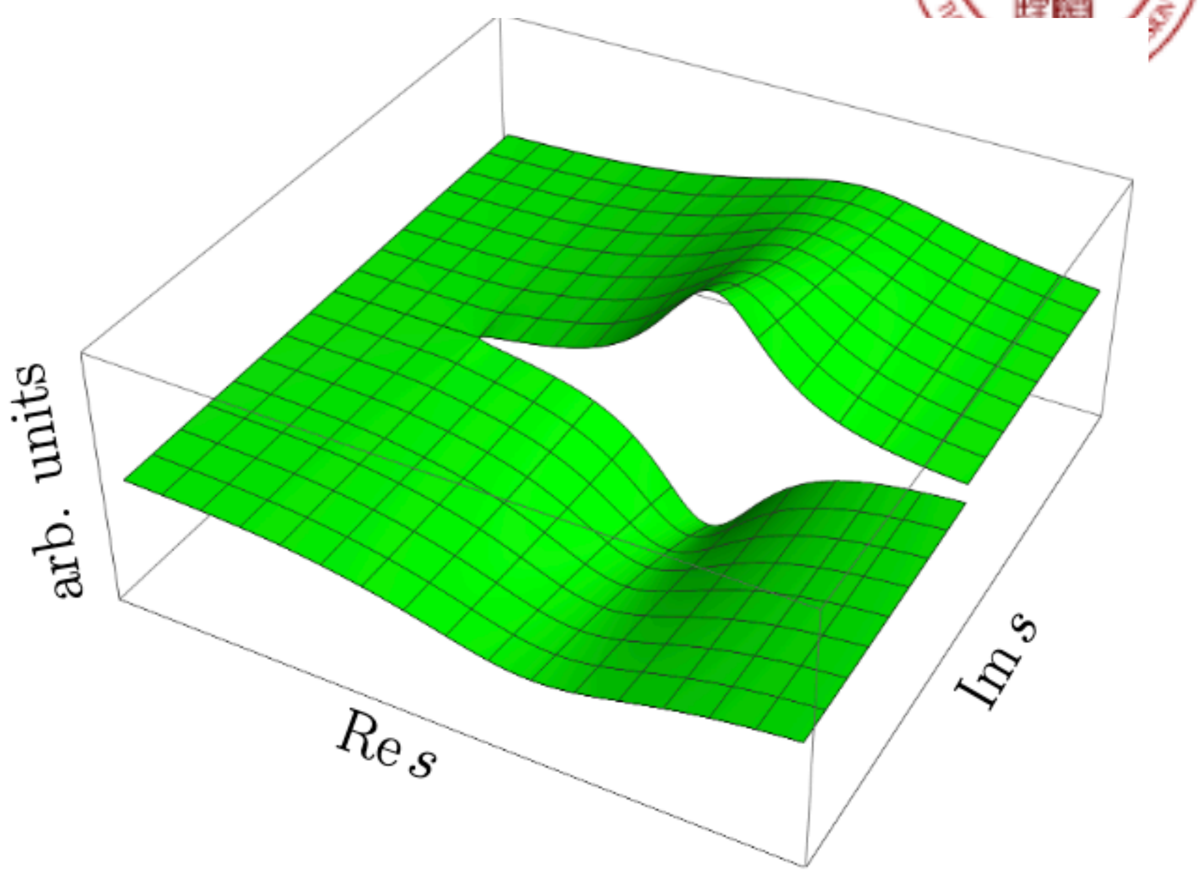
Dispersion theory (on-shell)

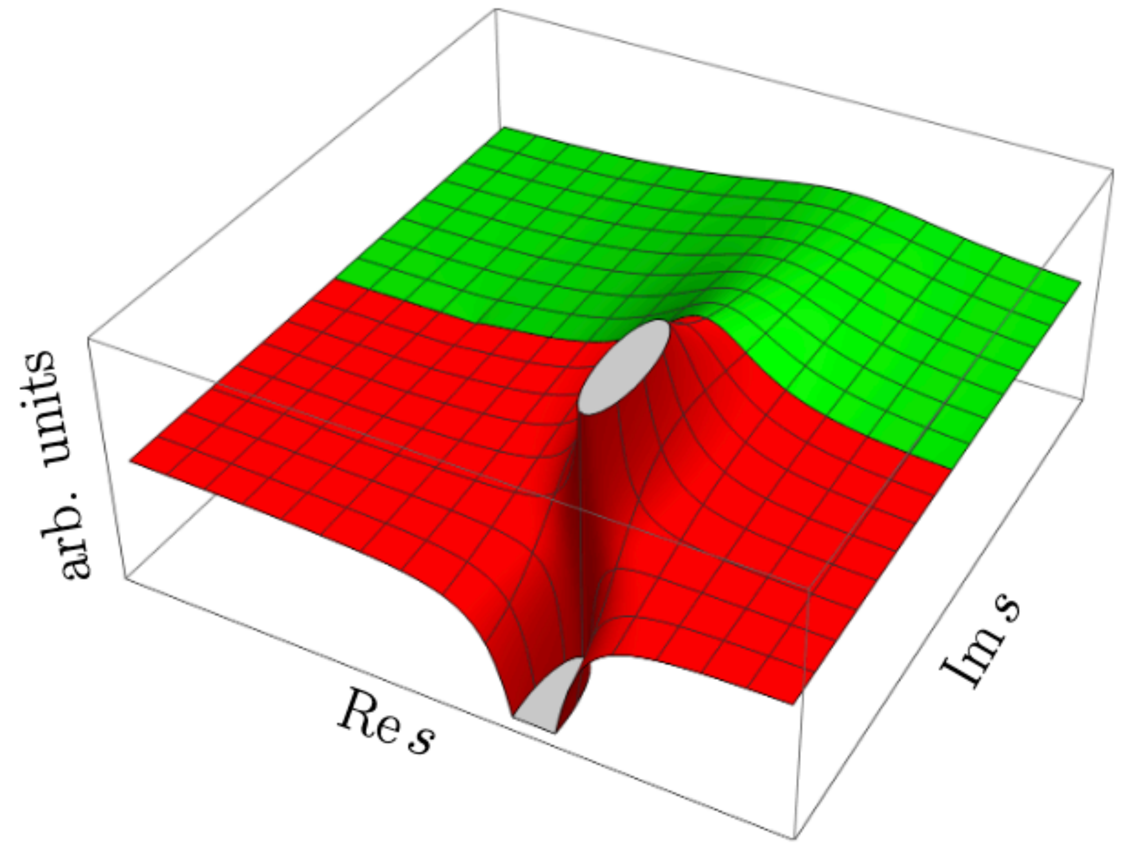
✓ Unitarity, Analyticity, Crossing symmetry

✓ Sub-channel interactions determined from scattering data

✓ To gain the maximal analyticity and continuation

($|\mathbf{q}| > 1$GeV)





Our goals:

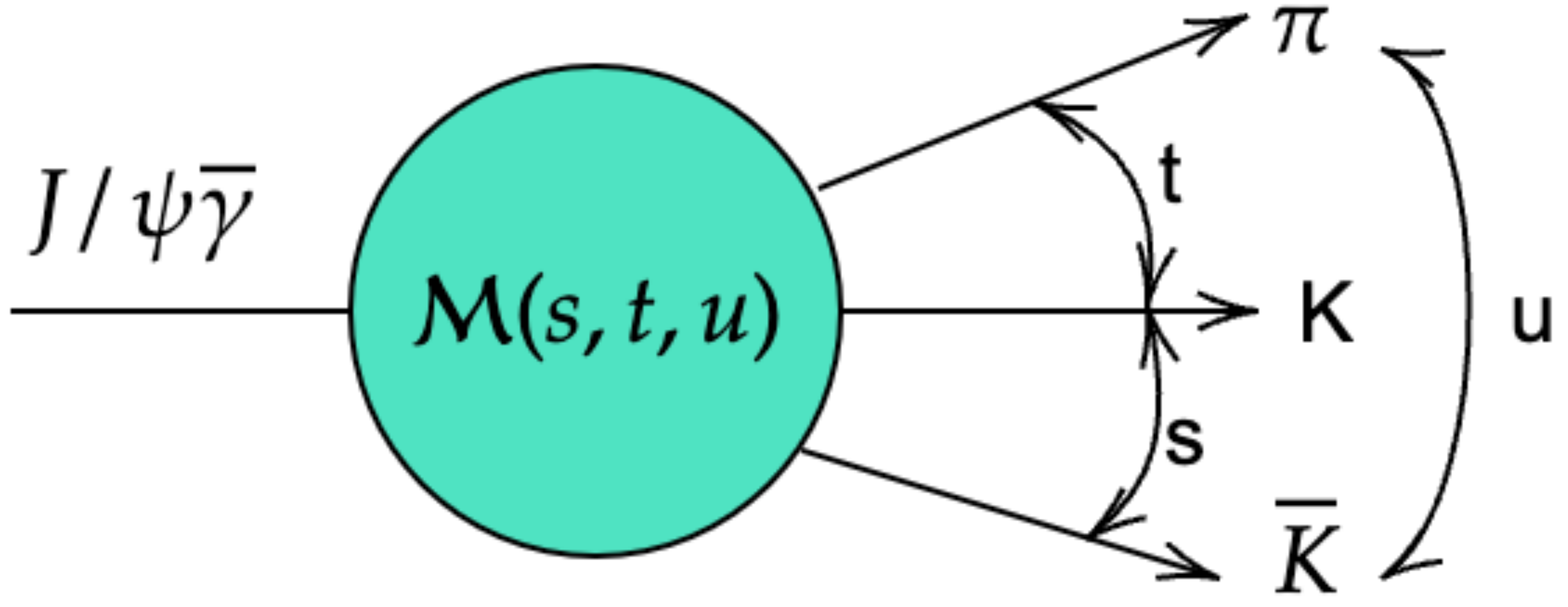
- To provide the generic $K\bar{K}\pi, \eta\pi\pi, 3\pi$ FSIs below 1.6GeV;
- To understand the "distortion" of the three-body unitarity over the two-body one and the triangle singularity mechanism;
- To provide the systematic prescriptions for iso-scalar pseudo-scalar spectra.

Muskhelishvili-Omnès Framework

Amplitudes on the Mandelstam plane

The LO amplitude of $J/\psi \to \gamma 0^{-+} \to \gamma K^0 \bar{K}^0 \pi^0$ can be factorized as,

$$\mathcal{M}_{J/\psi \to \gamma K^0 \bar{K}^0 \pi^0} \propto \epsilon_{\mu\nu\alpha\beta} \epsilon^{\mu}_{J/\psi} P^{\nu} \epsilon_{\gamma}^{*,\alpha} q_1^{\beta} \cdot \mathcal{M}_{0^{-+} \to K^0 \bar{K}^0 \pi^0}(s, t, u)$$

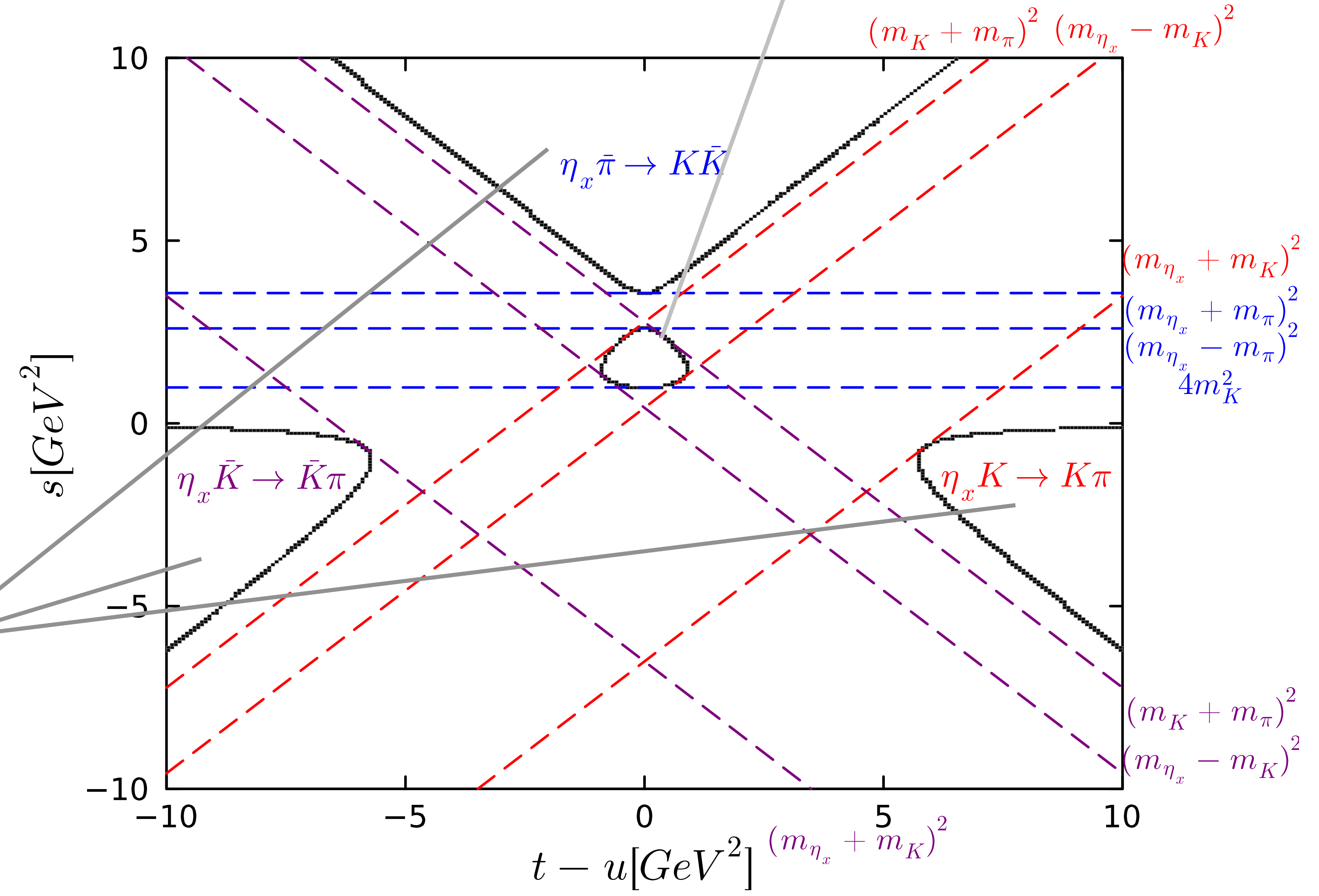


κ & $K^*(892)$

$a_0(980)$

Decay region

Scattering region



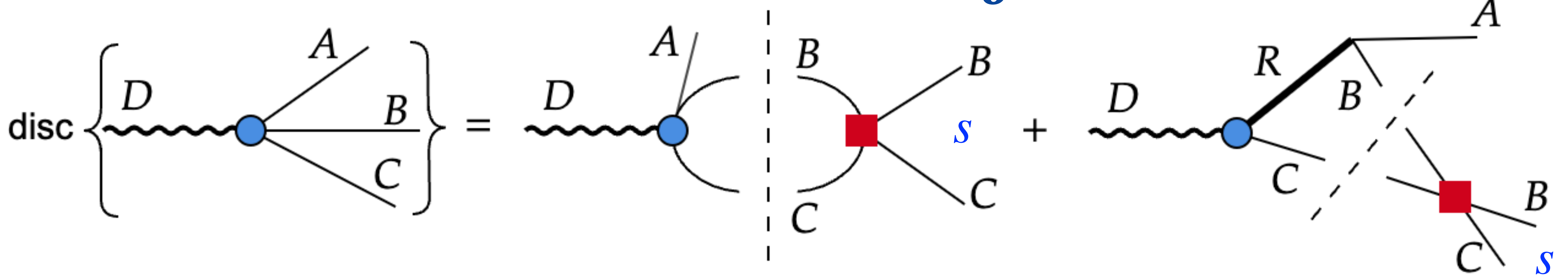
Gram$(s, t, m_{\bar{K}}^2, m_{\eta_x}^2, m_\pi^2, m_K^2) \leq 0$

By virtue of crossing symmetry and reconstruction theorem,

$$\mathcal{M}(s,t,u) = \frac{1}{\sqrt{2}} \mathcal{F}_0^1(s) + [(-\frac{1}{\sqrt{3}}) \mathcal{F}_0^{1/2}(t) + (-\frac{1}{\sqrt{3}})(t(s-u) - \Delta) \mathcal{F}_1^{1/2}(t)] + [t \leftrightarrow u]$$

The single-variable amplitudes $\mathcal{F}_J^I(x)$ are then all what we desire!

Single-variable amplitudes $\mathscr{F}^I_J(x)$



Dispersion relation for two-body scattering,

$$\text{disc}\mathscr{F}^I_J(s) = 2iT^{I*}_J(s+i\epsilon)\Sigma(s)(\mathscr{F}^I_J(s+i\epsilon)+\hat{\mathscr{F}}^I_J(s+i\epsilon))$$

The most general solution (inhomogeneous Omnès problem),

$$x = s, t$$

$$\mathscr{F}^I_J(x) = \Omega^I_J(x)(a_0 + a_1 x + \cdots + a_n x^n + \frac{x^{n+1}}{\pi}\int_{x_{th}}^{\infty}\frac{dx'}{x'^{n+1}}\frac{\Omega^{-1}(x')T^{I*}_J(x')\Sigma(x')\hat{\mathscr{F}}^I_J(x')}{x'-x-i\epsilon})$$

FSIs — Sub-channel interaction — Crossed-channel projection

With $\Omega^I_J(x) = \exp(\frac{x}{\pi}\int_{x_{th}}\frac{\delta^I_J(x')dx'}{x'(x'-x-i\epsilon)}) \to x^{-\delta^I_J(\infty)/\pi}$.

