FB23

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Constraining the hidden-charm pentaquarks prediction and discriminating the spins of Pc(4440) and Pc(4457) through Effective Range Expansion

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Outlines

1. Introduction of exotic hadrons and Pc states

2. Effective range expansion and Weinberg compositeness

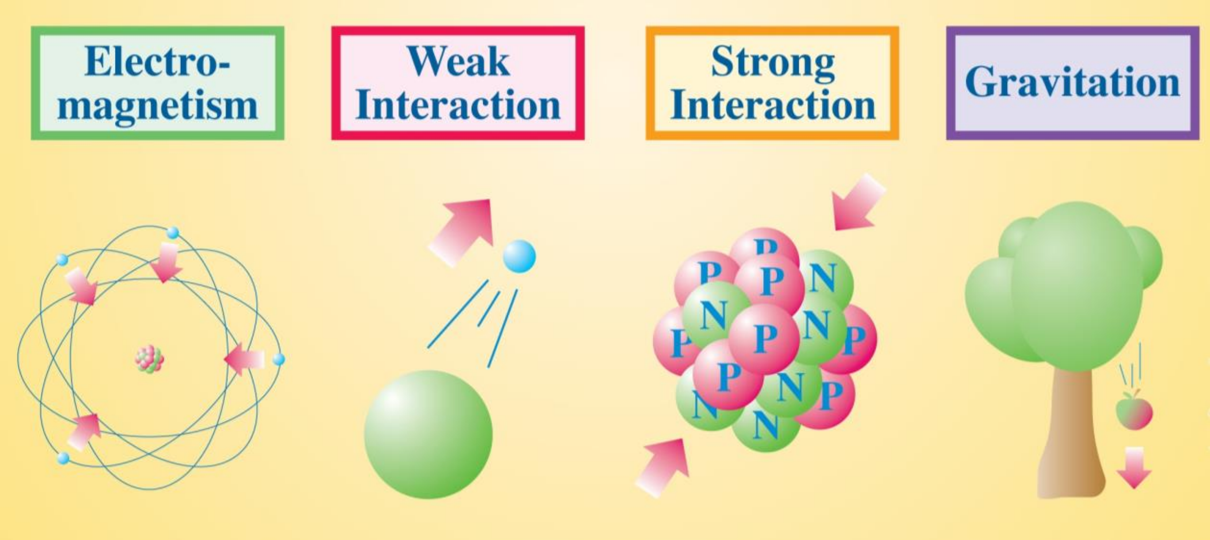
3. Molecule descriptions with contact effective field theory

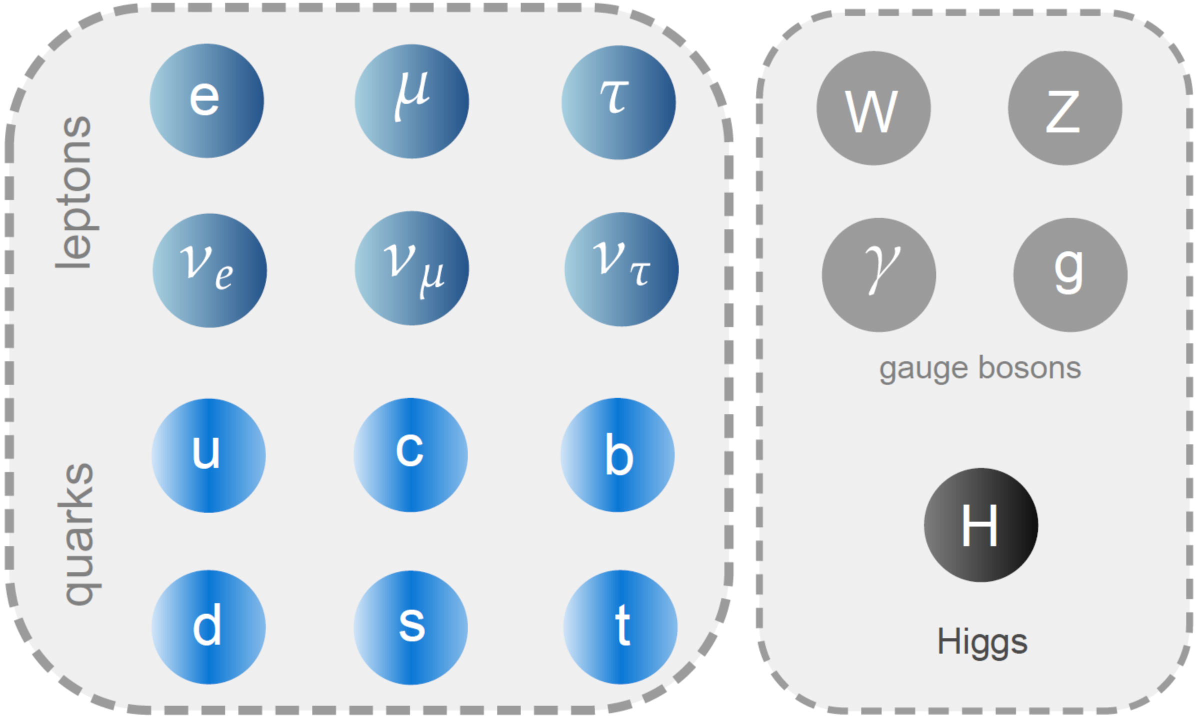
4. Results of the predicted effective range and scattering length

5. Summary and discussion

QCD and exotic hadrons

- **Quantum chromodynamics (QCD) --- $SU(3)_c$ gauge symmetry**





$$\mathcal{L} = \sum_{i,j=1}^{N_f} \bar{\psi}_i (i\gamma^\mu D_{\mu ij} - m_{ij})\psi_j - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

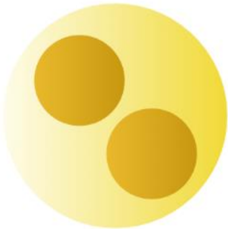
1. Asymptotic freedom

2. Quark confinement

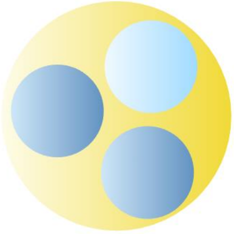
Matter made by quarks and gluons should be color singlet !

