

Few-body effects in heavy flavour hadronic molecules

A.V. Nefediev

HISKP, Bonn Uni., Germany

Few Body 23, Beijing, 26 September 2024

Hadronic physics before and after 2003

Consensus before 2003:

- Quark model provides a **decent description** of **low-lying** hadrons
- Quark model works surprisingly well even for **light flavours**
- Heavy flavours (c and b) comply with **nonrelativistic** theory
- Relativistic corrections somewhat **improve** the description
- Experiment gradually **fills** “missing states”
- Lattice provides additional/alternative **source of information**

Hadronic physics before and after 2003

Consensus before 2003:

- Quark model provides a **decent description** of **low-lying** hadrons
- Quark model works surprisingly well even for **light flavours**
- Heavy flavours (c and b) comply with **nonrelativistic** theory
- Relativistic corrections somewhat **improve** the description
- Experiment gradually **fills** “missing states”
- Lattice provides additional/alternative **source of information**

Situation after 2003:

- $X(3872)$ observed by Belle with properties **at odds** with quark model
- Number of such **unconventional** hadrons with heavy quarks **grows fast**
- **New branch** of hadrons spectroscopy — **exotic XYZ states**

“Exotic” versus “ordinary”

- “Ordinary” hadron = quark-antiquark mesons or 3-quark baryons
- “Exotic” hadron = not ordinary hadron
- Simplest exotic hadron = tetraquark ($Q\bar{Q}q\bar{q}$)



Compact tetraquarks (bound by confinement)



Hadro-Quarkonium (compact $\bar{Q}Q$ core plus light-quark cloud)



Hadronic molecule (extended object)

“Exotic” versus “ordinary”

- “Ordinary” hadron = quark-antiquark mesons or 3-quark baryons
- “Exotic” hadron = not ordinary hadron
- Simplest exotic hadron = tetraquark ($Q\bar{Q}q\bar{q}$)



Compact tetraquarks (bound by confinement)



Hadro-Quarkonium (compact $\bar{Q}Q$ core plus light-quark cloud)



Hadronic molecule (extended object)

Molecule = large probability to observe
physical state in hadron-hadron channel

“Exotic” versus “ordinary”

- “Ordinary” hadron = quark-antiquark mesons or 3-quark baryons
- “Exotic” hadron = not ordinary hadron

- 3S_1 NN system with $I = 0$:

Pole on RS-I with $E_B = 2.23$ MeV \implies deuteron

- 1S_0 NN system with $I = 1$:

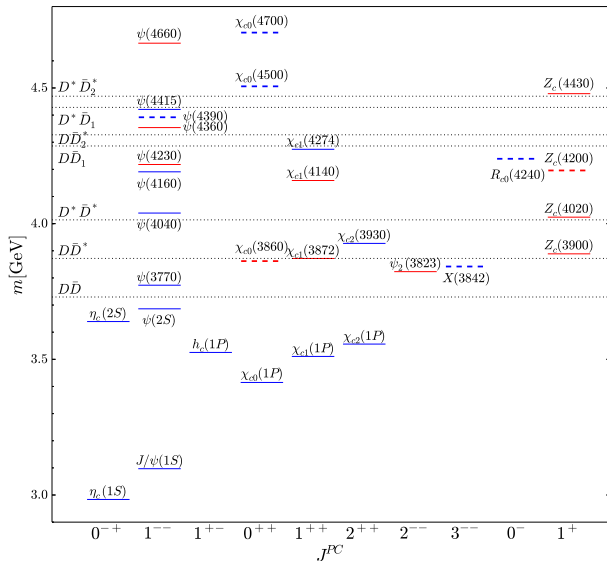
Pole on RS-II with $E_B = 0.067$ MeV \implies virtual state



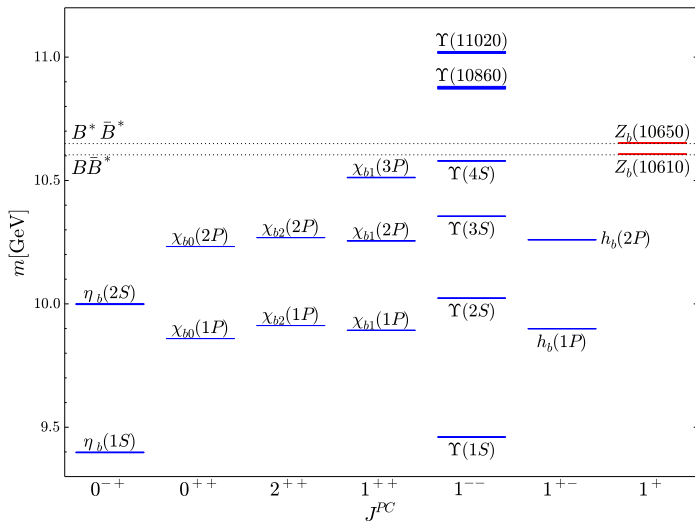
Hadronic molecule (extended object)