Froissart-Martin bound: $\mathscr{F}(x) \lesssim x\log^2(x)$

$$\Rightarrow \begin{cases} \delta^1_0(\infty) = \pi \to P_2(s) \\ \delta^{\frac{1}{2}}_0(\infty) = 2\pi \to P_3(t) \\ \delta^{\frac{1}{2}}_1(\infty) = \pi \to P_0(t) \end{cases}$$

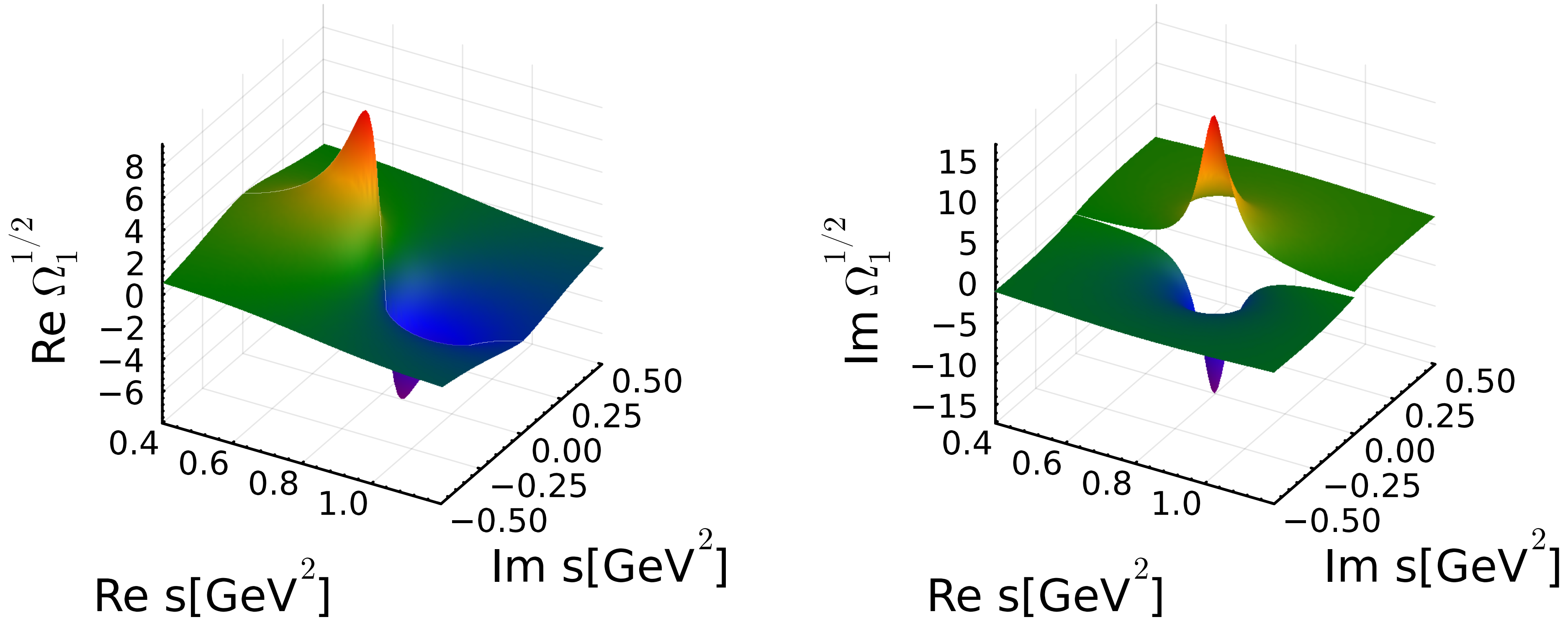
$[t(s-u)-\Delta] \asymp \mathcal{O}(t^2)$

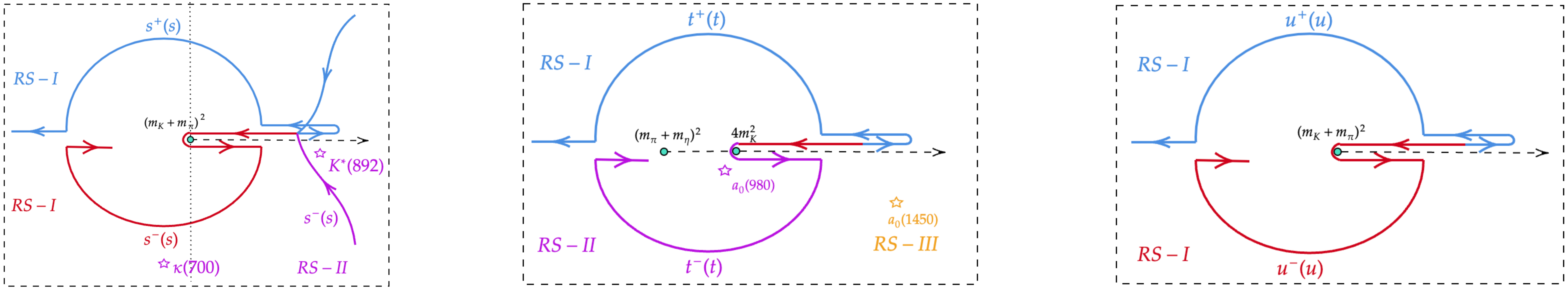
Inhomogeneities $\hat{\mathcal{F}}^I_J(x)$: p.w.a of $\mathcal{F}^I_J(x)$

$$\hat{\mathcal{F}}^1_0(s) = \left(-\sqrt{\frac{2}{3}}\right)2\langle \mathcal{F}^{1/2}_0\rangle_{t_s} + \left(-\sqrt{\frac{2}{3}}\right)\left[\frac{1}{2}(\Sigma_0 - s)(3s-\Sigma_0) - 2\Delta\right]\langle \mathcal{F}^{1/2}_1\rangle_{t_s} + \left(-\sqrt{\frac{2}{3}}\right)s\cdot\kappa_{K\bar{K}}\langle z_s \mathcal{F}^{1/2}_1\rangle_{t_s} + \left(-\sqrt{\frac{2}{3}}\right)\frac{\kappa^2_{K\bar{K}}}{2}\langle z_s^2\mathcal{F}^{1/2}_1\rangle_{t_s}$$

$$\langle f\rangle_{x_y} = \int_{-1}^{1} dz_y f(x(y,z_y)) \longrightarrow \int_{s^-(s)}^{s^+} ds', \quad \int_{t^-(s)}^{t^+} dt', \quad \int_{u^-(s)}^{u^+} du'$$

- elastic $K\pi \to K\pi$ $\longrightarrow$ Riemann sheet 1 only
- inelastic $K\bar{K} \to K\pi,\ K\pi \to K\bar{K}$ $\longrightarrow$ Riemann sheet 1&2
- $\mathcal{F}$ has the right-hand-cut as Ω



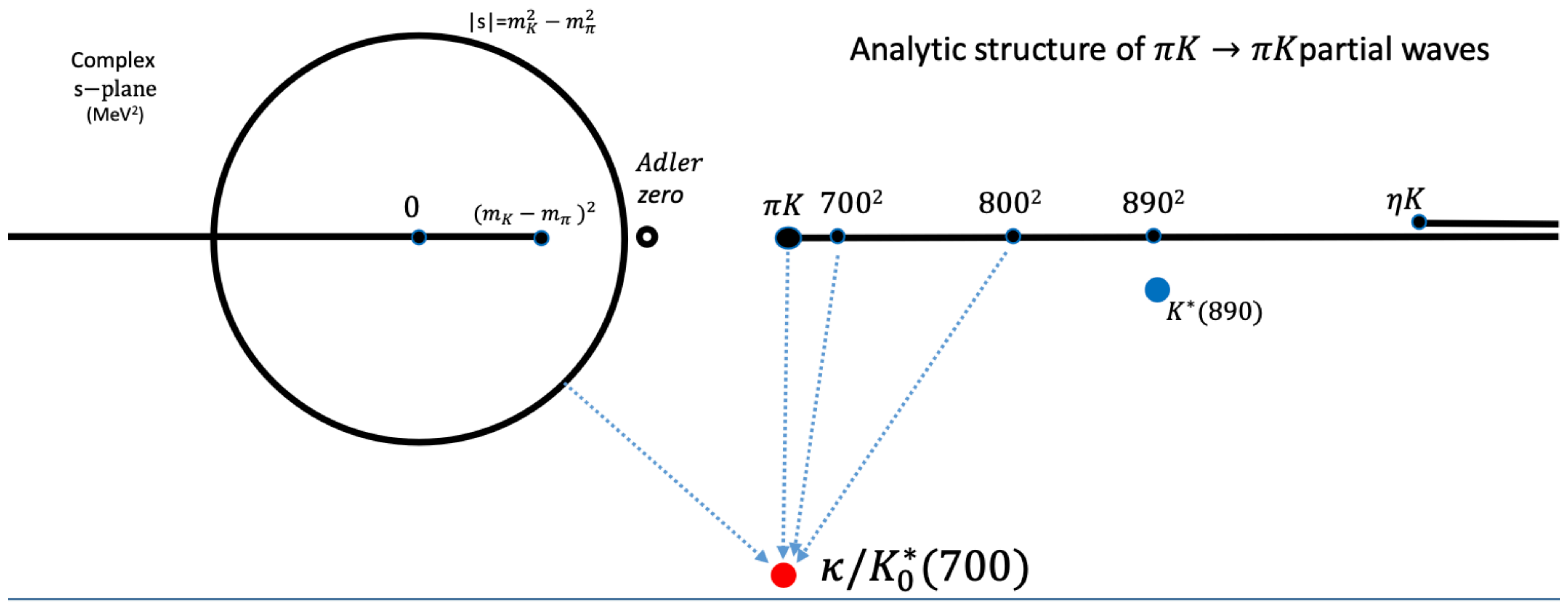


The integration needs to be continued on unphysical sheet!

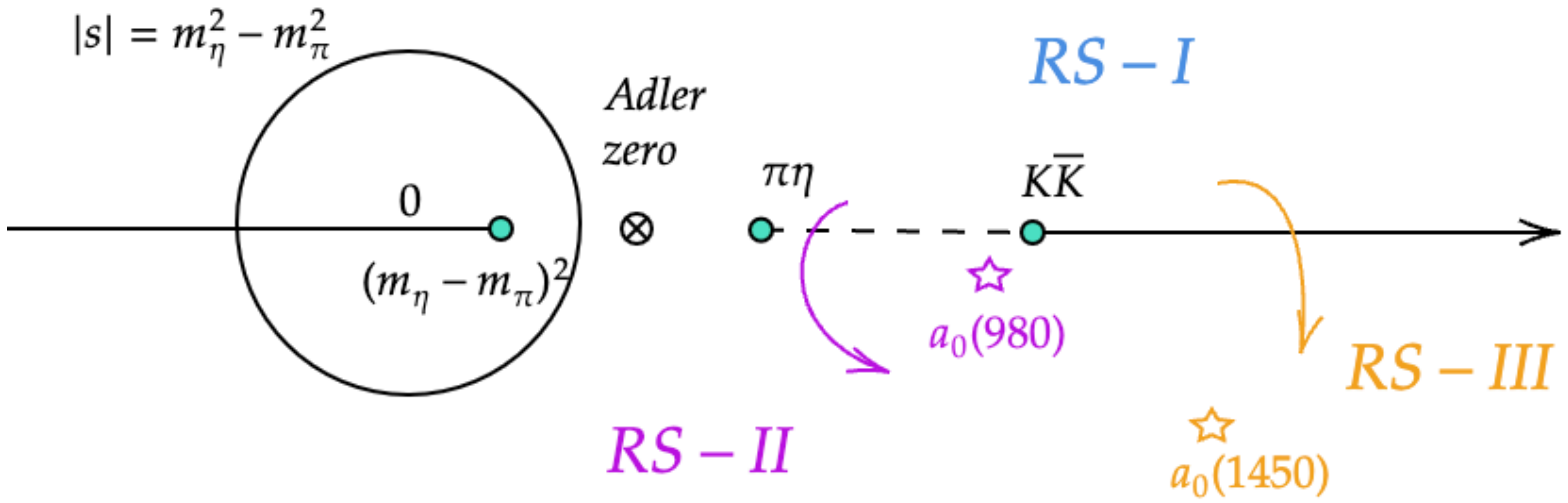
Analytical structure and continuation

M.Albaladejo et al., EPJC(2015)75:488

$K\pi$ scattering J.R.Peláez, j.physrep.2022.03.004



$\pi\eta - K\bar{K}$ scattering



- Single-channel continuation: $\Omega^{II}(s) = \Omega^{I}(s)/\hat{S}(s)$.
- Model-independent accesses to $\hat{S}(s)$:

(Elastical, narrow)

A. Conformal expansion: $T_J^I(s) = \frac{1}{\sigma(s)} \frac{1}{\cot \delta_J^I(s) - i}$, $\cot \delta_J^I(s) = \frac{\sqrt{s}}{2q^{2J+1}} F(s) \sum_n B_n \omega(s)^n$;

(Smooth, wide)

B. Schlesinger fraction method: $C_N(s) = F_1(s)/(1 + \frac{a_1(s-s_1)}{1+\frac{a_2(s-s_2)}{\cdots a_{N-1}(s-s_{N-1})}})$;

(Semi-determined)

C. Padé series: $P_M^N(s, s_0) = \frac{Q_N(s, s_0)}{R_M(s, s_0)}$

These methods give consistent results!

$(I, J) = (1/2, 0\ \&\ 1)$ $K\pi$ scattering

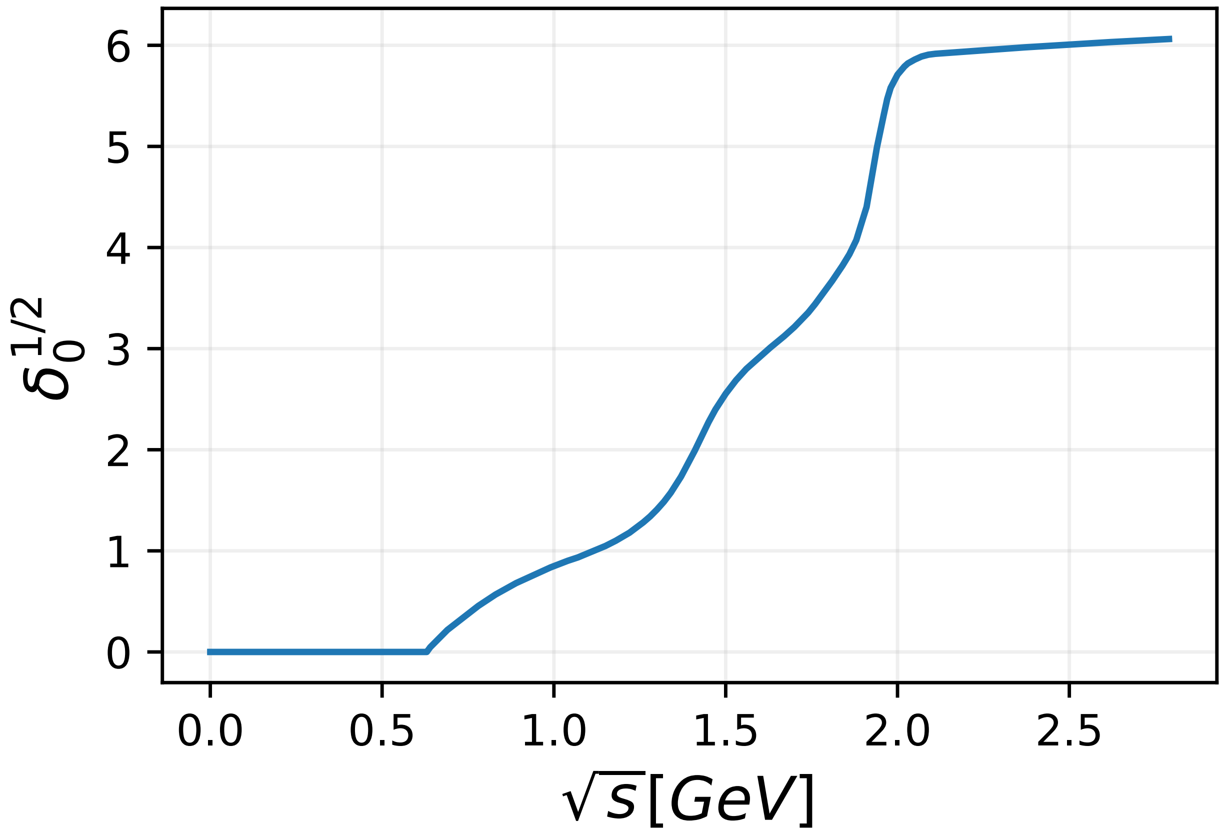
- Elastic up til $K\eta'$ threshold; L. von Detten et al., EPJC(2021) 81:420
- Treatment: conformal expansion ($\sim K\eta$ threshold)

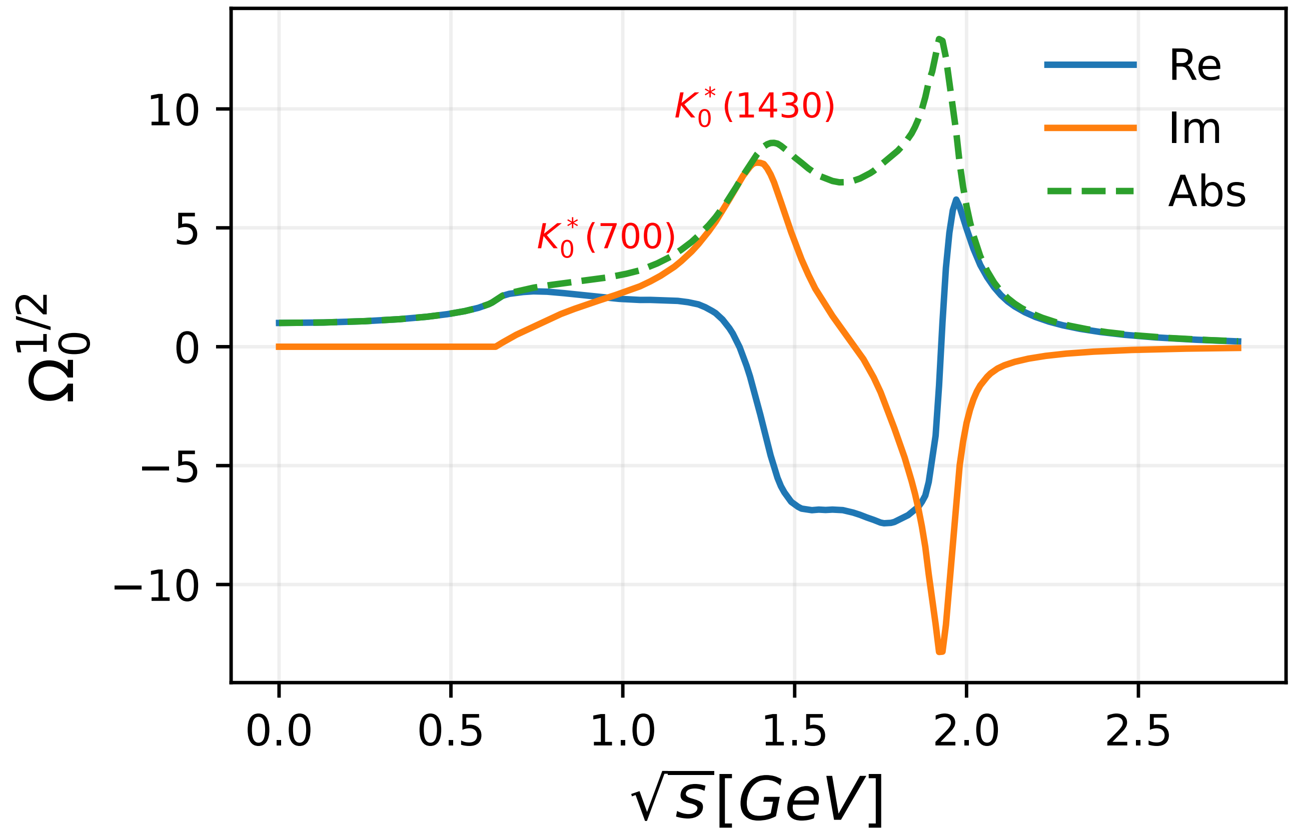
$\Rightarrow$ Schlesinger fraction (~ 1.3GeV).