Exotic hadrons

Traditional hadrons

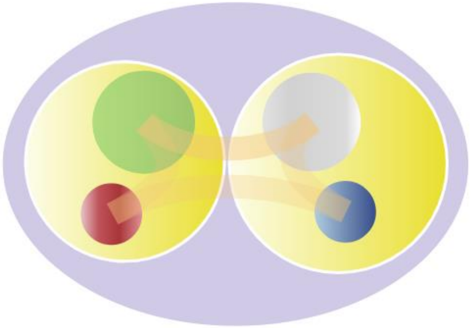


mesons

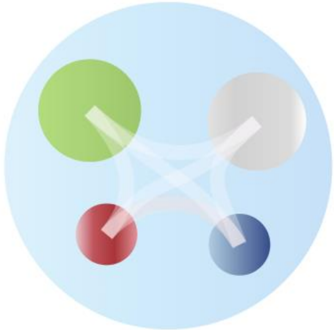


baryons

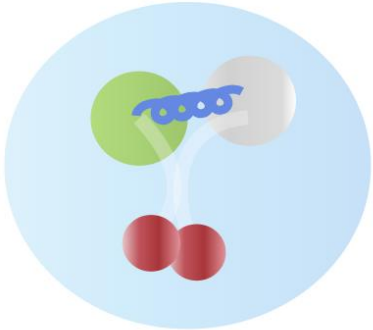
Exotic hadrons



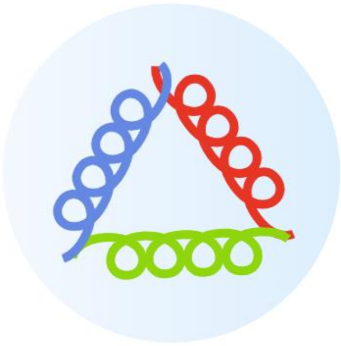
molecules



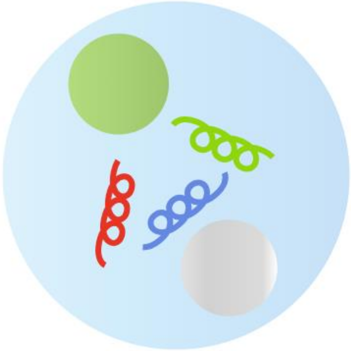
compact quarks



quarkonium
+ light clouds



glueballs



hybrids

Tetraquarks $q\bar{q}q\bar{q}$

Pentaquarks $qqqq\bar{q}$

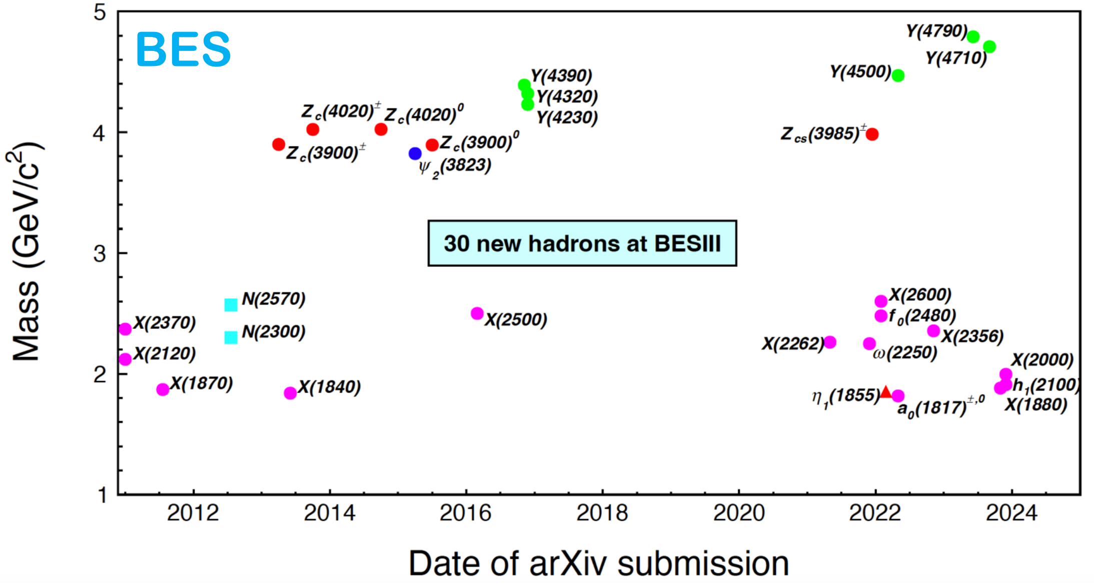
Hexaquarks $q\bar{q}q\bar{q}q\bar{q}$

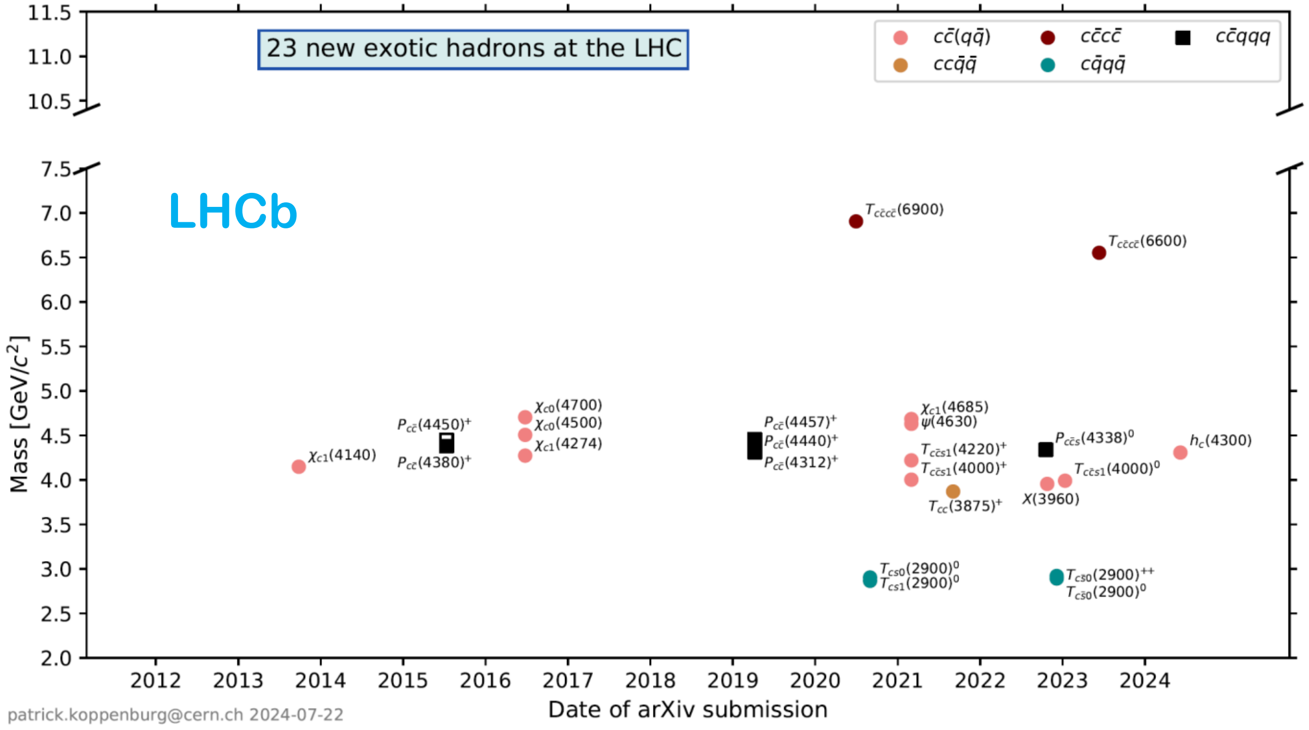
$qqqqqq$

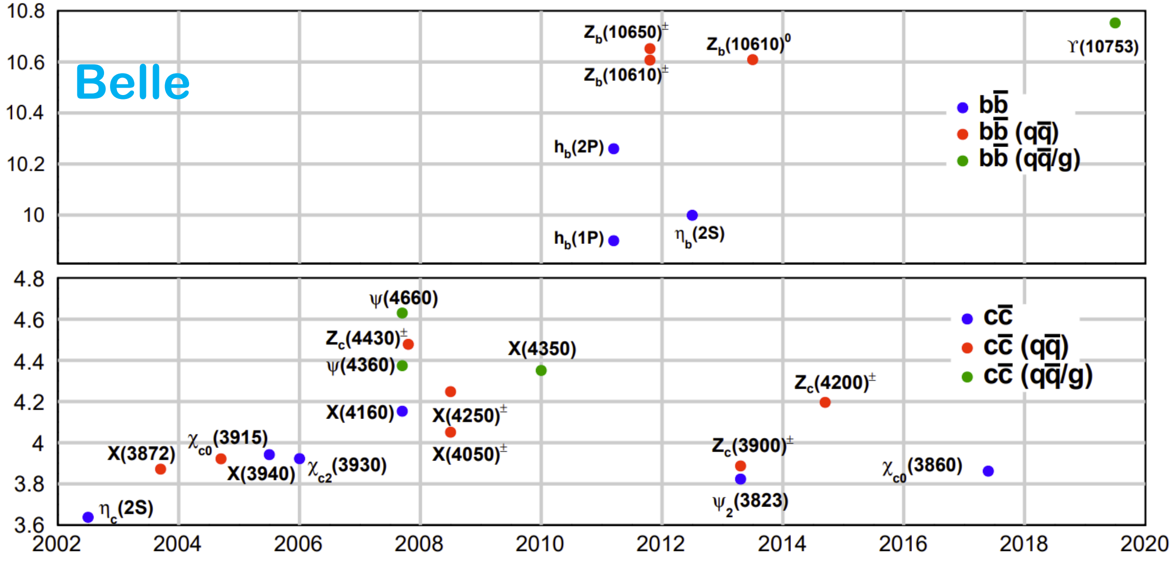
…

Exotic hadrons

- **Many new exotic hadrons discovered by BES, Belle, LHCb …**





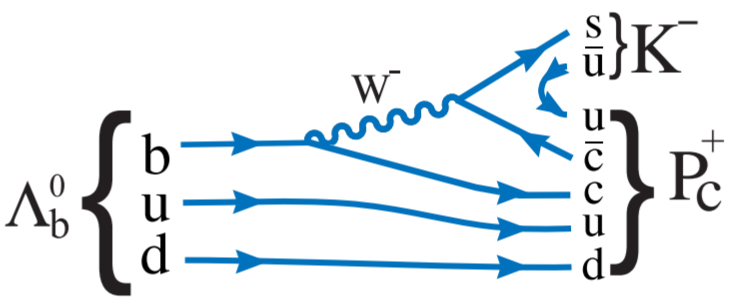


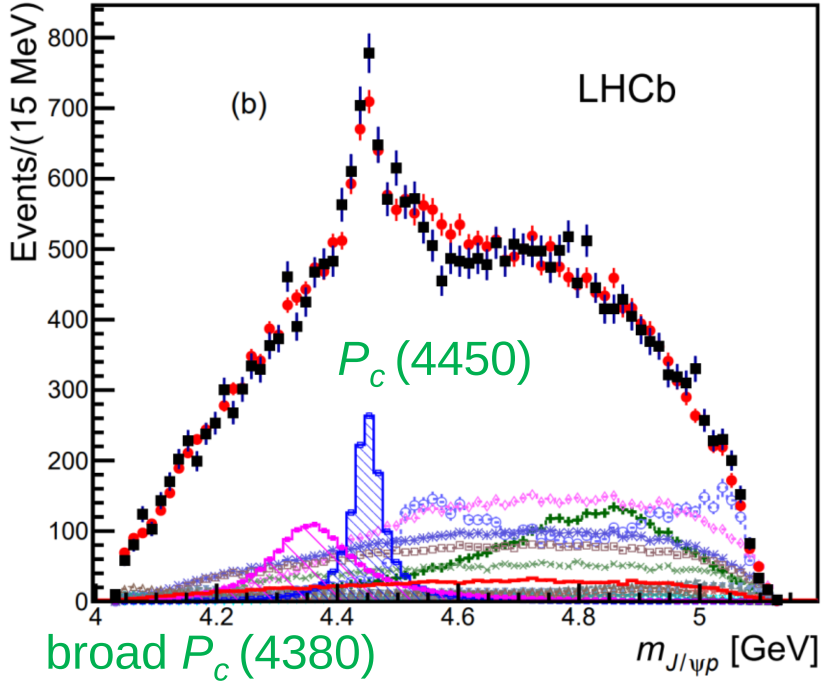
http://english.ihep.cas.cn/bes/re/pu/NewParticles/
Chinese Physics Letters 40, 121301; Cheng-Ping Shen et. al., 2023
https://www.nikhef.nl/~pkoppenb/particles.html

Pentaquarks and their HQSS partners

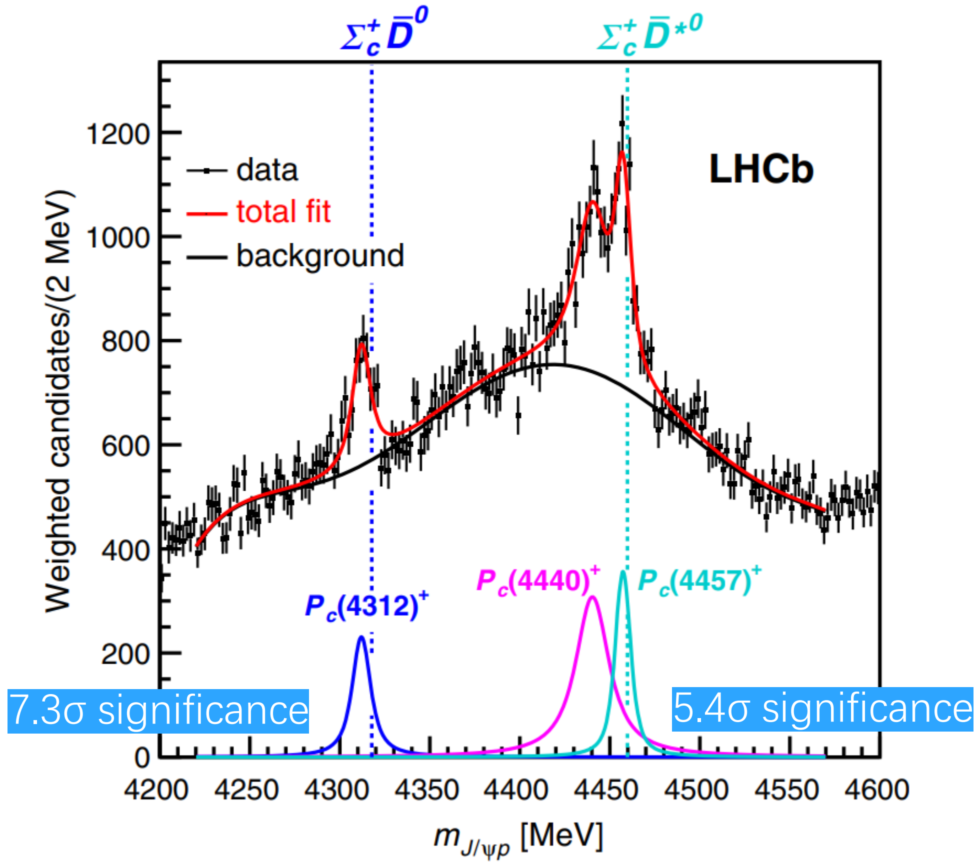
[PhysRevLett. 115,072001, LHCb Collaboration, 2015]