Molecule = large probability to observe physical state in hadron-hadron channel

Spectrum of charmonium

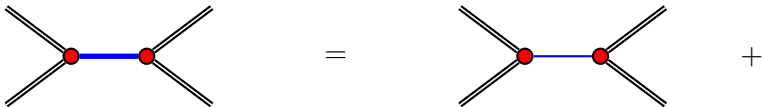


Spectrum of bottomonium



Effect of hadronic loops

$$|X\rangle = \begin{pmatrix} \lambda|\psi\rangle \\ \chi(\mathbf{p})|H_1H_2\rangle \end{pmatrix}$$

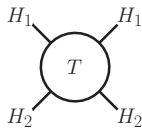


Flatté:

$$\frac{1}{E - E_f + \frac{i}{2}(gk + \Gamma_0)}$$



parameterised

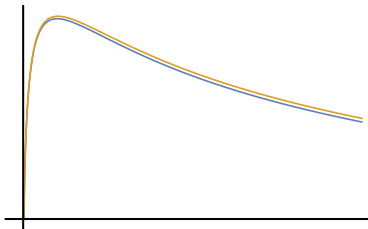


Γ_0 parameterises decay modes not related to H_1H_2 channel

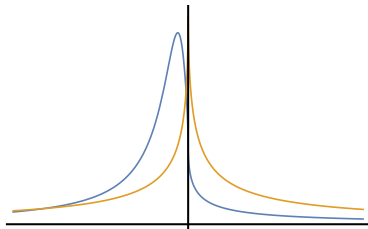
Examples of line shapes

$$\left. \begin{aligned} k(E) &= \sqrt{2\mu E} \Theta(E) \\ \kappa(E) &= \sqrt{-2\mu E} \Theta(-E) \end{aligned} \right\} \Rightarrow \text{Threshold phenomena}$$

$$\frac{gk(E)}{(E-E_f - \frac{1}{2}g\kappa(E))^2 + \frac{1}{4}(\Gamma_0 + gk(E))^2}$$

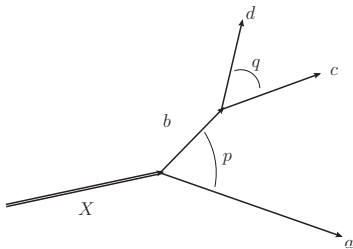


$$\frac{\Gamma_0}{(E-E_f - \frac{1}{2}g\kappa(E))^2 + \frac{1}{4}(\Gamma_0 + gk(E))^2}$$



- Blue curve — bound state (pole on RS-I)
- Yellow curve — virtual state (pole on RS-II)

Unstable constituent: Line shape



$$E_{\text{th}}^{(2)} = m_a + m_b$$

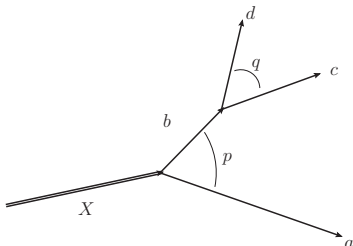
$$E_{\text{th}}^{(3)} = m_a + m_c + m_d < E_{\text{th}}^{(2)}$$

$$E_X = M_X - (m_a + m_c + m_d)$$

$$E_R = m_b - (m_c + m_d)$$

$$\Gamma_R = \Gamma(b \rightarrow c + d)$$

Unstable constituent: Line shape



$$E_{\text{th}}^{(2)} = m_a + m_b$$

$$E_{\text{th}}^{(3)} = m_a + m_c + m_d < E_{\text{th}}^{(2)}$$

$$E_X = M_X - (m_a + m_c + m_d)$$

$$E_R = m_b - (m_c + m_d)$$

$$\Gamma_R = \Gamma(b \rightarrow c + d)$$

$$\frac{d\text{Br}(a[cd])}{dE} = \frac{gk(E)}{(E - E_f - \frac{1}{2}g\kappa(E))^2 + \frac{1}{4}(\Gamma_0 + gk(E))^2}$$