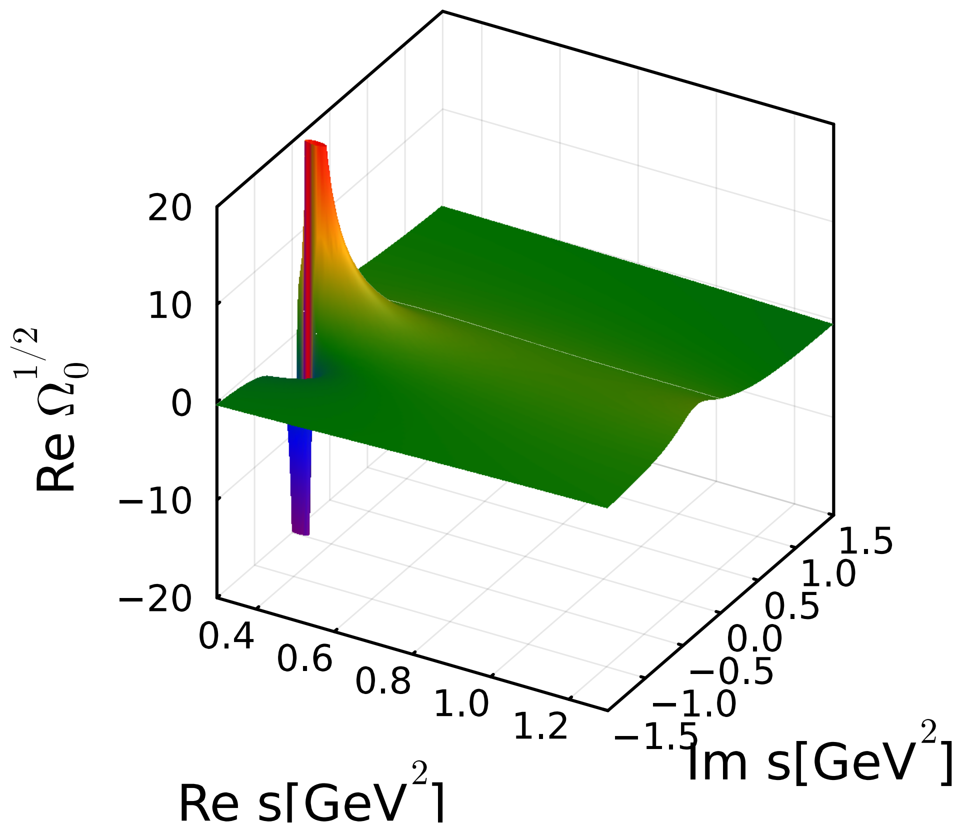
Continued to lower-half plane on RS-II

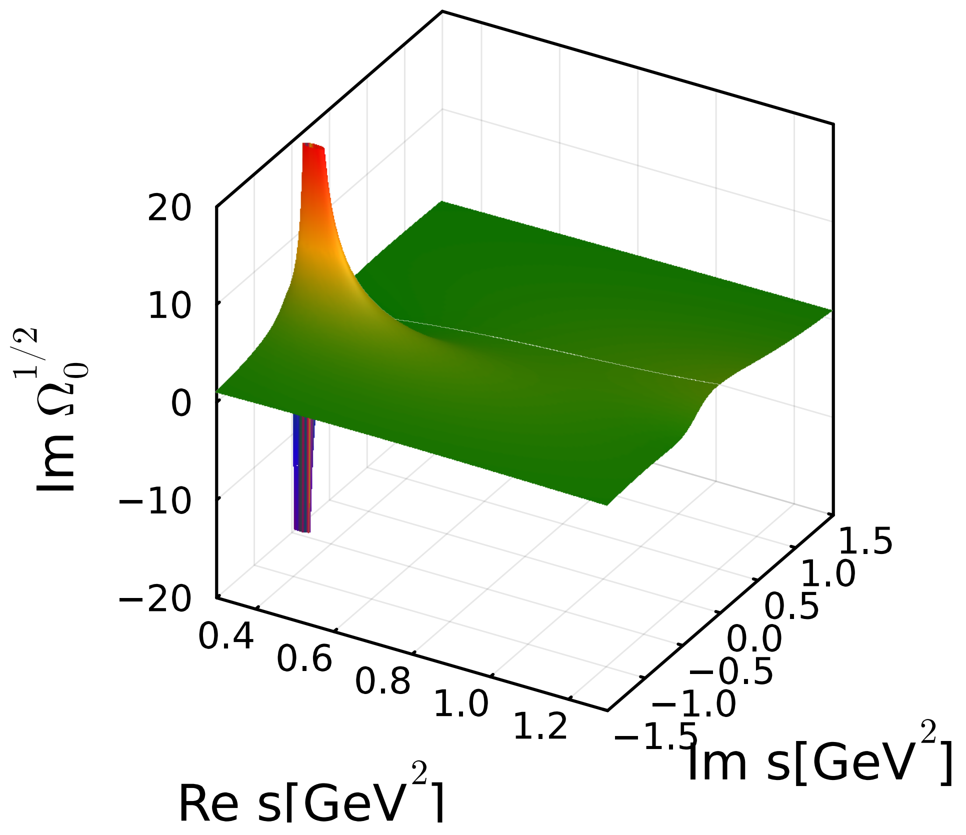
Pole positions (MeV):

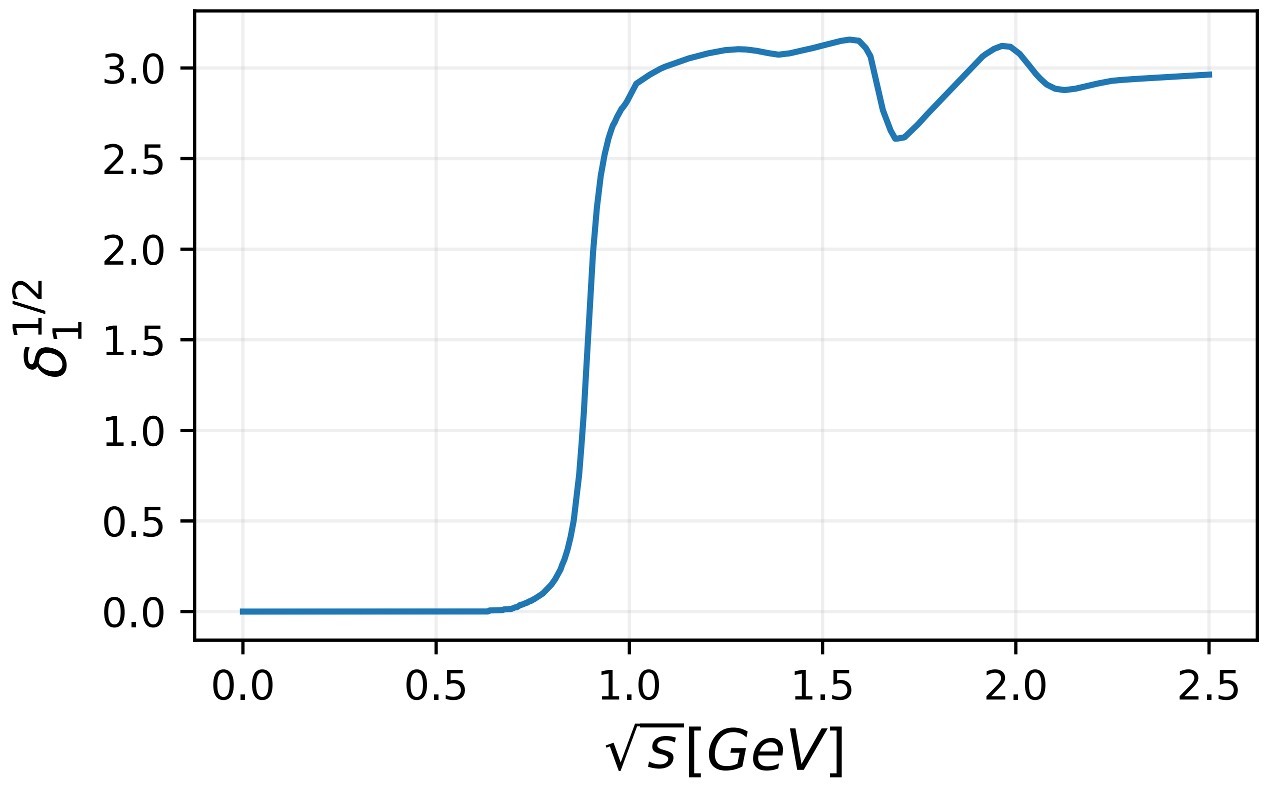
- κ: $667 - i335$;
- $K^*(892)$: $892 - i28$