LHCb : $\Lambda_b^0 \to J/\Psi p K^-$





[PhysRevLett. 122,222001, LHCb Collaboration, 2019]



$$
\begin{aligned}
m_{P_{c1}} &= 4311.9 \pm 0.7^{+6.8}_{-0.6}, & \Gamma_{P_{c1}} &= 9.8 \pm 2.7^{+3.7}_{-4.5} \\
m_{P_{c2}} &= 4440.3 \pm 1.3^{+4.1}_{-4.7}, & \Gamma_{P_{c2}} &= 20.6 \pm 4.9^{+8.7}_{-10.1} \\
m_{P_{c3}} &= 4457.3 \pm 0.6^{+4.1}_{-1.7}, & \Gamma_{P_{c3}} &= 6.4 \pm 2.0^{+5.7}_{-1.9}
\end{aligned}
$$

Pentaquarks and their HQSS partners

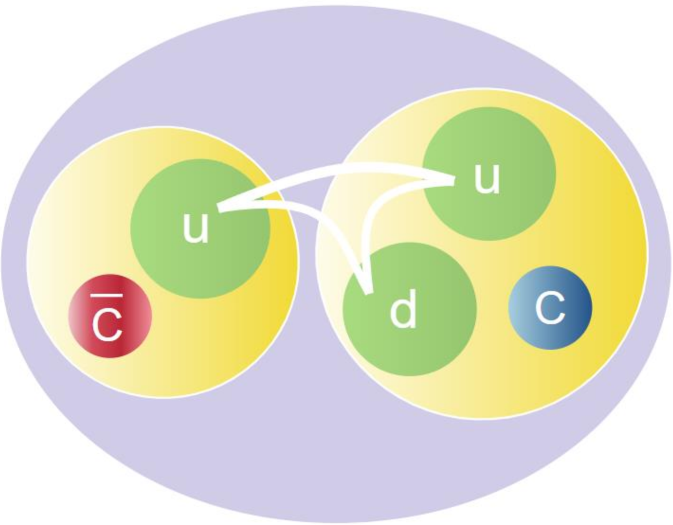
- **Molecular description**

[J.-J. Wu, R. Molina, E. Oset, and B. S. Zou, Phys. Rev. Lett.105, 232001, 2010]
[W. L. Wang, F. Huang, Z. Y. Zhang, and B. S. Zou, Phys. Rev.C 84, 015203, 2011]
[C. W. Xiao, J. Nieves, and E. Oset, Phys. Rev. D 88, 056012, 2013]
[T. Uchino, W.-H. Liang, and E. Oset, Eur. Phys. J. A 52, 43, 2016]
[M.-L. Du, V. Baru, F.-K. Guo, C. Hanhart, U.-G. Meisner, J. A.Oller, and Q. Wang, JHEP 08, 157, 2021]

Since the masses of the above pentaquarks are close to the thresholds, it's natural to regard them as meson-baryon molecules