$$E_f = E_X - \frac{1}{2}g\kappa(E_X)$$

Evaluating $k(E)$ and $\kappa(E)$

- For $\Gamma_R = 0$ the standard Flatté is recovered with

$$k(E) = \sqrt{2\mu E} \Theta(E) \quad \kappa(E) = \sqrt{-2\mu E} \Theta(-E)$$

Evaluating $k(E)$ and $\kappa(E)$

- For $\Gamma_R = 0$ the standard Flatté is recovered with

$$k(E) = \sqrt{2\mu E} \Theta(E) \quad \kappa(E) = \sqrt{-2\mu E} \Theta(-E)$$

- For $\Gamma_R/E_R \ll 1$ a simple form can be used with
(Nauenberg and Pais'62, Braaten and Lu'07)

$$k(E) = \sqrt{\mu \left(\sqrt{E^2 + \Gamma_R^2/4} + E \right)}$$
$$\kappa(E) = \sqrt{\mu \left(\sqrt{E^2 + \Gamma_R^2/4} - E \right)}$$

Evaluating $k(E)$ and $\kappa(E)$

- For $\Gamma_R = 0$ the standard Flatté is recovered with

$$k(E) = \sqrt{2\mu E} \Theta(E) \quad \kappa(E) = \sqrt{-2\mu E} \Theta(-E)$$

- For $\Gamma_R/E_R \ll 1$ a simple form can be used with
(Nauenberg and Pais'62, Braaten and Lu'07)

$$k(E) = \sqrt{\mu \left(\sqrt{E^2 + \Gamma_R^2/4} + E \right)}$$
$$\kappa(E) = \sqrt{\mu \left(\sqrt{E^2 + \Gamma_R^2/4} - E \right)}$$

Note: behaviour near the 3-body threshold is screwed up

Evaluating $k(E)$ and $\kappa(E)$

- For $\Gamma_R = 0$ the standard Flatté is recovered with

$$k(E) = \sqrt{2\mu E} \Theta(E) \quad \kappa(E) = \sqrt{-2\mu E} \Theta(-E)$$

- For $\Gamma_R/E_R \ll 1$ a simple form can be used with
(Nauenberg and Pais'62, Braaten and Lu'07)

$$k(E) = \sqrt{\mu \left(\sqrt{E^2 + \Gamma_R^2/4} + E \right)}$$
$$\kappa(E) = \sqrt{\mu \left(\sqrt{E^2 + \Gamma_R^2/4} - E \right)}$$

Note: behaviour near the 3-body threshold is screwed up

- In general case the full three-channel problem for $\{\psi_0, ab, a[cd]\}$ has to be solved (Hanhart, Kalashnikova, AN'10)

Coupled-channel problem

$$|X\rangle = \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi(\mathbf{p})|ab\rangle \\ \varphi(\mathbf{p}, \mathbf{q})|a[cd]\rangle \end{pmatrix} \quad \langle X|X\rangle = 1 \quad (H_0 + \mathbf{V})|X\rangle = E|X\rangle$$

Potential \mathbf{V} describes transitions $\psi_0 \leftrightarrow \{ab\}$ and $\{ab\} \leftrightarrow \{a[cd]\}$

Solution for the S -wave vertex $b \rightarrow c + d$:

$$k(E) = \Gamma_R \sqrt{\frac{\mu}{2E_R}} \left[\frac{E_R + \sqrt{E^2 + \frac{\Gamma_R^2}{4} \left(1 + \frac{E}{E_R}\right)}}{\sqrt{2E_R \left(-E + \frac{\Gamma_R^2}{8E_R} \left(1 + \frac{E}{E_R}\right) + \sqrt{E^2 + \frac{\Gamma_R^2}{4} \left(1 + \frac{E}{E_R}\right)}\right)}} - 1 \right]$$

$$\kappa(E) = \sqrt{\mu} \frac{E + \frac{\Gamma_R^2}{4E_R} - \sqrt{E^2 + \frac{\Gamma_R^2}{4} \left(1 + \frac{E}{E_R}\right)}}{\sqrt{\sqrt{E^2 + \frac{\Gamma_R^2}{4} \left(1 + \frac{E}{E_R}\right)} - E - \frac{\Gamma_R^2}{8E_R}}$$