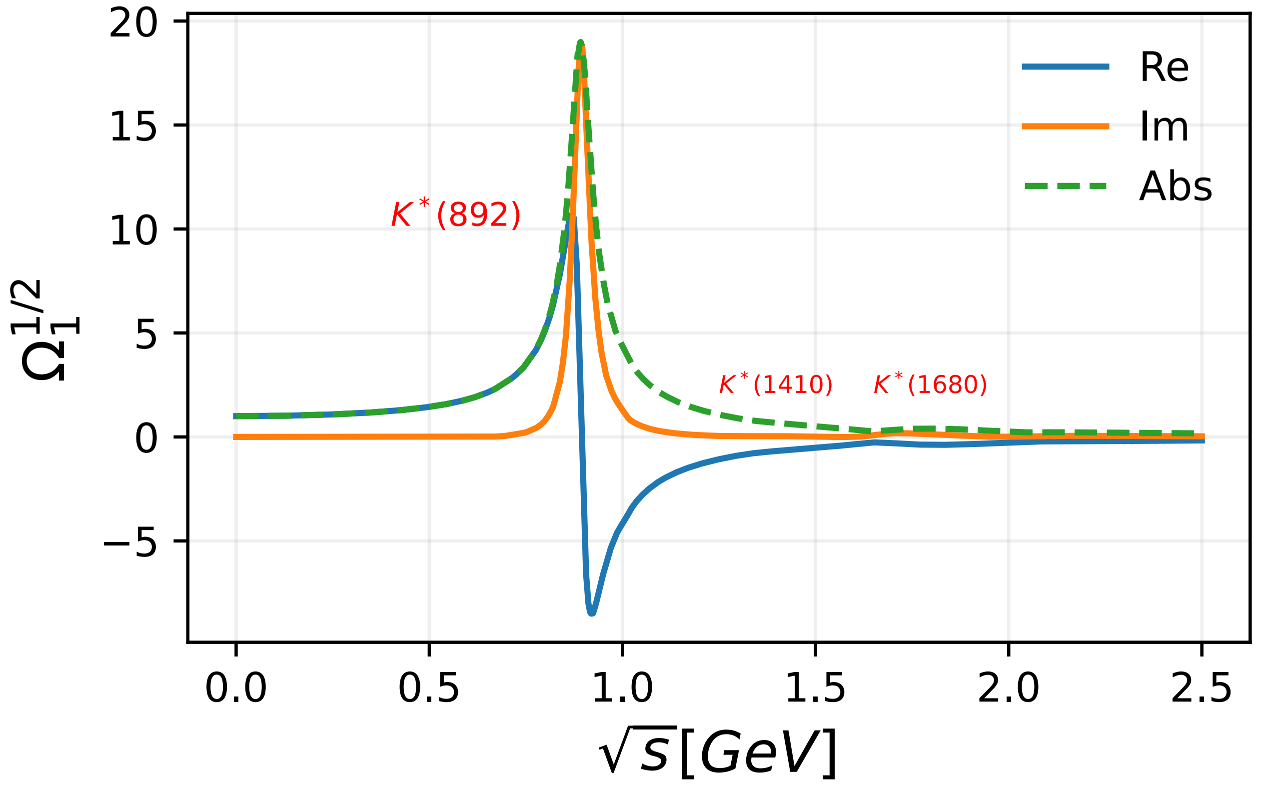


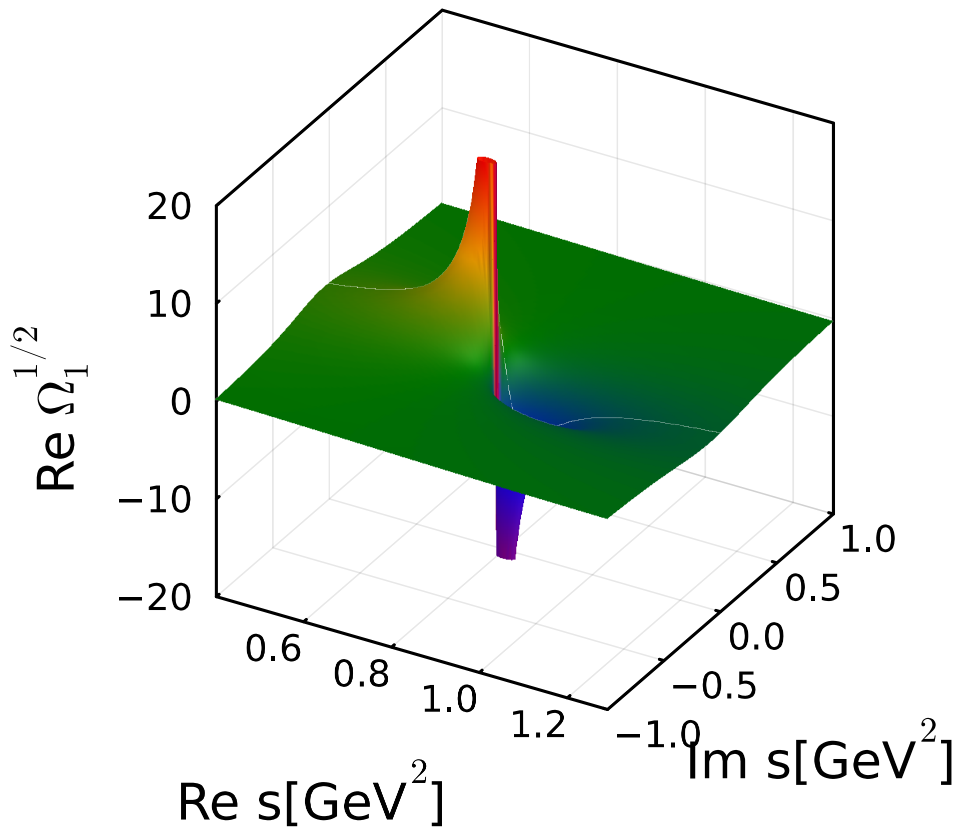


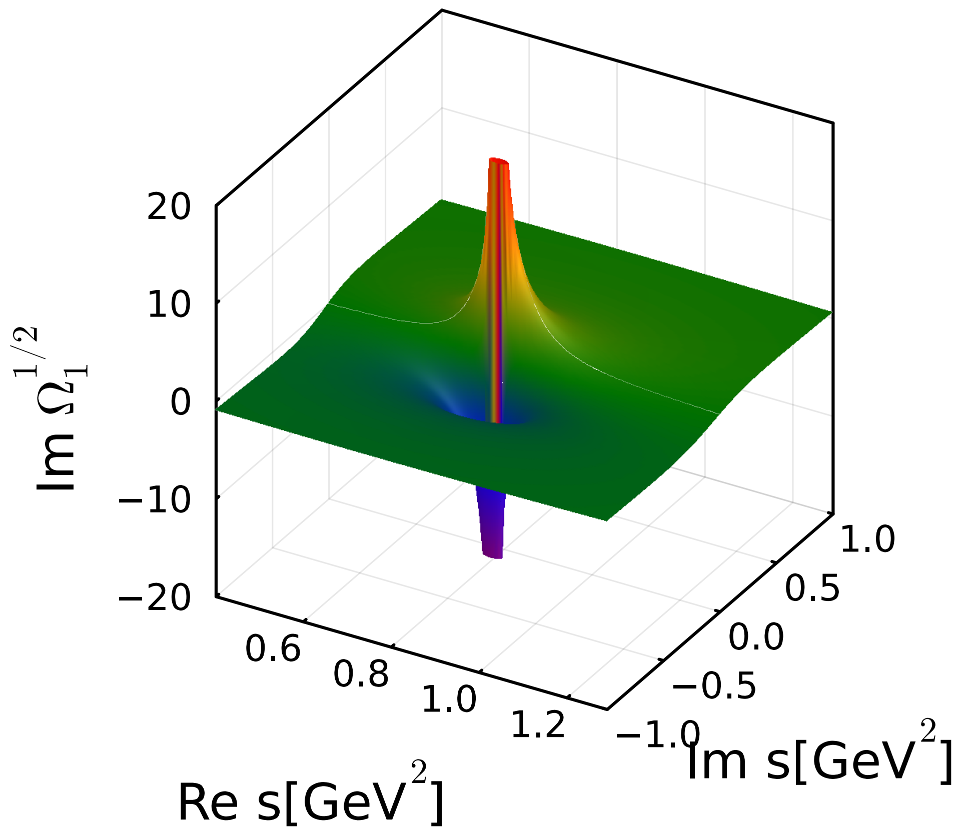










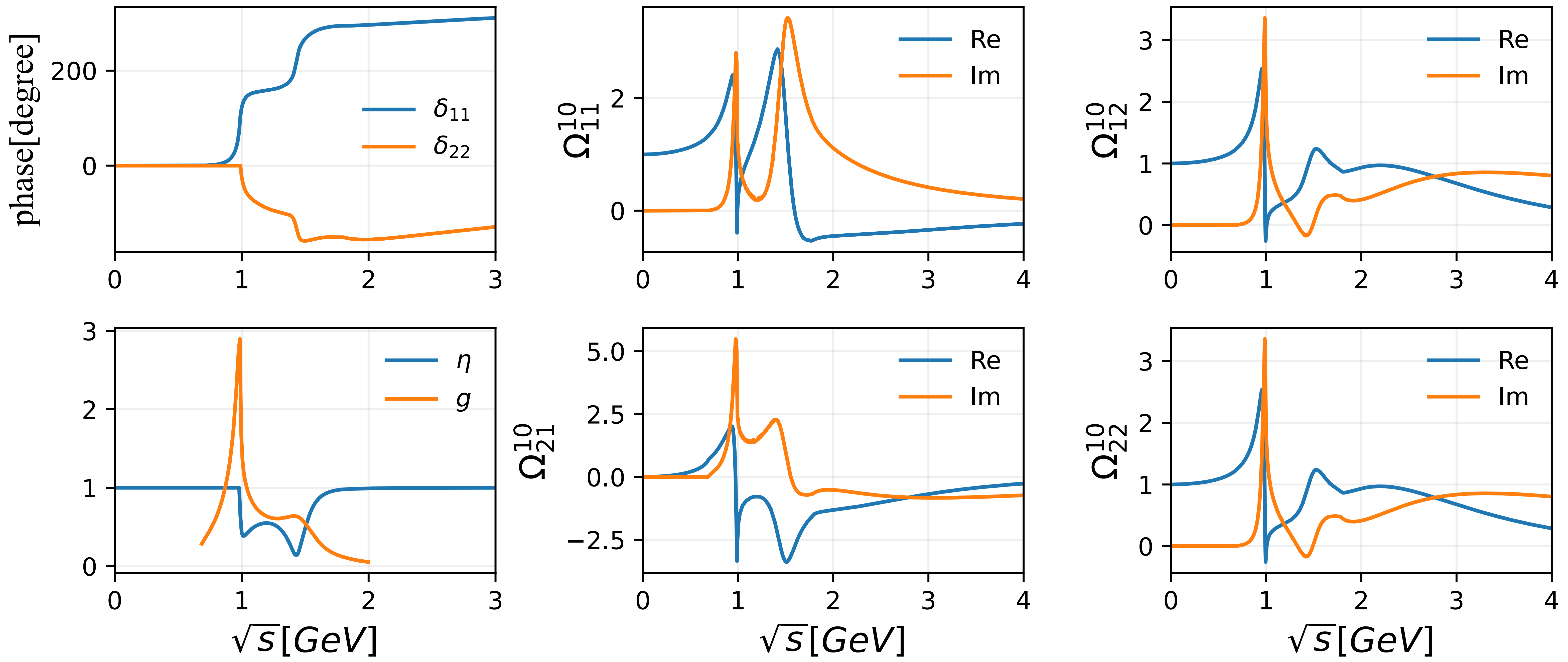


$(I, J) = (1,0)\ \pi\eta - K\bar{K}$ scattering

The isovector $\pi\eta - K\bar{K}$ coupling has a significant inelastic effect due to the onset of $a_0(980)$ and $a_0(1450)$. We adopt the following δ, η, g which satisfies the most ($5 \sim 6$) chiral constraints,

$$\Omega(s) = \frac{1}{\pi} \int_{s_{th}}^{\infty} ds' \frac{T^*(s')\Sigma(s')\Omega(s')}{s' - s - i\epsilon}$$

B.Moussallam, EPJC14,111–122(2000)
J.F.Donoghue, NPB343(1990)
M.Doring,JHEP10(2013)011



$$T(s) = \begin{pmatrix} \frac{\eta e^{2i\delta_{11}} - 1}{2i\sigma_1} & g e^{i\phi_{12}} \\ g^{i\phi_{12}} & \frac{\eta e^{2i\delta_{22}} - 1}{2i\sigma_2} \end{pmatrix}$$

M.Albaladejo et al.,EPJC(2015)75:488
M.Albaladejo et al., EPJC(2017)77:508

The form factors $F_S^{\pi\eta, K\bar{K}}(s)$ are evaluated to be the same with that in the above refs.

“Effective” elastic $K\bar{K}$ scattering

The 2×2 Omnès matrix describes the coupling between the production amplitudes $J/\psi\bar{\gamma}\bar{\pi} \to \pi\eta$ and $J/\psi\bar{\gamma}\bar{\pi} \to K\bar{K}$,

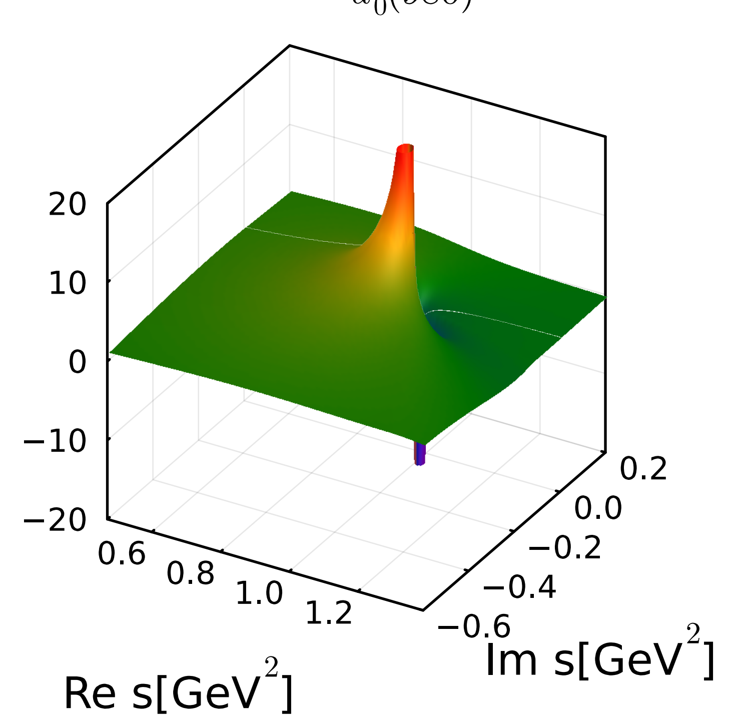
$$\begin{pmatrix} \mathscr{M}^{\pi\eta} \\ \mathscr{M}^{K\bar{K}} \end{pmatrix} = \begin{pmatrix} \Omega_0^1(s)_{\pi\eta\to\pi\eta} & \Omega_0^1(s)_{K\bar{K}\to\pi\eta} \\ \Omega_0^1(s)_{\pi\eta\to K\bar{K}} & \Omega_0^1(s)_{K\bar{K}\to K\bar{K}} \end{pmatrix} \begin{pmatrix} \mathscr{M}^{\chi,\pi\eta} \\ \mathscr{M}^{\chi,K\bar{K}} \end{pmatrix}.$$

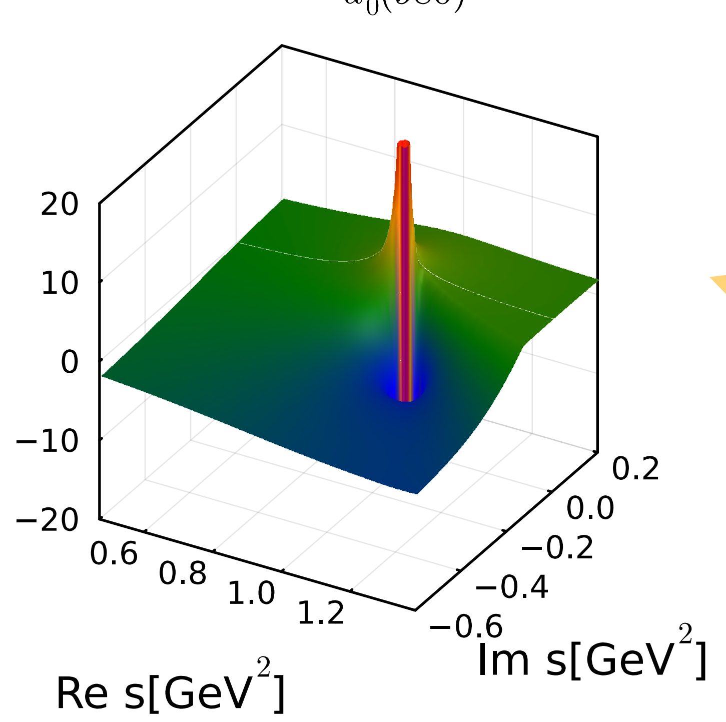
T.Isken et al., EPJC(2017)77:489;E.Kou et al. ,JHEP12(2023)17

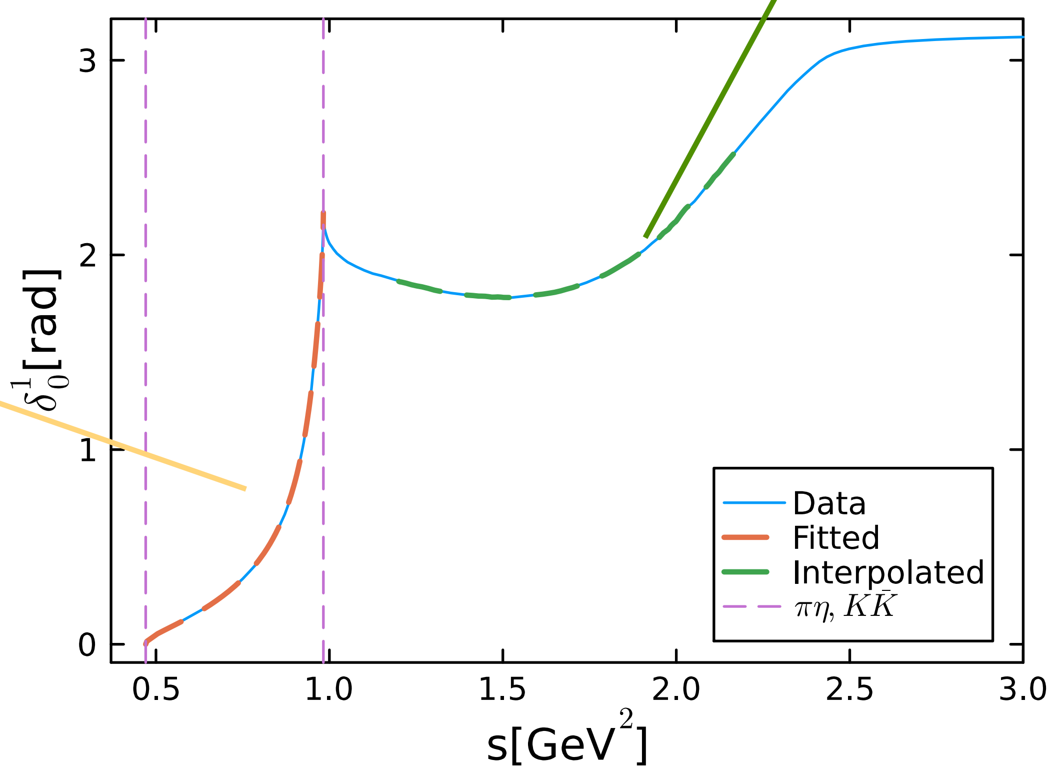
To simplify the problem, we adopt the idea of “effective phase shift” (LO approximation of production form factors),

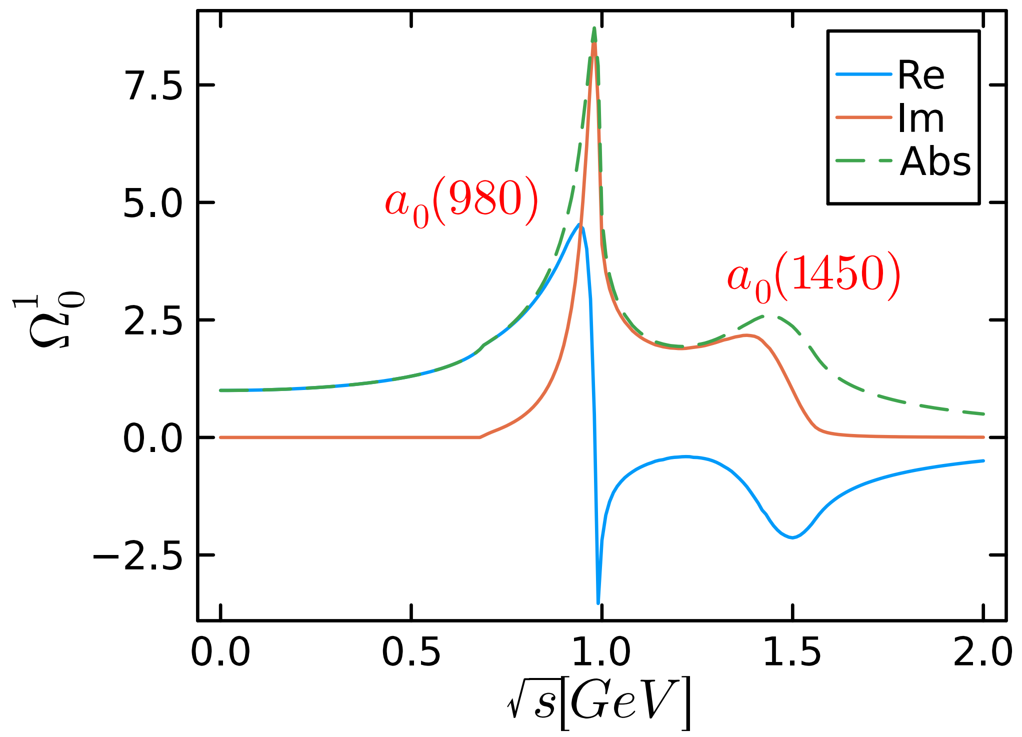
$$\mathscr{M}^{K\bar{K}} = \Omega(s)_{\pi\eta\to K\bar{K}}P_1(s) + \Omega(s)_{K\bar{K}\to K\bar{K}}P_2(s) = (\xi\cdot\Omega_{\pi\eta\to K\bar{K}}(s) + \Omega_{K\bar{K}\to K\bar{K}}(s))P_{eff}(s) =: \Omega_{eff}(s)P_{eff}(s).$$

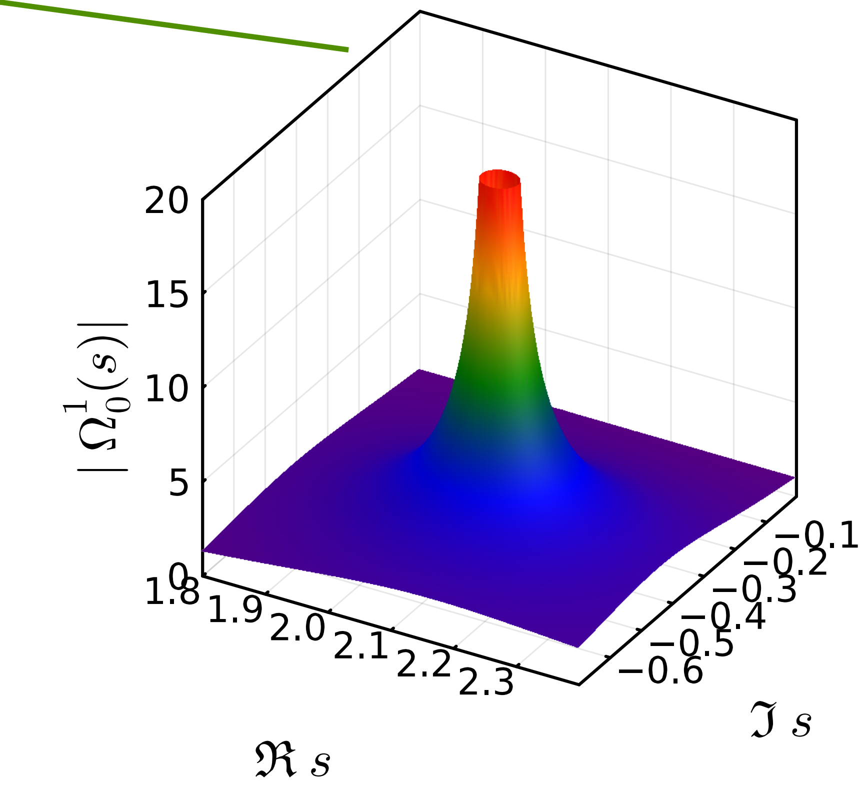
When $\xi = 1(F_S^\pi(0) = 0.816, F^{K\bar{K}}(0) = 1$ @NLO therein),











The ambiguity of ξ shall not affect a lot!