- **heavy quark spin symmetry (HQSS)**

$\overline{D}^{(*)}\ \Sigma_c^{(*)}$



Heavy mesons / **Heavy baryons**

$$H_c = \frac{1}{\sqrt{2}}[D + \vec{D}^* \cdot \vec{\sigma}] \qquad \vec{S}_c = \frac{1}{\sqrt{3}}\vec{\sigma}\Sigma_c + \vec{\Sigma}_c^*$$

$$\mathcal{L} = C_a \vec{S}_c^\dagger \cdot \vec{S}_c \mathrm{Tr}[\bar{H}_c^\dagger \bar{H}_c] + C_b \sum_{i=1}^{3} \vec{S}_c^\dagger \cdot (J_i \vec{S}_c) \mathrm{Tr}[\bar{H}_c^\dagger \sigma_i \bar{H}_c]$$

$$V = C_a + C_b\ \vec{\sigma}_L \cdot \vec{J}_L$$

Pentaquarks and their HQSS partners

Potentials

Molecule	J^P	V
$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	C_a
$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	C_a
$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	$C_a - \frac{4}{3}C_b$
$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	$C_a + \frac{2}{3}C_b$
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	$C_a - \frac{5}{3}C_b$
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	$C_a - \frac{2}{3}C_b$
$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	$C_a + C_b$

Scenario A

Pc (4440) -- $J = \frac{1}{2}$ $\bar{D}^*\Sigma_c$

Pc (4457) -- $J = \frac{3}{2}$ $\bar{D}^*\Sigma_c$

Scenario B

Pc (4440) -- $J = \frac{3}{2}$ $\bar{D}^*\Sigma_c$

Pc (4457) -- $J = \frac{1}{2}$ $\bar{D}^*\Sigma_c$

$$T = V + VGT$$

$$\Rightarrow \quad T = \frac{V}{1 - VG}$$

nonrelativistic propagator

$$G = \int \frac{d^3q}{(2\pi)^3} \frac{1}{B + \frac{q^2}{2\mu}} \exp[2(\frac{q}{\Lambda})^2]$$

$\Lambda \quad 0.5 - 1$ GeV

Results :

Scenario	Molecule	J^P	B (MeV)	M (MeV)
A	$\bar{D}\Sigma_c$	$(1/2)^-$	7.8–9.0	4311.8–4313.0
A	$\bar{D}\Sigma_c^*$	$(3/2)^-$	8.3–9.2	4376.1–4377.0
A	$\bar{D}^*\Sigma_c$	$(1/2)^-$	Input	4440.3
A	$\bar{D}^*\Sigma_c$	$(3/2)^-$	Input	4457.3
A	$\bar{D}^*\Sigma_c^*$	$(1/2)^-$	25.7–26.5	4500.2–4501.0
A	$\bar{D}^*\Sigma_c^*$	$(3/2)^-$	15.9–16.1	4510.6–4510.8
A	$\bar{D}^*\Sigma_c^*$	$(5/2)^-$	3.2–3.5	4523.3–4523.6
B	$\bar{D}\Sigma_c$	$(1/2)^-$	13.1–14.5	4306.3–4307.7
B	$\bar{D}\Sigma_c^*$	$(3/2)^-$	13.6–14.8	4370.5–4371.7
B	$\bar{D}^*\Sigma_c$	$(1/2)^-$	Input	4457.3
B	$\bar{D}^*\Sigma_c$	$(3/2)^-$	Input	4440.3
B	$\bar{D}^*\Sigma_c^*$	$(1/2)^-$	3.1–3.5	4523.2–4523.6
B	$\bar{D}^*\Sigma_c^*$	$(3/2)^-$	10.1–10.2	4516.5–4516.6
B	$\bar{D}^*\Sigma_c^*$	$(5/2)^-$	25.7–26.5	4500.2–4501.0

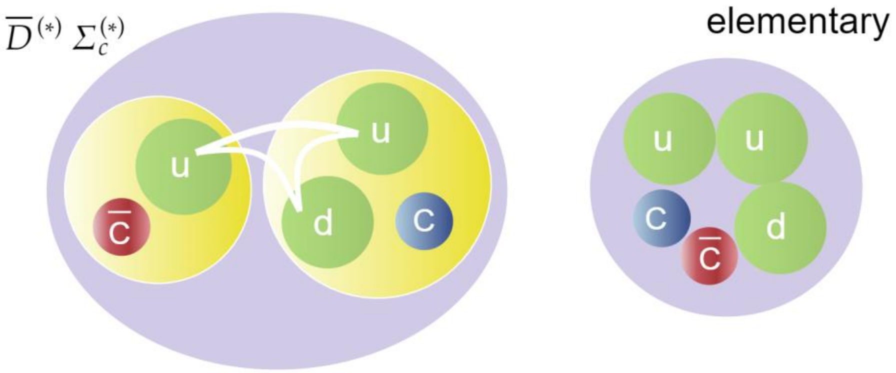
Weinberg compositness criterion

- **Many works have been done to study the spin problem of Pc(4440) and Pc(4457)**

  - mass spectrum and decays
  - production & line shape
  - one boson exchange model
  - machine learning on line shape
  - femtoscopic correlation functions
  - ...

[R. Chen, Z.F. Sun, X. Liu, and S.L. Zhu, Phys. Rev. D 100, 011502(R), 2019]
[M.-L. Du, V. Baru, F.-K. Guo, C. Hanhart, U.-G. Meisner, J. A. Oller, and Q. Wang, Phys. Rev. Lett. 124, 072001, 2020]
[M.Z. Liu, L.S. Geng, M.V. Valderrama, J.J. Xie, Phys. Rev. D 103, 054004, 2021]
[N. Yalikun, Y.H. Lin, F.K. Guo, Y. Kamiya, and B.S. Zou, Phys. Rev. D 104, 094039, 2021]
[Z.Y. Zhang, J.H. Liu, J.F. Hu, Q. Wang, U.-G. Meißner, j.scib.2023.04.018., 2023]
...

To determine which scenario of the Pc(4440) and Pc(4457) spins should be correct under two-body hadronic molecular $\bar{D}^{*}\Sigma_c$ description, it's helpful to reconsider the nature of "molecule" or "composite" picture.



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Effective range expansion

- **Low energy effective range expansion**

[H. W. Hammer, S. K¨onig, and U. van Kolck, Rev. Mod. Phys.92, 025004, 2020]
[Baru, and X.K. Dong, M.L. Du, Filin, F.K. Guo, Hanhart, Nefediev, Nieves and Q. Wang, j.physletb.2022.137290, 2022]

$$p^{2l+1}\cot\delta_l = -\frac{1}{a_l} + \frac{1}{2}r_l p^2 + \cdots \qquad \mathcal{T} = \frac{2\pi}{\mu}\frac{1}{p\cot\delta_0 - ip} = \frac{2\pi}{\mu}\frac{1}{-\frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \cdots}$$

- **Weinberg compositeness**

[Y. Li, F.K. Guo, J.Y. Pang, and J.J. Wu, Phys.Rev.D 105. 7, L071502, 2022]

$$g \simeq \langle h_1 h_2(\mathbf{k})|\, V\, |\Phi\rangle$$

$$|\Phi\rangle = \sqrt{Z}\,|\phi\rangle + \int \frac{d^3\mathbf{k}}{(2\pi)^3}\lambda(\mathbf{k})\,|h_1 h_2(\mathbf{k})\rangle$$

$$1 - Z = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{\langle h_1 h_2(\mathbf{k})|\, V\, |\Phi\rangle}{(E_B + \frac{\mathbf{k}^2}{2\mu})^2}(1 + \mathcal{O}(\frac{\mathbf{k}^2}{\beta^2}))$$

$$a_0 = 2\frac{(1-Z)}{\gamma(2-Z)} + \mathcal{O}(\frac{1}{m_{ex}}),$$
$$r_0 = -\frac{Z}{\gamma(1-Z)} + \mathcal{O}(\frac{1}{m_{ex}}).$$

Weinberg compositness criterion

- Weinberg compositness criterion was originally proposed on studying deuteron to discriminate between elementary particle and composite molecular state

Evidence That the Deuteron Is Not an Elementary Particle*

STEVEN WEINBERG†
Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 30 September 1964)

$$a_0 = 2\frac{(1-Z)}{\gamma(2-Z)} + \mathcal{O}(\frac{1}{m_{ex}}),$$
$$r_0 = -\frac{Z}{\gamma(1-Z)} + \mathcal{O}(\frac{1}{m_{ex}}).$$

