Few-body effects in heavy flavour hadronic molecules

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Hadronic physics before and after 2003

Consensus before 2003:

- Quark model provides a decent description of low-lying hadrons
- Quark model works surprisingly well even for light flavours
- Heavy flavours (c and b) comply with nonrelativistic theory
- Relativistic corrections somewhat improve the description
- Experiment gradually fills "missing states"
- Lattice provides additional/alternative source of information

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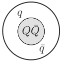
Situation after 2003:

- $X(3872)$ observed by Belle with properties at odds with quark model
- Number of such unconventional hadrons with heavy quarks grows fast
- New branch of hadrons spectroscopy — exotic XYZ states

"Exotic" versus "ordinary"

- "Ordinary" hadron = quark-antiquark mesons or 3-quark baryons
- "Exotic" hadron = not ordinary hadron
- Simplest exotic hadron = tetraquark ($Q\bar{Q}q\bar{q}$)

Compact tetraquarks (bound by confinement)



Hadro-Quarkonium (compact $\bar{Q}Q$ core plus light-quark cloud)

Hadronic molecule (extended object)

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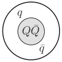

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Molecule = large probability to observe
physical state in hadron-hadron channel

“Exotic” versus “ordinary”

- “Ordinary” hadron = quark-antiquark mesons or 3-quark baryons
- “Exotic” hadron = not ordinary hadron

- $^3S_1$ NN system with $I = 0$:

  Pole on RS-I with $E_B = 2.23$ MeV $\Longrightarrow$ deuteron

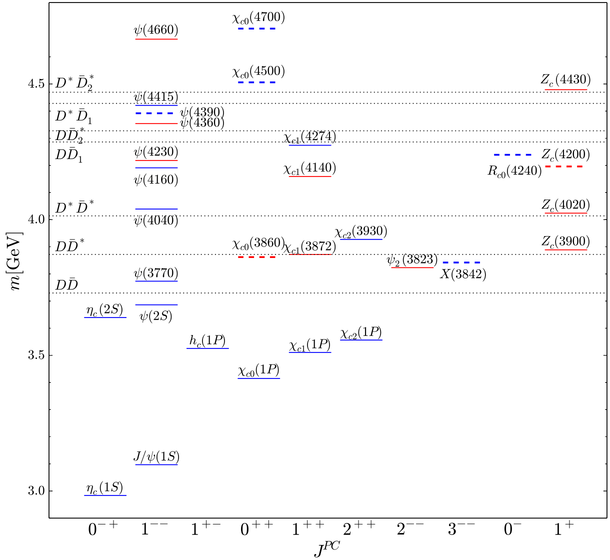
- $^1S_0$ NN system with $I = 1$:

  Pole on RS-II with $E_B = 0.067$ MeV $\Longrightarrow$ virtual state

Hadronic molecule (extended object)

Molecule = large probability to observe physical state in hadron-hadron channel

Spectrum of charmonium



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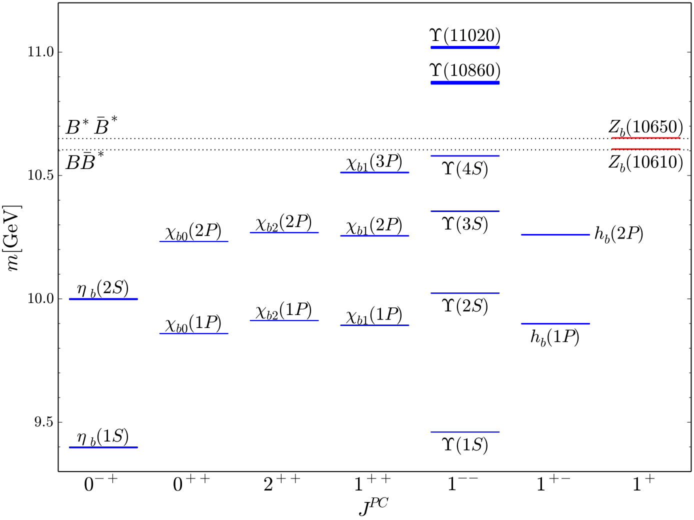
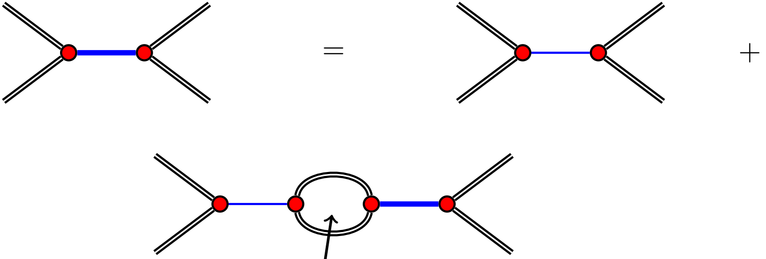
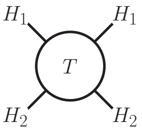