- Only changes high-energy behaviour
- Shifted into elastic region by contour deformation

$$\sqrt{s^{II}_{a_0(980)}} : 997.1 - i26.1 \qquad \sqrt{s^{III}_{a_0(1450)}} : 1465 - i137$$

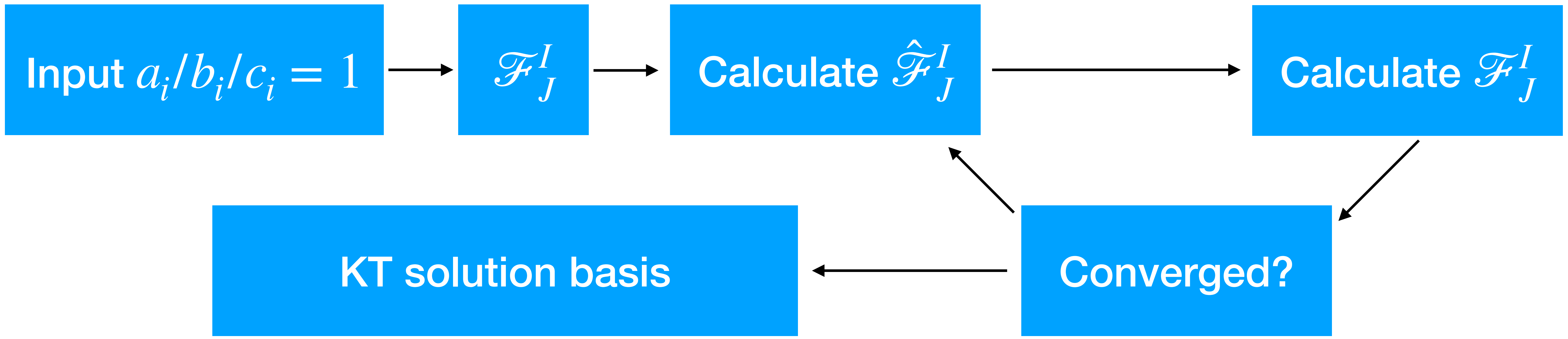
Khrui-Trieman Framework

Khrui-Trieman equation: $1 \to 3$ decaying

$$\mathcal{F}_J^I(x) = \Omega_J^I(x)\{P_n(x) + \frac{x^{n+1}}{\pi}\int \frac{dx'}{x'^{x+1}}\frac{\hat{\mathcal{F}}_J^I(x')\sin\delta_J^I(x')}{|\Omega_J^I(x')|(x'-x)}\}$$

$$\hat{\mathcal{F}}_J^I(x) \propto \int_{x-}^{x+} dx' \mathcal{F}_J^I(x')$$

Iteration



J.Gasser and A.Rusetsky,EPJC(2018)78:906

- The pseudo-threshold singularity $\hat{\mathcal{F}}_1^{1/2} \propto \frac{1}{\sqrt{(m_{\eta_x} - m_\pi)^2 - t}^{\,3}}$ can be avoided by contour deformation;

  e.g. $\gamma\gamma$ collisions, $p\bar{p}$ annihilation…

- The solutions are linearly-independent of subtractions $a_i/b_i/c_i$ $\to$ Generic & Reusable;

$$\mathcal{M}(s,t,u;m_{\eta_x}) = \sum_i C_i \mathcal{M}_i(s,t,u;m_{\eta_x})$$

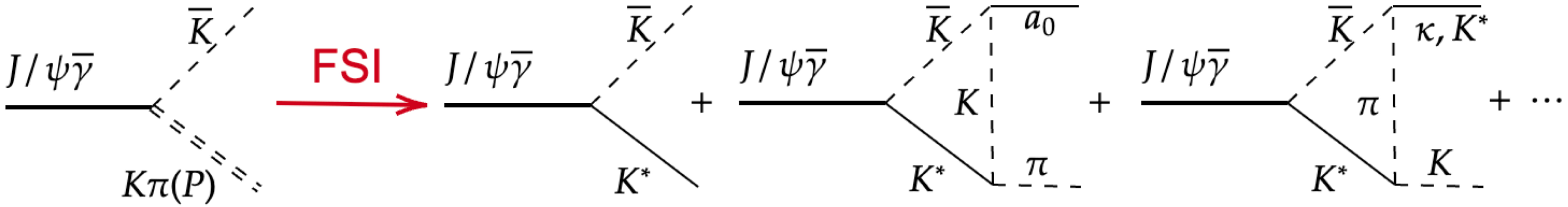
- The subtractions are calculated from CHPT / fitted by experimental data;

KT basis function: $c_0 = 1$

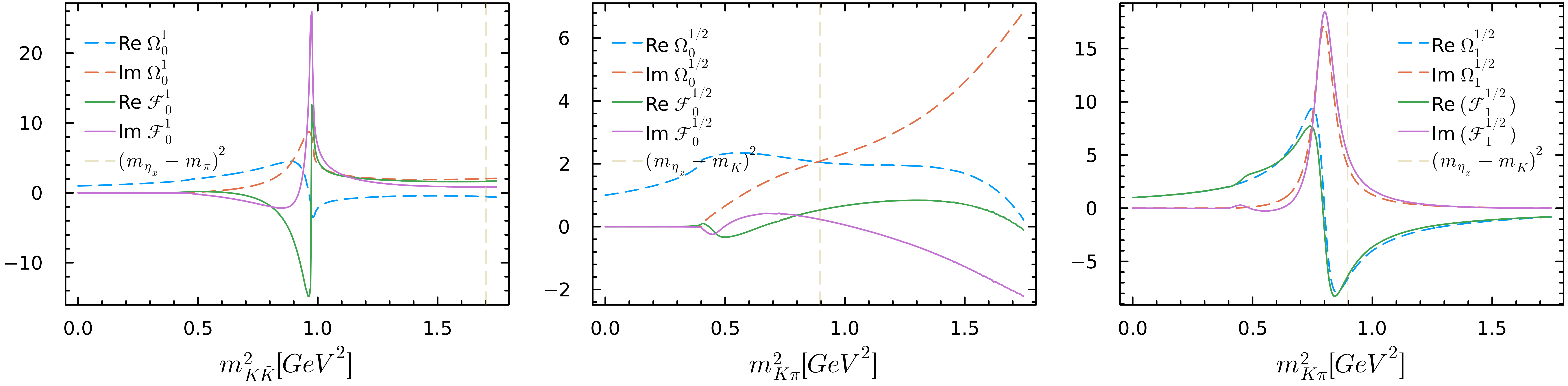
$$\mathscr{F}_0^1(s) = \Omega_0^1(s)\frac{s^3}{\pi}\int\frac{ds'}{s'^3}\frac{\hat{\mathscr{F}}_0^1(s')\sin\delta_0^1(s')}{|\Omega_0^1(s')|(s'-s)}\qquad \mathscr{F}_0^{1/2}(t) = \Omega_0^{1/2}(t)\frac{t^4}{\pi}\int\frac{dt'}{t'^4}\frac{\hat{\mathscr{F}}_0^{1/2}(t')\sin\delta_0^{1/2}(t')}{|\Omega_0^{1/2}(t')|(t'-t)}\qquad \mathscr{F}_1^{1/2}(t) = \Omega_1^{1/2}(t)\{1+\frac{t}{\pi}\int\frac{dt'}{t'}\frac{\hat{\mathscr{F}}_1^{1/2}(t')\sin\delta_1^{1/2}(t')}{|\Omega_1^{1/2}(t')|(t'-t)}\}$$

Dominant channel to $\eta(1440)$!

Triangle singularity



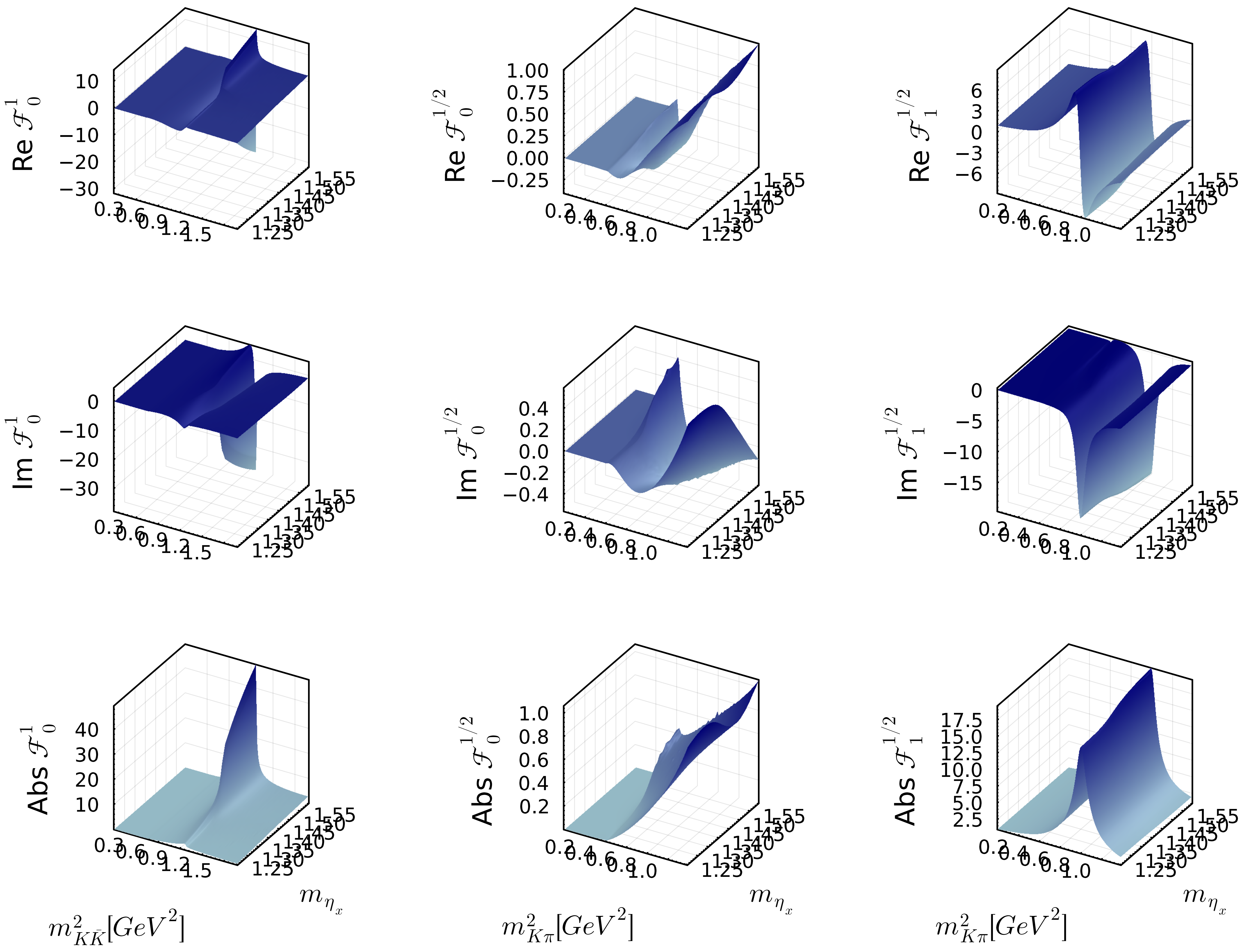
When $m_{K_S^0K_S^0\pi^0} \sim 1.44GeV$,



KT basis function: $c_0 = 1$

And for each $K_S^0 K_S^0 \pi^0$ bin in $1.24 \sim 1.6$ GeV…

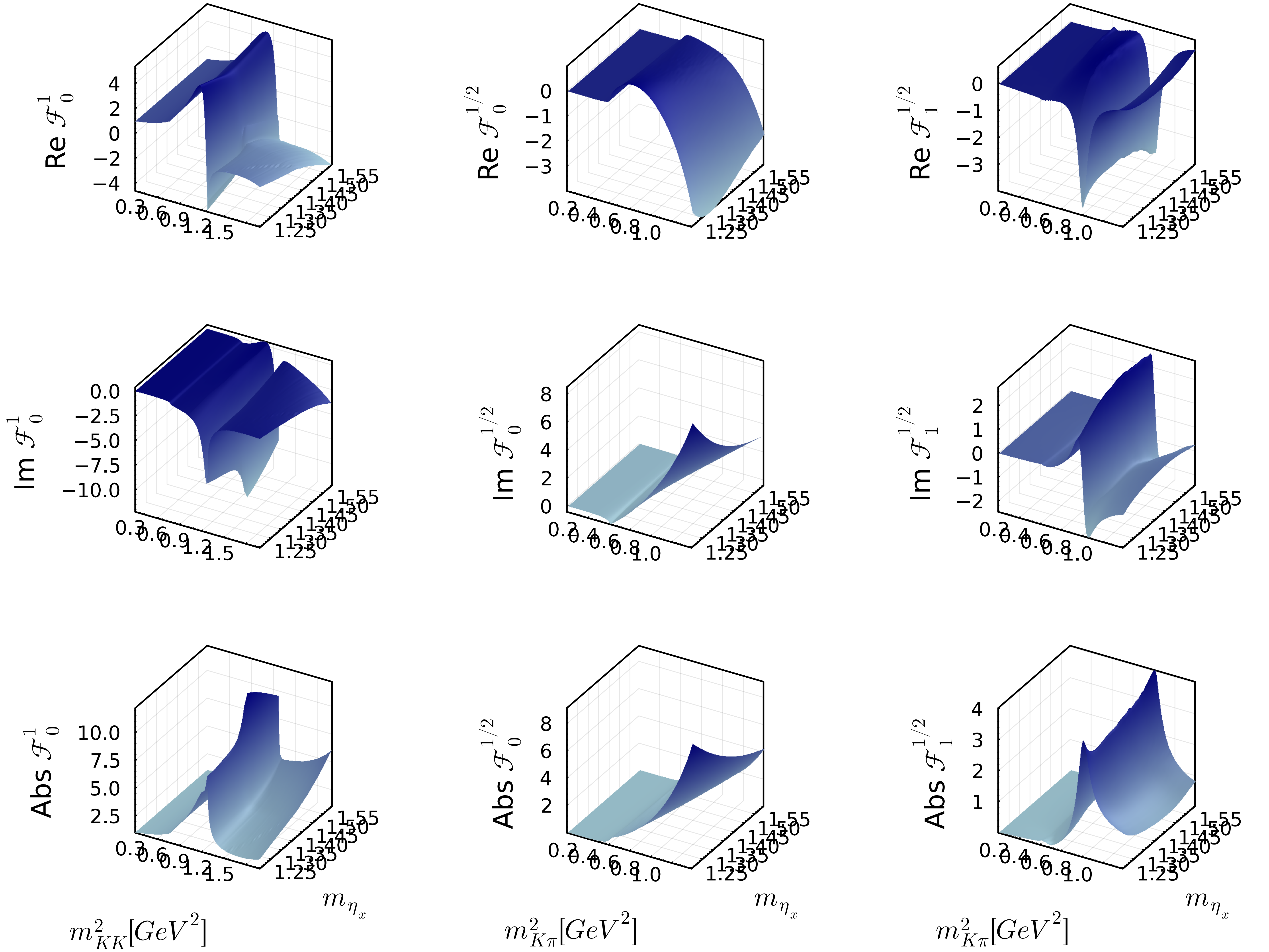
- $K^*K \to a_0\pi$: Triangle singularity
- $K^*K \to \kappa K$: weak coupling
- $K^*K \to K^*K$: vertex renormalization



KT basis function: $a_0 = 1$ or else

And for the case $a_0 = 1$...

- $a_0\pi \to \kappa K$: sizable coupling



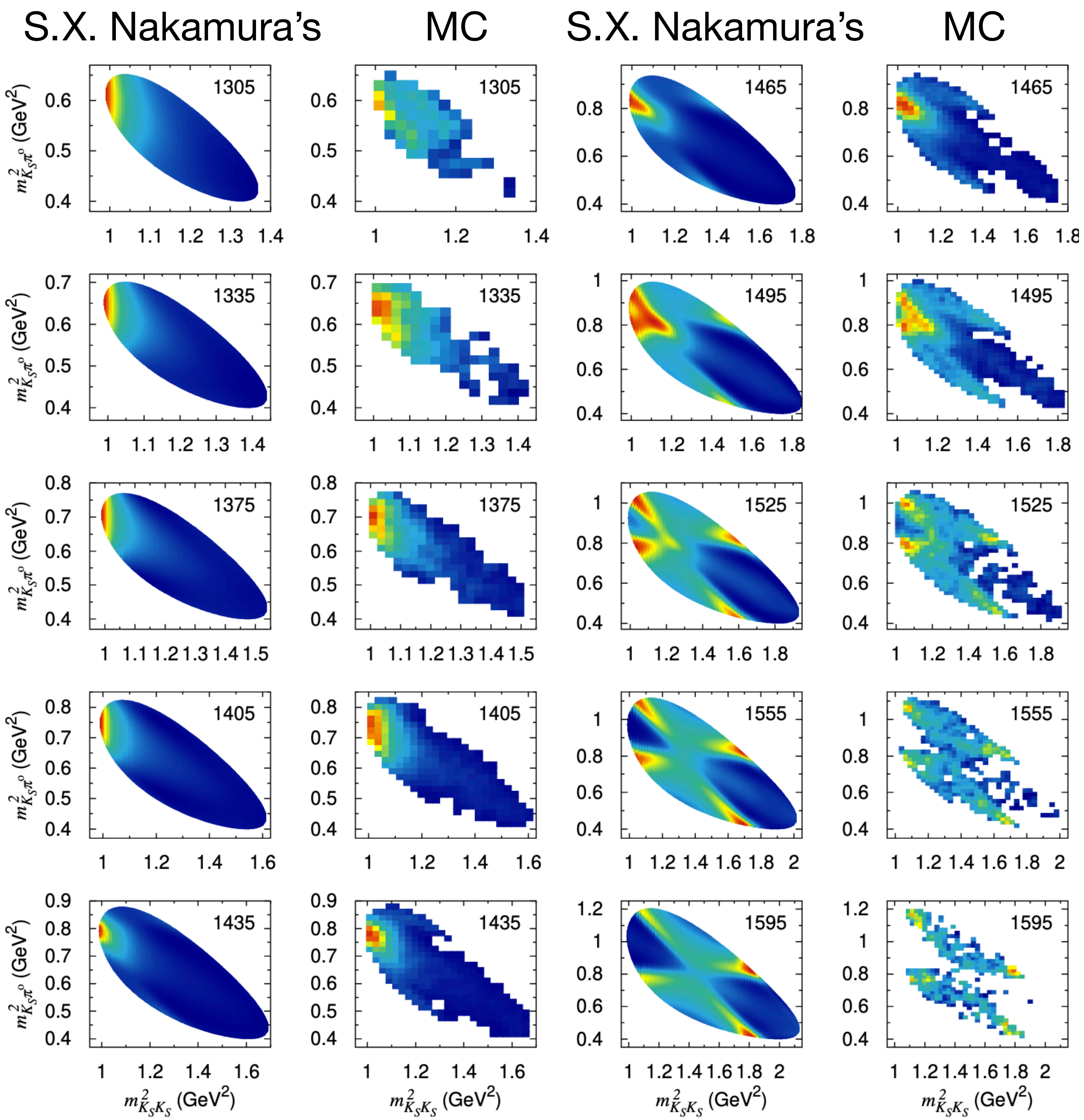
Each case $a_i, b_i, c_i = 1$ corresponds to such a basis!