→

$B = -2.22$ MeV, $\frac{1}{\gamma} = 4.31$ fm,
$a_0 = -5.42$ fm, $r_0 = 1.77$ fm

Z & 1-Z are sensitive to a and r due to the pole of r=a/2

$$1 - Z = \sqrt{\frac{a}{a+2r}} =: X$$

1-Z= 1.68 > 1 **?**

$$1 - Z = \sqrt{\frac{1}{1+2|\frac{r_0}{a_0}|}}$$

[Matuschek, V. Baru, F.-K. Guo, and C. Hanhart, Eur. Phys. J. A 57, 101, 2021]
[Y. Li, F.K. Guo, J.Y. Pang, and J.J. Wu, Phys.Rev.D 105. 7, L071502, 2022]
[Y.-B. Shen, M.-Z. Liu, Z.-W. Liu, and L.-S. Geng, 2024]

Scattering length &Effective range

[Landau and Lifshits, Quantum Mechanics: Non-Relativistic Theory, Course of Theoretical Physics, Vol. v.3]
[A. Esposito, L. Maiani, A. Pilloni, A. D. Polosa, and V. Riquer, Phys. Rev. D 105, L031503, 2022]
[vanKolck, arxiv: 2209.08432, 2022]

- **Weinberg compositeness**

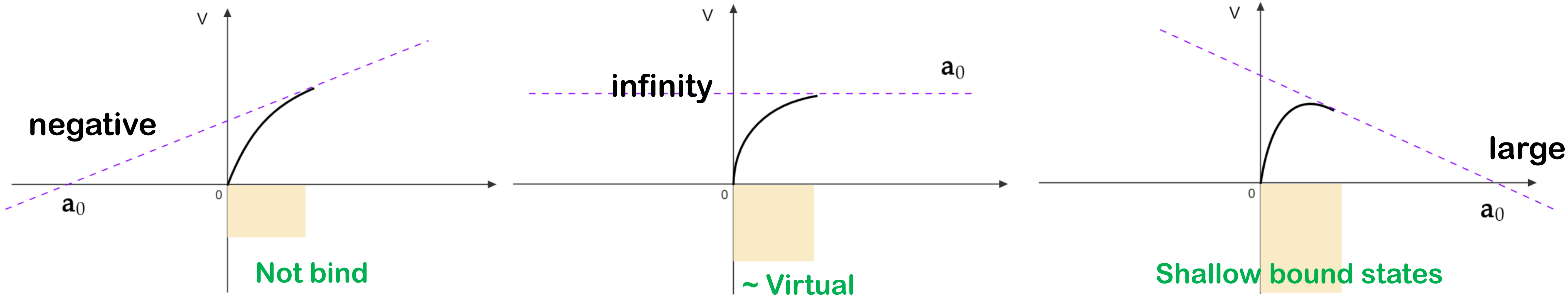
$$a_0 = 2\frac{(1-Z)}{\gamma(2-Z)} + \mathcal{O}(\frac{1}{m_{ex}}),$$
$$r_0 = -\frac{Z}{\gamma(1-Z)} + \mathcal{O}(\frac{1}{m_{ex}}).$$

$$Z = 0$$

the effective range $r_0 > 0$ is positive with the value around $O\left(\frac{1}{m_{ex}}\right)$, while $a_0 \sim \frac{1}{\gamma} + O\left(\frac{1}{m_{ex}}\right) \gg O\left(\frac{1}{m_{ex}}\right)$

- **The unnatural large scattering length of a0 can be easily verified from the emergence of shallow bound states**

e.g. toy model with square potential $\phi(x) \sim \sin(kx+\delta_0) \sim 1 - \frac{x}{a_0}$



Scattering length &Effective range

- **Weinberg compositeness**

[Landau and Lifshits, Quantum Mechanics: Non-Relativistic Theory, Course of Theoretical Physics, Vol. v.3]
[A. Esposito, L. Maiani, A. Pilloni, A. D. Polosa, and V. Riquer, Phys. Rev. D 105, L031503, 2022]
[vanKolck, arxiv: 2209.08432, 2022]

$$a_0 = 2\frac{(1-Z)}{\gamma(2-Z)} + \mathcal{O}(\frac{1}{m_{ex}}),$$
$$r_0 = -\frac{Z}{\gamma(1-Z)} + \mathcal{O}(\frac{1}{m_{ex}}).$$

$$Z = 0$$

the effective range r_0 >0 is positive with the value around $O\left(\frac{1}{m_{ex}}\right)$

- **2 assumptions for *positive* natural r0** $\sim O\left(\frac{1}{m_{ex}}\right)$

□ bound states, not virtual states or resonances

[T. Hyodo, Phys. Rev. Lett. 111, 132002, 2013]
[Matuschek, V. Baru, F.-K. Guo, and C. Hanhart, Eur. Phys. J. A 57, 101, 2021]
[Y. Li, F.K. Guo, J.Y. Pang, and J.J. Wu, Phys.Rev.D 105. 7, L071502, 2022]
[L. Meng, B. Wang, G.-J. Wang, and S.-L. Zhu, Phys. Rept.1019,1, 2023]
[Y.-B. Shen, M.-Z. Liu, Z.-W. Liu, and L.-S. Geng, 2024]

□ pure molecular states with *Z*~ 0, which consistent well with the 3 Pc states

e.g.

pseudo-meson σ and vector mesons of ρ and ω

$$\frac{1}{m_{ex}} < \frac{1-Z}{Z}\gamma$$

m_{ex} ~ 500 MeV, γ ~ 100 MeV

~~$Z \geq 0.2$~~

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Contact effective field theory up to NLO with spins

[E. Epelbaum, W. Glöckleb, U.-G. Meißner, Nuclear Physics A 747, 2005]

[J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, j.nuclphysa.2013.06.008, 2013]

$$\mathcal{L} = \psi^\dagger(i\partial_t + \frac{\nabla^2}{2m})\psi + C_0\psi^\dagger\psi\psi^\dagger\psi + C_2[(\psi^\dagger\psi^\dagger)(\psi\overleftrightarrow{\nabla}^2\psi) + H.c.] + \ldots$$

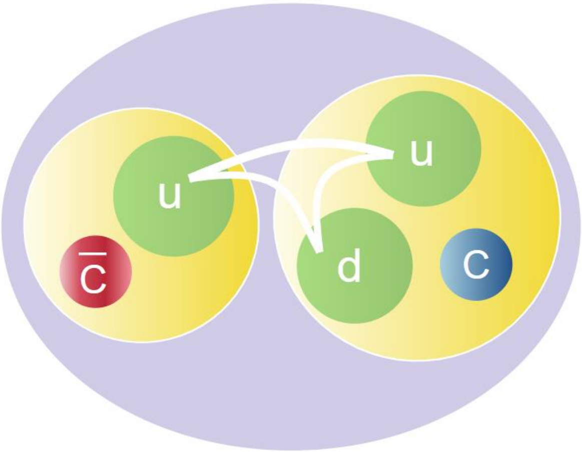
$$\begin{aligned} V = {} & C_0 + C_2^1 q^2 + C_2^2 k^2 + (C_2^3 q^2 + C_2^4 k^2)\vec{\sigma}_{1L}\cdot\vec{S}_{2L} \\ & + \frac{i}{2}C_2^5(\vec{\sigma}_{1L}+\vec{S}_{2L})\cdot(q\times k) + C_2^6(q\cdot\vec{\sigma}_{1L})(q\cdot\vec{S}_{2L}) \\ & + C_2^7(k\cdot\vec{\sigma}_{1L})(k\cdot\vec{S}_{2L}) + \frac{i}{2}C_2^8(\vec{\sigma}_{1L}-\vec{S}_{2L})\cdot(q\times k) \\ & + \cdots \end{aligned}$$

on-shell approximation for momentum

$$p_{cm} = \frac{\sqrt{[s-(m_1+m_2)^2][s-(m_1-m_2)^2]}}{2\sqrt{s}}$$

$$\begin{aligned} V(\bar{D}\Sigma_c) &= C_a + 2D_a p_{cm1}^2\,, \\ V(\bar{D}^*\Sigma_c\,, \tfrac{1}{2}^-) &= C_a - \frac{4}{3}C_b + (2D_a - 2D_b)p_{cm2}^2 \\ V(\bar{D}^*\Sigma_c\,, \tfrac{3}{2}^-) &= C_a + \frac{2}{3}C_b + (2D_a + D_b)p_{cm2}^2 \end{aligned}$$

$\bar{D}^{(*)}\ \Sigma_c^{(*)}$



S-wave Molecules

Ca, Cb, Da, Db

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Matching effective range for Pc states