Effect of hadronic loops

$$|X\rangle = \begin{pmatrix} \lambda|\psi\rangle \\ \chi(\boldsymbol{p})|H_1 H_2\rangle \end{pmatrix}$$



Flatté: $$\frac{1}{E - E_f + \frac{i}{2}(g{\color{red}k} + \Gamma_0)} \Longrightarrow$$ parameterised

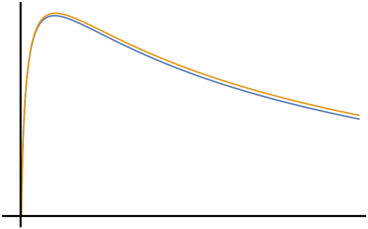


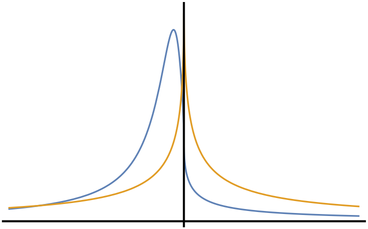
Γ_0 parameterises decay modes not related to $H_1 H_2$ channel

Examples of line shapes

$$\left.\begin{array}{l} k(E) = \sqrt{2\mu E}\,\Theta(E) \\ \kappa(E) = \sqrt{-2\mu E}\,\Theta(-E) \end{array}\right\} \Longrightarrow$$ Threshold phenomena

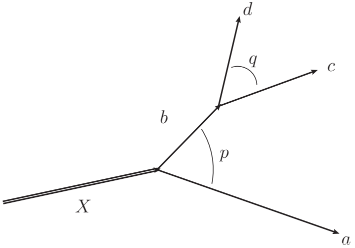
$$\frac{gk(E)}{\left(E-E_f-\frac{1}{2}g\kappa(E)\right)^2+\frac{1}{4}(\Gamma_0+gk(E))^2} \qquad \frac{\Gamma_0}{\left(E-E_f-\frac{1}{2}g\kappa(E)\right)^2+\frac{1}{4}(\Gamma_0+gk(E))^2}$$





- Blue curve — bound state (pole on RS-I)
- Yellow curve — virtual state (pole on RS-II)

Unstable constituent: Line shape



$$E_{\text{th}}^{(2)} = m_a + m_b$$

$$E_{\text{th}}^{(3)} = m_a + m_c + m_d \;{\color{red}<}\; E_{\text{th}}^{(2)}$$

$$E_X = M_X - (m_a + m_c + m_d)$$

$$E_R = m_b - (m_c + m_d)$$

$$\Gamma_R = \Gamma(b \to c + d)$$

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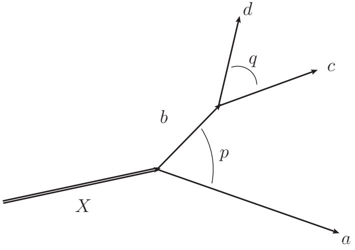

$$E_{\rm th}^{(2)} = m_a + m_b$$

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$$E_X = M_X - (m_a + m_c + m_d)$$

$$E_R = m_b - (m_c + m_d)$$

$$\Gamma_R = \Gamma(b \to c + d)$$

$$\frac{d\mathrm{Br}(a[cd])}{dE} = \frac{gk(E)}{\left(E - E_f - \frac{1}{2}g\kappa(E)\right)^2 + \frac{1}{4}\left(\Gamma_0 + gk(E)\right)^2}$$

$$E_f = E_X - \frac{1}{2}g\kappa(E_X)$$

Evaluating $k(E)$ and $\kappa(E)$

- For $\Gamma_R = 0$ the standard Flatté is recovered with

$$k(E) = \sqrt{2\mu E}\,\Theta(E) \qquad \kappa(E) = \sqrt{-2\mu E}\,\Theta(-E)$$