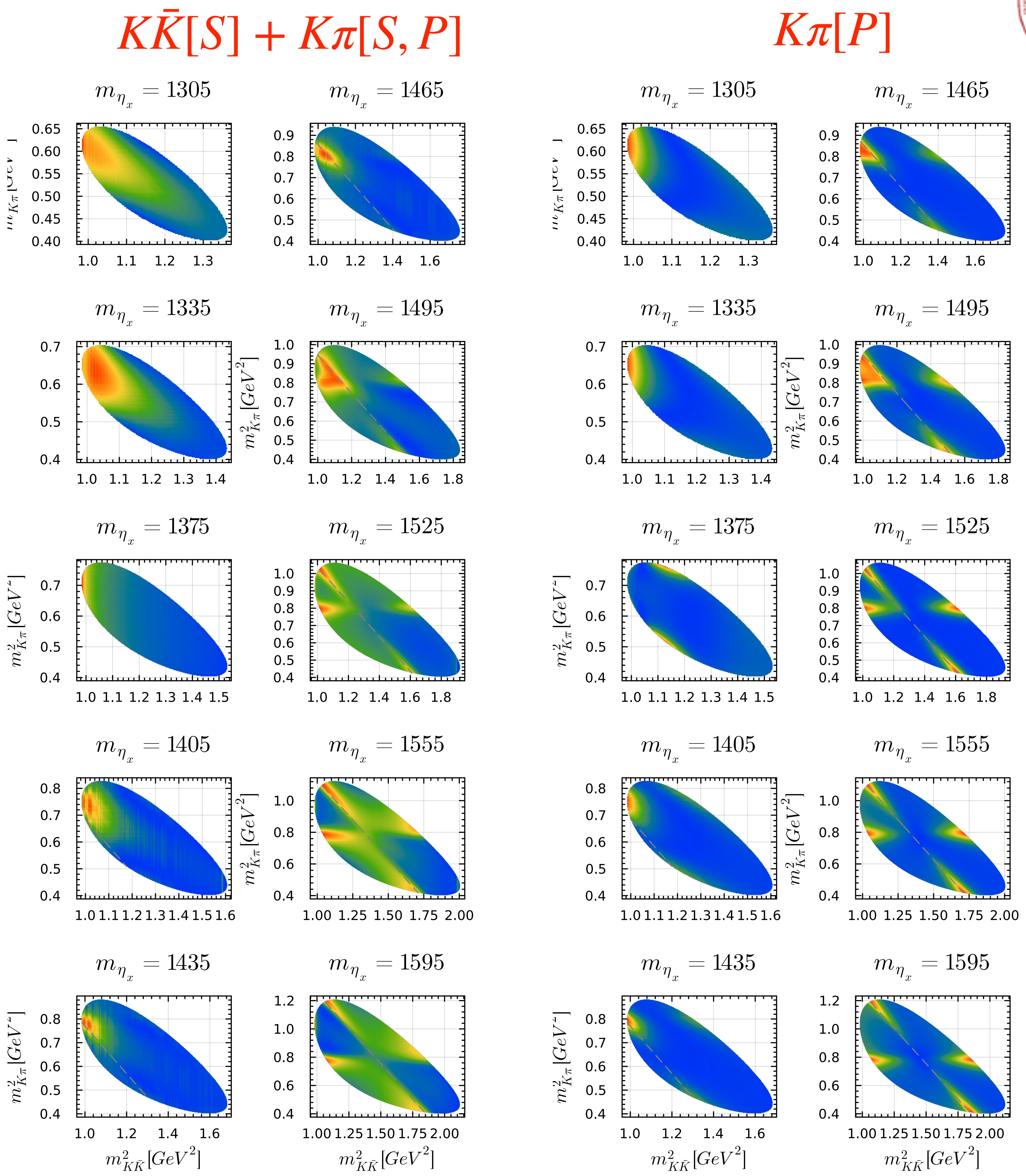
Discussion

Monte-Carlo Dalitz plots

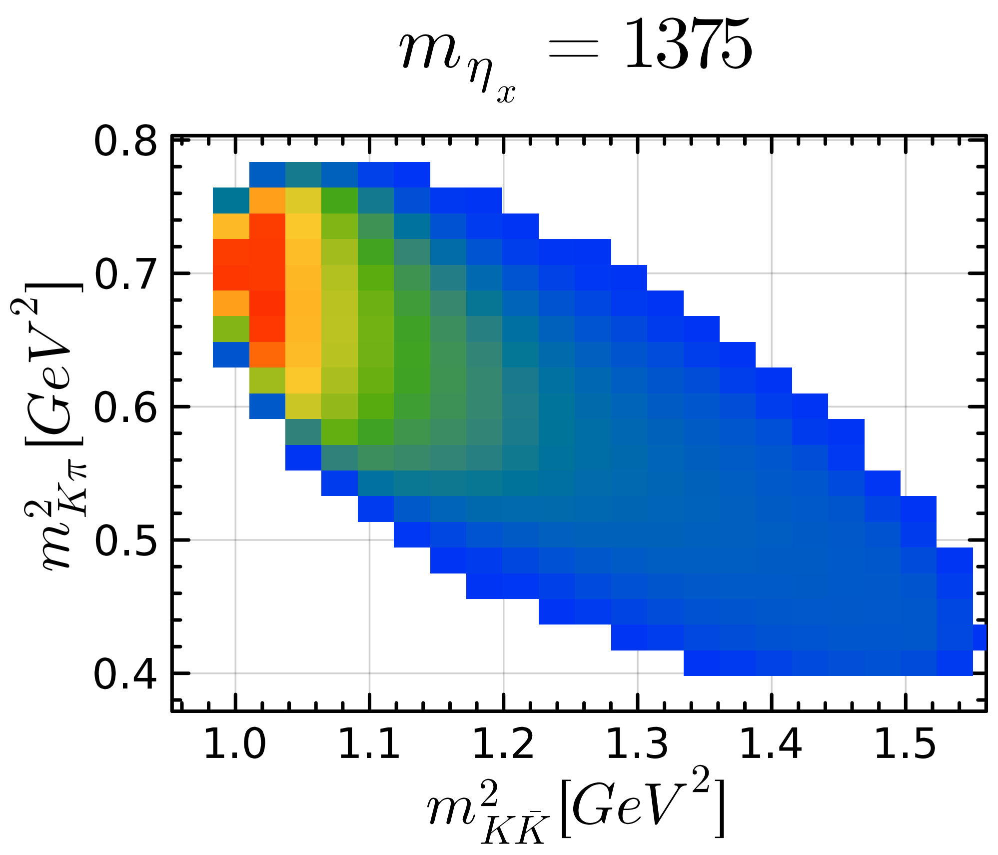
S.X.Nakamura et al., PRD.109.014021
;PRD.107.L091505

BESIII, JHEP03(2023)121



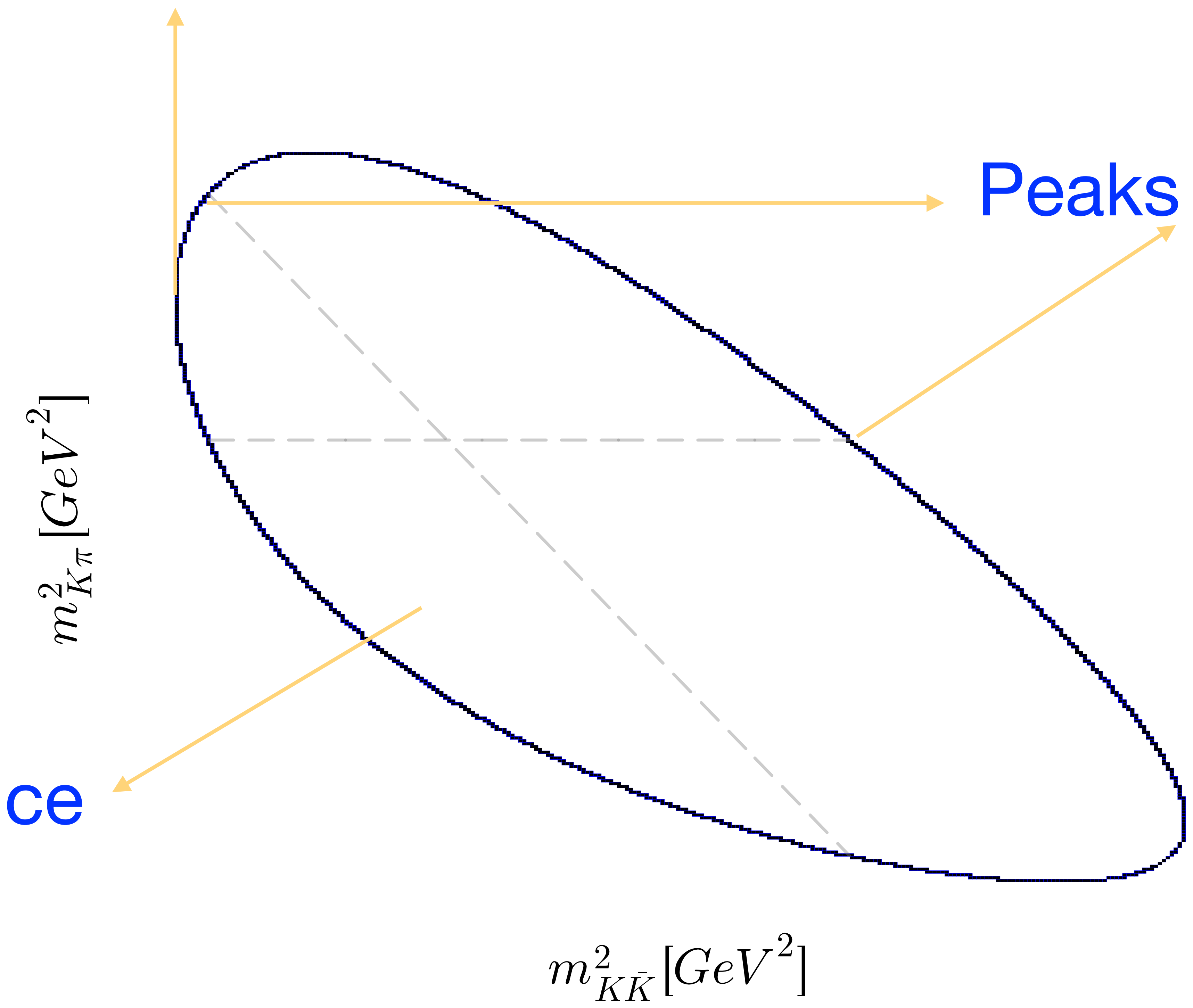


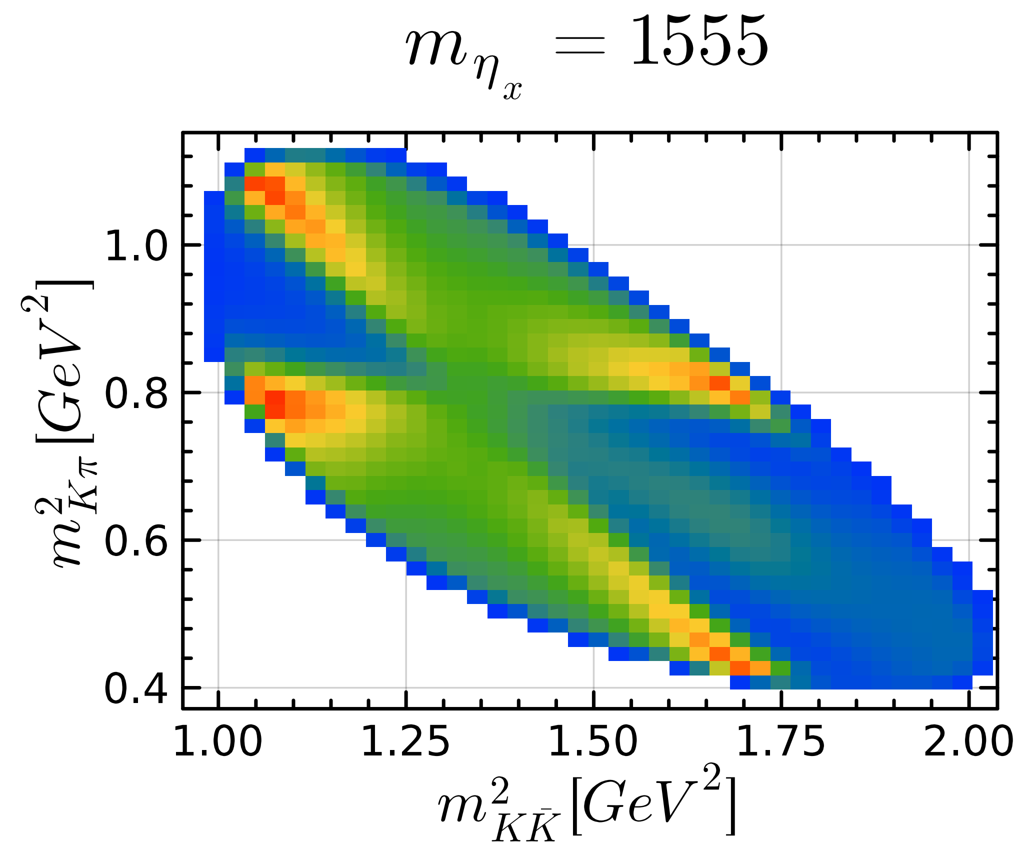
Analysis from Monte-Carlo data



$a_0\pi$ from tree-level and TS

Peaks and kinematic reflection of K^*K





Constructive/destructive interference

between $a_0\pi$ & κK

On real axis (Data), our model is consistent with S.X. Nakamura, PRD109.014021;107.L091505

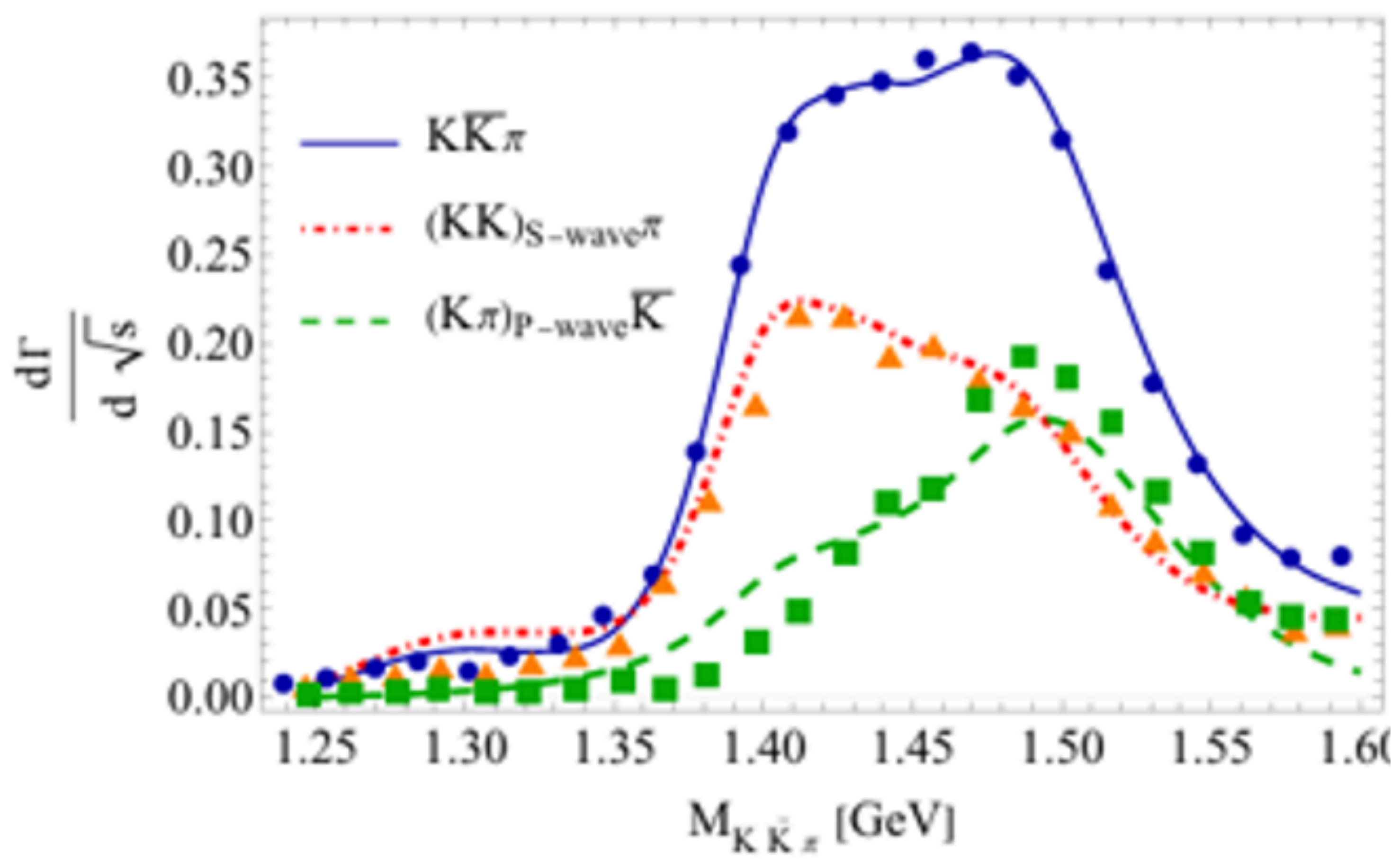
On complex plane (analytical continuation), it remains questionable.