Coupled-channel problem

$$|X\rangle = \begin{pmatrix} \lambda|\psi_0\rangle \\ \chi(\mathbf{p})|ab\rangle \\ \varphi(\mathbf{p}, \mathbf{q})|a[cd]\rangle \end{pmatrix} \quad \langle X|X\rangle = 1 \quad (H_0 + \mathbf{V})|X\rangle = E|X\rangle$$

Potential

- Analytic formulae with correct threshold behaviour

Solution

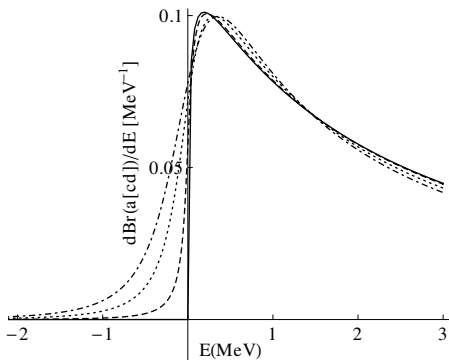
- Controlled by dimensionless ratio Γ_R/E_R

$$k(E) = \Gamma_R \sqrt{\frac{\mu}{2E_R}} \left[\frac{E_R + \sqrt{E^2 + \frac{\Gamma_R^2}{4} \left(1 + \frac{E}{E_R}\right)}}{\sqrt{2E_R \left(-E + \frac{\Gamma_R^2}{8E_R} \left(1 + \frac{E}{E_R}\right) + \sqrt{E^2 + \frac{\Gamma_R^2}{4} \left(1 + \frac{E}{E_R}\right)}\right)}} - 1 \right]$$

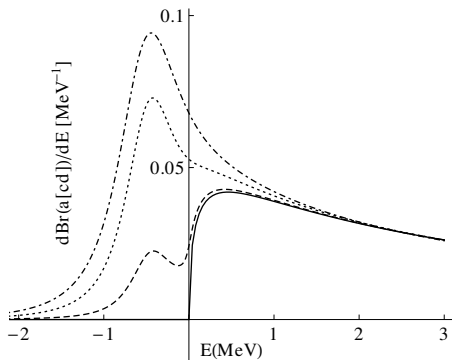
$$\kappa(E) = \sqrt{\mu} \frac{E + \frac{\Gamma_R^2}{4E_R} - \sqrt{E^2 + \frac{\Gamma_R^2}{4} \left(1 + \frac{E}{E_R}\right)}}{\sqrt{\sqrt{E^2 + \frac{\Gamma_R^2}{4} \left(1 + \frac{E}{E_R}\right)} - E - \frac{\Gamma_R^2}{8E_R}}$$

Examples of the line shapes

Virtual state ($E_X > 0$)



Bound state ($E_X < 0$)



Solid line:

$$\Gamma_R = 0$$

Dashed line:

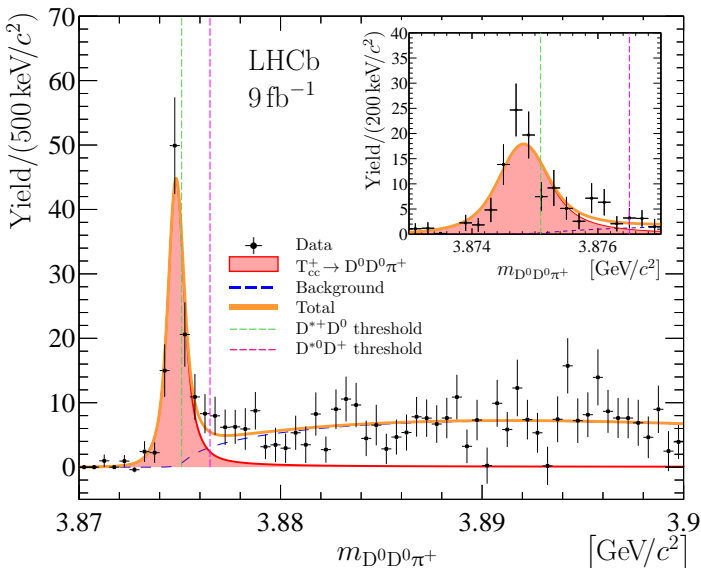
$$\Gamma_R/E_R = 0.01$$

Dotted line:

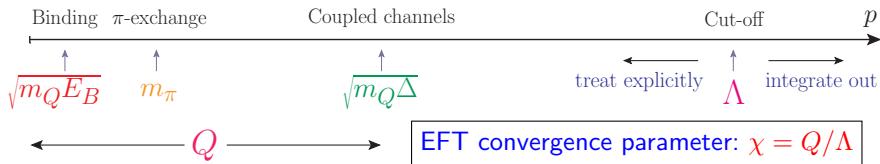
$$\Gamma_R/E_R = 0.07$$

Dash-dotted line:

$$\Gamma_R/E_R = 0.1$$

$T_{cc}^+(cc\bar{u}\bar{d})$ @ LHCb (Nature Phys. 18 (2022) 7, 751)

Effective field theory for hadronic molecules

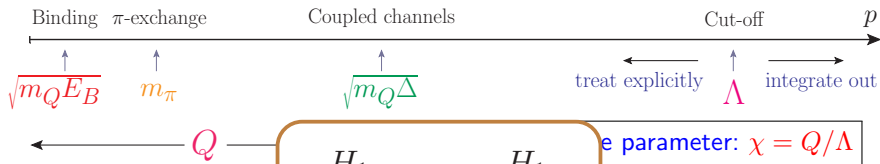


Interaction potential between heavy hadrons:

- Includes all **relevant interactions** $\times + \text{---} \pi \text{---} + \dots$
- Complies with **relevant symmetries** (chiral, HQSS, etc)
- Incorporates **coupled-channel dynamics**
- **Expanded** in powers of p^2/Λ^2 and **truncated** at necessary order (LO, NLO...)
- **Iterated** to all orders via (multichannel) Lippmann-Schwinger equation

$$T = V - VGT$$

Effective field theory for hadronic molecules

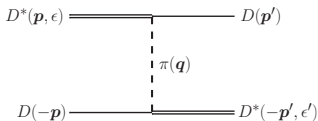


Interaction potential between

- Includes all **relevant** interactions
- Complies with **relevant** symmetries
- Incorporates **coupled-channels**
- **Expanded** in powers of p^2/Λ^2 and **truncated** at necessary order (LO, NLO...)
- **Iterated** to all orders via (multichannel) Lippmann-Schwinger equation

$$T = V - VGT$$

Pion exchange in $I = 0$ DD^* system



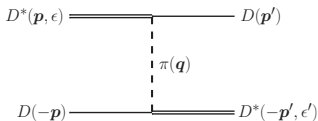
$$V_\pi(\mathbf{p}, \mathbf{p}') = \left(\frac{g_c}{2f_\pi} \right)^2 \langle \boldsymbol{\tau} \cdot \boldsymbol{\tau} \rangle \frac{(\boldsymbol{\epsilon} \cdot \mathbf{q})(\mathbf{q} \cdot \boldsymbol{\epsilon}'^*)}{u - m_\pi^2}$$

⇒
 $I = 0$
central
~~recoil~~

$$\left(\frac{g_c}{2f_\pi} \right)^2 \left(-1 + \overbrace{\frac{\mu_\pi^2}{\mathbf{q}^2 + [m_\pi^2 - (m_{D^*} - m_D)^2]}}^{\text{Long-range OPE}} \right)$$

Effective mass μ_π^2

Pion exchange in $I = 0$ DD^* system



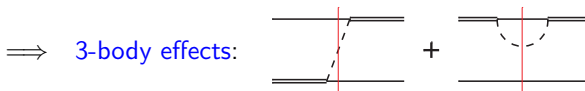
$$V_\pi(\mathbf{p}, \mathbf{p}') = \left(\frac{g_c}{2f_\pi} \right)^2 \langle \boldsymbol{\tau} \cdot \boldsymbol{\tau} \rangle \frac{(\boldsymbol{\epsilon} \cdot \mathbf{q})(\mathbf{q} \cdot \boldsymbol{\epsilon}'^*)}{u - m_\pi^2}$$

Long-range OPE

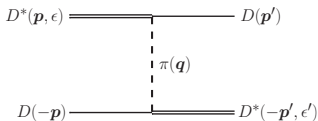
$$\Rightarrow \left(\frac{g_c}{2f_\pi} \right)^2 \left(-1 + \frac{\mu_\pi^2}{q^2 + \underbrace{[m_\pi^2 - (m_{D^*} - m_D)^2]}_{\text{Effective mass } \mu_\pi^2}} \right)$$