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$$k(E) = \sqrt{\mu\left(\sqrt{E^2 + \Gamma_R^2/4} + E\right)}$$

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Note: behaviour near the 3-body threshold is screwed up

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- In general case the full three-channel problem for $\{\psi_0, ab, a[cd]\}$ has to be solved (Hanhart,Kalashnikova,AN'10)

Coupled-channel problem

$$|X\rangle = \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi(\boldsymbol{p})|ab\rangle \\ \varphi(\boldsymbol{p},\boldsymbol{q})|a[cd]\rangle \end{pmatrix} \qquad \langle X|X\rangle = 1 \qquad (H_0 + V)|X\rangle = E|X\rangle$$

Potential V describes transitions $\psi_0 \leftrightarrow \{ab\}$ and $\{ab\} \leftrightarrow \{a[cd]\}$

Solution for the S-wave vertex $b \to c + d$:

$$k(E) = \Gamma_R\sqrt{\frac{\mu}{2E_R}}\left[\frac{E_R + \sqrt{E^2 + \frac{\Gamma_R^2}{4}\left(1 + \frac{E}{E_R}\right)}}{\sqrt{2E_R\left(-E + \frac{\Gamma_R^2}{8E_R}\left(1 + \frac{E}{E_R}\right) + \sqrt{E^2 + \frac{\Gamma_R^2}{4}\left(1 + \frac{E}{E_R}\right)}\right)}} - 1\right]$$

$$\kappa(E) = \sqrt{\mu}\frac{E + \frac{\Gamma_R^2}{4E_R} - \sqrt{E^2 + \frac{\Gamma_R^2}{4}\left(1 + \frac{E}{E_R}\right)}}{\sqrt{\sqrt{E^2 + \frac{\Gamma_R^2}{4}\left(1 + \frac{E}{E_R}\right)} - E - \frac{\Gamma_R^2}{8E_R}}}$$

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Potentia

Solution

- Analytic formulae with correct threshold behaviour
- Controlled by dimensionless ratio Γ_R/E_R

$$k(E) = \Gamma_R\sqrt{\frac{\mu}{2E_R}}\left[\frac{E_R + \sqrt{E^2 + \frac{\Gamma_R^2}{4}\left(1+\frac{E}{E_R}\right)}}{\sqrt{2E_R\left(-E + \frac{\Gamma_R^2}{8E_R}\left(1+\frac{E}{E_R}\right) + \sqrt{E^2 + \frac{\Gamma_R^2}{4}\left(1+\frac{E}{E_R}\right)}\right)}} - 1\right]$$