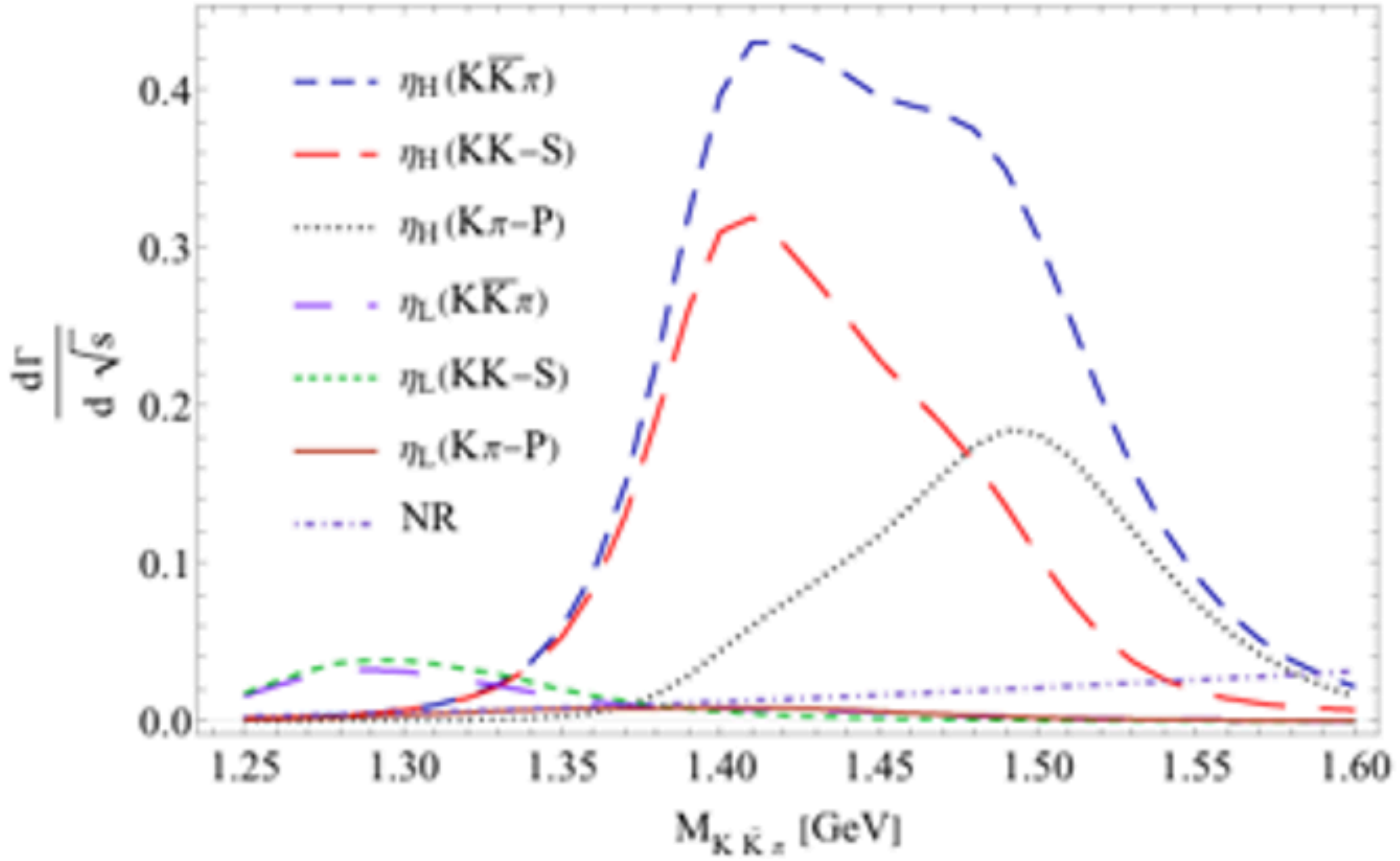
Fitting scheme up to one-loop level

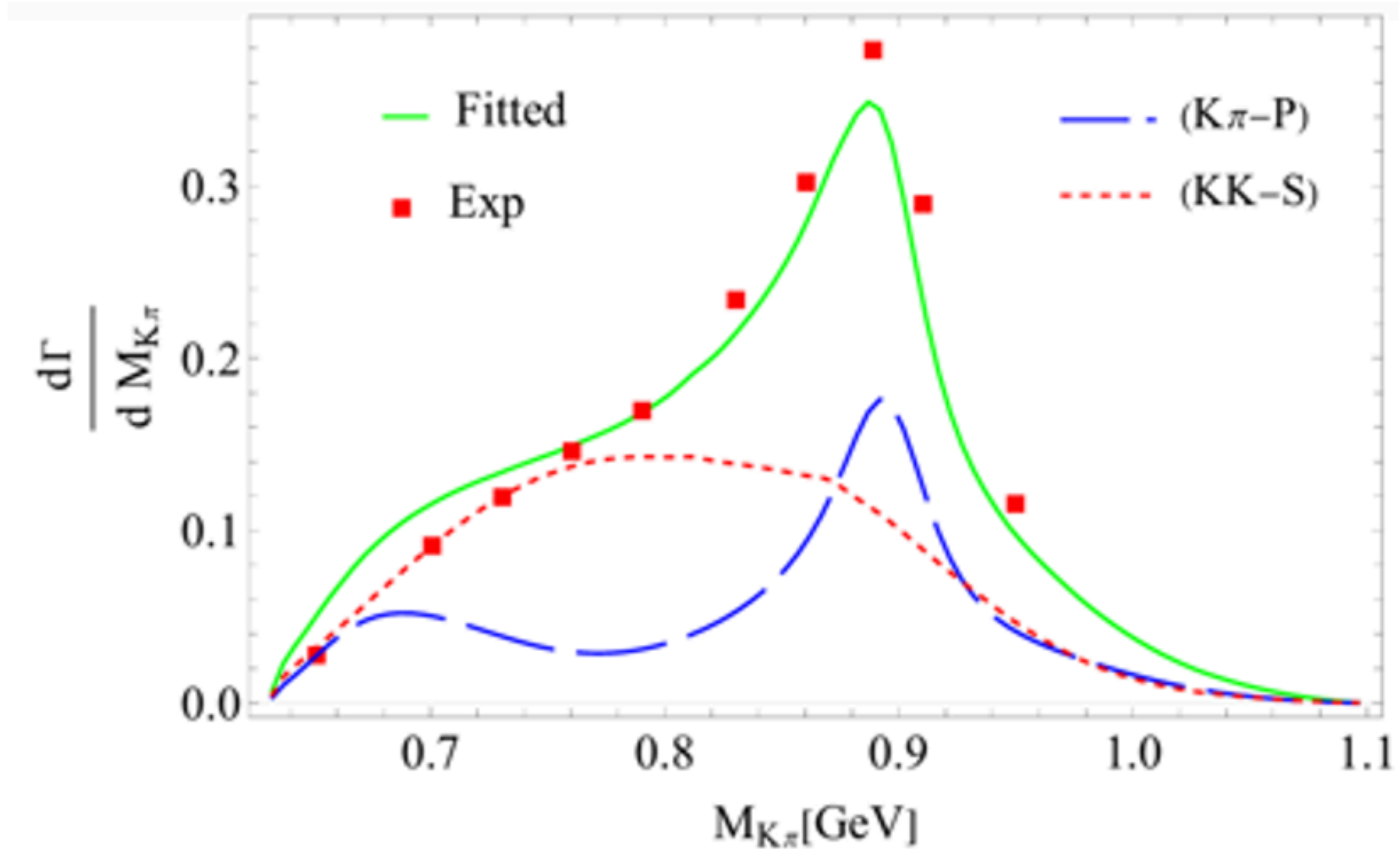
Y.Cheng et al., arXiv:2407.10234

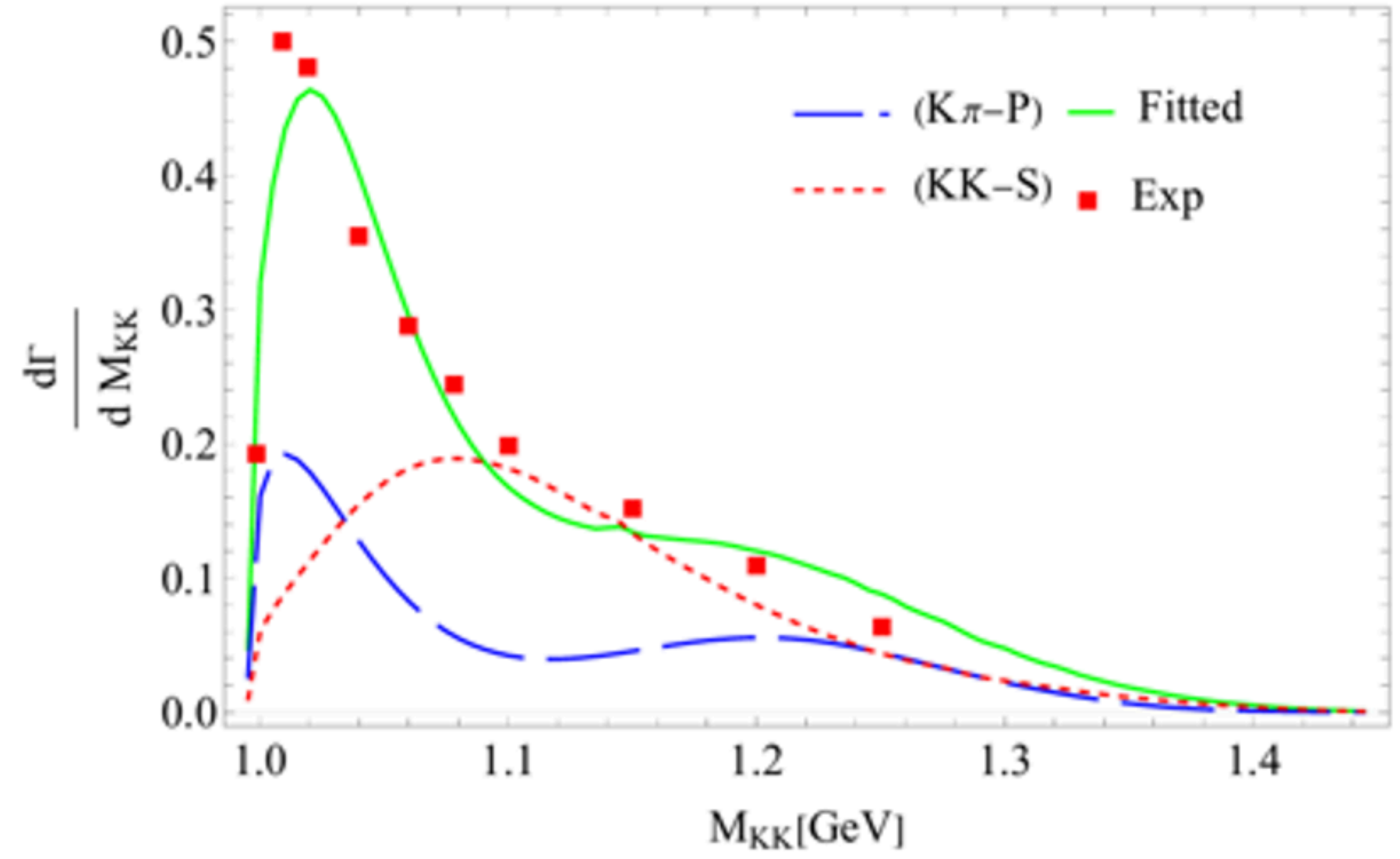
The preliminary Khrui-Trieman study implies that

- Most corrections above two-loops shall be able to be absorbed into the vertex, propagators…
- The one-loop approximation (TS mechanism) may be reasonable









Isobaric approach

- $\eta(1295)$ & $\eta(1440)$ intermediators

🙂 3-body spectrum
🙂 2-body spectra } BESIII spectra
🙂 Dalitz plots } BESIII MC

A comprehensive dispersive analysis is on the way!

Summary & Outlook

- The nature of iso-scalar pseudo-scalar states and their dynamics involved are still beyond our knowledge;

- The 2-body $K\bar{K}\pi$ FSIs have been established dispersively (almost model-independently) and the 3-body ones are almost ready $\Rightarrow$ $\eta\pi\pi, 3\pi$ etc

- The above treatment proceeds similarly for generic 3-body scatterings in a modern & sophisticated perspective $\Rightarrow$ $f_1(1285), f_1(1420), a_1(1260)$

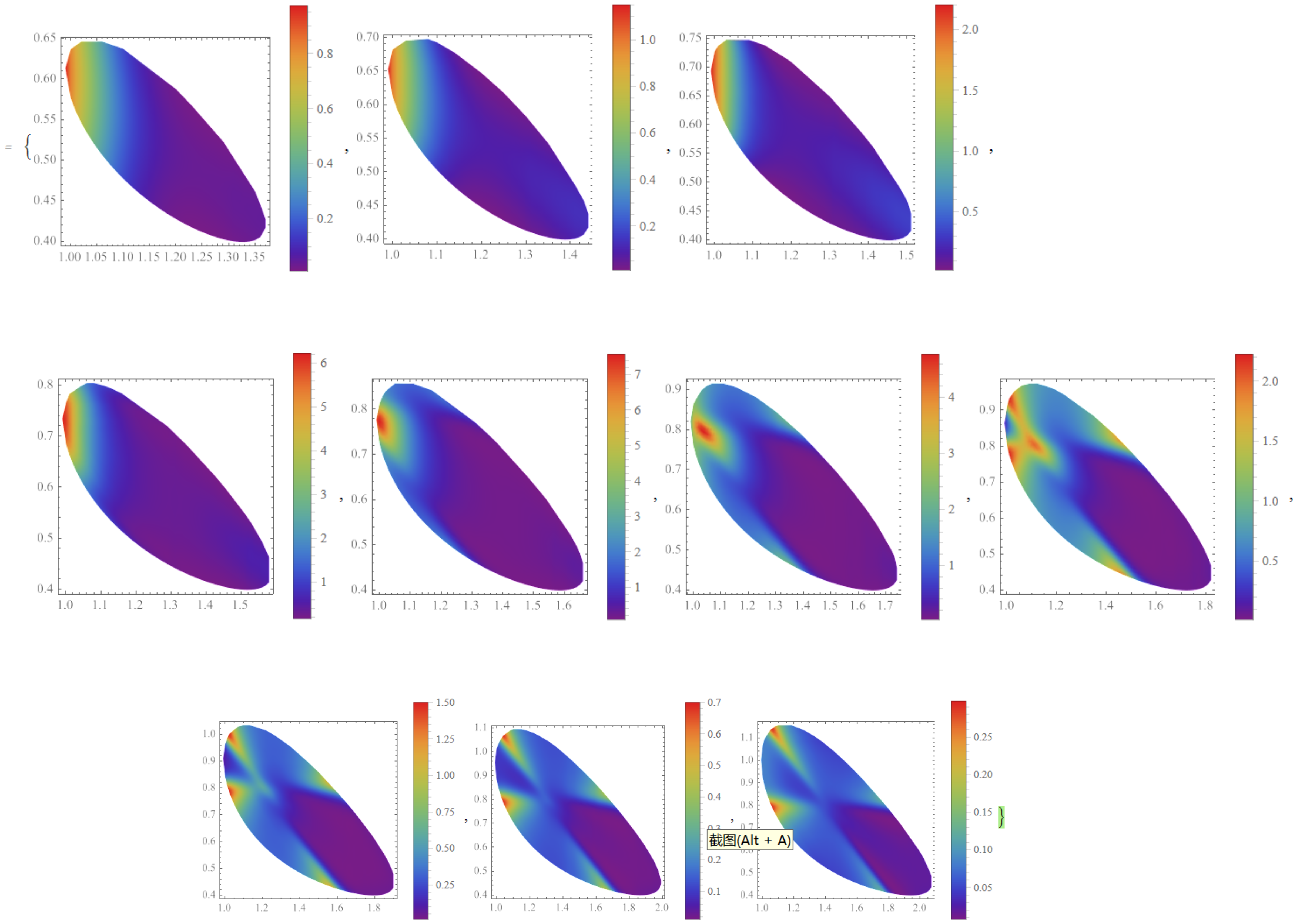
- The comprehensive understanding of those states relies on the inclusions of more robust experimental data (upcoming) and more fundamental theories such as χPT (setting up)

Thank you!

Spares

MC fitting in isobaric approach

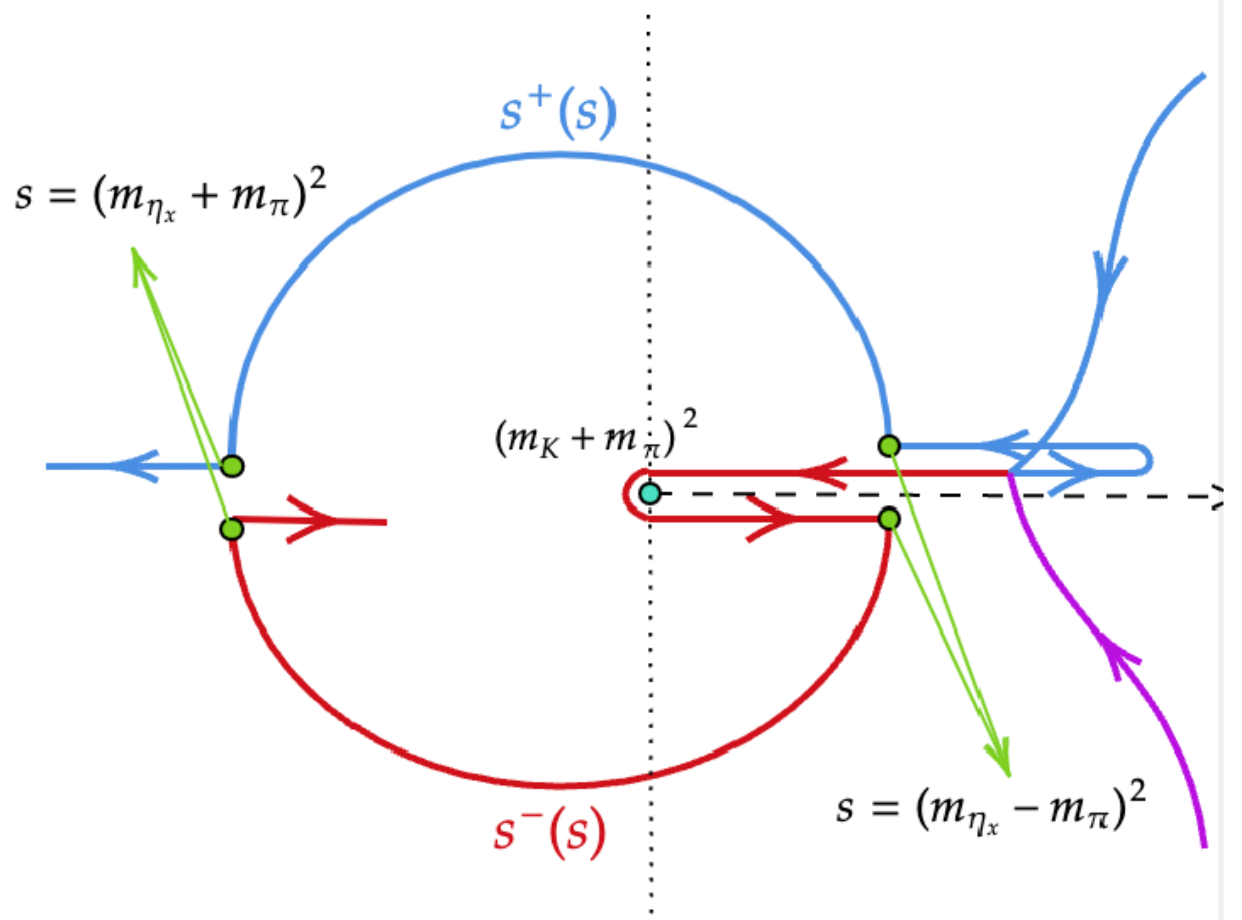
1300~30~1600 MeV



Pseudo-threshold singularity and its nature

$$\kappa_{K\bar{K}}(s) = \frac{\sqrt{\lambda(s,m_K^2,m_K^2)}\sqrt{(m_{\eta_x}-m_\pi)^2-s+i\epsilon}\sqrt{(m_{\eta_x}+m_\pi)^2-s+i\epsilon}}{s}$$

$$\kappa_{\pi K}(t) = \frac{\sqrt{\lambda(t,m_\pi^2,m_K^2)}\sqrt{(m_{\eta_x}-m_K)^2-t+i\epsilon}\sqrt{(m_{\eta_x}+m_K)^2-t+i\epsilon}}{t}$$