- **Finding the poles of T to get the couplings**

$$\mathcal{T} = V[1 + V\,G(q,\Lambda) + (V\,G(q,\Lambda))^2 + \cdots]$$

$$G(E_B,\Lambda) = \frac{\mu}{\pi^2}\int_0^{+\infty} dq \frac{q^2}{2\mu E_B + q^2} e^{-\frac{q^2}{\Lambda^2}}$$

r_0 <0 from Wigner bound $\quad r \leq 2\left[R - \frac{R^2}{a} + \frac{R^3}{3a^2}\right]$

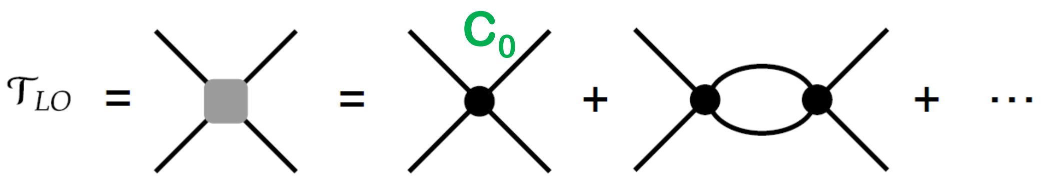
- **Expanding T while $a_0 \sim \frac{1}{\gamma} > \frac{1}{\Lambda} \sim O\left(\frac{1}{m_{ex}}\right)$**

$$\mathcal{T} = \frac{2\pi}{\mu}\frac{1}{p\,\cot\delta_0 - ip} = \frac{2\pi}{\mu}\frac{1}{-\frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \cdots}$$

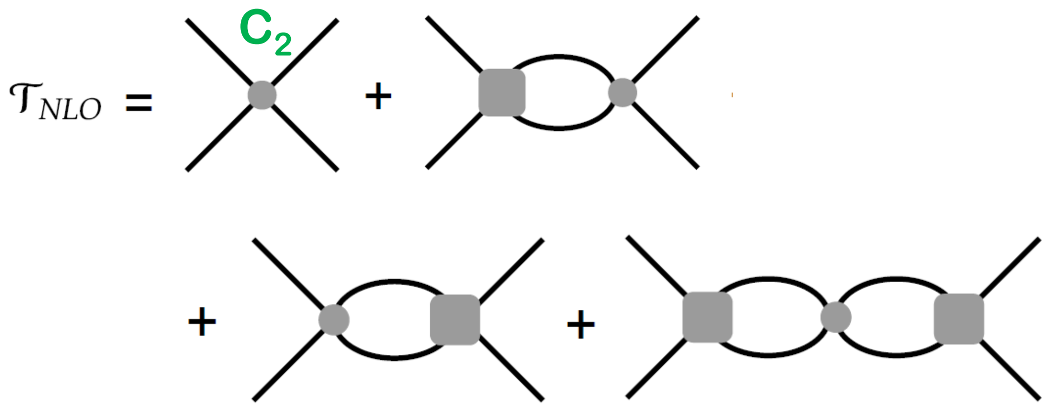
$$\mathcal{T} = -\frac{2\pi}{\mu}\frac{1}{\frac{1}{a_0} + ip}\left[1 + \frac{r_0}{2(\frac{1}{a_0} + ip)}p^2 + \cdots\right]$$

LO NLO

LO from the C_0 interaction



NLO from the C_2 interaction



Matching effective range for Pc states

[D.B. Kaplan, J.M. Savage, and M.B. Wise, Nucl.Phys.B 534, 1998]

$$C_0 = \frac{2\pi}{\mu}\frac{1}{\frac{1}{a_0} - \alpha\Lambda},$$
$$C_2 = \frac{2\pi}{\mu}\frac{1}{(\frac{1}{a_0} - \alpha\Lambda)^2}\frac{r_0}{2}$$

$$\alpha\Lambda = \tfrac{2\pi}{\mu}G(0,\Lambda) \sim 0.4\,\Lambda$$

$$r_0 = \frac{4\pi}{\mu}\frac{C_2}{C_0^2}$$

- **power counting analysis**

[E. Epelbaum, arxiv: 1001.3229, 2010]

LO

$$\mathcal{T} = -\frac{2\pi}{\mu}\frac{1}{\frac{1}{a_0}+ik}\left[1 + \frac{r_0}{2(\frac{1}{a_0}+ik)}k^2 + \frac{r_0^2}{4(\frac{1}{a_0}+ik)^2}k^4 + \cdots\right]$$

NLO

☐ Weinberg power counting

naive dimensional analysis (NDA), dictates a condition for naturalness, suggesting that the dimensionless parameters in OPE should be of O(1).

$$\frac{C_2}{C_0} \sim \Lambda^{-2}$$

☐ KSW counting

$$\frac{C_2}{C_0} \sim \frac{1}{\Lambda(\alpha\Lambda)}$$

$\alpha\Lambda \to \Lambda$

$\alpha\Lambda \to p$

LO, $O(k^{-1})$

NLO, $O(k^0)$

$$R = \Lambda^2\left|\frac{C_2}{C_0}\right| \sim \frac{1}{\alpha} \simeq 2.5$$

Calculations for the prediction

- **What do we expect for the molecular assumption for Pc states ?**

- ☐ **the positive natural effective range** $r_0 \sim O\left(\frac{1}{m_{ex}}\right) \sim 0.5$ **fm**

  short range saturated by pseudo-meson σ and vector mesons of ρ and ω

- ☐ **the unnatural large scattering length** $a_0 \gg O\left(\frac{1}{m_{ex}}\right)$

- ☐ **the reasonable R~2.5 from power counting of the contact field theory**

- **How to deal with 4 couplings with 3 inputs from Pc(4312), Pc(4440) and Pc(4457) ?**

- **Scheme A: neglecting the spin-spin interaction relevant term, namely, setting Db=0**
- **Scheme B: bring in the Pc(4380) discovered by LHCb in 2015, we now have 4 mass inputs of the Pc(4312), Pc(4440), Pc(4457) and Pc(4380) states**
- **Scheme C: the power counting of the low energy couplings in effective field theory can be used to determine the Db term**

Outlines

1. Introduction of exotic hadrons and Pc states
2. Effective range expansion and compositeness
3. Molecule descriptions of Pc states with contact effective field theory
4. Matching effective range with NLO contact potential
5. Results and discussion

Results from Scheme A & B

Scenario	Λ(GeV)	$a_{0P_{c1}}$(fm)	$a_{0P_{c2}}$(fm)	$a_{0P_{c3}}$(fm)	$r_{0P_{c1}}$(fm)	$r_{0P_{c2}}$(fm)	$r_{0P_{c3}}$(fm)	$R_{P_{c1}}$	$R_{P_{c2}}$	$R_{P_{c3}}$
A	0.5	2.09	1.58	2.57	-0.04	-0.02	-0.06	0.3	0.2	0.3
B	0.5	2.88	2.45	3.40	0.61	0.53	0.75	3.0	3.4	2.8
A	1	1.42	-0.34	-0.07	-0.54	-0.31	-0.72	5.3	4.1	6.2
B	1	2.24	1.74	2.75	0.50	0.44	0.58	4.1	4.5	4.0