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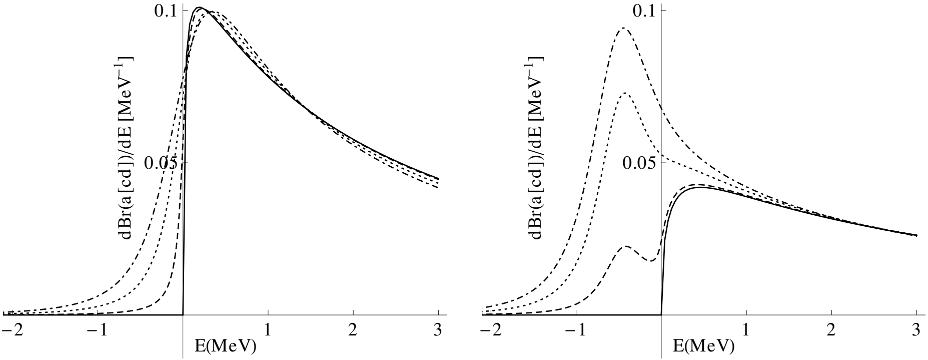
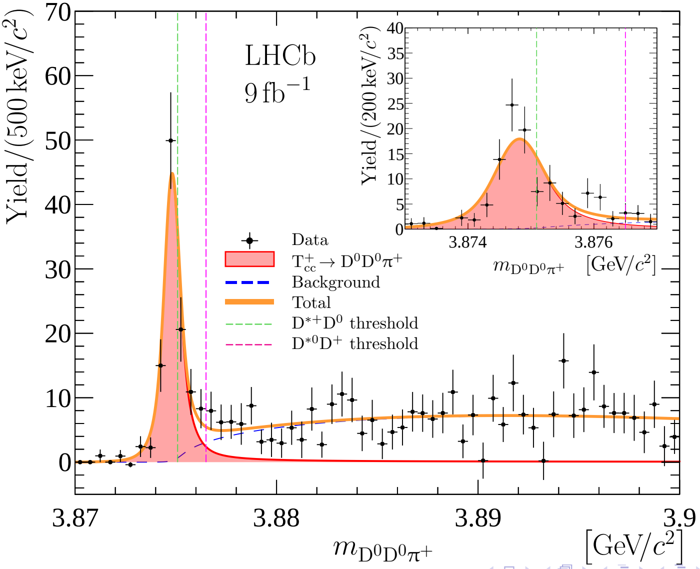
Examples of the line shapes

Virtual state ($E_X > 0$) Bound state ($E_X < 0$)



Solid line:	$\Gamma_R = 0$
Dashed line:	$\Gamma_R/E_R = 0.01$
Dotted line:	$\Gamma_R/E_R = 0.07$
Dash-dotted line:	$\Gamma_R/E_R = 0.1$

$T_{cc}^{+}(cc\bar{u}\bar{d})$ @ LHCb (Nature Phys. 18 (2022) 7, 751)



Simple Flatté fit $(\chi^2/N_{\rm dof} \approx 1)$

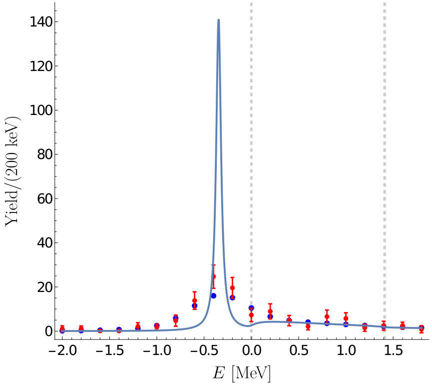
$$\mathcal{A} = \frac{\sqrt{\mathcal{N}}}{E - E_f + \frac{i}{2}\left[g(k_1 + k_2) + \Gamma_0\right]}$$

$$k = \sqrt{2\mu\left[E - m_D - \left(m_{D^*} - \frac{i}{2}\Gamma_{D^*}\right)\right]}$$

$\Gamma_0^{\rm fit} = 0 \implies$ No compact component

Pole position:

$$E_{\rm pole} = (-347 - i31)\ \text{keV}$$



In neglect of D^* width:

$$X_1 = \frac{\sqrt{E_B + \Delta}}{\sqrt{E_B} + \sqrt{E_B + \Delta}} \qquad X_2 = \frac{\sqrt{E_B}}{\sqrt{E_B} + \sqrt{E_B + \Delta}}$$

For $E_B = 347$ keV and $\Delta = 1.41$ MeV: $X_1 = 0.7 \quad X_2 = 0.3$

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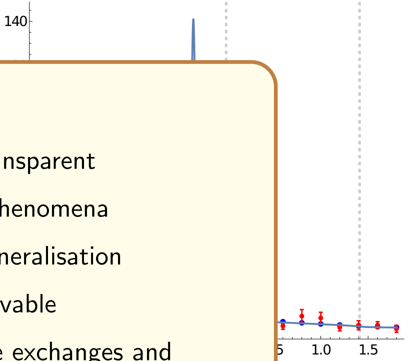
$k = \sqrt{2\mu [}$

$\Gamma_0^{\rm fit} = 0$

Pole posi

$E_{\rm pol}$



Flatté parametrisation:

- $+$ Simple and physically transparent
- $+$ Accounts for threshold phenomena
- $-$ Difficult multichannel generalisation
- $-$ Not systematically improvable
- $-$ Obscure effect of particle exchanges and few-body dynamics

In neglect

$$X_1 = \frac{\sqrt{E_B + \Delta}}{\sqrt{E_B} + \sqrt{E_B + \Delta}} \qquad X_2 = \frac{\sqrt{E_B}}{\sqrt{E_B} + \sqrt{E_B + \Delta}}$$

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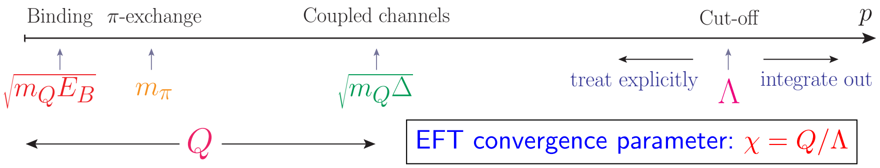
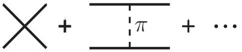
Effective field theory for hadronic molecules



Interaction potential between heavy hadrons:

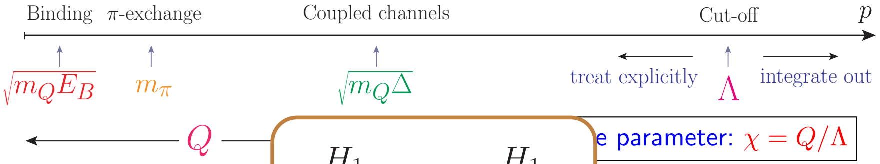
- Includes all relevant interactions

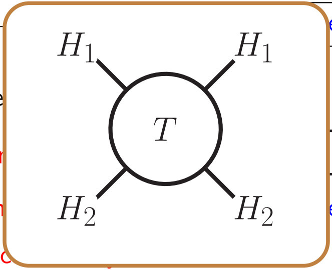


- Complies with relevant symmetries (chiral, HQSS, etc)
- Incorporates coupled-channel dynamics
- Expanded in powers of p^2/Λ^2 and truncated at necessary order (LO, NLO...)
- Iterated to all orders via (multichannel) Lippmann-Schwinger equation

$$T = V - VGT$$

Effective field theory for hadronic molecules





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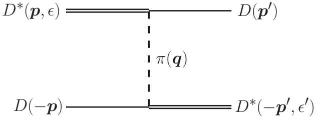
Pion exchange in $I = 0$ DD^* system



$$V_\pi(\boldsymbol{p}, \boldsymbol{p}') = \left(\frac{g_c}{2 f_\pi}\right)^2 \langle \boldsymbol{\tau} \cdot \boldsymbol{\tau} \rangle \frac{(\boldsymbol{\epsilon} \cdot \boldsymbol{q})(\boldsymbol{q} \cdot \boldsymbol{\epsilon}'^*)}{u - m_\pi^2}$$

$$\xRightarrow[\substack{I\,=\,0 \\ \text{central} \\ \cancel{\text{recoil}}}]{} \quad \left(\frac{g_c}{2 f_\pi}\right)^2 \left(-1 + \overbrace{\frac{\mu_\pi^2}{\boldsymbol{q}^2 + \underbrace{[m_\pi^2 - (m_{D^*} - m_D)^2]}_{\text{Effective mass } \mu_\pi^2}}}^{\text{Long-range OPE}}\right)$$

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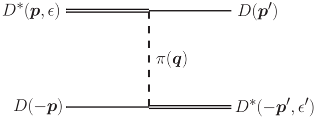

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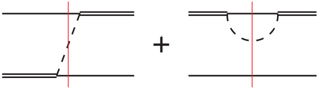
Physical quark masses ($m_\pi < m_{D^*} - m_D \Longrightarrow \mu_\pi^2 < 0$):

$\Longrightarrow$ 3-body effects:



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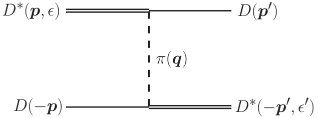

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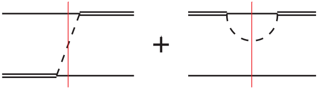

Lattice quark masses ($m_\pi > m_{D^*} - m_D \Longrightarrow \mu_\pi^2 > 0$):

$\Longrightarrow$ Different pion physics!