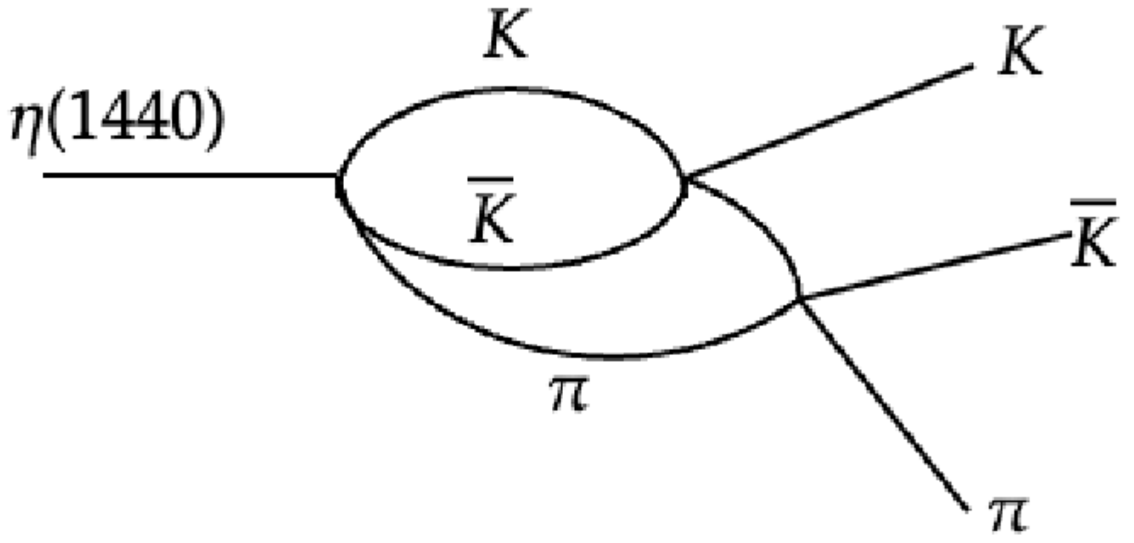


The singular behaviour of $\hat{\mathcal{F}}_J^I(x)$ at pseudo-threshold is $\frac{\tilde{\mathcal{F}}_J^I(x)}{\kappa^{2J+1}(x)} \propto \frac{1}{\sqrt{a_x-x}^{2J+1}}$:

1. manifests both when solving $\mathcal{F}_J^I(x)$ and $\hat{\mathcal{F}}_J^I(x)$
2. S-wave $(J=0)$ $\Rightarrow$ integrable numerically
3. above S-wave $(J>0)$ $\Rightarrow$ very hard to integrate numerically

The integral $H(x) = \frac{x^n}{\pi}\int \frac{dx'}{x'^n}\frac{\hat{\mathcal{F}}(x')\sin\delta(x')}{|\Omega(x')|(x'-x)}$ J.Gasser,NPB850(2011)96-147

1. is finite on physical sheet, i.e., $H(a_x+i\epsilon)$
2. disc $H(a_x) = H(a_x+i\epsilon) - H(a_x-i\epsilon) = \infty$
3. can be evaluated both analytically and numerically



Avoiding the pseudo-threshold singularity

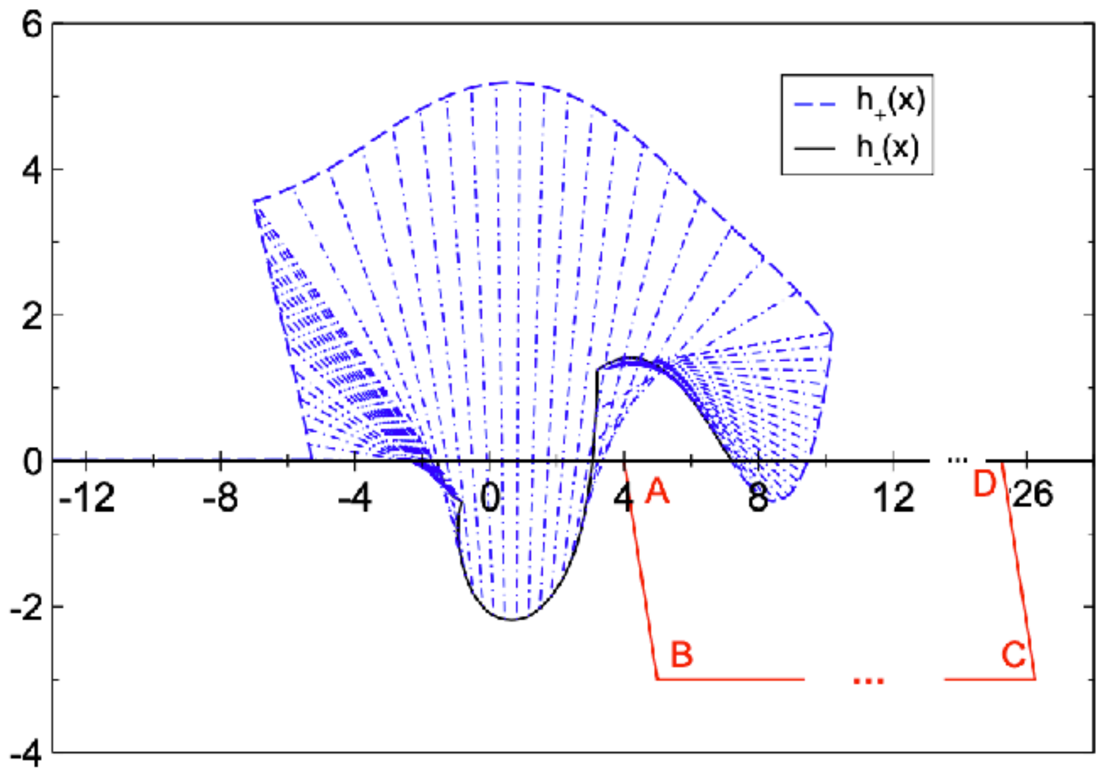
$$H(x+i\epsilon)=\frac{x^n}{\pi}\int\frac{dx'}{x'^n}\frac{\tilde{\mathcal{F}}_J^I(x')}{\kappa^{2J+1}(x')(x'-x-i\epsilon)}\frac{\sin\delta_J^I(x')}{|\Omega_J^I(x')|}$$

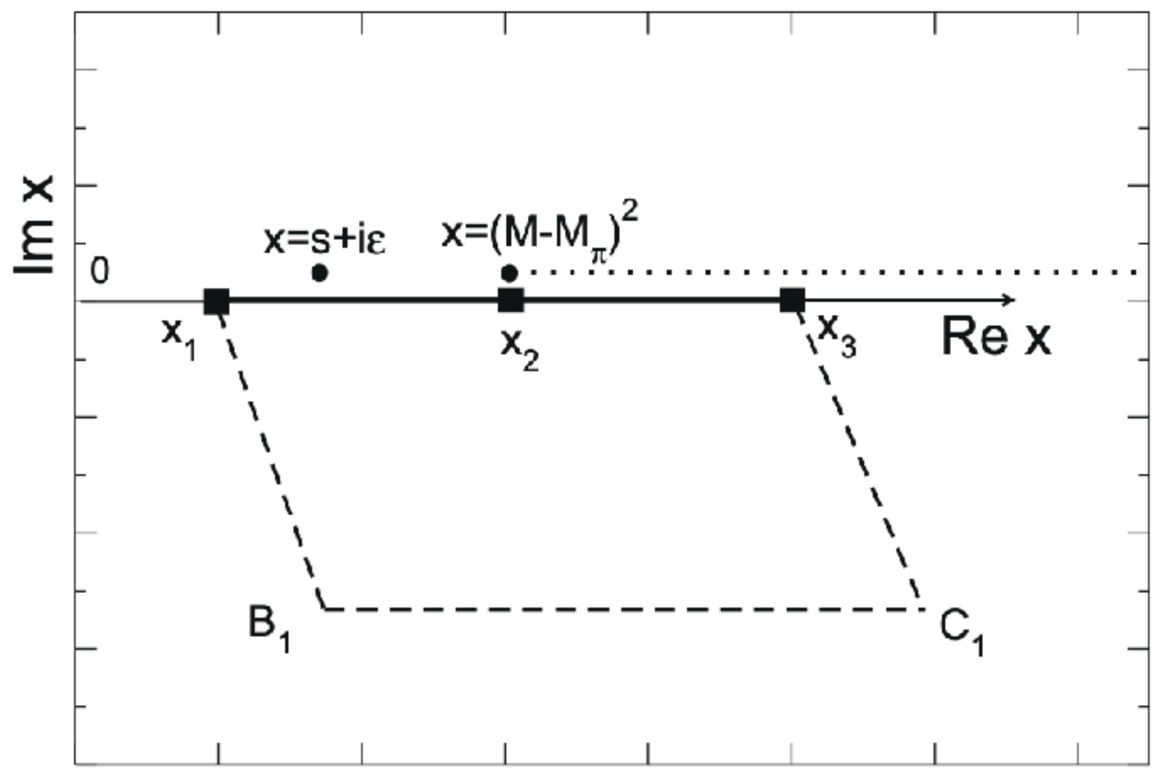
- Analytical approach G.Colangelo et al.,EPJC(2018)78:947

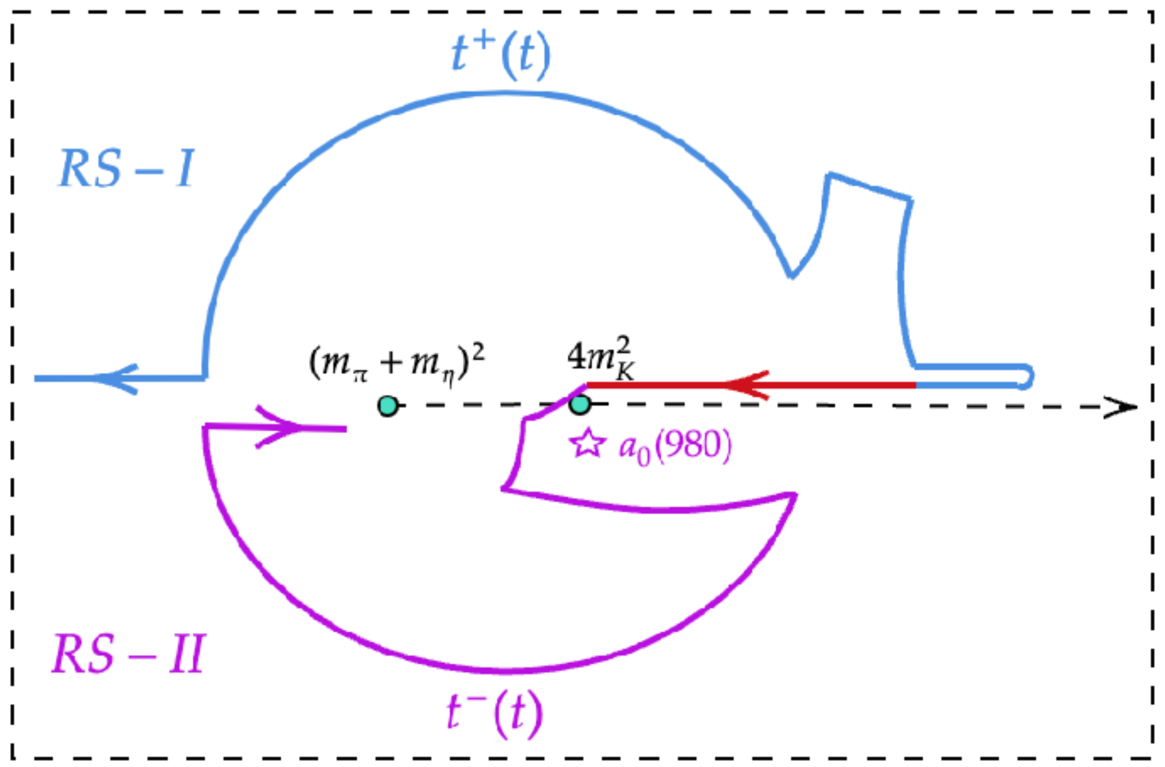
$$\mathcal{M}_1^H(s)=\Omega_1(s)\{\int_{s_1}^{s_3}ds'\frac{\bar{\phi}(s')H_1(s')-h(s')\bar{\phi}(s_2)H_1(s_2)}{(s'-s-i\epsilon)(s_2-s')^{3/2}}+\bar{\phi}(s_2)H_1(s_2)G(s)\}$$

J.Gasser and A.Rusetsky,EPJC(2018)78:906

- Contour deformation without crossing the pole positions (δ_J^I diverges at pole)







- Contour deformation even crossing the pole positions:

$\frac{\sin\delta_J^I(x')}{|\Omega_J^I(x')|}$ is free of the singularities $\Rightarrow$ { the singularity is avoided; integrate on elastic complex region now!

Analytical continuation of $\sin\delta_J^I/|\Omega_J^I|$ (1)

For 2-body elastic scattering,

$$f_l(s)=\frac{e^{i2\delta_l(s)}-1}{2i\sigma(s)}=\frac{1}{\sigma(s)}\cdot\frac{1}{\cot\delta_l(s)-i}$$

with $\cot\delta_l(s)$ real and satisfying Schwartz reflection theorem and can be expanded by conformal polynomials on a certain analytical region.
The S-matrix is then,

$$\hat{S}(s)=\begin{cases}1+2i\sigma f_l(s)=\frac{\cot\delta_l(s)+i}{\cot\delta_l(s)-i}, & \Im s\geq 0\\ [\frac{\cot\delta_l(s^*)+i}{\cot\delta_l(s^*)-i}]^*=\frac{\cot\delta_l(s)-i}{\cot\delta_l(s)+i}, & \Im s<0.\end{cases}$$

By utilizing $\frac{\sin\delta_l(s)}{|\Omega_l(s)|}=\frac{e^{i\delta_l(s)}\sin\delta_l(s)}{\Omega_l(s)}=\frac{1}{\Omega_l(s)}\cdot\frac{1}{\cot\delta_l(s)-i}$ and $\Omega_l^{(II)}(s)=\frac{\Omega^{(I)}(s)}{\hat{S}(s)}$, one derives,

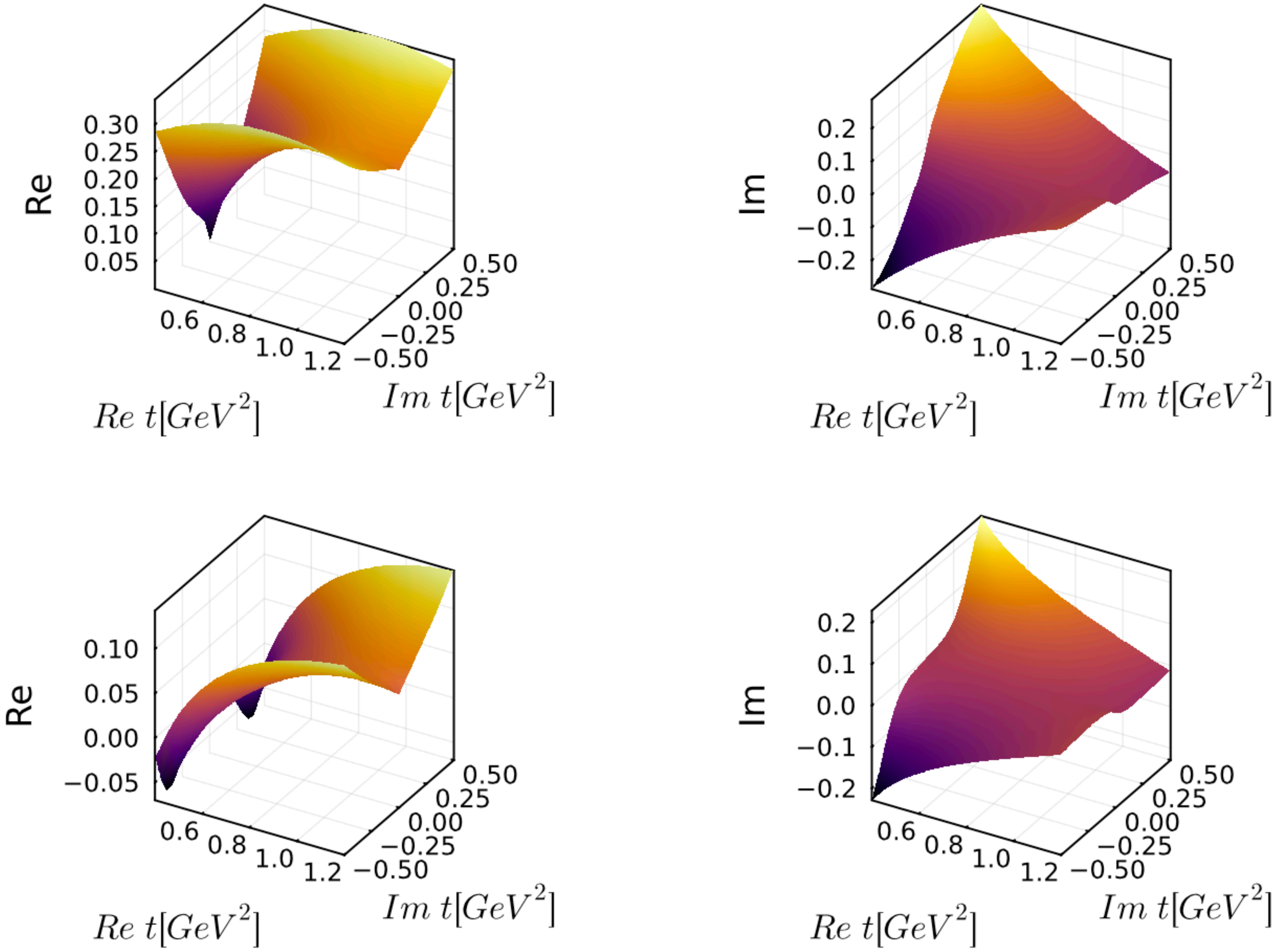
$$\frac{\sin\delta_l(s)}{|\Omega_l(s)|}=\begin{cases}\frac{1}{\Omega_l^{(I)}(s)}\frac{1}{\cot\delta_l(s)-i}, & \Im s\geq 0\\ \frac{1}{\Omega_l^{(I)}(s)}\frac{1}{\cot\delta_l(s)+i}, & \Im s<0\end{cases}.$$

The convention of $\cot\delta_l(s)$ may differentiate from the literature by an extra minus sign on the lower half plane but the conclusion shall not change!

Analytical continuation of $\sin\delta_I^I / |\Omega_I^I|$ *(2)*

The complex function $\dfrac{\sin\delta_{0,1}^{1/2}(s)}{|\Omega_{0,1}^{1/2}(s)|}$ of $K\pi$ scatterings are plotted below,

$$\frac{\sin\delta_{0,1}^{1/2}(t)}{|\Omega_{0,1}^{1/2}(t)|}$$



The dispersive integral on any deformed integral-path on the lower half plane (<u>even crossing the pole position</u>) has been checked to be consistent with that integrated from the real axis!