Scenario A ✗

Pc (4440) -- $J = \frac{1}{2}$ $\bar{D}^*\Sigma_c$

Pc (4457) -- $J = \frac{3}{2}$ $\bar{D}^*\Sigma_c$

Pc (4440) -- $J = \frac{3}{2}$ $\bar{D}^*\Sigma_c$

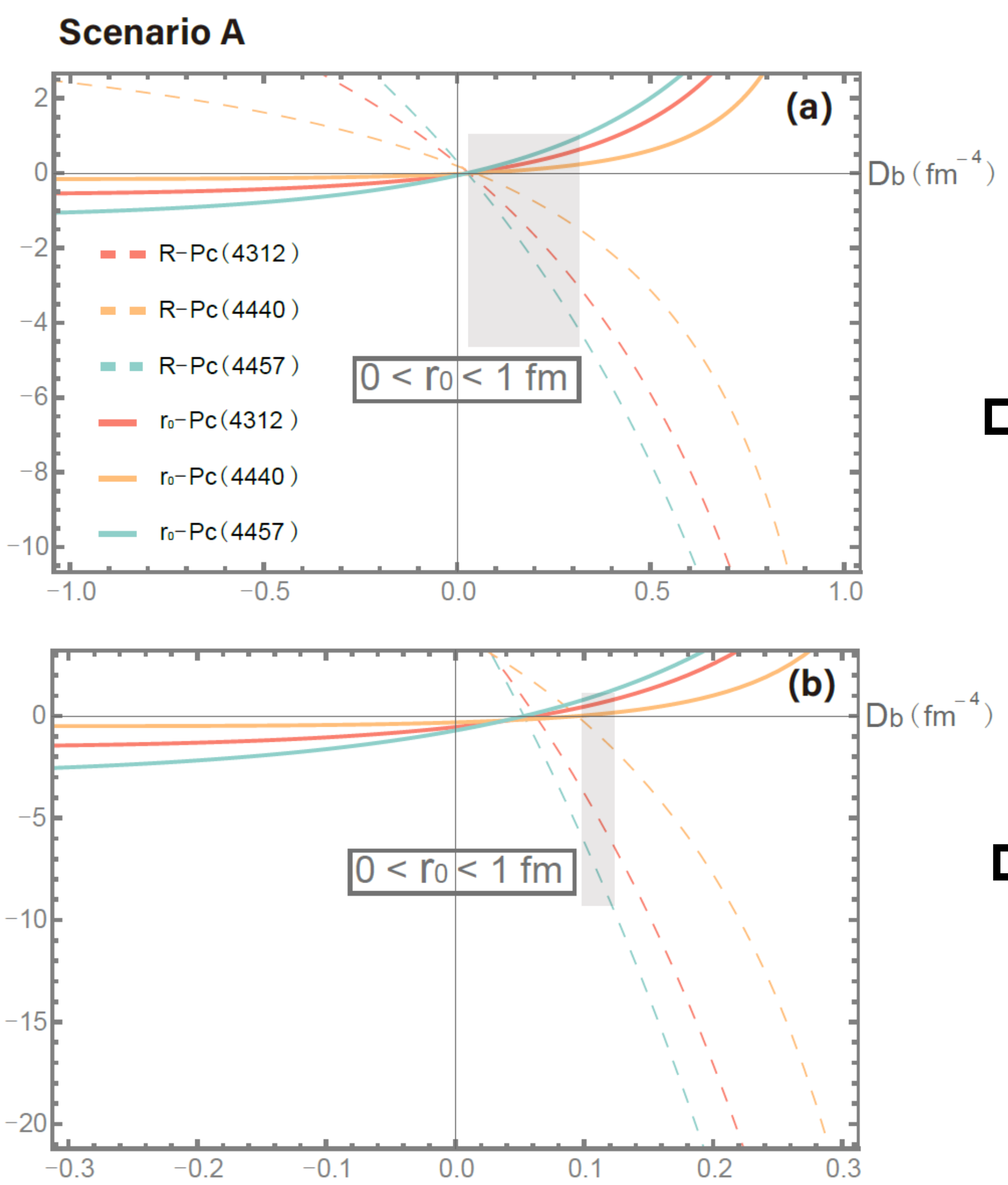
Pc (4457) -- $J = \frac{1}{2}$ $\bar{D}^*\Sigma_c$

Scenario B √

Scenario	Λ(GeV)	$a_{0P_{c1}}$(fm)	$a_{0P_{c2}}$(fm)	$a_{0P_{c3}}$(fm)	$a_{0P_{c4}}$(fm)	$r_{0P_{c1}}$(fm)	$r_{0P_{c2})}$(fm)	$r_{0P_{c3}}$(fm)	$r_{0P_{c4}}$(fm)	$R_{P_{c1}}$	$R_{P_{c2}}$	$R_{P_{c3}}$	$R_{P_{c4}}$
A	0.5	5.03	2.88	6.37	4.81	1.68	0.75	2.28	1.66	6.7	3.6	8.6	6.7
B	0.5	5.01	3.29	32.82	4.81	1.68	0.93	4.10	1.66	6.7	13.5	4.2	6.7
A	1	5.01	2.07	7.00	4.55	1.87	0.65	2.62	1.85	13.2	5.5	17.9	13.2
B	1	5.01	2.54	-27.30	4.55	1.87	0.89	4.62	1.85	13.2	28.9	7.0	13.2

These abnormal a0, r0 and R got from both spin configurations indicate that Pc(4380) might not be suitable to be regarded as a molecular state together with Pc(4312), Pc(4440) and Pc(4457); however …

Results from Scheme C



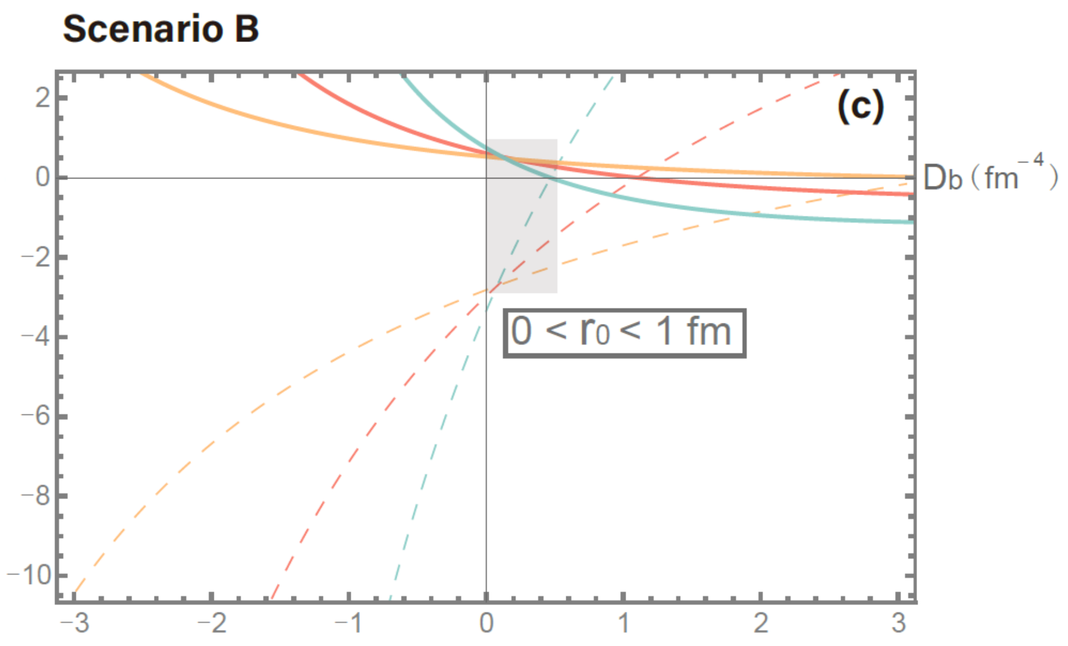
$$N \sim 10$$

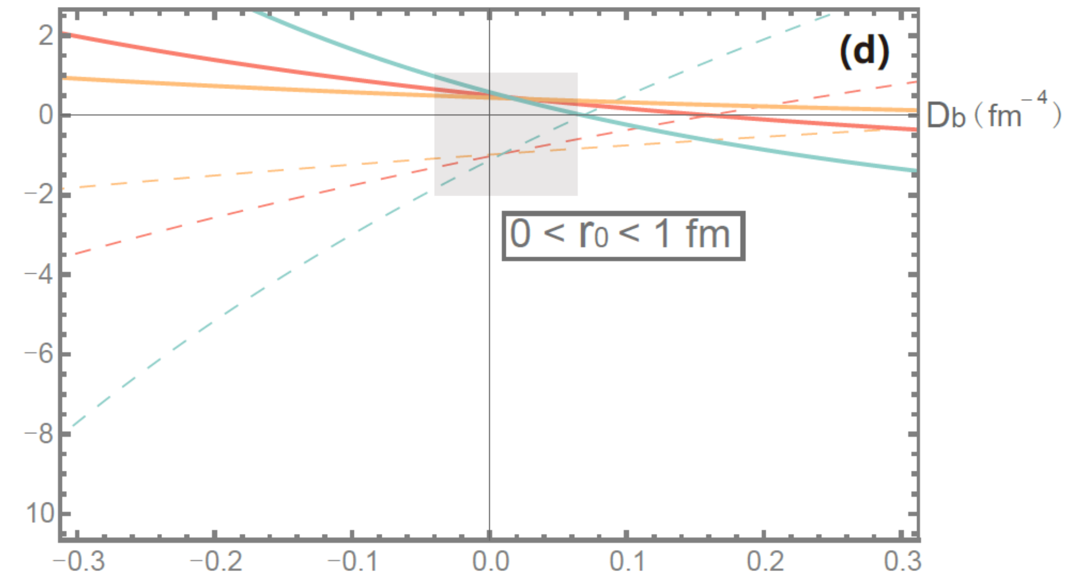
$$D_b \sim N \times \frac{1}{\Lambda^2} \times [-C, C]$$