EFT approach to T_{cc}^{+}

$$\left.\begin{aligned} \gamma_B &= \sqrt{m_D E_B} \simeq 25\ \text{MeV} \\ p_{\text{data}}^{\text{max}} &= \sqrt{m_D\, \Delta E_{\text{data}}} \simeq 100\ \text{MeV} \\ p_{\text{coupl.ch.}} &= \sqrt{m_D (m_{D^*} - m_D)} \simeq 500\ \text{MeV} \end{aligned}\right\} \Longrightarrow$$

$\Lambda = 500$ MeV
Potential at LO
OPE included
No couple channels

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No couple channels

- Lippmann-Schwinger equation for scattering amplitude (v_0 — free parameter)

$$T = V - VGT$$

$$V = v_0 + V_\pi$$

- Production amplitude (P — free parameter = overall normalisation)

$$U = P - PGT$$

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$\Longrightarrow$

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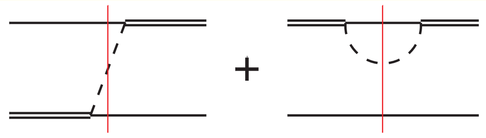
3-body effects:



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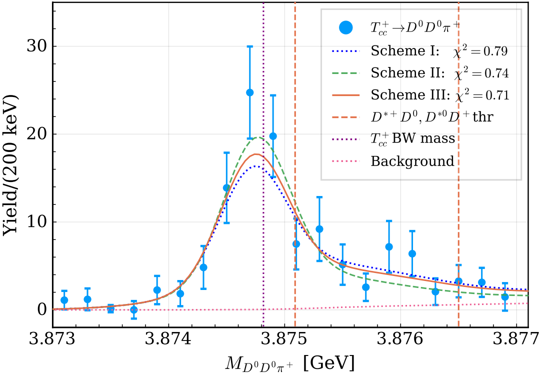
Fitting schemes, results, and conclusions

	Γ_{D^*}=const, $\cancel{\text{OPE}}$	$\Gamma_{D^*}(p,M)$, $\cancel{\text{OPE}}$	$\Gamma_{D^*}(p,M)$, OPE
$\chi^2/\mathrm{d.o.f.}$	0.79	0.74	0.71
v_0 [GeV^{-2}]	-23.34 ± 0.08	$-22.88^{+0.08}_{-0.06}$	$-5.04^{+0.10}_{-0.08}$
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$



- (Quasi)bound state just below $D^{*+}D^0$ threshold
- Compositeness: 70% & 30%

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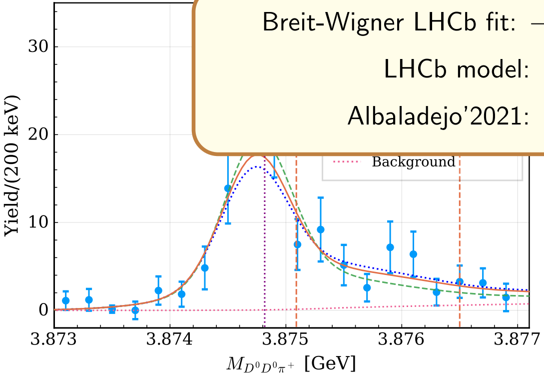
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$



Breit-Wigner LHCb fit: $-273 - i410$ keV

LHCb model: $-360 - i24$ keV

Albaladejo'2021: $-356 - i39$ keV

- state just below $D^{*+}D^0$ threshold
- Compositeness: 70% & 30%

Conclusions

- Collider experiments at energies above open-flavour threshold started new era in hadronic spectroscopy
- Many discovered hadrons do not fit simple quark model scheme and qualify as exotic states
- Many exotic hadrons are strong candidates to hadronic molecules
- Instability of molecule constituents brings about few-body effects
- Such effects contain essential information on the nature of near-threshold states
- Violation of few-body unitarity may result in substantial deviation of extracted properties of exotic hadrons from their actual nature

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Thank you for your attention!