$I = 0$
 central
~~recoil~~

Physical quark masses ($m_\pi < m_{D^*} - m_D \Rightarrow \mu_\pi^2 < 0$):



Pion exchange in $I = 0$ DD^* system

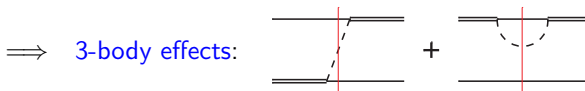


$$V_\pi(\mathbf{p}, \mathbf{p}') = \left(\frac{g_c}{2f_\pi} \right)^2 \langle \boldsymbol{\tau} \cdot \boldsymbol{\tau} \rangle \frac{(\boldsymbol{\epsilon} \cdot \mathbf{q})(\mathbf{q} \cdot \boldsymbol{\epsilon}'^*)}{u - m_\pi^2}$$

Long-range OPE

$$\Rightarrow_{\substack{I=0 \\ \text{central} \\ \text{recoil}}} \left(\frac{g_c}{2f_\pi} \right)^2 \left(-1 + \frac{\mu_\pi^2}{q^2 + \underbrace{[m_\pi^2 - (m_{D^*} - m_D)^2]}_{\text{Effective mass } \mu_\pi^2}} \right)$$

Physical quark masses ($m_\pi < m_{D^*} - m_D \Rightarrow \mu_\pi^2 < 0$):



Lattice quark masses ($m_\pi > m_{D^*} - m_D \Rightarrow \mu_\pi^2 > 0$):

\Rightarrow Different pion physics!

EFT approach to T_{cc}^+

(Baru et al.'2022)

$$\gamma_B = \sqrt{m_D E_B} \simeq 25 \text{ MeV}$$

$$|\mu_\pi| = \sqrt{(m_{D^*} - m_D)^2 - m_\pi^2} \simeq 40 \text{ MeV}$$

$$p_{\text{coupl.ch.}} = \sqrt{m_D(m_{D^*} - m_D)} \simeq 500 \text{ MeV}$$

$$p_{\text{data}}^{\text{max}} = \sqrt{m_D \Delta E_{\text{data}}} \simeq 100 \text{ MeV}$$



$$\Lambda = 500 \text{ MeV}$$

Potential at LO

OPE included

No couple channels

EFT approach to T_{cc}^+

(Baru et al.'2022)

$$\left. \begin{aligned}
 \gamma_B &= \sqrt{m_D E_B} \simeq 25 \text{ MeV} \\
 |\mu_\pi| &= \sqrt{(m_{D^*} - m_D)^2 - m_\pi^2} \simeq 40 \text{ MeV} \\
 p_{\text{coupl.ch.}} &= \sqrt{m_D(m_{D^*} - m_D)} \simeq 500 \text{ MeV} \\
 p_{\text{data}}^{\text{max}} &= \sqrt{m_D \Delta E_{\text{data}}} \simeq 100 \text{ MeV}
 \end{aligned} \right\} \Rightarrow \begin{aligned}
 \Lambda &= 500 \text{ MeV} \\
 \text{Potential at LO} \\
 \text{OPE included} \\
 \text{No couple channels}
 \end{aligned}$$

- Lippmann-Schwinger equation for scattering amplitude (v_0 — free parameter)

$$T = V - VGT$$

$$V = v_0 + V_\pi$$

- Production amplitude (P — free parameter = overall normalisation)

$$U = P - PGT$$

EFT approach to T_{cc}^+

(Baru et al.'2022)

$$\gamma_B = \sqrt{m_D E_B} \simeq 25 \text{ MeV}$$

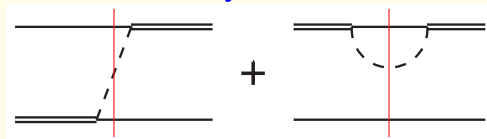
$$|\mu_\pi| = \sqrt{(m_{D^*} - m_D)^2 - m_\pi^2} \simeq 40 \text{ MeV}$$

$$p_{\text{coupl.ch.}} = \sqrt{m_D(m_{D^*} - m_D)} \simeq 500 \text{ MeV}$$

$$p_{\text{data}}^{\text{max}} = \sqrt{m_D(m_{D^*} - m_D)}$$

$\Lambda = 500 \text{ MeV}$
 Potential at LO
 OPE included
 No couple channels

3-body effects:



- Lippmann-S

(free parameter)

- Production amplitude (P — free parameter = overall normalisation)

$$U = P - PGT$$

Fitting schemes, results, and conclusions

$$\Gamma_{D^*} = \text{const}, \text{OPE}$$

$$\Gamma_{D^*}(p, M), \text{OPE}$$

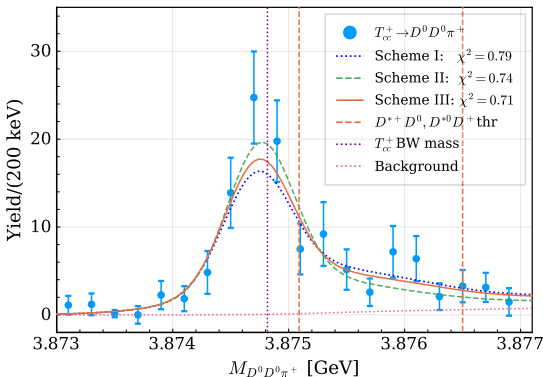
$$\Gamma_{D^*}(p, M), \text{OPE}$$

 $\chi^2/\text{d.o.f.}$

0.79

0.74

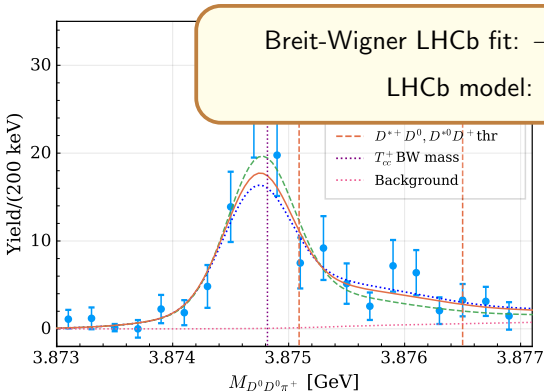
0.71

 $v_0 [\text{GeV}^{-2}]$
 -23.34 ± 0.08
 $-22.88^{+0.08}_{-0.06}$
 $-5.04^{+0.10}_{-0.08}$
 $\text{Pole} [\text{keV}]$
 $-368^{+43}_{-42} - i(37 \pm 0)$
 $-333^{+41}_{-36} - i(18 \pm 1)$
 $-356^{+39}_{-38} - i(28 \pm 1)$


- (Quasi)bound state just below $D^{*+} D^0$ threshold
- Compositeness: 70% & 30%

Fitting schemes, results, and conclusions

	$\Gamma_{D^*} = \text{const}, \text{OPE}$	$\Gamma_{D^*}(p, M), \text{OPE}$	$\Gamma_{D^*}(p, M), \text{OPE}$
$\chi^2/\text{d.o.f.}$	0.79	0.74	0.71
$v_0 [\text{GeV}^{-2}]$	-23.34 ± 0.08	$-22.88^{+0.08}_{-0.06}$	$-5.04^{+0.10}_{-0.08}$
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$



- (Quasi)bound state just below $D^{*+} D^0$ threshold
- Compositeness: 70% & 30%

Conclusions

- Collider experiments at energies above **open-flavour threshold** started **new era** in hadronic spectroscopy
- Many discovered hadrons **do not fit** simple **quark model scheme** and qualify as **exotic states**
- Many exotic hadrons are strong **candidates** to **hadronic molecules**
- **Instability** of molecule constituents brings about **few-body effects**
- Such effects contain **essential information** on the **nature** of near-threshold states
- Violation of **few-body unitarity** may result in substantial **deviation** of extracted properties of exotic hadrons from their **actual nature**

Conclusions

- Collider experiments at energies above **open-flavour threshold** started **new era** in hadronic spectroscopy
- Many discovered hadrons **do not fit** simple **quark model scheme** and qualify as **exotic states**
- Many exotic hadrons are strong **candidates** to **hadronic molecules**
- **Instability** of molecule constituents brings about **few-body effects**
- Such effects contain **essential information** on the **nature** of near-threshold states
- Violation of **few-body unitarity** may result in substantial **deviation** of extracted properties of exotic hadrons from their **actual nature**

Thank you for your attention!