- For both cutoffs with Λ = 0.5 GeV and Λ =1 GeV, the effective range r0 can not simultaneously exhibit a natural positive value around O(1/m_{ex}) ~ 0.5 fm

- Also, the ratio R for these pentaquarks exceed the natural range of O(1/α) ≃ 2.5 while taking r0 around O(1/m_{ex})

Results from Scheme C

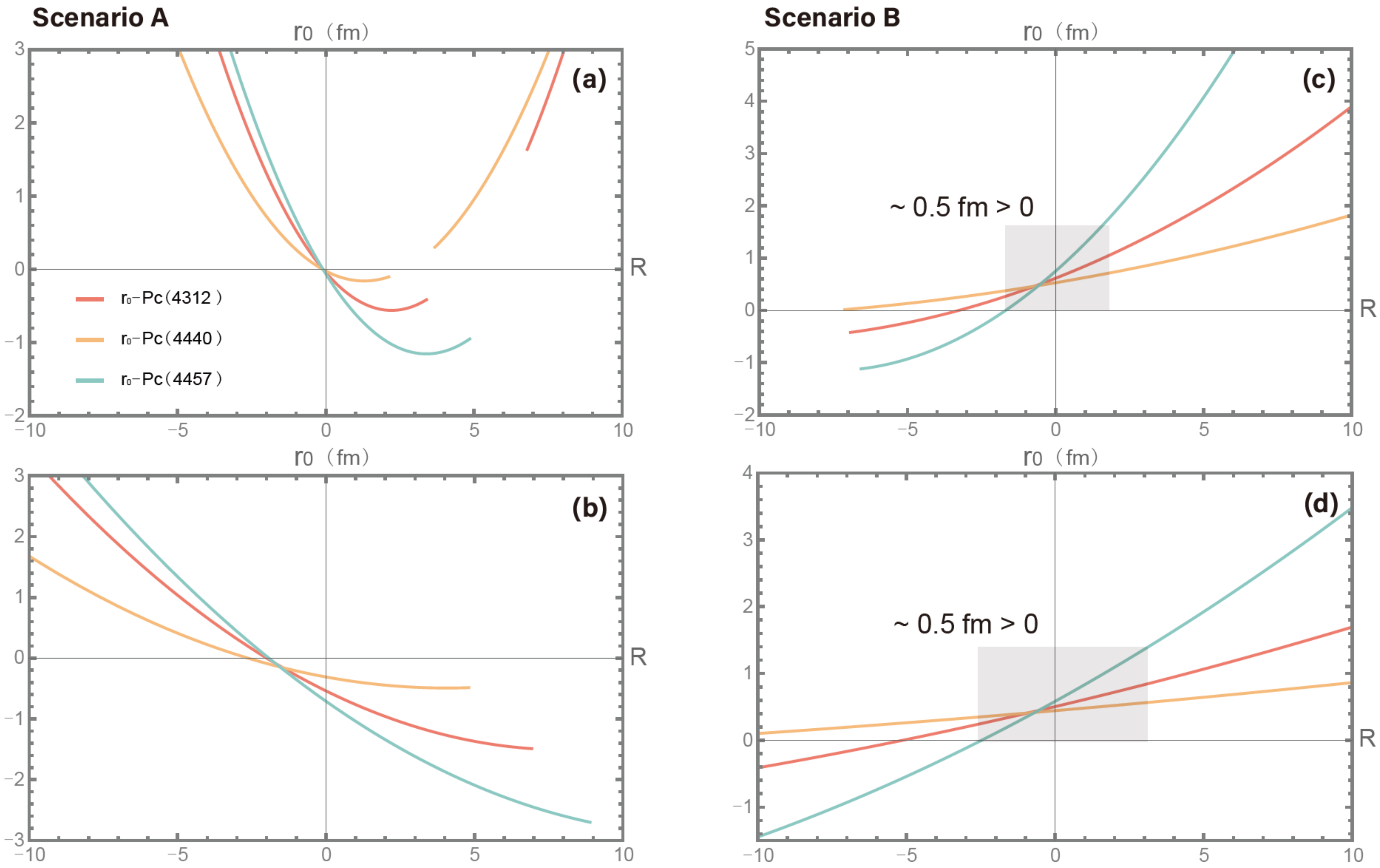




- We get the reasonable R~2.5 for the r_0 ~ O(1/mex)~ 0.5 fm area
- All the 3 pentaquarks get a same r_0 ~ 0.5 fm with the nearly the same R

Results from Scheme C

$$R_b = \Lambda^2 \frac{D_b}{C_0}$$



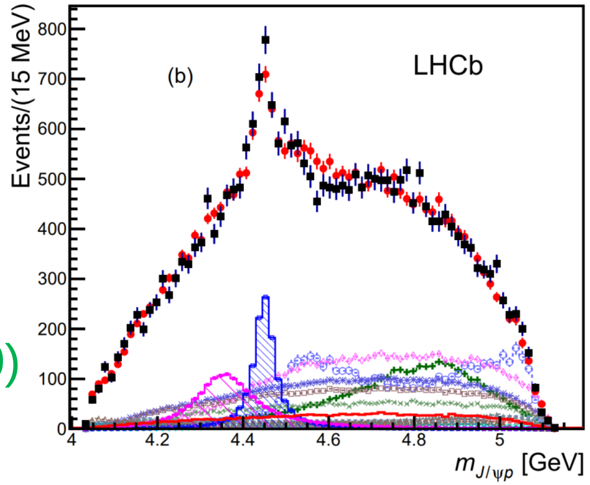
Results from Scheme C

$D_b \sim 0.15\ \text{fm}^{-4}$ and $\sim 0.03\ \text{fm}^{-4}$

Λ(GeV)	$m_{\bar{D}\Sigma_c^*}$(MeV)	$a_{0P_{c1}}$(fm)	$a_{0P_{c2}}$(fm)	$a_{0P_{c3}}$(fm)	$a_{0P_{c4}}$(fm)	$r_{0P_{c1}}$(fm)	$r_{0P_{c2})}$(fm)	$r_{0P_{c3}}$(fm)	$r_{0P_{c4}}$(fm)	$R_{P_{c1}}$	$R_{P_{c2}}$	$R_{P_{c3}}$	$R_{P_{c4}}$
0.5	4375.84	2.73	2.37	3.08	2.67	0.50	0.48	0.46	0.49	2.50	2.63	2.19	2.50
1	4375.6	2.13	1.69	2.52	2.06	0.40	0.41	0.31	0.39	3.31	3.67	2.49	3.31

$$m_{\bar{D}\Sigma_c^*} = 4375.84\ \text{MeV}\ (\Lambda = 0.5\ \text{GeV}),$$
$$m_{\bar{D}\Sigma_c^*} = 4375.6\ \text{MeV}\ (\Lambda = 1\ \text{GeV}).$$

broad $P_c(4380)$



- the broad Pc(4380) cannot yet be excluded as the HQSS molecular partner of Pc(4312), Pc(4440) and Pc(4457)
- meanwhile, the predicted mass of Pc(4380) is constrained to ~ 4375.6 – 4375.84 MeV based on the naturalness from power counting

Conclusions

- To summarize, matching the effective range expansion from scattering amplitude, we conclude it is more natural to assign the quantum numbers $J^P = \frac{3}{2}^-$ to Pc(4440) and $J^P = \frac{1}{2}^-$ to Pc(4457) in molecular $\bar{D}^*\Sigma_c$ states within 3 schemes

- Within the molecular states composed of the same components, the pentaquarks with higher spin might prefer lower mass

- Besides, the results presented from scheme (B) suggest that if the Pc(4380) is to be considered as part of the HQSS molecular system alongside Pc(4312), Pc(4440) and Pc(4457) states, it's mass should be constrained to $\sim$ 4375.6 − 4375.84 MeV based on the naturalness from power counting

- Single-channel & pure molecular bound states have been assumed

- The above findings may hold significant importance for future experimental investigations and further theoretical research on pentaquarks' internal structure

Thank you!